

# EPDE for D2Q5 with constant velocities, supplementary material for Equivalent Finite Difference Equations and Equivalent Partial Differential Equations for the Lattice Boltzmann Method

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## 1 Global definitions

In  $\mathbb{R}^2$ , the position and velocity vectors are given by  $\mathbf{x} = (x, y)$  and  $\mathbf{u} = (u, v)$ , respectively. Discrete velocity vectors:

$$\{\mathbf{c}_i\}_{i=1}^5 = \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix} \right).$$

Equilibrium DF vector  $\mathbf{f}^{eq}$ :

$$\mathbf{f}^{eq} = \begin{pmatrix} 1 - u^2 - 2c_s^2 - v^2 \\ \frac{1}{2}u + \frac{1}{2}u^2 + \frac{1}{2}c_s^2 \\ \frac{1}{2}v + \frac{1}{2}c_s^2 + \frac{1}{2}v^2 \\ -\frac{1}{2}u + \frac{1}{2}u^2 + \frac{1}{2}c_s^2 \\ -\frac{1}{2}v + \frac{1}{2}c_s^2 + \frac{1}{2}v^2 \end{pmatrix}.$$

Lattice speed of sound:  $c_s = \frac{1}{\sqrt{3}}$ .

Moments  $\boldsymbol{\mu} = (\mu_1, \mu_2, \dots, \mu_5)^T$  are given by

$$\boldsymbol{\mu} = \tilde{\mathbf{M}}\mathbf{f},$$

where  $\mathbf{f} = (f_1, f_2, \dots, f_5)^T$  and Matrix  $\mathbf{M}$ :

$$\tilde{\mathbf{M}} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}.$$

## 2 SRT

### 2.1 Definitions

Matrix  $\mathbf{A} = \mathbf{S}$ :

$$\mathbf{A} = \begin{pmatrix} \omega & 0 & 0 & 0 & 0 \\ 0 & \omega & 0 & 0 & 0 \\ 0 & 0 & \omega & 0 & 0 \\ 0 & 0 & 0 & \omega & 0 \\ 0 & 0 & 0 & 0 & \omega \end{pmatrix}.$$

where

$$\mathbf{S} = \text{diag}(\omega, \omega, \omega, \omega, \omega).$$

Matrix  $\mathbf{B}$ :

$$\mathbf{B} = \begin{pmatrix} 0 & -1 + \omega & -1 + \omega & -1 + \omega & -1 + \omega \\ -1 + \omega & 0 & -1 + \omega & -1 + \omega & -1 + \omega \\ -1 + \omega & -1 + \omega & 0 & -1 + \omega & -1 + \omega \\ -1 + \omega & -1 + \omega & -1 + \omega & 0 & -1 + \omega \\ -1 + \omega & -1 + \omega & -1 + \omega & -1 + \omega & 0 \end{pmatrix}.$$

### 2.2 EPDE for $\mu_1$

$$\begin{aligned} & \gamma_{[t]}^{[\mu_1]} \delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[x]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[y]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[t^2]}^{[\mu_1]} \delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[tx]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} \\ & + \gamma_{[ty]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[x^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[y^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} = 0, \end{aligned}$$

where

$$\gamma_{[t]}^{[\mu_1]} = -\omega^4,$$

$$\gamma_{[x]}^{[\mu_1]} = -u\omega^4,$$

$$\gamma_{[y]}^{[\mu_1]} = -\omega^4 v,$$

$$\gamma_{[t^2]}^{[\mu_1]} = -4\omega^3 + \frac{7}{2}\omega^4,$$

$$\gamma_{[tx]}^{[\mu_1]} = 3u\omega^4 - 3u\omega^3,$$

$$\gamma_{[ty]}^{[\mu_1]} = 3\omega^4 v - 3\omega^3 v,$$

$$\gamma_{[x^2]}^{[\mu_1]} = u^2 \omega^3 - \frac{1}{2} \omega^4 c_s^2 + \omega^3 c_s^2 - \frac{1}{2} u^2 \omega^4.$$

$$\gamma_{[y^2]}^{[\mu_1]} = -\frac{1}{2} \omega^4 c_s^2 - \frac{1}{2} \omega^4 v^2 + \omega^3 v^2 + \omega^3 c_s^2,$$

### 2.3 EPDE for $\mu_2$

$$\begin{aligned} & \gamma_{[1]}^{[\mu_1]} \mu_1 + \gamma_{[1]}^{[\mu_2]} \mu_2 + \gamma_{[t]}^{[\mu_1]} \delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_2]} \delta_t \frac{\partial \mu_2}{\partial t} + \gamma_{[x]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[t^2]}^{[\mu_1]} \delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_2]} \delta_t^2 \frac{\partial^2 \mu_2}{\partial t^2} \\ & + \gamma_{[tx]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} + \gamma_{[x^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_2]} \delta_l^2 \frac{\partial^2 \mu_2}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x \partial y} + \gamma_{[y^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_2]} \delta_l^2 \frac{\partial^2 \mu_2}{\partial y^2} = 0, \end{aligned}$$

where

$$\gamma_{[1]}^{[\mu_1]} = -4u\omega^5 + 5u\omega^4,$$

$$\gamma_{[1]}^{[\mu_2]} = 4\omega^5 - 5\omega^4,$$

$$\gamma_{[t]}^{[\mu_1]} = 16u\omega^5 - 31u\omega^4 + 15u\omega^3,$$

$$\gamma_{[t]}^{[\mu_2]} = -20\omega^3 - 16\omega^5 + 35\omega^4,$$

$$\gamma_{[x]}^{[\mu_1]} = -5u^2\omega^3 + 4\omega^4 c_s^2 - 5\omega^3 c_s^2 + 4u^2\omega^4,$$

$$\gamma_{[t^2]}^{[\mu_1]} = -32u\omega^5 + \frac{157}{2}u\omega^4 - \frac{123}{2}u\omega^3 + 15u\omega^2,$$

$$\gamma_{[t^2]}^{[\mu_2]} = -30\omega^2 + 90\omega^3 + 32\omega^5 - \frac{185}{2}\omega^4,$$

$$\gamma_{[tx]}^{[\mu_1]} = -10u^2\omega^2 + 22u^2\omega^3 - 12\omega^4 c_s^2 - 2\omega^2 + 2\omega^3 - 10\omega^2 c_s^2 + 22\omega^3 c_s^2 - 12u^2\omega^4,$$

$$\gamma_{[x^2]}^{[\mu_1]} = -u\omega^4 + \frac{3}{2}u\omega^3,$$

$$\gamma_{[x^2]}^{[\mu_2]} = 3\omega^2 - 6\omega^3 + 3\omega^4,$$

$$\gamma_{[xy]}^{[\mu_1]} = -2\omega^2 v + 2\omega^3 v,$$

$$\gamma_{[y^2]}^{[\mu_1]} = -3u\omega^4 + 6u\omega^3 - 3u\omega^2.$$

$$\gamma_{[y^2]}^{[\mu_2]} = 3\omega^2 - 6\omega^3 + 3\omega^4,$$

### 2.4 EPDE for $\mu_3$

$$\begin{aligned} & \gamma_{[1]}^{[\mu_1]} \mu_1 + \gamma_{[1]}^{[\mu_3]} \mu_3 + \gamma_{[t]}^{[\mu_1]} \delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_3]} \delta_t \frac{\partial \mu_3}{\partial t} + \gamma_{[y]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[t^2]}^{[\mu_1]} \delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_3]} \delta_t^2 \frac{\partial^2 \mu_3}{\partial t^2} \\ & + \gamma_{[ty]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[x^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_3]} \delta_l^2 \frac{\partial^2 \mu_3}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x \partial y} + \gamma_{[y^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_3]} \delta_l^2 \frac{\partial^2 \mu_3}{\partial y^2} = 0, \end{aligned}$$

where

$$\gamma_{[1]}^{[\mu_1]} = 5\omega^4 v - 4\omega^5 v,$$

$$\gamma_{[1]}^{[\mu_3]} = 4\omega^5 - 5\omega^4,$$

$$\gamma_{[t]}^{[\mu_1]} = -31\omega^4 v + 16\omega^5 v + 15\omega^3 v,$$

$$\gamma_{[t]}^{[\mu_3]} = -20\omega^3 - 16\omega^5 + 35\omega^4,$$

$$\gamma_{[y]}^{[\mu_1]} = 4\omega^4 c_s^2 + 4\omega^4 v^2 - 5\omega^3 v^2 - 5\omega^3 c_s^2,$$

$$\gamma_{[t^2]}^{[\mu_1]} = \frac{157}{2}\omega^4 v - 32\omega^5 v + 15\omega^2 v - \frac{123}{2}\omega^3 v,$$

$$\gamma_{[t^2]}^{[\mu_3]} = -30\omega^2 + 90\omega^3 + 32\omega^5 - \frac{185}{2}\omega^4,$$

$$\gamma_{[ty]}^{[\mu_1]} = -12\omega^4 c_s^2 - 2\omega^2 + 2\omega^3 - 12\omega^4 v^2 + 22\omega^3 v^2 - 10\omega^2 c_s^2 - 10\omega^2 v^2 + 22\omega^3 c_s^2,$$

$$\gamma_{[x^2]}^{[\mu_1]} = -3\omega^4 v - 3\omega^2 v + 6\omega^3 v,$$

$$\gamma_{[x^2]}^{[\mu_3]} = 3\omega^2 - 6\omega^3 + 3\omega^4,$$

$$\gamma_{[xy]}^{[\mu_1]} = 2u\omega^3 - 2u\omega^2,$$

$$\gamma_{[y^2]}^{[\mu_1]} = -\omega^4 v + \frac{3}{2}\omega^3 v.$$

$$\gamma_{[y^2]}^{[\mu_3]} = 3\omega^2 - 6\omega^3 + 3\omega^4,$$

## 2.5 EPDE for $\mu_4$

$$\begin{aligned} & \gamma_{[1]}^{[\mu_1]} \mu_1 + \gamma_{[1]}^{[\mu_4]} \mu_4 + \gamma_{[t]}^{[\mu_1]} \delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_4]} \delta_t \frac{\partial \mu_4}{\partial t} + \gamma_{[x]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[y]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[t^2]}^{[\mu_1]} \delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_4]} \delta_t^2 \frac{\partial^2 \mu_4}{\partial t^2} \\ & + \gamma_{[tx]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} + \gamma_{[ty]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[x^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_4]} \delta_l^2 \frac{\partial^2 \mu_4}{\partial x^2} + \gamma_{[y^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_4]} \delta_l^2 \frac{\partial^2 \mu_4}{\partial y^2} = 0, \end{aligned}$$

where

$$\gamma_{[1]}^{[\mu_1]} = -4\omega^5 c_s^2 + 5\omega^4 c_s^2 - 4u^2 \omega^5 + 5u^2 \omega^4,$$

$$\gamma_{[1]}^{[\mu_4]} = 4\omega^5 - 5\omega^4,$$

$$\begin{aligned}
\gamma_{[t]}^{[\mu_1]} &= 16\omega^5 c_s^2 + 15u^2\omega^3 - 31\omega^4 c_s^2 + 2\omega^3 - 2\omega^4 + 16u^2\omega^5 + 15\omega^3 c_s^2 - 31u^2\omega^4, \\
\gamma_{[t]}^{[\mu_4]} &= -20\omega^3 - 16\omega^5 + 35\omega^4, \\
\gamma_{[x]}^{[\mu_1]} &= 2u\omega^4 - 3u\omega^3, \\
\gamma_{[y]}^{[\mu_1]} &= -2\omega^4 v + 2\omega^3 v, \\
\gamma_{[t^2]}^{[\mu_1]} &= 15u^2\omega^2 - 32\omega^5 c_s^2 - \frac{123}{2}u^2\omega^3 + \frac{157}{2}\omega^4 c_s^2 + 6\omega^2 - 13\omega^3 + 15\omega^2 c_s^2 + 7\omega^4 - 32u^2\omega^5 - \frac{123}{2}\omega^3 c_s^2 + \frac{157}{2}u^2\omega^4, \\
\gamma_{[t^2]}^{[\mu_4]} &= -30\omega^2 + 90\omega^3 + 32\omega^5 - \frac{185}{2}\omega^4, \\
\gamma_{[tx]}^{[\mu_1]} &= -6u\omega^4 + 12u\omega^3 - 6u\omega^2, \\
\gamma_{[ty]}^{[\mu_1]} &= 6\omega^4 v + 4\omega^2 v - 10\omega^3 v, \\
\gamma_{[x^2]}^{[\mu_1]} &= \frac{5}{2}u^2\omega^3 - 2\omega^4 c_s^2 + \frac{5}{2}\omega^3 c_s^2 - 2u^2\omega^4, \\
\gamma_{[x^2]}^{[\mu_4]} &= 3\omega^2 - 6\omega^3 + 3\omega^4, \\
\gamma_{[y^2]}^{[\mu_1]} &= -3u^2\omega^2 + 6u^2\omega^3 - 4\omega^4 c_s^2 - \omega^4 v^2 + 3\omega^3 v^2 - 5\omega^2 c_s^2 - 2\omega^2 v^2 + 9\omega^3 c_s^2 - 3u^2\omega^4. \\
\gamma_{[y^2]}^{[\mu_4]} &= 3\omega^2 - 6\omega^3 + 3\omega^4,
\end{aligned}$$

## 2.6 EPDE for $\mu_5$

$$\begin{aligned}
&\gamma_{[1]}^{[\mu_1]}\mu_1 + \gamma_{[1]}^{[\mu_5]}\mu_5 + \gamma_{[t]}^{[\mu_1]}\delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_5]}\delta_t \frac{\partial \mu_5}{\partial t} + \gamma_{[x]}^{[\mu_1]}\delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[y]}^{[\mu_1]}\delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[t^2]}^{[\mu_1]}\delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_5]}\delta_t^2 \frac{\partial^2 \mu_5}{\partial t^2} \\
&+ \gamma_{[tx]}^{[\mu_1]}\delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} + \gamma_{[ty]}^{[\mu_1]}\delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[x^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_5]}\delta_l^2 \frac{\partial^2 \mu_5}{\partial x^2} + \gamma_{[y^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_5]}\delta_l^2 \frac{\partial^2 \mu_5}{\partial y^2} = 0,
\end{aligned}$$

where

$$\begin{aligned}
\gamma_{[1]}^{[\mu_1]} &= -4\omega^5 c_s^2 + 5\omega^4 c_s^2 - 4\omega^5 v^2 + 5\omega^4 v^2, \\
\gamma_{[1]}^{[\mu_5]} &= 4\omega^5 - 5\omega^4, \\
\gamma_{[t]}^{[\mu_1]} &= 16\omega^5 c_s^2 - 31\omega^4 c_s^2 + 16\omega^5 v^2 + 2\omega^3 - 31\omega^4 v^2 + 15\omega^3 v^2 - 2\omega^4 + 15\omega^3 c_s^2, \\
\gamma_{[t]}^{[\mu_5]} &= -20\omega^3 - 16\omega^5 + 35\omega^4, \\
\gamma_{[x]}^{[\mu_1]} &= -2u\omega^4 + 2u\omega^3, \\
\gamma_{[y]}^{[\mu_1]} &= 2\omega^4 v - 3\omega^3 v, \\
\gamma_{[t^2]}^{[\mu_1]} &= -32\omega^5 c_s^2 + \frac{157}{2}\omega^4 c_s^2 + 6\omega^2 - 32\omega^5 v^2 - 13\omega^3 + \frac{157}{2}\omega^4 v^2 - \frac{123}{2}\omega^3 v^2 + 15\omega^2 c_s^2 + 7\omega^4 + 15\omega^2 v^2 - \frac{123}{2}\omega^3 c_s^2, \\
\gamma_{[t^2]}^{[\mu_5]} &= -30\omega^2 + 90\omega^3 + 32\omega^5 - \frac{185}{2}\omega^4,
\end{aligned}$$

$$\begin{aligned}
\gamma_{[tx]}^{[\mu_1]} &= 6u\omega^4 - 10u\omega^3 + 4u\omega^2, \\
\gamma_{[ty]}^{[\mu_1]} &= -6\omega^4v - 6\omega^2v + 12\omega^3v, \\
\gamma_{[x^2]}^{[\mu_1]} &= -2u^2\omega^2 + 3u^2\omega^3 - 4\omega^4c_s^2 - 3\omega^4v^2 + 6\omega^3v^2 - 5\omega^2c_s^2 - 3\omega^2v^2 + 9\omega^3c_s^2 - u^2\omega^4, \\
\gamma_{[x^2]}^{[\mu_5]} &= 3\omega^2 - 6\omega^3 + 3\omega^4, \\
\gamma_{[y^2]}^{[\mu_1]} &= -2\omega^4c_s^2 - 2\omega^4v^2 + \frac{5}{2}\omega^3v^2 + \frac{5}{2}\omega^3c_s^2, \\
\gamma_{[y^2]}^{[\mu_5]} &= 3\omega^2 - 6\omega^3 + 3\omega^4,
\end{aligned}$$

### 3 MRT 1: relaxation of $m_{00}$ , $m_{10}$ , $m_{01}$ , $m_{20}$ , $m_{02}$

#### 3.1 Definitions

Matrix  $\mathbf{A} = \mathbf{M}^{-1}\mathbf{SM}$ :

$$\mathbf{A} = \begin{pmatrix} \omega_0 & -\omega_3 + \omega_0 & \omega_0 - \omega_4 & -\omega_3 + \omega_0 & \omega_0 - \omega_4 \\ 0 & \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 & 0 & -\frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 & 0 \\ 0 & 0 & \frac{1}{2}\omega_2 + \frac{1}{2}\omega_4 & 0 & -\frac{1}{2}\omega_2 + \frac{1}{2}\omega_4 \\ 0 & -\frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 & 0 & \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 & 0 \\ 0 & 0 & -\frac{1}{2}\omega_2 + \frac{1}{2}\omega_4 & 0 & \frac{1}{2}\omega_2 + \frac{1}{2}\omega_4 \end{pmatrix}.$$

where

$$\mathbf{S} = \text{diag}(\omega_0, \omega_1, \omega_2, \omega_3, \omega_4)$$

and

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}.$$

Matrix  $\mathbf{B}$ :

$$\mathbf{B} = \begin{pmatrix} 0 & -1 + \omega_3 & -1 + \omega_4 & -1 + \omega_3 & -1 + \omega_4 \\ -1 + \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 & 0 & -1 + \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 & -1 + \omega_1 & -1 + \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 \\ -1 + \frac{1}{2}\omega_2 + \frac{1}{2}\omega_4 & -1 + \frac{1}{2}\omega_2 + \frac{1}{2}\omega_4 & 0 & -1 + \frac{1}{2}\omega_2 + \frac{1}{2}\omega_4 & -1 + \omega_2 \\ -1 + \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 & -1 + \omega_1 & -1 + \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 & 0 & -1 + \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 \\ -1 + \frac{1}{2}\omega_2 + \frac{1}{2}\omega_4 & -1 + \frac{1}{2}\omega_2 + \frac{1}{2}\omega_4 & -1 + \omega_2 & -1 + \frac{1}{2}\omega_2 + \frac{1}{2}\omega_4 & 0 \end{pmatrix}.$$

#### 3.2 EPDE for $\mu_1$

$$\begin{aligned}
&\gamma_{[t]}^{[\mu_1]}\delta_t\frac{\partial\mu_1}{\partial t} + \gamma_{[x]}^{[\mu_1]}\delta_l\frac{\partial\mu_1}{\partial x} + \gamma_{[y]}^{[\mu_1]}\delta_l\frac{\partial\mu_1}{\partial y} + \gamma_{[t^2]}^{[\mu_1]}\delta_t^2\frac{\partial^2\mu_1}{\partial t^2} + \gamma_{[tx]}^{[\mu_1]}\delta_t\delta_l\frac{\partial^2\mu_1}{\partial t\partial x} \\
&+ \gamma_{[ty]}^{[\mu_1]}\delta_t\delta_l\frac{\partial^2\mu_1}{\partial t\partial y} + \gamma_{[x^2]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial x^2} + \gamma_{[y^2]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial y^2} = 0,
\end{aligned}$$

where

$$\gamma_{[t]}^{[\mu_1]} = -\omega_4\omega_1\omega_2\omega_3,$$

$$\gamma_{[x]}^{[\mu_1]} = -\omega_4\omega_1\omega_2u\omega_3,$$

$$\gamma_{[y]}^{[\mu_1]} = -\omega_4\omega_1\omega_2\omega_3v,$$

$$\gamma_{[t^2]}^{[\mu_1]} = -\omega_1\omega_2\omega_3 - \omega_4\omega_1\omega_3 - \omega_4\omega_1\omega_2 + \frac{7}{2}\omega_4\omega_1\omega_2\omega_3 - \omega_4\omega_2\omega_3,$$

$$\gamma_{[tx]}^{[\mu_1]} = -\omega_4\omega_1\omega_2u - \omega_4\omega_1u\omega_3 + 3\omega_4\omega_1\omega_2u\omega_3 - \omega_1\omega_2u\omega_3,$$

$$\gamma_{[ty]}^{[\mu_1]} = -\omega_4\omega_2\omega_3v + 3\omega_4\omega_1\omega_2\omega_3v - \omega_1\omega_2\omega_3v - \omega_4\omega_1\omega_2v,$$

$$\gamma_{[x^2]}^{[\mu_1]} = -\frac{1}{2}\omega_4\omega_1\omega_2u^2\omega_3 - \frac{1}{2}\omega_4\omega_1c_s^2\omega_2\omega_3 + \omega_4c_s^2\omega_2\omega_3 + \omega_4\omega_2u^2\omega_3.$$

$$\gamma_{[y^2]}^{[\mu_1]} = -\frac{1}{2}\omega_4\omega_1c_s^2\omega_2\omega_3 + \omega_4\omega_1c_s^2\omega_3 - \frac{1}{2}\omega_4\omega_1\omega_2\omega_3v^2 + \omega_4\omega_1\omega_3v^2,$$

### 3.3 EPDE for $\mu_2$

$$\begin{aligned} & \gamma_{[1]}^{[\mu_1]}\mu_1 + \gamma_{[1]}^{[\mu_2]}\mu_2 + \gamma_{[t]}^{[\mu_1]}\delta_t\frac{\partial\mu_1}{\partial t} + \gamma_{[t]}^{[\mu_2]}\delta_t\frac{\partial\mu_2}{\partial t} + \gamma_{[x]}^{[\mu_1]}\delta_l\frac{\partial\mu_1}{\partial x} + \gamma_{[t^2]}^{[\mu_1]}\delta_t^2\frac{\partial^2\mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_2]}\delta_t^2\frac{\partial^2\mu_2}{\partial t^2} \\ & + \gamma_{[tx]}^{[\mu_1]}\delta_t\delta_l\frac{\partial^2\mu_1}{\partial t\partial x} + \gamma_{[x^2]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_2]}\delta_l^2\frac{\partial^2\mu_2}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial x\partial y} + \gamma_{[y^2]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_2]}\delta_l^2\frac{\partial^2\mu_2}{\partial y^2} = 0, \end{aligned}$$

where

$$\gamma_{[1]}^{[\mu_1]} = -\omega_4\omega_1\omega_2^2u\omega_3 - \omega_4\omega_1^2\omega_2u\omega_3 - \omega_4\omega_1\omega_2u\omega_3^2 - \omega_4^2\omega_1\omega_2u\omega_3 + 5\omega_4\omega_1\omega_2u\omega_3,$$

$$\gamma_{[1]}^{[\mu_2]} = \omega_4^2\omega_1\omega_2\omega_3 + \omega_4\omega_1\omega_2^2\omega_3 + \omega_4\omega_1^2\omega_2\omega_3 - 5\omega_4\omega_1\omega_2\omega_3 + \omega_4\omega_1\omega_2\omega_3^2,$$

$$\begin{aligned} \gamma_{[t]}^{[\mu_1]} &= -\omega_4\omega_1^2u\omega_3 + 4\omega_4\omega_1\omega_2^2u\omega_3 + 5\omega_4\omega_1\omega_2u - \omega_1\omega_2^2u\omega_3 - \omega_4^2\omega_1u\omega_3 - \omega_1^2\omega_2u\omega_3 + 4\omega_4\omega_1^2\omega_2u\omega_3 - \omega_1\omega_2u\omega_3^2 + \\ & 4\omega_4\omega_1\omega_2u\omega_3^2 - \omega_4\omega_1u\omega_3^2 - \omega_4^2\omega_1\omega_2u + 4\omega_4^2\omega_1\omega_2u\omega_3 - \omega_4\omega_1\omega_2^2u + 5\omega_4\omega_1u\omega_3 - 22\omega_4\omega_1\omega_2u\omega_3 + \\ & 5\omega_1\omega_2u\omega_3 - \omega_4\omega_1^2\omega_2u, \end{aligned}$$

$$\begin{aligned} \gamma_{[t]}^{[\mu_2]} &= \omega_4^2\omega_2\omega_3 - 5\omega_1\omega_2\omega_3 + \omega_4\omega_2^2\omega_3 - 4\omega_4^2\omega_1\omega_2\omega_3 - 5\omega_4\omega_1\omega_3 + \omega_4\omega_1\omega_3^2 - 4\omega_4\omega_1\omega_2^2\omega_3 + \omega_1\omega_2\omega_3^2 + \omega_4^2\omega_1\omega_2 - \\ & 4\omega_4\omega_1^2\omega_2\omega_3 + \omega_4\omega_1^2\omega_2 - 5\omega_4\omega_1\omega_2 + 23\omega_4\omega_1\omega_2\omega_3 + \omega_4\omega_2\omega_3^2 + \omega_1\omega_2^2\omega_3 - 5\omega_4\omega_2\omega_3 + \omega_4\omega_1^2\omega_3 - 4\omega_4\omega_1\omega_2\omega_3^2 + \\ & \omega_4\omega_1\omega_2^2 + \omega_1^2\omega_2\omega_3 + \omega_4^2\omega_1\omega_3, \end{aligned}$$

$$\begin{aligned} \gamma_{[x]}^{[\mu_1]} &= \omega_4\omega_1\omega_2u^2\omega_3 + \omega_4\omega_1c_s^2\omega_2\omega_3 + \omega_4\omega_2^2u^2\omega_3 + \omega_4c_s^2\omega_2^2\omega_3 + \omega_4^2c_s^2\omega_2\omega_3 + \omega_4^2\omega_2u^2\omega_3 + \omega_4\omega_2u^2\omega_3^2 + \\ & \omega_4c_s^2\omega_2\omega_3^2 - 5\omega_4c_s^2\omega_2\omega_3 - 5\omega_4\omega_2u^2\omega_3, \end{aligned}$$

$$\begin{aligned} \gamma_{[t^2]}^{[\mu_1]} &= \frac{7}{2}\omega_4\omega_1^2u\omega_3 - \omega_1u\omega_3^2 - 8\omega_4\omega_1\omega_2^2u\omega_3 - \frac{37}{2}\omega_4\omega_1\omega_2u + \frac{7}{2}\omega_1\omega_2^2u\omega_3 + \frac{7}{2}\omega_4^2\omega_1u\omega_3 - \omega_4\omega_1^2u + 5\omega_1u\omega_3 - \omega_1\omega_2^2u - \\ & \omega_4^2\omega_1u + \frac{7}{2}\omega_1^2\omega_2u\omega_3 - \omega_1^2\omega_2u - 8\omega_4\omega_1^2\omega_2u\omega_3 + \frac{7}{2}\omega_1\omega_2u\omega_3^2 - 8\omega_4\omega_1\omega_2u\omega_3^2 - \omega_1^2u\omega_3 + \frac{7}{2}\omega_4\omega_1u\omega_3^2 + \frac{7}{2}\omega_4^2\omega_1\omega_2u + \\ & 5\omega_4\omega_1u + 5\omega_1\omega_2u - 8\omega_4^2\omega_1\omega_2u\omega_3 + \frac{7}{2}\omega_4\omega_1\omega_2^2u - \frac{37}{2}\omega_4\omega_1u\omega_3 + 47\omega_4\omega_1\omega_2u\omega_3 - \frac{37}{2}\omega_1\omega_2u\omega_3 + \frac{7}{2}\omega_4\omega_1^2\omega_2u, \end{aligned}$$

$$\begin{aligned} \gamma_{[t^2]}^{[\mu_2]} &= -\frac{7}{2}\omega_4^2\omega_2\omega_3 - 5\omega_2\omega_3 + \omega_1^2\omega_3 + \frac{39}{2}\omega_1\omega_2\omega_3 + \omega_1\omega_2^2 - 5\omega_4\omega_2 + \omega_4\omega_1^2 - \frac{7}{2}\omega_4\omega_2^2\omega_3 + 8\omega_4^2\omega_1\omega_2\omega_3 + \frac{39}{2}\omega_4\omega_1\omega_3 - \\ & \frac{7}{2}\omega_4\omega_1\omega_3^2 + 8\omega_4\omega_1\omega_2^2\omega_3 - 5\omega_1\omega_2 - \frac{7}{2}\omega_1\omega_2\omega_3^2 + \omega_4\omega_2^2 - \frac{7}{2}\omega_4^2\omega_1\omega_2 - 5\omega_4\omega_1 + \omega_4^2\omega_3 + 8\omega_4\omega_1^2\omega_2\omega_3 - \frac{7}{2}\omega_4\omega_1^2\omega_2 + \end{aligned}$$



$$\begin{aligned}
& \omega_2\omega_3^2 + \omega_1\omega_3^2 - 5\omega_4\omega_3 + \frac{39}{2}\omega_4\omega_1\omega_2 - \frac{101}{2}\omega_4\omega_1\omega_2\omega_3 + \omega_1^2\omega_2 - \frac{7}{2}\omega_4\omega_2\omega_3^2 + \omega_4^2\omega_1 + \omega_4^2\omega_2 - \frac{7}{2}\omega_1\omega_2^2\omega_3 + \\
& \frac{39}{2}\omega_4\omega_2\omega_3 - \frac{7}{2}\omega_4\omega_1^2\omega_3 + 8\omega_4\omega_1\omega_2\omega_3^2 - \frac{7}{2}\omega_4\omega_1\omega_2^2 + \omega_2^2\omega_3 - \frac{7}{2}\omega_1^2\omega_2\omega_3 - 5\omega_1\omega_3 + \omega_4\omega_3^2 - \frac{7}{2}\omega_4^2\omega_1\omega_3, \\
\gamma_{[tx]}^{[\mu_1]} &= -5\omega_4c_s^2\omega_3 - 3\omega_4\omega_1\omega_2u^2\omega_3 - 3\omega_4\omega_1c_s^2\omega_2\omega_3 - 3\omega_4\omega_2^2u^2\omega_3 + \omega_1c_s^2\omega_2\omega_3 + \omega_1\omega_2u^2\omega_3 - 3\omega_4c_s^2\omega_2^2\omega_3 - \\
& 2\omega_4\omega_2 + \omega_4u^2\omega_3^2 + \omega_2u^2\omega_3^2 + c_s^2\omega_2\omega_3^2 + \omega_4\omega_1u^2\omega_3 - 5c_s^2\omega_2\omega_3 - 5\omega_2u^2\omega_3 - 5\omega_4u^2\omega_3 - 3\omega_4^2c_s^2\omega_2\omega_3 - \\
& 3\omega_4^2\omega_2u^2\omega_3 + \omega_4\omega_1c_s^2\omega_3 + \omega_4c_s^2\omega_3^2 + \omega_2^2u^2\omega_3 + \omega_4^2u^2\omega_3 + \omega_4\omega_1\omega_2 + c_s^2\omega_2^2\omega_3 - 3\omega_4\omega_2u^2\omega_3^2 - \\
& 3\omega_4c_s^2\omega_2\omega_3^2 + 16\omega_4c_s^2\omega_2\omega_3 + 16\omega_4\omega_2u^2\omega_3 + \omega_4\omega_2\omega_3 + \omega_4^2c_s^2\omega_3, \\
\gamma_{[x^2]}^{[\mu_1]} &= \frac{3}{2}\omega_4\omega_1\omega_2u - \frac{1}{2}\omega_4^2\omega_1\omega_2u - \frac{1}{2}\omega_4\omega_1\omega_2^2u, \\
\gamma_{[x^2]}^{[\mu_2]} &= \frac{1}{2}\omega_4^2\omega_2\omega_3 + 3\omega_4\omega_2 + \frac{1}{2}\omega_4\omega_2^2\omega_3 - \omega_4\omega_2^2 + \frac{1}{2}\omega_4^2\omega_1\omega_2 - \frac{3}{2}\omega_4\omega_1\omega_2 + \frac{1}{2}\omega_4\omega_1\omega_2\omega_3 + \frac{1}{2}\omega_4\omega_2\omega_3^2 - \omega_4^2\omega_2 - \\
& \frac{5}{2}\omega_4\omega_2\omega_3 + \frac{1}{2}\omega_4\omega_1\omega_2^2, \\
\gamma_{[xy]}^{[\mu_1]} &= \omega_4\omega_2\omega_3v - 2\omega_4\omega_2v + \omega_4\omega_1\omega_2v, \\
\gamma_{[y^2]}^{[\mu_1]} &= -\frac{1}{2}\omega_4\omega_1^2u\omega_3 + \omega_1u\omega_3^2 - \frac{1}{2}\omega_4^2\omega_1u\omega_3 - 3\omega_1u\omega_3 - \frac{1}{2}\omega_1^2\omega_2u\omega_3 - \frac{1}{2}\omega_1\omega_2u\omega_3^2 + \omega_1^2u\omega_3 - \frac{1}{2}\omega_4\omega_1u\omega_3^2 + \\
& \frac{5}{2}\omega_4\omega_1u\omega_3 - \frac{1}{2}\omega_4\omega_1\omega_2u\omega_3 + \frac{3}{2}\omega_1\omega_2u\omega_3, \\
\gamma_{[y^2]}^{[\mu_2]} &= -\omega_1^2\omega_3 - \frac{3}{2}\omega_1\omega_2\omega_3 - \frac{5}{2}\omega_4\omega_1\omega_3 + \frac{1}{2}\omega_4\omega_1\omega_3^2 + \frac{1}{2}\omega_1\omega_2\omega_3^2 - \omega_1\omega_3^2 + \frac{1}{2}\omega_4\omega_1\omega_2\omega_3 + \frac{1}{2}\omega_4\omega_1^2\omega_3 + \frac{1}{2}\omega_1^2\omega_2\omega_3 + \\
& 3\omega_1\omega_3 + \frac{1}{2}\omega_4^2\omega_1\omega_3,
\end{aligned}$$

### 3.4 EPDE for $\mu_3$

$$\begin{aligned}
& \gamma_{[1]}^{[\mu_1]}\mu_1 + \gamma_{[1]}^{[\mu_3]}\mu_3 + \gamma_{[t]}^{[\mu_1]}\delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_3]}\delta_t \frac{\partial \mu_3}{\partial t} + \gamma_{[y]}^{[\mu_1]}\delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[t^2]}^{[\mu_1]}\delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_3]}\delta_t^2 \frac{\partial^2 \mu_3}{\partial t^2} \\
& + \gamma_{[ty]}^{[\mu_1]}\delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[x^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_3]}\delta_l^2 \frac{\partial^2 \mu_3}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial x \partial y} + \gamma_{[y^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_3]}\delta_l^2 \frac{\partial^2 \mu_3}{\partial y^2} = 0,
\end{aligned}$$

where

$$\begin{aligned}
\gamma_{[1]}^{[\mu_1]} &= -\omega_4\omega_1^2\omega_2\omega_3v - \omega_4\omega_1\omega_2\omega_3^2v - \omega_4\omega_1\omega_2^2\omega_3v - \omega_4^2\omega_1\omega_2\omega_3v + 5\omega_4\omega_1\omega_2\omega_3v, \\
\gamma_{[1]}^{[\mu_3]} &= \omega_4^2\omega_1\omega_2\omega_3 + \omega_4\omega_1\omega_2^2\omega_3 + \omega_4\omega_1^2\omega_2\omega_3 - 5\omega_4\omega_1\omega_2\omega_3 + \omega_4\omega_1\omega_2\omega_3^2, \\
\gamma_{[t]}^{[\mu_1]} &= -\omega_4\omega_1^2\omega_2v - \omega_1^2\omega_2\omega_3v + 4\omega_4\omega_1^2\omega_2\omega_3v - \omega_4\omega_1\omega_2^2v - \omega_4^2\omega_1\omega_2v + 4\omega_4\omega_1\omega_2\omega_3^2v - \omega_1\omega_2\omega_3^2v + 4\omega_4\omega_1\omega_2^2\omega_3v + \\
& 5\omega_4\omega_2\omega_3v - \omega_1\omega_2^2\omega_3v - \omega_4\omega_2\omega_3^2v + 4\omega_4^2\omega_1\omega_2\omega_3v - \omega_4\omega_2^2\omega_3v - 22\omega_4\omega_1\omega_2\omega_3v + 5\omega_1\omega_2\omega_3v + 5\omega_4\omega_1\omega_2v - \omega_4^2\omega_2\omega_3v, \\
\gamma_{[t]}^{[\mu_3]} &= \omega_4^2\omega_2\omega_3 - 5\omega_1\omega_2\omega_3 + \omega_4\omega_2^2\omega_3 - 4\omega_4^2\omega_1\omega_2\omega_3 - 5\omega_4\omega_1\omega_3 + \omega_4\omega_1\omega_3^2 - 4\omega_4\omega_1\omega_2^2\omega_3 + \omega_1\omega_2\omega_3^2 + \omega_4^2\omega_1\omega_2 - \\
& 4\omega_4\omega_1^2\omega_2\omega_3 + \omega_4\omega_1^2\omega_2 - 5\omega_4\omega_1\omega_2 + 23\omega_4\omega_1\omega_2\omega_3 + \omega_4\omega_2\omega_3^2 + \omega_1\omega_2^2\omega_3 - 5\omega_4\omega_2\omega_3 + \omega_4\omega_1^2\omega_3 - 4\omega_4\omega_1\omega_2\omega_3^2 + \\
& \omega_4\omega_1\omega_2^2 + \omega_1^2\omega_2\omega_3 + \omega_4^2\omega_1\omega_3, \\
\gamma_{[y]}^{[\mu_1]} &= \omega_4\omega_1\omega_3^2v^2 + \omega_4\omega_1c_s^2\omega_2\omega_3 + \omega_4\omega_1c_s^2\omega_3^2 + \omega_4\omega_1^2\omega_3v^2 - 5\omega_4\omega_1c_s^2\omega_3 + \omega_4^2\omega_1\omega_3v^2 + \omega_4\omega_1^2c_s^2\omega_3 + \\
& \omega_4\omega_1\omega_2\omega_3v^2 - 5\omega_4\omega_1\omega_3v^2 + \omega_4^2\omega_1c_s^2\omega_3, \\
\gamma_{[t^2]}^{[\mu_1]} &= -\omega_2\omega_3^2v + \frac{7}{2}\omega_4\omega_1^2\omega_2v + \frac{7}{2}\omega_1^2\omega_2\omega_3v - \omega_2^2\omega_3v - 8\omega_4\omega_1^2\omega_2\omega_3v + \frac{7}{2}\omega_4\omega_1\omega_2^2v + \frac{7}{2}\omega_4^2\omega_1\omega_2v - 8\omega_4\omega_1\omega_2\omega_3^2v - \omega_4\omega_2^2v + \\
& 5\omega_1\omega_2v + \frac{7}{2}\omega_1\omega_2\omega_3^2v - 8\omega_4\omega_1\omega_2^2\omega_3v - \frac{37}{2}\omega_4\omega_2\omega_3v + \frac{7}{2}\omega_1\omega_2^2\omega_3v - \omega_4^2\omega_2v + \frac{7}{2}\omega_4\omega_2\omega_3^2v - \omega_1^2\omega_2v - 8\omega_4^2\omega_1\omega_2\omega_3v + \\
& \frac{7}{2}\omega_4\omega_2^2\omega_3v + 47\omega_4\omega_1\omega_2\omega_3v + 5\omega_4\omega_2v - \frac{37}{2}\omega_1\omega_2\omega_3v - \omega_1\omega_2^2v - \frac{37}{2}\omega_4\omega_1\omega_2v + 5\omega_2\omega_3v + \frac{7}{2}\omega_4^2\omega_2\omega_3v,
\end{aligned}$$

$$\begin{aligned}
\gamma_{[t^2]}^{[\mu_3]} &= -\frac{7}{2}\omega_4^2\omega_2\omega_3 - 5\omega_2\omega_3 + \omega_1^2\omega_3 + \frac{39}{2}\omega_1\omega_2\omega_3 + \omega_1\omega_2^2 - 5\omega_4\omega_2 + \omega_4\omega_1^2 - \frac{7}{2}\omega_4\omega_2^2\omega_3 + 8\omega_4^2\omega_1\omega_2\omega_3 + \frac{39}{2}\omega_4\omega_1\omega_3 - \\
&\quad \frac{7}{2}\omega_4\omega_1\omega_3^2 + 8\omega_4\omega_1\omega_2^2\omega_3 - 5\omega_1\omega_2 - \frac{7}{2}\omega_1\omega_2\omega_3^2 + \omega_4\omega_2^2 - \frac{7}{2}\omega_4^2\omega_1\omega_2 - 5\omega_4\omega_1 + \omega_4^2\omega_3 + 8\omega_4\omega_1^2\omega_2\omega_3 - \frac{7}{2}\omega_4\omega_1^2\omega_2 + \\
&\quad \omega_2\omega_3^2 + \omega_1\omega_3^2 - 5\omega_4\omega_3 + \frac{39}{2}\omega_4\omega_1\omega_2 - \frac{101}{2}\omega_4\omega_1\omega_2\omega_3 + \omega_1^2\omega_2 - \frac{7}{2}\omega_4\omega_2\omega_3^2 + \omega_4^2\omega_1 + \omega_4^2\omega_2 - \frac{7}{2}\omega_1\omega_2^2\omega_3 + \\
&\quad \frac{39}{2}\omega_4\omega_2\omega_3 - \frac{7}{2}\omega_4\omega_1^2\omega_3 + 8\omega_4\omega_1\omega_2\omega_3^2 - \frac{7}{2}\omega_4\omega_1\omega_2^2 + \omega_2^2\omega_3 - \frac{7}{2}\omega_1^2\omega_2\omega_3 - 5\omega_1\omega_3 + \omega_4\omega_3^2 - \frac{7}{2}\omega_4^2\omega_1\omega_3, \\
\gamma_{[ty]}^{[\mu_1]} &= -5\omega_4c_s^2\omega_3 - 3\omega_4\omega_1\omega_3^2v^2 - 3\omega_4\omega_1c_s^2\omega_2\omega_3 - 3\omega_4\omega_1c_s^2\omega_3^2 + \omega_4\omega_2\omega_3v^2 + \omega_1\omega_2\omega_3 - 3\omega_4\omega_1^2\omega_3v^2 + \omega_4\omega_1\omega_3 - \\
&\quad 5\omega_4\omega_1v^2 + \omega_4^2\omega_3v^2 + \omega_4\omega_3^2v^2 + 16\omega_4\omega_1c_s^2\omega_3 - 5\omega_4\omega_1c_s^2 + \omega_4c_s^2\omega_3^2 - 3\omega_4^2\omega_1\omega_3v^2 - 5\omega_4\omega_3v^2 + \omega_4\omega_1^2c_s^2 + \\
&\quad \omega_4\omega_1\omega_2v^2 - 3\omega_4\omega_1^2c_s^2\omega_3 + \omega_4^2\omega_1c_s^2 + \omega_4\omega_1c_s^2\omega_2 + \omega_4c_s^2\omega_2\omega_3 + \omega_4^2c_s^2\omega_3 - 3\omega_4\omega_1\omega_2\omega_3v^2 + \omega_4\omega_1^2v^2 + \\
&\quad 16\omega_4\omega_1\omega_3v^2 + \omega_4^2\omega_1v^2 - 2\omega_1\omega_3 - 3\omega_4^2\omega_1c_s^2\omega_3, \\
\gamma_{[x^2]}^{[\mu_1]} &= -\frac{1}{2}\omega_4\omega_1\omega_2^2v - \frac{1}{2}\omega_4^2\omega_1\omega_2v + \omega_4\omega_2^2v + \frac{5}{2}\omega_4\omega_2\omega_3v + \omega_4^2\omega_2v - \frac{1}{2}\omega_4\omega_2\omega_3^2v - \frac{1}{2}\omega_4\omega_2^2\omega_3v - \frac{1}{2}\omega_4\omega_1\omega_2\omega_3v - \\
&\quad 3\omega_4\omega_2v + \frac{3}{2}\omega_4\omega_1\omega_2v - \frac{1}{2}\omega_4^2\omega_2\omega_3v, \\
\gamma_{[x^2]}^{[\mu_3]} &= \frac{1}{2}\omega_4^2\omega_2\omega_3 + 3\omega_4\omega_2 + \frac{1}{2}\omega_4\omega_2^2\omega_3 - \omega_4\omega_2^2 + \frac{1}{2}\omega_4^2\omega_1\omega_2 - \frac{3}{2}\omega_4\omega_1\omega_2 + \frac{1}{2}\omega_4\omega_1\omega_2\omega_3 + \frac{1}{2}\omega_4\omega_2\omega_3^2 - \omega_4^2\omega_2 - \\
&\quad \frac{5}{2}\omega_4\omega_2\omega_3 + \frac{1}{2}\omega_4\omega_1\omega_2^2, \\
\gamma_{[xy]}^{[\mu_1]} &= -2\omega_1u\omega_3 + \omega_4\omega_1u\omega_3 + \omega_1\omega_2u\omega_3, \\
\gamma_{[y^2]}^{[\mu_1]} &= -\frac{1}{2}\omega_1^2\omega_2\omega_3v - \frac{1}{2}\omega_1\omega_2\omega_3^2v + \frac{3}{2}\omega_1\omega_2\omega_3v. \\
\gamma_{[y^2]}^{[\mu_3]} &= -\omega_1^2\omega_3 - \frac{3}{2}\omega_1\omega_2\omega_3 - \frac{5}{2}\omega_4\omega_1\omega_3 + \frac{1}{2}\omega_4\omega_1\omega_3^2 + \frac{1}{2}\omega_1\omega_2\omega_3^2 - \omega_1\omega_3^2 + \frac{1}{2}\omega_4\omega_1\omega_2\omega_3 + \frac{1}{2}\omega_4\omega_1^2\omega_3 + \frac{1}{2}\omega_1^2\omega_2\omega_3 + \\
&\quad 3\omega_1\omega_3 + \frac{1}{2}\omega_4^2\omega_1\omega_3,
\end{aligned}$$

### 3.5 EPDE for $\mu_4$

$$\begin{aligned}
&\gamma_{[1]}^{[\mu_1]}\mu_1 + \gamma_{[1]}^{[\mu_4]}\mu_4 + \gamma_{[t]}^{[\mu_1]}\delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_4]}\delta_t \frac{\partial \mu_4}{\partial t} + \gamma_{[x]}^{[\mu_1]}\delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[y]}^{[\mu_1]}\delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[t^2]}^{[\mu_1]}\delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_4]}\delta_t^2 \frac{\partial^2 \mu_4}{\partial t^2} \\
&+ \gamma_{[tx]}^{[\mu_1]}\delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} + \gamma_{[ty]}^{[\mu_1]}\delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[x^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_4]}\delta_l^2 \frac{\partial^2 \mu_4}{\partial x^2} + \gamma_{[y^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_4]}\delta_l^2 \frac{\partial^2 \mu_4}{\partial y^2} = 0,
\end{aligned}$$

where

$$\begin{aligned}
\gamma_{[1]}^{[\mu_1]} &= 5\omega_4\omega_1\omega_2u^2\omega_3 + 5\omega_4\omega_1c_s^2\omega_2\omega_3 - \omega_4^2\omega_1c_s^2\omega_2\omega_3 - \omega_4^2\omega_1\omega_2u^2\omega_3 - \omega_4\omega_1c_s^2\omega_2\omega_3^2 - \omega_4\omega_1\omega_2u^2\omega_3^2 - \\
&\quad \omega_4\omega_1^2\omega_2u^2\omega_3 - \omega_4\omega_1^2c_s^2\omega_2\omega_3 - \omega_4\omega_1\omega_2^2u^2\omega_3 - \omega_4\omega_1c_s^2\omega_2^2\omega_3, \\
\gamma_{[1]}^{[\mu_4]} &= \omega_4^2\omega_1\omega_2\omega_3 + \omega_4\omega_1\omega_2^2\omega_3 + \omega_4\omega_1^2\omega_2\omega_3 - 5\omega_4\omega_1\omega_2\omega_3 + \omega_4\omega_1\omega_2\omega_3^2, \\
\gamma_{[t]}^{[\mu_1]} &= -22\omega_4\omega_1\omega_2u^2\omega_3 - 22\omega_4\omega_1c_s^2\omega_2\omega_3 - \omega_4\omega_2^2u^2\omega_3 - \omega_4\omega_1c_s^2\omega_3^2 + 5\omega_1c_s^2\omega_2\omega_3 + 5\omega_1\omega_2u^2\omega_3 - \omega_4c_s^2\omega_2^2\omega_3 + \\
&\quad 4\omega_4^2\omega_1c_s^2\omega_2\omega_3 + 4\omega_4^2\omega_1\omega_2u^2\omega_3 + 5\omega_4\omega_1u^2\omega_3 - \omega_4\omega_1u^2\omega_3^2 - \omega_4^2c_s^2\omega_2\omega_3 - \omega_4^2\omega_2u^2\omega_3 - \omega_1\omega_2u^2\omega_3^2 - \\
&\quad \omega_1c_s^2\omega_2\omega_3^2 + 5\omega_4\omega_1c_s^2\omega_3 + 4\omega_4\omega_1c_s^2\omega_2\omega_3^2 - \omega_4\omega_1^2\omega_2 + 4\omega_4\omega_1\omega_2u^2\omega_3^2 - \omega_1^2c_s^2\omega_2\omega_3 - \omega_1^2\omega_2u^2\omega_3 + 2\omega_4\omega_1\omega_2 + \\
&\quad 4\omega_4\omega_1^2\omega_2u^2\omega_3 + 4\omega_4\omega_1^2c_s^2\omega_2\omega_3 - \omega_4^2\omega_1u^2\omega_3 - \omega_4\omega_1\omega_2\omega_3 - \omega_4\omega_1^2c_s^2\omega_3 - \omega_4\omega_2u^2\omega_3^2 - \omega_4c_s^2\omega_2\omega_3^2 + \\
&\quad 5\omega_4c_s^2\omega_2\omega_3 + 5\omega_4\omega_2u^2\omega_3 + 4\omega_4\omega_1\omega_2^2u^2\omega_3 - \omega_1\omega_2^2u^2\omega_3 + 4\omega_4\omega_1c_s^2\omega_2^2\omega_3 - \omega_4\omega_1^2u^2\omega_3 - \omega_1c_s^2\omega_2^2\omega_3 - \omega_4^2\omega_1c_s^2\omega_3, \\
\gamma_{[t]}^{[\mu_4]} &= \omega_4^2\omega_2\omega_3 - 5\omega_1\omega_2\omega_3 + \omega_4\omega_2^2\omega_3 - 4\omega_4^2\omega_1\omega_2\omega_3 - 5\omega_4\omega_1\omega_3 + \omega_4\omega_1\omega_3^2 - 4\omega_4\omega_1\omega_2^2\omega_3 + \omega_1\omega_2\omega_3^2 + \omega_4^2\omega_1\omega_2 - \\
&\quad 4\omega_4\omega_1^2\omega_2\omega_3 + \omega_4\omega_1^2\omega_2 - 5\omega_4\omega_1\omega_2 + 23\omega_4\omega_1\omega_2\omega_3 + \omega_4\omega_2\omega_3^2 + \omega_1\omega_2^2\omega_3 - 5\omega_4\omega_2\omega_3 + \omega_4\omega_1^2\omega_3 - 4\omega_4\omega_1\omega_2\omega_3^2 + \\
&\quad \omega_4\omega_1\omega_2^2 + \omega_1^2\omega_2\omega_3 + \omega_4^2\omega_1\omega_3,
\end{aligned}$$

$$\begin{aligned}
\gamma_{[x]}^{[\mu_1]} &= -3\omega_4\omega_1\omega_2u + \omega_4^2\omega_1\omega_2u + \omega_4\omega_1\omega_2^2u, \\
\gamma_{[y]}^{[\mu_1]} &= -\omega_4\omega_1^2\omega_2v - \omega_4\omega_1\omega_2\omega_3v + 2\omega_4\omega_1\omega_2v, \\
\gamma_{[t^2]}^{[\mu_1]} &= 5\omega_4c_s^2\omega_3 + 47\omega_4\omega_1\omega_2u^2\omega_3 + 47\omega_4\omega_1c_s^2\omega_2\omega_3 + \frac{7}{2}\omega_4\omega_2^2u^2\omega_3 + \frac{7}{2}\omega_4\omega_1c_s^2\omega_3^2 - \frac{37}{2}\omega_1c_s^2\omega_2\omega_3 - \frac{37}{2}\omega_1\omega_2u^2\omega_3 - \\
&\quad \omega_1\omega_2\omega_3 + \frac{7}{2}\omega_4c_s^2\omega_2^2\omega_3 + 2\omega_4\omega_2 - \omega_1^2c_s^2\omega_3 - \omega_4\omega_1^2 - \omega_4u^2\omega_3^2 - 8\omega_4^2\omega_1c_s^2\omega_2\omega_3 - 8\omega_4^2\omega_1\omega_2u^2\omega_3 - \omega_4\omega_1\omega_3 - \\
&\quad \omega_2u^2\omega_3^2 - c_s^2\omega_2\omega_3^2 - \frac{37}{2}\omega_4\omega_1u^2\omega_3 + \frac{7}{2}\omega_4\omega_1u^2\omega_3^2 + 5c_s^2\omega_2\omega_3 + 5\omega_2u^2\omega_3 + 2\omega_1\omega_2 + 2\omega_4\omega_1 + 5\omega_4u^2\omega_3 + \\
&\quad \frac{7}{2}\omega_4^2c_s^2\omega_2\omega_3 + \frac{7}{2}\omega_4^2\omega_2u^2\omega_3 + \frac{7}{2}\omega_1\omega_2u^2\omega_3^2 + \frac{7}{2}\omega_1c_s^2\omega_2\omega_3^2 - \frac{37}{2}\omega_4\omega_1c_s^2\omega_3 - 8\omega_4\omega_1c_s^2\omega_2\omega_3^2 + \frac{7}{2}\omega_4\omega_1^2\omega_2 - \\
&\quad \omega_4c_s^2\omega_3^2 - 8\omega_4\omega_1\omega_2u^2\omega_3 - \omega_1^2u^2\omega_3 - \omega_2^2u^2\omega_3 + \frac{7}{2}\omega_1^2c_s^2\omega_2\omega_3 + \frac{7}{2}\omega_1^2\omega_2u^2\omega_3 - \omega_4^2u^2\omega_3 - 8\omega_4\omega_1\omega_2 - \omega_1c_s^2\omega_3^2 - \\
&\quad 8\omega_4\omega_1^2\omega_2u^2\omega_3 - 8\omega_4\omega_1^2c_s^2\omega_2\omega_3 + \frac{7}{2}\omega_4^2\omega_1u^2\omega_3 + \frac{7}{2}\omega_4\omega_1\omega_2\omega_3 - c_s^2\omega_2^2\omega_3 + \frac{7}{2}\omega_4\omega_1^2c_s^2\omega_3 + \frac{7}{2}\omega_4\omega_2u^2\omega_3^2 + \\
&\quad 5\omega_1u^2\omega_3 - \omega_1^2\omega_2 + \frac{7}{2}\omega_4c_s^2\omega_2\omega_3^2 - \frac{37}{2}\omega_4c_s^2\omega_2\omega_3 - \omega_1u^2\omega_3^2 - \frac{37}{2}\omega_4\omega_2u^2\omega_3 - \omega_4\omega_2\omega_3 - 8\omega_4\omega_1\omega_2^2u^2\omega_3 - \\
&\quad \omega_4^2c_s^2\omega_3 + \frac{7}{2}\omega_1\omega_2^2u^2\omega_3 - 8\omega_4\omega_1c_s^2\omega_2^2\omega_3 + 5\omega_1c_s^2\omega_3 + \frac{7}{2}\omega_4\omega_1^2u^2\omega_3 + \frac{7}{2}\omega_1c_s^2\omega_2^2\omega_3 + \frac{7}{2}\omega_4^2\omega_1c_s^2\omega_3, \\
\gamma_{[t^2]}^{[\mu_4]} &= -\frac{7}{2}\omega_4^2\omega_2\omega_3 - 5\omega_2\omega_3 + \omega_1^2\omega_3 + \frac{39}{2}\omega_1\omega_2\omega_3 + \omega_1\omega_2^2 - 5\omega_4\omega_2 + \omega_4\omega_1^2 - \frac{7}{2}\omega_4\omega_2^2\omega_3 + 8\omega_4^2\omega_1\omega_2\omega_3 + \frac{39}{2}\omega_4\omega_1\omega_3 - \\
&\quad \frac{7}{2}\omega_4\omega_1\omega_3^2 + 8\omega_4\omega_1\omega_2^2\omega_3 - 5\omega_1\omega_2 - \frac{7}{2}\omega_1\omega_2\omega_3^2 + \omega_4\omega_2^2 - \frac{7}{2}\omega_1^2\omega_1\omega_2 - 5\omega_4\omega_1 + \omega_4^2\omega_3 + 8\omega_4\omega_1^2\omega_2\omega_3 - \frac{7}{2}\omega_4\omega_1^2\omega_2 + \\
&\quad \omega_2\omega_3^2 + \omega_1\omega_3^2 - 5\omega_4\omega_3 + \frac{39}{2}\omega_4\omega_1\omega_2 - \frac{101}{2}\omega_4\omega_1\omega_2\omega_3 + \omega_1^2\omega_2 - \frac{7}{2}\omega_4\omega_2\omega_3^2 + \omega_4^2\omega_1 + \omega_4^2\omega_2 - \frac{7}{2}\omega_1\omega_2^2\omega_3 + \\
&\quad \frac{39}{2}\omega_4\omega_2\omega_3 - \frac{7}{2}\omega_4\omega_1^2\omega_3 + 8\omega_4\omega_1\omega_2\omega_3^2 - \frac{7}{2}\omega_4\omega_1\omega_2^2 + \omega_2^2\omega_3 - \frac{7}{2}\omega_1^2\omega_2\omega_3 - 5\omega_1\omega_3 + \omega_4\omega_3^2 - \frac{7}{2}\omega_4^2\omega_1\omega_3, \\
\gamma_{[tx]}^{[\mu_1]} &= 10\omega_4\omega_1\omega_2u + \omega_1\omega_2^2u + \omega_4^2\omega_1u - 3\omega_4^2\omega_1\omega_2u - 3\omega_4\omega_1u - 3\omega_1\omega_2u - 3\omega_4\omega_1\omega_2^2u, \\
\gamma_{[ty]}^{[\mu_1]} &= 3\omega_4\omega_1^2\omega_2v + 2\omega_1\omega_2v - \omega_4\omega_2\omega_3v - \omega_1^2\omega_2v + 3\omega_4\omega_1\omega_2\omega_3v + 2\omega_4\omega_2v - \omega_1\omega_2\omega_3v - 7\omega_4\omega_1\omega_2v, \\
\gamma_{[x^2]}^{[\mu_1]} &= -\frac{1}{2}\omega_4\omega_1\omega_2u^2\omega_3 - \frac{1}{2}\omega_4\omega_1c_s^2\omega_2\omega_3 - \frac{1}{2}\omega_4\omega_2^2u^2\omega_3 - \frac{1}{2}\omega_4c_s^2\omega_2^2\omega_3 - \frac{1}{2}\omega_4^2c_s^2\omega_2\omega_3 - \frac{1}{2}\omega_4^2\omega_2u^2\omega_3 - \\
&\quad \frac{1}{2}\omega_4\omega_2u^2\omega_3^2 - \frac{1}{2}\omega_4c_s^2\omega_2\omega_3^2 + \frac{5}{2}\omega_4c_s^2\omega_2\omega_3 + \frac{5}{2}\omega_4\omega_2u^2\omega_3, \\
\gamma_{[x^2]}^{[\mu_4]} &= \frac{1}{2}\omega_4^2\omega_2\omega_3 + 3\omega_4\omega_2 + \frac{1}{2}\omega_4\omega_2^2\omega_3 - \omega_4\omega_2^2 + \frac{1}{2}\omega_4^2\omega_1\omega_2 - \frac{3}{2}\omega_4\omega_1\omega_2 + \frac{1}{2}\omega_4\omega_1\omega_2\omega_3 + \frac{1}{2}\omega_4\omega_2\omega_3^2 - \omega_4^2\omega_2 - \\
&\quad \frac{5}{2}\omega_4\omega_2\omega_3 + \frac{1}{2}\omega_4\omega_1\omega_2^2, \\
\gamma_{[y^2]}^{[\mu_1]} &= -\frac{1}{2}\omega_4\omega_1\omega_2u^2\omega_3 - \omega_4\omega_1c_s^2\omega_2\omega_3 - \frac{1}{2}\omega_4\omega_1^2c_s^2\omega_2 - \frac{1}{2}\omega_4\omega_1c_s^2\omega_3^2 + \frac{3}{2}\omega_1c_s^2\omega_2\omega_3 + \frac{3}{2}\omega_1\omega_2u^2\omega_3 + \omega_1^2c_s^2\omega_3 - \\
&\quad 2\omega_4\omega_1v^2 + \frac{5}{2}\omega_4\omega_1u^2\omega_3 - \frac{1}{2}\omega_4\omega_1u^2\omega_3^2 - \frac{1}{2}\omega_1\omega_2u^2\omega_3^2 - \frac{1}{2}\omega_1c_s^2\omega_2\omega_3^2 + \frac{7}{2}\omega_4\omega_1c_s^2\omega_3 - \frac{1}{2}\omega_4\omega_1^2\omega_2v^2 - 2\omega_4\omega_1c_s^2 + \\
&\quad \omega_1^2u^2\omega_3 - \frac{1}{2}\omega_1^2c_s^2\omega_2\omega_3 - \frac{1}{2}\omega_1^2\omega_2u^2\omega_3 + \omega_4\omega_1^2c_s^2 + \omega_1c_s^2\omega_3^2 + \omega_4\omega_1\omega_2v^2 - \frac{1}{2}\omega_4^2\omega_1u^2\omega_3 - \frac{1}{2}\omega_4\omega_1^2c_s^2\omega_3 - \\
&\quad 3\omega_1u^2\omega_3 + \omega_4\omega_1c_s^2\omega_2 + \omega_1u^2\omega_3^2 - \frac{1}{2}\omega_4\omega_1\omega_2\omega_3v^2 + \omega_4\omega_1^2v^2 + \omega_4\omega_1\omega_3v^2 - 3\omega_1c_s^2\omega_3 - \frac{1}{2}\omega_4\omega_1^2u^2\omega_3 - \frac{1}{2}\omega_4^2\omega_1c_s^2\omega_3. \\
\gamma_{[y^2]}^{[\mu_4]} &= -\omega_1^2\omega_3 - \frac{3}{2}\omega_1\omega_2\omega_3 - \frac{5}{2}\omega_4\omega_1\omega_3 + \frac{1}{2}\omega_4\omega_1\omega_3^2 + \frac{1}{2}\omega_1\omega_2\omega_3^2 - \omega_1\omega_3^2 + \frac{1}{2}\omega_4\omega_1\omega_2\omega_3 + \frac{1}{2}\omega_4\omega_1^2\omega_3 + \frac{1}{2}\omega_1^2\omega_2\omega_3 + \\
&\quad 3\omega_1\omega_3 + \frac{1}{2}\omega_4^2\omega_1\omega_3,
\end{aligned}$$

### 3.6 EPDE for $\mu_5$

$$\begin{aligned}
&\gamma_{[1]}^{[\mu_1]}\mu_1 + \gamma_{[1]}^{[\mu_5]}\mu_5 + \gamma_{[t]}^{[\mu_1]}\delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_5]}\delta_t \frac{\partial \mu_5}{\partial t} + \gamma_{[x]}^{[\mu_1]}\delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[y]}^{[\mu_1]}\delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[t^2]}^{[\mu_1]}\delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_5]}\delta_t^2 \frac{\partial^2 \mu_5}{\partial t^2} \\
&+ \gamma_{[tx]}^{[\mu_1]}\delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} + \gamma_{[ty]}^{[\mu_1]}\delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[x^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_5]}\delta_l^2 \frac{\partial^2 \mu_5}{\partial x^2} + \gamma_{[y^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_5]}\delta_l^2 \frac{\partial^2 \mu_5}{\partial y^2} = 0,
\end{aligned}$$

where

$$\begin{aligned}
\gamma_{[1]}^{[\mu_1]} &= 5\omega_4\omega_1c_s^2\omega_2\omega_3 - \omega_4\omega_1\omega_2^2\omega_3v^2 - \omega_4^2\omega_1c_s^2\omega_2\omega_3 - \omega_4\omega_1\omega_2\omega_3^2v^2 - \omega_4\omega_1^2\omega_2\omega_3v^2 - \omega_4\omega_1c_s^2\omega_2\omega_3^2 - \\
&\quad \omega_4\omega_1^2c_s^2\omega_2\omega_3 + 5\omega_4\omega_1\omega_2\omega_3v^2 - \omega_4\omega_1c_s^2\omega_2^2\omega_3 - \omega_4^2\omega_1\omega_2\omega_3v^2, \\
\gamma_{[1]}^{[\mu_5]} &= \omega_4^2\omega_1\omega_2\omega_3 + \omega_4\omega_1\omega_2^2\omega_3 + \omega_4\omega_1^2\omega_2\omega_3 - 5\omega_4\omega_1\omega_2\omega_3 + \omega_4\omega_1\omega_2\omega_3^2, \\
\gamma_{[t]}^{[\mu_1]} &= -\omega_4\omega_1\omega_3^2v^2 - 22\omega_4\omega_1c_s^2\omega_2\omega_3 - \omega_4\omega_1^2c_s^2\omega_2 - \omega_4\omega_1c_s^2\omega_3^2 + 4\omega_4\omega_1\omega_2^2\omega_3v^2 + 5\omega_4\omega_2\omega_3v^2 + 2\omega_1\omega_2\omega_3 - \\
&\quad \omega_4c_s^2\omega_2^2\omega_3 - \omega_4\omega_1^2\omega_3v^2 + 4\omega_4^2\omega_1c_s^2\omega_2\omega_3 - \omega_4^2\omega_1\omega_2v^2 + 4\omega_4\omega_1\omega_2\omega_3^2v^2 + 4\omega_4\omega_1^2\omega_2\omega_3v^2 - \omega_4\omega_1\omega_2^2v^2 - \\
&\quad \omega_4^2\omega_1c_s^2\omega_2 - \omega_4^2c_s^2\omega_2\omega_3 + 5\omega_4\omega_1c_s^2\omega_3 - \omega_4\omega_1^2\omega_2v^2 + 4\omega_4\omega_1c_s^2\omega_2\omega_3^2 - \omega_4^2\omega_1\omega_3v^2 - \omega_4^2\omega_2\omega_3v^2 + 4\omega_4\omega_1^2c_s^2\omega_2\omega_3 + \\
&\quad 5\omega_4\omega_1\omega_2v^2 - \omega_4\omega_1\omega_2\omega_3 - \omega_4\omega_1^2c_s^2\omega_3 - \omega_4\omega_1c_s^2\omega_2^2 - \omega_4c_s^2\omega_2\omega_3^2 + 5\omega_4\omega_1c_s^2\omega_2 + 5\omega_4c_s^2\omega_2\omega_3 - \omega_1\omega_2^2\omega_3 - \\
&\quad \omega_4\omega_2^2\omega_3v^2 - 22\omega_4\omega_1\omega_2\omega_3v^2 + 4\omega_4\omega_1c_s^2\omega_2^2\omega_3 + 5\omega_4\omega_1\omega_3v^2 + 4\omega_4^2\omega_1\omega_2\omega_3v^2 - \omega_4\omega_2\omega_3^2v^2 - \omega_4^2\omega_1c_s^2\omega_3, \\
\gamma_{[t]}^{[\mu_5]} &= \omega_4^2\omega_2\omega_3 - 5\omega_1\omega_2\omega_3 + \omega_4\omega_2^2\omega_3 - 4\omega_4^2\omega_1\omega_2\omega_3 - 5\omega_4\omega_1\omega_3 + \omega_4\omega_1\omega_3^2 - 4\omega_4\omega_1\omega_2^2\omega_3 + \omega_1\omega_2\omega_3^2 + \omega_4^2\omega_1\omega_2 - \\
&\quad 4\omega_4\omega_1^2\omega_2\omega_3 + \omega_4\omega_1^2\omega_2 - 5\omega_4\omega_1\omega_2 + 23\omega_4\omega_1\omega_2\omega_3 + \omega_4\omega_2\omega_3^2 + \omega_1\omega_2^2\omega_3 - 5\omega_4\omega_2\omega_3 + \omega_4\omega_1^2\omega_3 - 4\omega_4\omega_1\omega_2\omega_3^2 + \\
&\quad \omega_4\omega_1\omega_2^2 + \omega_1^2\omega_2\omega_3 + \omega_4^2\omega_1\omega_3, \\
\gamma_{[x]}^{[\mu_1]} &= -\omega_1\omega_2^2u\omega_3 - \omega_4\omega_1\omega_2u\omega_3 + 2\omega_1\omega_2u\omega_3, \\
\gamma_{[y]}^{[\mu_1]} &= \omega_1^2\omega_2\omega_3v + \omega_1\omega_2\omega_3^2v - 3\omega_1\omega_2\omega_3v, \\
\gamma_{[t^2]}^{[\mu_1]} &= 5\omega_4c_s^2\omega_3 + \frac{7}{2}\omega_4\omega_1\omega_3^2v^2 + 47\omega_4\omega_1c_s^2\omega_2\omega_3 - \omega_4^2\omega_2v^2 + \frac{7}{2}\omega_4\omega_1^2c_s^2\omega_2 + 2\omega_2\omega_3 + \frac{7}{2}\omega_4\omega_1c_s^2\omega_3^2 - \\
&\quad 8\omega_4\omega_1\omega_2^2\omega_3v^2 - \frac{37}{2}\omega_4\omega_2\omega_3v^2 - 8\omega_1\omega_2\omega_3 + \frac{7}{2}\omega_4c_s^2\omega_2^2\omega_3 - \omega_1\omega_2^2 - \omega_4\omega_2^2v^2 + \frac{7}{2}\omega_4\omega_1^2\omega_3v^2 - 8\omega_4^2\omega_1c_s^2\omega_2\omega_3 - \\
&\quad \omega_4\omega_1\omega_3 + \frac{7}{2}\omega_4^2\omega_1\omega_2v^2 + 5\omega_4\omega_1v^2 - 8\omega_4\omega_1\omega_2\omega_3^2v^2 - 8\omega_4\omega_1^2\omega_2\omega_3v^2 + \frac{7}{2}\omega_4\omega_1\omega_2^2v^2 - \omega_4^2\omega_3v^2 + 2\omega_1\omega_2 + \\
&\quad \frac{7}{2}\omega_4^2\omega_1c_s^2\omega_2 + \frac{7}{2}\omega_4^2c_s^2\omega_2\omega_3 - \omega_4\omega_3^2v^2 - \frac{37}{2}\omega_4\omega_1c_s^2\omega_3 - \omega_4^2c_s^2\omega_2 + \frac{7}{2}\omega_4\omega_1^2\omega_2v^2 - 8\omega_4\omega_1c_s^2\omega_2\omega_3^2 + 5\omega_4\omega_1c_s^2 - \\
&\quad \omega_4c_s^2\omega_3^2 + \frac{7}{2}\omega_4^2\omega_1\omega_3v^2 + 5\omega_4\omega_3v^2 + \frac{7}{2}\omega_4^2\omega_2\omega_3v^2 - \omega_4\omega_1\omega_2 - \omega_4\omega_1^2c_s^2 - 8\omega_4\omega_1^2c_s^2\omega_2\omega_3 - \frac{37}{2}\omega_4\omega_1\omega_2v^2 + \\
&\quad 5\omega_4c_s^2\omega_2 + \frac{7}{2}\omega_4\omega_1\omega_2\omega_3 + \frac{7}{2}\omega_4\omega_1^2c_s^2\omega_3 + \frac{7}{2}\omega_4\omega_1c_s^2\omega_2^2 + \frac{7}{2}\omega_4c_s^2\omega_2\omega_3^2 - \omega_4^2\omega_1c_s^2 - \frac{37}{2}\omega_4\omega_1c_s^2\omega_2 + 5\omega_4\omega_2v^2 - \\
&\quad \frac{37}{2}\omega_4c_s^2\omega_2\omega_3 + \frac{7}{2}\omega_1\omega_2^2\omega_3 - \omega_4\omega_2\omega_3 - \omega_4^2c_s^2\omega_3 - \omega_4c_s^2\omega_2^2 + \frac{7}{2}\omega_4\omega_2^2\omega_3v^2 + 47\omega_4\omega_1\omega_2\omega_3v^2 - \omega_4\omega_1^2v^2 - \\
&\quad 8\omega_4\omega_1c_s^2\omega_2^2\omega_3 - \frac{37}{2}\omega_4\omega_1\omega_3v^2 - \omega_2^2\omega_3 - 8\omega_4^2\omega_1\omega_2\omega_3v^2 - \omega_4^2\omega_1v^2 + 2\omega_1\omega_3 + \frac{7}{2}\omega_4\omega_2\omega_3^2v^2 + \frac{7}{2}\omega_4^2\omega_1c_s^2\omega_3, \\
\gamma_{[t^2]}^{[\mu_5]} &= -\frac{7}{2}\omega_4^2\omega_2\omega_3 - 5\omega_2\omega_3 + \omega_1^2\omega_3 + \frac{39}{2}\omega_1\omega_2\omega_3 + \omega_1\omega_2^2 - 5\omega_4\omega_2 + \omega_4\omega_1^2 - \frac{7}{2}\omega_4\omega_2^2\omega_3 + 8\omega_4^2\omega_1\omega_2\omega_3 + \frac{39}{2}\omega_4\omega_1\omega_3 - \\
&\quad \frac{7}{2}\omega_4\omega_1\omega_3^2 + 8\omega_4\omega_1\omega_2^2\omega_3 - 5\omega_1\omega_2 - \frac{7}{2}\omega_1\omega_2\omega_3^2 + \omega_4\omega_2^2 - \frac{7}{2}\omega_4^2\omega_1\omega_2 - 5\omega_4\omega_1 + \omega_4^2\omega_3 + 8\omega_4\omega_1^2\omega_2\omega_3 - \frac{7}{2}\omega_4\omega_1^2\omega_2 + \\
&\quad \omega_2\omega_3^2 + \omega_1\omega_3^2 - 5\omega_4\omega_3 + \frac{39}{2}\omega_4\omega_1\omega_2 - \frac{101}{2}\omega_4\omega_1\omega_2\omega_3 + \omega_1^2\omega_2 - \frac{7}{2}\omega_4\omega_2\omega_3^2 + \omega_4^2\omega_1 + \omega_4^2\omega_2 - \frac{7}{2}\omega_1\omega_2^2\omega_3 + \\
&\quad \frac{39}{2}\omega_4\omega_2\omega_3 - \frac{7}{2}\omega_4\omega_1^2\omega_3 + 8\omega_4\omega_1\omega_2\omega_3^2 - \frac{7}{2}\omega_4\omega_1\omega_2^2 + \omega_2^2\omega_3 - \frac{7}{2}\omega_1^2\omega_2\omega_3 - 5\omega_1\omega_3 + \omega_4\omega_3^2 - \frac{7}{2}\omega_4^2\omega_1\omega_3, \\
\gamma_{[tx]}^{[\mu_1]} &= -\omega_4\omega_1\omega_2u + 3\omega_1\omega_2^2u\omega_3 + 2\omega_1u\omega_3 - \omega_1\omega_2^2u + 2\omega_1\omega_2u - \omega_4\omega_1u\omega_3 + 3\omega_4\omega_1\omega_2u\omega_3 - 7\omega_1\omega_2u\omega_3, \\
\gamma_{[ty]}^{[\mu_1]} &= \omega_2\omega_3^2v - 3\omega_1^2\omega_2\omega_3v - 3\omega_1\omega_2v - 3\omega_1\omega_2\omega_3^2v + \omega_1^2\omega_2v + 10\omega_1\omega_2\omega_3v - 3\omega_2\omega_3v, \\
\gamma_{[x^2]}^{[\mu_1]} &= -\frac{1}{2}\omega_4\omega_1\omega_2u^2\omega_3 - \omega_4\omega_1c_s^2\omega_2\omega_3 + \omega_4^2\omega_2v^2 + \omega_1c_s^2\omega_2\omega_3 + \omega_1\omega_2u^2\omega_3 + \frac{5}{2}\omega_4\omega_2\omega_3v^2 - \frac{1}{2}\omega_4c_s^2\omega_2^2\omega_3 + \omega_4\omega_2^2v^2 - \\
&\quad \frac{1}{2}\omega_4^2\omega_1\omega_2v^2 - \frac{1}{2}\omega_4\omega_1\omega_2^2v^2 - 2c_s^2\omega_2\omega_3 - 2\omega_2u^2\omega_3 - \frac{1}{2}\omega_4^2\omega_1c_s^2\omega_2 - \frac{1}{2}\omega_4^2c_s^2\omega_2\omega_3 + \omega_4^2c_s^2\omega_2 + \omega_2^2u^2\omega_3 - \\
&\quad \frac{1}{2}\omega_4^2\omega_2\omega_3v^2 + \frac{3}{2}\omega_4\omega_1\omega_2v^2 - 3\omega_4c_s^2\omega_2 + c_s^2\omega_2^2\omega_3 - \frac{1}{2}\omega_4\omega_1c_s^2\omega_2^2 - \frac{1}{2}\omega_4c_s^2\omega_2\omega_3^2 + \frac{3}{2}\omega_4\omega_1c_s^2\omega_2 - 3\omega_4\omega_2v^2 + \\
&\quad \frac{7}{2}\omega_4c_s^2\omega_2\omega_3 + \omega_4\omega_2u^2\omega_3 + \omega_4c_s^2\omega_2^2 - \frac{1}{2}\omega_4\omega_2^2\omega_3v^2 - \frac{1}{2}\omega_4\omega_1\omega_2\omega_3v^2 - \frac{1}{2}\omega_1\omega_2^2u^2\omega_3 - \frac{1}{2}\omega_1c_s^2\omega_2^2\omega_3 - \frac{1}{2}\omega_4\omega_2\omega_3^2v^2, \\
\gamma_{[x^2]}^{[\mu_5]} &= \frac{1}{2}\omega_4^2\omega_2\omega_3 + 3\omega_4\omega_2 + \frac{1}{2}\omega_4\omega_2^2\omega_3 - \omega_4\omega_2^2 + \frac{1}{2}\omega_4^2\omega_1\omega_2 - \frac{3}{2}\omega_4\omega_1\omega_2 + \frac{1}{2}\omega_4\omega_1\omega_2\omega_3 + \frac{1}{2}\omega_4\omega_2\omega_3^2 - \omega_4^2\omega_2 - \\
&\quad \frac{5}{2}\omega_4\omega_2\omega_3 + \frac{1}{2}\omega_4\omega_1\omega_2^2, \\
\gamma_{[y^2]}^{[\mu_1]} &= -\frac{1}{2}\omega_4\omega_1\omega_3^2v^2 - \frac{1}{2}\omega_4\omega_1c_s^2\omega_2\omega_3 - \frac{1}{2}\omega_4\omega_1c_s^2\omega_3^2 - \frac{1}{2}\omega_4\omega_1^2\omega_3v^2 + \frac{5}{2}\omega_4\omega_1c_s^2\omega_3 - \frac{1}{2}\omega_4^2\omega_1\omega_3v^2 - \frac{1}{2}\omega_4\omega_1^2c_s^2\omega_3 - \\
&\quad \frac{1}{2}\omega_4\omega_1\omega_2\omega_3v^2 + \frac{5}{2}\omega_4\omega_1\omega_3v^2 - \frac{1}{2}\omega_4^2\omega_1c_s^2\omega_3, \\
\gamma_{[y^2]}^{[\mu_5]} &= -\omega_1^2\omega_3 - \frac{3}{2}\omega_1\omega_2\omega_3 - \frac{5}{2}\omega_4\omega_1\omega_3 + \frac{1}{2}\omega_4\omega_1\omega_3^2 + \frac{1}{2}\omega_1\omega_2\omega_3^2 - \omega_1\omega_3^2 + \frac{1}{2}\omega_4\omega_1\omega_2\omega_3 + \frac{1}{2}\omega_4\omega_1^2\omega_3 + \frac{1}{2}\omega_1^2\omega_2\omega_3 + \\
&\quad 3\omega_1\omega_3 + \frac{1}{2}\omega_4^2\omega_1\omega_3,
\end{aligned}$$

## 4 MRT 2: relaxation of $m_{00}$ , $m_{10}$ , $m_{01}$ , $m_{20} + m_{02}$ , $m_{20} - m_{02}$

### 4.1 Definitions

Matrix  $\mathbf{A} = \mathbf{M}^{-1}\mathbf{S}\mathbf{M}$ :

$$\mathbf{A} = \begin{pmatrix} \omega_0 & -\omega_3 + \omega_0 & -\omega_3 + \omega_0 & -\omega_3 + \omega_0 & -\omega_3 + \omega_0 \\ 0 & \frac{1}{4}\omega_4 + \frac{1}{2}\omega_1 + \frac{1}{4}\omega_3 & -\frac{1}{4}\omega_4 + \frac{1}{4}\omega_3 & \frac{1}{4}\omega_4 - \frac{1}{2}\omega_1 + \frac{1}{4}\omega_3 & -\frac{1}{4}\omega_4 + \frac{1}{4}\omega_3 \\ 0 & -\frac{1}{4}\omega_4 + \frac{1}{4}\omega_3 & \frac{1}{4}\omega_4 + \frac{1}{4}\omega_3 + \frac{1}{2}\omega_2 & -\frac{1}{4}\omega_4 + \frac{1}{4}\omega_3 & \frac{1}{4}\omega_4 + \frac{1}{4}\omega_3 - \frac{1}{2}\omega_2 \\ 0 & \frac{1}{4}\omega_4 - \frac{1}{2}\omega_1 + \frac{1}{4}\omega_3 & -\frac{1}{4}\omega_4 + \frac{1}{4}\omega_3 & \frac{1}{4}\omega_4 + \frac{1}{2}\omega_1 + \frac{1}{4}\omega_3 & -\frac{1}{4}\omega_4 + \frac{1}{4}\omega_3 \\ 0 & -\frac{1}{4}\omega_4 + \frac{1}{4}\omega_3 & \frac{1}{4}\omega_4 + \frac{1}{4}\omega_3 - \frac{1}{2}\omega_2 & \frac{1}{4}\omega_4 + \frac{1}{4}\omega_3 & \frac{1}{4}\omega_4 + \frac{1}{4}\omega_3 + \frac{1}{2}\omega_2 \end{pmatrix}.$$

where

$$\mathbf{S} = \text{diag}(\omega_0, \omega_1, \omega_2, \omega_3, \omega_4)$$

and

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & 1 & -1 \end{pmatrix}$$

Matrix  $\mathbf{B}$ :

$$\mathbf{B} = \begin{pmatrix} 0 & -1 + \omega_3 & -1 + \omega_3 & -1 + \omega_3 & -1 + \omega_3 \\ -1 + \frac{1}{4}\omega_4 + \frac{1}{2}\omega_1 + \frac{1}{4}\omega_3 & 0 & -1 + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_1 & -1 + \omega_1 & -1 + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_1 \\ -1 + \frac{1}{4}\omega_4 + \frac{1}{4}\omega_3 + \frac{1}{2}\omega_2 & -1 + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_2 & 0 & -1 + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_2 & -1 + \omega_2 \\ -1 + \frac{1}{4}\omega_4 + \frac{1}{2}\omega_1 + \frac{1}{4}\omega_3 & -1 + \omega_1 & -1 + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_1 & 0 & -1 + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_1 \\ -1 + \frac{1}{4}\omega_4 + \frac{1}{4}\omega_3 + \frac{1}{2}\omega_2 & -1 + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_2 & -1 + \omega_2 & -1 + \frac{1}{2}\omega_2 & 0 \end{pmatrix}.$$

### 4.2 EPDE for $\mu_1$

$$\begin{aligned} & \gamma_{[t]}^{[\mu_1]} \delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[x]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[y]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[t^2]}^{[\mu_1]} \delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[tx]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} \\ & + \gamma_{[ty]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[x^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x \partial y} + \gamma_{[y^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} = 0, \end{aligned}$$

where

$$\begin{aligned} \gamma_{[t]}^{[\mu_1]} &= -\frac{1}{8}\omega_2 v^2 \omega_3 \omega_4 \omega_1 + \frac{1}{8}\omega_2 \omega_3 \omega_4 \omega_1 u^2 + \frac{1}{8}\omega_2 v^2 \omega_4^2 \omega_1 + \frac{1}{8}\omega_2 \omega_4^2 \omega_1 u^2 - \frac{1}{8}\omega_2 \omega_4^2 \omega_1 + \frac{1}{4}\omega_2 \omega_4^2 c_s^2 \omega_1 - \frac{1}{4}\omega_2 \omega_3 \omega_4^2 u - \frac{7}{8}\omega_2 \omega_3 \omega_4 \omega_1, \\ \gamma_{[x]}^{[\mu_1]} &= -\frac{3}{4}\omega_2 \omega_3 \omega_4 \omega_1 u - \frac{1}{4}\omega_2 \omega_3 \omega_4^2 c_s^2 - \frac{1}{4}\omega_2 \omega_3 \omega_4^2 u^2, \\ \gamma_{[y]}^{[\mu_1]} &= -\omega_2 v \omega_3 \omega_4 \omega_1, \\ \gamma_{[t^2]}^{[\mu_1]} &= \frac{1}{4}\omega_2 \omega_4^2 c_s^2 + \frac{7}{16}\omega_2 v^2 \omega_3 \omega_4 \omega_1 - \frac{1}{8}\omega_2 v^2 \omega_3 \omega_4 - \frac{7}{16}\omega_2 \omega_3 \omega_4 \omega_1 u^2 - \frac{7}{16}\omega_2 v^2 \omega_4^2 \omega_1 - \frac{7}{16}\omega_2 \omega_4^2 \omega_1 u^2 + \frac{1}{8}\omega_2 \omega_3 \omega_4 u^2 - \frac{7}{8}\omega_3 \omega_4 \omega_1 - \frac{7}{8}\omega_2 \omega_3 \omega_4 + \frac{7}{16}\omega_2 \omega_4^2 \omega_1 - \frac{7}{8}\omega_2 \omega_4^2 c_s^2 \omega_1 - \frac{1}{4}\omega_3 \omega_4^2 u - \frac{1}{4}\omega_2 \omega_3 \omega_4 u - \omega_2 \omega_3 \omega_1 - \frac{1}{8}\omega_2 \omega_4^2 + \end{aligned}$$

$$\begin{aligned}
& \frac{1}{4}\omega_2\omega_4c_s^2\omega_1 - \omega_2\omega_4\omega_1 + \frac{1}{4}\omega_4^2c_s^2\omega_1 + \frac{1}{8}\omega_2\omega_4^2u^2 + \frac{1}{8}\omega_2v^2\omega_4^2 - \frac{1}{8}\omega_4^2\omega_1 + \frac{7}{8}\omega_2\omega_3\omega_4^2u + \frac{1}{8}\omega_4^2\omega_1u^2 + \\
& \frac{49}{16}\omega_2\omega_3\omega_4\omega_1 + \frac{1}{8}v^2\omega_4^2\omega_1 - \frac{1}{8}v^2\omega_3\omega_4\omega_1 - \frac{1}{4}\omega_2\omega_4^2u + \frac{1}{4}\omega_2\omega_4\omega_1u^2 + \frac{1}{8}\omega_3\omega_4\omega_1u^2, \\
\gamma_{[tx]}^{[\mu_1]} &= -\omega_2\omega_3\omega_1u - \frac{1}{8}\omega_2v^2\omega_3\omega_4 - \frac{1}{4}\omega_3\omega_4^2u^2 - \frac{1}{8}\omega_2\omega_3\omega_4u^2 + \frac{1}{8}\omega_2\omega_3\omega_4 + \frac{9}{4}\omega_2\omega_3\omega_4\omega_1u + \frac{3}{4}\omega_2\omega_3\omega_4^2c_s^2 - \frac{1}{8}\omega_2\omega_4^2 - \\
& \frac{3}{4}\omega_3\omega_4\omega_1u - \frac{1}{4}\omega_2\omega_3\omega_4c_s^2 - \frac{1}{4}\omega_3\omega_4^2c_s^2 - \frac{3}{4}\omega_2\omega_4\omega_1u - \frac{1}{8}\omega_2\omega_4^2u^2 + \frac{1}{8}\omega_2v^2\omega_4^2 + \frac{3}{4}\omega_2\omega_3\omega_4^2u^2, \\
\gamma_{[ty]}^{[\mu_1]} &= -\omega_2v\omega_4\omega_1 + \frac{1}{8}\omega_3\omega_4\omega_1 - \frac{1}{4}\omega_3\omega_4^2u + 3\omega_2v\omega_3\omega_4\omega_1 + \frac{1}{4}\omega_4^2c_s^2\omega_1 - \frac{1}{8}\omega_4^2\omega_1 + \frac{1}{8}\omega_4^2\omega_1u^2 - \omega_2v\omega_3\omega_1 + \\
& \frac{1}{8}v^2\omega_4^2\omega_1 - \frac{1}{8}v^2\omega_3\omega_4\omega_1 - \omega_2v\omega_3\omega_4 + \frac{1}{8}\omega_3\omega_4\omega_1u^2, \\
\gamma_{[x^2]}^{[\mu_1]} &= -\frac{3}{8}\omega_2\omega_3\omega_4\omega_1u^2 + \frac{3}{4}\omega_2\omega_3\omega_4u^2 - \frac{3}{8}\omega_2\omega_3\omega_4c_s^2\omega_1 + \frac{1}{4}\omega_2\omega_3\omega_4u + \frac{3}{4}\omega_2\omega_3\omega_4c_s^2 - \frac{1}{8}\omega_2\omega_3\omega_4^2u, \\
\gamma_{[xy]}^{[\mu_1]} &= -\frac{1}{4}\omega_3\omega_4^2u^2 + \frac{1}{4}\omega_3\omega_4\omega_1u - \frac{1}{4}\omega_3\omega_4^2c_s^2, \\
\gamma_{[y^2]}^{[\mu_1]} &= -\frac{1}{2}\omega_2v^2\omega_3\omega_4\omega_1 - \frac{1}{2}\omega_2\omega_3\omega_4c_s^2\omega_1 + \omega_3\omega_4c_s^2\omega_1 + v^2\omega_3\omega_4\omega_1,
\end{aligned}$$

### 4.3 EPDE for $\mu_2$

$$\begin{aligned}
& \gamma_{[1]}^{[\mu_1]}\mu_1 + \gamma_{[1]}^{[\mu_2]}\mu_2 + \gamma_{[t]}^{[\mu_1]}\delta_t \frac{\partial\mu_1}{\partial t} + \gamma_{[t]}^{[\mu_2]}\delta_t \frac{\partial\mu_2}{\partial t} + \gamma_{[x]}^{[\mu_1]}\delta_l \frac{\partial\mu_1}{\partial x} + \gamma_{[x]}^{[\mu_2]}\delta_l \frac{\partial\mu_2}{\partial x} + \gamma_{[y]}^{[\mu_1]}\delta_l \frac{\partial\mu_1}{\partial y} + \gamma_{[y]}^{[\mu_2]}\delta_l \frac{\partial\mu_2}{\partial y} \\
& + \gamma_{[t^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2\mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_2]}\delta_l^2 \frac{\partial^2\mu_2}{\partial t^2} + \gamma_{[tx]}^{[\mu_1]}\delta_l \delta_t \frac{\partial^2\mu_1}{\partial t \partial x} + \gamma_{[tx]}^{[\mu_2]}\delta_l \delta_t \frac{\partial^2\mu_2}{\partial t \partial x} + \gamma_{[ty]}^{[\mu_1]}\delta_l \delta_t \frac{\partial^2\mu_1}{\partial t \partial y} + \gamma_{[ty]}^{[\mu_2]}\delta_l \delta_t \frac{\partial^2\mu_2}{\partial t \partial y} \\
& + \gamma_{[x^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2\mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_2]}\delta_l^2 \frac{\partial^2\mu_2}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]}\delta_l^2 \frac{\partial^2\mu_1}{\partial x \partial y} + \gamma_{[xy]}^{[\mu_2]}\delta_l^2 \frac{\partial^2\mu_2}{\partial x \partial y} + \gamma_{[y^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2\mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_2]}\delta_l^2 \frac{\partial^2\mu_2}{\partial y^2} = 0,
\end{aligned}$$

where

$$\begin{aligned}
\gamma_{[1]}^{[\mu_1]} &= \frac{35}{8}\omega_2\omega_3\omega_4\omega_1u + \frac{1}{8}\omega_2\omega_4^2\omega_1u - \omega_2^2\omega_3\omega_4\omega_1u - \frac{7}{8}\omega_2\omega_3\omega_4^2\omega_1u - \frac{3}{4}\omega_2\omega_3\omega_4\omega_1^2u - \frac{7}{8}\omega_2\omega_3^2\omega_4\omega_1u, \\
\gamma_{[1]}^{[\mu_2]} &= \frac{7}{8}\omega_2\omega_3^2\omega_4\omega_1 + \frac{7}{8}\omega_2\omega_3\omega_4^2\omega_1 - \frac{1}{8}\omega_2\omega_4^2\omega_1 + \omega_2^2\omega_3\omega_4\omega_1 + \frac{3}{4}\omega_2\omega_3\omega_4\omega_1^2 - \frac{35}{8}\omega_2\omega_3\omega_4\omega_1, \\
\gamma_{[t]}^{[\mu_1]} &= -\omega_2^2\omega_4\omega_1u + 5\omega_2\omega_3\omega_1u - \frac{7}{8}\omega_3^2\omega_4\omega_1u - \frac{7}{8}\omega_3\omega_4^2\omega_1u - \frac{39}{2}\omega_2\omega_3\omega_4\omega_1u - \frac{5}{4}\omega_2\omega_4^2\omega_1u - \frac{3}{4}\omega_3\omega_4\omega_1^2u + \\
& 4\omega_2^2\omega_3\omega_4\omega_1u - \frac{3}{4}\omega_2\omega_4\omega_1^2u + \frac{35}{8}\omega_3\omega_4\omega_1u - \omega_2^2\omega_3\omega_1u + \frac{9}{2}\omega_2\omega_4\omega_1u + \frac{7}{2}\omega_2\omega_3\omega_4^2\omega_1u + \frac{1}{8}\omega_4^2\omega_1u + \\
& 3\omega_2\omega_3\omega_4\omega_1^2u - \omega_2\omega_3^2\omega_1u + \frac{7}{2}\omega_2\omega_3^2\omega_4\omega_1u - \omega_2\omega_3\omega_1^2u, \\
\gamma_{[t]}^{[\mu_2]} &= -\frac{7}{2}\omega_2\omega_3^2\omega_4\omega_1 + \omega_2\omega_3\omega_1^2 - \frac{7}{2}\omega_2\omega_3\omega_4^2\omega_1 - \frac{35}{8}\omega_3\omega_4\omega_1 + \frac{7}{8}\omega_2\omega_3\omega_4^2 - \frac{35}{8}\omega_2\omega_3\omega_4 + \frac{5}{4}\omega_2\omega_4^2\omega_1 + \frac{3}{4}\omega_3\omega_4\omega_1^2 - \\
& 4\omega_2^2\omega_3\omega_4\omega_1 - 5\omega_2\omega_3\omega_1 + \omega_2^2\omega_4\omega_1 - \frac{1}{8}\omega_2\omega_4^2 + \omega_2^2\omega_3\omega_4 + \omega_2\omega_3^2\omega_1 - \frac{9}{2}\omega_2\omega_4\omega_1 + \omega_2^2\omega_3\omega_1 + \frac{7}{8}\omega_2\omega_3^2\omega_4 - \\
& \frac{1}{8}\omega_4^2\omega_1 - 3\omega_2\omega_3\omega_4\omega_1^2 + \frac{81}{4}\omega_2\omega_3\omega_4\omega_1 + \frac{7}{8}\omega_3\omega_4^2\omega_1 + \frac{3}{4}\omega_2\omega_4\omega_1^2 + \frac{7}{8}\omega_3^2\omega_4\omega_1, \\
\gamma_{[x]}^{[\mu_1]} &= -\frac{1}{8}\omega_2\omega_4^2c_s^2 + \omega_2^2\omega_3\omega_4c_s^2 + \frac{3}{4}\omega_2\omega_3\omega_4\omega_1u^2 - \frac{35}{8}\omega_2\omega_3\omega_4u^2 + \frac{3}{4}\omega_2\omega_3\omega_4c_s^2\omega_1 + \frac{7}{8}\omega_2\omega_3^2\omega_4c_s^2 + \\
& \frac{1}{4}\omega_2\omega_3\omega_4\omega_1u - \frac{5}{8}\omega_2\omega_3\omega_4u + \frac{7}{8}\omega_2\omega_3\omega_4^2c_s^2 - \frac{35}{8}\omega_2\omega_3\omega_4c_s^2 + \frac{1}{8}\omega_2\omega_3^2\omega_4u - \frac{1}{8}\omega_2\omega_4^2u^2 + \omega_2^2\omega_3\omega_4u^2 + \\
& \frac{1}{8}\omega_2\omega_3\omega_4^2u + \frac{7}{8}\omega_2\omega_3\omega_4^2u^2 + \frac{1}{8}\omega_2\omega_4^2u + \frac{7}{8}\omega_2\omega_3^2\omega_4u^2, \\
\gamma_{[x]}^{[\mu_2]} &= -\frac{1}{8}\omega_2\omega_3\omega_4^2 + \frac{5}{8}\omega_2\omega_3\omega_4 - \frac{1}{8}\omega_2\omega_4^2 - \frac{1}{8}\omega_2\omega_3^2\omega_4 - \frac{1}{4}\omega_2\omega_3\omega_4\omega_1, \\
\gamma_{[y]}^{[\mu_1]} &= \frac{1}{8}\omega_3^2\omega_4\omega_1u + \frac{1}{8}\omega_3\omega_4^2\omega_1u + \frac{1}{4}\omega_3\omega_4\omega_1^2u - \frac{5}{8}\omega_3\omega_4\omega_1u + \frac{1}{8}\omega_4^2\omega_1u, \\
\gamma_{[y]}^{[\mu_2]} &= \frac{5}{8}\omega_3\omega_4\omega_1 - \frac{1}{4}\omega_3\omega_4\omega_1^2 - \frac{1}{8}\omega_4^2\omega_1 - \frac{1}{8}\omega_3\omega_4^2\omega_1 - \frac{1}{8}\omega_3^2\omega_4\omega_1,
\end{aligned}$$

$$\begin{aligned}
\gamma_{[t^2]}^{[\mu_1]} &= -\omega_3\omega_1^2u + \frac{7}{2}\omega_2^2\omega_4\omega_1u - \frac{37}{2}\omega_2\omega_3\omega_1u + \frac{49}{16}\omega_3^2\omega_4\omega_1u - \omega_3^2\omega_1u + \frac{49}{16}\omega_3\omega_4^2\omega_1u + 42\omega_2\omega_3\omega_4\omega_1u + \frac{29}{8}\omega_2\omega_4^2\omega_1u + \\
&\quad \frac{21}{8}\omega_3\omega_4\omega_1^2u - 8\omega_2^2\omega_3\omega_4\omega_1u + 5\omega_2\omega_1u + \frac{21}{8}\omega_2\omega_4\omega_1^2u + \frac{9}{2}\omega_4\omega_1u - \frac{261}{16}\omega_3\omega_4\omega_1u - \omega_2\omega_1^2u + \frac{7}{2}\omega_2^2\omega_3\omega_1u - \\
&\quad \frac{67}{4}\omega_2\omega_4\omega_1u - \frac{3}{4}\omega_4\omega_1^2u - 7\omega_2\omega_3\omega_4^2\omega_1u - \omega_2^2\omega_1u - \frac{19}{16}\omega_4^2\omega_1u - 6\omega_2\omega_3\omega_4\omega_1^2u + \frac{7}{2}\omega_2\omega_3^2\omega_1u - 7\omega_2\omega_3^2\omega_4\omega_1u + \\
&\quad 5\omega_3\omega_1u + \frac{7}{2}\omega_2\omega_3\omega_1^2u, \\
\gamma_{[t^2]}^{[\mu_2]} &= 7\omega_2\omega_3^2\omega_4\omega_1 - 5\omega_2\omega_3 + \omega_2^2\omega_4 - \frac{7}{2}\omega_2\omega_3\omega_1^2 + \frac{3}{4}\omega_4\omega_1^2 + 7\omega_2\omega_3\omega_4^2\omega_1 + \frac{273}{16}\omega_3\omega_4\omega_1 - \frac{49}{16}\omega_2\omega_3\omega_4^2 + \omega_2^2\omega_1 + \\
&\quad \frac{277}{16}\omega_2\omega_3\omega_4 - \frac{29}{8}\omega_2\omega_4^2\omega_1 - \frac{9}{2}\omega_4\omega_1 + \frac{7}{8}\omega_3^2\omega_4 - \frac{21}{8}\omega_3\omega_4\omega_1^2 + 8\omega_2^2\omega_3\omega_4\omega_1 + \omega_2\omega_3^2 + \frac{39}{2}\omega_2\omega_3\omega_1 - \frac{7}{2}\omega_2^2\omega_4\omega_1 + \omega_3^2\omega_1 + \\
&\quad \frac{19}{16}\omega_2\omega_4^2 - \frac{7}{2}\omega_2^2\omega_3\omega_4 - \frac{7}{2}\omega_2\omega_3^2\omega_1 - \frac{1}{8}\omega_4^2 - 5\omega_3\omega_1 + \frac{35}{2}\omega_2\omega_4\omega_1 - \frac{7}{2}\omega_2^2\omega_3\omega_1 - \frac{49}{16}\omega_2\omega_3^2\omega_4 + \omega_2\omega_1^2 - \frac{35}{8}\omega_3\omega_4 + \frac{19}{16}\omega_4^2\omega_1 + \\
&\quad 6\omega_2\omega_3\omega_4\omega_1^2 - \frac{357}{8}\omega_2\omega_3\omega_4\omega_1 + \frac{7}{8}\omega_3\omega_4^2 - \frac{49}{16}\omega_3\omega_4^2\omega_1 - \frac{21}{8}\omega_2\omega_4\omega_1^2 - 5\omega_2\omega_1 - \frac{49}{16}\omega_3^2\omega_4\omega_1 + \omega_3\omega_1^2 - \frac{9}{2}\omega_2\omega_4 + \omega_2^2\omega_3, \\
\gamma_{[tx]}^{[\mu_1]} &= \frac{1}{8}\omega_2\omega_4^2c_s^2 + \frac{1}{2}\omega_2v^2\omega_3\omega_1 - \frac{1}{2}\omega_2^2v^2\omega_4 + \frac{7}{8}\omega_3^2\omega_4u^2 + \omega_2\omega_3c_s^2\omega_1 + \frac{1}{2}\omega_2^2\omega_4 - 3\omega_2^2\omega_3\omega_4c_s^2 - \frac{9}{4}\omega_2\omega_3\omega_4\omega_1u^2 + \\
&\quad \frac{7}{8}\omega_3\omega_4^2u^2 - \frac{5}{2}\omega_2v^2\omega_3 + \frac{113}{8}\omega_2\omega_3\omega_4u^2 - 5\omega_2\omega_3c_s^2 + \frac{1}{8}\omega_4^2u - \frac{1}{2}\omega_2\omega_4u + \frac{1}{2}\omega_2\omega_3\omega_4 - \frac{9}{4}\omega_2\omega_3\omega_4c_s^2\omega_1 - \\
&\quad 2\omega_2\omega_4u^2 + \frac{1}{2}\omega_2^2\omega_3u^2 - \frac{21}{8}\omega_2\omega_3^2\omega_4c_s^2 - \frac{3}{4}\omega_2\omega_3\omega_4\omega_1u + \frac{1}{2}\omega_2v^2\omega_3^2 + \frac{1}{8}\omega_3\omega_4^2u - \frac{1}{8}\omega_4^2u^2 + \frac{15}{8}\omega_2\omega_3\omega_4u - \\
&\quad \frac{21}{8}\omega_2\omega_3\omega_4^2c_s^2 + \frac{1}{2}\omega_2\omega_3^2u^2 + \frac{1}{2}\omega_2\omega_3\omega_1 - \frac{35}{8}\omega_3\omega_4c_s^2 + \frac{1}{8}\omega_3^2\omega_4u + \frac{1}{2}\omega_2\omega_3\omega_1u^2 + \frac{1}{2}\omega_2\omega_4^2 + \frac{1}{4}\omega_3\omega_4\omega_1u - \\
&\quad \frac{5}{2}\omega_2\omega_3u^2 + \frac{113}{8}\omega_2\omega_3\omega_4c_s^2 + \frac{1}{2}\omega_2^2\omega_4u^2 + \frac{7}{8}\omega_3\omega_4^2c_s^2 - \frac{1}{4}\omega_2\omega_4c_s^2\omega_1 - \frac{3}{8}\omega_2\omega_3^2\omega_4u + \frac{1}{4}\omega_2\omega_4\omega_1u + \\
&\quad \frac{3}{4}\omega_3\omega_4c_s^2\omega_1 + \frac{7}{8}\omega_3\omega_4c_s^2 + \frac{1}{2}\omega_2\omega_4\omega_1 + \frac{5}{8}\omega_2\omega_4^2u^2 - 3\omega_2^2\omega_3\omega_4u^2 - \frac{5}{8}\omega_3\omega_4u - \frac{1}{2}\omega_2v^2\omega_4^2 - \frac{3}{8}\omega_2\omega_3\omega_4^2u + \\
&\quad \frac{5}{2}\omega_2v^2\omega_4 - \frac{35}{8}\omega_3\omega_4u^2 - \frac{1}{2}\omega_2v^2\omega_4\omega_1 + \omega_2\omega_3^2c_s^2 - \frac{21}{8}\omega_2\omega_3\omega_4^2u^2 - \frac{1}{8}\omega_2\omega_4^2u - \frac{21}{8}\omega_2\omega_3^2\omega_4u^2 + \omega_2^2\omega_3c_s^2 + \\
&\quad \frac{1}{2}\omega_2\omega_4c_s^2 + \frac{1}{4}\omega_2\omega_4\omega_1u^2 + \frac{1}{2}\omega_2^2v^2\omega_3 - 2\omega_2\omega_4 - \frac{1}{2}\omega_2^2\omega_3 + \frac{3}{4}\omega_3\omega_4\omega_1u^2 - \frac{1}{8}\omega_4^2c_s^2, \\
\gamma_{[tx]}^{[\mu_2]} &= -\frac{1}{4}\omega_3\omega_4\omega_1 + \frac{3}{8}\omega_2\omega_3\omega_4^2 - \frac{15}{8}\omega_2\omega_3\omega_4 - \frac{1}{8}\omega_3^2\omega_4 + \frac{1}{8}\omega_2\omega_4^2 - \frac{1}{8}\omega_4^2 - \frac{1}{4}\omega_2\omega_4\omega_1 + \frac{3}{8}\omega_2\omega_3^2\omega_4 + \frac{5}{8}\omega_3\omega_4 + \\
&\quad \frac{3}{4}\omega_2\omega_3\omega_4\omega_1 - \frac{1}{8}\omega_3\omega_4^2 + \frac{1}{2}\omega_2\omega_4, \\
\gamma_{[ty]}^{[\mu_1]} &= -\frac{3}{8}\omega_3^2\omega_4\omega_1u - \frac{3}{8}\omega_3\omega_4^2\omega_1u - \frac{3}{4}\omega_3\omega_4\omega_1^2u - \frac{1}{2}\omega_4\omega_1u + \frac{15}{8}\omega_3\omega_4\omega_1u + \frac{1}{4}\omega_4\omega_1^2u - \frac{1}{8}\omega_4^2\omega_1u, \\
\gamma_{[ty]}^{[\mu_2]} &= -\frac{1}{4}\omega_4\omega_1^2 - \frac{17}{8}\omega_3\omega_4\omega_1 + \frac{1}{2}\omega_4\omega_1 - \frac{1}{8}\omega_3^2\omega_4 + \frac{3}{4}\omega_3\omega_4\omega_1^2 - \frac{1}{8}\omega_4^2 + \frac{5}{8}\omega_3\omega_4 + \frac{1}{8}\omega_4^2\omega_1 - \frac{1}{8}\omega_3\omega_4^2 + \frac{3}{8}\omega_3\omega_4^2\omega_1 + \frac{3}{8}\omega_3^2\omega_4\omega_1, \\
\gamma_{[x^2]}^{[\mu_1]} &= \frac{5}{4}\omega_2\omega_3\omega_1u - \frac{1}{4}\omega_2\omega_3\omega_4\omega_1u - \frac{1}{2}\omega_2^2\omega_3\omega_1u + \frac{1}{4}\omega_2\omega_4\omega_1u - \frac{1}{4}\omega_2\omega_3^2\omega_1u, \\
\gamma_{[x^2]}^{[\mu_2]} &= \frac{5}{2}\omega_2\omega_3 + \frac{7}{16}\omega_2\omega_3\omega_4^2 - \frac{43}{16}\omega_2\omega_3\omega_4 - \frac{1}{2}\omega_2\omega_3^2 - \frac{5}{4}\omega_2\omega_3\omega_1 - \frac{1}{16}\omega_2\omega_4^2 + \frac{1}{2}\omega_2^2\omega_3\omega_4 + \frac{1}{4}\omega_2\omega_3^2\omega_1 - \frac{1}{4}\omega_2\omega_4\omega_1 + \\
&\quad \frac{1}{2}\omega_2^2\omega_3\omega_1 + \frac{7}{16}\omega_2\omega_3^2\omega_4 + \frac{5}{8}\omega_2\omega_3\omega_4\omega_1 + \frac{1}{2}\omega_2\omega_4 - \omega_2^2\omega_3, \\
\gamma_{[xy]}^{[\mu_1]} &= -\frac{1}{8}\omega_3^2\omega_4u^2 - \frac{1}{8}\omega_3\omega_4^2u^2 + \frac{1}{2}\omega_2v\omega_4 + \frac{1}{8}\omega_4^2u + \frac{1}{8}\omega_3\omega_4^2u - \frac{1}{8}\omega_4^2u^2 + \frac{5}{8}\omega_3\omega_4c_s^2 + \frac{1}{8}\omega_3^2\omega_4u + \frac{1}{4}\omega_3\omega_4\omega_1u - \\
&\quad \frac{1}{8}\omega_3\omega_4^2c_s^2 - \frac{1}{4}\omega_3\omega_4c_s^2\omega_1 - \frac{1}{8}\omega_3^2\omega_4c_s^2 - \frac{5}{8}\omega_3\omega_4u + \frac{1}{2}\omega_2v\omega_3^2 - \frac{5}{2}\omega_2v\omega_3 + \frac{5}{8}\omega_3\omega_4u^2 + \omega_2v\omega_3\omega_1 + \frac{1}{2}\omega_2v\omega_3\omega_4 - \\
&\quad \frac{1}{4}\omega_3\omega_4\omega_1u^2 - \frac{1}{8}\omega_4^2c_s^2, \\
\gamma_{[xy]}^{[\mu_2]} &= -\frac{1}{4}\omega_3\omega_4\omega_1 - \frac{1}{8}\omega_3^2\omega_4 - \frac{1}{8}\omega_4^2 + \frac{5}{8}\omega_3\omega_4 - \frac{1}{8}\omega_3\omega_4^2, \\
\gamma_{[y^2]}^{[\mu_1]} &= \omega_3\omega_1^2u + \frac{5}{4}\omega_2\omega_3\omega_1u - \frac{7}{16}\omega_3^2\omega_4\omega_1u + \frac{1}{2}\omega_3^2\omega_1u - \frac{7}{16}\omega_3\omega_4^2\omega_1u - \frac{3}{4}\omega_2\omega_3\omega_4\omega_1u - \frac{3}{8}\omega_3\omega_4\omega_1^2u - \frac{1}{2}\omega_4\omega_1u + \\
&\quad \frac{43}{16}\omega_3\omega_4\omega_1u + \frac{1}{4}\omega_2\omega_4\omega_1u + \frac{1}{16}\omega_4^2\omega_1u - \frac{1}{4}\omega_2\omega_3^2\omega_1u - \frac{5}{2}\omega_3\omega_1u - \frac{1}{2}\omega_2\omega_3\omega_1^2u, \\
\gamma_{[y^2]}^{[\mu_2]} &= \frac{1}{2}\omega_2\omega_3\omega_1^2 - \frac{43}{16}\omega_3\omega_4\omega_1 + \frac{1}{2}\omega_4\omega_1 + \frac{3}{8}\omega_3\omega_4\omega_1^2 - \frac{5}{4}\omega_2\omega_3\omega_1 - \frac{1}{2}\omega_3^2\omega_1 + \frac{1}{4}\omega_2\omega_3^2\omega_1 + \frac{5}{2}\omega_3\omega_1 - \frac{1}{4}\omega_2\omega_4\omega_1 - \\
&\quad \frac{1}{16}\omega_4^2\omega_1 + \frac{3}{4}\omega_2\omega_3\omega_4\omega_1 + \frac{7}{16}\omega_3\omega_4^2\omega_1 + \frac{7}{16}\omega_3^2\omega_4\omega_1 - \omega_3\omega_1^2,
\end{aligned}$$

#### 4.4 EPDE for $\mu_3$

$$\begin{aligned}
& \gamma_{[1]}^{[\mu_1]} \mu_1 + \gamma_{[1]}^{[\mu_3]} \mu_3 + \gamma_{[t]}^{[\mu_1]} \delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_3]} \delta_t \frac{\partial \mu_3}{\partial t} + \gamma_{[x]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[x]}^{[\mu_3]} \delta_l \frac{\partial \mu_3}{\partial x} + \gamma_{[y]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[y]}^{[\mu_3]} \delta_l \frac{\partial \mu_3}{\partial y} \\
& + \gamma_{[t^2]}^{[\mu_1]} \delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_3]} \delta_t^2 \frac{\partial^2 \mu_3}{\partial t^2} + \gamma_{[tx]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} + \gamma_{[tx]}^{[\mu_3]} \delta_t \delta_l \frac{\partial^2 \mu_3}{\partial t \partial x} + \gamma_{[ty]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[ty]}^{[\mu_3]} \delta_t \delta_l \frac{\partial^2 \mu_3}{\partial t \partial y} \\
& + \gamma_{[x^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_3]} \delta_l^2 \frac{\partial^2 \mu_3}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x \partial y} + \gamma_{[xy]}^{[\mu_3]} \delta_l^2 \frac{\partial^2 \mu_3}{\partial x \partial y} + \gamma_{[y^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_3]} \delta_l^2 \frac{\partial^2 \mu_3}{\partial y^2} = 0,
\end{aligned}$$

where

$$\begin{aligned}
\gamma_{[1]}^{[\mu_1]} &= -\frac{7}{8}\omega_2 v \omega_3^2 \omega_4 \omega_1 - \frac{7}{8}\omega_2 v \omega_3 \omega_4^2 \omega_1 + \frac{35}{8}\omega_2 v \omega_3 \omega_4 \omega_1 - \omega_2^2 v \omega_3 \omega_4 \omega_1 + \frac{1}{8}\omega_2 v \omega_4^2 \omega_1 - \frac{3}{4}\omega_2 v \omega_3 \omega_4 \omega_1^2, \\
\gamma_{[1]}^{[\mu_3]} &= \frac{7}{8}\omega_2 \omega_3^2 \omega_4 \omega_1 + \frac{7}{8}\omega_2 \omega_3 \omega_4^2 \omega_1 - \frac{1}{8}\omega_2 \omega_4^2 \omega_1 + \omega_2^2 \omega_3 \omega_4 \omega_1 + \frac{3}{4}\omega_2 \omega_3 \omega_4 \omega_1^2 - \frac{35}{8}\omega_2 \omega_3 \omega_4 \omega_1, \\
\gamma_{[t]}^{[\mu_1]} &= -\frac{1}{8}\omega_2 v^2 \omega_3 \omega_4 \omega_1 + \frac{9}{2}\omega_2 v \omega_4 \omega_1 - \frac{7}{8}\omega_2 v \omega_3^2 \omega_4 - \frac{1}{4}\omega_3 \omega_4^3 u + \frac{1}{8}\omega_3 \omega_4 \omega_1^2 u^2 - \frac{1}{8}v^2 \omega_3^2 \omega_4 \omega_1 + \frac{1}{4}\omega_4^3 c_s^2 \omega_1 + \\
& \frac{1}{8}\omega_2 \omega_3 \omega_4 \omega_1 u^2 - \omega_2 v \omega_3^2 \omega_1 + \frac{1}{8}\omega_2 v^2 \omega_4^2 \omega_1 + \frac{1}{8}\omega_4^2 \omega_1^2 u^2 + \frac{1}{8}\omega_2 \omega_4^2 \omega_1 u^2 + \frac{1}{4}\omega_3 \omega_4^2 \omega_1 u^2 - \frac{1}{8}\omega_2 \omega_1^2 \omega_1 + \\
& \frac{1}{4}\omega_2 \omega_4^2 c_s^2 \omega_1 - \frac{1}{4}\omega_3 \omega_4^2 \omega_1 u + \frac{5}{2}\omega_2 v \omega_3^2 \omega_4 \omega_1 + \frac{1}{4}\omega_3 \omega_4^2 c_s^2 \omega_1 + \frac{1}{8}\omega_2 v \omega_4^2 + \frac{5}{4}\omega_3 \omega_4^2 u - \frac{1}{8}\omega_3 \omega_4 \omega_1^2 - \omega_2^2 v \omega_3 \omega_1 + \\
& \frac{1}{8}\omega_3^2 \omega_4 \omega_1 u^2 - \omega_2^2 v \omega_3 \omega_4 + \frac{1}{8}\omega_4^3 \omega_1 u^2 - \frac{3}{4}\omega_2 v \omega_4 \omega_1^2 + \frac{7}{2}\omega_2 v \omega_3 \omega_4^2 \omega_1 + \frac{1}{8}v^2 \omega_4^3 \omega_1 - \omega_2^2 v \omega_4 \omega_1 - \frac{1}{4}\omega_3^2 \omega_1^2 u - \\
& \frac{7}{4}\omega_2 v \omega_3 \omega_4 \omega_1 - \frac{7}{8}\omega_2 v \omega_3 \omega_4^2 - \frac{1}{8}v^2 \omega_3 \omega_4 \omega_1^2 - \frac{5}{4}\omega_4^2 c_s^2 \omega_1 + 4\omega_2^2 v \omega_3 \omega_4 \omega_1 - \omega_2 v \omega_3 \omega_1^2 + \frac{1}{2}\omega_4^2 \omega_1 + \frac{1}{8}v^2 \omega_4^2 \omega_1^2 - \\
& \frac{1}{4}\omega_2 \omega_3 \omega_4^2 u - \frac{5}{8}\omega_4^2 \omega_1 u^2 + \frac{1}{8}\omega_2 \omega_3 \omega_4 \omega_1 + 5\omega_2 v \omega_3 \omega_1 - \frac{1}{8}\omega_2^2 \omega_1^2 - \frac{1}{8}\omega_3 \omega_4^2 \omega_1 - \frac{5}{8}v^2 \omega_4^2 \omega_1 + \frac{5}{8}v^2 \omega_3 \omega_4 \omega_1 + \\
& \frac{1}{4}\omega_4^2 c_s^2 \omega_1^2 + \frac{35}{8}\omega_2 v \omega_3 \omega_4 - \frac{5}{4}\omega_2 v \omega_4^2 \omega_1 + 3\omega_2 v \omega_3 \omega_4 \omega_1^2 - \frac{1}{8}\omega_4^3 \omega_1 - \frac{5}{8}\omega_3 \omega_4 \omega_1 u^2, \\
\gamma_{[t]}^{[\mu_3]} &= -\frac{7}{2}\omega_2 \omega_3^2 \omega_4 \omega_1 + \omega_2 \omega_3 \omega_4^2 \omega_1 - \frac{7}{2}\omega_2 \omega_3 \omega_4^2 \omega_1 - \frac{35}{8}\omega_3 \omega_4 \omega_1 + \frac{7}{8}\omega_2 \omega_3 \omega_4^2 - \frac{35}{8}\omega_2 \omega_3 \omega_4 + \frac{5}{4}\omega_2 \omega_4^2 \omega_1 + \frac{3}{4}\omega_3 \omega_4 \omega_1^2 - \\
& 4\omega_2^2 \omega_3 \omega_4 \omega_1 - 5\omega_2 \omega_3 \omega_1 + \omega_2^2 \omega_4 \omega_1 - \frac{1}{8}\omega_2 \omega_4^2 + \omega_2^2 \omega_3 \omega_4 + \omega_2 \omega_3^2 \omega_1 - \frac{9}{2}\omega_2 \omega_4 \omega_1 + \omega_2^2 \omega_3 \omega_1 + \frac{7}{8}\omega_2 \omega_3^2 \omega_4 - \\
& \frac{1}{8}\omega_4^2 \omega_1 - 3\omega_2 \omega_3 \omega_4 \omega_1^2 + \frac{81}{4}\omega_2 \omega_3 \omega_4 \omega_1 + \frac{7}{8}\omega_3 \omega_4^2 \omega_1 + \frac{3}{4}\omega_2 \omega_4 \omega_1^2 + \frac{7}{8}\omega_3^2 \omega_4 \omega_1, \\
\gamma_{[x]}^{[\mu_1]} &= \frac{1}{8}\omega_2 v \omega_3^2 \omega_4 + \frac{1}{8}\omega_3^2 \omega_4 \omega_1 u + \frac{5}{4}\omega_3 \omega_4^2 u^2 - \frac{1}{4}\omega_3 \omega_4^2 \omega_1 u^2 - \frac{1}{4}\omega_3^2 \omega_4^2 c_s^2 + \frac{1}{8}\omega_3 \omega_4^2 \omega_1 u - \frac{1}{4}\omega_3 \omega_4^2 c_s^2 \omega_1 + \\
& \frac{1}{4}\omega_2 \omega_3 \omega_4 \omega_1 u + \frac{1}{8}\omega_2 v \omega_4^2 - \frac{1}{4}\omega_2 \omega_3 \omega_4^2 c_s^2 - \frac{1}{4}\omega_3 \omega_4^3 u^2 - \frac{5}{8}\omega_3 \omega_4 \omega_1 u + \frac{5}{4}\omega_3 \omega_4^2 c_s^2 + \frac{1}{4}\omega_2 v \omega_3 \omega_4 \omega_1 + \frac{1}{8}\omega_2 v \omega_3 \omega_4^2 - \\
& \frac{1}{8}\omega_4^2 \omega_1 u - \frac{1}{4}\omega_2 \omega_3 \omega_4^2 u^2 - \frac{1}{4}\omega_3 \omega_4^3 c_s^2 - \frac{1}{4}\omega_3^2 \omega_4^2 u^2 - \frac{5}{8}\omega_2 v \omega_3 \omega_4, \\
\gamma_{[x]}^{[\mu_3]} &= -\frac{1}{8}\omega_2 \omega_3 \omega_4^2 + \frac{5}{8}\omega_2 \omega_3 \omega_4 - \frac{1}{8}\omega_2 \omega_4^2 - \frac{1}{8}\omega_2 \omega_3^2 \omega_4 - \frac{1}{4}\omega_2 \omega_3 \omega_4 \omega_1, \\
\gamma_{[y]}^{[\mu_1]} &= \omega_2 v^2 \omega_3 \omega_4 \omega_1 + \frac{7}{8}\omega_3^2 \omega_4 c_s^2 \omega_1 + \frac{7}{8}v^2 \omega_3^2 \omega_4 \omega_1 + \frac{7}{8}v^2 \omega_3 \omega_4^2 \omega_1 + \omega_2 \omega_3 \omega_4 c_s^2 \omega_1 + \frac{7}{8}\omega_3 \omega_4^2 c_s^2 \omega_1 - \frac{35}{8}\omega_3 \omega_4 c_s^2 \omega_1 + \\
& \frac{3}{4}v^2 \omega_3 \omega_4 \omega_1^2 - \frac{1}{8}\omega_4^2 c_s^2 \omega_1 - \frac{1}{8}v^2 \omega_4^2 \omega_1 - \frac{35}{8}v^2 \omega_3 \omega_4 \omega_1 + \frac{3}{4}\omega_3 \omega_4 c_s^2 \omega_1^2, \\
\gamma_{[y]}^{[\mu_3]} &= \frac{5}{8}\omega_3 \omega_4 \omega_1 - \frac{1}{4}\omega_3 \omega_4 \omega_1^2 - \frac{1}{8}\omega_4^2 \omega_1 - \frac{1}{8}\omega_3 \omega_4^2 \omega_1 - \frac{1}{8}\omega_3^2 \omega_4 \omega_1, \\
\gamma_{[t^2]}^{[\mu_1]} &= \frac{1}{8}\omega_4^3 u^2 + \frac{1}{4}\omega_2 \omega_4^2 c_s^2 + \frac{7}{16}\omega_2 v^2 \omega_3 \omega_4 \omega_1 - \frac{33}{2}\omega_2 v \omega_4 \omega_1 + \frac{49}{16}\omega_2 v \omega_3^2 \omega_4 + 5\omega_2 v \omega_1 - \omega_2^2 v \omega_3 + \frac{7}{8}\omega_3 \omega_4^3 u + \\
& \frac{1}{8}\omega_3^2 \omega_4 u^2 - \frac{5}{4}\omega_4 \omega_1 u^2 - \frac{7}{16}\omega_3 \omega_4 \omega_1^2 u^2 + \frac{1}{4}\omega_4 c_s^2 \omega_1^2 + \frac{7}{16}v^2 \omega_3^2 \omega_4 \omega_1 - \frac{7}{8}\omega_4^3 c_s^2 \omega_1 - \frac{1}{4}\omega_4 \omega_1^2 - \frac{1}{8}\omega_2 v^2 \omega_3 \omega_4 - \\
& \frac{7}{16}\omega_2 \omega_3 \omega_4 \omega_1 u^2 + \frac{7}{2}\omega_2 v \omega_3^2 \omega_1 - \frac{7}{16}\omega_2 v^2 \omega_4^2 \omega_1 - \frac{7}{16}\omega_4^2 \omega_1^2 u^2 + \frac{1}{4}\omega_3 \omega_4^2 u^2 + \frac{9}{2}\omega_2 v \omega_4 - \frac{7}{16}\omega_2 \omega_4^2 \omega_1 u^2 + \\
& \frac{1}{8}\omega_2 \omega_3 \omega_4 u^2 + \frac{5}{4}\omega_4^2 u - \frac{1}{8}v^2 \omega_3^2 \omega_4 - \frac{1}{4}\omega_3 \omega_4 \omega_1 - \frac{7}{8}\omega_3 \omega_4^2 \omega_1 u^2 + \frac{1}{8}\omega_2 \omega_3 \omega_4 + \frac{7}{16}\omega_2 \omega_4^2 \omega_1 - \frac{7}{8}\omega_2 \omega_4^2 c_s^2 \omega_1 - \\
& \frac{5}{8}v^2 \omega_4^2 + \frac{7}{8}\omega_3 \omega_4^3 \omega_1 u - 7\omega_2 v \omega_3^2 \omega_4 \omega_1 - \frac{7}{8}\omega_3 \omega_4^2 c_s^2 \omega_1 + \frac{1}{2}\omega_4 \omega_1 - \frac{19}{16}\omega_2 v \omega_4^2 - \frac{37}{8}\omega_3 \omega_4^2 u - \frac{5}{8}\omega_4^2 u^2 - \frac{1}{4}\omega_2 \omega_3 \omega_4 u + \\
& \frac{7}{16}\omega_3 \omega_4 \omega_1^2 + \frac{7}{2}\omega_2^2 v \omega_3 \omega_1 - \frac{5}{4}\omega_4 c_s^2 \omega_1 + \frac{1}{8}v^2 \omega_4^3 - \frac{7}{16}\omega_3^2 \omega_4 \omega_1 u^2 - \frac{1}{4}\omega_4^3 u + \frac{7}{2}\omega_2^2 v \omega_3 \omega_4 - \frac{7}{16}\omega_4^3 \omega_1 u^2 + \\
& \frac{21}{8}\omega_2 v \omega_4 \omega_1^2 - 7\omega_2 v \omega_3 \omega_4^2 \omega_1 - \frac{1}{4}\omega_3^2 \omega_4 u - \omega_2 v \omega_1^2 - \frac{1}{8}\omega_2 \omega_4^2 - \frac{1}{4}\omega_3 \omega_4 \omega_1 u + \frac{1}{4}\omega_3 \omega_4^2 c_s^2 + \frac{1}{4}\omega_2 \omega_4 c_s^2 \omega_1 + \frac{1}{2}\omega_4^2 - \\
& \frac{7}{16}v^2 \omega_4^3 \omega_1 + \frac{7}{2}\omega_2^2 v \omega_4 \omega_1 + \frac{7}{8}\omega_3^2 \omega_4^2 u + \frac{329}{8}\omega_2 v \omega_3 \omega_4 \omega_1 + \frac{1}{4}\omega_3 \omega_4 c_s^2 \omega_1 + \frac{49}{16}\omega_2 v \omega_3 \omega_4^2 - \frac{1}{8}\omega_4^3 + \frac{7}{16}v^2 \omega_3 \omega_4 \omega_1^2 + \\
& \frac{37}{8}\omega_4^2 c_s^2 \omega_1 + \frac{1}{4}\omega_4^3 c_s^2 + \frac{1}{8}\omega_2 \omega_4^2 u^2 - 8\omega_2^2 v \omega_3 \omega_4 \omega_1 - \frac{1}{4}\omega_4^2 \omega_1 u + \frac{5}{4}\omega_3 \omega_4 u - \omega_2 v \omega_3^2 + \frac{1}{8}\omega_2 v^2 \omega_4^2 + \frac{7}{2}\omega_2 v \omega_3 \omega_1^2 - \\
& 2\omega_4^2 \omega_1 - \frac{7}{16}v^2 \omega_4^2 \omega_1^2 + \frac{7}{8}\omega_2 \omega_3 \omega_4^2 u - \omega_2^2 v \omega_1 + 5\omega_2 v \omega_3 + \frac{39}{16}\omega_4^2 \omega_1 u^2 - \frac{5}{8}\omega_3 \omega_4 u^2 - \frac{7}{16}\omega_2 \omega_3 \omega_4 \omega_1 - \frac{37}{2}\omega_2 v \omega_3 \omega_1 - \\
& \frac{1}{8}\omega_3 \omega_4^2 + \frac{7}{16}\omega_4^2 \omega_1^2 + \frac{7}{16}\omega_3 \omega_4^2 \omega_1 + \frac{35}{16}v^2 \omega_4^2 \omega_1 + \frac{5}{8}v^2 \omega_3 \omega_4 - \frac{35}{16}v^2 \omega_3 \omega_4 \omega_1 - \frac{7}{8}\omega_4^2 c_s^2 \omega_1^2 - \frac{1}{4}\omega_2 \omega_4^2 u + \frac{1}{4}\omega_2 \omega_4 \omega_1 u^2 + \\
& \frac{1}{4}\omega_4 \omega_1^2 u^2 - \omega_2^2 v \omega_4 - \frac{261}{16}\omega_2 v \omega_3 \omega_4 + \frac{29}{8}\omega_2 v \omega_4^2 \omega_1 - 6\omega_2 v \omega_3 \omega_4 \omega_1^2 + \frac{7}{16}\omega_4^3 \omega_1 + \frac{39}{16}\omega_3 \omega_4 \omega_1 u^2 - \frac{5}{4}\omega_4^2 c_s^2,
\end{aligned}$$



$$\begin{aligned}
\gamma_{[t^2]}^{[\mu_3]} &= 7\omega_2\omega_3^2\omega_4\omega_1 - 5\omega_2\omega_3 + \omega_2^2\omega_4 - \frac{7}{2}\omega_2\omega_3\omega_1^2 + \frac{3}{4}\omega_4\omega_1^2 + 7\omega_2\omega_3\omega_4^2\omega_1 + \frac{273}{16}\omega_3\omega_4\omega_1 - \frac{49}{16}\omega_2\omega_3\omega_4^2 + \omega_2^2\omega_1 + \\
&\quad \frac{277}{16}\omega_2\omega_3\omega_4 - \frac{29}{8}\omega_2\omega_4^2\omega_1 - \frac{9}{2}\omega_4\omega_1 + \frac{7}{8}\omega_3^2\omega_4 - \frac{21}{8}\omega_3\omega_4\omega_1^2 + 8\omega_2^2\omega_3\omega_4\omega_1 + \omega_2\omega_3^2 + \frac{39}{2}\omega_2\omega_3\omega_1 - \frac{7}{2}\omega_2^2\omega_4\omega_1 + \omega_2^2\omega_1 + \\
&\quad \frac{19}{16}\omega_2\omega_4^2 - \frac{7}{2}\omega_2^2\omega_3\omega_4 - \frac{7}{2}\omega_2\omega_3^2\omega_1 - \frac{1}{8}\omega_4^2 - 5\omega_3\omega_1 + \frac{35}{2}\omega_2\omega_4\omega_1 - \frac{7}{2}\omega_2^2\omega_3\omega_1 - \frac{49}{16}\omega_2\omega_3^2\omega_4 + \omega_2\omega_1^2 - \frac{35}{8}\omega_3\omega_4 + \frac{19}{16}\omega_4^2\omega_1 + \\
&\quad 6\omega_2\omega_3\omega_4\omega_1^2 - \frac{357}{8}\omega_2\omega_3\omega_4\omega_1 + \frac{7}{8}\omega_3\omega_4^2 - \frac{49}{16}\omega_3\omega_4^2\omega_1 - \frac{21}{8}\omega_2\omega_4\omega_1^2 - 5\omega_2\omega_1 - \frac{49}{16}\omega_2^2\omega_4\omega_1 + \omega_3\omega_1^2 - \frac{9}{2}\omega_2\omega_4 + \omega_2^2\omega_3, \\
\gamma_{[tx]}^{[\mu_1]} &= -\frac{1}{8}\omega_4^3u^2 + \frac{1}{4}\omega_2v\omega_4\omega_1 - \frac{3}{8}\omega_2v\omega_3\omega_4 - \frac{1}{8}\omega_3^2\omega_4u^2 - \frac{3}{8}\omega_3^2\omega_4\omega_1u - \frac{1}{8}\omega_2v^2\omega_3\omega_4 - \frac{15}{4}\omega_3\omega_4^2u^2 - \frac{1}{2}\omega_2v\omega_4 - \\
&\quad \frac{1}{4}\omega_2\omega_3\omega_4u^2 - \frac{1}{8}v^2\omega_3^2\omega_4 - \frac{1}{8}\omega_3\omega_4\omega_1 + \frac{3}{4}\omega_3\omega_4^2\omega_1u^2 + \frac{1}{8}\omega_2\omega_3\omega_4 + \frac{3}{4}\omega_3^2\omega_4^2c_s^2 - \frac{5}{8}v^2\omega_4^2 - \frac{3}{8}\omega_3\omega_4^2\omega_1u + \\
&\quad \frac{1}{4}\omega_3\omega_4^2c_s^2\omega_1 - \frac{3}{4}\omega_2\omega_3\omega_4\omega_1u - \frac{1}{8}\omega_2v\omega_4^2 + \frac{5}{8}\omega_4^2u^2 + \frac{3}{4}\omega_2\omega_3\omega_4^2c_s^2 + \frac{1}{8}v^2\omega_4^3 + \frac{5}{4}\omega_3\omega_4c_s^2 - \frac{3}{4}\omega_4\omega_1u + \frac{3}{4}\omega_3\omega_4^3u^2 - \\
&\quad \frac{1}{4}\omega_2\omega_4^2 + \frac{15}{8}\omega_3\omega_4\omega_1u - \frac{1}{4}\omega_2\omega_3\omega_4c_s^2 - \frac{15}{4}\omega_3\omega_4^2c_s^2 + \frac{1}{2}\omega_4^2 + \frac{1}{4}\omega_2\omega_4\omega_1u - \frac{3}{4}\omega_2v\omega_3\omega_4\omega_1 - \frac{1}{4}\omega_3\omega_4c_s^2\omega_1 - \\
&\quad \frac{1}{8}\omega_2v\omega_3\omega_4^2 - \frac{1}{4}\omega_3^2\omega_4c_s^2 - \frac{1}{8}\omega_4^3 - \frac{1}{8}\omega_2\omega_4^2u^2 + \frac{3}{8}\omega_4^2\omega_1u + \frac{1}{8}\omega_2v^2\omega_4^2 - \frac{1}{8}\omega_4^2\omega_1 - \frac{1}{8}\omega_1^2\omega_4u^2 + \frac{5}{8}\omega_3\omega_4u^2 - \frac{1}{8}\omega_3\omega_4^2 + \\
&\quad \frac{1}{8}v^2\omega_4^2\omega_1 + \frac{5}{8}v^2\omega_3\omega_4 + \frac{3}{4}\omega_2\omega_3\omega_4^2u^2 - \frac{1}{8}v^2\omega_3\omega_4\omega_1 + \frac{3}{4}\omega_3\omega_4^3c_s^2 + \frac{3}{4}\omega_3^2\omega_4^2u^2 + \frac{15}{8}\omega_2v\omega_3\omega_4 - \frac{1}{8}\omega_3\omega_4\omega_1u^2, \\
\gamma_{[tx]}^{[\mu_3]} &= -\frac{1}{4}\omega_3\omega_4\omega_1 + \frac{3}{8}\omega_2\omega_3\omega_4^2 - \frac{15}{8}\omega_2\omega_3\omega_4 - \frac{1}{8}\omega_3^2\omega_4 + \frac{1}{8}\omega_2\omega_4^2 - \frac{1}{8}\omega_4^2 - \frac{1}{4}\omega_2\omega_4\omega_1 + \frac{3}{8}\omega_2\omega_3^2\omega_4 + \frac{5}{8}\omega_3\omega_4 + \\
&\quad \frac{3}{4}\omega_2\omega_3\omega_4\omega_1 - \frac{1}{8}\omega_3\omega_4^2 + \frac{1}{2}\omega_2\omega_4, \\
\gamma_{[ty]}^{[\mu_1]} &= -3\omega_2v^2\omega_3\omega_4\omega_1 + \frac{1}{2}v^2\omega_3^2\omega_1 - \frac{21}{8}\omega_3^2\omega_4c_s^2\omega_1 + \frac{1}{2}\omega_2v^2\omega_3\omega_1 + \frac{7}{4}\omega_4\omega_1u^2 + \omega_2\omega_3c_s^2\omega_1 + \frac{1}{4}\omega_4c_s^2\omega_1^2 - \\
&\quad \frac{21}{8}v^2\omega_3^2\omega_4\omega_1 + \frac{1}{4}\omega_4\omega_1^2 + \omega_2v^2\omega_3\omega_4 + \frac{1}{8}\omega_4^2u + \frac{7}{8}v^2\omega_3^2\omega_4 - \frac{5}{2}v^2\omega_4\omega_1 + \frac{3}{8}\omega_3\omega_4\omega_1 + \omega_3^2c_s^2\omega_1 - \frac{21}{8}v^2\omega_3\omega_4^2\omega_1 - \\
&\quad \frac{1}{8}v^2\omega_4^2 - 3\omega_2\omega_3\omega_4c_s^2\omega_1 - \frac{21}{8}\omega_3\omega_4^2c_s^2\omega_1 - \frac{3}{2}\omega_4\omega_1 - \frac{1}{8}\omega_3\omega_4^2u + \frac{1}{2}\omega_3^2\omega_1u^2 + \frac{1}{2}v^2\omega_4\omega_1^2 - \frac{3}{4}\omega_4c_s^2\omega_1 + \\
&\quad \frac{1}{2}\omega_2\omega_3\omega_1 - \frac{35}{8}\omega_3\omega_4c_s^2 + \frac{1}{2}\omega_3\omega_1u^2 - \frac{1}{8}\omega_3^2\omega_4u + \frac{1}{2}\omega_2\omega_3\omega_1u^2 - \frac{1}{4}\omega_3\omega_4\omega_1u + \omega_2\omega_3\omega_4c_s^2 + \frac{7}{8}\omega_3\omega_4^2c_s^2 + \\
&\quad \omega_3c_s^2\omega_1^2 + \frac{1}{2}v^2\omega_3\omega_1^2 - \frac{5}{2}\omega_3\omega_1u^2 + \frac{113}{8}\omega_3\omega_4c_s^2\omega_1 + \frac{7}{8}\omega_3^2\omega_4c_s^2 - \frac{9}{4}v^2\omega_3\omega_4\omega_1^2 + \frac{1}{2}\omega_2\omega_4\omega_1 + \frac{3}{8}\omega_4^2c_s^2\omega_1 + \\
&\quad \frac{5}{8}\omega_3\omega_4u + \frac{3}{8}\omega_4^2\omega_1 + \frac{7}{8}v^2\omega_3\omega_4^2 - \frac{3}{8}\omega_4^2\omega_1u^2 + \frac{1}{2}\omega_2v^2\omega_4\omega_1 + \frac{3}{2}v^2\omega_4^2\omega_1 - \frac{35}{8}v^2\omega_3\omega_4 + 14v^2\omega_3\omega_4\omega_1 - \\
&\quad \frac{5}{2}v^2\omega_3\omega_1 - \frac{1}{2}\omega_2\omega_4\omega_1u^2 - \frac{1}{4}\omega_4\omega_1^2u^2 - \frac{1}{2}\omega_3\omega_1^2 - \frac{9}{4}\omega_3\omega_4c_s^2\omega_1^2 + \frac{1}{8}\omega_3\omega_4\omega_1u^2 - \frac{1}{8}\omega_4^2c_s^2 - 5\omega_3c_s^2\omega_1, \\
\gamma_{[ty]}^{[\mu_3]} &= -\frac{1}{4}\omega_4\omega_1^2 - \frac{17}{8}\omega_3\omega_4\omega_1 + \frac{1}{2}\omega_4\omega_1 - \frac{1}{8}\omega_3^2\omega_4 + \frac{3}{4}\omega_3\omega_4\omega_1^2 - \frac{1}{8}\omega_4^2 + \frac{5}{8}\omega_3\omega_4 + \frac{1}{8}\omega_4^2\omega_1 - \frac{1}{8}\omega_3\omega_4^2 + \frac{3}{8}\omega_3\omega_4^2\omega_1 + \frac{3}{8}\omega_3^2\omega_4\omega_1, \\
\gamma_{[x^2]}^{[\mu_1]} &= \frac{1}{16}\omega_3^2\omega_4c_s^2\omega_1 + \frac{1}{4}\omega_2v\omega_4\omega_1 - \frac{7}{16}\omega_2v\omega_3^2\omega_4 + \omega_2^2v\omega_3 - \frac{1}{8}\omega_3\omega_4^3u - \frac{1}{8}\omega_3^2\omega_4u^2 + \frac{1}{8}\omega_2\omega_3\omega_4\omega_1u^2 - \frac{1}{4}\omega_2v\omega_3^2\omega_1 - \\
&\quad \frac{1}{8}\omega_3\omega_4^2u^2 - \frac{1}{2}\omega_2v\omega_4 - \frac{1}{4}\omega_2\omega_3\omega_4u^2 - \frac{1}{8}\omega_4^2u + \frac{1}{16}\omega_3\omega_4^2\omega_1u^2 + \frac{1}{8}\omega_2\omega_3\omega_4c_s^2\omega_1 - \frac{1}{8}\omega_3\omega_4^2\omega_1u + \frac{1}{16}\omega_3\omega_4^2c_s^2\omega_1 + \\
&\quad \frac{1}{16}\omega_2v\omega_4^2 + \frac{3}{4}\omega_3\omega_4^2u + \frac{1}{8}\omega_4^2u^2 + \frac{1}{4}\omega_2\omega_3\omega_4u - \frac{1}{2}\omega_2^2v\omega_3\omega_1 + \frac{1}{16}\omega_3^2\omega_4\omega_1u^2 + \frac{5}{8}\omega_3\omega_4c_s^2 - \frac{1}{2}\omega_2^2v\omega_3\omega_4 + \\
&\quad \frac{1}{8}\omega_3^2\omega_4u - \frac{1}{4}\omega_2\omega_3\omega_4c_s^2 - \frac{1}{8}\omega_3\omega_4^2c_s^2 - \frac{1}{8}\omega_3^2\omega_4^2u - \frac{5}{8}\omega_2v\omega_3\omega_4\omega_1 - \frac{5}{16}\omega_3\omega_4c_s^2\omega_1 - \frac{7}{16}\omega_2v\omega_3\omega_4^2 - \frac{1}{8}\omega_3^2\omega_4c_s^2 - \\
&\quad \frac{1}{16}\omega_4^2c_s^2\omega_1 - \frac{5}{8}\omega_3\omega_4u + \frac{1}{2}\omega_2v\omega_3^2 - \frac{1}{8}\omega_2\omega_3\omega_4^2u - \frac{5}{2}\omega_2v\omega_3 - \frac{1}{16}\omega_4^2\omega_1u^2 + \frac{5}{8}\omega_3\omega_4u^2 + \frac{5}{4}\omega_2v\omega_3\omega_1 + \\
&\quad \frac{43}{16}\omega_2v\omega_3\omega_4 - \frac{5}{16}\omega_3\omega_4\omega_1u^2 + \frac{1}{8}\omega_4^2c_s^2, \\
\gamma_{[x^2]}^{[\mu_3]} &= \frac{5}{2}\omega_2\omega_3 + \frac{7}{16}\omega_2\omega_3\omega_4^2 - \frac{43}{16}\omega_2\omega_3\omega_4 - \frac{1}{2}\omega_2\omega_3^2 - \frac{5}{4}\omega_2\omega_3\omega_1 - \frac{1}{16}\omega_2\omega_4^2 + \frac{1}{2}\omega_2^2\omega_3\omega_4 + \frac{1}{4}\omega_2\omega_3^2\omega_1 - \frac{1}{4}\omega_2\omega_4\omega_1 + \\
&\quad \frac{1}{2}\omega_2^2\omega_3\omega_1 + \frac{7}{16}\omega_2\omega_3^2\omega_4 + \frac{5}{8}\omega_2\omega_3\omega_4\omega_1 + \frac{1}{2}\omega_2\omega_4 - \omega_2^2\omega_3, \\
\gamma_{[xy]}^{[\mu_1]} &= -\frac{1}{8}\omega_3^2\omega_4u^2 + \omega_2\omega_3\omega_1u - \frac{1}{8}\omega_3\omega_4^2u^2 + \frac{1}{2}\omega_3^2\omega_1u - \frac{1}{8}v^2\omega_3^2\omega_4 - \frac{1}{8}v^2\omega_4^2 + \frac{1}{8}\omega_4^2u^2 + \frac{5}{4}\omega_3\omega_4c_s^2 + \frac{1}{4}\omega_4\omega_1u + \frac{1}{2}\omega_3\omega_4\omega_1u - \\
&\quad \frac{1}{4}\omega_3\omega_4^2c_s^2 - \frac{1}{2}\omega_3\omega_4c_s^2\omega_1 - \frac{1}{4}\omega_3^2\omega_4c_s^2 - \frac{1}{8}v^2\omega_3\omega_4^2 + \frac{5}{8}\omega_3\omega_4u^2 + \frac{5}{8}v^2\omega_3\omega_4 - \frac{1}{4}v^2\omega_3\omega_4\omega_1 - \frac{5}{2}\omega_3\omega_1u - \frac{1}{4}\omega_3\omega_4\omega_1u^2, \\
\gamma_{[xy]}^{[\mu_3]} &= -\frac{1}{4}\omega_3\omega_4\omega_1 - \frac{1}{8}\omega_3^2\omega_4 - \frac{1}{8}\omega_4^2 + \frac{5}{8}\omega_3\omega_4 - \frac{1}{8}\omega_3\omega_4^2, \\
\gamma_{[y^2]}^{[\mu_1]} &= -\frac{1}{16}\omega_3^2\omega_4c_s^2\omega_1 + \frac{1}{4}\omega_2v\omega_4\omega_1 - \frac{1}{16}v^2\omega_3^2\omega_4\omega_1 - \frac{1}{4}\omega_2v\omega_3^2\omega_1 - \frac{1}{16}v^2\omega_3\omega_4^2\omega_1 - \frac{1}{16}\omega_3\omega_4^2c_s^2\omega_1 - \frac{1}{4}\omega_2v\omega_3\omega_4\omega_1 + \\
&\quad \frac{5}{16}\omega_3\omega_4c_s^2\omega_1 - \frac{1}{8}v^2\omega_3\omega_4\omega_1^2 - \frac{1}{16}\omega_4^2c_s^2\omega_1 - \frac{1}{2}\omega_2v\omega_3\omega_1^2 + \frac{5}{4}\omega_2v\omega_3\omega_1 - \frac{1}{16}v^2\omega_4^2\omega_1 + \frac{5}{16}v^2\omega_3\omega_4\omega_1 - \frac{1}{8}\omega_3\omega_4c_s^2\omega_1^2. \\
\gamma_{[y^2]}^{[\mu_3]} &= \frac{1}{2}\omega_2\omega_3\omega_1^2 - \frac{43}{16}\omega_3\omega_4\omega_1 + \frac{1}{2}\omega_4\omega_1 + \frac{3}{8}\omega_3\omega_4\omega_1^2 - \frac{5}{4}\omega_2\omega_3\omega_1 - \frac{1}{2}\omega_3^2\omega_1 + \frac{1}{4}\omega_2\omega_3^2\omega_1 + \frac{5}{2}\omega_3\omega_1 - \frac{1}{4}\omega_2\omega_4\omega_1 - \\
&\quad \frac{1}{16}\omega_4^2\omega_1 + \frac{3}{4}\omega_2\omega_3\omega_4\omega_1 + \frac{7}{16}\omega_3\omega_4^2\omega_1 + \frac{7}{16}\omega_3^2\omega_4\omega_1 - \omega_3\omega_1^2,
\end{aligned}$$

#### 4.5 EPDE for $\mu_4$

$$\begin{aligned}
& \gamma_{[1]}^{[\mu_1]} \mu_1 + \gamma_{[1]}^{[\mu_4]} \mu_4 + \gamma_{[t]}^{[\mu_1]} \delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_4]} \delta_t \frac{\partial \mu_4}{\partial t} + \gamma_{[x]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[x]}^{[\mu_4]} \delta_l \frac{\partial \mu_4}{\partial x} + \gamma_{[y]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial y} \\
& + \gamma_{[y]}^{[\mu_4]} \delta_l \frac{\partial \mu_4}{\partial y} + \gamma_{[t^2]}^{[\mu_1]} \delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_4]} \delta_t^2 \frac{\partial^2 \mu_4}{\partial t^2} + \gamma_{[tx]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} + \gamma_{[tx]}^{[\mu_4]} \delta_t \delta_l \frac{\partial^2 \mu_4}{\partial t \partial x} + \gamma_{[ty]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} \\
& + \gamma_{[ty]}^{[\mu_4]} \delta_t \delta_l \frac{\partial^2 \mu_4}{\partial t \partial y} + \gamma_{[x^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_4]} \delta_l^2 \frac{\partial^2 \mu_4}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x \partial y} + \gamma_{[xy]}^{[\mu_4]} \delta_l^2 \frac{\partial^2 \mu_4}{\partial x \partial y} = 0,
\end{aligned}$$

where

$$\begin{aligned}
\gamma_{[1]}^{[\mu_1]} &= -\omega_2^2 \omega_3 \omega_4 \omega_1 u^2 + \frac{35}{8} \omega_2 \omega_3 \omega_4 \omega_1 u^2 + \frac{1}{8} \omega_2 \omega_4^2 \omega_1 u^2 - \frac{3}{4} \omega_2 \omega_3 \omega_4 c_s^2 \omega_1^2 + \frac{1}{8} \omega_2 \omega_4^2 c_s^2 \omega_1 + \frac{35}{8} \omega_2 \omega_3 \omega_4 c_s^2 \omega_1 - \\
& \omega_2^2 \omega_3 \omega_4 c_s^2 \omega_1 - \frac{7}{8} \omega_2 \omega_3 \omega_4^2 c_s^2 \omega_1 - \frac{7}{8} \omega_2 \omega_3^2 \omega_4 \omega_1 u^2 - \frac{3}{4} \omega_2 \omega_3 \omega_4 \omega_1^2 u^2 - \frac{7}{8} \omega_2 \omega_3^2 \omega_4 c_s^2 \omega_1 - \frac{7}{8} \omega_2 \omega_3 \omega_4^2 \omega_1 u^2, \\
\gamma_{[1]}^{[\mu_4]} &= \frac{7}{8} \omega_2 \omega_3^2 \omega_4 \omega_1 + \frac{7}{8} \omega_2 \omega_3 \omega_4^2 \omega_1 - \frac{1}{8} \omega_2 \omega_4^2 \omega_1 + \omega_2^2 \omega_3 \omega_4 \omega_1 + \frac{3}{4} \omega_2 \omega_3 \omega_4 \omega_1^2 - \frac{35}{8} \omega_2 \omega_3 \omega_4 \omega_1, \\
\gamma_{[t]}^{[\mu_1]} &= \frac{1}{8} \omega_2 \omega_4^2 c_s^2 - \frac{1}{4} \omega_2 \omega_4 \omega_1^2 u^2 - \frac{7}{8} \omega_3^2 \omega_4 c_s^2 \omega_1 + \frac{5}{2} \omega_2 v^2 \omega_3 \omega_1 - \frac{3}{4} \omega_3 \omega_4 \omega_1^2 u^2 + 4 \omega_2^2 \omega_3 \omega_4 \omega_1 u^2 + 5 \omega_2 \omega_3 c_s^2 \omega_1 - \\
& \frac{1}{2} \omega_2 \omega_3 \omega_1^2 - \omega_2^2 \omega_3 \omega_4 c_s^2 - \frac{77}{4} \omega_2 \omega_3 \omega_4 \omega_1 u^2 + \frac{1}{2} \omega_2 v^2 \omega_4^2 \omega_1 - \frac{3}{4} \omega_2 \omega_4^2 \omega_1 u^2 + \frac{35}{8} \omega_2 \omega_3 \omega_4 u^2 - \frac{7}{8} \omega_3 \omega_4^2 \omega_1 u^2 + \\
& 3 \omega_2 \omega_3 \omega_4 c_s^2 \omega_1^2 - \frac{1}{2} \omega_2 \omega_4^2 \omega_1 - \frac{1}{4} \omega_2 \omega_4^2 c_s^2 \omega_1 - \frac{77}{4} \omega_2 \omega_3 \omega_4 c_s^2 \omega_1 - \frac{7}{8} \omega_2 \omega_3^2 \omega_4 c_s^2 - \frac{7}{8} \omega_3 \omega_4^2 c_s^2 \omega_1 - \frac{1}{4} \omega_2 \omega_3 \omega_4 \omega_1 u + \\
& \frac{5}{8} \omega_2 \omega_3 \omega_4 u + \frac{1}{2} \omega_2^2 v^2 \omega_4 \omega_1 - \frac{7}{8} \omega_2 \omega_3 \omega_4^2 c_s^2 - \omega_2 \omega_3 c_s^2 \omega_1^2 - \frac{1}{2} \omega_2^2 \omega_4 \omega_1 - \frac{7}{8} \omega_3^2 \omega_4 \omega_1 u^2 + 4 \omega_2^2 \omega_3 \omega_4 c_s^2 \omega_1 - \\
& \frac{1}{2} \omega_2 v^2 \omega_3 \omega_1^2 - \frac{1}{2} \omega_2^2 \omega_4 \omega_1 u^2 + \frac{5}{2} \omega_2 \omega_3 \omega_1 u^2 + \frac{7}{2} \omega_2 \omega_3 \omega_4^2 c_s^2 \omega_1 + \frac{35}{8} \omega_2 \omega_3 \omega_4 c_s^2 - \frac{1}{2} \omega_2 \omega_4 c_s^2 \omega_1 - \omega_2^2 \omega_3 c_s^2 \omega_1 - \\
& \frac{1}{2} \omega_2^2 v^2 \omega_3 \omega_1 - \frac{1}{8} \omega_2 \omega_3^2 \omega_4 u - \frac{1}{2} \omega_2 \omega_3 \omega_1^2 u^2 + \frac{35}{8} \omega_3 \omega_4 c_s^2 \omega_1 + 2 \omega_2 \omega_4 \omega_1 + \frac{1}{2} \omega_2^2 \omega_3 \omega_1 + \frac{1}{8} \omega_4^2 c_s^2 \omega_1 + \frac{1}{8} \omega_2 \omega_4^2 u^2 - \\
& \frac{1}{2} \omega_2 \omega_3^2 \omega_1 u^2 - \omega_2^2 \omega_3 \omega_4 u^2 + \frac{1}{2} \omega_2 v^2 \omega_4 \omega_1^2 + \frac{7}{2} \omega_2 \omega_3^2 \omega_4 \omega_1 u^2 - \frac{1}{8} \omega_2 \omega_3 \omega_4^2 u + \frac{1}{8} \omega_4^2 \omega_1 u^2 - \frac{1}{2} \omega_2 \omega_3 \omega_4 \omega_1 - \frac{5}{2} \omega_2 v^2 \omega_4 \omega_1 + \\
& 3 \omega_2 \omega_3 \omega_4 \omega_1^2 u^2 - \frac{7}{8} \omega_2 \omega_3 \omega_4^2 u^2 + \frac{7}{2} \omega_2 \omega_3^2 \omega_4 c_s^2 \omega_1 - \frac{1}{2} \omega_2 \omega_4 \omega_1^2 - \omega_2 \omega_3^2 c_s^2 \omega_1 - \frac{1}{8} \omega_2 \omega_4^2 u - \frac{7}{8} \omega_2 \omega_3^2 \omega_4 u^2 - \\
& \frac{1}{2} \omega_2^2 \omega_3 \omega_1 u^2 + 2 \omega_2 \omega_4 \omega_1 u^2 + \frac{7}{2} \omega_2 \omega_3 \omega_4^2 \omega_1 u^2 - \frac{1}{2} \omega_2 v^2 \omega_3 \omega_1 - \frac{3}{4} \omega_3 \omega_4 c_s^2 \omega_1^2 + \frac{35}{8} \omega_3 \omega_4 \omega_1 u^2 + \frac{1}{4} \omega_2 \omega_4 c_s^2 \omega_1^2, \\
\gamma_{[t]}^{[\mu_4]} &= -\frac{7}{2} \omega_2 \omega_3^2 \omega_4 \omega_1 + \omega_2 \omega_3 \omega_1^2 - \frac{7}{2} \omega_2 \omega_3 \omega_4^2 \omega_1 - \frac{35}{8} \omega_3 \omega_4 \omega_1 + \frac{7}{8} \omega_2 \omega_3 \omega_4^2 - \frac{35}{8} \omega_2 \omega_3 \omega_4 + \frac{5}{4} \omega_2 \omega_4^2 \omega_1 + \frac{3}{4} \omega_3 \omega_4 \omega_1^2 - \\
& 4 \omega_2^2 \omega_3 \omega_4 \omega_1 - 5 \omega_2 \omega_3 \omega_1 + \omega_2^2 \omega_4 \omega_1 - \frac{1}{8} \omega_2 \omega_4^2 + \omega_2^2 \omega_3 \omega_4 + \omega_2 \omega_3^2 \omega_1 - \frac{9}{2} \omega_2 \omega_4 \omega_1 + \omega_2^2 \omega_3 \omega_1 + \frac{7}{8} \omega_2 \omega_3^2 \omega_4 - \\
& \frac{1}{8} \omega_4^2 \omega_1 - 3 \omega_2 \omega_3 \omega_4 \omega_1^2 + \frac{81}{4} \omega_2 \omega_3 \omega_4 \omega_1 + \frac{7}{8} \omega_3 \omega_4^2 \omega_1 + \frac{3}{4} \omega_2 \omega_4 \omega_1^2 + \frac{7}{8} \omega_3^2 \omega_4 \omega_1, \\
\gamma_{[x]}^{[\mu_1]} &= -\frac{5}{2} \omega_2 \omega_3 \omega_1 u + \frac{1}{2} \omega_2 \omega_3 \omega_4 \omega_1 u + \omega_2^2 \omega_3 \omega_1 u - \frac{1}{2} \omega_2 \omega_4 \omega_1 u + \frac{1}{2} \omega_2 \omega_3^2 \omega_1 u, \\
\gamma_{[x]}^{[\mu_4]} &= -\frac{1}{8} \omega_2 \omega_3 \omega_4^2 + \frac{5}{8} \omega_2 \omega_3 \omega_4 - \frac{1}{8} \omega_2 \omega_4^2 - \frac{1}{8} \omega_2 \omega_3^2 \omega_4 - \frac{1}{4} \omega_2 \omega_3 \omega_4 \omega_1, \\
\gamma_{[y]}^{[\mu_1]} &= \frac{1}{8} \omega_3^2 \omega_4 c_s^2 \omega_1 - \frac{1}{2} \omega_2 v \omega_4 \omega_1 + \frac{1}{4} \omega_3 \omega_4 \omega_1^2 u^2 - \frac{1}{2} \omega_2 v \omega_3^2 \omega_1 + \frac{1}{8} \omega_3 \omega_4^2 \omega_1 u^2 + \frac{1}{8} \omega_3 \omega_4^2 c_s^2 \omega_1 + \frac{1}{8} \omega_3^2 \omega_4 \omega_1 u^2 - \\
& \frac{1}{2} \omega_2 v \omega_3 \omega_4 \omega_1 - \frac{5}{8} \omega_3 \omega_4 c_s^2 \omega_1 + \frac{1}{8} \omega_4^2 c_s^2 \omega_1 - \omega_2 v \omega_3 \omega_1^2 + \frac{1}{8} \omega_4^2 \omega_1 u^2 + \frac{5}{2} \omega_2 v \omega_3 \omega_1 + \frac{1}{4} \omega_3 \omega_4 c_s^2 \omega_1^2 - \frac{5}{8} \omega_3 \omega_4 \omega_1 u^2, \\
\gamma_{[y]}^{[\mu_4]} &= \frac{5}{8} \omega_3 \omega_4 \omega_1 - \frac{1}{4} \omega_3 \omega_4 \omega_1^2 - \frac{1}{8} \omega_4^2 \omega_1 - \frac{1}{8} \omega_3 \omega_4^2 \omega_1 - \frac{1}{8} \omega_3^2 \omega_4 \omega_1, \\
\gamma_{[t^2]}^{[\mu_1]} &= -\frac{3}{16} \omega_2 \omega_4^2 c_s^2 + \frac{7}{8} \omega_2 \omega_4 \omega_1^2 u^2 - \frac{1}{2} v^2 \omega_3^2 \omega_1 + \frac{49}{16} \omega_3^2 \omega_4 c_s^2 \omega_1 - \frac{37}{4} \omega_2 v^2 \omega_3 \omega_1 + \frac{1}{2} \omega_2^2 v^2 \omega_4 - \frac{7}{8} \omega_3^2 \omega_4 u^2 + 2 \omega_4 \omega_1 u^2 + \\
& \frac{21}{8} \omega_3 \omega_4 \omega_1^2 u^2 - 8 \omega_2^2 \omega_3 \omega_4 \omega_1 u^2 - \frac{37}{2} \omega_2 \omega_3 c_s^2 \omega_1 + \frac{1}{4} \omega_4 c_s^2 \omega_1^2 - \frac{1}{2} \omega_2^2 \omega_4 + \frac{7}{4} \omega_2 \omega_3 \omega_1^2 + \frac{7}{2} \omega_2^2 \omega_3 \omega_4 c_s^2 - \frac{1}{2} \omega_4 \omega_1^2 + \\
& \frac{329}{8} \omega_2 \omega_3 \omega_4 \omega_1 u^2 - \frac{7}{4} \omega_2 v^2 \omega_4^2 \omega_1 - \frac{7}{8} \omega_3 \omega_4^2 u^2 + \frac{5}{2} \omega_2 v^2 \omega_3 + \frac{15}{8} \omega_2 \omega_4^2 \omega_1 u^2 - \frac{261}{16} \omega_2 \omega_3 \omega_4 u^2 + 5 \omega_2 \omega_3 c_s^2 - \frac{1}{8} \omega_4^2 u - \\
& \frac{5}{2} v^2 \omega_4 \omega_1 - \frac{1}{2} \omega_3 \omega_4 \omega_1 - \omega_2^2 c_s^2 \omega_1 + \frac{1}{2} \omega_2 \omega_4 u + \frac{49}{16} \omega_3 \omega_4^2 \omega_1 u^2 - 6 \omega_2 \omega_3 \omega_4 c_s^2 \omega_1^2 - \frac{1}{2} \omega_2 \omega_3 \omega_4 + \frac{7}{4} \omega_2 \omega_4^2 \omega_1 + \\
& \frac{7}{8} \omega_2 \omega_4^2 c_s^2 \omega_1 + \frac{329}{8} \omega_2 \omega_3 \omega_4 c_s^2 \omega_1 + 2 \omega_2 \omega_4 u^2 - \frac{1}{2} \omega_2^2 \omega_3 u^2 + \frac{49}{16} \omega_2 \omega_3^2 \omega_4 c_s^2 + \frac{49}{16} \omega_3 \omega_4^2 c_s^2 \omega_1 + \frac{7}{8} \omega_2 \omega_3 \omega_4 \omega_1 u + \\
& 2 \omega_4 \omega_1 - \frac{1}{2} \omega_2 v^2 \omega_3^2 - \frac{1}{8} \omega_3 \omega_4^2 u + \frac{1}{8} \omega_4^2 u^2 - \frac{1}{2} \omega_3^2 \omega_1 u^2 - \frac{35}{16} \omega_2 \omega_3 \omega_4 u - \frac{7}{4} \omega_2^2 v^2 \omega_4 \omega_1 + \frac{1}{2} v^2 \omega_4 \omega_1^2 + \\
& \frac{49}{16} \omega_2 \omega_3 \omega_4^2 c_s^2 - \frac{1}{2} \omega_2 \omega_3^2 u^2 + \frac{7}{2} \omega_2 \omega_3 c_s^2 \omega_1^2 - \frac{1}{2} \omega_4 c_s^2 \omega_1 - \frac{1}{2} \omega_2 \omega_3 \omega_1 + \frac{7}{4} \omega_2^2 \omega_4 \omega_1 + \frac{49}{16} \omega_3^2 \omega_4 \omega_1 u^2 + \frac{35}{8} \omega_3 \omega_4 c_s^2 - \\
& \frac{1}{2} \omega_3 \omega_1^2 u^2 - 8 \omega_2^2 \omega_3 \omega_4 c_s^2 \omega_1 + \frac{7}{4} \omega_2 v^2 \omega_3 \omega_1^2 - \frac{1}{8} \omega_3^2 \omega_4 u + \frac{7}{4} \omega_2^2 \omega_4 \omega_1 u^2 - \frac{37}{4} \omega_2 \omega_3 \omega_1 u^2 - \frac{1}{2} \omega_2 \omega_4^2 - \\
& 7 \omega_2 \omega_3 \omega_1^2 c_s^2 \omega_1 - \frac{1}{4} \omega_3 \omega_4 \omega_1 u + \frac{5}{2} \omega_2 \omega_3 u^2 - \frac{261}{16} \omega_2 \omega_3 \omega_4 c_s^2 - \frac{1}{2} \omega_2^2 \omega_4 u^2 - \frac{7}{8} \omega_3 \omega_4^2 c_s^2 + 2 \omega_2 \omega_4 c_s^2 \omega_1 - \\
& \omega_3 c_s^2 \omega_1^2 + \frac{7}{2} \omega_2^2 \omega_3 c_s^2 \omega_1 + \frac{7}{4} \omega_2^2 v^2 \omega_3 \omega_1 - \frac{1}{2} v^2 \omega_3 \omega_1^2 + \frac{7}{16} \omega_2 \omega_3^2 \omega_4 u + \frac{5}{2} \omega_3 \omega_1 u^2 - \frac{1}{4} \omega_2 \omega_4 \omega_1 u + \frac{7}{4} \omega_2 \omega_3 \omega_1^2 u^2 - \\
& \frac{257}{16} \omega_3 \omega_4 c_s^2 \omega_1 - \frac{7}{8} \omega_3^2 \omega_4 c_s^2 - \frac{17}{2} \omega_2 \omega_4 \omega_1 - \frac{7}{4} \omega_2^2 \omega_3 \omega_1 - \omega_2 \omega_1^2 - \frac{3}{16} \omega_4^2 c_s^2 \omega_1 - \frac{11}{16} \omega_2 \omega_4^2 u^2 + \frac{7}{4} \omega_2 \omega_3^2 \omega_1 u^2 +
\end{aligned}$$

$$\begin{aligned}
& \frac{7}{2}\omega_2^2\omega_3\omega_4u^2 + \frac{5}{8}\omega_3\omega_4u + \frac{1}{2}\omega_2v^2\omega_4^2 - \frac{7}{4}\omega_2v^2\omega_4\omega_1^2 - \frac{1}{2}\omega_4^2\omega_1 - 7\omega_2\omega_3^2\omega_4\omega_1u^2 + \frac{7}{16}\omega_2\omega_3\omega_4^2u - \frac{5}{2}\omega_2v^2\omega_4 - \\
& \frac{11}{16}\omega_4^2\omega_1u^2 + \frac{35}{8}\omega_3\omega_4u^2 + \frac{7}{4}\omega_2\omega_3\omega_4\omega_1 + \frac{37}{4}\omega_2v^2\omega_4\omega_1 - 6\omega_2\omega_3\omega_4\omega_1^2u^2 + \frac{1}{2}v^2\omega_4^2\omega_1 - \omega_2\omega_3^2c_s^2 + \\
& \frac{49}{16}\omega_2\omega_3\omega_4^2u^2 - 7\omega_2\omega_3^2\omega_4c_s^2\omega_1 + \frac{7}{4}\omega_2\omega_4\omega_1^2 + \frac{7}{2}\omega_2\omega_3^2c_s^2\omega_1 + 2\omega_2\omega_1 + \frac{3}{16}\omega_2\omega_4^2u + \frac{5}{2}v^2\omega_3\omega_1 + \frac{49}{16}\omega_2\omega_3^2\omega_4u^2 - \\
& \omega_2^2\omega_3c_s^2 - \frac{1}{2}\omega_2\omega_4c_s^2 + \frac{7}{4}\omega_2^2\omega_3\omega_1u^2 - \frac{29}{4}\omega_2\omega_4\omega_1u^2 - \frac{1}{4}\omega_4\omega_1^2u^2 - \frac{1}{5}\omega_2^2v^2\omega_3 - \frac{1}{2}\omega_3\omega_1^2 - 7\omega_2\omega_3\omega_4^2\omega_1u^2 + \\
& \frac{7}{4}\omega_2v^2\omega_3^2\omega_1 + \frac{21}{8}\omega_3\omega_4c_s^2\omega_1^2 + 2\omega_2\omega_4 + \frac{1}{2}\omega_2^2\omega_3 - \frac{257}{16}\omega_3\omega_4\omega_1u^2 + \frac{1}{8}\omega_4^2c_s^2 - \frac{7}{8}\omega_2\omega_4c_s^2\omega_1^2 + 5\omega_3c_s^2\omega_1, \\
\gamma_{[t^2]}^{[\mu_4]} &= 7\omega_2\omega_3^2\omega_4\omega_1 - 5\omega_2\omega_3 + \omega_2^2\omega_4 - \frac{7}{2}\omega_2\omega_3\omega_1^2 + \frac{3}{4}\omega_4\omega_1^2 + 7\omega_2\omega_3\omega_4^2\omega_1 + \frac{273}{16}\omega_3\omega_4\omega_1 - \frac{49}{16}\omega_2\omega_3\omega_4^2 + \omega_2^2\omega_1 + \\
& \frac{277}{16}\omega_2\omega_3\omega_4 - \frac{29}{8}\omega_2\omega_4^2\omega_1 - \frac{9}{2}\omega_4\omega_1 + \frac{7}{8}\omega_3^2\omega_4 - \frac{21}{8}\omega_3\omega_4\omega_1^2 + 8\omega_2^2\omega_3\omega_4\omega_1 + \omega_2\omega_3^2 + \frac{39}{2}\omega_2\omega_3\omega_1 - \frac{7}{2}\omega_2^2\omega_4\omega_1 + \omega_3^2\omega_1 + \\
& \frac{19}{16}\omega_2\omega_4^2 - \frac{7}{2}\omega_2^2\omega_3\omega_4 - \frac{7}{2}\omega_2\omega_3^2\omega_1 - \frac{1}{8}\omega_4^2 - 5\omega_3\omega_1 + \frac{35}{2}\omega_2\omega_4\omega_1 - \frac{7}{2}\omega_2^2\omega_3\omega_1 - \frac{49}{16}\omega_2\omega_3^2\omega_4 + \omega_2\omega_1^2 - \frac{35}{8}\omega_3\omega_4 + \frac{19}{16}\omega_4^2\omega_1 + \\
& 6\omega_2\omega_3\omega_4\omega_1^2 - \frac{357}{8}\omega_2\omega_3\omega_4\omega_1 + \frac{7}{8}\omega_3\omega_4^2 - \frac{49}{16}\omega_3\omega_4\omega_1 - \frac{21}{8}\omega_2\omega_4\omega_1^2 - 5\omega_2\omega_1 - \frac{49}{16}\omega_3^2\omega_4\omega_1 + \omega_3\omega_1^2 - \frac{9}{2}\omega_2\omega_4 + \omega_2^2\omega_3, \\
\gamma_{[tx]}^{[\mu_1]} &= \frac{17}{2}\omega_2\omega_3\omega_1u + \frac{1}{2}\omega_3^2\omega_1u - \frac{3}{2}\omega_2\omega_3\omega_4\omega_1u - 3\omega_2\omega_1u - \frac{1}{2}\omega_4\omega_1u + \frac{1}{2}\omega_3\omega_4\omega_1u - 3\omega_2^2\omega_3\omega_1u + \frac{3}{2}\omega_2\omega_4\omega_1u + \\
& \omega_2^2\omega_1u - \frac{3}{2}\omega_2\omega_3^2\omega_1u - \frac{5}{2}\omega_3\omega_1u, \\
\gamma_{[tx]}^{[\mu_4]} &= -\frac{1}{4}\omega_3\omega_4\omega_1 + \frac{3}{8}\omega_2\omega_3\omega_4^2 - \frac{15}{8}\omega_2\omega_3\omega_4 - \frac{1}{8}\omega_3^2\omega_4 + \frac{1}{8}\omega_2\omega_4^2 - \frac{1}{8}\omega_4^2 - \frac{1}{4}\omega_2\omega_4\omega_1 + \frac{3}{8}\omega_2\omega_3^2\omega_4 + \frac{5}{8}\omega_3\omega_4 + \\
& \frac{3}{4}\omega_2\omega_3\omega_4\omega_1 - \frac{1}{8}\omega_3\omega_4^2 + \frac{1}{2}\omega_2\omega_4, \\
\gamma_{[ty]}^{[\mu_1]} &= -\frac{3}{8}\omega_3^2\omega_4c_s^2\omega_1 + \frac{1}{2}\omega_2v\omega_4\omega_1 + 2\omega_2v\omega_1 + \frac{1}{8}\omega_3^2\omega_4u^2 - \frac{1}{2}\omega_4\omega_1u^2 - \frac{3}{4}\omega_3\omega_4\omega_1^2u^2 + \frac{1}{4}\omega_4c_s^2\omega_1^2 + \frac{3}{2}\omega_2v\omega_3^2\omega_1 + \\
& \frac{1}{8}\omega_3\omega_4^2u^2 - \frac{1}{2}\omega_2v\omega_4 - \frac{1}{8}\omega_4^2u - \frac{3}{8}\omega_3\omega_4^2\omega_1u^2 - \frac{3}{8}\omega_3\omega_4^2c_s^2\omega_1 - \frac{1}{8}\omega_3\omega_4^2u + \frac{1}{8}\omega_4^2u^2 - \frac{1}{2}\omega_4c_s^2\omega_1 - \frac{3}{8}\omega_3^2\omega_4\omega_1u^2 - \\
& \frac{5}{8}\omega_3\omega_4c_s^2 - \frac{1}{8}\omega_3^2\omega_4u - \omega_2v\omega_1^2 - \frac{1}{4}\omega_3\omega_4\omega_1u + \frac{1}{8}\omega_3\omega_4^2c_s^2 + \frac{3}{2}\omega_2v\omega_3\omega_4\omega_1 + \frac{17}{8}\omega_3\omega_4c_s^2\omega_1 + \frac{1}{8}\omega_3^2\omega_4c_s^2 - \\
& \frac{1}{8}\omega_4^2c_s^2\omega_1 + \frac{5}{8}\omega_3\omega_4u - \frac{1}{2}\omega_2v\omega_3^2 + 3\omega_2v\omega_3\omega_1^2 + \frac{5}{2}\omega_2v\omega_3 - \frac{1}{8}\omega_4^2\omega_1u^2 - \frac{5}{8}\omega_3\omega_4u^2 - \frac{17}{2}\omega_2v\omega_3\omega_1 + \frac{1}{4}\omega_4\omega_1^2u^2 - \\
& \frac{1}{2}\omega_2v\omega_3\omega_4 - \frac{3}{4}\omega_3\omega_4c_s^2\omega_1^2 + \frac{17}{8}\omega_3\omega_4\omega_1u^2 + \frac{1}{8}\omega_4^2c_s^2, \\
\gamma_{[ty]}^{[\mu_4]} &= -\frac{1}{4}\omega_4\omega_1^2 - \frac{17}{8}\omega_3\omega_4\omega_1 + \frac{1}{2}\omega_4\omega_1 - \frac{1}{8}\omega_3^2\omega_4 + \frac{3}{4}\omega_3\omega_4\omega_1^2 - \frac{1}{8}\omega_4^2 + \frac{5}{8}\omega_3\omega_4 + \frac{1}{8}\omega_4^2\omega_1 - \frac{1}{8}\omega_3\omega_4^2 + \frac{3}{8}\omega_3\omega_4^2\omega_1 + \frac{3}{8}\omega_3^2\omega_4\omega_1, \\
\gamma_{[x^2]}^{[\mu_1]} &= \frac{1}{16}\omega_2\omega_4^2c_s^2 - \frac{1}{2}\omega_2^2\omega_3\omega_4c_s^2 - \frac{3}{8}\omega_2\omega_3\omega_4\omega_1u^2 + \frac{35}{16}\omega_2\omega_3\omega_4u^2 - \frac{3}{8}\omega_2\omega_3\omega_4c_s^2\omega_1 - \frac{7}{16}\omega_2\omega_3^2\omega_4c_s^2 - \\
& \frac{1}{16}\omega_2\omega_3\omega_4\omega_1u + \frac{5}{16}\omega_2\omega_3\omega_4u - \frac{7}{16}\omega_2\omega_3\omega_4^2c_s^2 + \frac{35}{16}\omega_2\omega_3\omega_4c_s^2 - \frac{1}{16}\omega_2\omega_3^2\omega_4u + \frac{1}{16}\omega_2\omega_4^2u^2 - \frac{1}{2}\omega_2^2\omega_3\omega_4u^2 - \\
& \frac{1}{16}\omega_2\omega_3\omega_4^2u - \frac{7}{16}\omega_2\omega_3\omega_4^2u^2 - \frac{1}{16}\omega_2\omega_4^2u - \frac{7}{16}\omega_2\omega_3^2\omega_4u^2, \\
\gamma_{[x^2]}^{[\mu_4]} &= \frac{5}{2}\omega_2\omega_3 + \frac{7}{16}\omega_2\omega_3\omega_4^2 - \frac{43}{16}\omega_2\omega_3\omega_4 - \frac{1}{2}\omega_2\omega_3^2 - \frac{5}{4}\omega_2\omega_3\omega_1 - \frac{1}{16}\omega_2\omega_4^2 + \frac{1}{2}\omega_2^2\omega_3\omega_4 + \frac{1}{4}\omega_2\omega_3^2\omega_1 - \frac{1}{4}\omega_2\omega_4\omega_1 + \\
& \frac{1}{2}\omega_2^2\omega_3\omega_1 + \frac{7}{16}\omega_2\omega_3^2\omega_4 + \frac{5}{8}\omega_2\omega_3\omega_4\omega_1 + \frac{1}{2}\omega_2\omega_4 - \omega_2^2\omega_3, \\
\gamma_{[xy]}^{[\mu_4]} &= -\frac{1}{4}\omega_3\omega_4\omega_1 - \frac{1}{8}\omega_3^2\omega_4 - \frac{1}{8}\omega_4^2 + \frac{5}{8}\omega_3\omega_4 - \frac{1}{8}\omega_3\omega_4^2, \\
\gamma_{[y^2]}^{[\mu_1]} &= -\frac{1}{4}\omega_2v^2\omega_3\omega_4\omega_1 + \frac{1}{2}v^2\omega_3^2\omega_1 - \frac{7}{16}\omega_3^2\omega_4c_s^2\omega_1 + \frac{5}{4}\omega_2v^2\omega_3\omega_1 - \frac{1}{2}\omega_4\omega_1u^2 - \frac{3}{8}\omega_3\omega_4\omega_1^2u^2 + \frac{5}{2}\omega_2\omega_3c_s^2\omega_1 - \\
& \frac{3}{4}\omega_2\omega_3\omega_4\omega_1u^2 + \frac{1}{2}v^2\omega_4\omega_1 + \omega_3^2c_s^2\omega_1 - \frac{7}{16}\omega_3\omega_4^2\omega_1u^2 - \omega_2\omega_3\omega_4c_s^2\omega_1 - \frac{7}{16}\omega_3\omega_4^2c_s^2\omega_1 + \frac{1}{2}\omega_3^2\omega_1u^2 - \\
& \omega_2\omega_3c_s^2\omega_1^2 - \frac{7}{16}\omega_3^2\omega_4\omega_1u^2 + \omega_3\omega_1^2u^2 - \frac{1}{2}\omega_2v^2\omega_3\omega_1^2 + \frac{5}{4}\omega_2\omega_3\omega_1u^2 + 2\omega_3c_s^2\omega_1^2 + v^2\omega_3\omega_1^2 - \frac{5}{2}\omega_3\omega_1u^2 - \\
& \frac{1}{2}\omega_2\omega_3\omega_1^2u^2 + \frac{51}{16}\omega_3\omega_4c_s^2\omega_1 + \frac{1}{16}\omega_4^2c_s^2\omega_1 - \frac{1}{4}\omega_2\omega_3^2\omega_1u^2 + \frac{1}{16}\omega_4^2\omega_1u^2 - \frac{1}{4}\omega_2v^2\omega_4\omega_1 + \frac{1}{2}v^2\omega_3\omega_4\omega_1 - \\
& \frac{1}{2}\omega_2\omega_3^2c_s^2\omega_1 - \frac{5}{2}v^2\omega_3\omega_1 + \frac{1}{4}\omega_2\omega_4\omega_1u^2 - \frac{1}{4}\omega_2v^2\omega_3^2\omega_1 - \frac{3}{8}\omega_3\omega_4c_s^2\omega_1^2 + \frac{43}{16}\omega_3\omega_4\omega_1u^2 - 5\omega_3c_s^2\omega_1. \\
\gamma_{[y^2]}^{[\mu_4]} &= \frac{1}{2}\omega_2\omega_3\omega_1^2 - \frac{43}{16}\omega_3\omega_4\omega_1 + \frac{1}{2}\omega_4\omega_1 + \frac{3}{8}\omega_3\omega_4\omega_1^2 - \frac{5}{4}\omega_2\omega_3\omega_1 - \frac{1}{2}\omega_3^2\omega_1 + \frac{1}{4}\omega_2\omega_3^2\omega_1 + \frac{5}{2}\omega_3\omega_1 - \frac{1}{4}\omega_2\omega_4\omega_1 - \\
& \frac{1}{16}\omega_4^2\omega_1 + \frac{3}{4}\omega_2\omega_3\omega_4\omega_1 + \frac{7}{16}\omega_3\omega_4^2\omega_1 + \frac{7}{16}\omega_3^2\omega_4\omega_1 - \omega_3\omega_1^2,
\end{aligned}$$

#### 4.6 EPDE for $\mu_5$

$$\begin{aligned}
& \gamma_{[1]}^{[\mu_1]} \mu_1 + \gamma_{[1]}^{[\mu_5]} \mu_5 + \gamma_{[t]}^{[\mu_1]} \delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_5]} \delta_t \frac{\partial \mu_5}{\partial t} + \gamma_{[x]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[x]}^{[\mu_5]} \delta_l \frac{\partial \mu_5}{\partial x} + \gamma_{[y]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[y]}^{[\mu_5]} \delta_l \frac{\partial \mu_5}{\partial y} \\
& + \gamma_{[t^2]}^{[\mu_1]} \delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_5]} \delta_t^2 \frac{\partial^2 \mu_5}{\partial t^2} + \gamma_{[tx]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} + \gamma_{[tx]}^{[\mu_5]} \delta_t \delta_l \frac{\partial^2 \mu_5}{\partial t \partial x} + \gamma_{[ty]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[ty]}^{[\mu_5]} \delta_t \delta_l \frac{\partial^2 \mu_5}{\partial t \partial y} \\
& + \gamma_{[x^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_5]} \delta_l^2 \frac{\partial^2 \mu_5}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x \partial y} + \gamma_{[xy]}^{[\mu_5]} \delta_l^2 \frac{\partial^2 \mu_5}{\partial x \partial y} + \gamma_{[y^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_5]} \delta_l^2 \frac{\partial^2 \mu_5}{\partial y^2} = 0,
\end{aligned}$$

where

$$\begin{aligned}
\gamma_{[1]}^{[\mu_1]} &= \frac{35}{8} \omega_2 v^2 \omega_3 \omega_4 \omega_1 + \frac{1}{8} \omega_2 v^2 \omega_4^2 \omega_1 - \omega_2^2 v^2 \omega_3 \omega_4 \omega_1 - \frac{3}{4} \omega_2 \omega_3 \omega_4 c_s^2 \omega_1^2 + \frac{1}{8} \omega_2 \omega_4^2 c_s^2 \omega_1 + \frac{35}{8} \omega_2 \omega_3 \omega_4 c_s^2 \omega_1 - \\
& \frac{3}{4} \omega_2 v^2 \omega_3 \omega_4 \omega_1^2 - \omega_2^2 \omega_3 \omega_4 c_s^2 \omega_1 - \frac{7}{8} \omega_2 \omega_3 \omega_4^2 c_s^2 \omega_1 - \frac{7}{8} \omega_2 v^2 \omega_3^2 \omega_4 \omega_1 - \frac{7}{8} \omega_2 \omega_3^2 \omega_4 c_s^2 \omega_1 - \frac{7}{8} \omega_2 v^2 \omega_3 \omega_4^2 \omega_1,
\end{aligned}$$

$$\gamma_{[1]}^{[\mu_5]} = \frac{7}{8} \omega_2 \omega_3^2 \omega_4 \omega_1 + \frac{7}{8} \omega_2 \omega_3 \omega_4^2 \omega_1 - \frac{1}{8} \omega_2 \omega_4^2 \omega_1 + \omega_2^2 \omega_3 \omega_4 \omega_1 + \frac{3}{4} \omega_2 \omega_3 \omega_4 \omega_1^2 - \frac{35}{8} \omega_2 \omega_3 \omega_4 \omega_1,$$

$$\begin{aligned}
\gamma_{[t]}^{[\mu_1]} &= \frac{1}{8} \omega_2 \omega_4^2 c_s^2 + \frac{1}{4} \omega_2 \omega_4 \omega_1^2 u^2 - \frac{155}{8} \omega_2 v^2 \omega_3 \omega_4 \omega_1 - \frac{7}{8} \omega_3^2 \omega_4 c_s^2 \omega_1 + \frac{5}{2} \omega_2 v^2 \omega_3 \omega_1 + 5 \omega_2 \omega_3 c_s^2 \omega_1 + \frac{1}{2} \omega_2 \omega_3 \omega_1^2 - \\
& \frac{7}{8} v^2 \omega_3^2 \omega_4 \omega_1 - \omega_2^2 \omega_3 \omega_4 c_s^2 + \frac{35}{8} \omega_2 v^2 \omega_3 \omega_4 - \frac{1}{8} \omega_2 \omega_3 \omega_4 \omega_1 u^2 - \frac{7}{8} \omega_2 v^2 \omega_4^2 \omega_1 + \frac{3}{8} \omega_2 \omega_4^2 \omega_1 u^2 + 4 \omega_2^2 v^2 \omega_3 \omega_4 \omega_1 - \\
& \frac{7}{8} v^2 \omega_3 \omega_4^2 \omega_1 + 3 \omega_2 \omega_3 \omega_4 c_s^2 \omega_1^2 - \frac{3}{8} \omega_2 \omega_4^2 \omega_1 - \frac{1}{2} \omega_2 \omega_4^2 c_s^2 \omega_1 - \frac{39}{2} \omega_2 \omega_3 \omega_4 c_s^2 \omega_1 - \frac{7}{8} \omega_2 \omega_3^2 \omega_4 c_s^2 - \frac{7}{8} \omega_3 \omega_4^2 c_s^2 \omega_1 + \\
& \frac{1}{4} \omega_2 \omega_3 \omega_4 \omega_1 u - \frac{7}{8} \omega_2 v^2 \omega_3 \omega_4^2 - \frac{5}{8} \omega_2 \omega_3 \omega_4 u - \frac{1}{2} \omega_2^2 v^2 \omega_4 \omega_1 - \frac{7}{8} \omega_2 \omega_3 \omega_4^2 c_s^2 - \omega_2 \omega_3 c_s^2 \omega_1^2 - \frac{1}{2} \omega_2^2 \omega_4 \omega_1 + \\
& 3 \omega_2 v^2 \omega_3 \omega_4 \omega_1^2 + 4 \omega_2^2 \omega_3 \omega_4 c_s^2 \omega_1 - \frac{1}{2} \omega_2 v^2 \omega_3 \omega_1^2 + \frac{1}{2} \omega_2^2 \omega_4 \omega_1 u^2 + \frac{5}{2} \omega_2 \omega_3 \omega_1 u^2 + \frac{7}{2} \omega_2 \omega_3 \omega_4^2 c_s^2 \omega_1 + \frac{35}{8} \omega_2 \omega_3 \omega_4 c_s^2 + \\
& \frac{3}{4} \omega_2 \omega_4 c_s^2 \omega_1 - \omega_2^2 \omega_3 c_s^2 \omega_1 - \frac{1}{2} \omega_2^2 v^2 \omega_3 \omega_1 + \frac{1}{8} \omega_2 \omega_3^2 \omega_4 u - \frac{1}{2} \omega_2 \omega_3 \omega_1^2 u^2 + \frac{35}{8} \omega_3 \omega_4 c_s^2 \omega_1 - \frac{3}{4} v^2 \omega_3 \omega_4 \omega_1^2 + \frac{3}{2} \omega_2 \omega_4 \omega_1 - \\
& \frac{1}{2} \omega_2^2 \omega_3 \omega_1 + \frac{1}{8} \omega_4^2 c_s^2 \omega_1 - \frac{1}{2} \omega_2 \omega_3^2 \omega_1 u^2 + \frac{1}{8} \omega_2 v^2 \omega_4^2 - \frac{1}{2} \omega_2 v^2 \omega_4 \omega_1^2 - \omega_2^2 v^2 \omega_3 \omega_4 + \frac{1}{8} \omega_2 \omega_3 \omega_4^2 u - \frac{3}{8} \omega_2 \omega_3 \omega_4 \omega_1 + \\
& \frac{5}{2} \omega_2 v^2 \omega_4 \omega_1 - \frac{7}{8} \omega_2 v^2 \omega_3^2 \omega_4 + \frac{7}{2} \omega_2 v^2 \omega_3^2 \omega_4 \omega_1 + \frac{1}{8} v^2 \omega_4^2 \omega_1 + \frac{35}{8} v^2 \omega_3 \omega_4 \omega_1 + \frac{7}{2} \omega_2 \omega_3^2 \omega_4 c_s^2 \omega_1 - \frac{1}{4} \omega_2 \omega_4 \omega_1^2 - \\
& \omega_2 \omega_3^2 c_s^2 \omega_1 - \frac{1}{8} \omega_2 \omega_4^2 u + \frac{7}{2} \omega_2 v^2 \omega_3 \omega_4^2 \omega_1 - \frac{1}{2} \omega_2^2 \omega_3 \omega_1 u^2 - \frac{7}{4} \omega_2 \omega_4 \omega_1 u^2 - \frac{1}{2} \omega_2 v^2 \omega_3^2 \omega_1 - \frac{3}{4} \omega_3 \omega_4 c_s^2 \omega_1^2 - \frac{1}{4} \omega_2 \omega_4 c_s^2 \omega_1^2,
\end{aligned}$$

$$\begin{aligned}
\gamma_{[t]}^{[\mu_5]} &= -\frac{7}{2} \omega_2 \omega_3^2 \omega_4 \omega_1 + \omega_2 \omega_3 \omega_1^2 - \frac{7}{2} \omega_2 \omega_3 \omega_4^2 \omega_1 - \frac{35}{8} \omega_3 \omega_4 \omega_1 + \frac{7}{8} \omega_2 \omega_3 \omega_4^2 - \frac{35}{8} \omega_2 \omega_3 \omega_4 + \frac{5}{4} \omega_2 \omega_4^2 \omega_1 + \frac{3}{4} \omega_3 \omega_4 \omega_1^2 - \\
& 4 \omega_2^2 \omega_3 \omega_4 \omega_1 - 5 \omega_2 \omega_3 \omega_1 + \omega_2^2 \omega_4 \omega_1 - \frac{1}{8} \omega_2 \omega_4^2 + \omega_2^2 \omega_3 \omega_4 + \omega_2 \omega_3^2 \omega_1 - \frac{9}{2} \omega_2 \omega_4 \omega_1 + \omega_2^2 \omega_3 \omega_1 + \frac{7}{8} \omega_2 \omega_3^2 \omega_4 - \\
& \frac{1}{8} \omega_4^2 \omega_1 - 3 \omega_2 \omega_3 \omega_4 \omega_1^2 + \frac{81}{4} \omega_2 \omega_3 \omega_4 \omega_1 + \frac{7}{8} \omega_3 \omega_4^2 \omega_1 + \frac{3}{4} \omega_2 \omega_4 \omega_1^2 + \frac{7}{8} \omega_3^2 \omega_4 \omega_1,
\end{aligned}$$

$$\begin{aligned}
\gamma_{[x]}^{[\mu_1]} &= \frac{1}{4} \omega_2 v^2 \omega_3 \omega_4 \omega_1 + \frac{5}{2} \omega_2 \omega_3 \omega_1 u - \frac{5}{8} \omega_2 v^2 \omega_3 \omega_4 + \frac{1}{4} \omega_2 \omega_3 \omega_4 \omega_1 u^2 - \frac{5}{8} \omega_2 \omega_3 \omega_4 u^2 + \frac{1}{2} \omega_2 \omega_3 \omega_4 c_s^2 \omega_1 + \\
& \frac{1}{4} \omega_2 \omega_3^2 \omega_4 c_s^2 - \frac{1}{2} \omega_2 \omega_3 \omega_4 \omega_1 u + \frac{1}{8} \omega_2 v^2 \omega_3 \omega_4^2 + \frac{1}{4} \omega_2 \omega_3 \omega_4^2 c_s^2 - \frac{5}{4} \omega_2 \omega_3 \omega_4 c_s^2 - \omega_2^2 \omega_3 \omega_1 u - \frac{1}{4} \omega_2 \omega_4 \omega_1 u - \\
& \frac{1}{8} \omega_2 \omega_4^2 u^2 + \frac{1}{8} \omega_2 v^2 \omega_4^2 - \frac{1}{2} \omega_2 \omega_3^2 \omega_1 u + \frac{1}{8} \omega_2 v^2 \omega_3^2 \omega_4 + \frac{1}{8} \omega_2 \omega_3 \omega_4^2 u^2 + \frac{1}{8} \omega_2 \omega_3^2 \omega_4 u^2,
\end{aligned}$$

$$\gamma_{[x]}^{[\mu_5]} = -\frac{1}{8} \omega_2 \omega_3 \omega_4^2 + \frac{5}{8} \omega_2 \omega_3 \omega_4 - \frac{1}{8} \omega_2 \omega_4^2 - \frac{1}{8} \omega_2 \omega_3^2 \omega_4 - \frac{1}{4} \omega_2 \omega_3 \omega_4 \omega_1,$$

$$\begin{aligned}
\gamma_{[y]}^{[\mu_1]} &= \frac{1}{8} \omega_3^2 \omega_4 c_s^2 \omega_1 - \frac{1}{2} \omega_2 v \omega_4 \omega_1 + \frac{1}{8} v^2 \omega_3^2 \omega_4 \omega_1 + \frac{1}{2} \omega_2 v \omega_3^2 \omega_1 + \frac{1}{8} v^2 \omega_3 \omega_4^2 \omega_1 + \frac{1}{8} \omega_3 \omega_4^2 c_s^2 \omega_1 + \frac{1}{2} \omega_2 v \omega_3 \omega_4 \omega_1 - \\
& \frac{5}{8} \omega_3 \omega_4 c_s^2 \omega_1 + \frac{1}{4} v^2 \omega_3 \omega_4 \omega_1^2 + \frac{1}{8} \omega_4^2 c_s^2 \omega_1 + \omega_2 v \omega_3 \omega_1^2 - \frac{5}{2} \omega_2 v \omega_3 \omega_1 + \frac{1}{8} v^2 \omega_4^2 \omega_1 - \frac{5}{8} v^2 \omega_3 \omega_4 \omega_1 + \frac{1}{4} \omega_3 \omega_4 c_s^2 \omega_1^2,
\end{aligned}$$

$$\gamma_{[y]}^{[\mu_5]} = \frac{5}{8} \omega_3 \omega_4 \omega_1 - \frac{1}{4} \omega_3 \omega_4 \omega_1^2 - \frac{1}{8} \omega_4^2 \omega_1 - \frac{1}{8} \omega_3 \omega_4^2 \omega_1 - \frac{1}{8} \omega_3^2 \omega_4 \omega_1,$$

$$\begin{aligned}
\gamma_{[t^2]}^{[\mu_1]} &= -\frac{7}{16} \omega_2 \omega_4^2 c_s^2 - \frac{7}{8} \omega_2 \omega_4 \omega_1^2 u^2 + \frac{665}{16} \omega_2 v^2 \omega_3 \omega_4 \omega_1 - \frac{1}{2} v^2 \omega_3^2 \omega_1 + \frac{49}{16} \omega_3^2 \omega_4 c_s^2 \omega_1 - \frac{37}{4} \omega_2 v^2 \omega_3 \omega_1 - \frac{1}{2} \omega_2^2 v^2 \omega_4 - \\
& \frac{7}{4} \omega_4 \omega_1 u^2 - \frac{37}{2} \omega_2 \omega_3 c_s^2 \omega_1 - \frac{1}{4} \omega_4 c_s^2 \omega_1^2 - \frac{1}{2} \omega_2^2 \omega_4 - \frac{7}{4} \omega_2 \omega_3 \omega_1^2 + \frac{49}{16} v^2 \omega_3^2 \omega_4 \omega_1 + \frac{7}{2} \omega_2^2 \omega_3 \omega_4 c_s^2 - \frac{1}{4} \omega_4 \omega_1^2 - \\
& \frac{263}{16} \omega_2 v^2 \omega_3 \omega_4 + \frac{7}{16} \omega_2 \omega_3 \omega_4 \omega_1 u^2 + \frac{37}{16} \omega_2 v^2 \omega_4^2 \omega_1 + \frac{5}{2} \omega_2 v^2 \omega_3 - \frac{21}{16} \omega_2 \omega_4^2 \omega_1 u^2 - \frac{1}{8} \omega_2 \omega_3 \omega_4 u^2 + 5 \omega_2 \omega_3 c_s^2 - \frac{1}{8} \omega_4^2 u - \\
& \frac{7}{8} v^2 \omega_3^2 \omega_4 + \frac{5}{2} v^2 \omega_4 \omega_1 - \frac{3}{8} \omega_3 \omega_4 \omega_1 - \omega_3^2 c_s^2 \omega_1 - 8 \omega_2^2 v^2 \omega_3 \omega_4 \omega_1 + \frac{49}{16} v^2 \omega_3 \omega_4^2 \omega_1 - \frac{3}{4} \omega_2 \omega_4 u - 6 \omega_2 \omega_3 \omega_4 c_s^2 \omega_1^2 - \\
& \omega_2^2 \omega_1 - \frac{3}{8} \omega_2 \omega_3 \omega_4 + \frac{21}{16} \omega_2 \omega_4^2 \omega_1 + \omega_2 \omega_4^2 c_s^2 \omega_1 + \frac{1}{8} v^2 \omega_4^2 + 42 \omega_2 \omega_3 \omega_4 c_s^2 \omega_1 - \frac{7}{4} \omega_2 \omega_4 u^2 - \frac{1}{2} \omega_2^2 \omega_3 u^2 + \\
& \frac{49}{16} \omega_2 \omega_3^2 \omega_4 c_s^2 + \frac{49}{16} \omega_3 \omega_4^2 c_s^2 \omega_1 - \frac{7}{8} \omega_2 \omega_3 \omega_4 \omega_1 u + \frac{49}{16} \omega_2 v^2 \omega_3 \omega_4^2 + \frac{3}{2} \omega_4 \omega_1 - \frac{1}{2} \omega_2 v^2 \omega_3^2 + \frac{1}{8} \omega_3 \omega_4^2 u - \frac{1}{2} \omega_3^2 \omega_1 u^2 + \\
& \frac{35}{16} \omega_2 \omega_3 \omega_4 u + \frac{7}{4} \omega_2^2 v^2 \omega_4 \omega_1 - \frac{1}{2} v^2 \omega_4 \omega_1^2 + \frac{49}{16} \omega_2 \omega_3 \omega_4^2 c_s^2 - \frac{1}{2} \omega_2 \omega_3^2 u^2 + \frac{7}{2} \omega_2 \omega_3 c_s^2 \omega_1^2 + \frac{3}{4} \omega_4 c_s^2 \omega_1 - \frac{1}{2} \omega_2 \omega_3 \omega_1 + \\
& \frac{1}{4} \omega_2^2 \omega_4 \omega_1 + \frac{35}{8} \omega_3 \omega_4 c_s^2 - \frac{1}{2} \omega_3 \omega_1^2 u^2 - 6 \omega_2 v^2 \omega_3 \omega_4 \omega_1^2 - 8 \omega_2^2 \omega_3 \omega_4 c_s^2 \omega_1 + \frac{7}{4} \omega_2 v^2 \omega_3 \omega_1^2 + \frac{1}{8} \omega_3^2 \omega_4 u - \frac{7}{4} \omega_2^2 \omega_4 \omega_1 u^2 - \\
& \frac{37}{4} \omega_2 \omega_3 \omega_1 u^2 - \frac{3}{8} \omega_2 \omega_4^2 - \frac{7}{2} \omega_2 \omega_3 \omega_4^2 c_s^2 \omega_1 + \frac{1}{4} \omega_3 \omega_4 \omega_1 u + \frac{5}{2} \omega_2 \omega_3 u^2 - \frac{265}{16} \omega_2 \omega_3 \omega_4 c_s^2 + \frac{1}{2} \omega_2^2 \omega_4 u^2 - \frac{7}{8} \omega_3 \omega_4^2 c_s^2 -
\end{aligned}$$

$$\begin{aligned}
& \frac{21}{8}\omega_2\omega_4c_s^2\omega_1 - \omega_3c_s^2\omega_1^2 + \frac{7}{2}\omega_2^2\omega_3c_s^2\omega_1 + \frac{7}{4}\omega_2^2v^2\omega_3\omega_1 - \frac{1}{2}v^2\omega_3\omega_1^2 - \frac{7}{16}\omega_2\omega_3^2\omega_4u + \frac{5}{2}\omega_3\omega_1u^2 + \frac{1}{4}\omega_2\omega_4\omega_1u + \\
& \frac{7}{4}\omega_2\omega_3\omega_1^2u^2 - \frac{261}{16}\omega_3\omega_4c_s^2\omega_1 - \frac{7}{8}\omega_3^2\omega_4c_s^2 + \frac{21}{8}v^2\omega_3\omega_4\omega_1^2 - \frac{13}{2}\omega_2\omega_4\omega_1 + \frac{7}{4}\omega_2^2\omega_3\omega_1 - \frac{7}{16}\omega_4^2c_s^2\omega_1 + \frac{3}{8}\omega_2\omega_4^2u^2 + \\
& \frac{7}{4}\omega_2\omega_3^2\omega_1u^2 - \frac{5}{8}\omega_3\omega_4u - \frac{13}{16}\omega_2v^2\omega_4^2 + \frac{7}{4}\omega_2v^2\omega_4\omega_1^2 - \frac{3}{8}\omega_4^2\omega_1 - \frac{7}{8}v^2\omega_3\omega_4^2 + \frac{7}{2}\omega_2^2v^2\omega_3\omega_4 - \frac{7}{16}\omega_2\omega_3\omega_4^2u + \\
& \frac{5}{2}\omega_2v^2\omega_4 + \frac{3}{8}\omega_4^2\omega_1u^2 + \frac{21}{16}\omega_2\omega_3\omega_4\omega_1 - \frac{37}{4}\omega_2v^2\omega_4\omega_1 + \frac{49}{16}\omega_2v^2\omega_3^2\omega_4 - 7\omega_2v^2\omega_3^2\omega_4\omega_1 - \frac{13}{16}v^2\omega_4^2\omega_1 + \\
& \frac{35}{8}v^2\omega_3\omega_4 - \omega_2\omega_3^2c_s^2 - \frac{259}{16}v^2\omega_3\omega_4\omega_1 - 7\omega_2\omega_3^2\omega_4c_s^2\omega_1 + \frac{7}{8}\omega_2\omega_4\omega_1^2 + \frac{7}{2}\omega_2\omega_3^2c_s^2\omega_1 + 2\omega_2\omega_1 + \frac{7}{16}\omega_2\omega_4^2u + \\
& \frac{5}{2}v^2\omega_3\omega_1 - 7\omega_2v^2\omega_3\omega_4^2\omega_1 - \omega_2^2\omega_3c_s^2 + \frac{3}{4}\omega_2\omega_4c_s^2 + \frac{7}{4}\omega_2^2\omega_3\omega_1u^2 + \frac{53}{8}\omega_2\omega_4\omega_1u^2 + \frac{1}{4}\omega_4\omega_1^2u^2 - \frac{1}{2}\omega_2^2v^2\omega_3 + \\
& \frac{1}{2}\omega_3\omega_1^2 + \frac{7}{4}\omega_2v^2\omega_3^2\omega_1 + \frac{21}{8}\omega_3\omega_4c_s^2\omega_1^2 + \frac{3}{2}\omega_2\omega_4 - \frac{1}{2}\omega_2^2\omega_3 - \frac{1}{8}\omega_3\omega_4\omega_1u^2 + \frac{1}{8}\omega_4^2c_s^2 + \frac{7}{8}\omega_2\omega_4c_s^2\omega_1^2 + 5\omega_3c_s^2\omega_1, \\
\gamma_{[t^2]}^{[\mu_5]} &= 7\omega_2\omega_3^2\omega_4\omega_1 - 5\omega_2\omega_3 + \omega_2^2\omega_4 - \frac{7}{2}\omega_2\omega_3\omega_1^2 + \frac{3}{4}\omega_4\omega_1^2 + 7\omega_2\omega_3\omega_4^2\omega_1 + \frac{273}{16}\omega_3\omega_4\omega_1 - \frac{49}{16}\omega_2\omega_3\omega_4^2 + \omega_2^2\omega_1 + \\
& \frac{277}{16}\omega_2\omega_3\omega_4 - \frac{29}{8}\omega_2\omega_4^2\omega_1 - \frac{9}{2}\omega_4\omega_1 + \frac{7}{8}\omega_3^2\omega_4 - \frac{21}{8}\omega_3\omega_4\omega_1^2 + 8\omega_2^2\omega_3\omega_4\omega_1 + \omega_2\omega_3^2 + \frac{39}{2}\omega_2\omega_3\omega_1 - \frac{7}{2}\omega_2^2\omega_4\omega_1 + \omega_3^2\omega_1 + \\
& \frac{19}{16}\omega_2\omega_4^2 - \frac{7}{2}\omega_2^2\omega_3\omega_4 - \frac{7}{2}\omega_2\omega_3^2\omega_1 - \frac{1}{8}\omega_4^2 - 5\omega_3\omega_1 + \frac{35}{2}\omega_2\omega_4\omega_1 - \frac{7}{2}\omega_2^2\omega_3\omega_1 - \frac{49}{16}\omega_2\omega_3^2\omega_4 + \omega_2\omega_1^2 - \frac{35}{8}\omega_3\omega_4 + \frac{19}{16}\omega_4^2\omega_1 + \\
& 6\omega_2\omega_3\omega_4\omega_1^2 - \frac{357}{8}\omega_2\omega_3\omega_4\omega_1 + \frac{7}{8}\omega_3\omega_4^2 - \frac{49}{16}\omega_3\omega_4^2\omega_1 - \frac{21}{8}\omega_2\omega_4\omega_1^2 - 5\omega_2\omega_1 - \frac{49}{16}\omega_3^2\omega_4\omega_1 + \omega_3\omega_1^2 - \frac{9}{2}\omega_2\omega_4 + \omega_2^2\omega_3, \\
\gamma_{[x^1]}^{[\mu_1]} &= -\frac{3}{4}\omega_2v^2\omega_3\omega_4\omega_1 + \frac{1}{8}\omega_3^2\omega_4u^2 - \frac{17}{2}\omega_2\omega_3\omega_1u + \frac{7}{4}\omega_2v^2\omega_3\omega_4 - \frac{3}{4}\omega_2\omega_3\omega_4\omega_1u^2 + \frac{1}{8}\omega_3\omega_4^2u^2 - \frac{1}{2}\omega_3^2\omega_1u + \\
& \frac{7}{4}\omega_2\omega_3\omega_4u^2 + \frac{1}{8}v^2\omega_3^2\omega_4 + \frac{1}{8}\omega_2\omega_3\omega_4 + \frac{1}{8}v^2\omega_4^2 - \frac{3}{2}\omega_2\omega_3\omega_4c_s^2\omega_1 - \frac{3}{4}\omega_2\omega_3^2\omega_4c_s^2 + \frac{3}{2}\omega_2\omega_3\omega_4\omega_1u - \frac{3}{8}\omega_2v^2\omega_3\omega_4^2 - \\
& \frac{1}{8}\omega_4^2u^2 - \frac{3}{4}\omega_2\omega_3\omega_4^2c_s^2 + 2\omega_2\omega_1u - \frac{5}{4}\omega_3\omega_4c_s^2 - \frac{1}{4}\omega_4\omega_1u + \frac{1}{8}\omega_2\omega_4^2 - \frac{1}{2}\omega_3\omega_4\omega_1u + \frac{7}{2}\omega_2\omega_3\omega_4c_s^2 + \frac{1}{4}\omega_3\omega_4^2c_s^2 + \\
& 3\omega_2^2\omega_3\omega_1u + \frac{1}{2}\omega_3\omega_4c_s^2\omega_1 + \frac{1}{4}\omega_3^2\omega_4c_s^2 - \omega_2^2\omega_1u + \frac{1}{4}\omega_2\omega_4\omega_1 + \frac{1}{4}\omega_2\omega_4^2u^2 - \frac{1}{4}\omega_2v^2\omega_4^2 + \frac{1}{8}v^2\omega_3\omega_4^2 + \frac{3}{2}\omega_2\omega_3^2\omega_1u - \\
& \frac{5}{8}\omega_3\omega_4u^2 - \frac{3}{8}\omega_2v^2\omega_3^2\omega_4 - \frac{5}{8}v^2\omega_3\omega_4 - \frac{3}{8}\omega_2\omega_3\omega_4^2u^2 + \frac{1}{4}v^2\omega_3\omega_4\omega_1 - \frac{3}{8}\omega_2\omega_3^2\omega_4u^2 + \frac{5}{2}\omega_3\omega_1u - \frac{1}{2}\omega_2\omega_4 + \frac{1}{4}\omega_3\omega_4\omega_1u^2, \\
\gamma_{[tx]}^{[\mu_5]} &= -\frac{1}{4}\omega_3\omega_4\omega_1 + \frac{3}{8}\omega_2\omega_3\omega_4^2 - \frac{15}{8}\omega_2\omega_3\omega_4 - \frac{1}{8}\omega_3^2\omega_4 + \frac{1}{8}\omega_2\omega_4^2 - \frac{1}{8}\omega_4^2 - \frac{1}{4}\omega_2\omega_4\omega_1 + \frac{3}{8}\omega_2\omega_3^2\omega_4 + \frac{5}{8}\omega_3\omega_4 + \\
& \frac{3}{4}\omega_2\omega_3\omega_4\omega_1 - \frac{1}{8}\omega_3\omega_4^2 + \frac{1}{2}\omega_2\omega_4, \\
\gamma_{[ty]}^{[\mu_1]} &= -\frac{3}{8}\omega_3^2\omega_4c_s^2\omega_1 + \frac{3}{2}\omega_2v\omega_4\omega_1 - 3\omega_2v\omega_1 + \frac{3}{4}\omega_4\omega_1u^2 - \frac{1}{4}\omega_4c_s^2\omega_1^2 - \frac{3}{8}v^2\omega_3^2\omega_4\omega_1 + \frac{1}{4}\omega_4\omega_1^2 - \frac{3}{2}\omega_2v\omega_3^2\omega_1 - \\
& \frac{1}{2}\omega_2v\omega_4 - \frac{1}{8}\omega_4^2u + \frac{1}{8}v^2\omega_3^2\omega_4 + \frac{1}{8}\omega_3\omega_4\omega_1 - \frac{3}{8}v^2\omega_3\omega_4^2\omega_1 + \frac{1}{8}v^2\omega_4^2 - \frac{3}{8}\omega_3\omega_4^2c_s^2\omega_1 - \frac{1}{2}\omega_4\omega_1 + \frac{1}{8}\omega_3\omega_4^2u + \frac{3}{4}\omega_4c_s^2\omega_1 - \\
& \frac{5}{8}\omega_3\omega_4c_s^2 + \frac{1}{8}\omega_3^2\omega_4u + \omega_2v\omega_1^2 + \frac{1}{4}\omega_3\omega_4\omega_1u + \frac{1}{8}\omega_3\omega_4^2c_s^2 - \frac{3}{2}\omega_2v\omega_3\omega_4\omega_1 + \frac{15}{8}\omega_3\omega_4c_s^2\omega_1 + \frac{1}{8}\omega_2^2\omega_4c_s^2 - \\
& \frac{3}{4}v^2\omega_3\omega_4\omega_1^2 - \frac{3}{8}\omega_4^2c_s^2\omega_1 - \frac{5}{8}\omega_3\omega_4u + \frac{1}{2}\omega_2v\omega_3^2 - 3\omega_2v\omega_3\omega_1^2 + \frac{1}{8}\omega_4^2\omega_1 + \frac{1}{8}v^2\omega_3\omega_4^2 - \frac{5}{2}\omega_2v\omega_3 - \frac{1}{8}\omega_4^2\omega_1u^2 + \\
& \frac{17}{2}\omega_2v\omega_3\omega_1 - \frac{1}{4}v^2\omega_4^2\omega_1 - \frac{5}{8}v^2\omega_3\omega_4 + 2v^2\omega_3\omega_4\omega_1 - \frac{1}{4}\omega_4\omega_1^2u^2 + \frac{1}{2}\omega_2v\omega_3\omega_4 - \frac{3}{4}\omega_3\omega_4c_s^2\omega_1^2 - \frac{1}{8}\omega_3\omega_4\omega_1u^2 + \frac{1}{8}\omega_4^2c_s^2, \\
\gamma_{[ty]}^{[\mu_5]} &= -\frac{1}{4}\omega_4\omega_1^2 - \frac{17}{8}\omega_3\omega_4\omega_1 + \frac{1}{2}\omega_4\omega_1 - \frac{1}{8}\omega_3^2\omega_4 + \frac{3}{4}\omega_3\omega_4\omega_1^2 - \frac{1}{8}\omega_4^2 + \frac{5}{8}\omega_3\omega_4 + \frac{1}{8}\omega_4^2\omega_1 - \frac{1}{8}\omega_3\omega_4^2 + \frac{3}{8}\omega_3\omega_4^2\omega_1 + \frac{3}{8}\omega_3^2\omega_4\omega_1, \\
\gamma_{[x^2]}^{[\mu_1]} &= \frac{1}{16}\omega_2\omega_4^2c_s^2 - \frac{5}{8}\omega_2v^2\omega_3\omega_4\omega_1 + \frac{5}{4}\omega_2v^2\omega_3\omega_1 + \frac{5}{2}\omega_2\omega_3c_s^2\omega_1 - \frac{1}{2}\omega_2^2\omega_3\omega_4c_s^2 + \frac{43}{16}\omega_2v^2\omega_3\omega_4 - \frac{1}{4}\omega_2\omega_3\omega_4\omega_1u^2 - \\
& \frac{5}{2}\omega_2v^2\omega_3 + \frac{1}{2}\omega_2\omega_3\omega_4u^2 - 5\omega_2\omega_3c_s^2 + \frac{1}{4}\omega_2\omega_4u - \frac{7}{8}\omega_2\omega_3\omega_4c_s^2\omega_1 + \frac{1}{4}\omega_2\omega_4u^2 + \omega_2^2\omega_3u^2 - \frac{7}{16}\omega_2\omega_3^2\omega_4c_s^2 + \\
& \frac{1}{8}\omega_2\omega_3\omega_4\omega_1u - \frac{7}{16}\omega_2v^2\omega_3\omega_4^2 + \frac{1}{2}\omega_2v^2\omega_3^2 - \frac{5}{16}\omega_2\omega_3\omega_4u - \frac{7}{16}\omega_2\omega_3\omega_4^2c_s^2 + \frac{1}{2}\omega_2\omega_3^2u^2 + \frac{5}{4}\omega_2\omega_3\omega_1u^2 - \\
& \frac{5}{2}\omega_2\omega_3u^2 + \frac{51}{16}\omega_2\omega_3\omega_4c_s^2 + \frac{1}{8}\omega_2\omega_4c_s^2\omega_1 - \omega_2^2\omega_3c_s^2\omega_1 - \frac{1}{2}\omega_2^2v^2\omega_3\omega_1 + \frac{1}{16}\omega_2\omega_3^2\omega_4u - \frac{1}{4}\omega_2\omega_3^2\omega_1u^2 + \\
& \frac{1}{16}\omega_2v^2\omega_4^2 - \frac{1}{2}\omega_2^2v^2\omega_3\omega_4 + \frac{1}{16}\omega_2\omega_3\omega_4^2u - \frac{1}{2}\omega_2v^2\omega_4 + \frac{1}{4}\omega_2v^2\omega_4\omega_1 - \frac{7}{16}\omega_2v^2\omega_3^2\omega_4 + \omega_2\omega_3^2c_s^2 - \frac{1}{2}\omega_2\omega_3^2c_s^2\omega_1 - \\
& \frac{1}{16}\omega_2\omega_4^2u + 2\omega_2^2\omega_3c_s^2 - \frac{1}{4}\omega_2\omega_4c_s^2 - \frac{1}{2}\omega_2^2\omega_3\omega_1u^2 - \frac{1}{8}\omega_2\omega_4\omega_1u^2 + \omega_2^2v^2\omega_3 - \frac{1}{4}\omega_2v^2\omega_3^2\omega_1, \\
\gamma_{[x^5]}^{[\mu_5]} &= \frac{5}{2}\omega_2\omega_3 + \frac{7}{16}\omega_2\omega_3\omega_4^2 - \frac{43}{16}\omega_2\omega_3\omega_4 - \frac{1}{2}\omega_2\omega_3^2 - \frac{5}{4}\omega_2\omega_3\omega_1 - \frac{1}{16}\omega_2\omega_4^2 + \frac{1}{2}\omega_2^2\omega_3\omega_4 + \frac{1}{4}\omega_2\omega_3^2\omega_1 - \frac{1}{4}\omega_2\omega_4\omega_1 + \\
& \frac{1}{2}\omega_2^2\omega_3\omega_1 + \frac{7}{16}\omega_2\omega_3^2\omega_4 + \frac{5}{8}\omega_2\omega_3\omega_4\omega_1 + \frac{1}{2}\omega_2\omega_4 - \omega_2^2\omega_3, \\
\gamma_{[xy]}^{[\mu_1]} &= \frac{1}{8}\omega_3^2\omega_4u^2 + \frac{1}{8}\omega_3\omega_4^2u^2 + \frac{1}{8}v^2\omega_3^2\omega_4 + \frac{1}{8}v^2\omega_4^2 - \frac{1}{8}\omega_4^2u^2 - \frac{5}{4}\omega_3\omega_4c_s^2 + \frac{1}{4}\omega_4\omega_1u + \frac{1}{4}\omega_3\omega_4^2c_s^2 + \frac{1}{2}\omega_3\omega_4c_s^2\omega_1 + \\
& \frac{1}{4}\omega_3^2\omega_4c_s^2 + \frac{1}{8}v^2\omega_3\omega_4^2 - \frac{5}{8}\omega_3\omega_4u^2 - \frac{5}{8}v^2\omega_3\omega_4 + \frac{1}{4}v^2\omega_3\omega_4\omega_1 + \frac{1}{4}\omega_3\omega_4\omega_1u^2, \\
\gamma_{[xy]}^{[\mu_5]} &= -\frac{1}{4}\omega_3\omega_4\omega_1 - \frac{1}{8}\omega_3^2\omega_4 - \frac{1}{8}\omega_4^2 + \frac{5}{8}\omega_3\omega_4 - \frac{1}{8}\omega_3\omega_4^2, \\
\gamma_{[y^2]}^{[\mu_1]} &= -\frac{1}{2}\omega_2v^2\omega_3\omega_4\omega_1 - \frac{7}{16}\omega_3^2\omega_4c_s^2\omega_1 - \frac{7}{16}v^2\omega_3^2\omega_4\omega_1 - \frac{7}{16}v^2\omega_3\omega_4^2\omega_1 - \frac{1}{2}\omega_2\omega_3\omega_4c_s^2\omega_1 - \frac{7}{16}\omega_3\omega_4^2c_s^2\omega_1 + \\
& \frac{35}{16}\omega_3\omega_4c_s^2\omega_1 - \frac{3}{8}v^2\omega_3\omega_4\omega_1^2 + \frac{1}{16}\omega_4^2c_s^2\omega_1 + \frac{1}{16}v^2\omega_4^2\omega_1 + \frac{35}{16}v^2\omega_3\omega_4\omega_1 - \frac{3}{8}\omega_3\omega_4c_s^2\omega_1^2, \\
\gamma_{[y^2]}^{[\mu_5]} &= \frac{1}{2}\omega_2\omega_3\omega_1^2 - \frac{43}{16}\omega_3\omega_4\omega_1 + \frac{1}{2}\omega_4\omega_1 + \frac{3}{8}\omega_3\omega_4\omega_1^2 - \frac{5}{4}\omega_2\omega_3\omega_1 - \frac{1}{2}\omega_3^2\omega_1 + \frac{1}{4}\omega_2\omega_3^2\omega_1 + \frac{5}{2}\omega_3\omega_1 - \frac{1}{4}\omega_2\omega_4\omega_1 - \\
& \frac{1}{16}\omega_4^2\omega_1 + \frac{3}{4}\omega_2\omega_3\omega_4\omega_1 + \frac{7}{16}\omega_3\omega_4^2\omega_1 + \frac{7}{16}\omega_3^2\omega_4\omega_1 - \omega_3\omega_1^2,
\end{aligned}$$

## 5 MRT 3: with ortogonalization and relaxation of $m_{00}$ , $m_{10}$ , $m_{01}$ , $m_{20}$ , $m_{02}$

### 5.1 Definitions

Matrix  $\mathbf{A} = \mathbf{M}^{-1}\mathbf{S}\mathbf{M}$ :

$$\begin{aligned}
\mathbf{A}_{1,1} &= \frac{2}{3}\omega_4 + \frac{2}{15}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{1,2} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{1,3} &= -\frac{1}{3}\omega_4 + \frac{2}{15}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{1,4} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{1,5} &= -\frac{1}{3}\omega_4 + \frac{2}{15}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{2,1} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{2,2} &= \frac{1}{2}\omega_1 + \frac{3}{10}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{2,3} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{2,4} &= -\frac{1}{2}\omega_1 + \frac{3}{10}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{2,5} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{3,1} &= -\frac{1}{3}\omega_4 + \frac{2}{15}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{3,2} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{3,3} &= \frac{1}{6}\omega_4 + \frac{2}{15}\omega_3 + \frac{1}{5}\omega_0 + \frac{1}{2}\omega_2, \\
\mathbf{A}_{3,4} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{3,5} &= \frac{1}{6}\omega_4 + \frac{2}{15}\omega_3 + \frac{1}{5}\omega_0 - \frac{1}{2}\omega_2, \\
\mathbf{A}_{4,1} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{4,2} &= -\frac{1}{2}\omega_1 + \frac{3}{10}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{4,3} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{4,4} &= \frac{1}{2}\omega_1 + \frac{3}{10}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{4,5} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{5,1} &= -\frac{1}{3}\omega_4 + \frac{2}{15}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{5,2} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0,
\end{aligned}$$

$$\begin{aligned}\mathbf{A}_{5,3} &= \frac{1}{6}\omega_4 + \frac{2}{15}\omega_3 + \frac{1}{5}\omega_0 - \frac{1}{2}\omega_2, \\ \mathbf{A}_{5,4} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0, \\ \mathbf{A}_{5,5} &= \frac{1}{6}\omega_4 + \frac{2}{15}\omega_3 + \frac{1}{5}\omega_0 + \frac{1}{2}\omega_2.\end{aligned}$$

where

$$\mathbf{S} = \text{diag}(\omega_0, \omega_1, \omega_2, \omega_3, \omega_4)$$

and

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ -2 & 3 & -2 & 3 & -2 \\ -2 & 0 & 1 & 0 & 1 \end{pmatrix}$$

Matrix  $\mathbf{B}$ :

$$\mathbf{B} = \begin{pmatrix} 0 & -1 + \frac{2}{3}\omega_4 + \frac{1}{3}\omega_3 & -1 + \omega_4 & -1 + \frac{2}{3}\omega_4 + \frac{1}{3}\omega_3 & -1 + \omega_4 \\ -1 + \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 & 0 & -1 + \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 & -1 + \omega_1 & -1 + \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 \\ -1 + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_2 & -1 + \frac{1}{6}\omega_4 + \frac{1}{3}\omega_3 + \frac{1}{2}\omega_2 & 0 & -1 + \frac{1}{6}\omega_4 + \frac{1}{3}\omega_3 + \frac{1}{2}\omega_2 & -1 + \omega_2 \\ -1 + \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 & -1 + \omega_1 & -1 + \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 & 0 & -1 + \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 \\ -1 + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_2 & -1 + \frac{1}{6}\omega_4 + \frac{1}{3}\omega_3 + \frac{1}{2}\omega_2 & -1 + \omega_2 & -1 + \frac{1}{6}\omega_4 + \frac{1}{3}\omega_3 + \frac{1}{2}\omega_2 & 0 \end{pmatrix}.$$

## 5.2 EPDE for $\mu_1$

$$\begin{aligned}\gamma_{[t]}^{[\mu_1]} \delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[x]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[y]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[t^2]}^{[\mu_1]} \delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[tx]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} \\ + \gamma_{[ty]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[x^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[y^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} = 0,\end{aligned}$$

where

$$\gamma_{[t]}^{[\mu_1]} = -\omega_3 \omega_4 \omega_1 \omega_2,$$

$$\gamma_{[x]}^{[\mu_1]} = -\omega_3 u \omega_4 \omega_1 \omega_2,$$

$$\gamma_{[y]}^{[\mu_1]} = -\omega_3 \omega_4 \omega_1 v \omega_2,$$

$$\gamma_{[t^2]}^{[\mu_1]} = -\omega_3 \omega_4 \omega_2 - \omega_4 \omega_1 \omega_2 + \frac{7}{2} \omega_3 \omega_4 \omega_1 \omega_2 - \omega_3 \omega_4 \omega_1 - \omega_3 \omega_1 \omega_2,$$

$$\gamma_{[tx]}^{[\mu_1]} = -u \omega_4 \omega_1 \omega_2 - \omega_3 u \omega_1 \omega_2 - \omega_3 u \omega_4 \omega_1 + 3 \omega_3 u \omega_4 \omega_1 \omega_2,$$

$$\gamma_{[ty]}^{[\mu_1]} = -\omega_4 \omega_1 v \omega_2 - \omega_3 \omega_4 v \omega_2 + 3 \omega_3 \omega_4 \omega_1 v \omega_2 - \omega_3 \omega_1 v \omega_2,$$

$$\gamma_{[x^2]}^{[\mu_1]} = -\frac{1}{2} \omega_3 \omega_4 \omega_1 \omega_2 c_s^2 - \frac{1}{2} \omega_3 u^2 \omega_4 \omega_1 \omega_2 + \omega_3 \omega_4 \omega_2 c_s^2 + \omega_3 u^2 \omega_4 \omega_2,$$

$$\gamma_{[y^2]}^{[\mu_1]} = -\frac{1}{2} \omega_3 \omega_4 \omega_1 \omega_2 c_s^2 - \frac{1}{2} \omega_3 \omega_4 \omega_1 v^2 \omega_2 + \omega_3 \omega_4 \omega_1 v^2 + \omega_3 \omega_4 \omega_1 c_s^2,$$

### 5.3 EPDE for $\mu_2$

$$\begin{aligned} & \gamma_{[1]}^{[\mu_1]} \mu_1 + \gamma_{[1]}^{[\mu_2]} \mu_2 + \gamma_{[t]}^{[\mu_1]} \delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_2]} \delta_t \frac{\partial \mu_2}{\partial t} + \gamma_{[x]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[t^2]}^{[\mu_1]} \delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_2]} \delta_t^2 \frac{\partial^2 \mu_2}{\partial t^2} \\ & + \gamma_{[tx]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} + \gamma_{[x^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_2]} \delta_l^2 \frac{\partial^2 \mu_2}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x \partial y} + \gamma_{[y^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_2]} \delta_l^2 \frac{\partial^2 \mu_2}{\partial y^2} = 0, \end{aligned}$$

where

$$\gamma_{[1]}^{[\mu_1]} = -\omega_3 u \omega_4 \omega_1^2 \omega_2 - \omega_3 u \omega_4^2 \omega_1 \omega_2 - \omega_3 u \omega_4 \omega_1 \omega_2^2 - \omega_3^2 u \omega_4 \omega_1 \omega_2 + 5\omega_3 u \omega_4 \omega_1 \omega_2,$$

$$\gamma_{[1]}^{[\mu_2]} = \omega_3 \omega_4 \omega_1 \omega_2^2 - 5\omega_3 \omega_4 \omega_1 \omega_2 + \omega_3 \omega_4 \omega_1^2 \omega_2 + \omega_3 \omega_4^2 \omega_1 \omega_2 + \omega_3^2 \omega_4 \omega_1 \omega_2,$$

$$\begin{aligned} \gamma_{[t]}^{[\mu_1]} = & 4\omega_3 u \omega_4 \omega_1^2 \omega_2 - \omega_3^2 u \omega_1 \omega_2 + 4\omega_3 u \omega_4^2 \omega_1 \omega_2 - u \omega_4 \omega_1 \omega_2^2 - \omega_3 u \omega_4^2 \omega_1 - \omega_3^2 u \omega_4 \omega_1 + 5u \omega_4 \omega_1 \omega_2 - \omega_3 u \omega_1^2 \omega_2 + 5\omega_3 u \omega_1 \omega_2 - \\ & u \omega_4^2 \omega_1 \omega_2 + 4\omega_3 u \omega_4 \omega_1 \omega_2^2 - u \omega_4 \omega_1^2 \omega_2 - \omega_3 u \omega_4 \omega_1^2 + 5\omega_3 u \omega_4 \omega_1 + 4\omega_3^2 u \omega_4 \omega_1 \omega_2 - 22\omega_3 u \omega_4 \omega_1 \omega_2 - \omega_3 u \omega_1 \omega_2^2, \\ \gamma_{[t]}^{[\mu_2]} = & \omega_3^2 \omega_1 \omega_2 - 4\omega_3 \omega_4 \omega_1 \omega_2^2 + \omega_3 \omega_1^2 \omega_2 - 5\omega_3 \omega_4 \omega_2 - 5\omega_4 \omega_1 \omega_2 + \omega_4 \omega_1 \omega_2^2 + \omega_3 \omega_4 \omega_2^2 + \omega_3^2 \omega_4 \omega_1 + 23\omega_3 \omega_4 \omega_1 \omega_2 + \\ & \omega_3 \omega_4^2 \omega_1 - 4\omega_3 \omega_4 \omega_1^2 \omega_2 - 5\omega_3 \omega_4 \omega_1 - 4\omega_3 \omega_4^2 \omega_1 \omega_2 + \omega_3 \omega_1 \omega_2^2 - 5\omega_3 \omega_1 \omega_2 + \omega_4 \omega_1^2 \omega_2 + \omega_3 \omega_4^2 \omega_2 + \omega_3 \omega_4 \omega_1^2 + \\ & \omega_3^2 \omega_4 \omega_2 + \omega_4^2 \omega_1 \omega_2 - 4\omega_3^2 \omega_4 \omega_1 \omega_2, \end{aligned}$$

$$\gamma_{[x]}^{[\mu_1]} = \omega_3^2 \omega_4 \omega_2 c_s^2 + \omega_3^2 u^2 \omega_4 \omega_2 + \omega_3 \omega_4 \omega_1 \omega_2 c_s^2 + \omega_3 u^2 \omega_4^2 \omega_2 + \omega_3 \omega_4^2 \omega_2 c_s^2 + \omega_3 u^2 \omega_4 \omega_1 \omega_2 + \omega_3 \omega_4 \omega_2^2 c_s^2 - 5\omega_3 \omega_4 \omega_2 c_s^2 - 5\omega_3 u^2 \omega_4 \omega_2 + \omega_3 u^2 \omega_4 \omega_2^2,$$

$$\begin{aligned} \gamma_{[t^2]}^{[\mu_1]} = & -8\omega_3 u \omega_4 \omega_1^2 \omega_2 + \frac{7}{2} \omega_3^2 u \omega_1 \omega_2 - \omega_3 u \omega_1^2 - 8\omega_3 u \omega_4^2 \omega_1 \omega_2 - u \omega_1^2 \omega_2 + \frac{7}{2} u \omega_4 \omega_1 \omega_2^2 + \frac{7}{2} \omega_3 u \omega_4^2 \omega_1 + \frac{7}{2} \omega_3^2 u \omega_4 \omega_1 - \\ & \frac{37}{2} u \omega_4 \omega_1 \omega_2 + \frac{7}{2} \omega_3 u \omega_1^2 \omega_2 - u \omega_4^2 \omega_1 + 5\omega_3 u \omega_1 - \frac{37}{2} \omega_3 u \omega_1 \omega_2 + \frac{7}{2} u \omega_4^2 \omega_1 \omega_2 + 5u \omega_4 \omega_1 - 8\omega_3 u \omega_4 \omega_1 \omega_2^2 - \omega_3^2 u \omega_1 + \\ & \frac{7}{2} u \omega_4 \omega_1^2 \omega_2 - u \omega_1 \omega_2^2 + \frac{7}{2} \omega_3 u \omega_4 \omega_1^2 + 5u \omega_1 \omega_2 - \frac{37}{2} \omega_3 u \omega_4 \omega_1 - 8\omega_3^2 u \omega_4 \omega_1 \omega_2 + 47\omega_3 u \omega_4 \omega_1 \omega_2 + \frac{7}{2} \omega_3 u \omega_1 \omega_2^2 - u \omega_4 \omega_1^2, \end{aligned}$$

$$\begin{aligned} \gamma_{[t^2]}^{[\mu_2]} = & \omega_3^2 \omega_2 - \frac{7}{2} \omega_3^2 \omega_1 \omega_2 + \omega_1 \omega_2^2 + 8\omega_3 \omega_4 \omega_1 \omega_2^2 - \frac{7}{2} \omega_3 \omega_1^2 \omega_2 - 5\omega_4 \omega_1 - 5\omega_3 \omega_4 + \frac{39}{2} \omega_3 \omega_4 \omega_2 + \frac{39}{2} \omega_4 \omega_1 \omega_2 + \omega_3 \omega_1^2 + \\ & \omega_3 \omega_4^2 + \omega_4 \omega_1^2 - \frac{7}{2} \omega_4 \omega_1 \omega_2^2 - \frac{7}{2} \omega_3 \omega_4 \omega_2^2 - 5\omega_3 \omega_1 - \frac{7}{2} \omega_3^2 \omega_4 \omega_1 - 5\omega_1 \omega_2 - \frac{101}{2} \omega_3 \omega_4 \omega_1 \omega_2 + \omega_4^2 \omega_2 - \frac{7}{2} \omega_3 \omega_4^2 \omega_1 + \\ & 8\omega_3 \omega_4 \omega_1^2 \omega_2 + \omega_1^2 \omega_2 - 5\omega_4 \omega_2 + \frac{39}{2} \omega_3 \omega_4 \omega_1 + \omega_3 \omega_2^2 + 8\omega_3 \omega_4^2 \omega_1 \omega_2 + \omega_3^2 \omega_1 - \frac{7}{2} \omega_3 \omega_1 \omega_2^2 + \frac{39}{2} \omega_3 \omega_1 \omega_2 + \omega_4^2 \omega_1 + \\ & \omega_3^2 \omega_4 - \frac{7}{2} \omega_4 \omega_1^2 \omega_2 - \frac{7}{2} \omega_3 \omega_4^2 \omega_2 + \omega_4 \omega_2^2 - \frac{7}{2} \omega_3 \omega_4 \omega_1^2 - 5\omega_3 \omega_2 - \frac{7}{2} \omega_3^2 \omega_4 \omega_2 - \frac{7}{2} \omega_4^2 \omega_1 \omega_2 + 8\omega_3^2 \omega_4 \omega_1 \omega_2, \end{aligned}$$

$$\begin{aligned} \gamma_{[tx]}^{[\mu_1]} = & -3\omega_3^2 \omega_4 \omega_2 c_s^2 + \omega_3 u^2 \omega_4 \omega_1 - 5\omega_3 u^2 \omega_2 - 3\omega_3^2 u^2 \omega_4 \omega_2 - 3\omega_3 \omega_4 \omega_1 \omega_2 c_s^2 - 5\omega_3 \omega_2 c_s^2 + \omega_3^2 u^2 \omega_4 + \omega_3 \omega_4 \omega_2 + \\ & \omega_4 \omega_1 \omega_2 + \omega_3^2 \omega_4 c_s^2 + \omega_3 u^2 \omega_1 \omega_2 - 3\omega_3 u^2 \omega_4^2 \omega_2 - 3\omega_3 \omega_4^2 \omega_2 c_s^2 + \omega_3 u^2 \omega_2^2 + \omega_3 \omega_4^2 c_s^2 + \omega_3 \omega_1 \omega_2 c_s^2 - \\ & 3\omega_3 u^2 \omega_4 \omega_1 \omega_2 - 3\omega_3 \omega_4 \omega_2^2 c_s^2 + \omega_3 u^2 \omega_4^2 - 5\omega_3 \omega_4 c_s^2 - 2\omega_4 \omega_2 + 16\omega_3 \omega_4 \omega_2 c_s^2 + 16\omega_3 u^2 \omega_4 \omega_2 - 3\omega_3 u^2 \omega_4 \omega_2^2 + \\ & \omega_3 \omega_2^2 c_s^2 + \omega_3 \omega_4 \omega_1 c_s^2 + \omega_3^2 u^2 \omega_2 - 5\omega_3 u^2 \omega_4 + \omega_3^2 \omega_2 c_s^2, \end{aligned}$$

$$\gamma_{[x^2]}^{[\mu_1]} = -\frac{1}{2} u \omega_4 \omega_1 \omega_2^2 + \frac{3}{2} u \omega_4 \omega_1 \omega_2 - \frac{1}{2} u \omega_4^2 \omega_1 \omega_2,$$

$$\begin{aligned} \gamma_{[x^2]}^{[\mu_2]} = & -\frac{5}{2} \omega_3 \omega_4 \omega_2 - \frac{3}{2} \omega_4 \omega_1 \omega_2 + \frac{1}{2} \omega_4 \omega_1 \omega_2^2 + \frac{1}{2} \omega_3 \omega_4 \omega_2^2 + \frac{1}{2} \omega_3 \omega_4 \omega_1 \omega_2 - \omega_4^2 \omega_2 + 3\omega_4 \omega_2 + \frac{1}{2} \omega_3 \omega_4^2 \omega_2 - \omega_4 \omega_2^2 + \\ & \frac{1}{2} \omega_3^2 \omega_4 \omega_2 + \frac{1}{2} \omega_4^2 \omega_1 \omega_2, \end{aligned}$$

$$\gamma_{[xy]}^{[\mu_1]} = \omega_4 \omega_1 v \omega_2 + \omega_3 \omega_4 v \omega_2 - 2\omega_4 v \omega_2,$$

$$\begin{aligned} \gamma_{[y^2]}^{[\mu_1]} = & -\frac{1}{6} \omega_3^2 u \omega_1 \omega_2 + \frac{1}{3} \omega_3 u \omega_1^2 - \frac{1}{2} \omega_3 u \omega_4^2 \omega_1 - \frac{1}{2} \omega_3^2 u \omega_4 \omega_1 + \frac{2}{3} u \omega_4 \omega_1 \omega_2 - \frac{1}{6} \omega_3 u \omega_1^2 \omega_2 - \frac{5}{3} \omega_3 u \omega_1 + \frac{5}{6} \omega_3 u \omega_1 \omega_2 - \\ & \frac{4}{3} u \omega_4 \omega_1 + \frac{1}{3} \omega_3^2 u \omega_1 - \frac{1}{3} u \omega_4 \omega_1^2 \omega_2 - \frac{1}{2} \omega_3 u \omega_4 \omega_1^2 + \frac{19}{6} \omega_3 u \omega_4 \omega_1 - \frac{5}{6} \omega_3 u \omega_4 \omega_1 \omega_2 + \frac{2}{3} u \omega_4 \omega_1^2. \end{aligned}$$



$$\gamma_{[y^2]}^{[\mu_2]} = \frac{1}{6}\omega_3^2\omega_1\omega_2 + \frac{1}{6}\omega_3\omega_1^2\omega_2 + \frac{4}{3}\omega_4\omega_1 - \frac{2}{3}\omega_4\omega_1\omega_2 - \frac{1}{3}\omega_3\omega_1^2 - \frac{2}{3}\omega_4\omega_1^2 + \frac{5}{3}\omega_3\omega_1 + \frac{1}{2}\omega_3^2\omega_4\omega_1 + \frac{5}{6}\omega_3\omega_4\omega_1\omega_2 + \frac{1}{2}\omega_3\omega_4^2\omega_1 - \frac{19}{6}\omega_3\omega_4\omega_1 - \frac{1}{3}\omega_3^2\omega_1 - \frac{5}{6}\omega_3\omega_1\omega_2 + \frac{1}{3}\omega_4\omega_1^2\omega_2 + \frac{1}{2}\omega_3\omega_4\omega_1^2,$$

#### 5.4 EPDE for $\mu_3$

$$\begin{aligned} & \gamma_{[1]}^{[\mu_1]}\mu_1 + \gamma_{[1]}^{[\mu_3]}\mu_3 + \gamma_{[t]}^{[\mu_1]}\delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_3]}\delta_t \frac{\partial \mu_3}{\partial t} + \gamma_{[y]}^{[\mu_1]}\delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[t^2]}^{[\mu_1]}\delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_3]}\delta_t^2 \frac{\partial^2 \mu_3}{\partial t^2} \\ & + \gamma_{[ty]}^{[\mu_1]}\delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[x^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_3]}\delta_l^2 \frac{\partial^2 \mu_3}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial x \partial y} + \gamma_{[y^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_3]}\delta_l^2 \frac{\partial^2 \mu_3}{\partial y^2} = 0, \end{aligned}$$

where

$$\gamma_{[1]}^{[\mu_1]} = -\omega_3\omega_4^2\omega_1v\omega_2 - \omega_3\omega_4\omega_1^2v\omega_2 + 5\omega_3\omega_4\omega_1v\omega_2 - \omega_3^2\omega_4\omega_1v\omega_2 - \omega_3\omega_4\omega_1v\omega_2^2,$$

$$\gamma_{[1]}^{[\mu_3]} = \omega_3\omega_4\omega_1\omega_2^2 - 5\omega_3\omega_4\omega_1\omega_2 + \omega_3\omega_4\omega_1^2\omega_2 + \omega_3\omega_4^2\omega_1\omega_2 + \omega_3^2\omega_4\omega_1\omega_2,$$

$$\begin{aligned} \gamma_{[t]}^{[\mu_1]} &= -\omega_3\omega_1^2v\omega_2 + 4\omega_3\omega_4^2\omega_1v\omega_2 + 5\omega_4\omega_1v\omega_2 + 5\omega_3\omega_4v\omega_2 - \omega_3^2\omega_1v\omega_2 + 4\omega_3\omega_4\omega_1^2v\omega_2 - \omega_4\omega_1v\omega_2^2 - \omega_3\omega_4v\omega_2^2 - \\ & 22\omega_3\omega_4\omega_1v\omega_2 - \omega_3\omega_1v\omega_2^2 - \omega_4^2\omega_1v\omega_2 + 4\omega_3^2\omega_4\omega_1v\omega_2 - \omega_3^2\omega_4v\omega_2 + 5\omega_3\omega_1v\omega_2 - \omega_3\omega_4^2v\omega_2 - \omega_4\omega_1^2v\omega_2 + \\ & 4\omega_3\omega_4\omega_1v\omega_2^2, \end{aligned}$$

$$\begin{aligned} \gamma_{[t]}^{[\mu_3]} &= \omega_3^2\omega_1\omega_2 - 4\omega_3\omega_4\omega_1\omega_2^2 + \omega_3\omega_1^2\omega_2 - 5\omega_3\omega_4\omega_2 - 5\omega_4\omega_1\omega_2 + \omega_4\omega_1\omega_2^2 + \omega_3\omega_4\omega_2^2 + \omega_3^2\omega_4\omega_1 + 23\omega_3\omega_4\omega_1\omega_2 + \\ & \omega_3\omega_4^2\omega_1 - 4\omega_3\omega_4\omega_1^2\omega_2 - 5\omega_3\omega_4\omega_1 - 4\omega_3\omega_4^2\omega_1\omega_2 + \omega_3\omega_1\omega_2^2 - 5\omega_3\omega_1\omega_2 + \omega_4\omega_1^2\omega_2 + \omega_3\omega_4^2\omega_2 + \omega_3\omega_4\omega_1^2 + \\ & \omega_3^2\omega_4\omega_2 + \omega_4^2\omega_1\omega_2 - 4\omega_3^2\omega_4\omega_1\omega_2, \end{aligned}$$

$$\begin{aligned} \gamma_{[y]}^{[\mu_1]} &= \omega_3^2\omega_4\omega_1v^2 + \omega_3\omega_4^2\omega_1c_s^2 + \omega_3\omega_4\omega_1\omega_2c_s^2 + \omega_3\omega_4\omega_1^2c_s^2 + \omega_3^2\omega_4\omega_1c_s^2 + \omega_3\omega_4^2\omega_1v^2 + \omega_3\omega_4\omega_1v^2\omega_2 + \\ & \omega_3\omega_4\omega_1^2v^2 - 5\omega_3\omega_4\omega_1v^2 - 5\omega_3\omega_4\omega_1c_s^2, \end{aligned}$$

$$\begin{aligned} \gamma_{[t^2]}^{[\mu_1]} &= -\omega_4v\omega_2^2 + \frac{7}{2}\omega_3\omega_1^2v\omega_2 - 8\omega_3\omega_4^2\omega_1v\omega_2 - \frac{37}{2}\omega_4\omega_1v\omega_2 - \frac{37}{2}\omega_3\omega_4v\omega_2 + \frac{7}{2}\omega_3^2\omega_1v\omega_2 - \omega_1^2v\omega_2 - 8\omega_3\omega_4\omega_1^2v\omega_2 - \\ & \omega_3^2v\omega_2 + \frac{7}{2}\omega_4\omega_1v\omega_2^2 + \frac{7}{2}\omega_3\omega_4v\omega_2^2 + 5\omega_4v\omega_2 + 47\omega_3\omega_4\omega_1v\omega_2 + 5\omega_3v\omega_2 + 5\omega_1v\omega_2 + \frac{7}{2}\omega_3\omega_1v\omega_2^2 + \frac{7}{2}\omega_4^2\omega_1v\omega_2 - \\ & 8\omega_3^2\omega_4\omega_1v\omega_2 + \frac{7}{2}\omega_3^2\omega_4v\omega_2 - \omega_4^2v\omega_2 - \frac{37}{2}\omega_3\omega_1v\omega_2 + \frac{7}{2}\omega_3\omega_4^2v\omega_2 + \frac{7}{2}\omega_4\omega_1^2v\omega_2 - \omega_3v\omega_2^2 - 8\omega_3\omega_4\omega_1v\omega_2^2 - \omega_1v\omega_2^2, \end{aligned}$$

$$\begin{aligned} \gamma_{[t^2]}^{[\mu_3]} &= \omega_3^2\omega_2 - \frac{7}{2}\omega_3^2\omega_1\omega_2 + \omega_1\omega_2^2 + 8\omega_3\omega_4\omega_1\omega_2^2 - \frac{7}{2}\omega_3\omega_1^2\omega_2 - 5\omega_4\omega_1 - 5\omega_3\omega_4 + \frac{39}{2}\omega_3\omega_4\omega_2 + \frac{39}{2}\omega_4\omega_1\omega_2 + \omega_3\omega_1^2 + \\ & \omega_3\omega_4^2 + \omega_4\omega_1^2 - \frac{7}{2}\omega_4\omega_1\omega_2^2 - \frac{7}{2}\omega_3\omega_4\omega_2^2 - 5\omega_3\omega_1 - \frac{7}{2}\omega_3^2\omega_4\omega_1 - 5\omega_1\omega_2 - \frac{101}{2}\omega_3\omega_4\omega_1\omega_2 + \omega_4^2\omega_2 - \frac{7}{2}\omega_3\omega_4^2\omega_1 + \\ & 8\omega_3\omega_4\omega_1^2\omega_2 + \omega_1^2\omega_2 - 5\omega_4\omega_2 + \frac{39}{2}\omega_3\omega_4\omega_1 + \omega_3\omega_2^2 + 8\omega_3\omega_4^2\omega_1\omega_2 + \omega_3^2\omega_1 - \frac{7}{2}\omega_3\omega_1\omega_2^2 + \frac{39}{2}\omega_3\omega_1\omega_2 + \omega_4^2\omega_1 + \\ & \omega_3^2\omega_4 - \frac{7}{2}\omega_4\omega_1^2\omega_2 - \frac{7}{2}\omega_3\omega_4^2\omega_2 + \omega_4\omega_2^2 - \frac{7}{2}\omega_3\omega_4\omega_1^2 - 5\omega_3\omega_2 - \frac{7}{2}\omega_3^2\omega_4\omega_2 - \frac{7}{2}\omega_4^2\omega_1\omega_2 + 8\omega_3^2\omega_4\omega_1\omega_2, \end{aligned}$$

$$\begin{aligned} \gamma_{[ty]}^{[\mu_1]} &= \omega_4^2\omega_1v^2 - 3\omega_3^2\omega_4\omega_1v^2 + \omega_3^2\omega_4v^2 - 3\omega_3\omega_4^2\omega_1c_s^2 + \omega_3\omega_4^2v^2 + \omega_4\omega_1^2v^2 + \frac{4}{3}\omega_4\omega_1 + \frac{2}{3}u^2\omega_4\omega_1^2 - \\ & 3\omega_3\omega_4\omega_1\omega_2c_s^2 + \frac{2}{3}\omega_3\omega_1^2 - 3\omega_3\omega_4\omega_1^2c_s^2 + \frac{5}{3}\omega_4^2\omega_1c_s^2 - \frac{2}{3}\omega_4\omega_1^2 - \frac{10}{3}u^2\omega_4\omega_1 - 3\omega_3^2\omega_4\omega_1c_s^2 + \omega_3^2\omega_4c_s^2 - \\ & \frac{2}{3}\omega_3^2u^2\omega_1 - \frac{10}{3}\omega_3\omega_1 - \frac{2}{3}\omega_3u^2\omega_1\omega_2 - 3\omega_3\omega_4^2\omega_1v^2 - 3\omega_3\omega_4\omega_1v^2\omega_2 + \omega_3\omega_4^2c_s^2 + \frac{5}{3}\omega_4\omega_1^2c_s^2 - \frac{2}{3}\omega_3\omega_1\omega_2c_s^2 + \\ & \frac{10}{3}\omega_3\omega_1c_s^2 - 3\omega_3\omega_4\omega_1^2v^2 + \frac{10}{3}\omega_3u^2\omega_1 - \frac{25}{3}\omega_4\omega_1c_s^2 - 5\omega_3\omega_4c_s^2 - \frac{2}{3}\omega_3\omega_1^2c_s^2 + \frac{1}{3}\omega_3\omega_4\omega_1 + \omega_3\omega_4\omega_2c_s^2 + \\ & 16\omega_3\omega_4\omega_1v^2 + \frac{5}{3}\omega_4\omega_1\omega_2c_s^2 + \frac{2}{3}u^2\omega_4^2\omega_1 + \frac{2}{3}\omega_3^2\omega_1 - \frac{2}{3}\omega_3^2\omega_1c_s^2 + \omega_3\omega_1\omega_2 - 5\omega_4\omega_1v^2 + \omega_3\omega_4v^2\omega_2 - \\ & 5\omega_3\omega_4v^2 + \omega_4\omega_1v^2\omega_2 + 16\omega_3\omega_4\omega_1c_s^2 + \frac{2}{3}u^2\omega_4\omega_1\omega_2 - \frac{2}{3}\omega_3u^2\omega_1^2, \end{aligned}$$

$$\begin{aligned} \gamma_{[x^2]}^{[\mu_1]} &= \omega_4v\omega_2^2 + \frac{3}{2}\omega_4\omega_1v\omega_2 + \frac{5}{2}\omega_3\omega_4v\omega_2 - \frac{1}{2}\omega_4\omega_1v\omega_2^2 - \frac{1}{2}\omega_3\omega_4v\omega_2^2 - 3\omega_4v\omega_2 - \frac{1}{2}\omega_3\omega_4\omega_1v\omega_2 - \frac{1}{2}\omega_4^2\omega_1v\omega_2 - \\ & \frac{1}{2}\omega_3^2\omega_4v\omega_2 + \omega_4^2v\omega_2 - \frac{1}{2}\omega_3\omega_4^2v\omega_2, \end{aligned}$$

$$\begin{aligned}
\gamma_{[x^2]}^{[\mu_3]} &= -\frac{5}{2}\omega_3\omega_4\omega_2 - \frac{3}{2}\omega_4\omega_1\omega_2 + \frac{1}{2}\omega_4\omega_1\omega_2^2 + \frac{1}{2}\omega_3\omega_4\omega_2^2 + \frac{1}{2}\omega_3\omega_4\omega_1\omega_2 - \omega_4^2\omega_2 + 3\omega_4\omega_2 + \frac{1}{2}\omega_3\omega_4^2\omega_2 - \omega_4\omega_2^2 + \\
&\quad \frac{1}{2}\omega_3^2\omega_4\omega_2 + \frac{1}{2}\omega_4^2\omega_1\omega_2, \\
\gamma_{[xy]}^{[\mu_1]} &= \frac{2}{3}u\omega_4\omega_1\omega_2 + \frac{2}{3}u\omega_4^2\omega_1 + \frac{1}{3}\omega_3u\omega_1\omega_2 - 2u\omega_4\omega_1 + \frac{1}{3}\omega_3u\omega_4\omega_1, \\
\gamma_{[y^2]}^{[\mu_1]} &= -\frac{1}{6}\omega_3\omega_1^2v\omega_2 + \frac{2}{3}\omega_4\omega_1v\omega_2 - \frac{1}{6}\omega_3^2\omega_1v\omega_2 - \frac{1}{3}\omega_3\omega_4\omega_1v\omega_2 + \frac{5}{6}\omega_3\omega_1v\omega_2 - \frac{1}{3}\omega_4\omega_1^2v\omega_2. \\
\gamma_{[y^2]}^{[\mu_3]} &= \frac{1}{6}\omega_3^2\omega_1\omega_2 + \frac{1}{6}\omega_3\omega_1^2\omega_2 + \frac{4}{3}\omega_4\omega_1 - \frac{2}{3}\omega_4\omega_1\omega_2 - \frac{1}{3}\omega_3\omega_1^2 - \frac{2}{3}\omega_4\omega_1^2 + \frac{5}{3}\omega_3\omega_1 + \frac{1}{2}\omega_3^2\omega_4\omega_1 + \frac{5}{6}\omega_3\omega_4\omega_1\omega_2 + \\
&\quad \frac{1}{2}\omega_3\omega_4^2\omega_1 - \frac{19}{6}\omega_3\omega_4\omega_1 - \frac{1}{3}\omega_3^2\omega_1 - \frac{5}{6}\omega_3\omega_1\omega_2 + \frac{1}{3}\omega_4\omega_1^2\omega_2 + \frac{1}{2}\omega_3\omega_4\omega_1^2,
\end{aligned}$$

## 5.5 EPDE for $\mu_4$

$$\begin{aligned}
&\gamma_{[1]}^{[\mu_1]}\mu_1 + \gamma_{[1]}^{[\mu_4]}\mu_4 + \gamma_{[t]}^{[\mu_1]}\delta_t\frac{\partial\mu_1}{\partial t} + \gamma_{[t]}^{[\mu_4]}\delta_t\frac{\partial\mu_4}{\partial t} + \gamma_{[x]}^{[\mu_1]}\delta_l\frac{\partial\mu_1}{\partial x} + \gamma_{[y]}^{[\mu_1]}\delta_l\frac{\partial\mu_1}{\partial y} + \gamma_{[t^2]}^{[\mu_1]}\delta_t^2\frac{\partial^2\mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_4]}\delta_t^2\frac{\partial^2\mu_4}{\partial t^2} \\
&\quad + \gamma_{[tx]}^{[\mu_1]}\delta_t\delta_l\frac{\partial^2\mu_1}{\partial t\partial x} + \gamma_{[ty]}^{[\mu_1]}\delta_t\delta_l\frac{\partial^2\mu_1}{\partial t\partial y} + \gamma_{[x^2]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_4]}\delta_l^2\frac{\partial^2\mu_4}{\partial x^2} + \gamma_{[y^2]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_4]}\delta_l^2\frac{\partial^2\mu_4}{\partial y^2} = 0,
\end{aligned}$$

where

$$\begin{aligned}
\gamma_{[1]}^{[\mu_1]} &= -\omega_3^2\omega_4\omega_1\omega_2c_s^2 - \omega_3u^2\omega_4\omega_1\omega_2^2 + 5\omega_3\omega_4\omega_1\omega_2c_s^2 - \omega_3^2u^2\omega_4\omega_1\omega_2 + 5\omega_3u^2\omega_4\omega_1\omega_2 - \omega_3u^2\omega_4\omega_1^2\omega_2 - \\
&\quad \omega_3\omega_4^2\omega_1\omega_2c_s^2 - \omega_3u^2\omega_4^2\omega_1\omega_2 - \omega_3\omega_4\omega_1\omega_2^2c_s^2 - \omega_3\omega_4\omega_1^2\omega_2c_s^2, \\
\gamma_{[1]}^{[\mu_4]} &= \omega_3\omega_4\omega_1\omega_2^2 - 5\omega_3\omega_4\omega_1\omega_2 + \omega_3\omega_4\omega_1^2\omega_2 + \omega_3\omega_4^2\omega_1\omega_2 + \omega_3^2\omega_4\omega_1\omega_2, \\
\gamma_{[t]}^{[\mu_1]} &= -\omega_3^2\omega_4\omega_2c_s^2 + 4\omega_3^2\omega_4\omega_1\omega_2c_s^2 + 4\omega_3u^2\omega_4\omega_1\omega_2^2 - \omega_3\omega_4^2\omega_1c_s^2 + 5\omega_3u^2\omega_4\omega_1 - \omega_3^2u^2\omega_4\omega_2 - 22\omega_3\omega_4\omega_1\omega_2c_s^2 + \\
&\quad 2\omega_4\omega_1\omega_2 - \omega_3u^2\omega_1\omega_2^2 - \omega_3\omega_4\omega_1^2c_s^2 - \omega_3^2\omega_4\omega_1c_s^2 + 4\omega_3^2u^2\omega_4\omega_1\omega_2 + 5\omega_3u^2\omega_1\omega_2 - \omega_3u^2\omega_4^2\omega_2 - \omega_3\omega_4^2\omega_2c_s^2 - \\
&\quad \omega_3u^2\omega_4\omega_1^2 + 5\omega_3\omega_1\omega_2c_s^2 - \omega_3\omega_4\omega_1\omega_2 - 22\omega_3u^2\omega_4\omega_1\omega_2 - \omega_3\omega_4\omega_2^2c_s^2 - \omega_3\omega_1\omega_2^2c_s^2 - \omega_3^2u^2\omega_4\omega_1 + \\
&\quad 4\omega_3u^2\omega_4\omega_1^2\omega_2 + 5\omega_3\omega_4\omega_2c_s^2 - \omega_3\omega_1^2\omega_2c_s^2 - \omega_3u^2\omega_1^2\omega_2 + 4\omega_3\omega_4^2\omega_1\omega_2c_s^2 + 5\omega_3u^2\omega_4\omega_2 + 4\omega_3u^2\omega_4^2\omega_1\omega_2 - \\
&\quad \omega_3u^2\omega_4\omega_2^2 + 4\omega_3\omega_4\omega_1\omega_2^2c_s^2 - \omega_3^2u^2\omega_1\omega_2 + 5\omega_3\omega_4\omega_1c_s^2 - \omega_4\omega_1^2\omega_2 - \omega_3u^2\omega_4^2\omega_1 - \omega_3^2\omega_1\omega_2c_s^2 + 4\omega_3\omega_4\omega_1^2\omega_2c_s^2, \\
\gamma_{[t]}^{[\mu_4]} &= \omega_3^2\omega_1\omega_2 - 4\omega_3\omega_4\omega_1\omega_2^2 + \omega_3\omega_1^2\omega_2 - 5\omega_3\omega_4\omega_2 - 5\omega_4\omega_1\omega_2 + \omega_4\omega_1\omega_2^2 + \omega_3\omega_4\omega_2^2 + \omega_3^2\omega_4\omega_1 + 23\omega_3\omega_4\omega_1\omega_2 + \\
&\quad \omega_3\omega_4^2\omega_1 - 4\omega_3\omega_4\omega_1^2\omega_2 - 5\omega_3\omega_4\omega_1 - 4\omega_3\omega_4^2\omega_1\omega_2 + \omega_3\omega_1\omega_2^2 - 5\omega_3\omega_1\omega_2 + \omega_4\omega_1^2\omega_2 + \omega_3\omega_4^2\omega_2 + \omega_3\omega_4\omega_1^2 + \\
&\quad \omega_3^2\omega_4\omega_2 + \omega_4^2\omega_1\omega_2 - 4\omega_3^2\omega_4\omega_1\omega_2, \\
\gamma_{[x]}^{[\mu_1]} &= u\omega_4\omega_1\omega_2^2 - 3u\omega_4\omega_1\omega_2 + u\omega_4^2\omega_1\omega_2, \\
\gamma_{[y]}^{[\mu_1]} &= 2\omega_4\omega_1v\omega_2 - \omega_3\omega_4\omega_1v\omega_2 - \omega_4\omega_1^2v\omega_2, \\
\gamma_{[t^2]}^{[\mu_1]} &= \frac{7}{2}\omega_3^2\omega_4\omega_2c_s^2 - 8\omega_3^2\omega_4\omega_1\omega_2c_s^2 - 8\omega_3u^2\omega_4\omega_1\omega_2^2 + \frac{7}{2}\omega_3\omega_4^2\omega_1c_s^2 - \frac{37}{2}\omega_3u^2\omega_4\omega_1 + 5\omega_3u^2\omega_2 + \frac{7}{2}\omega_3^2u^2\omega_4\omega_2 + \\
&\quad 2\omega_4\omega_1 + 47\omega_3\omega_4\omega_1\omega_2c_s^2 + 5\omega_3\omega_2c_s^2 - \omega_3^2u^2\omega_4 - \omega_3\omega_4\omega_2 - 8\omega_4\omega_1\omega_2 + \frac{7}{2}\omega_3u^2\omega_1\omega_2^2 + \frac{7}{2}\omega_3\omega_4\omega_1^2c_s^2 - \omega_4\omega_1^2 + \\
&\quad \frac{7}{2}\omega_3^2\omega_4\omega_1c_s^2 - \omega_3^2\omega_4c_s^2 - \omega_3^2u^2\omega_1 - 8\omega_3^2u^2\omega_4\omega_1\omega_2 - \frac{37}{2}\omega_3u^2\omega_1\omega_2 + \frac{7}{2}\omega_3u^2\omega_4^2\omega_2 + \frac{7}{2}\omega_3\omega_4^2\omega_2c_s^2 - \omega_3u^2\omega_2^2 + \\
&\quad \frac{7}{2}\omega_3u^2\omega_4\omega_1^2 + 2\omega_1\omega_2 - \omega_3\omega_4^2c_s^2 - \frac{37}{2}\omega_3\omega_1\omega_2c_s^2 + \frac{7}{2}\omega_3\omega_4\omega_1\omega_2 + 47\omega_3u^2\omega_4\omega_1\omega_2 + 5\omega_3\omega_1c_s^2 + \frac{7}{2}\omega_3\omega_4\omega_2^2c_s^2 - \\
&\quad \omega_3u^2\omega_4^2 + 5\omega_3u^2\omega_1 + \frac{7}{2}\omega_3\omega_1\omega_2^2c_s^2 + \frac{7}{2}\omega_3^2u^2\omega_4\omega_1 - \omega_1^2\omega_2 + 5\omega_3\omega_4c_s^2 - 8\omega_3u^2\omega_4\omega_1^2\omega_2 - \omega_3\omega_1^2c_s^2 + 2\omega_4\omega_2 - \\
&\quad \omega_3\omega_4\omega_1 - \frac{37}{2}\omega_3\omega_4\omega_2c_s^2 + \frac{7}{2}\omega_3\omega_1^2\omega_2c_s^2 + \frac{7}{2}\omega_3u^2\omega_1^2\omega_2 - 8\omega_3\omega_4^2\omega_1\omega_2c_s^2 - \frac{37}{2}\omega_3u^2\omega_4\omega_2 - \omega_3^2\omega_1c_s^2 - \\
&\quad 8\omega_3u^2\omega_4^2\omega_1\omega_2 - \omega_3\omega_1\omega_2 + \frac{7}{2}\omega_3u^2\omega_4\omega_2^2 - \omega_3\omega_2^2c_s^2 - 8\omega_3\omega_4\omega_1\omega_2^2c_s^2 + \frac{7}{2}\omega_3^2u^2\omega_1\omega_2 - \frac{37}{2}\omega_3\omega_4\omega_1c_s^2 + \\
&\quad \frac{7}{2}\omega_4\omega_1^2\omega_2 - \omega_3^2u^2\omega_2 + \frac{7}{2}\omega_3u^2\omega_4^2\omega_1 + \frac{7}{2}\omega_3^2\omega_1\omega_2c_s^2 + 5\omega_3u^2\omega_4 - 8\omega_3\omega_4\omega_1^2\omega_2c_s^2 - \omega_3^2\omega_2c_s^2 - \omega_3u^2\omega_1^2,
\end{aligned}$$

$$\begin{aligned}
\gamma_{[t^2]}^{[\mu_4]} &= \omega_3^2 \omega_2 - \frac{7}{2} \omega_3^2 \omega_1 \omega_2 + \omega_1 \omega_2^2 + 8 \omega_3 \omega_4 \omega_1 \omega_2^2 - \frac{7}{2} \omega_3 \omega_1^2 \omega_2 - 5 \omega_4 \omega_1 - 5 \omega_3 \omega_4 + \frac{39}{2} \omega_3 \omega_4 \omega_2 + \frac{39}{2} \omega_4 \omega_1 \omega_2 + \omega_3 \omega_1^2 + \\
&\quad \omega_3 \omega_4^2 + \omega_4 \omega_1^2 - \frac{7}{2} \omega_4 \omega_1 \omega_2^2 - \frac{7}{2} \omega_3 \omega_4 \omega_2^2 - 5 \omega_3 \omega_1 - \frac{7}{2} \omega_3^2 \omega_4 \omega_1 - 5 \omega_1 \omega_2 - \frac{101}{2} \omega_3 \omega_4 \omega_1 \omega_2 + \omega_4^2 \omega_2 - \frac{7}{2} \omega_3 \omega_4^2 \omega_1 + \\
&\quad 8 \omega_3 \omega_4 \omega_1^2 \omega_2 + \omega_1^2 \omega_2 - 5 \omega_4 \omega_2 + \frac{39}{2} \omega_3 \omega_4 \omega_1 + \omega_3 \omega_2^2 + 8 \omega_3 \omega_4^2 \omega_1 \omega_2 + \omega_3^2 \omega_1 - \frac{7}{2} \omega_3 \omega_1 \omega_2^2 + \frac{39}{2} \omega_3 \omega_1 \omega_2 + \omega_4^2 \omega_1 + \\
&\quad \omega_3^2 \omega_4 - \frac{7}{2} \omega_4 \omega_1^2 \omega_2 - \frac{7}{2} \omega_3 \omega_4^2 \omega_2 + \omega_4 \omega_2^2 - \frac{7}{2} \omega_3 \omega_4 \omega_1^2 - 5 \omega_3 \omega_2 - \frac{7}{2} \omega_3^2 \omega_4 \omega_2 - \frac{7}{2} \omega_4^2 \omega_1 \omega_2 + 8 \omega_3^2 \omega_4 \omega_1 \omega_2, \\
\gamma_{[tx]}^{[\mu_1]} &= -3u\omega_4\omega_1\omega_2^2 + 10u\omega_4\omega_1\omega_2 + u\omega_4^2\omega_1 - 3u\omega_4^2\omega_1\omega_2 - 3u\omega_4\omega_1 + u\omega_1\omega_2^2 - 3u\omega_1\omega_2, \\
\gamma_{[ty]}^{[\mu_1]} &= -7\omega_4\omega_1v\omega_2 - \omega_3\omega_4v\omega_2 - \omega_1^2v\omega_2 + 2\omega_4v\omega_2 + 3\omega_3\omega_4\omega_1v\omega_2 + 2\omega_1v\omega_2 - \omega_3\omega_1v\omega_2 + 3\omega_4\omega_1^2v\omega_2, \\
\gamma_{[x^2]}^{[\mu_1]} &= -\frac{1}{2}\omega_3^2\omega_4\omega_2c_s^2 - \frac{1}{2}\omega_3^2u^2\omega_4\omega_2 - \frac{1}{2}\omega_3\omega_4\omega_1\omega_2c_s^2 - \frac{1}{2}\omega_3u^2\omega_4^2\omega_2 - \frac{1}{2}\omega_3\omega_4^2\omega_2c_s^2 - \frac{1}{2}\omega_3u^2\omega_4\omega_1\omega_2 - \\
&\quad \frac{1}{2}\omega_3\omega_4\omega_2^2c_s^2 + \frac{5}{2}\omega_3\omega_4\omega_2c_s^2 + \frac{5}{2}\omega_3u^2\omega_4\omega_2 - \frac{1}{2}\omega_3u^2\omega_4\omega_2^2, \\
\gamma_{[x^2]}^{[\mu_4]} &= -\frac{5}{2}\omega_3\omega_4\omega_2 - \frac{3}{2}\omega_4\omega_1\omega_2 + \frac{1}{2}\omega_4\omega_1\omega_2^2 + \frac{1}{2}\omega_3\omega_4\omega_2^2 + \frac{1}{2}\omega_3\omega_4\omega_1\omega_2 - \omega_4^2\omega_2 + 3\omega_4\omega_2 + \frac{1}{2}\omega_3\omega_4^2\omega_2 - \omega_4\omega_2^2 + \\
&\quad \frac{1}{2}\omega_3^2\omega_4\omega_2 + \frac{1}{2}\omega_4^2\omega_1\omega_2, \\
\gamma_{[y^2]}^{[\mu_1]} &= -\frac{1}{2}\omega_4\omega_1^2v^2\omega_2 - \frac{1}{2}\omega_3\omega_4^2\omega_1c_s^2 + \frac{19}{6}\omega_3u^2\omega_4\omega_1 + \omega_4\omega_1^2v^2 + \frac{2}{3}u^2\omega_4\omega_1^2 - \frac{4}{3}\omega_3\omega_4\omega_1\omega_2c_s^2 - \frac{1}{3}u^2\omega_4\omega_1^2\omega_2 - \\
&\quad \frac{1}{2}\omega_3\omega_4\omega_1^2c_s^2 - \frac{4}{3}u^2\omega_4\omega_1 - \frac{1}{2}\omega_3^2\omega_4\omega_1c_s^2 + \frac{1}{3}\omega_3^2u^2\omega_1 + \frac{5}{6}\omega_3u^2\omega_1\omega_2 - \frac{5}{6}\omega_4\omega_1^2\omega_2c_s^2 - \frac{1}{2}\omega_3u^2\omega_4\omega_1^2 - \\
&\quad \frac{1}{2}\omega_3\omega_4\omega_1v^2\omega_2 + \frac{5}{3}\omega_4\omega_1^2c_s^2 + \frac{5}{6}\omega_3\omega_1\omega_2c_s^2 - \frac{5}{6}\omega_3u^2\omega_4\omega_1\omega_2 - \frac{5}{3}\omega_3\omega_1c_s^2 - \frac{5}{3}\omega_3u^2\omega_1 - \frac{10}{3}\omega_4\omega_1c_s^2 - \\
&\quad \frac{1}{2}\omega_3^2u^2\omega_4\omega_1 + \frac{1}{3}\omega_3\omega_1^2c_s^2 + \omega_3\omega_4\omega_1v^2 + \frac{5}{3}\omega_4\omega_1\omega_2c_s^2 - \frac{1}{6}\omega_3\omega_1^2\omega_2c_s^2 - \frac{1}{6}\omega_3u^2\omega_1^2\omega_2 + \frac{1}{3}\omega_3^2\omega_1c_s^2 - 2\omega_4\omega_1v^2 + \\
&\quad \omega_4\omega_1v^2\omega_2 - \frac{1}{6}\omega_3^2u^2\omega_1\omega_2 + \frac{25}{6}\omega_3\omega_4\omega_1c_s^2 + \frac{2}{3}u^2\omega_4\omega_1\omega_2 - \frac{1}{2}\omega_3u^2\omega_4^2\omega_1 - \frac{1}{6}\omega_3^2\omega_1\omega_2c_s^2 + \frac{1}{3}\omega_3u^2\omega_1^2. \\
\gamma_{[y^2]}^{[\mu_4]} &= \frac{1}{6}\omega_3^2\omega_1\omega_2 + \frac{1}{6}\omega_3\omega_1^2\omega_2 + \frac{4}{3}\omega_4\omega_1 - \frac{2}{3}\omega_4\omega_1\omega_2 - \frac{1}{3}\omega_3\omega_1^2 - \frac{2}{3}\omega_4\omega_1^2 + \frac{5}{3}\omega_3\omega_1 + \frac{1}{2}\omega_3^2\omega_4\omega_1 + \frac{5}{6}\omega_3\omega_4\omega_1\omega_2 + \\
&\quad \frac{1}{2}\omega_3\omega_4^2\omega_1 - \frac{19}{6}\omega_3\omega_4\omega_1 - \frac{1}{3}\omega_3^2\omega_1 - \frac{5}{6}\omega_3\omega_1\omega_2 + \frac{1}{3}\omega_4\omega_1^2\omega_2 + \frac{1}{2}\omega_3\omega_4\omega_1^2,
\end{aligned}$$

## 5.6 EPDE for $\mu_5$

$$\begin{aligned}
&\gamma_{[1]}^{[\mu_1]}\mu_1 + \gamma_{[1]}^{[\mu_5]}\mu_5 + \gamma_{[t]}^{[\mu_1]}\delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_5]}\delta_t \frac{\partial \mu_5}{\partial t} + \gamma_{[x]}^{[\mu_1]}\delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[y]}^{[\mu_1]}\delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[t^2]}^{[\mu_1]}\delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_5]}\delta_t^2 \frac{\partial^2 \mu_5}{\partial t^2} \\
&+ \gamma_{[tx]}^{[\mu_1]}\delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} + \gamma_{[ty]}^{[\mu_1]}\delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[x^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_5]}\delta_l^2 \frac{\partial^2 \mu_5}{\partial x^2} + \gamma_{[y^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_5]}\delta_l^2 \frac{\partial^2 \mu_5}{\partial y^2} = 0,
\end{aligned}$$

where

$$\begin{aligned}
\gamma_{[1]}^{[\mu_1]} &= -\omega_3^2\omega_4\omega_1\omega_2c_s^2 - \omega_3\omega_4\omega_1v^2\omega_2^2 + 5\omega_3\omega_4\omega_1\omega_2c_s^2 - \omega_3^2\omega_4\omega_1v^2\omega_2 + 5\omega_3\omega_4\omega_1v^2\omega_2 - \omega_3\omega_4\omega_1^2v^2\omega_2 - \\
&\quad \omega_3\omega_4^2\omega_1\omega_2c_s^2 - \omega_3\omega_4\omega_1\omega_2^2c_s^2 - \omega_3\omega_4\omega_1^2\omega_2c_s^2 - \omega_3\omega_4^2\omega_1v^2\omega_2, \\
\gamma_{[1]}^{[\mu_5]} &= \omega_3\omega_4\omega_1\omega_2^2 - 5\omega_3\omega_4\omega_1\omega_2 + \omega_3\omega_4\omega_1^2\omega_2 + \omega_3\omega_4^2\omega_1\omega_2 + \omega_3^2\omega_4\omega_1\omega_2, \\
\gamma_{[t]}^{[\mu_1]} &= -\omega_4\omega_1^2v^2\omega_2 - \omega_3^2\omega_4\omega_2c_s^2 - \omega_3^2\omega_4\omega_1v^2 - \frac{5}{3}\omega_4^2\omega_1\omega_2c_s^2 - \omega_3\omega_4^2v^2\omega_2 + 4\omega_3^2\omega_4\omega_1\omega_2c_s^2 - \frac{2}{3}\omega_3^2\omega_1\omega_2 - \frac{2}{3}u^2\omega_4^2\omega_1\omega_2 - \\
&\quad \omega_3\omega_4^2\omega_1c_s^2 + 4\omega_3\omega_4\omega_1v^2\omega_2^2 - \frac{2}{3}\omega_3\omega_1^2\omega_2 - 22\omega_3\omega_4\omega_1\omega_2c_s^2 - \frac{4}{3}\omega_4\omega_1\omega_2 - \frac{2}{3}u^2\omega_4\omega_1^2\omega_2 + \frac{2}{3}\omega_3u^2\omega_1\omega_2^2 - \omega_3\omega_4\omega_1^2c_s^2 - \\
&\quad \omega_3^2\omega_4\omega_1c_s^2 - \frac{10}{3}\omega_3u^2\omega_1\omega_2 - \frac{5}{3}\omega_4\omega_1^2\omega_2c_s^2 - \omega_3^2\omega_4v^2\omega_2 - \omega_3\omega_4^2\omega_1v^2 + 4\omega_3^2\omega_4\omega_1v^2\omega_2 - \omega_4^2\omega_1v^2\omega_2 - \omega_3\omega_4^2\omega_2c_s^2 - \\
&\quad 22\omega_3\omega_4\omega_1v^2\omega_2 - \frac{10}{3}\omega_3\omega_1\omega_2c_s^2 - \frac{1}{3}\omega_3\omega_4\omega_1\omega_2 - \omega_3\omega_4\omega_1^2v^2 - \frac{5}{3}\omega_4\omega_1\omega_2^2c_s^2 - \omega_3\omega_4\omega_2^2c_s^2 + \frac{2}{3}\omega_3\omega_1\omega_2^2c_s^2 + \\
&\quad 5\omega_3\omega_4\omega_2c_s^2 + 5\omega_3\omega_4\omega_1v^2 + \frac{25}{3}\omega_4\omega_1\omega_2c_s^2 + \frac{2}{3}\omega_3\omega_1^2\omega_2c_s^2 + \frac{2}{3}\omega_3u^2\omega_1^2\omega_2 + 4\omega_3\omega_4\omega_1^2v^2\omega_2 + 4\omega_3\omega_4^2\omega_1\omega_2c_s^2 - \\
&\quad \frac{2}{3}u^2\omega_4\omega_1\omega_2^2 - \omega_3\omega_1\omega_2^2 - \omega_3\omega_4v^2\omega_2^2 - \omega_4\omega_1v^2\omega_2^2 + \frac{10}{3}\omega_3\omega_1\omega_2 + 5\omega_3\omega_4v^2\omega_2 + 5\omega_4\omega_1v^2\omega_2 + 4\omega_3\omega_4\omega_1\omega_2^2c_s^2 + \\
&\quad \frac{2}{3}\omega_3^2u^2\omega_1\omega_2 + 5\omega_3\omega_4\omega_1c_s^2 + \frac{2}{3}\omega_4\omega_1^2\omega_2 + \frac{10}{3}u^2\omega_4\omega_1\omega_2 + \frac{2}{3}\omega_3^2\omega_1\omega_2c_s^2 + 4\omega_3\omega_4\omega_1^2\omega_2c_s^2 + 4\omega_3\omega_4^2\omega_1v^2\omega_2,
\end{aligned}$$

$$\begin{aligned}
\gamma_{[t]}^{[\mu_5]} &= \omega_3^2 \omega_1 \omega_2 - 4\omega_3 \omega_4 \omega_1 \omega_2^2 + \omega_3 \omega_1^2 \omega_2 - 5\omega_3 \omega_4 \omega_2 - 5\omega_4 \omega_1 \omega_2 + \omega_4 \omega_1 \omega_2^2 + \omega_3 \omega_4 \omega_2^2 + \omega_3^2 \omega_4 \omega_1 + 23\omega_3 \omega_4 \omega_1 \omega_2 + \\
&\quad \omega_3 \omega_4^2 \omega_1 - 4\omega_3 \omega_4 \omega_1^2 \omega_2 - 5\omega_3 \omega_4 \omega_1 - 4\omega_3 \omega_4^2 \omega_1 \omega_2 + \omega_3 \omega_1 \omega_2^2 - 5\omega_3 \omega_1 \omega_2 + \omega_4 \omega_1^2 \omega_2 + \omega_3 \omega_4^2 \omega_2 + \omega_3 \omega_4 \omega_1^2 + \\
&\quad \omega_3^2 \omega_4 \omega_2 + \omega_4^2 \omega_1 \omega_2 - 4\omega_3^2 \omega_4 \omega_1 \omega_2, \\
\gamma_{[x]}^{[\mu_1]} &= -\frac{2}{3} u \omega_4 \omega_1 \omega_2^2 + 2u \omega_4 \omega_1 \omega_2 - \frac{2}{3} u \omega_4^2 \omega_1 \omega_2 - \frac{1}{3} \omega_3 u \omega_4 \omega_1 \omega_2 - \frac{1}{3} \omega_3 u \omega_1 \omega_2^2, \\
\gamma_{[y]}^{[\mu_1]} &= \frac{1}{3} \omega_3 \omega_1^2 v \omega_2 - \frac{4}{3} \omega_4 \omega_1 v \omega_2 + \frac{1}{3} \omega_3^2 \omega_1 v \omega_2 + \frac{2}{3} \omega_3 \omega_4 \omega_1 v \omega_2 - \frac{5}{3} \omega_3 \omega_1 v \omega_2 + \frac{2}{3} \omega_4 \omega_1^2 v \omega_2, \\
\gamma_{[t^2]}^{[\mu_1]} &= \frac{7}{2} \omega_4 \omega_1^2 v^2 \omega_2 + \frac{7}{2} \omega_3^2 \omega_4 \omega_2 c_s^2 - \frac{5}{3} \omega_4^2 \omega_2 c_s^2 - \omega_4^2 \omega_1 v^2 + \frac{7}{2} \omega_3^2 \omega_4 \omega_1 v^2 + \frac{35}{6} \omega_4^2 \omega_1 \omega_2 c_s^2 + \frac{7}{2} \omega_3 \omega_4^2 v^2 \omega_2 - \\
&\quad \frac{2}{3} u^2 \omega_4^2 \omega_2 - 8\omega_3^2 \omega_4 \omega_1 \omega_2 c_s^2 - \omega_3^2 \omega_4 v^2 - \frac{2}{3} \omega_3^2 \omega_2 + \frac{7}{3} \omega_3^2 \omega_1 \omega_2 + \frac{7}{3} u^2 \omega_4^2 \omega_1 \omega_2 + \frac{7}{2} \omega_3 \omega_4^2 \omega_1 c_s^2 - 8\omega_3 \omega_4 \omega_1 v^2 \omega_2^2 - \\
&\quad \omega_1 \omega_2^2 + \frac{7}{3} \omega_3 \omega_1^2 \omega_2 - \frac{10}{3} \omega_3 u^2 \omega_2 - \frac{5}{3} \omega_4 \omega_2^2 c_s^2 - \omega_3 \omega_4^2 v^2 - \omega_4 \omega_1^2 v^2 - \frac{4}{3} \omega_4 \omega_1 - \frac{2}{3} u^2 \omega_4 \omega_1^2 + 47\omega_3 \omega_4 \omega_1 \omega_2 c_s^2 - \\
&\quad \frac{10}{3} \omega_3 \omega_2 c_s^2 - \frac{1}{3} \omega_3 \omega_4 \omega_2 + \frac{13}{3} \omega_4 \omega_1 \omega_2 + \frac{7}{3} u^2 \omega_4 \omega_1^2 \omega_2 - \frac{7}{3} \omega_3 u^2 \omega_1 \omega_2^2 - \frac{2}{3} \omega_3 \omega_1^2 + \frac{7}{2} \omega_3 \omega_4 \omega_1 c_s^2 - \frac{5}{3} \omega_4^2 \omega_1 c_s^2 + \\
&\quad \frac{2}{3} \omega_4 \omega_1^2 + \frac{10}{3} u^2 \omega_4 \omega_1 + \frac{7}{2} \omega_3^2 \omega_4 \omega_1 c_s^2 - \omega_3^2 \omega_4 c_s^2 + \frac{2}{3} \omega_3^2 u^2 \omega_1 + \frac{10}{3} \omega_3 \omega_1 + \frac{37}{3} \omega_3 u^2 \omega_1 \omega_2 + \frac{35}{6} \omega_4 \omega_1^2 \omega_2 c_s^2 + \\
&\quad \frac{7}{2} \omega_3 \omega_4 v^2 \omega_2 + \frac{7}{2} \omega_3 \omega_4 \omega_1 v^2 - \omega_4^2 v^2 \omega_2 - 8\omega_3^2 \omega_4 \omega_1 v^2 \omega_2 + \frac{7}{2} \omega_1^2 \omega_1 v^2 \omega_2 + \frac{7}{2} \omega_3 \omega_4^2 \omega_2 c_s^2 + \frac{2}{3} \omega_3 u^2 \omega_2^2 + \\
&\quad 47\omega_3 \omega_4 \omega_1 v^2 \omega_2 + 2\omega_1 \omega_2 - \omega_3 \omega_4^2 c_s^2 - \frac{5}{3} \omega_4 \omega_1^2 c_s^2 + \frac{37}{3} \omega_3 \omega_1 \omega_2 c_s^2 + \frac{7}{6} \omega_3 \omega_4 \omega_1 \omega_2 - \frac{10}{3} \omega_3 \omega_1 c_s^2 + \frac{7}{2} \omega_3 \omega_4 \omega_1^2 v^2 + \\
&\quad \frac{35}{6} \omega_4 \omega_1 \omega_2^2 c_s^2 + \frac{7}{2} \omega_3 \omega_4 \omega_2^2 c_s^2 - \frac{10}{3} \omega_3 u^2 \omega_1 - \frac{7}{3} \omega_3 \omega_1 \omega_2^2 c_s^2 + \frac{25}{3} \omega_4 \omega_1 c_s^2 + 5\omega_3 \omega_4 c_s^2 + \frac{2}{3} \omega_3 \omega_1^2 c_s^2 - \frac{2}{3} u^2 \omega_4 \omega_2^2 - \\
&\quad \frac{4}{3} \omega_4 \omega_2 - \frac{1}{3} \omega_3 \omega_4 \omega_1 - \frac{37}{3} \omega_3 \omega_4 \omega_2 c_s^2 - \frac{37}{2} \omega_3 \omega_4 \omega_1 v^2 - \omega_3 \omega_2^2 - \frac{185}{6} \omega_4 \omega_1 \omega_2 c_s^2 + 5\omega_4 v^2 \omega_2 - \frac{7}{3} \omega_3 \omega_1^2 \omega_2 c_s^2 - \\
&\quad \frac{2}{3} u^2 \omega_4^2 \omega_1 - \frac{2}{3} \omega_3^2 \omega_1 - \frac{7}{3} \omega_3 u^2 \omega_1^2 \omega_2 - 8\omega_3 \omega_4 \omega_1^2 v^2 \omega_2 - 8\omega_3 \omega_4^2 \omega_1 \omega_2 c_s^2 + \frac{7}{3} u^2 \omega_4 \omega_1 \omega_2^2 + \frac{7}{2} \omega_3 \omega_1 \omega_2^2 + \frac{2}{3} \omega_3^2 \omega_1 c_s^2 + \\
&\quad \frac{7}{2} \omega_3 \omega_4 v^2 \omega_2^2 + \frac{7}{2} \omega_4 \omega_1 v^2 \omega_2^2 - \frac{40}{3} \omega_3 \omega_1 \omega_2 + 5\omega_4 \omega_1 v^2 - \frac{37}{2} \omega_3 \omega_4 v^2 \omega_2 + 5\omega_3 \omega_4 v^2 - \frac{37}{2} \omega_4 \omega_1 v^2 \omega_2 + \frac{25}{3} \omega_4 \omega_2 c_s^2 + \\
&\quad \frac{2}{3} \omega_3 \omega_2^2 c_s^2 - 8\omega_3 \omega_4 \omega_1 \omega_2^2 c_s^2 - \frac{7}{3} \omega_3^2 u^2 \omega_1 \omega_2 - \frac{37}{2} \omega_3 \omega_4 \omega_1 c_s^2 - \frac{7}{3} \omega_4 \omega_1^2 \omega_2 - \frac{37}{3} u^2 \omega_4 \omega_1 \omega_2 + \frac{10}{3} u^2 \omega_4 \omega_2 + \\
&\quad \frac{2}{3} \omega_3 u^2 \omega_2 - \omega_4 v^2 \omega_2^2 + \frac{10}{3} \omega_3 \omega_2 - \frac{7}{3} \omega_3^2 \omega_1 \omega_2 c_s^2 - 8\omega_3 \omega_4 \omega_1^2 \omega_2 c_s^2 + \frac{2}{3} \omega_3^2 \omega_2 c_s^2 + \frac{2}{3} \omega_3 u^2 \omega_1^2 - 8\omega_3 \omega_4^2 \omega_1 v^2 \omega_2, \\
\gamma_{[t^2]}^{[\mu_5]} &= \omega_3^2 \omega_2 - \frac{7}{2} \omega_3^2 \omega_1 \omega_2 + \omega_1 \omega_2^2 + 8\omega_3 \omega_4 \omega_1 \omega_2^2 - \frac{7}{2} \omega_3 \omega_1^2 \omega_2 - 5\omega_4 \omega_1 - 5\omega_3 \omega_4 + \frac{39}{2} \omega_3 \omega_4 \omega_2 + \frac{39}{2} \omega_4 \omega_1 \omega_2 + \omega_3 \omega_1^2 + \\
&\quad \omega_3 \omega_4^2 + \omega_4 \omega_1^2 - \frac{7}{2} \omega_4 \omega_1 \omega_2^2 - \frac{7}{2} \omega_3 \omega_4 \omega_2^2 - 5\omega_3 \omega_1 - \frac{7}{2} \omega_3^2 \omega_4 \omega_1 - 5\omega_1 \omega_2 - \frac{101}{2} \omega_3 \omega_4 \omega_1 \omega_2 + \omega_4^2 \omega_2 - \frac{7}{2} \omega_3 \omega_4^2 \omega_1 + \\
&\quad 8\omega_3 \omega_4 \omega_1^2 \omega_2 + \omega_1^2 \omega_2 - 5\omega_4 \omega_2 + \frac{39}{2} \omega_3 \omega_4 \omega_1 + \omega_3 \omega_2^2 + 8\omega_3 \omega_4^2 \omega_1 \omega_2 + \omega_3^2 \omega_1 - \frac{7}{2} \omega_3 \omega_1 \omega_2^2 + \frac{39}{2} \omega_3 \omega_1 \omega_2 + \omega_4^2 \omega_1 + \\
&\quad \omega_3^2 \omega_4 - \frac{7}{2} \omega_4 \omega_1^2 \omega_2 - \frac{7}{2} \omega_3 \omega_4^2 \omega_2 + \omega_4 \omega_2^2 - \frac{7}{2} \omega_3 \omega_4 \omega_1^2 - 5\omega_3 \omega_2 - \frac{7}{2} \omega_3^2 \omega_4 \omega_2 - \frac{7}{2} \omega_4^2 \omega_1 \omega_2 + 8\omega_3^2 \omega_4 \omega_1 \omega_2, \\
\gamma_{[tx]}^{[\mu_1]} &= 2u \omega_4 \omega_1 \omega_2^2 - 7u \omega_4 \omega_1 \omega_2 - \frac{2}{3} u \omega_4^2 \omega_1 - \omega_3 u \omega_1 \omega_2 + 2u \omega_4^2 \omega_1 \omega_2 + 2u \omega_4 \omega_1 - u \omega_1 \omega_2^2 + 2u \omega_1 \omega_2 - \frac{1}{3} \omega_3 u \omega_4 \omega_1 + \\
&\quad \omega_3 u \omega_4 \omega_1 \omega_2 + \omega_3 u \omega_1 \omega_2^2, \\
\gamma_{[ty]}^{[\mu_1]} &= -\omega_3 \omega_1^2 v \omega_2 + \frac{14}{3} \omega_4 \omega_1 v \omega_2 + \frac{2}{3} \omega_3 \omega_4 v \omega_2 - \omega_3^2 \omega_1 v \omega_2 + \omega_1^2 v \omega_2 + \frac{1}{3} \omega_3^2 v \omega_2 - \frac{4}{3} \omega_4 v \omega_2 - 2\omega_3 \omega_4 \omega_1 v \omega_2 - \\
&\quad \frac{5}{3} \omega_3 v \omega_2 - 3\omega_1 v \omega_2 + \frac{16}{3} \omega_3 \omega_1 v \omega_2 - 2\omega_4 \omega_1^2 v \omega_2, \\
\gamma_{[x^2]}^{[\mu_1]} &= -\frac{1}{2} \omega_3^2 \omega_4 \omega_2 c_s^2 + \frac{5}{3} \omega_4^2 \omega_2 c_s^2 - \frac{5}{6} \omega_4^2 \omega_1 \omega_2 c_s^2 - \frac{1}{2} \omega_3 \omega_4^2 v^2 \omega_2 + \frac{2}{3} u^2 \omega_4^2 \omega_2 - \frac{1}{3} u^2 \omega_4^2 \omega_1 \omega_2 + \frac{5}{3} \omega_4 \omega_2^2 c_s^2 - \\
&\quad \frac{2}{3} \omega_3 \omega_4 \omega_1 \omega_2 c_s^2 - \frac{1}{6} \omega_3 u^2 \omega_1 \omega_2^2 - \frac{1}{2} \omega_3^2 \omega_4 v^2 \omega_2 + \omega_4^2 v^2 \omega_2 - \frac{1}{3} \omega_4^2 \omega_1 v^2 \omega_2 - \frac{1}{3} \omega_3 \omega_4^2 \omega_2 c_s^2 + \frac{1}{3} \omega_3 u^2 \omega_2^2 - \\
&\quad \frac{1}{2} \omega_3 \omega_4 \omega_1 v^2 \omega_2 - \frac{1}{6} \omega_3 u^2 \omega_4 \omega_1 \omega_2 - \frac{5}{6} \omega_4 \omega_1 \omega_2^2 c_s^2 - \frac{1}{2} \omega_3 \omega_4 \omega_2^2 c_s^2 - \frac{1}{6} \omega_3 \omega_1 \omega_2^2 c_s^2 + \frac{2}{3} u^2 \omega_4 \omega_2^2 + \frac{17}{6} \omega_3 \omega_4 \omega_2 c_s^2 + \\
&\quad \frac{5}{2} \omega_4 \omega_1 \omega_2 c_s^2 - 3\omega_4 v^2 \omega_2 - \frac{1}{3} u^2 \omega_4 \omega_1 \omega_2^2 + \frac{1}{3} \omega_3 u^2 \omega_4 \omega_2 - \frac{1}{2} \omega_3 \omega_4 v^2 \omega_2^2 - \frac{1}{3} \omega_4 \omega_1 v^2 \omega_2^2 + \frac{5}{2} \omega_3 \omega_4 v^2 \omega_2 + \\
&\quad \frac{3}{2} \omega_4 \omega_1 v^2 \omega_2 - 5\omega_4 \omega_2 c_s^2 + \frac{1}{3} \omega_3 \omega_2^2 c_s^2 + u^2 \omega_4 \omega_1 \omega_2 - 2u^2 \omega_4 \omega_2 + \omega_4 v^2 \omega_2^2, \\
\gamma_{[x^2]}^{[\mu_5]} &= -\frac{5}{2} \omega_3 \omega_4 \omega_2 - \frac{3}{2} \omega_4 \omega_1 \omega_2 + \frac{1}{2} \omega_4 \omega_1 \omega_2^2 + \frac{1}{2} \omega_3 \omega_4 \omega_2^2 + \frac{1}{2} \omega_3 \omega_4 \omega_1 \omega_2 - \omega_4^2 \omega_2 + 3\omega_4 \omega_2 + \frac{1}{2} \omega_3 \omega_4^2 \omega_2 - \omega_4 \omega_2^2 + \\
&\quad \frac{1}{2} \omega_3^2 \omega_4 \omega_2 + \frac{1}{2} \omega_4^2 \omega_1 \omega_2, \\
\gamma_{[y^2]}^{[\mu_1]} &= -\frac{1}{2} \omega_3^2 \omega_4 \omega_1 v^2 - \frac{1}{2} \omega_3 \omega_4^2 \omega_1 c_s^2 - \frac{1}{2} \omega_3 \omega_4 \omega_1 \omega_2 c_s^2 - \frac{1}{2} \omega_3 \omega_4 \omega_1^2 c_s^2 - \frac{1}{2} \omega_3^2 \omega_4 \omega_1 c_s^2 - \frac{1}{2} \omega_3 \omega_4^2 \omega_1 v^2 - \\
&\quad \frac{1}{2} \omega_3 \omega_4 \omega_1 v^2 \omega_2 - \frac{1}{2} \omega_3 \omega_4 \omega_1^2 v^2 + \frac{5}{2} \omega_3 \omega_4 \omega_1 v^2 + \frac{5}{2} \omega_3 \omega_4 \omega_1 c_s^2, \\
\gamma_{[y^2]}^{[\mu_5]} &= \frac{1}{6} \omega_3^2 \omega_1 \omega_2 + \frac{1}{6} \omega_3 \omega_1^2 \omega_2 + \frac{4}{3} \omega_4 \omega_1 - \frac{2}{3} \omega_4 \omega_1 \omega_2 - \frac{1}{3} \omega_3 \omega_1^2 - \frac{2}{3} \omega_4 \omega_1^2 + \frac{5}{3} \omega_3 \omega_1 + \frac{1}{2} \omega_3^2 \omega_4 \omega_1 + \frac{5}{6} \omega_3 \omega_4 \omega_1 \omega_2 + \\
&\quad \frac{1}{2} \omega_3 \omega_4^2 \omega_1 - \frac{19}{6} \omega_3 \omega_4 \omega_1 - \frac{1}{3} \omega_3^2 \omega_1 - \frac{5}{6} \omega_3 \omega_1 \omega_2 + \frac{1}{3} \omega_4 \omega_1^2 \omega_2 + \frac{1}{2} \omega_3 \omega_4 \omega_1^2,
\end{aligned}$$

## 6 MRT 4: with ortogonalization and relaxation of $m_{00}$ , $m_{10}$ , $m_{01}$ , $m_{20} + m_{02}$ , $m_{20} - m_{02}$

### 6.1 Definitions

Matrix  $\mathbf{A} = \mathbf{M}^{-1}\mathbf{SM}$ :

$$\begin{aligned}
\mathbf{A}_{1,1} &= \frac{4}{5}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{1,2} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{1,3} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{1,4} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{1,5} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{2,1} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{2,2} &= \frac{1}{4}\omega_4 + \frac{1}{2}\omega_1 + \frac{1}{20}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{2,3} &= -\frac{1}{4}\omega_4 + \frac{1}{20}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{2,4} &= \frac{1}{4}\omega_4 - \frac{1}{2}\omega_1 + \frac{1}{20}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{2,5} &= -\frac{1}{4}\omega_4 + \frac{1}{20}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{3,1} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{3,2} &= -\frac{1}{4}\omega_4 + \frac{1}{20}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{3,3} &= \frac{1}{4}\omega_4 + \frac{1}{20}\omega_3 + \frac{1}{5}\omega_0 + \frac{1}{2}\omega_2, \\
\mathbf{A}_{3,4} &= -\frac{1}{4}\omega_4 + \frac{1}{20}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{3,5} &= \frac{1}{4}\omega_4 + \frac{1}{20}\omega_3 + \frac{1}{5}\omega_0 - \frac{1}{2}\omega_2, \\
\mathbf{A}_{4,1} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{4,2} &= \frac{1}{4}\omega_4 - \frac{1}{2}\omega_1 + \frac{1}{20}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{4,3} &= -\frac{1}{4}\omega_4 + \frac{1}{20}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{4,4} &= \frac{1}{4}\omega_4 + \frac{1}{2}\omega_1 + \frac{1}{20}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{4,5} &= -\frac{1}{4}\omega_4 + \frac{1}{20}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{5,1} &= -\frac{1}{5}\omega_3 + \frac{1}{5}\omega_0, \\
\mathbf{A}_{5,2} &= -\frac{1}{4}\omega_4 + \frac{1}{20}\omega_3 + \frac{1}{5}\omega_0,
\end{aligned}$$

$$\begin{aligned}\mathbf{A}_{5,3} &= \frac{1}{4}\omega_4 + \frac{1}{20}\omega_3 + \frac{1}{5}\omega_0 - \frac{1}{2}\omega_2, \\ \mathbf{A}_{5,4} &= -\frac{1}{4}\omega_4 + \frac{1}{20}\omega_3 + \frac{1}{5}\omega_0, \\ \mathbf{A}_{5,5} &= \frac{1}{4}\omega_4 + \frac{1}{20}\omega_3 + \frac{1}{5}\omega_0 + \frac{1}{2}\omega_2.\end{aligned}$$

where

$$\mathbf{S} = \text{diag}(\omega_0, \omega_1, \omega_2, \omega_3, \omega_4)$$

and

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ -2^2 & 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & 1 & -1 \end{pmatrix}$$

Matrix  $\mathbf{B}$ :

$$\mathbf{B} = \begin{pmatrix} 0 & -1 + \omega_3 & -1 + \omega_3 & -1 + \omega_3 & -1 + \omega_3 \\ -1 + \frac{1}{4}\omega_4 + \frac{1}{2}\omega_1 + \frac{1}{4}\omega_3 & 0 & -1 + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_1 & -1 + \omega_1 & -1 + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_1 \\ -1 + \frac{1}{4}\omega_4 + \frac{1}{4}\omega_3 + \frac{1}{2}\omega_2 & -1 + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_2 & 0 & -1 + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_2 & -1 + \omega_2 \\ -1 + \frac{1}{4}\omega_4 + \frac{1}{2}\omega_1 + \frac{1}{4}\omega_3 & -1 + \omega_1 & -1 + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_1 & 0 & -1 + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_1 \\ -1 + \frac{1}{4}\omega_4 + \frac{1}{4}\omega_3 + \frac{1}{2}\omega_2 & -1 + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_2 & -1 + \omega_2 & -1 + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_2 & 0 \end{pmatrix}.$$

## 6.2 EPDE for $\mu_1$

$$\begin{aligned}\gamma_{[t]}^{[\mu_1]} \delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[x]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[y]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[t^2]}^{[\mu_1]} \delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[tx]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} \\ + \gamma_{[ty]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[x^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[y^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} = 0,\end{aligned}$$

where

$$\gamma_{[t]}^{[\mu_1]} = -\omega_3 \omega_4 \omega_1 \omega_2,$$

$$\gamma_{[x]}^{[\mu_1]} = -\omega_3 \omega_4 \omega_1 u \omega_2,$$

$$\gamma_{[y]}^{[\mu_1]} = -v \omega_3 \omega_4 \omega_1 \omega_2,$$

$$\gamma_{[t^2]}^{[\mu_1]} = -\omega_3 \omega_4 \omega_2 - \omega_3 \omega_1 \omega_2 + \frac{7}{2} \omega_3 \omega_4 \omega_1 \omega_2 - \omega_3 \omega_4 \omega_1 - \omega_4 \omega_1 \omega_2,$$

$$\gamma_{[tx]}^{[\mu_1]} = -\omega_3 \omega_4 \omega_1 u - \omega_3 \omega_1 u \omega_2 - \omega_4 \omega_1 u \omega_2 + 3 \omega_3 \omega_4 \omega_1 u \omega_2,$$

$$\gamma_{[ty]}^{[\mu_1]} = -v \omega_3 \omega_4 \omega_2 + 3 v \omega_3 \omega_4 \omega_1 \omega_2 - v \omega_3 \omega_1 \omega_2 - v \omega_4 \omega_1 \omega_2,$$

$$\gamma_{[x^2]}^{[\mu_1]} = -\frac{1}{2} \omega_3 \omega_4 \omega_1 u^2 \omega_2 - \frac{1}{2} \omega_3 \omega_4 c_s^2 \omega_1 \omega_2 + \omega_3 \omega_4 u^2 \omega_2 + \omega_3 \omega_4 c_s^2 \omega_2,$$

$$\gamma_{[y^2]}^{[\mu_1]} = -\frac{1}{2} v^2 \omega_3 \omega_4 \omega_1 \omega_2 - \frac{1}{2} \omega_3 \omega_4 c_s^2 \omega_1 \omega_2 + \omega_3 \omega_4 c_s^2 \omega_1 + v^2 \omega_3 \omega_4 \omega_1,$$

### 6.3 EPDE for $\mu_2$

$$\begin{aligned} & \gamma_{[1]}^{[\mu_1]} \mu_1 + \gamma_{[1]}^{[\mu_2]} \mu_2 + \gamma_{[t]}^{[\mu_1]} \delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_2]} \delta_t \frac{\partial \mu_2}{\partial t} + \gamma_{[x]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[t^2]}^{[\mu_1]} \delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_2]} \delta_t^2 \frac{\partial^2 \mu_2}{\partial t^2} \\ & + \gamma_{[tx]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} + \gamma_{[x^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_2]} \delta_l^2 \frac{\partial^2 \mu_2}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x \partial y} + \gamma_{[y^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_2]} \delta_l^2 \frac{\partial^2 \mu_2}{\partial y^2} = 0, \end{aligned}$$

where

$$\gamma_{[1]}^{[\mu_1]} = -\omega_3 \omega_4 \omega_1^2 u \omega_2 - \omega_3 \omega_4^2 \omega_1 u \omega_2 - \omega_3 \omega_4 \omega_1 u \omega_2^2 + 5\omega_3 \omega_4 \omega_1 u \omega_2 - \omega_3^2 \omega_4 \omega_1 u \omega_2,$$

$$\gamma_{[1]}^{[\mu_2]} = \omega_3 \omega_4 \omega_1 \omega_2^2 - 5\omega_3 \omega_4 \omega_1 \omega_2 + \omega_3 \omega_4 \omega_1^2 \omega_2 + \omega_3 \omega_4^2 \omega_1 \omega_2 + \omega_3^2 \omega_4 \omega_1 \omega_2,$$

$$\begin{aligned} \gamma_{[t]}^{[\mu_1]} &= -\omega_3^2 \omega_1 u \omega_2 + 4\omega_3 \omega_4 \omega_1^2 u \omega_2 + 5\omega_3 \omega_4 \omega_1 u - \omega_4^2 \omega_1 u \omega_2 + 4\omega_3 \omega_4^2 \omega_1 u \omega_2 - \omega_3 \omega_1^2 u \omega_2 - \omega_4 \omega_1^2 u \omega_2 - \omega_3^2 \omega_4 \omega_1 u + \\ & 5\omega_3 \omega_1 u \omega_2 + 5\omega_4 \omega_1 u \omega_2 + 4\omega_3 \omega_4 \omega_1 u \omega_2^2 - \omega_3 \omega_4^2 \omega_1 u - \omega_4 \omega_1 u \omega_2^2 - 22\omega_3 \omega_4 \omega_1 u \omega_2 - \omega_3 \omega_1 u \omega_2^2 + \\ & 4\omega_3^2 \omega_4 \omega_1 u \omega_2 - \omega_3 \omega_4 \omega_1^2 u, \end{aligned}$$

$$\begin{aligned} \gamma_{[t]}^{[\mu_2]} &= \omega_3^2 \omega_4 \omega_1 + \omega_4^2 \omega_1 \omega_2 - 4\omega_3 \omega_4 \omega_1 \omega_2^2 + \omega_3 \omega_4^2 \omega_1 + \omega_4 \omega_1^2 \omega_2 - 5\omega_3 \omega_4 \omega_2 + \omega_3 \omega_1 \omega_2^2 + \omega_3 \omega_4 \omega_2^2 - 5\omega_3 \omega_1 \omega_2 + \\ & 23\omega_3 \omega_4 \omega_1 \omega_2 + \omega_3^2 \omega_1 \omega_2 - 4\omega_3 \omega_4 \omega_1^2 \omega_2 - 4\omega_3 \omega_4^2 \omega_1 \omega_2 + \omega_4 \omega_1 \omega_2^2 + \omega_3 \omega_1^2 \omega_2 + \omega_3 \omega_4 \omega_1^2 - 5\omega_3 \omega_4 \omega_1 - 5\omega_4 \omega_1 \omega_2 + \\ & \omega_3 \omega_4^2 \omega_2 + \omega_3^2 \omega_4 \omega_2 - 4\omega_3^2 \omega_4 \omega_1 \omega_2, \end{aligned}$$

$$\begin{aligned} \gamma_{[x]}^{[\mu_1]} &= \omega_3^2 \omega_4 u^2 \omega_2 + \omega_3^2 \omega_4 c_s^2 \omega_2 + \omega_3 \omega_4 \omega_1 u^2 \omega_2 + \omega_3 \omega_4 c_s^2 \omega_1 \omega_2 + \omega_3 \omega_4^2 u^2 \omega_2 + \omega_3 \omega_4^2 c_s^2 \omega_2 - 5\omega_3 \omega_4 u^2 \omega_2 - \\ & 5\omega_3 \omega_4 c_s^2 \omega_2 + \omega_3 \omega_4 c_s^2 \omega_2^2 + \omega_3 \omega_4 u^2 \omega_2^2, \end{aligned}$$

$$\begin{aligned} \gamma_{[t^2]}^{[\mu_1]} &= \frac{7}{2} \omega_3^2 \omega_1 u \omega_2 - 8\omega_3 \omega_4 \omega_1^2 u \omega_2 - \frac{37}{2} \omega_3 \omega_4 \omega_1 u + 5\omega_1 u \omega_2 + \frac{7}{2} \omega_4^2 \omega_1 u \omega_2 + 5\omega_3 \omega_1 u - \omega_4 \omega_1^2 u - 8\omega_3 \omega_4^2 \omega_1 u \omega_2 - \omega_1 u \omega_2^2 + \\ & \frac{7}{2} \omega_3 \omega_1^2 u \omega_2 + \frac{7}{2} \omega_4 \omega_1^2 u \omega_2 - \omega_4^2 \omega_1 u + \frac{7}{2} \omega_3^2 \omega_4 \omega_1 u - \frac{37}{2} \omega_3 \omega_1 u \omega_2 + 5\omega_4 \omega_1 u - \frac{37}{2} \omega_4 \omega_1 u \omega_2 - 8\omega_3 \omega_4 \omega_1 u \omega_2^2 + \\ & \frac{7}{2} \omega_3 \omega_4^2 \omega_1 u + \frac{7}{2} \omega_4 \omega_1 u \omega_2^2 - \omega_3 \omega_1^2 u + 47\omega_3 \omega_4 \omega_1 u \omega_2 + \frac{7}{2} \omega_3 \omega_1 u \omega_2^2 - \omega_1^2 u \omega_2 - 8\omega_3^2 \omega_4 \omega_1 u \omega_2 - \omega_3^2 \omega_1 u + \frac{7}{2} \omega_3 \omega_4 \omega_1^2 u, \end{aligned}$$

$$\begin{aligned} \gamma_{[t^2]}^{[\mu_2]} &= -\frac{7}{2} \omega_3^2 \omega_4 \omega_1 - 5\omega_1 \omega_2 + \omega_4 \omega_2^2 - \frac{7}{2} \omega_4^2 \omega_1 \omega_2 + 8\omega_3 \omega_4 \omega_1 \omega_2^2 + \omega_3 \omega_4^2 - \frac{7}{2} \omega_3 \omega_4^2 \omega_1 + \omega_3 \omega_1^2 - \frac{7}{2} \omega_4 \omega_1^2 \omega_2 + \\ & \frac{39}{2} \omega_3 \omega_4 \omega_2 - \frac{7}{2} \omega_3 \omega_1 \omega_2^2 - \frac{7}{2} \omega_3 \omega_4 \omega_2^2 + \frac{39}{2} \omega_3 \omega_1 \omega_2 + \omega_3^2 \omega_2 - 5\omega_3 \omega_1 + \omega_1 \omega_2^2 - 5\omega_4 \omega_2 - 5\omega_3 \omega_4 - \frac{101}{2} \omega_3 \omega_4 \omega_1 \omega_2 + \\ & \omega_4^2 \omega_1 - \frac{7}{2} \omega_3^2 \omega_1 \omega_2 + \omega_4^2 \omega_2 + 8\omega_3 \omega_4 \omega_1^2 \omega_2 + \omega_3^2 \omega_4 - 5\omega_4 \omega_1 + 8\omega_3 \omega_4^2 \omega_1 \omega_2 - \frac{7}{2} \omega_4 \omega_1 \omega_2^2 - 5\omega_3 \omega_2 - \frac{7}{2} \omega_3 \omega_1^2 \omega_2 + \\ & \omega_3^2 \omega_1 - \frac{7}{2} \omega_3 \omega_4 \omega_1^2 + \frac{39}{2} \omega_3 \omega_4 \omega_1 + \frac{39}{2} \omega_4 \omega_1 \omega_2 + \omega_3 \omega_2^2 - \frac{7}{2} \omega_3 \omega_4^2 \omega_2 + \omega_4 \omega_1^2 - \frac{7}{2} \omega_3^2 \omega_4 \omega_2 + \omega_1^2 \omega_2 + 8\omega_3^2 \omega_4 \omega_1 \omega_2, \end{aligned}$$

$$\begin{aligned} \gamma_{[tx]}^{[\mu_1]} &= \omega_3 c_s^2 \omega_1 \omega_2 - 3\omega_3^2 \omega_4 u^2 \omega_2 + \frac{1}{2} \omega_4 \omega_2^2 + \frac{1}{2} v^2 \omega_3^2 \omega_2 - \frac{5}{2} \omega_4 u^2 \omega_2 - 3\omega_3^2 \omega_4 c_s^2 \omega_2 + \frac{5}{2} v^2 \omega_4 \omega_2 + \frac{1}{2} \omega_3 \omega_4 \omega_2 - \\ & 3\omega_3 \omega_4 \omega_1 u^2 \omega_2 - 5\omega_3 \omega_4 u^2 + \omega_3 \omega_4^2 c_s^2 - \frac{1}{2} v^2 \omega_4 \omega_2^2 + \omega_3^2 c_s^2 \omega_2 + \frac{1}{2} \omega_3 \omega_1 \omega_2 - 3\omega_3 \omega_4 c_s^2 \omega_1 \omega_2 + \frac{1}{2} \omega_4 \omega_1 u^2 \omega_2 + \\ & \omega_3 \omega_4 c_s^2 \omega_1 - 3\omega_3 \omega_4^2 u^2 \omega_2 + \frac{1}{2} \omega_4 u^2 \omega_2^2 - 3\omega_3 \omega_4^2 c_s^2 \omega_2 - 2\omega_4 \omega_2 + \omega_3^2 \omega_4 c_s^2 + \omega_3 \omega_4 \omega_1 u^2 - \frac{1}{2} v^2 \omega_4 \omega_1 \omega_2 + \\ & \frac{1}{2} \omega_3^2 u^2 \omega_2 + \frac{1}{2} \omega_3 \omega_1 u^2 \omega_2 + \frac{1}{2} \omega_4^2 \omega_2 + \frac{1}{2} v^2 \omega_3 \omega_2^2 - 5\omega_3 c_s^2 \omega_2 + 16\omega_3 \omega_4 u^2 \omega_2 - 5\omega_3 \omega_4 c_s^2 + 16\omega_3 \omega_4 c_s^2 \omega_2 - \\ & \frac{5}{2} \omega_3 u^2 \omega_2 + \frac{1}{2} \omega_3 u^2 \omega_2^2 - \frac{1}{2} v^2 \omega_4^2 \omega_2 + \frac{1}{2} \omega_4 \omega_1 \omega_2 - \frac{1}{2} \omega_3 \omega_2^2 - 3\omega_3 \omega_4 c_s^2 \omega_2^2 + \omega_3 \omega_4^2 u^2 + \frac{1}{2} \omega_4^2 u^2 \omega_2 - 3\omega_3 \omega_4 u^2 \omega_2^2 + \\ & \frac{1}{2} v^2 \omega_3 \omega_1 \omega_2 + \omega_3^2 \omega_4 u^2 + \omega_3 c_s^2 \omega_2^2 - \frac{5}{2} v^2 \omega_3 \omega_2, \end{aligned}$$

$$\gamma_{[x^2]}^{[\mu_1]} = -\frac{1}{4} \omega_3^2 \omega_1 u \omega_2 + \frac{5}{4} \omega_3 \omega_1 u \omega_2 + \frac{1}{4} \omega_4 \omega_1 u \omega_2 - \frac{1}{4} \omega_3 \omega_4 \omega_1 u \omega_2 - \frac{1}{2} \omega_3 \omega_1 u \omega_2^2,$$

$$\begin{aligned} \gamma_{[x^2]}^{[\mu_2]} &= -3\omega_3 \omega_4 \omega_2 + \frac{1}{2} \omega_3 \omega_1 \omega_2^2 + \frac{1}{2} \omega_3 \omega_4 \omega_2^2 - \frac{5}{4} \omega_3 \omega_1 \omega_2 - \frac{1}{2} \omega_3^2 \omega_2 + \frac{1}{2} \omega_4 \omega_2 + \frac{3}{4} \omega_3 \omega_4 \omega_1 \omega_2 + \frac{1}{4} \omega_3^2 \omega_1 \omega_2 + \frac{5}{2} \omega_3 \omega_2 - \\ & \frac{1}{4} \omega_4 \omega_1 \omega_2 - \omega_3 \omega_2^2 + \frac{1}{2} \omega_3 \omega_4^2 \omega_2 + \frac{1}{2} \omega_3^2 \omega_4 \omega_2, \end{aligned}$$

$$\gamma_{[xy]}^{[\mu_1]} = \frac{1}{2} v \omega_3 \omega_4 \omega_2 - \frac{5}{2} v \omega_3 \omega_2 + v \omega_3 \omega_1 \omega_2 + \frac{1}{2} v \omega_3^2 \omega_2 + \frac{1}{2} v \omega_4 \omega_2,$$

$$\gamma_{[y^2]}^{[\mu_1]} = -\frac{1}{4}\omega_3^2\omega_1u\omega_2 + 3\omega_3\omega_4\omega_1u - \frac{5}{2}\omega_3\omega_1u - \frac{1}{2}\omega_3\omega_1^2u\omega_2 - \frac{1}{2}\omega_3^2\omega_4\omega_1u + \frac{5}{4}\omega_3\omega_1u\omega_2 - \frac{1}{2}\omega_4\omega_1u + \frac{1}{4}\omega_4\omega_1u\omega_2 - \frac{1}{2}\omega_3\omega_4^2\omega_1u + \omega_3\omega_1^2u - \frac{3}{4}\omega_3\omega_4\omega_1u\omega_2 + \frac{1}{2}\omega_3^2\omega_1u - \frac{1}{2}\omega_3\omega_4\omega_1^2u.$$

$$\gamma_{[y^2]}^{[\mu_2]} = \frac{1}{2}\omega_3^2\omega_4\omega_1 + \frac{1}{2}\omega_3\omega_4^2\omega_1 - \omega_3\omega_1^2 - \frac{5}{4}\omega_3\omega_1\omega_2 + \frac{5}{2}\omega_3\omega_1 + \frac{3}{4}\omega_3\omega_4\omega_1\omega_2 + \frac{1}{4}\omega_3^2\omega_1\omega_2 + \frac{1}{2}\omega_4\omega_1 + \frac{1}{2}\omega_3\omega_1^2\omega_2 - \frac{1}{2}\omega_3^2\omega_1 + \frac{1}{2}\omega_3\omega_4\omega_1^2 - 3\omega_3\omega_4\omega_1 - \frac{1}{4}\omega_4\omega_1\omega_2,$$

## 6.4 EPDE for $\mu_3$

$$\begin{aligned} & \gamma_{[1]}^{[\mu_1]}\mu_1 + \gamma_{[1]}^{[\mu_3]}\mu_3 + \gamma_{[t]}^{[\mu_1]}\delta_t\frac{\partial\mu_1}{\partial t} + \gamma_{[t]}^{[\mu_3]}\delta_t\frac{\partial\mu_3}{\partial t} + \gamma_{[y]}^{[\mu_1]}\delta_l\frac{\partial\mu_1}{\partial y} + \gamma_{[t^2]}^{[\mu_1]}\delta_t^2\frac{\partial^2\mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_3]}\delta_t^2\frac{\partial^2\mu_3}{\partial t^2} \\ & + \gamma_{[ty]}^{[\mu_1]}\delta_t\delta_l\frac{\partial^2\mu_1}{\partial t\partial y} + \gamma_{[x^2]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_3]}\delta_l^2\frac{\partial^2\mu_3}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial x\partial y} + \gamma_{[y^2]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_3]}\delta_l^2\frac{\partial^2\mu_3}{\partial y^2} = 0, \end{aligned}$$

where

$$\gamma_{[1]}^{[\mu_1]} = 5v\omega_3\omega_4\omega_1\omega_2 - v\omega_3\omega_4\omega_1\omega_2^2 - v\omega_3^2\omega_4\omega_1\omega_2 - v\omega_3\omega_4^2\omega_1\omega_2 - v\omega_3\omega_4\omega_1^2\omega_2,$$

$$\gamma_{[1]}^{[\mu_3]} = \omega_3\omega_4\omega_1\omega_2^2 - 5\omega_3\omega_4\omega_1\omega_2 + \omega_3\omega_4\omega_1^2\omega_2 + \omega_3\omega_4^2\omega_1\omega_2 + \omega_3^2\omega_4\omega_1\omega_2,$$

$$\gamma_{[t]}^{[\mu_1]} = 5v\omega_3\omega_4\omega_2 - v\omega_3\omega_1\omega_2^2 - v\omega_4\omega_1^2\omega_2 - 22v\omega_3\omega_4\omega_1\omega_2 - v\omega_4^2\omega_1\omega_2 + 4v\omega_3\omega_4\omega_1\omega_2^2 - v\omega_3\omega_4\omega_2^2 + 5v\omega_3\omega_1\omega_2 + 4v\omega_3^2\omega_4\omega_1\omega_2 - v\omega_3\omega_1^2\omega_2 - v\omega_4\omega_1\omega_2^2 - v\omega_3^2\omega_1\omega_2 - v\omega_3^2\omega_4\omega_2 + 4v\omega_3\omega_4^2\omega_1\omega_2 - v\omega_3\omega_4^2\omega_2 + 5v\omega_4\omega_1\omega_2 + 4v\omega_3\omega_4\omega_1^2\omega_2,$$

$$\gamma_{[t]}^{[\mu_3]} = \omega_3^2\omega_4\omega_1 + \omega_4^2\omega_1\omega_2 - 4\omega_3\omega_4\omega_1\omega_2^2 + \omega_3\omega_4^2\omega_1 + \omega_4\omega_1^2\omega_2 - 5\omega_3\omega_4\omega_2 + \omega_3\omega_1\omega_2^2 + \omega_3\omega_4\omega_2^2 - 5\omega_3\omega_1\omega_2 + 23\omega_3\omega_4\omega_1\omega_2 + \omega_3^2\omega_1\omega_2 - 4\omega_3\omega_4\omega_1^2\omega_2 - 4\omega_3\omega_4^2\omega_1\omega_2 + \omega_4\omega_1\omega_2^2 + \omega_3\omega_1^2\omega_2 + \omega_3\omega_4\omega_1^2 - 5\omega_3\omega_4\omega_1 - 5\omega_4\omega_1\omega_2 + \omega_3\omega_4^2\omega_2 + \omega_3^2\omega_4\omega_2 - 4\omega_3^2\omega_4\omega_1\omega_2,$$

$$\gamma_{[y]}^{[\mu_1]} = v^2\omega_3\omega_4\omega_1^2 + \omega_3\omega_4c_s^2\omega_1^2 + v^2\omega_3\omega_4\omega_1\omega_2 + \omega_3\omega_4c_s^2\omega_1\omega_2 - 5\omega_3\omega_4c_s^2\omega_1 - 5v^2\omega_3\omega_4\omega_1 + v^2\omega_3\omega_4^2\omega_1 + \omega_3\omega_4^2c_s^2\omega_1 + v^2\omega_3^2\omega_4\omega_1 + \omega_3^2\omega_4c_s^2\omega_1,$$

$$\gamma_{[t^2]}^{[\mu_1]} = -\frac{37}{2}v\omega_3\omega_4\omega_2 + \frac{7}{2}v\omega_3\omega_1\omega_2^2 + \frac{7}{2}v\omega_4\omega_1^2\omega_2 + 47v\omega_3\omega_4\omega_1\omega_2 - v\omega_4^2\omega_2 + \frac{7}{2}v\omega_4^2\omega_1\omega_2 + 5v\omega_3\omega_2 - v\omega_3\omega_2^2 - v\omega_1^2\omega_2 - 8v\omega_3\omega_4\omega_1\omega_2^2 + \frac{7}{2}v\omega_3\omega_4\omega_2^2 - \frac{37}{2}v\omega_3\omega_1\omega_2 - 8v\omega_3^2\omega_4\omega_1\omega_2 - v\omega_4\omega_2^2 + 5v\omega_1\omega_2 + \frac{7}{2}v\omega_3\omega_1^2\omega_2 + \frac{7}{2}v\omega_4\omega_1\omega_2^2 + \frac{7}{2}v\omega_3^2\omega_1\omega_2 + \frac{7}{2}v\omega_3^2\omega_4\omega_2 - 8v\omega_3\omega_4^2\omega_1\omega_2 - v\omega_3^2\omega_2 + \frac{7}{2}v\omega_3\omega_4^2\omega_2 - \frac{37}{2}v\omega_4\omega_1\omega_2 + 5v\omega_4\omega_2 - 8v\omega_3\omega_4\omega_1^2\omega_2 - v\omega_1\omega_2^2,$$

$$\gamma_{[t^2]}^{[\mu_3]} = -\frac{7}{2}\omega_3^2\omega_4\omega_1 - 5\omega_1\omega_2 + \omega_4\omega_2^2 - \frac{7}{2}\omega_4^2\omega_1\omega_2 + 8\omega_3\omega_4\omega_1\omega_2^2 + \omega_3\omega_4^2 - \frac{7}{2}\omega_3\omega_4^2\omega_1 + \omega_3\omega_1^2 - \frac{7}{2}\omega_4\omega_1^2\omega_2 + \frac{39}{2}\omega_3\omega_4\omega_2 - \frac{7}{2}\omega_3\omega_1\omega_2^2 - \frac{7}{2}\omega_3\omega_4\omega_2^2 + \frac{39}{2}\omega_3\omega_1\omega_2 + \omega_3^2\omega_2 - 5\omega_3\omega_1 + \omega_1\omega_2^2 - 5\omega_4\omega_2 - 5\omega_3\omega_4 - \frac{101}{2}\omega_3\omega_4\omega_1\omega_2 + \omega_4^2\omega_1 - \frac{7}{2}\omega_3^2\omega_1\omega_2 + \omega_4^2\omega_2 + 8\omega_3\omega_4\omega_1^2\omega_2 + \omega_3^2\omega_4 - 5\omega_4\omega_1 + 8\omega_3\omega_4^2\omega_1\omega_2 - \frac{7}{2}\omega_4\omega_1\omega_2^2 - 5\omega_3\omega_2 - \frac{7}{2}\omega_3\omega_1^2\omega_2 + \omega_3^2\omega_1 - \frac{7}{2}\omega_3\omega_4\omega_1^2 + \frac{39}{2}\omega_3\omega_4\omega_1 + \frac{39}{2}\omega_4\omega_1\omega_2 + \omega_3\omega_2^2 - \frac{7}{2}\omega_3\omega_4^2\omega_2 + \omega_4\omega_1^2 - \frac{7}{2}\omega_3^2\omega_4\omega_2 + \omega_1^2\omega_2 + 8\omega_3^2\omega_4\omega_1\omega_2,$$

$$\begin{aligned} \gamma_{[ty]}^{[\mu_1]} = & -\frac{5}{2}v^2\omega_3\omega_1 + \omega_3c_s^2\omega_1^2 + \omega_3c_s^2\omega_1\omega_2 - 3v^2\omega_3\omega_4\omega_1^2 - \frac{1}{2}\omega_4^2\omega_1u^2 - \frac{1}{2}\omega_3\omega_1^2 - 3\omega_3\omega_4c_s^2\omega_1^2 + \frac{1}{2}v^2\omega_4^2\omega_1 - 3v^2\omega_3\omega_4\omega_1\omega_2 - \frac{1}{2}\omega_4\omega_1^2u^2 - 5v^2\omega_3\omega_4 + \omega_3\omega_4^2c_s^2 + \frac{1}{2}\omega_3\omega_1\omega_2 - \frac{5}{2}\omega_3\omega_1u^2 + v^2\omega_3\omega_4^2 - 3\omega_3\omega_4c_s^2\omega_1\omega_2 - \frac{1}{2}\omega_4\omega_1u^2\omega_2 + 16\omega_3\omega_4c_s^2\omega_1 + 16v^2\omega_3\omega_4\omega_1 + \omega_3^2\omega_4c_s^2 - 5\omega_3c_s^2\omega_1 + \frac{1}{2}v^2\omega_3\omega_1^2 + \frac{1}{2}v^2\omega_4\omega_1\omega_2 + \frac{1}{2}\omega_4^2\omega_1 + \frac{1}{2}\omega_3\omega_1u^2\omega_2 - 3v^2\omega_3\omega_4^2\omega_1 + \frac{1}{2}\omega_3^2\omega_1u^2 + v^2\omega_3\omega_4\omega_2 - 3\omega_3\omega_4^2c_s^2\omega_1 - 2\omega_4\omega_1 - 5\omega_3\omega_4c_s^2 + \omega_3\omega_4c_s^2\omega_2 + \frac{1}{2}\omega_3\omega_1^2u^2 - 3v^2\omega_3^2\omega_4\omega_1 + \frac{1}{2}v^2\omega_4\omega_1^2 + \omega_3^2c_s^2\omega_1 + \frac{1}{2}\omega_3\omega_4\omega_1 - \frac{5}{2}v^2\omega_4\omega_1 - 3\omega_3^2\omega_4c_s^2\omega_1 + \frac{1}{2}\omega_4\omega_1\omega_2 + \frac{5}{2}\omega_4\omega_1u^2 + v^2\omega_3^2\omega_4 + \frac{1}{2}v^2\omega_3^2\omega_1 + \frac{1}{2}\omega_4\omega_1^2 + \frac{1}{2}v^2\omega_3\omega_1\omega_2, \end{aligned}$$



$$\begin{aligned}
\gamma_{[x^2]}^{[\mu_1]} &= 3v\omega_3\omega_4\omega_2 - \frac{1}{2}v\omega_3\omega_1\omega_2^2 - \frac{3}{4}v\omega_3\omega_4\omega_1\omega_2 - \frac{5}{2}v\omega_3\omega_2 + v\omega_3\omega_2^2 - \frac{1}{2}v\omega_3\omega_4\omega_2^2 + \frac{5}{4}v\omega_3\omega_1\omega_2 - \frac{1}{4}v\omega_3^2\omega_1\omega_2 - \\
&\quad \frac{1}{2}v\omega_3^2\omega_4\omega_2 + \frac{1}{2}v\omega_3^2\omega_2 - \frac{1}{2}v\omega_3\omega_4^2\omega_2 + \frac{1}{4}v\omega_4\omega_1\omega_2 - \frac{1}{2}v\omega_4\omega_2, \\
\gamma_{[x^2]}^{[\mu_3]} &= -3\omega_3\omega_4\omega_2 + \frac{1}{2}\omega_3\omega_1\omega_2^2 + \frac{1}{2}\omega_3\omega_4\omega_2^2 - \frac{5}{4}\omega_3\omega_1\omega_2 - \frac{1}{2}\omega_3^2\omega_2 + \frac{1}{2}\omega_4\omega_2 + \frac{3}{4}\omega_3\omega_4\omega_1\omega_2 + \frac{1}{4}\omega_3^2\omega_1\omega_2 + \frac{5}{2}\omega_3\omega_2 - \\
&\quad \frac{1}{4}\omega_4\omega_1\omega_2 - \omega_3\omega_2^2 + \frac{1}{2}\omega_3\omega_4^2\omega_2 + \frac{1}{2}\omega_3^2\omega_4\omega_2, \\
\gamma_{[xy]}^{[\mu_1]} &= \frac{1}{2}\omega_3\omega_4\omega_1u - \frac{5}{2}\omega_3\omega_1u + \omega_3\omega_1u\omega_2 + \frac{1}{2}\omega_4\omega_1u + \frac{1}{2}\omega_3^2\omega_1u, \\
\gamma_{[y^2]}^{[\mu_1]} &= -\frac{1}{4}v\omega_3\omega_4\omega_1\omega_2 + \frac{5}{4}v\omega_3\omega_1\omega_2 - \frac{1}{2}v\omega_3\omega_1^2\omega_2 - \frac{1}{4}v\omega_3^2\omega_1\omega_2 + \frac{1}{4}v\omega_4\omega_1\omega_2. \\
\gamma_{[y^2]}^{[\mu_3]} &= \frac{1}{2}\omega_3^2\omega_4\omega_1 + \frac{1}{2}\omega_3\omega_4^2\omega_1 - \omega_3\omega_1^2 - \frac{5}{4}\omega_3\omega_1\omega_2 + \frac{5}{2}\omega_3\omega_1 + \frac{3}{4}\omega_3\omega_4\omega_1\omega_2 + \frac{1}{4}\omega_3^2\omega_1\omega_2 + \frac{1}{2}\omega_4\omega_1 + \frac{1}{2}\omega_3\omega_1^2\omega_2 - \\
&\quad \frac{1}{2}\omega_3^2\omega_1 + \frac{1}{2}\omega_3\omega_4\omega_1^2 - 3\omega_3\omega_4\omega_1 - \frac{1}{4}\omega_4\omega_1\omega_2,
\end{aligned}$$

## 6.5 EPDE for $\mu_4$

$$\begin{aligned}
&\gamma_{[1]}^{[\mu_1]}\mu_1 + \gamma_{[1]}^{[\mu_4]}\mu_4 + \gamma_{[t]}^{[\mu_1]}\delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_4]}\delta_t \frac{\partial \mu_4}{\partial t} + \gamma_{[x]}^{[\mu_1]}\delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[y]}^{[\mu_1]}\delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[t^2]}^{[\mu_1]}\delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_4]}\delta_t^2 \frac{\partial^2 \mu_4}{\partial t^2} \\
&\quad + \gamma_{[tx]}^{[\mu_1]}\delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} + \gamma_{[ty]}^{[\mu_1]}\delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[x^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_4]}\delta_l^2 \frac{\partial^2 \mu_4}{\partial x^2} + \gamma_{[y^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_4]}\delta_l^2 \frac{\partial^2 \mu_4}{\partial y^2} = 0,
\end{aligned}$$

where

$$\begin{aligned}
\gamma_{[1]}^{[\mu_1]} &= -\omega_3^2\omega_4\omega_1u^2\omega_2 + 5\omega_3\omega_4\omega_1u^2\omega_2 - \omega_3\omega_4c_s^2\omega_1\omega_2^2 - \omega_3\omega_4\omega_1u^2\omega_2^2 + 5\omega_3\omega_4c_s^2\omega_1\omega_2 - \omega_3^2\omega_4c_s^2\omega_1\omega_2 - \\
&\quad \omega_3\omega_4^2\omega_1u^2\omega_2 - \omega_3\omega_4c_s^2\omega_1^2\omega_2 - \omega_3\omega_4^2c_s^2\omega_1\omega_2 - \omega_3\omega_4\omega_1^2u^2\omega_2, \\
\gamma_{[1]}^{[\mu_4]} &= \omega_3\omega_4\omega_1\omega_2^2 - 5\omega_3\omega_4\omega_1\omega_2 + \omega_3\omega_4\omega_1^2\omega_2 + \omega_3\omega_4^2\omega_1\omega_2 + \omega_3^2\omega_4\omega_1\omega_2, \\
\gamma_{[t]}^{[\mu_1]} &= \frac{1}{2}v^2\omega_4\omega_1\omega_2^2 + 4\omega_3^2\omega_4\omega_1u^2\omega_2 - \frac{1}{2}v^2\omega_3\omega_1^2\omega_2 + 5\omega_3c_s^2\omega_1\omega_2 - \frac{1}{2}\omega_3\omega_1u^2\omega_2^2 - \omega_3^2\omega_4u^2\omega_2 - \frac{1}{2}\omega_4^2\omega_1\omega_2 - \frac{1}{2}v^2\omega_3^2\omega_1\omega_2 - \\
&\quad \omega_3\omega_4c_s^2\omega_1^2 - \omega_3^2\omega_4c_s^2\omega_2 - \frac{1}{2}\omega_4\omega_1^2\omega_2 - 22\omega_3\omega_4\omega_1u^2\omega_2 - \frac{1}{2}\omega_4\omega_1u^2\omega_2^2 + 4\omega_3\omega_4c_s^2\omega_1\omega_2^2 + \frac{1}{2}\omega_3\omega_1\omega_2^2 + \\
&\quad 4\omega_3\omega_4\omega_1u^2\omega_2^2 - 22\omega_3\omega_4c_s^2\omega_1\omega_2 + \frac{5}{2}\omega_4\omega_1u^2\omega_2 + 5\omega_3\omega_4c_s^2\omega_1 - \omega_3\omega_4^2u^2\omega_2 - \omega_3\omega_4^2c_s^2\omega_2 + 5\omega_3\omega_4\omega_1u^2 - \\
&\quad \frac{1}{2}\omega_3\omega_4\omega_1\omega_2 - \frac{5}{2}v^2\omega_4\omega_1\omega_2 - \omega_3c_s^2\omega_1\omega_2^2 + 4\omega_3^2\omega_4c_s^2\omega_1\omega_2 + \frac{5}{2}\omega_3\omega_1u^2\omega_2 - \frac{1}{2}\omega_4\omega_1^2u^2\omega_2 - \omega_3\omega_4\omega_1^2u^2 - \\
&\quad \omega_3^2c_s^2\omega_1\omega_2 + \frac{1}{2}v^2\omega_4\omega_1^2\omega_2 + 5\omega_3\omega_4u^2\omega_2 - \omega_3\omega_4^2c_s^2\omega_1 - \frac{1}{2}v^2\omega_3\omega_1\omega_2^2 - \omega_3\omega_4^2\omega_1u^2 + 4\omega_3\omega_4^2\omega_1u^2\omega_2 - \frac{1}{2}\omega_4\omega_1\omega_2^2 + \\
&\quad 5\omega_3\omega_4c_s^2\omega_2 + 4\omega_3\omega_4c_s^2\omega_1^2\omega_2 - \frac{1}{2}\omega_3\omega_1^2\omega_2 - \frac{1}{2}\omega_3\omega_1^2u^2\omega_2 + \frac{1}{2}v^2\omega_4^2\omega_1\omega_2 - \omega_3^2\omega_4c_s^2\omega_1 + 2\omega_4\omega_1\omega_2 - \frac{1}{2}\omega_4^2\omega_1u^2\omega_2 + \\
&\quad 4\omega_3\omega_4^2c_s^2\omega_1\omega_2 - \omega_3\omega_4c_s^2\omega_2^2 - \frac{1}{2}\omega_3^2\omega_1u^2\omega_2 - \omega_3\omega_4u^2\omega_2^2 - \omega_3c_s^2\omega_1^2\omega_2 + \frac{5}{2}v^2\omega_3\omega_1\omega_2 - \omega_3^2\omega_4\omega_1u^2 + 4\omega_3\omega_4\omega_1^2u^2\omega_2, \\
\gamma_{[t]}^{[\mu_4]} &= \omega_3^2\omega_4\omega_1 + \omega_4^2\omega_1\omega_2 - 4\omega_3\omega_4\omega_1\omega_2^2 + \omega_3\omega_4^2\omega_1 + \omega_4\omega_1^2\omega_2 - 5\omega_3\omega_4\omega_2 + \omega_3\omega_1\omega_2^2 + \omega_3\omega_4\omega_2^2 - 5\omega_3\omega_1\omega_2 + \\
&\quad 23\omega_3\omega_4\omega_1\omega_2 + \omega_3^2\omega_1\omega_2 - 4\omega_3\omega_4\omega_1^2\omega_2 - 4\omega_3\omega_4^2\omega_1\omega_2 + \omega_4\omega_1\omega_2^2 + \omega_3\omega_1^2\omega_2 + \omega_3\omega_4\omega_1^2 - 5\omega_3\omega_4\omega_1 - 5\omega_4\omega_1\omega_2 + \\
&\quad \omega_3\omega_4^2\omega_2 + \omega_3^2\omega_4\omega_2 - 4\omega_3^2\omega_4\omega_1\omega_2, \\
\gamma_{[x]}^{[\mu_1]} &= \frac{1}{2}\omega_3^2\omega_1u\omega_2 - \frac{5}{2}\omega_3\omega_1u\omega_2 - \frac{1}{2}\omega_4\omega_1u\omega_2 + \frac{1}{2}\omega_3\omega_4\omega_1u\omega_2 + \omega_3\omega_1u\omega_2^2, \\
\gamma_{[y]}^{[\mu_1]} &= -\frac{1}{2}v\omega_3\omega_4\omega_1\omega_2 + \frac{5}{2}v\omega_3\omega_1\omega_2 - v\omega_3\omega_1^2\omega_2 - \frac{1}{2}v\omega_3^2\omega_1\omega_2 - \frac{1}{2}v\omega_4\omega_1\omega_2, \\
\gamma_{[t^2]}^{[\mu_1]} &= -\frac{7}{4}v^2\omega_4\omega_1\omega_2^2 + \frac{5}{2}v^2\omega_3\omega_1 - \omega_3c_s^2\omega_1^2 - 8\omega_3^2\omega_4\omega_1u^2\omega_2 + \frac{7}{4}v^2\omega_3\omega_1^2\omega_2 - \frac{37}{2}\omega_3c_s^2\omega_1\omega_2 + \frac{7}{4}\omega_3\omega_1u^2\omega_2^2 + \\
&\quad \frac{7}{2}\omega_3^2\omega_4u^2\omega_2 + 2\omega_1\omega_2 - \frac{1}{2}\omega_4\omega_2^2 + \frac{7}{4}\omega_4^2\omega_1\omega_2 - \frac{1}{2}\omega_4^2\omega_1u^2 - \frac{1}{2}v^2\omega_3^2\omega_2 + \frac{7}{4}v^2\omega_3^2\omega_1\omega_2 - \frac{1}{2}\omega_3\omega_1^2 + \frac{7}{2}\omega_3\omega_4c_s^2\omega_1^2 + \\
&\quad \frac{1}{2}v^2\omega_4^2\omega_1 + \frac{5}{2}\omega_4u^2\omega_2 - \frac{1}{2}\omega_4\omega_1^2u^2 + \frac{7}{2}\omega_3^2\omega_4c_s^2\omega_2 + \frac{7}{4}\omega_4\omega_1^2\omega_2 - \frac{5}{2}v^2\omega_4\omega_2 - \frac{1}{2}\omega_3\omega_4\omega_2 + 47\omega_3\omega_4\omega_1u^2\omega_2 +
\end{aligned}$$

$$\begin{aligned}
& 5\omega_3\omega_4u^2 + \frac{7}{4}\omega_4\omega_1u^2\omega_2^2 - 8\omega_3\omega_4c_s^2\omega_1\omega_2^2 - \frac{7}{4}\omega_3\omega_1\omega_2^2 - \omega_3\omega_4^2c_s^2 + \frac{1}{2}v^2\omega_4\omega_2^2 - \omega_3^2c_s^2\omega_2 - 8\omega_3\omega_4\omega_1u^2\omega_2^2 - \\
& \frac{1}{2}\omega_3\omega_1\omega_2 + \frac{5}{2}\omega_3\omega_1u^2 + 47\omega_3\omega_4c_s^2\omega_1\omega_2 - \frac{37}{4}\omega_4\omega_1u^2\omega_2 - \frac{37}{2}\omega_3\omega_4c_s^2\omega_1 + \frac{7}{2}\omega_3\omega_4^2u^2\omega_2 - \frac{1}{2}\omega_4u^2\omega_2^2 + \\
& \frac{7}{2}\omega_3\omega_4^2c_s^2\omega_2 + 2\omega_4\omega_2 - \omega_3^2\omega_4c_s^2 - \frac{37}{2}\omega_3\omega_4\omega_1u^2 + \frac{7}{4}\omega_3\omega_4\omega_1\omega_2 + 5\omega_3c_s^2\omega_1 - \frac{1}{2}v^2\omega_3\omega_1^2 + \frac{37}{4}v^2\omega_4\omega_1\omega_2 + \\
& \frac{7}{2}\omega_3c_s^2\omega_1\omega_2^2 - \frac{1}{2}\omega_3^2u^2\omega_2 - 8\omega_3^2\omega_4c_s^2\omega_1\omega_2 - \frac{1}{2}\omega_4^2\omega_1 - \frac{37}{4}\omega_3\omega_1u^2\omega_2 - \frac{1}{2}\omega_3^2\omega_1u^2 + \frac{7}{4}\omega_4\omega_1^2u^2\omega_2 - \frac{1}{2}\omega_4^2\omega_2 - \\
& \frac{1}{2}v^2\omega_3\omega_2^2 + 5\omega_3c_s^2\omega_2 + \frac{7}{2}\omega_3\omega_4\omega_1^2u^2 + \frac{7}{2}\omega_3^2c_s^2\omega_1\omega_2 - \frac{7}{4}v^2\omega_4\omega_1^2\omega_2 - \frac{37}{2}\omega_3\omega_4u^2\omega_2 + \frac{7}{2}\omega_3\omega_4^2c_s^2\omega_1 + 2\omega_4\omega_1 + \\
& \frac{7}{4}v^2\omega_3\omega_1\omega_2^2 + \frac{7}{2}\omega_3\omega_4^2\omega_1u^2 - 8\omega_3\omega_4^2\omega_1u^2\omega_2 + 5\omega_3\omega_4c_s^2 + \frac{7}{4}\omega_4\omega_1\omega_2^2 - \frac{37}{2}\omega_3\omega_4c_s^2\omega_2 - 8\omega_3\omega_4c_s^2\omega_1^2\omega_2 - \\
& \frac{1}{2}\omega_3\omega_1^2u^2 + \frac{7}{4}\omega_3\omega_1^2\omega_2 + \frac{7}{4}\omega_3\omega_1^2u^2\omega_2 + \frac{5}{2}\omega_3u^2\omega_2 + \frac{1}{2}v^2\omega_4\omega_1^2 - \omega_3^2c_s^2\omega_1 - \frac{7}{4}v^2\omega_4^2\omega_1\omega_2 - \frac{1}{2}\omega_3u^2\omega_2^2 - \\
& \frac{1}{2}\omega_3\omega_4\omega_1 - \frac{5}{2}v^2\omega_4\omega_1 + \frac{7}{2}\omega_3^2\omega_4c_s^2\omega_1 + \frac{1}{2}v^2\omega_4^2\omega_2 - \frac{17}{2}\omega_4\omega_1\omega_2 + \frac{5}{2}\omega_4\omega_1u^2 + \frac{7}{4}\omega_4^2\omega_1u^2\omega_2 - 8\omega_3\omega_4^2c_s^2\omega_1\omega_2 + \\
& \frac{1}{2}\omega_3\omega_2^2 + \frac{7}{2}\omega_3\omega_4c_s^2\omega_2^2 - \omega_3\omega_4^2u^2 - \frac{1}{2}v^2\omega_3^2\omega_1 - \frac{1}{2}\omega_4^2u^2\omega_2 + \frac{7}{4}\omega_3^2\omega_1u^2\omega_2 + \frac{7}{2}\omega_3\omega_4u^2\omega_2^2 - \frac{1}{2}\omega_4\omega_1^2 + \\
& \frac{7}{2}\omega_3c_s^2\omega_1^2\omega_2 - \frac{37}{4}v^2\omega_3\omega_1\omega_2 + \frac{7}{2}\omega_3^2\omega_4\omega_1u^2 - \omega_1^2\omega_2 - \omega_3^2\omega_4u^2 - 8\omega_3\omega_4\omega_1^2u^2\omega_2 - \omega_3c_s^2\omega_2^2 + \frac{5}{2}v^2\omega_3\omega_2, \\
\gamma_{[t^2]}^{[\mu_4]} &= -\frac{7}{2}\omega_3^2\omega_4\omega_1 - 5\omega_1\omega_2 + \omega_4\omega_2^2 - \frac{7}{2}\omega_4^2\omega_1\omega_2 + 8\omega_3\omega_4\omega_1\omega_2^2 + \omega_3\omega_4^2 - \frac{7}{2}\omega_3\omega_4^2\omega_1 + \omega_3\omega_1^2 - \frac{7}{2}\omega_4\omega_1^2\omega_2 + \\
& \frac{39}{2}\omega_3\omega_4\omega_2 - \frac{7}{2}\omega_3\omega_1\omega_2^2 - \frac{7}{2}\omega_3\omega_4\omega_2^2 + \frac{39}{2}\omega_3\omega_1\omega_2 + \omega_3^2\omega_2 - 5\omega_3\omega_1 + \omega_1\omega_2^2 - 5\omega_4\omega_2 - 5\omega_3\omega_4 - \frac{101}{2}\omega_3\omega_4\omega_1\omega_2 + \\
& \omega_4^2\omega_1 - \frac{7}{2}\omega_3^2\omega_1\omega_2 + \omega_3^2\omega_2 + 8\omega_3\omega_4\omega_1^2\omega_2 + \omega_3^2\omega_4 - 5\omega_4\omega_1 + 8\omega_3\omega_4^2\omega_1\omega_2 - \frac{7}{2}\omega_4\omega_1\omega_2^2 - 5\omega_3\omega_2 - \frac{7}{2}\omega_3\omega_1^2\omega_2 + \\
& \omega_3^2\omega_1 - \frac{7}{2}\omega_3\omega_4\omega_1^2 + \frac{39}{2}\omega_3\omega_4\omega_1 + \frac{39}{2}\omega_4\omega_1\omega_2 + \omega_3\omega_2^2 - \frac{7}{2}\omega_3\omega_4^2\omega_2 + \omega_4\omega_1^2 - \frac{7}{2}\omega_3^2\omega_4\omega_2 + \omega_1^2\omega_2 + 8\omega_3^2\omega_4\omega_1\omega_2, \\
\gamma_{[tx]}^{[\mu_1]} &= -\frac{3}{2}\omega_3^2\omega_1u\omega_2 + \frac{1}{2}\omega_3\omega_4\omega_1u - 3\omega_1u\omega_2 - \frac{5}{2}\omega_3\omega_1u + \omega_1u\omega_2^2 + \frac{17}{2}\omega_3\omega_1u\omega_2 - \frac{1}{2}\omega_4\omega_1u + \frac{3}{2}\omega_4\omega_1u\omega_2 - \\
& \frac{3}{2}\omega_3\omega_4\omega_1u\omega_2 - 3\omega_3\omega_1u\omega_2^2 + \frac{1}{2}\omega_3^2\omega_1u, \\
\gamma_{[ty]}^{[\mu_1]} &= -\frac{1}{2}v\omega_3\omega_4\omega_2 + \frac{3}{2}v\omega_3\omega_4\omega_1\omega_2 + \frac{5}{2}v\omega_3\omega_2 - v\omega_1^2\omega_2 - \frac{17}{2}v\omega_3\omega_1\omega_2 + 2v\omega_1\omega_2 + 3v\omega_3\omega_1^2\omega_2 + \frac{3}{2}v\omega_3^2\omega_1\omega_2 - \\
& \frac{1}{2}v\omega_3^2\omega_2 + \frac{1}{2}v\omega_4\omega_1\omega_2 - \frac{1}{2}v\omega_4\omega_2, \\
\gamma_{[x^2]}^{[\mu_1]} &= -\frac{1}{2}\omega_3^2\omega_4u^2\omega_2 - \frac{1}{2}\omega_3^2\omega_4c_s^2\omega_2 - \frac{1}{2}\omega_3\omega_4\omega_1u^2\omega_2 - \frac{1}{2}\omega_3\omega_4c_s^2\omega_1\omega_2 - \frac{1}{2}\omega_3\omega_4^2u^2\omega_2 - \frac{1}{2}\omega_3\omega_4^2c_s^2\omega_2 + \\
& \frac{5}{2}\omega_3\omega_4u^2\omega_2 + \frac{5}{2}\omega_3\omega_4c_s^2\omega_2 - \frac{1}{2}\omega_3\omega_4c_s^2\omega_2^2 - \frac{1}{2}\omega_3\omega_4u^2\omega_2^2, \\
\gamma_{[x^2]}^{[\mu_4]} &= -3\omega_3\omega_4\omega_2 + \frac{1}{2}\omega_3\omega_1\omega_2^2 + \frac{1}{2}\omega_3\omega_4\omega_2^2 - \frac{5}{4}\omega_3\omega_1\omega_2 - \frac{1}{2}\omega_3^2\omega_2 + \frac{1}{2}\omega_4\omega_2 + \frac{3}{4}\omega_3\omega_4\omega_1\omega_2 + \frac{1}{4}\omega_3^2\omega_1\omega_2 + \frac{5}{2}\omega_3\omega_2 - \\
& \frac{1}{4}\omega_4\omega_1\omega_2 - \omega_3\omega_2^2 + \frac{1}{2}\omega_3\omega_4^2\omega_2 + \frac{1}{2}\omega_3^2\omega_4\omega_2, \\
\gamma_{[y^2]}^{[\mu_1]} &= -\frac{5}{2}v^2\omega_3\omega_1 + 2\omega_3c_s^2\omega_1^2 - \frac{1}{2}v^2\omega_3\omega_1^2\omega_2 + \frac{5}{2}\omega_3c_s^2\omega_1\omega_2 - \frac{1}{4}v^2\omega_3^2\omega_1\omega_2 - \frac{1}{2}\omega_3\omega_4c_s^2\omega_1^2 - \frac{1}{4}v^2\omega_3\omega_4\omega_1\omega_2 - \\
& \frac{3}{4}\omega_3\omega_4\omega_1u^2\omega_2 - \frac{5}{2}\omega_3\omega_1u^2 - \omega_3\omega_4c_s^2\omega_1\omega_2 + \frac{1}{4}\omega_4\omega_1u^2\omega_2 + \frac{7}{2}\omega_3\omega_4c_s^2\omega_1 + \frac{1}{2}v^2\omega_3\omega_4\omega_1 + 3\omega_3\omega_4\omega_1u^2 - \\
& 5\omega_3c_s^2\omega_1 + v^2\omega_3\omega_1^2 - \frac{1}{4}v^2\omega_4\omega_1\omega_2 + \frac{5}{4}\omega_3\omega_1u^2\omega_2 + \frac{1}{2}\omega_3^2\omega_1u^2 - \frac{1}{2}\omega_3\omega_4\omega_1^2u^2 - \frac{1}{2}\omega_3^2c_s^2\omega_1\omega_2 - \frac{1}{2}\omega_3\omega_4^2c_s^2\omega_1 - \\
& \frac{1}{2}\omega_3\omega_4^2u^2\omega_1 + \omega_3\omega_1^2u^2 - \frac{1}{2}\omega_3\omega_1^2u^2\omega_2 + \omega_3^2c_s^2\omega_1 + \frac{1}{2}v^2\omega_4\omega_1 - \frac{1}{2}\omega_3^2\omega_4c_s^2\omega_1 - \frac{1}{2}\omega_4\omega_1u^2 + \frac{1}{2}v^2\omega_3^2\omega_1 - \\
& \frac{1}{4}\omega_3^2\omega_1u^2\omega_2 - \omega_3c_s^2\omega_1^2\omega_2 + \frac{5}{4}v^2\omega_3\omega_1\omega_2 - \frac{1}{2}\omega_3^2\omega_4\omega_1u^2. \\
\gamma_{[y^2]}^{[\mu_4]} &= \frac{1}{2}\omega_3^2\omega_4\omega_1 + \frac{1}{2}\omega_3\omega_4^2\omega_1 - \omega_3\omega_1^2 - \frac{5}{4}\omega_3\omega_1\omega_2 + \frac{5}{2}\omega_3\omega_1 + \frac{3}{4}\omega_3\omega_4\omega_1\omega_2 + \frac{1}{4}\omega_3^2\omega_1\omega_2 + \frac{1}{2}\omega_4\omega_1 + \frac{1}{2}\omega_3\omega_1^2\omega_2 - \\
& \frac{1}{2}\omega_3^2\omega_1 + \frac{1}{2}\omega_3\omega_4\omega_1^2 - 3\omega_3\omega_4\omega_1 - \frac{1}{4}\omega_4\omega_1\omega_2,
\end{aligned}$$

## 6.6 EPDE for $\mu_5$

$$\begin{aligned}
& \gamma_{[1]}^{[\mu_1]}\mu_1 + \gamma_{[1]}^{[\mu_5]}\mu_5 + \gamma_{[t]}^{[\mu_1]}\delta_t\frac{\partial\mu_1}{\partial t} + \gamma_{[t]}^{[\mu_5]}\delta_t\frac{\partial\mu_5}{\partial t} + \gamma_{[x]}^{[\mu_1]}\delta_l\frac{\partial\mu_1}{\partial x} + \gamma_{[y]}^{[\mu_1]}\delta_l\frac{\partial\mu_1}{\partial y} + \gamma_{[t^2]}^{[\mu_1]}\delta_t^2\frac{\partial^2\mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_5]}\delta_t^2\frac{\partial^2\mu_5}{\partial t^2} \\
& + \gamma_{[tx]}^{[\mu_1]}\delta_t\delta_l\frac{\partial^2\mu_1}{\partial t\partial x} + \gamma_{[ty]}^{[\mu_1]}\delta_t\delta_l\frac{\partial^2\mu_1}{\partial t\partial y} + \gamma_{[x^2]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_5]}\delta_l^2\frac{\partial^2\mu_5}{\partial x^2} + \gamma_{[y^2]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_5]}\delta_l^2\frac{\partial^2\mu_5}{\partial y^2} = 0,
\end{aligned}$$

where

$$\begin{aligned}
\gamma_{[1]}^{[\mu_1]} &= 5v^2\omega_3\omega_4\omega_1\omega_2 - \omega_3\omega_4c_s^2\omega_1\omega_2^2 + 5\omega_3\omega_4c_s^2\omega_1\omega_2 - v^2\omega_3\omega_4\omega_1\omega_2^2 - \omega_3^2\omega_4c_s^2\omega_1\omega_2 - \omega_3\omega_4c_s^2\omega_1^2\omega_2 - \\
&\quad v^2\omega_3^2\omega_4\omega_1\omega_2 - v^2\omega_3\omega_4\omega_1^2\omega_2 - \omega_3\omega_4^2c_s^2\omega_1\omega_2 - v^2\omega_3\omega_4^2\omega_1\omega_2, \\
\gamma_{[1]}^{[\mu_5]} &= \omega_3\omega_4\omega_1\omega_2^2 - 5\omega_3\omega_4\omega_1\omega_2 + \omega_3\omega_4\omega_1^2\omega_2 + \omega_3\omega_4^2\omega_1\omega_2 + \omega_3^2\omega_4\omega_1\omega_2, \\
\gamma_{[t]}^{[\mu_1]} &= -\frac{1}{2}v^2\omega_4\omega_1\omega_2^2 - \frac{1}{2}v^2\omega_3\omega_1^2\omega_2 + 5\omega_3c_s^2\omega_1\omega_2 - \frac{1}{2}\omega_3\omega_1u^2\omega_2^2 - v^2\omega_3\omega_4\omega_1^2 - \frac{1}{2}\omega_4^2\omega_1\omega_2 - \frac{1}{2}v^2\omega_3^2\omega_1\omega_2 - \omega_3\omega_4c_s^2\omega_1^2 - \\
&\quad 22v^2\omega_3\omega_4\omega_1\omega_2 - \omega_3^2\omega_4c_s^2\omega_2 - \frac{1}{2}\omega_4\omega_1^2\omega_2 + \frac{1}{2}\omega_4\omega_1u^2\omega_2^2 + 4\omega_3\omega_4c_s^2\omega_1\omega_2^2 - \frac{1}{2}\omega_3\omega_1\omega_2^2 - 22\omega_3\omega_4c_s^2\omega_1\omega_2 - \\
&\quad \frac{5}{2}\omega_4\omega_1u^2\omega_2 - v^2\omega_3^2\omega_4\omega_2 + 5\omega_3\omega_4c_s^2\omega_1 + 4v^2\omega_3\omega_4\omega_1\omega_2^2 - \omega_3\omega_4^2c_s^2\omega_2 + 5v^2\omega_3\omega_4\omega_1 - \frac{1}{2}\omega_3\omega_4\omega_1\omega_2 + \\
&\quad \frac{5}{2}v^2\omega_4\omega_1\omega_2 - \omega_3c_s^2\omega_1\omega_2^2 + 4\omega_3^2\omega_4c_s^2\omega_1\omega_2 + \frac{5}{2}\omega_3\omega_1u^2\omega_2 - v^2\omega_3\omega_4^2\omega_2 - v^2\omega_3\omega_4^2\omega_1 + \frac{1}{2}\omega_4\omega_1^2u^2\omega_2 - \omega_3^2c_s^2\omega_1\omega_2 - \\
&\quad \frac{1}{2}v^2\omega_4\omega_1^2\omega_2 + 5v^2\omega_3\omega_4\omega_2 - \omega_3\omega_4^2c_s^2\omega_1 - \frac{1}{2}v^2\omega_3\omega_1\omega_2^2 - \frac{1}{2}\omega_4\omega_1\omega_2^2 + 5\omega_3\omega_4c_s^2\omega_2 + 4\omega_3\omega_4c_s^2\omega_1^2\omega_2 - v^2\omega_3^2\omega_4\omega_1 + \\
&\quad \frac{1}{2}\omega_3\omega_1^2\omega_2 - \frac{1}{2}\omega_3\omega_1^2u^2\omega_2 + 4v^2\omega_3^2\omega_4\omega_1\omega_2 - \frac{1}{2}v^2\omega_4^2\omega_1\omega_2 + 4v^2\omega_3\omega_4\omega_1^2\omega_2 - \omega_3^2\omega_4c_s^2\omega_1 + 2\omega_4\omega_1\omega_2 + \frac{1}{2}\omega_4^2\omega_1u^2\omega_2 + \\
&\quad 4\omega_3\omega_4^2c_s^2\omega_1\omega_2 - \omega_3\omega_4c_s^2\omega_2 + 4v^2\omega_3\omega_4^2\omega_1\omega_2 - \frac{1}{2}\omega_3^2\omega_1u^2\omega_2 - v^2\omega_3\omega_4\omega_2^2 - \omega_3c_s^2\omega_1^2\omega_2 + \frac{5}{2}v^2\omega_3\omega_1\omega_2, \\
\gamma_{[t]}^{[\mu_5]} &= \omega_3^2\omega_4\omega_1 + \omega_4^2\omega_1\omega_2 - 4\omega_3\omega_4\omega_1\omega_2^2 + \omega_3\omega_4^2\omega_1 + \omega_4\omega_1^2\omega_2 - 5\omega_3\omega_4\omega_2 + \omega_3\omega_1\omega_2^2 + \omega_3\omega_4\omega_2^2 - 5\omega_3\omega_1\omega_2 + \\
&\quad 23\omega_3\omega_4\omega_1\omega_2 + \omega_3^2\omega_1\omega_2 - 4\omega_3\omega_4\omega_1^2\omega_2 - 4\omega_3\omega_4^2\omega_1\omega_2 + \omega_4\omega_1\omega_2^2 + \omega_3\omega_1^2\omega_2 + \omega_3\omega_4\omega_1^2 - 5\omega_3\omega_4\omega_1 - 5\omega_4\omega_1\omega_2 + \\
&\quad \omega_3\omega_4^2\omega_2 + \omega_3^2\omega_4\omega_2 - 4\omega_3^2\omega_4\omega_1\omega_2, \\
\gamma_{[x]}^{[\mu_1]} &= -\frac{1}{2}\omega_3^2\omega_1u\omega_2 + \frac{5}{2}\omega_3\omega_1u\omega_2 - \frac{1}{2}\omega_4\omega_1u\omega_2 - \frac{1}{2}\omega_3\omega_4\omega_1u\omega_2 - \omega_3\omega_1u\omega_2^2, \\
\gamma_{[y]}^{[\mu_1]} &= \frac{1}{2}v\omega_3\omega_4\omega_1\omega_2 - \frac{5}{2}v\omega_3\omega_1\omega_2 + v\omega_3\omega_1^2\omega_2 + \frac{1}{2}v\omega_3^2\omega_1\omega_2 - \frac{1}{2}v\omega_4\omega_1\omega_2, \\
\gamma_{[t^2]}^{[\mu_1]} &= \frac{7}{4}v^2\omega_4\omega_1\omega_2^2 + \frac{5}{2}v^2\omega_3\omega_1 - \omega_3c_s^2\omega_1^2 + \frac{7}{4}v^2\omega_3\omega_1^2\omega_2 - \frac{37}{2}\omega_3c_s^2\omega_1\omega_2 + \frac{7}{4}\omega_3\omega_1u^2\omega_2^2 + 2\omega_1\omega_2 + \frac{7}{2}v^2\omega_3\omega_4\omega_1^2 - \\
&\quad \frac{1}{2}\omega_4\omega_2^2 + \frac{7}{4}\omega_4^2\omega_1\omega_2 + \frac{1}{2}\omega_4^2\omega_1u^2 - \frac{1}{2}v^2\omega_3^2\omega_2 + \frac{7}{4}v^2\omega_3^2\omega_1\omega_2 + \frac{1}{2}\omega_3\omega_1^2 + \frac{7}{2}\omega_3\omega_4c_s^2\omega_1^2 - \frac{1}{2}v^2\omega_4^2\omega_1 - \frac{5}{2}\omega_4u^2\omega_2 + \\
&\quad 47v^2\omega_3\omega_4\omega_1\omega_2 + \frac{1}{2}\omega_4\omega_1^2u^2 + \frac{7}{2}\omega_3^2\omega_4c_s^2\omega_2 + \frac{7}{4}\omega_4\omega_1^2\omega_2 + \frac{5}{2}v^2\omega_4\omega_2 - \frac{1}{2}\omega_3\omega_4\omega_2 - \frac{7}{4}\omega_4\omega_1u^2\omega_2^2 - \\
&\quad 8\omega_3\omega_4c_s^2\omega_1\omega_2^2 + 5v^2\omega_3\omega_4 + \frac{7}{4}\omega_3\omega_1\omega_2^2 - \omega_3\omega_4^2c_s^2 - \frac{1}{2}v^2\omega_4\omega_2^2 - \omega_3^2c_s^2\omega_2 - \frac{1}{2}\omega_3\omega_1\omega_2 + \frac{5}{2}\omega_3\omega_1u^2 - v^2\omega_3\omega_4^2 + \\
&\quad 47\omega_3\omega_4c_s^2\omega_1\omega_2 + \frac{7}{4}\omega_4\omega_1u^2\omega_2 + \frac{7}{2}v^2\omega_3^2\omega_4\omega_2 - \frac{37}{2}\omega_3\omega_4c_s^2\omega_1 + \frac{1}{2}\omega_4u^2\omega_2^2 - 8v^2\omega_3\omega_4\omega_1\omega_2^2 - \omega_1\omega_2^2 + \\
&\quad \frac{7}{2}\omega_3\omega_4^2c_s^2\omega_2 + 2\omega_4\omega_2 - \frac{37}{2}v^2\omega_3\omega_4\omega_1 - \omega_3^2\omega_4c_s^2 + \frac{7}{4}\omega_3\omega_4\omega_1\omega_2 + 5\omega_3c_s^2\omega_1 - \frac{1}{2}v^2\omega_3\omega_1^2 - \frac{37}{4}v^2\omega_4\omega_1\omega_2 + \\
&\quad \frac{7}{2}\omega_3c_s^2\omega_1\omega_2^2 - \frac{1}{2}\omega_3^2u^2\omega_2 - 8\omega_3^2\omega_4c_s^2\omega_1\omega_2 - \frac{1}{2}\omega_4^2\omega_1 - \frac{37}{4}\omega_3\omega_1u^2\omega_2 + \frac{7}{2}v^2\omega_3\omega_4^2\omega_2 + \frac{7}{2}v^2\omega_3\omega_4^2\omega_1 - \\
&\quad \frac{1}{2}\omega_3^2\omega_1u^2 - \frac{7}{4}\omega_4\omega_1^2u^2\omega_2 - \frac{1}{2}\omega_4^2\omega_2 - \frac{1}{2}v^2\omega_3\omega_2^2 + 5\omega_3c_s^2\omega_2 + \frac{7}{2}\omega_3^2c_s^2\omega_1\omega_2 + \frac{7}{4}v^2\omega_4\omega_1^2\omega_2 - \frac{37}{2}v^2\omega_3\omega_4\omega_2 + \\
&\quad \frac{7}{2}\omega_3\omega_4^2c_s^2\omega_1 + 2\omega_4\omega_1 + \frac{7}{4}v^2\omega_3\omega_1\omega_2^2 + 5\omega_3\omega_4c_s^2 + \frac{7}{4}\omega_4\omega_1\omega_2^2 - \frac{37}{2}\omega_3\omega_4c_s^2\omega_2 - 8\omega_3\omega_4c_s^2\omega_1^2\omega_2 - \frac{1}{2}\omega_3\omega_1^2u^2 + \\
&\quad \frac{7}{2}v^2\omega_3^2\omega_4\omega_1 - \frac{7}{4}\omega_3\omega_1^2\omega_2 + \frac{7}{4}\omega_3\omega_1^2u^2\omega_2 + \frac{5}{2}\omega_3u^2\omega_2 - 8v^2\omega_3^2\omega_4\omega_1\omega_2 - \frac{1}{2}v^2\omega_4\omega_1^2 - \omega_3^2c_s^2\omega_1 + \frac{7}{4}v^2\omega_4^2\omega_1\omega_2 - \\
&\quad \frac{1}{2}\omega_3u^2\omega_2^2 - \frac{1}{2}\omega_3\omega_4\omega_1 - 8v^2\omega_3\omega_4\omega_1\omega_2^2 + \frac{5}{2}v^2\omega_4\omega_1 + \frac{7}{2}\omega_3^2\omega_4c_s^2\omega_1 - \frac{1}{2}v^2\omega_4^2\omega_2 - \frac{17}{2}\omega_4\omega_1\omega_2 - \frac{5}{2}\omega_4\omega_1u^2 - \\
&\quad \frac{7}{4}\omega_4^2\omega_1u^2\omega_2 - 8\omega_3\omega_4^2c_s^2\omega_1\omega_2 - \frac{1}{2}\omega_3\omega_2^2 - v^2\omega_3^2\omega_4 + \frac{7}{2}\omega_3\omega_4c_s^2\omega_2^2 - \frac{1}{2}v^2\omega_3^2\omega_1 - 8v^2\omega_3\omega_4^2\omega_1\omega_2 + \frac{1}{2}\omega_4^2u^2\omega_2 + \\
&\quad \frac{7}{4}\omega_3\omega_1u^2\omega_2 - \frac{1}{2}\omega_4\omega_1^2 + \frac{7}{2}v^2\omega_3\omega_4\omega_2^2 + \frac{7}{2}\omega_3c_s^2\omega_1^2\omega_2 - \frac{37}{4}v^2\omega_3\omega_1\omega_2 - \omega_3c_s^2\omega_2^2 + \frac{5}{2}v^2\omega_3\omega_2, \\
\gamma_{[t^2]}^{[\mu_5]} &= -\frac{7}{2}\omega_3^2\omega_4\omega_1 - 5\omega_1\omega_2 + \omega_4\omega_2^2 - \frac{7}{2}\omega_4^2\omega_1\omega_2 + 8\omega_3\omega_4\omega_1\omega_2^2 + \omega_3\omega_4^2 - \frac{7}{2}\omega_3\omega_4^2\omega_1 + \omega_3\omega_1^2 - \frac{7}{2}\omega_4\omega_1^2\omega_2 + \\
&\quad \frac{39}{2}\omega_3\omega_4\omega_2 - \frac{7}{2}\omega_3\omega_1\omega_2^2 - \frac{7}{2}\omega_3\omega_4\omega_2^2 + \frac{39}{2}\omega_3\omega_1\omega_2 + \omega_3^2\omega_2 - 5\omega_3\omega_1 + \omega_1\omega_2^2 - 5\omega_4\omega_2 - 5\omega_3\omega_4 - \frac{101}{2}\omega_3\omega_4\omega_1\omega_2 + \\
&\quad \omega_4^2\omega_1 - \frac{7}{2}\omega_3^2\omega_1\omega_2 + \omega_4^2\omega_2 + 8\omega_3\omega_4\omega_1^2\omega_2 + \omega_3^2\omega_4 - 5\omega_4\omega_1 + 8\omega_3\omega_4^2\omega_1\omega_2 - \frac{7}{2}\omega_4\omega_1\omega_2^2 - 5\omega_3\omega_2 - \frac{7}{2}\omega_3\omega_1^2\omega_2 + \\
&\quad \omega_3^2\omega_1 - \frac{7}{2}\omega_3\omega_4\omega_1^2 + \frac{39}{2}\omega_3\omega_4\omega_1 + \frac{39}{2}\omega_4\omega_1\omega_2 + \omega_3\omega_2^2 - \frac{7}{2}\omega_3\omega_4^2\omega_2 + \omega_4\omega_1^2 - \frac{7}{2}\omega_3^2\omega_4\omega_2 + \omega_1^2\omega_2 + 8\omega_3^2\omega_4\omega_1\omega_2, \\
\gamma_{[tx]}^{[\mu_1]} &= \frac{3}{2}\omega_3^2\omega_1u\omega_2 - \frac{1}{2}\omega_3\omega_4\omega_1u + 2\omega_1u\omega_2 + \frac{5}{2}\omega_3\omega_1u - \omega_1u\omega_2^2 - \frac{17}{2}\omega_3\omega_1u\omega_2 - \frac{1}{2}\omega_4\omega_1u + \frac{1}{2}\omega_4\omega_1u\omega_2 + \\
&\quad \frac{3}{2}\omega_3\omega_4\omega_1u\omega_2 + 3\omega_3\omega_1u\omega_2^2 - \frac{1}{2}\omega_3^2\omega_1u, \\
\gamma_{[ty]}^{[\mu_1]} &= \frac{1}{2}v\omega_3\omega_4\omega_2 - \frac{3}{2}v\omega_3\omega_4\omega_1\omega_2 - \frac{5}{2}v\omega_3\omega_2 + v\omega_1^2\omega_2 + \frac{17}{2}v\omega_3\omega_1\omega_2 - 3v\omega_1\omega_2 - 3v\omega_3\omega_1^2\omega_2 - \frac{3}{2}v\omega_3^2\omega_1\omega_2 + \\
&\quad \frac{1}{2}v\omega_3^2\omega_2 + \frac{3}{2}v\omega_4\omega_1\omega_2 - \frac{1}{2}v\omega_4\omega_2, \\
\gamma_{[x^2]}^{[\mu_1]} &= \frac{5}{2}\omega_3c_s^2\omega_1\omega_2 - \frac{1}{2}\omega_3\omega_1u^2\omega_2^2 + \frac{1}{2}v^2\omega_3^2\omega_2 - \frac{1}{4}v^2\omega_3^2\omega_1\omega_2 + \frac{1}{2}\omega_4u^2\omega_2 - \frac{3}{4}v^2\omega_3\omega_4\omega_1\omega_2 - \frac{1}{2}\omega_3^2\omega_4c_s^2\omega_2 - \\
&\quad \frac{1}{2}v^2\omega_4\omega_2 - \frac{1}{4}\omega_3\omega_4\omega_1u^2\omega_2 + \omega_3^2c_s^2\omega_2 - \omega_3\omega_4c_s^2\omega_1\omega_2 - \frac{1}{4}\omega_4\omega_1u^2\omega_2 - \frac{1}{2}v^2\omega_3^2\omega_4\omega_2 - \frac{1}{2}\omega_3\omega_4^2c_s^2\omega_2 + \\
&\quad \frac{1}{4}v^2\omega_4\omega_1\omega_2 - \omega_3c_s^2\omega_1\omega_2^2 + \frac{1}{2}\omega_3^2u^2\omega_2 + \frac{5}{4}\omega_3\omega_1u^2\omega_2 - \frac{1}{2}v^2\omega_3\omega_4^2\omega_2 + v^2\omega_3\omega_2^2 - 5\omega_3c_s^2\omega_2 - \frac{1}{2}\omega_3^2c_s^2\omega_1\omega_2 + \\
&\quad \frac{1}{2}\omega_3\omega_4u^2\omega_2 + 3v^2\omega_3\omega_4\omega_2 - \frac{1}{2}v^2\omega_3\omega_1\omega_2^2 + \frac{7}{2}\omega_3\omega_4c_s^2\omega_2 - \frac{5}{2}\omega_3u^2\omega_2 + \omega_3u^2\omega_2^2 - \frac{1}{2}\omega_3\omega_4c_s^2\omega_2^2 -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4}\omega_3^2\omega_1u^2\omega_2 - \frac{1}{2}v^2\omega_3\omega_4\omega_2^2 + \frac{5}{4}v^2\omega_3\omega_1\omega_2 + 2\omega_3c_s^2\omega_2^2 - \frac{5}{2}v^2\omega_3\omega_2, \\
\gamma_{[x^2]}^{[\mu_5]} &= -3\omega_3\omega_4\omega_2 + \frac{1}{2}\omega_3\omega_1\omega_2^2 + \frac{1}{2}\omega_3\omega_4\omega_2^2 - \frac{5}{4}\omega_3\omega_1\omega_2 - \frac{1}{2}\omega_3^2\omega_2 + \frac{1}{2}\omega_4\omega_2 + \frac{3}{4}\omega_3\omega_4\omega_1\omega_2 + \frac{1}{4}\omega_3^2\omega_1\omega_2 + \frac{5}{2}\omega_3\omega_2 - \\
& \frac{1}{4}\omega_4\omega_1\omega_2 - \omega_3\omega_2^2 + \frac{1}{2}\omega_3\omega_4^2\omega_2 + \frac{1}{2}\omega_3^2\omega_4\omega_2, \\
\gamma_{[y^2]}^{[\mu_1]} &= -\frac{1}{2}v^2\omega_3\omega_4\omega_1^2 - \frac{1}{2}\omega_3\omega_4c_s^2\omega_1^2 - \frac{1}{2}v^2\omega_3\omega_4\omega_1\omega_2 - \frac{1}{2}\omega_3\omega_4c_s^2\omega_1\omega_2 + \frac{5}{2}\omega_3\omega_4c_s^2\omega_1 + \frac{5}{2}v^2\omega_3\omega_4\omega_1 - \\
& \frac{1}{2}v^2\omega_3\omega_4^2\omega_1 - \frac{1}{2}\omega_3\omega_4^2c_s^2\omega_1 - \frac{1}{2}v^2\omega_3^2\omega_4\omega_1 - \frac{1}{2}\omega_3^2\omega_4c_s^2\omega_1, \\
\gamma_{[y^2]}^{[\mu_5]} &= \frac{1}{2}\omega_3^2\omega_4\omega_1 + \frac{1}{2}\omega_3\omega_4^2\omega_1 - \omega_3\omega_1^2 - \frac{5}{4}\omega_3\omega_1\omega_2 + \frac{5}{2}\omega_3\omega_1 + \frac{3}{4}\omega_3\omega_4\omega_1\omega_2 + \frac{1}{4}\omega_3^2\omega_1\omega_2 + \frac{1}{2}\omega_4\omega_1 + \frac{1}{2}\omega_3\omega_1^2\omega_2 - \\
& \frac{1}{2}\omega_3^2\omega_1 + \frac{1}{2}\omega_3\omega_4\omega_1^2 - 3\omega_3\omega_4\omega_1 - \frac{1}{4}\omega_4\omega_1\omega_2,
\end{aligned}$$

## 7 CLBM 1: relaxation of $k_{00}$ , $k_{10}$ , $k_{01}$ , $k_{20}$ , $k_{02}$

### 7.1 Definitions

Matrix  $\mathbf{A} = \mathbf{K}^{-1}\mathbf{S}\mathbf{K}$ :

$$\begin{aligned}
\mathbf{A}_{1,1} &= -\omega_4v^2 + 2v^2\omega_2 - u^2\omega_3 - u^2\omega_0 + \omega_0 - v^2\omega_0 + 2u^2\omega_1, \\
\mathbf{A}_{1,2} &= -2u\omega_1 - \omega_3 - \omega_4v^2 + 2v^2\omega_2 - u^2\omega_3 - u^2\omega_0 + \omega_0 + 2u\omega_3 - v^2\omega_0 + 2u^2\omega_1, \\
\mathbf{A}_{1,3} &= 2\omega_4v - \omega_4 - \omega_4v^2 + 2v^2\omega_2 - u^2\omega_3 - u^2\omega_0 + \omega_0 - 2v\omega_2 - v^2\omega_0 + 2u^2\omega_1, \\
\mathbf{A}_{1,4} &= 2u\omega_1 - \omega_3 - \omega_4v^2 + 2v^2\omega_2 - u^2\omega_3 - u^2\omega_0 + \omega_0 - 2u\omega_3 - v^2\omega_0 + 2u^2\omega_1, \\
\mathbf{A}_{1,5} &= -2\omega_4v - \omega_4 - \omega_4v^2 + 2v^2\omega_2 - u^2\omega_3 - u^2\omega_0 + \omega_0 + 2v\omega_2 - v^2\omega_0 + 2u^2\omega_1, \\
\mathbf{A}_{2,1} &= \frac{1}{2}u(\omega_0 - \omega_1(1 + 2u) + u(\omega_3 + \omega_0)), \\
\mathbf{A}_{2,2} &= \frac{1}{2}u\omega_1 + \frac{1}{2}\omega_1 + \frac{1}{2}u\omega_0 + \frac{1}{2}\omega_3 + \frac{1}{2}u^2\omega_3 + \frac{1}{2}u^2\omega_0 - u\omega_3 - u^2\omega_1, \\
\mathbf{A}_{2,3} &= \frac{1}{2}u(\omega_0 - \omega_1(1 + 2u) + u(\omega_3 + \omega_0)), \\
\mathbf{A}_{2,4} &= \frac{1}{2}(\omega_3 - \omega_1(1 + 2u) + u(\omega_3 + \omega_0))(1 + u), \\
\mathbf{A}_{2,5} &= \frac{1}{2}u(\omega_0 - \omega_1(1 + 2u) + u(\omega_3 + \omega_0)), \\
\mathbf{A}_{3,1} &= \frac{1}{2}v(\omega_4v + (\omega_0 - 2\omega_2)v + \omega_0 - \omega_2), \\
\mathbf{A}_{3,2} &= \frac{1}{2}v(\omega_4v + (\omega_0 - 2\omega_2)v + \omega_0 - \omega_2), \\
\mathbf{A}_{3,3} &= -\omega_4v + \frac{1}{2}\omega_4 + \frac{1}{2}v\omega_0 + \frac{1}{2}\omega_4v^2 - v^2\omega_2 + \frac{1}{2}v\omega_2 + \frac{1}{2}v^2\omega_0 + \frac{1}{2}\omega_2, \\
\mathbf{A}_{3,4} &= \frac{1}{2}v(\omega_4v + (\omega_0 - 2\omega_2)v + \omega_0 - \omega_2), \\
\mathbf{A}_{3,5} &= \frac{1}{2}(1 + v)((\omega_0 - 2\omega_2)v - \omega_2 + (1 + v)\omega_4), \\
\mathbf{A}_{4,1} &= -\frac{1}{2}u(\omega_0 - u(\omega_3 + \omega_0) + (-1 + 2u)\omega_1), \\
\mathbf{A}_{4,2} &= -\frac{1}{2}(\omega_3 - u(\omega_3 + \omega_0) + (-1 + 2u)\omega_1)(-1 + u), \\
\mathbf{A}_{4,3} &= -\frac{1}{2}u(\omega_0 - u(\omega_3 + \omega_0) + (-1 + 2u)\omega_1), \\
\mathbf{A}_{4,4} &= -\frac{1}{2}u\omega_1 + \frac{1}{2}\omega_1 - \frac{1}{2}u\omega_0 + \frac{1}{2}\omega_3 + \frac{1}{2}u^2\omega_3 + \frac{1}{2}u^2\omega_0 + u\omega_3 - u^2\omega_1,
\end{aligned}$$

$$\begin{aligned}
\mathbf{A}_{4,5} &= -\frac{1}{2}u(\omega_0 - u(\omega_3 + \omega_0) + (-1 + 2u)\omega_1), \\
\mathbf{A}_{5,1} &= \frac{1}{2}v(\omega_4v + (\omega_0 - 2\omega_2)v - \omega_0 + \omega_2), \\
\mathbf{A}_{5,2} &= \frac{1}{2}v(\omega_4v + (\omega_0 - 2\omega_2)v - \omega_0 + \omega_2), \\
\mathbf{A}_{5,3} &= \frac{1}{2}((\omega_0 - 2\omega_2)v + \omega_4(-1 + v) + \omega_2)(-1 + v), \\
\mathbf{A}_{5,4} &= \frac{1}{2}v(\omega_4v + (\omega_0 - 2\omega_2)v - \omega_0 + \omega_2), \\
\mathbf{A}_{5,5} &= \omega_4v + \frac{1}{2}\omega_4 - \frac{1}{2}v\omega_0 + \frac{1}{2}\omega_4v^2 - v^2\omega_2 - \frac{1}{2}v\omega_2 + \frac{1}{2}v^2\omega_0 + \frac{1}{2}\omega_2.
\end{aligned}$$

where

$$\mathbf{S} = \text{diag}(\omega_0, \omega_1, \omega_2, \omega_3, \omega_4)$$

and

$$\mathbf{K} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -u & 1-u & -u & -u-1 & -u \\ -v & -v & 1-v & -v & -v-1 \\ u^2 & (1-u)^2 & u^2 & (u+1)^2 & u^2 \\ v^2 & v^2 & (1-v)^2 & v^2 & (v+1)^2 \end{pmatrix}$$

Matrix  $\mathbf{B}$ :

$$\begin{aligned}
\mathbf{B}_{1,1} &= 0, \\
\mathbf{B}_{1,2} &= -1 + 2u\omega_1 + \omega_3 - 2u\omega_3, \\
\mathbf{B}_{1,3} &= -1 - 2\omega_4v + \omega_4 + 2v\omega_2, \\
\mathbf{B}_{1,4} &= -1 - 2u\omega_1 + \omega_3 + 2u\omega_3, \\
\mathbf{B}_{1,5} &= -1 + 2\omega_4v + \omega_4 - 2v\omega_2, \\
\mathbf{B}_{2,1} &= -1 + u\omega_1 + \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 - u\omega_3, \\
\mathbf{B}_{2,2} &= 0, \\
\mathbf{B}_{2,3} &= -1 + u\omega_1 + \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 - u\omega_3, \\
\mathbf{B}_{2,4} &= -1 + 2u\omega_1 + \omega_1 - 2u\omega_3, \\
\mathbf{B}_{2,5} &= -1 + u\omega_1 + \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 - u\omega_3, \\
\mathbf{B}_{3,1} &= -1 - \omega_4v + \frac{1}{2}\omega_4 + v\omega_2 + \frac{1}{2}\omega_2, \\
\mathbf{B}_{3,2} &= -1 - \omega_4v + \frac{1}{2}\omega_4 + v\omega_2 + \frac{1}{2}\omega_2, \\
\mathbf{B}_{3,3} &= 0, \\
\mathbf{B}_{3,4} &= -1 - \omega_4v + \frac{1}{2}\omega_4 + v\omega_2 + \frac{1}{2}\omega_2, \\
\mathbf{B}_{3,5} &= -1 - 2\omega_4v + 2v\omega_2 + \omega_2, \\
\mathbf{B}_{4,1} &= -1 - u\omega_1 + \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 + u\omega_3, \\
\mathbf{B}_{4,2} &= -1 - 2u\omega_1 + \omega_1 + 2u\omega_3, \\
\mathbf{B}_{4,3} &= -1 - u\omega_1 + \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 + u\omega_3, \\
\mathbf{B}_{4,4} &= 0,
\end{aligned}$$

$$\begin{aligned}
\mathbf{B}_{4,5} &= -1 - u\omega_1 + \frac{1}{2}\omega_1 + \frac{1}{2}\omega_3 + u\omega_3, \\
\mathbf{B}_{5,1} &= -1 + \omega_4v + \frac{1}{2}\omega_4 - v\omega_2 + \frac{1}{2}\omega_2, \\
\mathbf{B}_{5,2} &= -1 + \omega_4v + \frac{1}{2}\omega_4 - v\omega_2 + \frac{1}{2}\omega_2, \\
\mathbf{B}_{5,3} &= -1 + 2\omega_4v - 2v\omega_2 + \omega_2, \\
\mathbf{B}_{5,4} &= -1 + \omega_4v + \frac{1}{2}\omega_4 - v\omega_2 + \frac{1}{2}\omega_2, \\
\mathbf{B}_{5,5} &= 0.
\end{aligned}$$

## 7.2 EPDE for $\mu_1$

$$\begin{aligned}
&\gamma_{[t]}^{[\mu_1]}\delta_t\frac{\partial\mu_1}{\partial t} + \gamma_{[x]}^{[\mu_1]}\delta_l\frac{\partial\mu_1}{\partial x} + \gamma_{[y]}^{[\mu_1]}\delta_l\frac{\partial\mu_1}{\partial y} + \gamma_{[t^2]}^{[\mu_1]}\delta_t^2\frac{\partial^2\mu_1}{\partial t^2} + \gamma_{[tx]}^{[\mu_1]}\delta_t\delta_l\frac{\partial^2\mu_1}{\partial t\partial x} \\
&+ \gamma_{[ty]}^{[\mu_1]}\delta_t\delta_l\frac{\partial^2\mu_1}{\partial t\partial y} + \gamma_{[x^2]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial x\partial y} + \gamma_{[y^2]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial y^2} = 0,
\end{aligned}$$

where

$$\gamma_{[t]}^{[\mu_1]} = -\omega_2\omega_3\omega_4\omega_1,$$

$$\gamma_{[x]}^{[\mu_1]} = -\omega_2u\omega_3\omega_4\omega_1,$$

$$\gamma_{[y]}^{[\mu_1]} = -\omega_2\omega_3v\omega_4\omega_1,$$

$$\gamma_{[t^2]}^{[\mu_1]} = -\omega_2\omega_4\omega_1 + \frac{7}{2}\omega_2\omega_3\omega_4\omega_1 - \omega_3\omega_4\omega_1 - \omega_2\omega_3\omega_1 - \omega_2\omega_3\omega_4,$$

$$\gamma_{[tx]}^{[\mu_1]} = \omega_2u\omega_4\omega_1 - \omega_2u\omega_3\omega_1 - 2\omega_2u\omega_3\omega_4 + 3\omega_2u\omega_3\omega_4\omega_1 - u\omega_3\omega_4\omega_1,$$

$$\gamma_{[ty]}^{[\mu_1]} = \omega_2\omega_3v\omega_1 - \omega_2\omega_3v\omega_4 + 3\omega_2\omega_3v\omega_4\omega_1 - \omega_2v\omega_4\omega_1 - 2\omega_3v\omega_4\omega_1,$$

$$\gamma_{[x^2]}^{[\mu_1]} = -\frac{1}{2}\omega_2u^2\omega_3\omega_4\omega_1 - \frac{1}{2}c_s^2\omega_2\omega_3\omega_4\omega_1 + c_s^2\omega_2\omega_3\omega_4 - \omega_2u^2\omega_3\omega_4 + 2\omega_2u^2\omega_4\omega_1,$$

$$\gamma_{[xy]}^{[\mu_1]} = -2u\omega_3v\omega_4\omega_1 + 2\omega_2uv\omega_4\omega_1 + 2\omega_2u\omega_3v\omega_1 - 2\omega_2u\omega_3v\omega_4,$$

$$\gamma_{[y^2]}^{[\mu_1]} = -\omega_3v^2\omega_4\omega_1 + c_s^2\omega_3\omega_4\omega_1 - \frac{1}{2}c_s^2\omega_2\omega_3\omega_4\omega_1 - \frac{1}{2}\omega_2\omega_3v^2\omega_4\omega_1 + 2\omega_2\omega_3v^2\omega_1,$$

## 7.3 EPDE for $\mu_2$

$$\begin{aligned}
& \gamma_{[1]}^{[\mu_1]} \mu_1 + \gamma_{[1]}^{[\mu_2]} \mu_2 + \gamma_{[t]}^{[\mu_1]} \delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_2]} \delta_t \frac{\partial \mu_2}{\partial t} + \gamma_{[x]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[x]}^{[\mu_2]} \delta_l \frac{\partial \mu_2}{\partial x} + \gamma_{[y]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[y]}^{[\mu_2]} \delta_l \frac{\partial \mu_2}{\partial y} \\
& + \gamma_{[t^2]}^{[\mu_1]} \delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_2]} \delta_t^2 \frac{\partial^2 \mu_2}{\partial t^2} + \gamma_{[tx]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} + \gamma_{[tx]}^{[\mu_2]} \delta_t \delta_l \frac{\partial^2 \mu_2}{\partial t \partial x} + \gamma_{[ty]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[ty]}^{[\mu_2]} \delta_t \delta_l \frac{\partial^2 \mu_2}{\partial t \partial y} \\
& + \gamma_{[x^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_2]} \delta_l^2 \frac{\partial^2 \mu_2}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x \partial y} + \gamma_{[xy]}^{[\mu_2]} \delta_l^2 \frac{\partial^2 \mu_2}{\partial x \partial y} + \gamma_{[y^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_2]} \delta_l^2 \frac{\partial^2 \mu_2}{\partial y^2} = 0,
\end{aligned}$$

where

$$\begin{aligned}
\gamma_{[1]}^{[\mu_1]} &= -\omega_2 u \omega_3 \omega_4^2 \omega_1 - \omega_2 u \omega_3^2 \omega_4 \omega_1 - \omega_2^2 u \omega_3 \omega_4 \omega_1 + 5\omega_2 u \omega_3 \omega_4 \omega_1 - \omega_2 u \omega_3 \omega_4 \omega_1^2, \\
\gamma_{[1]}^{[\mu_2]} &= -5\omega_2 \omega_3 \omega_4 \omega_1 + \omega_2 \omega_3 \omega_4 \omega_1^2 + \omega_2^2 \omega_3 \omega_4 \omega_1 + \omega_2 \omega_3 \omega_4^2 \omega_1 + \omega_2 \omega_3^2 \omega_4 \omega_1, \\
\gamma_{[t]}^{[\mu_1]} &= -u \omega_3 \omega_4^2 \omega_1 - \omega_2^2 u \omega_3 \omega_1 - u \omega_3^2 \omega_4 \omega_1 - \omega_2 u \omega_4 \omega_1^2 + 5\omega_2 u \omega_4 \omega_1 + 2\omega_2 u \omega_3^2 \omega_4 + 4\omega_2 u \omega_3 \omega_4^2 \omega_1 - \omega_2 u \omega_3^2 \omega_1 + \\
& 4\omega_2 u \omega_3^2 \omega_4 \omega_1 + 5\omega_2 u \omega_3 \omega_1 + 4\omega_2^2 u \omega_3 \omega_4 \omega_1 - u \omega_3 \omega_4 \omega_1^2 - \omega_2 u \omega_4^2 \omega_1 - 24\omega_2 u \omega_3 \omega_4 \omega_1 + 4\omega_2 u \omega_3 \omega_4 \omega_1^2 - \\
& \omega_2^2 u \omega_4 \omega_1 + 5u \omega_3 \omega_4 \omega_1 - \omega_2 u \omega_3 \omega_1^2, \\
\gamma_{[t]}^{[\mu_2]} &= -5\omega_2 \omega_4 \omega_1 + \omega_2^2 \omega_3 \omega_1 + \omega_2 \omega_3^2 \omega_4 + \omega_2 \omega_3^2 \omega_1 + 23\omega_2 \omega_3 \omega_4 \omega_1 + \omega_2^2 \omega_3 \omega_4 + \omega_3^2 \omega_4 \omega_1 - 4\omega_2 \omega_3 \omega_4 \omega_1^2 + \omega_3 \omega_4^2 \omega_1 + \\
& \omega_2 \omega_4 \omega_1^2 - 5\omega_3 \omega_4 \omega_1 + \omega_2 \omega_3 \omega_4^2 - 4\omega_2^2 \omega_3 \omega_4 \omega_1 + \omega_2 \omega_3 \omega_1^2 - 5\omega_2 \omega_3 \omega_1 - 4\omega_2 \omega_3 \omega_4^2 \omega_1 + \omega_2^2 \omega_4 \omega_1 - 4\omega_2 \omega_3^2 \omega_4 \omega_1 + \\
& \omega_2 \omega_4^2 \omega_1 + \omega_3 \omega_4 \omega_1^2 - 5\omega_2 \omega_3 \omega_4, \\
\gamma_{[x]}^{[\mu_1]} &= c_s^2 \omega_2 \omega_3 \omega_4^2 - \omega_2 u^2 \omega_3 \omega_4^2 + 2\omega_2^2 u^2 \omega_4 \omega_1 + \omega_2 u^2 \omega_3 \omega_4 \omega_1 + c_s^2 \omega_2 \omega_3 \omega_4 \omega_1 + 2\omega_2 u^2 \omega_4^2 \omega_1 - 5c_s^2 \omega_2 \omega_3 \omega_4 + \\
& 5\omega_2 u^2 \omega_3 \omega_4 - 10\omega_2 u^2 \omega_4 \omega_1 - \omega_2 u^2 \omega_3^2 \omega_4 + c_s^2 \omega_2 \omega_3^2 \omega_4 + c_s^2 \omega_2^2 \omega_3 \omega_4 - \omega_2^2 u^2 \omega_3 \omega_4 + 2\omega_2 u^2 \omega_4 \omega_1^2, \\
\gamma_{[x]}^{[\mu_2]} &= 2\omega_2^2 u \omega_3 \omega_4 - 2\omega_2 u \omega_4 \omega_1^2 + 10\omega_2 u \omega_4 \omega_1 + 4\omega_2 u \omega_3^2 \omega_4 - 2\omega_2 u \omega_4^2 \omega_1 - 10\omega_2 u \omega_3 \omega_4 - 2\omega_2 u \omega_3 \omega_4 \omega_1 + \\
& 2\omega_2 u \omega_3 \omega_4^2 - 2\omega_2^2 u \omega_4 \omega_1, \\
\gamma_{[y]}^{[\mu_1]} &= 10u \omega_3 v \omega_4 \omega_1 + 2\omega_2 u \omega_3^2 v \omega_1 - 2u \omega_3 v \omega_4 \omega_1^2 + 2\omega_2 u \omega_3^2 v \omega_4 - 2u \omega_3^2 v \omega_4 \omega_1 - 10\omega_2 u \omega_3 v \omega_1 + 2\omega_2^2 u \omega_3 v \omega_1 - \\
& 4u \omega_3 v \omega_4^2 \omega_1 + 2\omega_2 u \omega_3 v \omega_1^2, \\
\gamma_{[y]}^{[\mu_2]} &= 10\omega_2 \omega_3 v \omega_1 + 4\omega_3 v \omega_4^2 \omega_1 + 2\omega_3^2 v \omega_4 \omega_1 - 2\omega_2 \omega_3 v \omega_1^2 - 2\omega_2^2 \omega_3 v \omega_1 - 2\omega_2 \omega_3 v \omega_4 \omega_1 + 2\omega_3 v \omega_4 \omega_1^2 - \\
& 10\omega_3 v \omega_4 \omega_1 - 2\omega_2 \omega_3^2 v \omega_1, \\
\gamma_{[t^2]}^{[\mu_1]} &= -u \omega_4^2 \omega_1 + \frac{7}{2} u \omega_3 \omega_4^2 \omega_1 + \frac{7}{2} \omega_2^2 u \omega_3 \omega_1 + \frac{7}{2} u \omega_3^2 \omega_4 \omega_1 + 5u \omega_3 \omega_1 + \frac{7}{2} \omega_2 u \omega_4 \omega_1^2 - \omega_2^2 u \omega_1 - \frac{41}{2} \omega_2 u \omega_4 \omega_1 - u \omega_3 \omega_1^2 - \\
& 7\omega_2 u \omega_3^2 \omega_4 - 8\omega_2 u \omega_3 \omega_4^2 \omega_1 + 2\omega_2 u \omega_3^2 + \frac{7}{2} \omega_2 u \omega_3^2 \omega_1 - 8\omega_2 u \omega_3^2 \omega_4 \omega_1 - \omega_2 u \omega_1^2 - \frac{41}{2} \omega_2 u \omega_3 \omega_1 - 8\omega_2^2 u \omega_3 \omega_4 \omega_1 - \\
& u \omega_4 \omega_1^2 + \frac{7}{2} u \omega_3 \omega_4 \omega_1^2 + \frac{7}{2} \omega_2 u \omega_4^2 \omega_1 + 2\omega_2 u \omega_3 \omega_4 + 54\omega_2 u \omega_3 \omega_4 \omega_1 - u \omega_3^2 \omega_1 - 8\omega_2 u \omega_3 \omega_4 \omega_1^2 + \frac{7}{2} \omega_2^2 u \omega_4 \omega_1 + \\
& 5u \omega_4 \omega_1 - \frac{41}{2} u \omega_3 \omega_4 \omega_1 + 5\omega_2 u \omega_1 + 2u \omega_3^2 \omega_4 + \frac{7}{2} \omega_2 u \omega_3 \omega_1^2, \\
\gamma_{[t^2]}^{[\mu_2]} &= \frac{39}{2} \omega_2 \omega_4 \omega_1 - \frac{7}{2} \omega_2^2 \omega_3 \omega_1 - 5\omega_3 \omega_1 + \omega_2 \omega_4^2 - \frac{7}{2} \omega_2 \omega_3^2 \omega_4 + \omega_2 \omega_1^2 - \frac{7}{2} \omega_2 \omega_3^2 \omega_1 - 5\omega_3 \omega_4 - \frac{101}{2} \omega_2 \omega_3 \omega_4 \omega_1 + \omega_4^2 \omega_1 - \\
& \frac{7}{2} \omega_2^2 \omega_3 \omega_4 - \frac{7}{2} \omega_3^2 \omega_4 \omega_1 - 5\omega_2 \omega_1 + 8\omega_2 \omega_3 \omega_4 \omega_1^2 + \omega_3 \omega_4^2 - \frac{7}{2} \omega_3 \omega_4^2 \omega_1 + \omega_3 \omega_1^2 + \omega_2^2 \omega_3 - 5\omega_2 \omega_4 - \frac{7}{2} \omega_2 \omega_4 \omega_1^2 + \\
& \frac{39}{2} \omega_3 \omega_4 \omega_1 + \omega_2^2 \omega_4 - \frac{7}{2} \omega_2 \omega_3 \omega_4^2 - 5\omega_2 \omega_3 + \omega_4 \omega_1^2 + \omega_2^2 \omega_1 + 8\omega_2^2 \omega_3 \omega_4 \omega_1 - \frac{7}{2} \omega_2 \omega_3 \omega_1^2 + \frac{39}{2} \omega_2 \omega_3 \omega_1 + \\
& 8\omega_2 \omega_3 \omega_4^2 \omega_1 + \omega_3^2 \omega_4 - \frac{7}{2} \omega_2^2 \omega_4 \omega_1 - 5\omega_4 \omega_1 + 8\omega_2 \omega_3^2 \omega_4 \omega_1 - \frac{7}{2} \omega_2 \omega_4^2 \omega_1 - \frac{7}{2} \omega_3 \omega_4 \omega_1^2 + \omega_3^2 \omega_1 + \frac{39}{2} \omega_2 \omega_3 \omega_4 + \omega_2 \omega_3^2, \\
\gamma_{[tx]}^{[\mu_1]} &= \omega_2 \omega_4 \omega_1 - \omega_2^2 u^2 \omega_3 + c_s^2 \omega_2 \omega_3 \omega_4 \omega_1 - 3c_s^2 \omega_2 \omega_3 \omega_4^2 + 3\omega_2 u^2 \omega_3 \omega_4^2 - 10\omega_2 u^2 \omega_1 - 6\omega_2^2 u^2 \omega_4 \omega_1 + c_s^2 \omega_2 \omega_3 \omega_4^2 - \\
& 3\omega_2 u^2 \omega_3 \omega_4 \omega_1 - 3c_s^2 \omega_2 \omega_3 \omega_4 \omega_1 + c_s^2 \omega_2^2 \omega_3 - 10u^2 \omega_4 \omega_1 - u^2 \omega_3^2 \omega_4 + \omega_2 u^2 \omega_3 \omega_1 + c_s^2 \omega_2 \omega_3 \omega_1 + 2u^2 \omega_4 \omega_1^2 + \\
& 2\omega_2 u^2 \omega_1^2 - 6\omega_2 u^2 \omega_4^2 \omega_1 - 5c_s^2 \omega_3 \omega_4 - 2\omega_2 \omega_4 + u^2 \omega_3 \omega_4 \omega_1 + 16c_s^2 \omega_2 \omega_3 \omega_4 - 16\omega_2 u^2 \omega_3 \omega_4 + 32\omega_2 u^2 \omega_4 \omega_1 + \\
& c_s^2 \omega_3^2 \omega_4 + 3\omega_2 u^2 \omega_3^2 \omega_4 - 3c_s^2 \omega_2 \omega_3^2 \omega_4 - u^2 \omega_3 \omega_4^2 - 3c_s^2 \omega_2^2 \omega_3 \omega_4 - \omega_2 u^2 \omega_3^2 + c_s^2 \omega_2 \omega_3^2 + 2u^2 \omega_4^2 \omega_1 - \\
& 5c_s^2 \omega_2 \omega_3 + 5\omega_2 u^2 \omega_3 + 5u^2 \omega_3 \omega_4 + 2\omega_2^2 u^2 \omega_1 + 3\omega_2^2 u^2 \omega_3 \omega_4 - 6\omega_2 u^2 \omega_4 \omega_1^2 + \omega_2 \omega_3 \omega_4, \\
\gamma_{[tx]}^{[\mu_2]} &= -2u \omega_4^2 \omega_1 - 10u \omega_3 \omega_4 - 6\omega_2^2 u \omega_3 \omega_4 - 10\omega_2 u \omega_3 + 6\omega_2 u \omega_4 \omega_1^2 - 2\omega_2^2 u \omega_1 - 34\omega_2 u \omega_4 \omega_1 - 12\omega_2 u \omega_3^2 \omega_4 +
\end{aligned}$$

$$\begin{aligned}
& 4\omega_2 u \omega_3^2 + 2u \omega_3 \omega_4^2 - 2\omega_2 u \omega_1^2 - 2\omega_2 u \omega_3 \omega_1 - 2u \omega_4 \omega_1^2 + 6\omega_2 u \omega_4^2 \omega_1 + 34\omega_2 u \omega_3 \omega_4 + 6\omega_2 u \omega_3 \omega_4 \omega_1 - \\
& 6\omega_2 u \omega_3 \omega_4^2 + 6\omega_2^2 u \omega_4 \omega_1 + 10u \omega_4 \omega_1 + 2\omega_2^2 u \omega_3 - 2u \omega_3 \omega_4 \omega_1 + 10\omega_2 u \omega_1 + 4u \omega_3^2 \omega_4, \\
\gamma_{[ty]}^{[\mu_1]} &= 10uv \omega_4 \omega_1 + 2\omega_2 uv \omega_1^2 - 36u \omega_3 v \omega_4 \omega_1 + 4u \omega_3^2 v \omega_4 - 6\omega_2 u \omega_3^2 v \omega_1 + 6u \omega_3 v \omega_4 \omega_1^2 - 2uv \omega_4 \omega_1^2 - 10\omega_2 uv \omega_1 - \\
& 6\omega_2 u \omega_3^2 v \omega_4 + 6u \omega_3^2 v \omega_4 \omega_1 + 34\omega_2 u \omega_3 v \omega_1 + 2\omega_2 u \omega_3 v \omega_4 - 6\omega_2^2 u \omega_3 v \omega_1 - 2\omega_2 u \omega_3^2 v + 2\omega_2^2 uv \omega_1 - 4uv \omega_4^2 \omega_1 + \\
& 12u \omega_3 v \omega_4^2 \omega_1 - 6\omega_2 u \omega_3 v \omega_1^2, \\
\gamma_{[ty]}^{[\mu_2]} &= 2v \omega_4 \omega_1^2 - 34\omega_2 \omega_3 v \omega_1 - 2\omega_2 \omega_3 v \omega_4 - 12\omega_3 v \omega_4^2 \omega_1 + 10\omega_2 v \omega_1 - 2\omega_2 v \omega_1^2 + 2\omega_3^2 v \omega_4 - 6\omega_3^2 v \omega_4 \omega_1 + 10\omega_2 \omega_3 v - \\
& 10v \omega_4 \omega_1 + 6\omega_2 \omega_3 v \omega_1^2 + 6\omega_2^2 \omega_3 v \omega_1 - 2\omega_2 \omega_3^2 v - 10\omega_3 v \omega_4 + 4v \omega_4^2 \omega_1 + 6\omega_2 \omega_3 v \omega_4 \omega_1 - 2\omega_2^2 \omega_3 v - 6\omega_3 v \omega_4 \omega_1^2 - \\
& 2\omega_2 v \omega_4 \omega_1 + 34\omega_3 v \omega_4 \omega_1 + 6\omega_2 \omega_3^2 v \omega_1 + 4\omega_3 v \omega_4^2 - 2\omega_2^2 v \omega_1, \\
\gamma_{[x^2]}^{[\mu_1]} &= -\omega_2 u^3 \omega_3^2 \omega_4 - 2\omega_2 u^3 \omega_4 \omega_1^2 + \frac{3}{2}\omega_2 u \omega_4 \omega_1 + 3\omega_2 u^3 \omega_3 \omega_4 \omega_1 + c_s^2 \omega_2 u \omega_3^2 \omega_4 - \frac{1}{2}\omega_2 u \omega_4^2 \omega_1 - c_s^2 \omega_2 u \omega_3 \omega_4 \omega_1 - \\
& \frac{1}{2}\omega_2^2 u \omega_4 \omega_1, \\
\gamma_{[x^2]}^{[\mu_2]} &= -\frac{3}{2}\omega_2 \omega_4 \omega_1 - \omega_2 \omega_4^2 + \frac{1}{2}\omega_2 \omega_3^2 \omega_4 - 4\omega_2 u^2 \omega_3 \omega_4 \omega_1 + \frac{1}{2}\omega_2 \omega_3 \omega_4 \omega_1 + \frac{1}{2}\omega_2^2 \omega_3 \omega_4 + 3\omega_2 \omega_4 - \omega_2^2 \omega_4 + \frac{1}{2}\omega_2 \omega_3 \omega_4^2 + \\
& 2\omega_2 u^2 \omega_3^2 \omega_4 + \frac{1}{2}\omega_2^2 \omega_4 \omega_1 + \frac{1}{2}\omega_2 \omega_4^2 \omega_1 + 2\omega_2 u^2 \omega_4 \omega_1^2 - \frac{5}{2}\omega_2 \omega_3 \omega_4, \\
\gamma_{[xy]}^{[\mu_1]} &= 2\omega_2 u^2 \omega_3^2 v - 2c_s^2 \omega_2 \omega_3^2 v - 10c_s^2 \omega_3 v \omega_4 - 4u^2 \omega_3 v \omega_4^2 - 2\omega_2 v \omega_4 - 2\omega_2 u^2 \omega_3 v \omega_1 - 2c_s^2 \omega_2 \omega_3 v \omega_1 + \omega_2 \omega_3 v \omega_4 - \\
& 2c_s^2 \omega_2 \omega_3 v \omega_4 + 2\omega_2 u^2 \omega_3 v \omega_4 + 2u^2 \omega_3 v \omega_4 \omega_1 - 4\omega_2^2 u^2 v \omega_1 - 2c_s^2 \omega_2^2 \omega_3 v + 10u^2 \omega_3 v \omega_4 + 4c_s^2 \omega_3 v \omega_4^2 + \\
& 2\omega_2^2 u^2 \omega_3 v + 8u^2 v \omega_4^2 \omega_1 - 20u^2 v \omega_4 \omega_1 - 2u^2 \omega_3^2 v \omega_4 + 20\omega_2 u^2 v \omega_1 + \omega_2 v \omega_4 \omega_1 - 4\omega_2 u^2 v \omega_1^2 + 2c_s^2 \omega_3^2 v \omega_4 - \\
& 4\omega_2 u^2 v \omega_4 \omega_1 + 2c_s^2 \omega_3 v \omega_4 \omega_1 + 10c_s^2 \omega_2 \omega_3 v - 10\omega_2 u^2 \omega_3 v + 4u^2 v \omega_4 \omega_1^2, \\
\gamma_{[xy]}^{[\mu_2]} &= 20uv \omega_4 \omega_1 + 4\omega_2 uv \omega_1^2 - 4u \omega_3 v \omega_4 \omega_1 + 20\omega_2 u \omega_3 v + 8u \omega_3^2 v \omega_4 + 4\omega_2 uv \omega_4 \omega_1 - 4uv \omega_4 \omega_1^2 - 20\omega_2 uv \omega_1 + \\
& 4\omega_2 u \omega_3 v \omega_1 + 8u \omega_3 v \omega_4^2 - 4\omega_2 u \omega_3 v \omega_4 - 4\omega_2^2 u \omega_3 v - 8\omega_2 u \omega_3^2 v - 20u \omega_3 v \omega_4 + 4\omega_2^2 uv \omega_1 - 8uv \omega_4^2 \omega_1, \\
\gamma_{[y^2]}^{[\mu_1]} &= -\frac{1}{2}u \omega_3 \omega_4^2 \omega_1 - \frac{1}{2}u \omega_3^2 \omega_4 \omega_1 - 3u \omega_3 \omega_1 - 2u \omega_3 v^2 \omega_4^2 \omega_1 + c_s^2 \omega_2 u \omega_3^2 \omega_4 + u \omega_3 \omega_1^2 - 2\omega_2^2 u \omega_3 v^2 \omega_1 - \frac{1}{2}\omega_2 u \omega_3^2 \omega_1 + \\
& 4\omega_2 u \omega_3 v^2 \omega_1 + 3\omega_2 u \omega_3 v^2 \omega_4 \omega_1 - 2c_s^2 u \omega_3^2 \omega_4 + \frac{3}{2}\omega_2 u \omega_3 \omega_1 + \omega_2 u \omega_3^2 v^2 \omega_4 - \frac{1}{2}u \omega_3 \omega_4 \omega_1^2 - c_s^2 \omega_2 u \omega_3 \omega_4 \omega_1 - \\
& 4\omega_2 u \omega_3^2 v^2 - \frac{1}{2}\omega_2 u \omega_3 \omega_4 \omega_1 + u \omega_3^2 \omega_1 + 2u \omega_3^2 v^2 \omega_4 - 2u \omega_3 v^2 \omega_4 \omega_1 + 2c_s^2 u \omega_3 \omega_4 \omega_1 + \frac{5}{2}u \omega_3 \omega_4 \omega_1 - \frac{1}{2}\omega_2 u \omega_3 \omega_1^2. \\
\gamma_{[y^2]}^{[\mu_2]} &= 3\omega_3 \omega_1 + \frac{1}{2}\omega_2 \omega_3^2 \omega_1 + \frac{1}{2}\omega_2 \omega_3 \omega_4 \omega_1 + \frac{1}{2}\omega_3^2 \omega_4 \omega_1 - 4\omega_2 \omega_3 v^2 \omega_4 \omega_1 + \frac{1}{2}\omega_3 \omega_4^2 \omega_1 + 2\omega_2^2 \omega_3 v^2 \omega_1 - \omega_3 \omega_1^2 - \\
& \frac{5}{2}\omega_3 \omega_4 \omega_1 + \frac{1}{2}\omega_2 \omega_3 \omega_1^2 - \frac{3}{2}\omega_2 \omega_3 \omega_1 + 2\omega_3 v^2 \omega_4^2 \omega_1 + \frac{1}{2}\omega_3 \omega_4 \omega_1^2 - \omega_3^2 \omega_1,
\end{aligned}$$

## 7.4 EPDE for $\mu_3$

$$\begin{aligned}
& \gamma_{[1]}^{[\mu_1]} \mu_1 + \gamma_{[1]}^{[\mu_3]} \mu_3 + \gamma_{[t]}^{[\mu_1]} \delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_3]} \delta_t \frac{\partial \mu_3}{\partial t} + \gamma_{[x]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[x]}^{[\mu_3]} \delta_l \frac{\partial \mu_3}{\partial x} + \gamma_{[y]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[y]}^{[\mu_3]} \delta_l \frac{\partial \mu_3}{\partial y} \\
& + \gamma_{[t^2]}^{[\mu_1]} \delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_3]} \delta_t^2 \frac{\partial^2 \mu_3}{\partial t^2} + \gamma_{[tx]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} + \gamma_{[tx]}^{[\mu_3]} \delta_t \delta_l \frac{\partial^2 \mu_3}{\partial t \partial x} + \gamma_{[ty]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[ty]}^{[\mu_3]} \delta_t \delta_l \frac{\partial^2 \mu_3}{\partial t \partial y} \\
& + \gamma_{[x^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_3]} \delta_l^2 \frac{\partial^2 \mu_3}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x \partial y} + \gamma_{[xy]}^{[\mu_3]} \delta_l^2 \frac{\partial^2 \mu_3}{\partial x \partial y} + \gamma_{[y^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_3]} \delta_l^2 \frac{\partial^2 \mu_3}{\partial y^2} = 0,
\end{aligned}$$

where

$$\begin{aligned}
\gamma_{[1]}^{[\mu_1]} &= -\omega_2 \omega_3^2 v \omega_4 \omega_1 - \omega_2 \omega_3 v \omega_4^2 \omega_1 + 5\omega_2 \omega_3 v \omega_4 \omega_1 - \omega_2^2 \omega_3 v \omega_4 \omega_1 - \omega_2 \omega_3 v \omega_4 \omega_1^2, \\
\gamma_{[1]}^{[\mu_3]} &= -5\omega_2 \omega_3 \omega_4 \omega_1 + \omega_2 \omega_3 \omega_4 \omega_1^2 + \omega_2^2 \omega_3 \omega_4 \omega_1 + \omega_2 \omega_3 \omega_4^2 \omega_1 + \omega_2 \omega_3^2 \omega_4 \omega_1,
\end{aligned}$$



$$\begin{aligned}
\gamma_{[t]}^{[\mu_1]} &= 4\omega_2\omega_3^2v\omega_4\omega_1 + 5\omega_2\omega_3v\omega_1 + 5\omega_2\omega_3v\omega_4 - \omega_2v\omega_4^2\omega_1 + 2\omega_3v\omega_4^2\omega_1 - \omega_2\omega_3v\omega_4^2 - \omega_2^2v\omega_4\omega_1 - \omega_2\omega_3v\omega_1^2 + \\
&\quad 4\omega_2\omega_3v\omega_4^2\omega_1 - \omega_2^2\omega_3v\omega_1 - 24\omega_2\omega_3v\omega_4\omega_1 + 4\omega_2^2\omega_3v\omega_4\omega_1 - \omega_2v\omega_4\omega_1^2 - \omega_2^2\omega_3v\omega_4 + 5\omega_2v\omega_4\omega_1 - \omega_2\omega_3^2v\omega_1 + \\
&\quad 4\omega_2\omega_3v\omega_4\omega_1^2 - \omega_2\omega_3^2v\omega_4, \\
\gamma_{[t]}^{[\mu_3]} &= -5\omega_2\omega_4\omega_1 + \omega_2^2\omega_3\omega_1 + \omega_2\omega_3^2\omega_4 + \omega_2\omega_3^2\omega_1 + 23\omega_2\omega_3\omega_4\omega_1 + \omega_2^2\omega_3\omega_4 + \omega_3^2\omega_4\omega_1 - 4\omega_2\omega_3\omega_4\omega_1^2 + \omega_3\omega_4^2\omega_1 + \\
&\quad \omega_2\omega_4\omega_1^2 - 5\omega_3\omega_4\omega_1 + \omega_2\omega_3\omega_4^2 - 4\omega_2^2\omega_3\omega_4\omega_1 + \omega_2\omega_3\omega_1^2 - 5\omega_2\omega_3\omega_1 - 4\omega_2\omega_3\omega_4^2\omega_1 + \omega_2^2\omega_4\omega_1 - 4\omega_2\omega_3^2\omega_4\omega_1 + \\
&\quad \omega_2\omega_4^2\omega_1 + \omega_3\omega_4\omega_1^2 - 5\omega_2\omega_3\omega_4, \\
\gamma_{[x]}^{[\mu_1]} &= 2\omega_2uv\omega_4\omega_1^2 - 10\omega_2uv\omega_4\omega_1 + 2\omega_2^2uv\omega_4\omega_1 - 4\omega_2u\omega_3^2v\omega_4 - 2\omega_2^2u\omega_3v\omega_4 + 10\omega_2u\omega_3v\omega_4 + 2\omega_2uv\omega_4^2\omega_1 - \\
&\quad 2\omega_2u\omega_3v\omega_4^2 + 2u\omega_3v\omega_4^2\omega_1, \\
\gamma_{[x]}^{[\mu_3]} &= 2\omega_2^2u\omega_3\omega_4 - 2\omega_2u\omega_4\omega_1^2 + 10\omega_2u\omega_4\omega_1 + 4\omega_2u\omega_3^2\omega_4 - 2\omega_2u\omega_4^2\omega_1 - 10\omega_2u\omega_3\omega_4 - 2\omega_2u\omega_3\omega_4\omega_1 + \\
&\quad 2\omega_2u\omega_3\omega_4^2 - 2\omega_2^2u\omega_4\omega_1, \\
\gamma_{[y]}^{[\mu_1]} &= 5\omega_3v^2\omega_4\omega_1 - 5c_s^2\omega_3\omega_4\omega_1 + c_s^2\omega_2\omega_3\omega_4\omega_1 + 2\omega_2\omega_3^2v^2\omega_1 + \omega_2\omega_3v^2\omega_4\omega_1 + 2\omega_2^2\omega_3v^2\omega_1 - \omega_3v^2\omega_4\omega_1^2 + \\
&\quad c_s^2\omega_3\omega_4\omega_1^2 + 2\omega_2\omega_3v^2\omega_1^2 - \omega_3^2v^2\omega_4\omega_1 + c_s^2\omega_2^2\omega_3\omega_4\omega_1 + c_s^2\omega_3\omega_4^2\omega_1 - \omega_3v^2\omega_4^2\omega_1 - 10\omega_2\omega_3v^2\omega_1, \\
\gamma_{[y]}^{[\mu_3]} &= 10\omega_2\omega_3v\omega_1 + 4\omega_3v\omega_4^2\omega_1 + 2\omega_3^2v\omega_4\omega_1 - 2\omega_2\omega_3v\omega_1^2 - 2\omega_2^2\omega_3v\omega_1 - 2\omega_2\omega_3v\omega_4\omega_1 + 2\omega_3v\omega_4\omega_1^2 - \\
&\quad 10\omega_3v\omega_4\omega_1 - 2\omega_2\omega_3^2v\omega_1, \\
\gamma_{[t^2]}^{[\mu_1]} &= -8\omega_2\omega_3^2v\omega_4\omega_1 + 5\omega_2v\omega_4 - \frac{41}{2}\omega_2\omega_3v\omega_1 - \frac{41}{2}\omega_2\omega_3v\omega_4 + \frac{7}{2}\omega_2v\omega_4^2\omega_1 - 7\omega_3v\omega_4^2\omega_1 + 5\omega_2v\omega_1 - \omega_2v\omega_1^2 + \\
&\quad \frac{7}{2}\omega_2\omega_3v\omega_4^2 + \frac{7}{2}\omega_2^2v\omega_4\omega_1 + 5\omega_2\omega_3v + \frac{7}{2}\omega_2\omega_3v\omega_1^2 - 8\omega_2\omega_3v\omega_4^2\omega_1 - \omega_2v\omega_4^2 + \frac{7}{2}\omega_2^2\omega_3v\omega_1 - \omega_2\omega_3^2v + 2v\omega_4^2\omega_1 + \\
&\quad 54\omega_2\omega_3v\omega_4\omega_1 - \omega_2^2\omega_3v - 8\omega_2^2\omega_3v\omega_4\omega_1 + \frac{7}{2}\omega_2v\omega_4\omega_1^2 + \frac{7}{2}\omega_2^2\omega_3v\omega_4 - \frac{41}{2}\omega_2v\omega_4\omega_1 - \omega_2^2v\omega_4 + 2\omega_3v\omega_4\omega_1 + \\
&\quad \frac{7}{2}\omega_2\omega_3^2v\omega_1 - 8\omega_2\omega_3v\omega_4\omega_1^2 + \frac{7}{2}\omega_2\omega_3^2v\omega_4 + 2\omega_3v\omega_4^2 - \omega_2^2v\omega_1, \\
\gamma_{[t^2]}^{[\mu_3]} &= \frac{39}{2}\omega_2\omega_4\omega_1 - \frac{7}{2}\omega_2^2\omega_3\omega_1 - 5\omega_3\omega_1 + \omega_2\omega_4^2 - \frac{7}{2}\omega_2\omega_3^2\omega_4 + \omega_2\omega_1^2 - \frac{7}{2}\omega_2\omega_3^2\omega_1 - 5\omega_3\omega_4 - \frac{101}{2}\omega_2\omega_3\omega_4\omega_1 + \omega_4^2\omega_1 - \\
&\quad \frac{7}{2}\omega_2^2\omega_3\omega_4 - \frac{7}{2}\omega_3^2\omega_4\omega_1 - 5\omega_2\omega_1 + 8\omega_2\omega_3\omega_4\omega_1^2 + \omega_3\omega_4^2 - \frac{7}{2}\omega_3\omega_4^2\omega_1 + \omega_3\omega_1^2 + \omega_2^2\omega_3 - 5\omega_2\omega_4 - \frac{7}{2}\omega_2\omega_4\omega_1^2 + \\
&\quad \frac{39}{2}\omega_3\omega_4\omega_1 + \omega_2^2\omega_4 - \frac{7}{2}\omega_2\omega_3\omega_4^2 - 5\omega_2\omega_3 + \omega_4\omega_1^2 + \omega_2^2\omega_1 + 8\omega_2^2\omega_3\omega_4\omega_1 - \frac{7}{2}\omega_2\omega_3\omega_1^2 + \frac{39}{2}\omega_2\omega_3\omega_1 + \\
&\quad 8\omega_2\omega_3\omega_4^2\omega_1 + \omega_3^2\omega_4 - \frac{7}{2}\omega_2^2\omega_4\omega_1 - 5\omega_4\omega_1 + 8\omega_2\omega_3^2\omega_4\omega_1 - \frac{7}{2}\omega_2\omega_4^2\omega_1 - \frac{7}{2}\omega_3\omega_4\omega_1^2 + \omega_3^2\omega_1 + \frac{39}{2}\omega_2\omega_3\omega_4 + \omega_2\omega_3^2, \\
\gamma_{[tx]}^{[\mu_1]} &= 2\omega_2uv\omega_1^2 + 2u\omega_3v\omega_4\omega_1 - 6\omega_2uv\omega_4\omega_1^2 + 10\omega_2u\omega_3v + 34\omega_2uv\omega_4\omega_1 - 6\omega_2^2uv\omega_4\omega_1 - 10\omega_2uv\omega_1 + \\
&\quad 12\omega_2u\omega_3^2v\omega_4 + 6\omega_2^2u\omega_3v\omega_4 + 4u\omega_3v\omega_4^2 - 36\omega_2u\omega_3v\omega_4 - 2\omega_2^2u\omega_3v - 6\omega_2uv\omega_4^2\omega_1 + 6\omega_2u\omega_3v\omega_4^2 - 4\omega_2u\omega_3^2v + \\
&\quad 2\omega_2^2uv\omega_1 - 2uv\omega_4^2\omega_1 - 6u\omega_3v\omega_4^2\omega_1, \\
\gamma_{[tx]}^{[\mu_3]} &= -2u\omega_4^2\omega_1 - 10u\omega_3\omega_4 - 6\omega_2^2u\omega_3\omega_4 - 10\omega_2u\omega_3 + 6\omega_2u\omega_4\omega_1^2 - 2\omega_2^2u\omega_1 - 34\omega_2u\omega_4\omega_1 - 12\omega_2u\omega_3^2\omega_4 + \\
&\quad 4\omega_2u\omega_3^2 + 2u\omega_3\omega_4^2 - 2\omega_2u\omega_1^2 - 2\omega_2u\omega_3\omega_1 - 2u\omega_4\omega_1^2 + 6\omega_2u\omega_4^2\omega_1 + 34\omega_2u\omega_3\omega_4 + 6\omega_2u\omega_3\omega_4\omega_1 - \\
&\quad 6\omega_2u\omega_3\omega_4^2 + 6\omega_2^2u\omega_4\omega_1 + 10u\omega_4\omega_1 + 2\omega_2^2u\omega_3 - 2u\omega_3\omega_4\omega_1 + 10\omega_2u\omega_1 + 4u\omega_3^2\omega_4, \\
\gamma_{[ty]}^{[\mu_1]} &= -16\omega_3v^2\omega_4\omega_1 + 16c_s^2\omega_3\omega_4\omega_1 + \omega_2v^2\omega_4\omega_1 - 2\omega_3\omega_1 - 10\omega_2\omega_3v^2 + 2\omega_2^2v^2\omega_1 - \omega_3v^2\omega_4^2 + c_s^2\omega_3\omega_4^2 - \\
&\quad 3c_s^2\omega_2\omega_3\omega_4\omega_1 - 6\omega_2\omega_3^2v^2\omega_1 - v^2\omega_4^2\omega_1 - 3\omega_2\omega_3v^2\omega_4\omega_1 - 6\omega_2^2\omega_3v^2\omega_1 - 5c_s^2\omega_3\omega_4 + 5\omega_3v^2\omega_4 + \\
&\quad 3\omega_3v^2\omega_4\omega_1^2 + c_s^2\omega_4^2\omega_1 - 3c_s^2\omega_3\omega_4\omega_1^2 + c_s^2\omega_2\omega_3\omega_4 + c_s^2\omega_2\omega_4\omega_1 + \omega_3\omega_4\omega_1 - 6\omega_2\omega_3v^2\omega_1^2 + c_s^2\omega_3^2\omega_4 - \\
&\quad 5c_s^2\omega_4\omega_1 + 3\omega_3^2v^2\omega_4\omega_1 - \omega_3^2v^2\omega_4 + 5v^2\omega_4\omega_1 + 2\omega_2v^2\omega_1^2 + \omega_2\omega_3\omega_1 + \omega_2\omega_3v^2\omega_4 - 3c_s^2\omega_3^2\omega_4\omega_1 - \\
&\quad 10\omega_2v^2\omega_1 - v^2\omega_4\omega_1^2 + 2\omega_2^2\omega_3v^2 - 3c_s^2\omega_3\omega_4^2\omega_1 + 3\omega_3v^2\omega_4^2\omega_1 + c_s^2\omega_4\omega_1^2 + 2\omega_2\omega_3^2v^2 + 32\omega_2\omega_3v^2\omega_1, \\
\gamma_{[ty]}^{[\mu_3]} &= 2v\omega_4\omega_1^2 - 34\omega_2\omega_3v\omega_1 - 2\omega_2\omega_3v\omega_4 - 12\omega_3v\omega_4^2\omega_1 + 10\omega_2v\omega_1 - 2\omega_2v\omega_1^2 + 2\omega_3^2v\omega_4 - 6\omega_3^2v\omega_4\omega_1 + 10\omega_2\omega_3v - \\
&\quad 10v\omega_4\omega_1 + 6\omega_2\omega_3v\omega_1^2 + 6\omega_2^2\omega_3v\omega_1 - 2\omega_2\omega_3^2v - 10\omega_3v\omega_4 + 4v\omega_4^2\omega_1 + 6\omega_2\omega_3v\omega_4\omega_1 - 2\omega_2^2\omega_3v - 6\omega_3v\omega_4\omega_1^2 - \\
&\quad 2\omega_2v\omega_4\omega_1 + 34\omega_3v\omega_4\omega_1 + 6\omega_2\omega_3^2v\omega_1 + 4\omega_3v\omega_4^2 - 2\omega_2^2v\omega_1, \\
\gamma_{[x^2]}^{[\mu_1]} &= 2u^2\omega_3v\omega_4^2 - 3\omega_2v\omega_4 + \frac{5}{2}\omega_2\omega_3v\omega_4 - \frac{1}{2}\omega_2v\omega_4^2\omega_1 + 2c_s^2\omega_2\omega_3v\omega_4 - 2\omega_2u^2\omega_3v\omega_4 + c_s^2\omega_3v\omega_4^2\omega_1 - \frac{1}{2}\omega_2\omega_3v\omega_4^2 - \\
&\quad \frac{1}{2}\omega_2^2v\omega_4\omega_1 - 2c_s^2\omega_3v\omega_4^2 + \omega_2v\omega_4^2 - 4u^2v\omega_4^2\omega_1 + 3\omega_2u^2\omega_3v\omega_4\omega_1 - c_s^2\omega_2\omega_3v\omega_4\omega_1 - \frac{1}{2}\omega_2\omega_3v\omega_4\omega_1 - \\
&\quad \frac{1}{2}\omega_2^2\omega_3v\omega_4 - 2\omega_2u^2v\omega_4\omega_1^2 + \frac{3}{2}\omega_2v\omega_4\omega_1 + \omega_2^2v\omega_4 + 4\omega_2u^2v\omega_4\omega_1 + u^2\omega_3v\omega_4^2\omega_1 - 2\omega_2u^2\omega_3^2v\omega_4 - \frac{1}{2}\omega_2\omega_3^2v\omega_4,
\end{aligned}$$

$$\begin{aligned}
\gamma_{[x^2]}^{[\mu_3]} &= -\frac{3}{2}\omega_2\omega_4\omega_1 - \omega_2\omega_4^2 + \frac{1}{2}\omega_2\omega_3^2\omega_4 - 4\omega_2u^2\omega_3\omega_4\omega_1 + \frac{1}{2}\omega_2\omega_3\omega_4\omega_1 + \frac{1}{2}\omega_2^2\omega_3\omega_4 + 3\omega_2\omega_4 - \omega_2^2\omega_4 + \frac{1}{2}\omega_2\omega_3\omega_4^2 + \\
&\quad 2\omega_2u^2\omega_3^2\omega_4 + \frac{1}{2}\omega_2^2\omega_4\omega_1 + \frac{1}{2}\omega_2\omega_4^2\omega_1 + 2\omega_2u^2\omega_4\omega_1^2 - \frac{5}{2}\omega_2\omega_3\omega_4, \\
\gamma_{[xy]}^{[\mu_1]} &= -4\omega_2^2uv^2\omega_1 + 10u\omega_3v^2\omega_4 - 2c_s^2u\omega_4\omega_1^2 - 2u\omega_3\omega_1 + 2uv^2\omega_4^2\omega_1 - 2c_s^2\omega_2u\omega_4\omega_1 + 2\omega_2u\omega_3v^2\omega_4 - \\
&\quad 20\omega_2u\omega_3v^2 + 10c_s^2u\omega_4\omega_1 - 4\omega_2u\omega_3v^2\omega_1 - 2u\omega_3v^2\omega_4^2 + 4c_s^2u\omega_3^2\omega_4 - 2c_s^2u\omega_4^2\omega_1 + \omega_2u\omega_3\omega_1 - \\
&\quad 10c_s^2u\omega_3\omega_4 - 2\omega_2uv^2\omega_4\omega_1 + 20\omega_2uv^2\omega_1 + 2c_s^2\omega_2u\omega_3\omega_4 + 8\omega_2u\omega_3^2v^2 + 2uv^2\omega_4\omega_1^2 + 4\omega_2^2u\omega_3v^2 - \\
&\quad 4u\omega_3^2v^2\omega_4 - 10uv^2\omega_4\omega_1 + 2u\omega_3v^2\omega_4\omega_1 + 2c_s^2u\omega_3\omega_4^2 - 2c_s^2u\omega_3\omega_4\omega_1 - 4\omega_2uv^2\omega_1^2 + u\omega_3\omega_4\omega_1, \\
\gamma_{[xy]}^{[\mu_3]} &= 20uv\omega_4\omega_1 + 4\omega_2uv\omega_1^2 - 4u\omega_3v\omega_4\omega_1 + 20\omega_2u\omega_3v + 8u\omega_3^2v\omega_4 + 4\omega_2uv\omega_4\omega_1 - 4uv\omega_4\omega_1^2 - 20\omega_2uv\omega_1 + \\
&\quad 4\omega_2u\omega_3v\omega_1 + 8u\omega_3v\omega_4^2 - 4\omega_2u\omega_3v\omega_4 - 4\omega_2^2u\omega_3v - 8\omega_2u\omega_3^2v - 20u\omega_3v\omega_4 + 4\omega_2^2uv\omega_1 - 8uv\omega_4^2\omega_1, \\
\gamma_{[y^2]}^{[\mu_1]} &= \frac{3}{2}\omega_2\omega_3v\omega_1 + c_s^2\omega_3v\omega_4^2\omega_1 - 2\omega_2^2\omega_3v^3\omega_1 - \omega_3v^3\omega_4^2\omega_1 - \frac{1}{2}\omega_2\omega_3v\omega_1^2 - c_s^2\omega_2\omega_3v\omega_4\omega_1 - \frac{1}{2}\omega_2\omega_3^2v\omega_1 + 3\omega_2\omega_3v^3\omega_4\omega_1, \\
\gamma_{[y^2]}^{[\mu_3]} &= 3\omega_3\omega_1 + \frac{1}{2}\omega_2\omega_3^2\omega_1 + \frac{1}{2}\omega_2\omega_3\omega_4\omega_1 + \frac{1}{2}\omega_3^2\omega_4\omega_1 - 4\omega_2\omega_3v^2\omega_4\omega_1 + \frac{1}{2}\omega_3\omega_4^2\omega_1 + 2\omega_2^2\omega_3v^2\omega_1 - \omega_3\omega_1^2 - \\
&\quad \frac{5}{2}\omega_3\omega_4\omega_1 + \frac{1}{2}\omega_2\omega_3\omega_1^2 - \frac{3}{2}\omega_2\omega_3\omega_1 + 2\omega_3v^2\omega_4^2\omega_1 + \frac{1}{2}\omega_3\omega_4\omega_1^2 - \omega_3^2\omega_1,
\end{aligned}$$

## 7.5 EPDE for $\mu_4$

$$\begin{aligned}
&\gamma_{[1]}^{[\mu_1]}\mu_1 + \gamma_{[1]}^{[\mu_4]}\mu_4 + \gamma_{[t]}^{[\mu_1]}\delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_4]}\delta_t \frac{\partial \mu_4}{\partial t} + \gamma_{[x]}^{[\mu_1]}\delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[x]}^{[\mu_4]}\delta_l \frac{\partial \mu_4}{\partial x} + \gamma_{[y]}^{[\mu_1]}\delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[y]}^{[\mu_4]}\delta_l \frac{\partial \mu_4}{\partial y} \\
&\quad + \gamma_{[t^2]}^{[\mu_1]}\delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_4]}\delta_t^2 \frac{\partial^2 \mu_4}{\partial t^2} + \gamma_{[tx]}^{[\mu_1]}\delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} + \gamma_{[tx]}^{[\mu_4]}\delta_t \delta_l \frac{\partial^2 \mu_4}{\partial t \partial x} + \gamma_{[ty]}^{[\mu_1]}\delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[ty]}^{[\mu_4]}\delta_t \delta_l \frac{\partial^2 \mu_4}{\partial t \partial y} \\
&\quad + \gamma_{[x^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_4]}\delta_l^2 \frac{\partial^2 \mu_4}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial x \partial y} + \gamma_{[xy]}^{[\mu_4]}\delta_l^2 \frac{\partial^2 \mu_4}{\partial x \partial y} + \gamma_{[y^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_4]}\delta_l^2 \frac{\partial^2 \mu_4}{\partial y^2} = 0,
\end{aligned}$$

where

$$\begin{aligned}
\gamma_{[1]}^{[\mu_1]} &= -\omega_2^2u^2\omega_3\omega_4\omega_1 + 5\omega_2u^2\omega_3\omega_4\omega_1 + 5c_s^2\omega_2\omega_3\omega_4\omega_1 - \omega_2u^2\omega_3\omega_4\omega_1^2 - c_s^2\omega_2\omega_3\omega_4\omega_1^2 - c_s^2\omega_2^2\omega_3\omega_4\omega_1 - \\
&\quad \omega_2u^2\omega_3\omega_4^2\omega_1 - c_s^2\omega_2\omega_3\omega_4^2\omega_1 - c_s^2\omega_2\omega_3^2\omega_4\omega_1 - \omega_2u^2\omega_3^2\omega_4\omega_1, \\
\gamma_{[1]}^{[\mu_4]} &= -5\omega_2\omega_3\omega_4\omega_1 + \omega_2\omega_3\omega_4\omega_1^2 + \omega_2^2\omega_3\omega_4\omega_1 + \omega_2\omega_3\omega_4^2\omega_1 + \omega_2\omega_3^2\omega_4\omega_1, \\
\gamma_{[t]}^{[\mu_1]} &= 2\omega_2\omega_4\omega_1 + 5c_s^2\omega_3\omega_4\omega_1 - u^2\omega_3\omega_4\omega_1^2 - c_s^2\omega_2\omega_3\omega_4^2 + \omega_2u^2\omega_3\omega_4^2 + 4\omega_2^2u^2\omega_3\omega_4\omega_1 - 2\omega_2^2u^2\omega_4\omega_1 - \\
&\quad 30\omega_2u^2\omega_3\omega_4\omega_1 - 22c_s^2\omega_2\omega_3\omega_4\omega_1 - \omega_2\omega_3\omega_4\omega_1 - \omega_2u^2\omega_3\omega_1^2 - c_s^2\omega_2\omega_3\omega_1^2 + 5\omega_2u^2\omega_3\omega_1 + 5c_s^2\omega_2\omega_3\omega_1 + \\
&\quad 4\omega_2u^2\omega_3\omega_4\omega_1^2 + 4c_s^2\omega_2\omega_3\omega_4\omega_1^2 - 2\omega_2u^2\omega_4^2\omega_1 - \omega_2\omega_4\omega_1^2 + 5u^2\omega_3\omega_4\omega_1 - c_s^2\omega_3\omega_4\omega_1^2 + 5c_s^2\omega_2\omega_3\omega_4 - \\
&\quad 5\omega_2u^2\omega_3\omega_4 + 10\omega_2u^2\omega_4\omega_1 - c_s^2\omega_2^2\omega_3\omega_1 - u^2\omega_3\omega_4^2\omega_1 + 5\omega_2u^2\omega_3^2\omega_4 - c_s^2\omega_2\omega_3^2\omega_4 - c_s^2\omega_2\omega_3^2\omega_1 - \\
&\quad \omega_2u^2\omega_3^2\omega_1 + 4c_s^2\omega_2^2\omega_3\omega_4\omega_1 - c_s^2\omega_2^2\omega_3\omega_4 - u^2\omega_3^2\omega_4\omega_1 + 4\omega_2u^2\omega_3\omega_4^2\omega_1 - c_s^2\omega_3^2\omega_4\omega_1 + 4c_s^2\omega_2\omega_3\omega_4^2\omega_1 - \\
&\quad \omega_2^2u^2\omega_3\omega_1 - c_s^2\omega_3\omega_4^2\omega_1 + \omega_2^2u^2\omega_3\omega_4 + 4c_s^2\omega_2\omega_3^2\omega_4\omega_1 + 4\omega_2u^2\omega_3^2\omega_4\omega_1 + 2\omega_2u^2\omega_4\omega_1^2, \\
\gamma_{[t]}^{[\mu_4]} &= -5\omega_2\omega_4\omega_1 + \omega_2^2\omega_3\omega_1 + \omega_2\omega_3^2\omega_4 + \omega_2\omega_3^2\omega_1 + 23\omega_2\omega_3\omega_4\omega_1 + \omega_2^2\omega_3\omega_4 + \omega_3^2\omega_4\omega_1 - 4\omega_2\omega_3\omega_4\omega_1^2 + \omega_3\omega_4^2\omega_1 + \\
&\quad \omega_2\omega_4\omega_1^2 - 5\omega_3\omega_4\omega_1 + \omega_2\omega_3\omega_4^2 - 4\omega_2^2\omega_3\omega_4\omega_1 + \omega_2\omega_3\omega_1^2 - 5\omega_2\omega_3\omega_1 - 4\omega_2\omega_3\omega_4^2\omega_1 + \omega_2^2\omega_4\omega_1 - 4\omega_2\omega_3^2\omega_4\omega_1 + \\
&\quad \omega_2\omega_4^2\omega_1 + \omega_3\omega_4\omega_1^2 - 5\omega_2\omega_3\omega_4, \\
\gamma_{[x]}^{[\mu_1]} &= \\
&\quad 2\omega_2u^3\omega_3^2\omega_4 + 4\omega_2u^3\omega_4\omega_1^2 - 3\omega_2u\omega_4\omega_1 - 6\omega_2u^3\omega_3\omega_4\omega_1 - 2c_s^2\omega_2u\omega_3^2\omega_4 + \omega_2u\omega_4^2\omega_1 + 2c_s^2\omega_2u\omega_3\omega_4\omega_1 + \omega_2^2u\omega_4\omega_1,
\end{aligned}$$

$$\begin{aligned}
\gamma_{[x]}^{[\mu_4]} &= 2\omega_2^2 u \omega_3 \omega_4 - 2\omega_2 u \omega_4 \omega_1^2 + 10\omega_2 u \omega_4 \omega_1 + 4\omega_2 u \omega_3^2 \omega_4 - 2\omega_2 u \omega_4^2 \omega_1 - 10\omega_2 u \omega_3 \omega_4 \omega_1 + \\
&\quad 2\omega_2 u \omega_3 \omega_4^2 - 2\omega_2^2 u \omega_4 \omega_1, \\
\gamma_{[y]}^{[\mu_1]} &= -10\omega_2 u^2 \omega_3 v \omega_1 - 10c_s^2 \omega_2 \omega_3 v \omega_1 + 2\omega_2^2 u^2 \omega_3 v \omega_1 + 10u^2 \omega_3 v \omega_4 \omega_1 - 4c_s^2 \omega_3 v \omega_4^2 \omega_1 - 2c_s^2 \omega_3^2 v \omega_4 \omega_1 - \\
&\quad 2u^2 \omega_3 v \omega_4 \omega_1^2 + 2\omega_2 u^2 \omega_3 v \omega_1^2 + 2c_s^2 \omega_2 \omega_3 v \omega_1^2 + 2c_s^2 \omega_2^2 \omega_3 v \omega_1 - 6\omega_2 u^2 \omega_3 v \omega_4 \omega_1 + 2c_s^2 \omega_2 \omega_3 v \omega_4 \omega_1 - \\
&\quad \omega_2 \omega_3 v \omega_4 \omega_1 - \omega_2 v \omega_4 \omega_1^2 - 2u^2 \omega_3^2 v \omega_4 \omega_1 + 4\omega_2 u^2 v \omega_4 \omega_1^2 - 2c_s^2 \omega_3 v \omega_4 \omega_1^2 + 2\omega_2 v \omega_4 \omega_1 + 10c_s^2 \omega_3 v \omega_4 \omega_1 - \\
&\quad 4u^2 \omega_3 v \omega_4^2 \omega_1 + 2c_s^2 \omega_2 \omega_3^2 v \omega_1 + 2\omega_2 u^2 \omega_3^2 v \omega_1 + 4\omega_2 u^2 \omega_3^2 v \omega_4, \\
\gamma_{[y]}^{[\mu_4]} &= 10\omega_2 \omega_3 v \omega_1 + 4\omega_3 v \omega_4^2 \omega_1 + 2\omega_3^2 v \omega_4 \omega_1 - 2\omega_2 \omega_3 v \omega_1^2 - 2\omega_2^2 \omega_3 v \omega_1 - 2\omega_2 \omega_3 v \omega_4 \omega_1 + 2\omega_3 v \omega_4 \omega_1^2 - \\
&\quad 10\omega_3 v \omega_4 \omega_1 - 2\omega_2 \omega_3^2 v \omega_1, \\
\gamma_{[t^2]}^{[\mu_1]} &= -8\omega_2 \omega_4 \omega_1 + \omega_2^2 u^2 \omega_3 - \frac{37}{2} c_s^2 \omega_3 \omega_4 \omega_1 + \frac{7}{2} u^2 \omega_3 \omega_4 \omega_1^2 + \frac{7}{2} c_s^2 \omega_2 \omega_3 \omega_4^2 - u^2 \omega_3^2 \omega_1 - \frac{7}{2} \omega_2 u^2 \omega_3 \omega_4^2 + 10\omega_2 u^2 \omega_1 - \\
&\quad 8\omega_2^2 u^2 \omega_3 \omega_4 \omega_1 + 7\omega_2^2 u^2 \omega_4 \omega_1 - c_s^2 \omega_3 \omega_4^2 - \omega_2 \omega_1^2 - c_s^2 \omega_3 \omega_1^2 + 75\omega_2 u^2 \omega_3 \omega_4 \omega_1 + 47c_s^2 \omega_2 \omega_3 \omega_4 \omega_1 - c_s^2 \omega_2^2 \omega_3 + \\
&\quad 10u^2 \omega_4 \omega_1 + \frac{7}{2} \omega_2 \omega_3 \omega_4 \omega_1 + \frac{7}{2} \omega_2 u^2 \omega_3 \omega_1^2 + 5u^2 \omega_3^2 \omega_4 + \frac{7}{2} c_s^2 \omega_2 \omega_3 \omega_1^2 - \frac{53}{2} \omega_2 u^2 \omega_3 \omega_1 - \frac{37}{2} c_s^2 \omega_2 \omega_3 \omega_1 + 5c_s^2 \omega_3 \omega_1 + \\
&\quad 2\omega_2 \omega_1 - 8\omega_2 u^2 \omega_3 \omega_4 \omega_1^2 - 8c_s^2 \omega_2 \omega_3 \omega_4 \omega_1^2 + 2u^2 \omega_4 \omega_1^2 + 2\omega_2 u^2 \omega_1^2 + 7\omega_2 u^2 \omega_4^2 \omega_1 + 5c_s^2 \omega_3 \omega_4 + 2\omega_2 \omega_4 + \\
&\quad \frac{7}{2} \omega_2 \omega_4 \omega_1^2 - \frac{53}{2} u^2 \omega_3 \omega_4 \omega_1 + \frac{7}{2} c_s^2 \omega_3 \omega_4 \omega_1^2 - \frac{37}{2} c_s^2 \omega_2 \omega_3 \omega_4 + \frac{37}{2} \omega_2 u^2 \omega_3 \omega_4 - 37\omega_2 u^2 \omega_4 \omega_1 - \omega_3 \omega_4 \omega_1 - c_s^2 \omega_3^2 \omega_4 + \\
&\quad \frac{7}{2} c_s^2 \omega_2^2 \omega_3 \omega_1 + \frac{7}{2} u^2 \omega_3 \omega_4^2 \omega_1 - \frac{35}{2} \omega_2 u^2 \omega_3^2 \omega_4 - u^2 \omega_3 \omega_1^2 + \frac{7}{2} c_s^2 \omega_2 \omega_3^2 \omega_4 + \frac{7}{2} c_s^2 \omega_2 \omega_3^2 \omega_1 + u^2 \omega_3 \omega_4^2 + \frac{7}{2} \omega_2 u^2 \omega_3^2 \omega_1 - \\
&\quad \omega_4 \omega_1^2 - 8c_s^2 \omega_2^2 \omega_3 \omega_4 \omega_1 + \frac{7}{2} c_s^2 \omega_2^2 \omega_3 \omega_4 - c_s^2 \omega_2^2 \omega_3 \omega_1 + \frac{7}{2} u^2 \omega_3^2 \omega_4 \omega_1 + 5\omega_2 u^2 \omega_3^2 - c_s^2 \omega_2 \omega_3^2 - 8\omega_2 u^2 \omega_3 \omega_4^2 \omega_1 - \\
&\quad \omega_2 \omega_3 \omega_1 + \frac{7}{2} c_s^2 \omega_3^2 \omega_4 \omega_1 - 2u^2 \omega_4^2 \omega_1 + 5c_s^2 \omega_2 \omega_3 - 5\omega_2 u^2 \omega_3 - 8c_s^2 \omega_2 \omega_3 \omega_4^2 \omega_1 + 2\omega_4 \omega_1 - 5u^2 \omega_3 \omega_4 + \frac{7}{2} \omega_2^2 u^2 \omega_3 \omega_1 - \\
&\quad 2\omega_2^2 u^2 \omega_1 + \frac{7}{2} c_s^2 \omega_3 \omega_4^2 \omega_1 - \frac{7}{2} \omega_2^2 u^2 \omega_3 \omega_4 - 8c_s^2 \omega_2 \omega_3^2 \omega_4 \omega_1 + 5u^2 \omega_3 \omega_1 - 8\omega_2 u^2 \omega_3^2 \omega_4 \omega_1 - 7\omega_2 u^2 \omega_4 \omega_1^2 - \omega_2 \omega_3 \omega_4, \\
\gamma_{[t^2]}^{[\mu_4]} &= \frac{39}{2} \omega_2 \omega_4 \omega_1 - \frac{7}{2} \omega_2^2 \omega_3 \omega_1 - 5\omega_3 \omega_1 + \omega_2 \omega_4^2 - \frac{7}{2} \omega_2 \omega_3^2 \omega_4 + \omega_2 \omega_1^2 - \frac{7}{2} \omega_2 \omega_3^2 \omega_1 - 5\omega_3 \omega_4 - \frac{101}{2} \omega_2 \omega_3 \omega_4 \omega_1 + \omega_4^2 \omega_1 - \\
&\quad \frac{7}{2} \omega_2^2 \omega_3 \omega_4 - \frac{7}{2} \omega_3^2 \omega_4 \omega_1 - 5\omega_2 \omega_1 + 8\omega_2 \omega_3 \omega_4 \omega_1^2 + \omega_3 \omega_4^2 - \frac{7}{2} \omega_3 \omega_4^2 \omega_1 + \omega_3 \omega_1^2 + \omega_2^2 \omega_3 - 5\omega_2 \omega_4 - \frac{7}{2} \omega_2 \omega_4 \omega_1^2 + \\
&\quad \frac{39}{2} \omega_3 \omega_4 \omega_1 + \omega_2^2 \omega_4 - \frac{7}{2} \omega_2 \omega_3 \omega_4^2 - 5\omega_2 \omega_3 + \omega_4 \omega_1^2 + \omega_2^2 \omega_1 + 8\omega_2^2 \omega_3 \omega_4 \omega_1 - \frac{7}{2} \omega_2 \omega_3 \omega_1^2 + \frac{39}{2} \omega_2 \omega_3 \omega_1 + \\
&\quad 8\omega_2 \omega_3 \omega_4^2 \omega_1 + \omega_3^2 \omega_4 - \frac{7}{2} \omega_2^2 \omega_4 \omega_1 - 5\omega_4 \omega_1 + 8\omega_2 \omega_3^2 \omega_4 \omega_1 - \frac{7}{2} \omega_2 \omega_4^2 \omega_1 - \frac{7}{2} \omega_3 \omega_4 \omega_1^2 + \omega_3^2 \omega_1 + \frac{39}{2} \omega_2 \omega_3 \omega_4 + \omega_2 \omega_3^2, \\
\gamma_{[tx]}^{[\mu_1]} &= u \omega_4^2 \omega_1 - 6u^3 \omega_3 \omega_4 \omega_1 - 6\omega_2 u^3 \omega_3^2 \omega_4 + 4\omega_2 u^3 \omega_1^2 + \omega_2^2 u \omega_1 - 12\omega_2 u^3 \omega_4 \omega_1^2 + 12\omega_2 u \omega_4 \omega_1 + 18\omega_2 u^3 \omega_3 \omega_4 \omega_1 + \\
&\quad 6c_s^2 \omega_2 u \omega_3^2 \omega_4 - 2c_s^2 u \omega_3^2 \omega_4 + 2u^3 \omega_3^2 \omega_4 + 2c_s^2 \omega_2 u \omega_3 \omega_1 - 3\omega_2 u \omega_4^2 \omega_1 - 2\omega_2 u \omega_3 \omega_4 - 6c_s^2 \omega_2 u \omega_3 \omega_4 \omega_1 - \\
&\quad 3\omega_2^2 u \omega_4 \omega_1 - 2c_s^2 \omega_2 u \omega_3^2 - 3u \omega_4 \omega_1 + 2\omega_2 u^3 \omega_3^2 + 4u^3 \omega_4 \omega_1^2 + 2c_s^2 u \omega_3 \omega_4 \omega_1 - 6\omega_2 u^3 \omega_3 \omega_1 - 3\omega_2 u \omega_1, \\
\gamma_{[tx]}^{[\mu_4]} &= -2u \omega_4^2 \omega_1 - 10u \omega_3 \omega_4 - 6\omega_2^2 u \omega_3 \omega_4 - 10\omega_2 u \omega_3 + 6\omega_2 u \omega_4 \omega_1^2 - 2\omega_2^2 u \omega_1 - 34\omega_2 u \omega_4 \omega_1 - 12\omega_2 u \omega_3^2 \omega_4 + \\
&\quad 4\omega_2 u \omega_3^2 + 2u \omega_3 \omega_4^2 - 2\omega_2 u \omega_1^2 - 2\omega_2 u \omega_3 \omega_1 - 2u \omega_4 \omega_1^2 + 6\omega_2 u \omega_4^2 \omega_1 + 34\omega_2 u \omega_3 \omega_4 + 6\omega_2 u \omega_3 \omega_4 \omega_1 - \\
&\quad 6\omega_2 u \omega_3 \omega_4^2 + 6\omega_2^2 u \omega_4 \omega_1 + 10u \omega_4 \omega_1 + 2\omega_2^2 u \omega_3 - 2u \omega_3 \omega_4 \omega_1 + 10\omega_2 u \omega_1 + 4u \omega_3^2 \omega_4, \\
\gamma_{[ty]}^{[\mu_1]} &= -6\omega_2 u^2 \omega_3^2 v + 2c_s^2 \omega_2 \omega_3^2 v + 10c_s^2 \omega_3 v \omega_4 + 4u^2 \omega_3 v \omega_4^2 + 2\omega_2 v \omega_4 + 40\omega_2 u^2 \omega_3 v \omega_1 + 32c_s^2 \omega_2 \omega_3 v \omega_1 - \\
&\quad 2v \omega_4 \omega_1^2 + \omega_2 \omega_3 v \omega_1 - \omega_2 \omega_3 v \omega_4 - 6\omega_3^2 u^2 \omega_3 v \omega_1 + 2c_s^2 \omega_2 \omega_3 v \omega_4 - 2\omega_2 u^2 \omega_3 v \omega_4 - 48u^2 \omega_3 v \omega_4 \omega_1 + 4\omega_2^2 u^2 v \omega_1 + \\
&\quad 12c_s^2 \omega_3 v \omega_4^2 \omega_1 + 2c_s^2 \omega_3^2 v \omega_3 v - 2\omega_2 v \omega_1 + \omega_2 v \omega_1^2 + 6c_s^2 \omega_3^2 v \omega_4 \omega_1 + 6u^2 \omega_3 v \omega_4 \omega_1^2 - 6\omega_2 u^2 \omega_3 v \omega_1^2 + 4v \omega_4 \omega_1 - \\
&\quad 6c_s^2 \omega_2 \omega_3 v \omega_1^2 - 10u^2 \omega_3 v \omega_4 - 4c_s^2 \omega_3 v \omega_4^2 - 2\omega_2^2 u^2 \omega_3 v - 8u^2 v \omega_4^2 \omega_1 - 6c_s^2 \omega_2^2 \omega_3 v \omega_1 + 18\omega_2 u^2 \omega_3 v \omega_4 \omega_1 - \\
&\quad 6c_s^2 \omega_2 \omega_3 v \omega_4 \omega_1 + 20u^2 v \omega_4 \omega_1 + 3\omega_2 \omega_3 v \omega_4 \omega_1 + 3\omega_2 v \omega_4 \omega_1^2 + 10u^2 \omega_3^2 v \omega_4 + 6u^2 \omega_3^2 v \omega_4 \omega_1 - 20\omega_2 u^2 v \omega_1 - \\
&\quad 12\omega_2 u^2 v \omega_4 \omega_1^2 + 6c_s^2 \omega_3 v \omega_4 \omega_1^2 - 7\omega_2 v \omega_4 \omega_1 - 2c_s^2 \omega_3^2 v \omega_4 + 4\omega_2 u^2 v \omega_4 \omega_1 - 32c_s^2 \omega_3 v \omega_4 \omega_1 + 12u^2 \omega_3 v \omega_4^2 \omega_1 - \\
&\quad 2\omega_3 v \omega_4 \omega_1 - 6c_s^2 \omega_2 \omega_3^2 v \omega_1 - 6\omega_2 u^2 \omega_3^2 v \omega_1 - 10c_s^2 \omega_2 \omega_3 v + 10\omega_2 u^2 \omega_3 v - 12\omega_2 u^2 \omega_3^2 v \omega_4 + 4u^2 v \omega_4 \omega_1^2, \\
\gamma_{[ty]}^{[\mu_4]} &= 2v \omega_4 \omega_1^2 - 34\omega_2 \omega_3 v \omega_1 - 2\omega_2 \omega_3 v \omega_4 - 12\omega_3 v \omega_4^2 \omega_1 + 10\omega_2 v \omega_1 - 2\omega_2 v \omega_1^2 + 2\omega_3^2 v \omega_4 - 6\omega_3^2 v \omega_4 \omega_1 + 10\omega_2 \omega_3 v - \\
&\quad 10v \omega_4 \omega_1 + 6\omega_2 \omega_3 v \omega_1^2 + 6\omega_2^2 \omega_3 v \omega_1 - 2\omega_2 \omega_3^2 v - 10\omega_3 v \omega_4 + 4v \omega_4^2 \omega_1 + 6\omega_2 \omega_3 v \omega_4 \omega_1 - 2\omega_2^2 \omega_3 v - 6\omega_3 v \omega_4 \omega_1^2 - \\
&\quad 2\omega_2 v \omega_4 \omega_1 + 34\omega_3 v \omega_4 \omega_1 + 6\omega_2 \omega_3^2 v \omega_1 + 4\omega_3 v \omega_4^2 - 2\omega_2^2 v \omega_1, \\
\gamma_{[x^2]}^{[\mu_1]} &= -\frac{1}{2} c_s^2 \omega_2 \omega_3 \omega_4^2 + \frac{1}{2} \omega_2 u^2 \omega_3 \omega_4^2 - \omega_2^2 u^2 \omega_4 \omega_1 - \frac{1}{2} \omega_2 u^2 \omega_3 \omega_4 \omega_1 - \frac{1}{2} c_s^2 \omega_2 \omega_3 \omega_4 \omega_1 - \omega_2 u^2 \omega_4^2 \omega_1 + \frac{5}{2} c_s^2 \omega_2 \omega_3 \omega_4 - \\
&\quad \frac{5}{2} \omega_2 u^2 \omega_3 \omega_4 + 5\omega_2 u^2 \omega_4 \omega_1 + \frac{1}{2} \omega_2 u^2 \omega_3^2 \omega_4 - \frac{1}{2} c_s^2 \omega_2 \omega_3^2 \omega_4 - \frac{1}{2} c_s^2 \omega_2^2 \omega_3 \omega_4 + \frac{1}{2} \omega_2^2 u^2 \omega_3 \omega_4 - \omega_2 u^2 \omega_4 \omega_1^2, \\
\gamma_{[x^2]}^{[\mu_4]} &= -\frac{3}{2} \omega_2 \omega_4 \omega_1 - \omega_2 \omega_4^2 + \frac{1}{2} \omega_2 \omega_3^2 \omega_4 - 4\omega_2 u^2 \omega_3 \omega_4 \omega_1 + \frac{1}{2} \omega_2 \omega_3 \omega_4 \omega_1 + \frac{1}{2} \omega_2^2 \omega_3 \omega_4 + 3\omega_2 \omega_4 - \omega_2^2 \omega_4 + \frac{1}{2} \omega_2 \omega_3 \omega_4^2 + \\
&\quad 2\omega_2 u^2 \omega_3^2 \omega_4 + \frac{1}{2} \omega_2^2 \omega_4 \omega_1 + \frac{1}{2} \omega_2 \omega_4^2 \omega_1 + 2\omega_2 u^2 \omega_4 \omega_1^2 - \frac{5}{2} \omega_2 \omega_3 \omega_4,
\end{aligned}$$

$$\begin{aligned}
\gamma_{[xy]}^{[\mu_1]} &= -6uv\omega_4\omega_1 + 4c_s^2u\omega_3v\omega_4\omega_1 - 4c_s^2u\omega_3^2v\omega_4 - 8\omega_2u^3v\omega_1^2 + 12\omega_2u^3\omega_3v\omega_1 + 6\omega_2uv\omega_1 - 4c_s^2\omega_2u\omega_3v\omega_1 + \\
&\quad 8u^3v\omega_4\omega_1^2 - 2\omega_2u\omega_3v\omega_4 - 4\omega_2u^3\omega_3^2v + 4c_s^2\omega_2u\omega_3^2v + 4u^3\omega_3^2v\omega_4 - 2\omega_2^2uv\omega_1 + 4uv\omega_4^2\omega_1 - 12u^3\omega_3v\omega_4\omega_1, \\
\gamma_{[xy]}^{[\mu_4]} &= 20uv\omega_4\omega_1 + 4\omega_2uv\omega_1^2 - 4u\omega_3v\omega_4\omega_1 + 20\omega_2u\omega_3v + 8u\omega_3^2v\omega_4 + 4\omega_2uv\omega_4\omega_1 - 4uv\omega_4\omega_1^2 - 20\omega_2uv\omega_1 + \\
&\quad 4\omega_2u\omega_3v\omega_1 + 8u\omega_3v\omega_4^2 - 4\omega_2u\omega_3v\omega_4 - 4\omega_2^2u\omega_3v - 8\omega_2u\omega_3^2v - 20u\omega_3v\omega_4 + 4\omega_2^2uv\omega_1 - 8uv\omega_4^2\omega_1, \\
\gamma_{[y^2]}^{[\mu_1]} &= -\omega_3v^2\omega_4\omega_1 + 2\omega_2u^2\omega_3^2v^2\omega_4 + \frac{7}{2}c_s^2\omega_3\omega_4\omega_1 - \frac{1}{2}u^2\omega_3\omega_4\omega_1^2 - 2u^2\omega_3v^2\omega_4^2\omega_1 + \omega_2v^2\omega_4\omega_1 + u^2\omega_3^2\omega_1 - \\
&\quad 8\omega_2u^2v^2\omega_1^2 + c_s^2\omega_3\omega_1^2 + 4u^2v^2\omega_4\omega_1^2 - \frac{1}{2}\omega_2u^2\omega_3\omega_4\omega_1 - c_s^2\omega_2\omega_3\omega_4\omega_1 - 4c_s^2\omega_2u^2\omega_3\omega_4\omega_1 - \frac{1}{2}\omega_2u^2\omega_3\omega_1^2 - \\
&\quad \frac{1}{2}c_s^2\omega_2\omega_3\omega_1^2 + \frac{3}{2}\omega_2u^2\omega_3\omega_1 + \frac{3}{2}c_s^2\omega_2\omega_3\omega_1 - 3c_s^2\omega_3\omega_1 + 4u^2\omega_3^2v^2\omega_4 - \frac{1}{2}\omega_2\omega_3v^2\omega_4\omega_1 + 4c_s^2\omega_2\omega_3v^2\omega_4\omega_1 - \\
&\quad 2c_s^2\omega_2^2\omega_3v^2\omega_1 + 8c_s^2u^2\omega_3\omega_4\omega_1 - 8\omega_2u^2\omega_3^2v^2 + \frac{5}{2}u^2\omega_3\omega_4\omega_1 + 2\omega_2u^2v^2\omega_4\omega_1^2 - \frac{1}{2}c_s^2\omega_3\omega_4\omega_1^2 - \frac{1}{2}\omega_2v^2\omega_4\omega_1^2 + \\
&\quad c_s^2\omega_2\omega_4\omega_1 - \frac{1}{2}u^2\omega_3\omega_4^2\omega_1 - 2c_s^2\omega_4\omega_1 + u^2\omega_3\omega_1^2 + 2c_s^2\omega_2u^2\omega_3^2\omega_4 - \frac{1}{2}c_s^2\omega_2\omega_3^2\omega_1 + 2v^2\omega_4\omega_1 + 2\omega_2v^2\omega_1^2 - \\
&\quad \frac{1}{2}\omega_2u^2\omega_3^2\omega_1 - 4c_s^2u^2\omega_3^2\omega_4 + c_s^2\omega_3^2\omega_1 - \frac{1}{2}u^2\omega_3^2\omega_4\omega_1 - 2\omega_2^2u^2\omega_3v^2\omega_1 - \frac{1}{2}c_s^2\omega_3^2\omega_4\omega_1 - 4\omega_2v^2\omega_1 - v^2\omega_4\omega_1^2 - \\
&\quad 4c_s^2u^2\omega_4\omega_1^2 - 8u^2\omega_3v^2\omega_4\omega_1 - 2c_s^2\omega_3v^2\omega_4^2\omega_1 - \frac{1}{2}c_s^2\omega_3\omega_4^2\omega_1 + c_s^2\omega_4\omega_1^2 - 3u^2\omega_3\omega_1 - \frac{1}{2}c_s^2\omega_2\omega_4\omega_1^2 + \\
&\quad 2c_s^2\omega_2u^2\omega_4\omega_1^2 + 16\omega_2u^2\omega_3v^2\omega_1 + 2\omega_2\omega_3v^2\omega_1. \\
\gamma_{[y^2]}^{[\mu_4]} &= 3\omega_3\omega_1 + \frac{1}{2}\omega_2\omega_3^2\omega_1 + \frac{1}{2}\omega_2\omega_3\omega_4\omega_1 + \frac{1}{2}\omega_3^2\omega_4\omega_1 - 4\omega_2\omega_3v^2\omega_4\omega_1 + \frac{1}{2}\omega_3\omega_4^2\omega_1 + 2\omega_2^2\omega_3v^2\omega_1 - \omega_3\omega_1^2 - \\
&\quad \frac{5}{2}\omega_3\omega_4\omega_1 + \frac{1}{2}\omega_2\omega_3\omega_1^2 - \frac{3}{2}\omega_2\omega_3\omega_1 + 2\omega_3v^2\omega_4^2\omega_1 + \frac{1}{2}\omega_3\omega_4\omega_1^2 - \omega_3^2\omega_1,
\end{aligned}$$

## 7.6 EPDE for $\mu_5$

$$\begin{aligned}
&\gamma_{[1]}^{[\mu_1]}\mu_1 + \gamma_{[1]}^{[\mu_5]}\mu_5 + \gamma_{[t]}^{[\mu_1]}\delta_t\frac{\partial\mu_1}{\partial t} + \gamma_{[t]}^{[\mu_5]}\delta_t\frac{\partial\mu_5}{\partial t} + \gamma_{[x]}^{[\mu_1]}\delta_l\frac{\partial\mu_1}{\partial x} + \gamma_{[x]}^{[\mu_5]}\delta_l\frac{\partial\mu_5}{\partial x} + \gamma_{[y]}^{[\mu_1]}\delta_l\frac{\partial\mu_1}{\partial y} + \gamma_{[y]}^{[\mu_5]}\delta_l\frac{\partial\mu_5}{\partial y} \\
&\quad + \gamma_{[t^2]}^{[\mu_1]}\delta_t^2\frac{\partial^2\mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_5]}\delta_t^2\frac{\partial^2\mu_5}{\partial t^2} + \gamma_{[tx]}^{[\mu_1]}\delta_t\delta_l\frac{\partial^2\mu_1}{\partial t\partial x} + \gamma_{[tx]}^{[\mu_5]}\delta_t\delta_l\frac{\partial^2\mu_5}{\partial t\partial x} + \gamma_{[ty]}^{[\mu_1]}\delta_t\delta_l\frac{\partial^2\mu_1}{\partial t\partial y} + \gamma_{[ty]}^{[\mu_5]}\delta_t\delta_l\frac{\partial^2\mu_5}{\partial t\partial y} \\
&\quad + \gamma_{[x^2]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_5]}\delta_l^2\frac{\partial^2\mu_5}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial x\partial y} + \gamma_{[xy]}^{[\mu_5]}\delta_l^2\frac{\partial^2\mu_5}{\partial x\partial y} + \gamma_{[y^2]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_5]}\delta_l^2\frac{\partial^2\mu_5}{\partial y^2} = 0,
\end{aligned}$$

where

$$\begin{aligned}
\gamma_{[1]}^{[\mu_1]} &= -\omega_2\omega_3v^2\omega_4\omega_1^2 + 5c_s^2\omega_2\omega_3\omega_4\omega_1 + 5\omega_2\omega_3v^2\omega_4\omega_1 - c_s^2\omega_2\omega_3\omega_4\omega_1^2 - \omega_2^2\omega_3v^2\omega_4\omega_1 - c_s^2\omega_2^2\omega_3\omega_4\omega_1 - \\
&\quad \omega_2\omega_3v^2\omega_4^2\omega_1 - \omega_2\omega_3^2v^2\omega_4\omega_1 - c_s^2\omega_2\omega_3\omega_4^2\omega_1 - c_s^2\omega_2\omega_3^2\omega_4\omega_1, \\
\gamma_{[1]}^{[\mu_5]} &= -5\omega_2\omega_3\omega_4\omega_1 + \omega_2\omega_3\omega_4\omega_1^2 + \omega_2^2\omega_3\omega_4\omega_1 + \omega_2\omega_3\omega_4^2\omega_1 + \omega_2\omega_3^2\omega_4\omega_1, \\
\gamma_{[t]}^{[\mu_1]} &= -5\omega_3v^2\omega_4\omega_1 + 5c_s^2\omega_3\omega_4\omega_1 - \omega_3^2\omega_3\omega_1 - c_s^2\omega_2\omega_3\omega_4^2 - \omega_2\omega_3^2v^2\omega_4 + 5\omega_2v^2\omega_4\omega_1 + 4\omega_2\omega_3v^2\omega_4\omega_1^2 - \\
&\quad 22c_s^2\omega_2\omega_3\omega_4\omega_1 - \omega_2\omega_3\omega_4\omega_1 - 2\omega_2\omega_3^2v^2\omega_1 - c_s^2\omega_2^2\omega_4\omega_1 - 30\omega_2\omega_3v^2\omega_4\omega_1 + 4c_s^2\omega_2\omega_3\omega_4\omega_1^2 - \omega_2^2\omega_3v^2\omega_4 + \\
&\quad 4\omega_2^2\omega_3v^2\omega_4\omega_1 + 2\omega_2^2\omega_3v^2\omega_1 - c_s^2\omega_2\omega_4^2\omega_1 + \omega_3v^2\omega_4\omega_1^2 - c_s^2\omega_3\omega_4\omega_1^2 + 5c_s^2\omega_2\omega_3\omega_4 - \omega_2v^2\omega_4\omega_1^2 + \\
&\quad 5c_s^2\omega_2\omega_4\omega_1 - 2\omega_2\omega_3v^2\omega_1^2 - \omega_2^2v^2\omega_4\omega_1 + \omega_3^2v^2\omega_4\omega_1 - c_s^2\omega_2\omega_3^2\omega_4 + 4c_s^2\omega_2^2\omega_3\omega_4\omega_1 - \omega_2\omega_3v^2\omega_4^2 + \\
&\quad 4\omega_2\omega_3v^2\omega_4^2\omega_1 - c_s^2\omega_2^2\omega_3\omega_4 + 2\omega_2\omega_3\omega_1 + 5\omega_2\omega_3v^2\omega_4 - c_s^2\omega_3^2\omega_4\omega_1 + 4\omega_2\omega_3^2v^2\omega_4\omega_1 + 4c_s^2\omega_2\omega_3\omega_4^2\omega_1 - \\
&\quad c_s^2\omega_3\omega_4^2\omega_1 + 5\omega_3v^2\omega_4^2\omega_1 + 4c_s^2\omega_2\omega_3^2\omega_4\omega_1 - \omega_2v^2\omega_4^2\omega_1 - c_s^2\omega_2\omega_4\omega_1^2 + 10\omega_2\omega_3v^2\omega_1, \\
\gamma_{[t]}^{[\mu_5]} &= -5\omega_2\omega_4\omega_1 + \omega_2^2\omega_3\omega_1 + \omega_2\omega_3^2\omega_4 + \omega_2\omega_3^2\omega_1 + 23\omega_2\omega_3\omega_4\omega_1 + \omega_2^2\omega_3\omega_4 + \omega_3^2\omega_4\omega_1 - 4\omega_2\omega_3\omega_4\omega_1^2 + \omega_3\omega_4^2\omega_1 + \\
&\quad \omega_2\omega_4\omega_1^2 - 5\omega_3\omega_4\omega_1 + \omega_2\omega_3\omega_4^2 - 4\omega_2^2\omega_3\omega_4\omega_1 + \omega_2\omega_3\omega_1^2 - 5\omega_2\omega_3\omega_1 - 4\omega_2\omega_3\omega_4^2\omega_1 + \omega_2^2\omega_4\omega_1 - 4\omega_2\omega_3^2\omega_4\omega_1 + \\
&\quad \omega_2\omega_4^2\omega_1 + \omega_3\omega_4\omega_1^2 - 5\omega_2\omega_3\omega_4, \\
\gamma_{[x]}^{[\mu_1]} &= -\omega_2^2u\omega_3\omega_1 - 2\omega_2u\omega_3v^2\omega_4^2 - 2c_s^2\omega_2^2u\omega_3\omega_4 + 4u\omega_3v^2\omega_4^2\omega_1 + 2c_s^2\omega_2u\omega_4\omega_1^2 - 10c_s^2\omega_2u\omega_4\omega_1 - \\
&\quad 4c_s^2\omega_2u\omega_3^2\omega_4 + 10\omega_2u\omega_3v^2\omega_4 + 4\omega_2^2u\omega_3v^2\omega_1 - 2\omega_2^2u\omega_3v^2\omega_4 - 6\omega_2u\omega_3v^2\omega_4\omega_1 + 2\omega_2uv^2\omega_4^2\omega_1 +
\end{aligned}$$

$$\begin{aligned}
& 2\omega_2 u \omega_3 \omega_1 - 4\omega_2 u \omega_3^2 v^2 \omega_4 - 10\omega_2 u v^2 \omega_4 \omega_1 + 2c_s^2 \omega_2 u \omega_4^2 \omega_1 + 2\omega_2^2 u v^2 \omega_4 \omega_1 + 2c_s^2 \omega_2 u \omega_3 \omega_4 \omega_1 + \\
& 10c_s^2 \omega_2 u \omega_3 \omega_4 - \omega_2 u \omega_3 \omega_4 \omega_1 - 2c_s^2 \omega_2 u \omega_3 \omega_4^2 + 2c_s^2 \omega_2^2 u \omega_4 \omega_1 + 2\omega_2 u v^2 \omega_4 \omega_1^2, \\
\gamma_{[x]}^{[\mu_5]} &= 2\omega_2^2 u \omega_3 \omega_4 - 2\omega_2 u \omega_4 \omega_1^2 + 10\omega_2 u \omega_4 \omega_1 + 4\omega_2 u \omega_3^2 \omega_4 - 2\omega_2 u \omega_4^2 \omega_1 - 10\omega_2 u \omega_3 \omega_4 - 2\omega_2 u \omega_3 \omega_4 \omega_1 + \\
& 2\omega_2 u \omega_3 \omega_4^2 - 2\omega_2^2 u \omega_4 \omega_1, \\
\gamma_{[y]}^{[\mu_1]} &= -3\omega_2 \omega_3 v \omega_1 - 2c_s^2 \omega_3 v \omega_4^2 \omega_1 + 4\omega_2^2 \omega_3 v^3 \omega_1 + 2\omega_3 v^3 \omega_4^2 \omega_1 + \omega_2 \omega_3 v \omega_1^2 + 2c_s^2 \omega_2 \omega_3 v \omega_4 \omega_1 + \omega_2 \omega_3^2 v \omega_1 - \\
& 6\omega_2 \omega_3 v^3 \omega_4 \omega_1, \\
\gamma_{[y]}^{[\mu_5]} &= 10\omega_2 \omega_3 v \omega_1 + 4\omega_3 v \omega_4^2 \omega_1 + 2\omega_3^2 v \omega_4 \omega_1 - 2\omega_2 \omega_3 v \omega_1^2 - 2\omega_2^2 \omega_3 v \omega_1 - 2\omega_2 \omega_3 v \omega_4 \omega_1 + 2\omega_3 v \omega_4 \omega_1^2 - \\
& 10\omega_3 v \omega_4 \omega_1 - 2\omega_2 \omega_3^2 v \omega_1, \\
\gamma_{[t^2]}^{[\mu_1]} &= -\omega_2 \omega_4 \omega_1 + \frac{37}{2} \omega_3^2 v^2 \omega_4 \omega_1 - \frac{37}{2} c_s^2 \omega_3 \omega_4 \omega_1 + \frac{7}{2} \omega_2^2 \omega_3 \omega_1 + \frac{7}{2} c_s^2 \omega_2 \omega_3 \omega_4^2 + \frac{7}{2} \omega_2 \omega_3^2 v^2 \omega_4 - \frac{53}{2} \omega_2 v^2 \omega_4 \omega_1 + 2\omega_3 \omega_1 + \\
& 10\omega_2 \omega_3 v^2 + 2\omega_2^2 v^2 \omega_1 + 5\omega_3 v^2 \omega_4^2 - c_s^2 \omega_3 \omega_4^2 - 8\omega_2 \omega_3 v^2 \omega_4 \omega_1^2 - \omega_2^2 v^2 \omega_4 + 47c_s^2 \omega_2 \omega_3 \omega_4 \omega_1 + 5c_s^2 \omega_2 \omega_4 + \\
& \frac{7}{2} \omega_2 \omega_3 \omega_4 \omega_1 + 7\omega_2 \omega_3^2 v^2 \omega_1 + 5v^2 \omega_4^2 \omega_1 + \frac{7}{2} c_s^2 \omega_2^2 \omega_4 \omega_1 + 2\omega_2 \omega_1 + 75\omega_2 \omega_3 v^2 \omega_4 \omega_1 - 8c_s^2 \omega_2 \omega_3 \omega_4 \omega_1^2 - c_s^2 \omega_2 \omega_4^2 + \\
& \frac{7}{2} \omega_2^2 \omega_3 v^2 \omega_4 - 8\omega_2^2 \omega_3 v^2 \omega_4 \omega_1 - 7\omega_2^2 \omega_3 v^2 \omega_1 + \frac{7}{2} c_s^2 \omega_2 \omega_4^2 \omega_1 - \omega_2^2 \omega_3 + 5c_s^2 \omega_3 \omega_4 - 5\omega_3 v^2 \omega_4 - \frac{7}{2} \omega_3 v^2 \omega_4 \omega_1^2 - \\
& c_s^2 \omega_4^2 \omega_1 + \frac{7}{2} c_s^2 \omega_3 \omega_4 \omega_1^2 - \frac{37}{2} c_s^2 \omega_2 \omega_3 \omega_4 + \frac{7}{2} \omega_2 v^2 \omega_4 \omega_1^2 - \frac{37}{2} c_s^2 \omega_2 \omega_4 \omega_1 - \omega_3 \omega_4 \omega_1 + 7\omega_2 \omega_3 v^2 \omega_1^2 - c_s^2 \omega_3^2 \omega_4 + \\
& 2\omega_2 \omega_3 + \frac{7}{2} \omega_2^2 v^2 \omega_4 \omega_1 + 5c_s^2 \omega_4 \omega_1 - \omega_2 v^2 \omega_4^2 - \frac{7}{2} \omega_3^2 v^2 \omega_4 \omega_1 + \frac{7}{2} c_s^2 \omega_2 \omega_3^2 \omega_4 + \omega_3^2 v^2 \omega_4 - 5v^2 \omega_4 \omega_1 - 2\omega_2 v^2 \omega_1^2 - \\
& 8c_s^2 \omega_2^2 \omega_3 \omega_4 \omega_1 + \frac{7}{2} \omega_2 \omega_3 v^2 \omega_4^2 - \omega_2^2 \omega_1 - 8\omega_2 \omega_3 v^2 \omega_4^2 \omega_1 + \frac{7}{2} c_s^2 \omega_2^2 \omega_3 \omega_4 - 8\omega_2 \omega_3 \omega_1 - \frac{53}{2} \omega_2 \omega_3 v^2 \omega_4 + \frac{7}{2} c_s^2 \omega_3^2 \omega_4 \omega_1 - \\
& c_s^2 \omega_2^2 \omega_4 - 8\omega_2 \omega_3^2 v^2 \omega_4 \omega_1 - 8c_s^2 \omega_2 \omega_3 \omega_4^2 \omega_1 + 10\omega_2 v^2 \omega_1 + v^2 \omega_4 \omega_1^2 + 2\omega_2^2 \omega_3 v^2 + \frac{7}{2} c_s^2 \omega_3 \omega_4^2 \omega_1 - \frac{35}{2} \omega_3 v^2 \omega_4^2 \omega_1 - \\
& 8c_s^2 \omega_2 \omega_3^2 \omega_4 \omega_1 - c_s^2 \omega_4 \omega_1^2 + 5\omega_2 v^2 \omega_4 + \frac{7}{2} \omega_2 v^2 \omega_4^2 \omega_1 + \frac{7}{2} c_s^2 \omega_2 \omega_4 \omega_1^2 - 2\omega_2 \omega_3^2 v^2 - \omega_2 \omega_3 \omega_4 - 37\omega_2 \omega_3 v^2 \omega_1, \\
\gamma_{[t^2]}^{[\mu_5]} &= \frac{39}{2} \omega_2 \omega_4 \omega_1 - \frac{7}{2} \omega_2^2 \omega_3 \omega_1 - 5\omega_3 \omega_1 + \omega_2 \omega_4^2 - \frac{7}{2} \omega_2 \omega_3^2 \omega_4 + \omega_2 \omega_1^2 - \frac{7}{2} \omega_2 \omega_3^2 \omega_1 - 5\omega_3 \omega_4 - \frac{101}{2} \omega_2 \omega_3 \omega_4 \omega_1 + \omega_4^2 \omega_1 - \\
& \frac{7}{2} \omega_2^2 \omega_3 \omega_4 - \frac{7}{2} \omega_3^2 \omega_4 \omega_1 - 5\omega_2 \omega_1 + 8\omega_2 \omega_3 \omega_4 \omega_1^2 + \omega_3 \omega_4^2 - \frac{7}{2} \omega_3 \omega_4^2 \omega_1 + \omega_3 \omega_1^2 + \omega_2^2 \omega_3 - 5\omega_2 \omega_4 - \frac{7}{2} \omega_2 \omega_4 \omega_1^2 + \\
& \frac{39}{2} \omega_3 \omega_4 \omega_1 + \omega_2^2 \omega_4 - \frac{7}{2} \omega_2 \omega_3 \omega_4^2 - 5\omega_2 \omega_3 + \omega_4 \omega_1^2 + \omega_2^2 \omega_1 + 8\omega_2^2 \omega_3 \omega_4 \omega_1 - \frac{7}{2} \omega_2 \omega_3 \omega_1^2 + \frac{39}{2} \omega_2 \omega_3 \omega_1 + \\
& 8\omega_2 \omega_3 \omega_4^2 \omega_1 + \omega_3^2 \omega_4 - \frac{7}{2} \omega_2^2 \omega_4 \omega_1 - 5\omega_4 \omega_1 + 8\omega_2 \omega_3^2 \omega_4 \omega_1 - \frac{7}{2} \omega_2 \omega_4^2 \omega_1 - \frac{7}{2} \omega_3 \omega_4 \omega_1^2 + \omega_3^2 \omega_1 + \frac{39}{2} \omega_2 \omega_3 \omega_4 + \omega_2 \omega_3^2, \\
\gamma_{[tx]}^{[\mu_1]} &= -10u \omega_3 v^2 \omega_4 + 2c_s^2 u \omega_4 \omega_1^2 + 3\omega_2^2 u \omega_3 \omega_1 + 2u \omega_3 \omega_1 + 6\omega_2 u \omega_3 v^2 \omega_4^2 + 4\omega_2 u \omega_3 - 6uv^2 \omega_4^2 \omega_1 + 6c_s^2 \omega_2^2 u \omega_3 \omega_4 - \\
& 12u \omega_3 v^2 \omega_4^2 \omega_1 + \omega_2^2 u \omega_1 - 6c_s^2 \omega_2 u \omega_4 \omega_1^2 + \omega_2 u \omega_4 \omega_1 + 32c_s^2 \omega_2 u \omega_4 \omega_1 + 12c_s^2 \omega_2 u \omega_3^2 \omega_4 - 48\omega_2 u \omega_3 v^2 \omega_4 - \\
& 12\omega_2^2 u \omega_3 v^2 \omega_1 + 20\omega_2 u \omega_3 v^2 - 10c_s^2 u \omega_4 \omega_1 + 6\omega_2^2 u \omega_3 v^2 \omega_4 + 4\omega_2 u \omega_3 v^2 \omega_1 + 18\omega_2 u \omega_3 v^2 \omega_4 \omega_1 + 10u \omega_3 v^2 \omega_4^2 - \\
& 6\omega_2 uv^2 \omega_4^2 \omega_1 - 4c_s^2 u \omega_2^2 \omega_4 + 2c_s^2 u \omega_4^2 \omega_1 - 7\omega_2 u \omega_3 \omega_1 + 10c_s^2 u \omega_3 \omega_4 + 12\omega_2 u \omega_3^2 v^2 \omega_4 + 40\omega_2 uv^2 \omega_4 \omega_1 - \\
& 20\omega_2 uv^2 \omega_1 - 6c_s^2 \omega_2 u \omega_4^2 \omega_1 - 6\omega_2^2 uv^2 \omega_4 \omega_1 - 2\omega_2 u \omega_3 \omega_4 - 6c_s^2 \omega_2 u \omega_3 \omega_4 \omega_1 - 32c_s^2 \omega_2 u \omega_3 \omega_4 - 8\omega_2 u \omega_3^2 v^2 + \\
& 3\omega_2 u \omega_3 \omega_4 \omega_1 - 2uv^2 \omega_4^2 \omega_1 + 4\omega_2^2 u \omega_3 v^2 + 6c_s^2 \omega_2 u \omega_3 \omega_4^2 + 4u \omega_3^2 v^2 \omega_4 + 10uv^2 \omega_4 \omega_1 - 2u \omega_3 v^2 \omega_4 \omega_1 - \\
& 6c_s^2 \omega_2^2 u \omega_4 \omega_1 - 2\omega_2^2 u \omega_3 - 2c_s^2 u \omega_3 \omega_4^2 + 2c_s^2 u \omega_3 \omega_4 \omega_1 - 6\omega_2 uv^2 \omega_4 \omega_1^2 + 4\omega_2 uv^2 \omega_1^2 - u \omega_3 \omega_4 \omega_1 - 2\omega_2 u \omega_1, \\
\gamma_{[tx]}^{[\mu_5]} &= -2u \omega_4^2 \omega_1 - 10u \omega_3 \omega_4 - 6\omega_2^2 u \omega_3 \omega_4 - 10\omega_2 u \omega_3 + 6\omega_2 u \omega_4 \omega_1^2 - 2\omega_2^2 u \omega_1 - 34\omega_2 u \omega_4 \omega_1 - 12\omega_2 u \omega_3^2 \omega_4 + \\
& 4\omega_2 u \omega_3^2 + 2u \omega_3 \omega_4^2 - 2\omega_2 u \omega_1^2 - 2\omega_2 u \omega_3 \omega_1 - 2u \omega_4 \omega_1^2 + 6\omega_2 u \omega_4^2 \omega_1 + 34\omega_2 u \omega_3 \omega_4 + 6\omega_2 u \omega_3 \omega_4 \omega_1 - \\
& 6\omega_2 u \omega_3 \omega_4^2 + 6\omega_2^2 u \omega_4 \omega_1 + 10u \omega_4 \omega_1 + 2\omega_2^2 u \omega_3 - 2u \omega_3 \omega_4 \omega_1 + 10\omega_2 u \omega_1 + 4u \omega_3^2 \omega_4, \\
\gamma_{[ty]}^{[\mu_1]} &= -2c_s^2 v \omega_4^2 \omega_1 + 12\omega_2 \omega_3 v \omega_1 + 2c_s^2 \omega_2 \omega_3 v \omega_4 + 6c_s^2 \omega_3 v \omega_4^2 \omega_1 - 3\omega_2 v \omega_1 - 12\omega_2^2 \omega_3 v^3 \omega_1 + \omega_2 v \omega_1^2 + 4\omega_2^2 v^3 \omega_1 + \\
& 2\omega_3 v^3 \omega_4^2 - 6\omega_3 v^3 \omega_4^2 \omega_1 - 3\omega_2 \omega_3 v - 3\omega_2 \omega_3 v \omega_1^2 - 2c_s^2 \omega_3 v \omega_4^2 + \omega_2 \omega_3^2 v - 6c_s^2 \omega_2 \omega_3 v \omega_4 \omega_1 - 6\omega_2 \omega_3 v^3 \omega_4 - \\
& 6\omega_2 v^3 \omega_4 \omega_1 + 2c_s^2 \omega_2 v \omega_4 \omega_1 - 2\omega_3 v \omega_4 \omega_1 - 3\omega_2 \omega_3^2 v \omega_1 + 18\omega_2 \omega_3 v^3 \omega_4 \omega_1 + 4\omega_2^2 \omega_3 v^3 + 2v^3 \omega_4^2 \omega_1, \\
\gamma_{[ty]}^{[\mu_5]} &= 2v \omega_4 \omega_1^2 - 34\omega_2 \omega_3 v \omega_1 - 2\omega_2 \omega_3 v \omega_4 - 12\omega_3 v \omega_4^2 \omega_1 + 10\omega_2 v \omega_1 - 2\omega_2 v \omega_1^2 + 2\omega_3^2 v \omega_4 - 6\omega_3^2 v \omega_4 \omega_1 + 10\omega_2 \omega_3 v - \\
& 10v \omega_4 \omega_1 + 6\omega_2 \omega_3 v \omega_1^2 + 6\omega_2^2 \omega_3 v \omega_1 - 2\omega_2 \omega_3^2 v - 10\omega_3 v \omega_4 + 4v \omega_4^2 \omega_1 + 6\omega_2 \omega_3 v \omega_4 \omega_1 - 2\omega_2^2 \omega_3 v - 6\omega_3 v \omega_4 \omega_1^2 - \\
& 2\omega_2 v \omega_4 \omega_1 + 34\omega_3 v \omega_4 \omega_1 + 6\omega_2 \omega_3^2 v \omega_1 + 4\omega_3 v \omega_4^2 - 2\omega_2^2 v \omega_1, \\
\gamma_{[x^2]}^{[\mu_1]} &= -\omega_2^2 u^2 \omega_3 - 2\omega_2 u^2 \omega_3^2 v^2 \omega_4 + 16\omega_2 u^2 v^2 \omega_4 \omega_1 + 2u^2 \omega_3 v^2 \omega_4^2 \omega_1 - \frac{1}{2} c_s^2 \omega_2 \omega_3 \omega_4^2 - \frac{1}{2} \omega_2 \omega_3^2 v^2 \omega_4 + \frac{3}{2} \omega_2 v^2 \omega_4 \omega_1 + \\
& 4\omega_2^2 u^2 \omega_3 v^2 - 4\omega_2 u^2 \omega_1 + \omega_2^2 v^2 \omega_4 - \frac{1}{2} \omega_2 u^2 \omega_3 \omega_4 \omega_1 - c_s^2 \omega_2 \omega_3 \omega_4 \omega_1 + 4c_s^2 \omega_2 u^2 \omega_3 \omega_4 \omega_1 + c_s^2 \omega_2^2 \omega_3 - \\
& 3c_s^2 \omega_2 \omega_4 + \omega_2 u^2 \omega_3 \omega_1 + c_s^2 \omega_2 \omega_3 \omega_1 - \frac{1}{2} c_s^2 \omega_2^2 \omega_4 \omega_1 - \frac{1}{2} \omega_2 \omega_3 v^2 \omega_4 \omega_1 - 4c_s^2 \omega_2^2 \omega_3 v^2 + c_s^2 \omega_2 \omega_4^2 - \\
& \frac{1}{2} \omega_2^2 \omega_3 v^2 \omega_4 - 4c_s^2 \omega_2 \omega_3 v^2 \omega_4 \omega_1 - \frac{1}{2} c_s^2 \omega_2 \omega_4^2 \omega_1 + 2c_s^2 \omega_2^2 \omega_3 v^2 \omega_1 - 2\omega_2 u^2 v^2 \omega_4 \omega_1^2 + \frac{7}{2} c_s^2 \omega_2 \omega_3 \omega_4 -
\end{aligned}$$

$$\begin{aligned}
& \omega_2 u^2 \omega_3 \omega_4 + \frac{3}{2} c_s^2 \omega_2 \omega_4 \omega_1 + 2 \omega_2 u^2 \omega_4 \omega_1 - \frac{1}{2} c_s^2 \omega_2^2 \omega_3 \omega_1 - \frac{1}{2} \omega_2^2 v^2 \omega_4 \omega_1 + \omega_2 v^2 \omega_4^2 - \frac{1}{2} c_s^2 \omega_2 \omega_3^2 \omega_4 - \\
& 2 c_s^2 \omega_2 u^2 \omega_3^2 \omega_4 - 4 c_s^2 \omega_3 v^2 \omega_4^2 - 8 u^2 v^2 \omega_4^2 \omega_1 - \frac{1}{2} \omega_2 \omega_3 v^2 \omega_4^2 - \frac{1}{2} c_s^2 \omega_2^2 \omega_3 \omega_4 + 2 \omega_2^2 u^2 \omega_3 v^2 \omega_1 + 8 c_s^2 \omega_2 \omega_3 v^2 \omega_4 + \\
& \frac{5}{2} \omega_2 \omega_3 v^2 \omega_4 - 8 \omega_2^2 u^2 v^2 \omega_1 + c_s^2 \omega_2^2 \omega_4 - 2 c_s^2 \omega_2 \omega_3 + 2 \omega_2 u^2 \omega_3 - 8 \omega_2 u^2 \omega_3 v^2 \omega_4 - \frac{1}{2} \omega_2^2 u^2 \omega_3 \omega_1 + 2 \omega_2^2 u^2 \omega_1 + \\
& 2 c_s^2 \omega_3 v^2 \omega_4^2 \omega_1 - 3 \omega_2 v^2 \omega_4 - \frac{1}{2} \omega_2 v^2 \omega_4^2 \omega_1 + 4 u^2 \omega_3 v^2 \omega_4^2 - 2 c_s^2 \omega_2 u^2 \omega_4 \omega_1^2, \\
\gamma_{[x^2]}^{[\mu_5]} &= -\frac{3}{2} \omega_2 \omega_4 \omega_1 - \omega_2 \omega_4^2 + \frac{1}{2} \omega_2 \omega_3^2 \omega_4 - 4 \omega_2 u^2 \omega_3 \omega_4 \omega_1 + \frac{1}{2} \omega_2 \omega_3 \omega_4 \omega_1 + \frac{1}{2} \omega_2^2 \omega_3 \omega_4 + 3 \omega_2 \omega_4 - \omega_2^2 \omega_4 + \frac{1}{2} \omega_2 \omega_3 \omega_4^2 + \\
& 2 \omega_2 u^2 \omega_3^2 \omega_4 + \frac{1}{2} \omega_2^2 \omega_4 \omega_1 + \frac{1}{2} \omega_2 \omega_4^2 \omega_1 + 2 \omega_2 u^2 \omega_4 \omega_1^2 - \frac{5}{2} \omega_2 \omega_3 \omega_4, \\
\gamma_{[xy]}^{[\mu_1]} &= -2 \omega_2 u v \omega_1^2 - 12 \omega_2 u \omega_3 v^3 \omega_4 - 2 u \omega_3 v \omega_4 \omega_1 - 6 \omega_2 u \omega_3 v + 12 \omega_2 u v^3 \omega_4 \omega_1 + 4 u \omega_3 v^3 \omega_4^2 - 4 c_s^2 \omega_2 u v \omega_4 \omega_1 - \\
& 8 \omega_2^2 u v^3 \omega_1 + 6 \omega_2 u v \omega_1 - 4 c_s^2 u \omega_3 v \omega_4^2 - 4 u v^3 \omega_4^2 \omega_1 + 4 c_s^2 \omega_2 u \omega_3 v \omega_4 + 4 \omega_2 u \omega_3^2 v + 4 c_s^2 u v \omega_4^2 \omega_1 + 8 \omega_2^2 u \omega_3 v^3, \\
\gamma_{[xy]}^{[\mu_5]} &= 20 u v \omega_4 \omega_1 + 4 \omega_2 u v \omega_1^2 - 4 u \omega_3 v \omega_4 \omega_1 + 20 \omega_2 u \omega_3 v + 8 u \omega_3^2 v \omega_4 + 4 \omega_2 u v \omega_4 \omega_1 - 4 u v \omega_4 \omega_1^2 - 20 \omega_2 u v \omega_1 + \\
& 4 \omega_2 u \omega_3 v \omega_1 + 8 u \omega_3 v \omega_4^2 - 4 \omega_2 u \omega_3 v \omega_4 - 4 \omega_2^2 u \omega_3 v - 8 \omega_2 u \omega_3^2 v - 20 u \omega_3 v \omega_4 + 4 \omega_2^2 u v \omega_1 - 8 u v \omega_4^2 \omega_1, \\
\gamma_{[y^2]}^{[\mu_1]} &= -\frac{5}{2} \omega_3 v^2 \omega_4 \omega_1 + \frac{5}{2} c_s^2 \omega_3 \omega_4 \omega_1 - \frac{1}{2} c_s^2 \omega_2 \omega_3 \omega_4 \omega_1 - \omega_2 \omega_3^2 v^2 \omega_1 - \frac{1}{2} \omega_2 \omega_3 v^2 \omega_4 \omega_1 - \omega_2^2 \omega_3 v^2 \omega_1 + \frac{1}{2} \omega_3 v^2 \omega_4 \omega_1^2 - \\
& \frac{1}{2} c_s^2 \omega_3 \omega_4 \omega_1^2 - \omega_2 \omega_3 v^2 \omega_1^2 + \frac{1}{2} \omega_3^2 v^2 \omega_4 \omega_1 - \frac{1}{2} c_s^2 \omega_3^2 \omega_4 \omega_1 - \frac{1}{2} c_s^2 \omega_3 \omega_4^2 \omega_1 + \frac{1}{2} \omega_3 v^2 \omega_4^2 \omega_1 + 5 \omega_2 \omega_3 v^2 \omega_1. \\
\gamma_{[y^2]}^{[\mu_5]} &= 3 \omega_3 \omega_1 + \frac{1}{2} \omega_2 \omega_3^2 \omega_1 + \frac{1}{2} \omega_2 \omega_3 \omega_4 \omega_1 + \frac{1}{2} \omega_3^2 \omega_4 \omega_1 - 4 \omega_2 \omega_3 v^2 \omega_4 \omega_1 + \frac{1}{2} \omega_3 \omega_4^2 \omega_1 + 2 \omega_2^2 \omega_3 v^2 \omega_1 - \omega_3 \omega_1^2 - \\
& \frac{5}{2} \omega_3 \omega_4 \omega_1 + \frac{1}{2} \omega_2 \omega_3 \omega_1^2 - \frac{3}{2} \omega_2 \omega_3 \omega_1 + 2 \omega_3 v^2 \omega_4^2 \omega_1 + \frac{1}{2} \omega_3 \omega_4 \omega_1^2 - \omega_3^2 \omega_1,
\end{aligned}$$

## 8 CLBM 2: relaxation of $k_{00}$ , $k_{10}$ , $k_{01}$ , $k_{20} + k_{02}$ , $k_{20} - k_{02}$

### 8.1 Definitions

Matrix  $\mathbf{A} = \mathbf{K}^{-1} \mathbf{S} \mathbf{K}$ :

$$\begin{aligned}
\mathbf{A}_{1,1} &= \omega_0 - \omega_3 u^2 + 2 u^2 \omega_1 + 2 v^2 \omega_2 - \omega_3 v^2 - \omega_0 u^2 - v^2 \omega_0, \\
\mathbf{A}_{1,2} &= 2 \omega_3 u - \omega_3 + \omega_0 - \omega_3 u^2 + 2 u^2 \omega_1 + 2 v^2 \omega_2 - 2 u \omega_1 - \omega_3 v^2 - \omega_0 u^2 - v^2 \omega_0, \\
\mathbf{A}_{1,3} &= -\omega_3 + \omega_0 - \omega_3 u^2 + 2 u^2 \omega_1 + 2 v^2 \omega_2 - 2 v \omega_2 - \omega_3 v^2 + 2 \omega_3 v - \omega_0 u^2 - v^2 \omega_0, \\
\mathbf{A}_{1,4} &= -2 \omega_3 u - \omega_3 + \omega_0 - \omega_3 u^2 + 2 u^2 \omega_1 + 2 v^2 \omega_2 + 2 u \omega_1 - \omega_3 v^2 - \omega_0 u^2 - v^2 \omega_0, \\
\mathbf{A}_{1,5} &= -\omega_3 + \omega_0 - \omega_3 u^2 + 2 u^2 \omega_1 + 2 v^2 \omega_2 + 2 v \omega_2 - \omega_3 v^2 - 2 \omega_3 v - \omega_0 u^2 - v^2 \omega_0, \\
\mathbf{A}_{2,1} &= \frac{1}{4} \omega_3 u^2 - u^2 \omega_1 - \frac{1}{2} u \omega_1 + \frac{1}{4} \omega_3 v^2 + \frac{1}{2} \omega_0 u + \frac{1}{2} \omega_0 u^2 - \frac{1}{4} v^2 \omega_4 + \frac{1}{4} \omega_4 u^2, \\
\mathbf{A}_{2,2} &= -\frac{1}{2} \omega_3 u + \frac{1}{4} \omega_3 + \frac{1}{4} \omega_3 u^2 - u^2 \omega_1 + \frac{1}{2} u \omega_1 - \frac{1}{2} \omega_4 u + \frac{1}{4} \omega_3 v^2 + \frac{1}{2} \omega_0 u + \frac{1}{4} \omega_4 + \frac{1}{2} \omega_0 u^2 - \frac{1}{4} v^2 \omega_4 + \frac{1}{4} \omega_4 u^2 + \frac{1}{2} \omega_1, \\
\mathbf{A}_{2,3} &= \frac{1}{2} v \omega_4 + \frac{1}{4} \omega_3 + \frac{1}{4} \omega_3 u^2 - u^2 \omega_1 - \frac{1}{2} u \omega_1 + \frac{1}{4} \omega_3 v^2 + \frac{1}{2} \omega_0 u - \frac{1}{2} \omega_3 v - \frac{1}{4} \omega_4 + \frac{1}{2} \omega_0 u^2 - \frac{1}{4} v^2 \omega_4 + \frac{1}{4} \omega_4 u^2, \\
\mathbf{A}_{2,4} &= \frac{1}{2} \omega_3 u + \frac{1}{4} \omega_3 + \frac{1}{4} \omega_3 u^2 - u^2 \omega_1 - \frac{3}{2} u \omega_1 + \frac{1}{2} \omega_4 u + \frac{1}{4} \omega_3 v^2 + \frac{1}{2} \omega_0 u + \frac{1}{4} \omega_4 + \frac{1}{2} \omega_0 u^2 - \frac{1}{4} v^2 \omega_4 + \frac{1}{4} \omega_4 u^2 - \frac{1}{2} \omega_1, \\
\mathbf{A}_{2,5} &= -\frac{1}{2} v \omega_4 + \frac{1}{4} \omega_3 + \frac{1}{4} \omega_3 u^2 - u^2 \omega_1 - \frac{1}{2} u \omega_1 + \frac{1}{4} \omega_3 v^2 + \frac{1}{2} \omega_0 u + \frac{1}{2} \omega_3 v - \frac{1}{4} \omega_4 + \frac{1}{2} \omega_0 u^2 - \frac{1}{4} v^2 \omega_4 + \frac{1}{4} \omega_4 u^2, \\
\mathbf{A}_{3,1} &= \frac{1}{2} v \omega_0 + \frac{1}{4} \omega_3 u^2 - v^2 \omega_2 - \frac{1}{2} v \omega_2 + \frac{1}{4} \omega_3 v^2 + \frac{1}{4} v^2 \omega_4 - \frac{1}{4} \omega_4 u^2 + \frac{1}{2} v^2 \omega_0, \\
\mathbf{A}_{3,2} &= \frac{1}{2} v \omega_0 - \frac{1}{2} \omega_3 u + \frac{1}{4} \omega_3 + \frac{1}{4} \omega_3 u^2 - v^2 \omega_2 - \frac{1}{2} v \omega_2 + \frac{1}{2} \omega_4 u + \frac{1}{4} \omega_3 v^2 - \frac{1}{4} \omega_4 + \frac{1}{4} v^2 \omega_4 - \frac{1}{4} \omega_4 u^2 + \frac{1}{2} v^2 \omega_0, \\
\mathbf{A}_{3,3} &= -\frac{1}{2} v \omega_4 + \frac{1}{2} v \omega_0 + \frac{1}{4} \omega_3 + \frac{1}{4} \omega_3 u^2 - v^2 \omega_2 + \frac{1}{2} \omega_2 + \frac{1}{2} v \omega_2 + \frac{1}{4} \omega_3 v^2 - \frac{1}{2} \omega_3 v + \frac{1}{4} \omega_4 + \frac{1}{4} v^2 \omega_4 - \frac{1}{4} \omega_4 u^2 + \frac{1}{2} v^2 \omega_0, \\
\mathbf{A}_{3,4} &= \frac{1}{2} v \omega_0 + \frac{1}{2} \omega_3 u + \frac{1}{4} \omega_3 + \frac{1}{4} \omega_3 u^2 - v^2 \omega_2 - \frac{1}{2} v \omega_2 - \frac{1}{2} \omega_4 u + \frac{1}{4} \omega_3 v^2 - \frac{1}{4} \omega_4 + \frac{1}{4} v^2 \omega_4 - \frac{1}{4} \omega_4 u^2 + \frac{1}{2} v^2 \omega_0,
\end{aligned}$$

$$\begin{aligned}
\mathbf{A}_{3,5} &= \frac{1}{2}v\omega_4 + \frac{1}{2}v\omega_0 + \frac{1}{4}\omega_3 + \frac{1}{4}\omega_3u^2 - v^2\omega_2 - \frac{1}{2}\omega_2 - \frac{3}{2}v\omega_2 + \frac{1}{4}\omega_3v^2 + \frac{1}{2}\omega_3v + \frac{1}{4}\omega_4 + \frac{1}{4}v^2\omega_4 - \frac{1}{4}\omega_4u^2 + \frac{1}{2}v^2\omega_0, \\
\mathbf{A}_{4,1} &= \frac{1}{4}\omega_3u^2 - u^2\omega_1 + \frac{1}{2}u\omega_1 + \frac{1}{4}\omega_3v^2 - \frac{1}{2}\omega_0u + \frac{1}{2}\omega_0u^2 - \frac{1}{4}v^2\omega_4 + \frac{1}{4}\omega_4u^2, \\
\mathbf{A}_{4,2} &= -\frac{1}{2}\omega_3u + \frac{1}{4}\omega_3 + \frac{1}{4}\omega_3u^2 - u^2\omega_1 + \frac{3}{2}u\omega_1 - \frac{1}{2}\omega_4u + \frac{1}{4}\omega_3v^2 - \frac{1}{2}\omega_0u + \frac{1}{4}\omega_4 + \frac{1}{2}\omega_0u^2 - \frac{1}{4}v^2\omega_4 + \frac{1}{4}\omega_4u^2 - \frac{1}{2}\omega_1, \\
\mathbf{A}_{4,3} &= \frac{1}{2}v\omega_4 + \frac{1}{4}\omega_3 + \frac{1}{4}\omega_3u^2 - u^2\omega_1 + \frac{1}{2}u\omega_1 + \frac{1}{4}\omega_3v^2 - \frac{1}{2}\omega_0u - \frac{1}{2}\omega_3v - \frac{1}{4}\omega_4 + \frac{1}{2}\omega_0u^2 - \frac{1}{4}v^2\omega_4 + \frac{1}{4}\omega_4u^2, \\
\mathbf{A}_{4,4} &= \frac{1}{2}\omega_3u + \frac{1}{4}\omega_3 + \frac{1}{4}\omega_3u^2 - u^2\omega_1 - \frac{1}{2}u\omega_1 + \frac{1}{2}\omega_4u + \frac{1}{4}\omega_3v^2 - \frac{1}{2}\omega_0u + \frac{1}{4}\omega_4 + \frac{1}{2}\omega_0u^2 - \frac{1}{4}v^2\omega_4 + \frac{1}{4}\omega_4u^2 + \frac{1}{2}\omega_1, \\
\mathbf{A}_{4,5} &= -\frac{1}{2}v\omega_4 + \frac{1}{4}\omega_3 + \frac{1}{4}\omega_3u^2 - u^2\omega_1 + \frac{1}{2}u\omega_1 + \frac{1}{4}\omega_3v^2 - \frac{1}{2}\omega_0u + \frac{1}{2}\omega_3v - \frac{1}{4}\omega_4 + \frac{1}{2}\omega_0u^2 - \frac{1}{4}v^2\omega_4 + \frac{1}{4}\omega_4u^2, \\
\mathbf{A}_{5,1} &= -\frac{1}{2}v\omega_0 + \frac{1}{4}\omega_3u^2 - v^2\omega_2 + \frac{1}{2}v\omega_2 + \frac{1}{4}\omega_3v^2 + \frac{1}{4}v^2\omega_4 - \frac{1}{4}\omega_4u^2 + \frac{1}{2}v^2\omega_0, \\
\mathbf{A}_{5,2} &= -\frac{1}{2}v\omega_0 - \frac{1}{2}\omega_3u + \frac{1}{4}\omega_3 + \frac{1}{4}\omega_3u^2 - v^2\omega_2 + \frac{1}{2}v\omega_2 + \frac{1}{2}\omega_4u + \frac{1}{4}\omega_3v^2 - \frac{1}{4}\omega_4 + \frac{1}{4}v^2\omega_4 - \frac{1}{4}\omega_4u^2 + \frac{1}{2}v^2\omega_0, \\
\mathbf{A}_{5,3} &= -\frac{1}{2}v\omega_4 - \frac{1}{2}v\omega_0 + \frac{1}{4}\omega_3 + \frac{1}{4}\omega_3u^2 - v^2\omega_2 - \frac{1}{2}\omega_2 + \frac{3}{2}v\omega_2 + \frac{1}{4}\omega_3v^2 - \frac{1}{2}\omega_3v + \frac{1}{4}\omega_4 + \frac{1}{4}v^2\omega_4 - \frac{1}{4}\omega_4u^2 + \frac{1}{2}v^2\omega_0, \\
\mathbf{A}_{5,4} &= -\frac{1}{2}v\omega_0 + \frac{1}{2}\omega_3u + \frac{1}{4}\omega_3 + \frac{1}{4}\omega_3u^2 - v^2\omega_2 + \frac{1}{2}v\omega_2 - \frac{1}{2}\omega_4u + \frac{1}{4}\omega_3v^2 - \frac{1}{4}\omega_4 + \frac{1}{4}v^2\omega_4 - \frac{1}{4}\omega_4u^2 + \frac{1}{2}v^2\omega_0, \\
\mathbf{A}_{5,5} &= \frac{1}{2}v\omega_4 - \frac{1}{2}v\omega_0 + \frac{1}{4}\omega_3 + \frac{1}{4}\omega_3u^2 - v^2\omega_2 + \frac{1}{2}\omega_2 - \frac{1}{2}v\omega_2 + \frac{1}{4}\omega_3v^2 + \frac{1}{2}\omega_3v + \frac{1}{4}\omega_4 + \frac{1}{4}v^2\omega_4 - \frac{1}{4}\omega_4u^2 + \frac{1}{2}v^2\omega_0.
\end{aligned}$$

where

$$\mathbf{S} = \text{diag}(\omega_0, \omega_1, \omega_2, \omega_3, \omega_4)$$

and

$$\mathbf{K} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -u & 1-u & -u & -u-1 & -u \\ -v & -v & 1-v & -v & -v-1 \\ v^2+u^2 & v^2+u^2-2u+1 & v^2-2v+u^2+1 & v^2+u^2+2u+1 & v^2+2v+u^2+1 \\ u^2-v^2 & -v^2+u^2-2u+1 & -v^2+2v+u^2-1 & -v^2+u^2+2u+1 & -v^2-2v+u^2-1 \end{pmatrix}$$

Matrix  $\mathbf{B}$ :

$$\begin{aligned}
\mathbf{B}_{1,1} &= 0, \\
\mathbf{B}_{1,2} &= -1 - 2\omega_3u + \omega_3 + 2u\omega_1, \\
\mathbf{B}_{1,3} &= -1 + \omega_3 + 2v\omega_2 - 2\omega_3v, \\
\mathbf{B}_{1,4} &= -1 + 2\omega_3u + \omega_3 - 2u\omega_1, \\
\mathbf{B}_{1,5} &= -1 + \omega_3 - 2v\omega_2 + 2\omega_3v, \\
\mathbf{B}_{2,1} &= -1 - \frac{1}{2}\omega_3u + \frac{1}{4}\omega_3 + u\omega_1 - \frac{1}{2}\omega_4u + \frac{1}{4}\omega_4 + \frac{1}{2}\omega_1, \\
\mathbf{B}_{2,2} &= 0, \\
\mathbf{B}_{2,3} &= -1 - \frac{1}{2}v\omega_4 - \frac{1}{2}\omega_3u + u\omega_1 - \frac{1}{2}\omega_4u + \frac{1}{2}\omega_3v + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_1, \\
\mathbf{B}_{2,4} &= -1 - \omega_3u + 2u\omega_1 - \omega_4u + \omega_1, \\
\mathbf{B}_{2,5} &= -1 + \frac{1}{2}v\omega_4 - \frac{1}{2}\omega_3u + u\omega_1 - \frac{1}{2}\omega_4u - \frac{1}{2}\omega_3v + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_1, \\
\mathbf{B}_{3,1} &= -1 - \frac{1}{2}v\omega_4 + \frac{1}{4}\omega_3 + \frac{1}{2}\omega_2 + v\omega_2 - \frac{1}{2}\omega_3v + \frac{1}{4}\omega_4, \\
\mathbf{B}_{3,2} &= -1 - \frac{1}{2}v\omega_4 + \frac{1}{2}\omega_3u + \frac{1}{2}\omega_2 + v\omega_2 - \frac{1}{2}\omega_4u - \frac{1}{2}\omega_3v + \frac{1}{2}\omega_4,
\end{aligned}$$

$$\begin{aligned}
\mathbf{B}_{3,3} &= 0, \\
\mathbf{B}_{3,4} &= -1 - \frac{1}{2}v\omega_4 - \frac{1}{2}\omega_3u + \frac{1}{2}\omega_2 + v\omega_2 + \frac{1}{2}\omega_4u - \frac{1}{2}\omega_3v + \frac{1}{2}\omega_4, \\
\mathbf{B}_{3,5} &= -1 - v\omega_4 + \omega_2 + 2v\omega_2 - \omega_3v, \\
\mathbf{B}_{4,1} &= -1 + \frac{1}{2}\omega_3u + \frac{1}{4}\omega_3 - u\omega_1 + \frac{1}{2}\omega_4u + \frac{1}{4}\omega_4 + \frac{1}{2}\omega_1, \\
\mathbf{B}_{4,2} &= -1 + \omega_3u - 2u\omega_1 + \omega_4u + \omega_1, \\
\mathbf{B}_{4,3} &= -1 - \frac{1}{2}v\omega_4 + \frac{1}{2}\omega_3u - u\omega_1 + \frac{1}{2}\omega_4u + \frac{1}{2}\omega_3v + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_1, \\
\mathbf{B}_{4,4} &= 0, \\
\mathbf{B}_{4,5} &= -1 + \frac{1}{2}v\omega_4 + \frac{1}{2}\omega_3u - u\omega_1 + \frac{1}{2}\omega_4u - \frac{1}{2}\omega_3v + \frac{1}{2}\omega_4 + \frac{1}{2}\omega_1, \\
\mathbf{B}_{5,1} &= -1 + \frac{1}{2}v\omega_4 + \frac{1}{4}\omega_3 + \frac{1}{2}\omega_2 - v\omega_2 + \frac{1}{2}\omega_3v + \frac{1}{4}\omega_4, \\
\mathbf{B}_{5,2} &= -1 + \frac{1}{2}v\omega_4 + \frac{1}{2}\omega_3u + \frac{1}{2}\omega_2 - v\omega_2 - \frac{1}{2}\omega_4u + \frac{1}{2}\omega_3v + \frac{1}{2}\omega_4, \\
\mathbf{B}_{5,3} &= -1 + v\omega_4 + \omega_2 - 2v\omega_2 + \omega_3v, \\
\mathbf{B}_{5,4} &= -1 + \frac{1}{2}v\omega_4 - \frac{1}{2}\omega_3u + \frac{1}{2}\omega_2 - v\omega_2 + \frac{1}{2}\omega_4u + \frac{1}{2}\omega_3v + \frac{1}{2}\omega_4, \\
\mathbf{B}_{5,5} &= 0.
\end{aligned}$$

## 8.2 EPDE for $\mu_1$

$$\begin{aligned}
&\gamma_{[t]}^{[\mu_1]} \delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[x]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[y]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[t^2]}^{[\mu_1]} \delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[tx]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} \\
&+ \gamma_{[ty]}^{[\mu_1]} \delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[x^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x \partial y} + \gamma_{[y^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} = 0,
\end{aligned}$$

where

$$\gamma_{[t]}^{[\mu_1]} = -\omega_3\omega_4\omega_1\omega_2,$$

$$\gamma_{[x]}^{[\mu_1]} = -\omega_3\omega_4\omega_1u\omega_2,$$

$$\gamma_{[y]}^{[\mu_1]} = -v\omega_3\omega_4\omega_1\omega_2,$$

$$\gamma_{[t^2]}^{[\mu_1]} = \frac{7}{2}\omega_3\omega_4\omega_1\omega_2 - \omega_3\omega_4\omega_2 - \omega_4\omega_1\omega_2 - \omega_3\omega_4\omega_1 - \omega_3\omega_1\omega_2,$$

$$\gamma_{[tx]}^{[\mu_1]} = -\omega_3\omega_4\omega_1u - 2\omega_3\omega_4u\omega_2 + 3\omega_3\omega_4\omega_1u\omega_2,$$

$$\gamma_{[ty]}^{[\mu_1]} = -2v\omega_3\omega_4\omega_1 + 3v\omega_3\omega_4\omega_1\omega_2 - v\omega_3\omega_4\omega_2,$$

$$\gamma_{[x^2]}^{[\mu_1]} = \omega_4\omega_1u^2\omega_2 + \omega_3\omega_1u^2\omega_2 - \omega_3\omega_4u^2\omega_2 - \frac{1}{2}\omega_3\omega_4c_s^2\omega_1\omega_2 - \frac{1}{2}\omega_3\omega_4\omega_1u^2\omega_2 + \omega_3\omega_4c_s^2\omega_2,$$

$$\gamma_{[xy]}^{[\mu_1]} = 2v\omega_4\omega_1u\omega_2 + 2v\omega_3\omega_1u\omega_2 - 2v\omega_3\omega_4u\omega_2 - 2v\omega_3\omega_4\omega_1u.$$

$$\gamma_{[y^2]}^{[\mu_1]} = v^2\omega_4\omega_1\omega_2 + \omega_3\omega_4c_s^2\omega_1 - \frac{1}{2}\omega_3\omega_4c_s^2\omega_1\omega_2 + v^2\omega_3\omega_1\omega_2 - v^2\omega_3\omega_4\omega_1 - \frac{1}{2}v^2\omega_3\omega_4\omega_1\omega_2,$$



### 8.3 EPDE for $\mu_2$

$$\begin{aligned} & \gamma_{[1]}^{[\mu_1]} \mu_1 + \gamma_{[1]}^{[\mu_2]} \mu_2 + \gamma_{[t]}^{[\mu_1]} \delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_2]} \delta_t \frac{\partial \mu_2}{\partial t} + \gamma_{[x]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[x]}^{[\mu_2]} \delta_l \frac{\partial \mu_2}{\partial x} + \gamma_{[y]}^{[\mu_1]} \delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[y]}^{[\mu_2]} \delta_l \frac{\partial \mu_2}{\partial y} \\ & + \gamma_{[t^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_2]} \delta_l^2 \frac{\partial^2 \mu_2}{\partial t^2} + \gamma_{[tx]}^{[\mu_1]} \delta_l \delta_t \frac{\partial^2 \mu_1}{\partial t \partial x} + \gamma_{[tx]}^{[\mu_2]} \delta_l \delta_t \frac{\partial^2 \mu_2}{\partial t \partial x} + \gamma_{[ty]}^{[\mu_1]} \delta_l \delta_t \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[ty]}^{[\mu_2]} \delta_l \delta_t \frac{\partial^2 \mu_2}{\partial t \partial y} \\ & + \gamma_{[x^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_2]} \delta_l^2 \frac{\partial^2 \mu_2}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial x \partial y} + \gamma_{[xy]}^{[\mu_2]} \delta_l^2 \frac{\partial^2 \mu_2}{\partial x \partial y} + \gamma_{[y^2]}^{[\mu_1]} \delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_2]} \delta_l^2 \frac{\partial^2 \mu_2}{\partial y^2} = 0, \end{aligned}$$

where

$$\begin{aligned} \gamma_{[1]}^{[\mu_1]} &= -\omega_3 \omega_4 \omega_1 u \omega_2^2 - \omega_3^2 \omega_4 \omega_1 u \omega_2 + 5\omega_3 \omega_4 \omega_1 u \omega_2 - \omega_3 \omega_4 \omega_1^2 u \omega_2 - \omega_3 \omega_4^2 \omega_1 u \omega_2, \\ \gamma_{[1]}^{[\mu_2]} &= -5\omega_3 \omega_4 \omega_1 \omega_2 + \omega_3 \omega_4 \omega_1 \omega_2^2 + \omega_3^2 \omega_4 \omega_1 \omega_2 + \omega_3 \omega_4^2 \omega_1 \omega_2 + \omega_3 \omega_4 \omega_1^2 \omega_2, \\ \gamma_{[t]}^{[\mu_1]} &= 4\omega_3 \omega_4 \omega_1 u \omega_2^2 + 5\omega_3 \omega_4 \omega_1 u - \omega_4 \omega_1^2 u \omega_2 - \omega_3 \omega_1^2 u \omega_2 + 4\omega_3^2 \omega_4 \omega_1 u \omega_2 - \omega_3^2 \omega_1 u \omega_2 - \omega_4^2 \omega_1 u \omega_2 - \\ & 24\omega_3 \omega_4 \omega_1 u \omega_2 - \omega_3^2 \omega_4 \omega_1 u - \omega_3 \omega_1 u \omega_2^2 - \omega_4 \omega_1 u \omega_2^2 + \omega_3 \omega_4^2 u \omega_2 + 4\omega_3 \omega_4 \omega_1^2 u \omega_2 - \omega_3 \omega_4^2 \omega_1 u + 4\omega_3 \omega_4^2 \omega_1 u \omega_2 + \\ & 5\omega_4 \omega_1 u \omega_2 - \omega_3 \omega_4 \omega_1^2 u + \omega_3^2 \omega_4 u \omega_2 + 5\omega_3 \omega_1 u \omega_2, \\ \gamma_{[t]}^{[\mu_2]} &= 23\omega_3 \omega_4 \omega_1 \omega_2 - 5\omega_3 \omega_4 \omega_2 + \omega_3^2 \omega_1 \omega_2 + \omega_3 \omega_1^2 \omega_2 + \omega_4 \omega_1 \omega_2^2 + \omega_3 \omega_4^2 \omega_1 - 5\omega_4 \omega_1 \omega_2 + \omega_3^2 \omega_4 \omega_1 - 4\omega_3 \omega_4 \omega_1 \omega_2^2 + \\ & \omega_3 \omega_4 \omega_2^2 + \omega_4^2 \omega_1 \omega_2 - 4\omega_3^2 \omega_4 \omega_1 \omega_2 - 5\omega_3 \omega_4 \omega_1 + \omega_4 \omega_1^2 \omega_2 + \omega_3 \omega_1 \omega_2^2 + \omega_3^2 \omega_4 \omega_2 - 5\omega_3 \omega_1 \omega_2 - 4\omega_3 \omega_4^2 \omega_1 \omega_2 + \\ & \omega_3 \omega_4 \omega_1^2 - 4\omega_3 \omega_4 \omega_1^2 \omega_2 + \omega_3 \omega_4^2 \omega_2, \\ \gamma_{[x]}^{[\mu_1]} &= \omega_3 \omega_4^2 c_s^2 \omega_2 - 5\omega_4 \omega_1 u^2 \omega_2 - 5\omega_3 \omega_1 u^2 \omega_2 - \omega_3^2 \omega_4 u^2 \omega_2 - \omega_3 \omega_4^2 u^2 \omega_2 + \omega_3^2 \omega_4 c_s^2 \omega_2 + \omega_3 \omega_1 u^2 \omega_2^2 + \omega_4 \omega_1 u^2 \omega_2^2 + \\ & \omega_3^2 \omega_1 u^2 \omega_2 + 5\omega_3 \omega_4 u^2 \omega_2 + \omega_3 \omega_4 c_s^2 \omega_1 \omega_2 + \omega_4^2 \omega_1 u^2 \omega_2 + \omega_3 \omega_4 \omega_1 u^2 \omega_2 + \omega_3 \omega_4 c_s^2 \omega_2^2 + \omega_4 \omega_1^2 u^2 \omega_2 - \\ & 5\omega_3 \omega_4 c_s^2 \omega_2 + \omega_3 \omega_1^2 u^2 \omega_2 - \omega_3 \omega_4 u^2 \omega_2^2, \\ \gamma_{[x]}^{[\mu_2]} &= 2\omega_3 \omega_4 u \omega_2^2 - \omega_4 \omega_1^2 u \omega_2 - \omega_3 \omega_1^2 u \omega_2 - 10\omega_3 \omega_4 u \omega_2 - \omega_3^2 \omega_1 u \omega_2 - \omega_4^2 \omega_1 u \omega_2 - 2\omega_3 \omega_4 \omega_1 u \omega_2 - \omega_3 \omega_1 u \omega_2^2 - \\ & \omega_4 \omega_1 u \omega_2^2 + 3\omega_3 \omega_4^2 u \omega_2 + 5\omega_4 \omega_1 u \omega_2 + 3\omega_3^2 \omega_4 u \omega_2 + 5\omega_3 \omega_1 u \omega_2, \\ \gamma_{[y]}^{[\mu_1]} &= -5v\omega_4 \omega_1 u \omega_2 - 5v\omega_3 \omega_1 u \omega_2 + v\omega_3^2 \omega_4 u \omega_2 - 2v\omega_3 \omega_4 \omega_1^2 u - 3v\omega_3 \omega_4^2 \omega_1 u + v\omega_3 \omega_4^2 u \omega_2 + v\omega_3 \omega_1 u \omega_2^2 - \\ & 3v\omega_3^2 \omega_4 \omega_1 u + v\omega_4 \omega_1 u \omega_2^2 + v\omega_3^2 \omega_1 u \omega_2 + v\omega_4^2 \omega_1 u \omega_2 + v\omega_4 \omega_1^2 u \omega_2 + v\omega_3 \omega_1^2 u \omega_2 + 10v\omega_3 \omega_4 \omega_1 u, \\ \gamma_{[y]}^{[\mu_2]} &= 2v\omega_3 \omega_4 \omega_1^2 + 5v\omega_3 \omega_1 \omega_2 - v\omega_3 \omega_1 \omega_2^2 - 10v\omega_3 \omega_4 \omega_1 - v\omega_4 \omega_1^2 \omega_2 - v\omega_4^2 \omega_1 \omega_2 + 3v\omega_3^2 \omega_4 \omega_1 + 3v\omega_3 \omega_4^2 \omega_1 + \\ & 5v\omega_4 \omega_1 \omega_2 - v\omega_4 \omega_1 \omega_2^2 - v\omega_3 \omega_1^2 \omega_2 - 2v\omega_3 \omega_4 \omega_1 \omega_2 - v\omega_3^2 \omega_1 \omega_2, \\ \gamma_{[t^2]}^{[\mu_1]} &= -8\omega_3 \omega_4 \omega_1 u \omega_2^2 - \frac{41}{2} \omega_3 \omega_4 \omega_1 u - \omega_3^2 \omega_1 u + \frac{7}{2} \omega_4 \omega_1^2 u \omega_2 + \frac{7}{2} \omega_3 \omega_1^2 u \omega_2 - \omega_3 \omega_1^2 u - 8\omega_3^2 \omega_4 \omega_1 u \omega_2 + 5\omega_4 \omega_1 u + \\ & 2\omega_3 \omega_4 u \omega_2 + \frac{7}{2} \omega_3^2 \omega_1 u \omega_2 + \omega_3^2 u \omega_2 + \frac{7}{2} \omega_4^2 \omega_1 u \omega_2 - \omega_1^2 u \omega_2 + 54\omega_3 \omega_4 \omega_1 u \omega_2 + \frac{7}{2} \omega_3^2 \omega_4 \omega_1 u + \frac{7}{2} \omega_3 \omega_1 u \omega_2^2 - \\ & \omega_4^2 \omega_1 u + \frac{7}{2} \omega_4 \omega_1 u \omega_2^2 - \frac{7}{2} \omega_3 \omega_4^2 u \omega_2 + 5\omega_1 u \omega_2 - 8\omega_3 \omega_4 \omega_1^2 u \omega_2 - \omega_4 \omega_1^2 u - \omega_1 u \omega_2^2 + \omega_3^2 \omega_4 u + 5\omega_3 \omega_1 u + \\ & \frac{7}{2} \omega_3 \omega_4^2 \omega_1 u - 8\omega_3 \omega_4^2 \omega_1 u \omega_2 + \omega_4^2 u \omega_2 - \frac{41}{2} \omega_4 \omega_1 u \omega_2 + \frac{7}{2} \omega_3 \omega_4 \omega_1^2 u + \omega_3 \omega_4^2 u - \frac{7}{2} \omega_3^2 \omega_4 u \omega_2 - \frac{41}{2} \omega_3 \omega_1 u \omega_2, \\ \gamma_{[t^2]}^{[\mu_2]} &= -\frac{101}{2} \omega_3 \omega_4 \omega_1 \omega_2 + \omega_3^2 \omega_2 - 5\omega_4 \omega_1 + \frac{39}{2} \omega_3 \omega_4 \omega_2 - \frac{7}{2} \omega_3^2 \omega_1 \omega_2 + \omega_1 \omega_2^2 - \frac{7}{2} \omega_3 \omega_1^2 \omega_2 - 5\omega_3 \omega_4 - \frac{7}{2} \omega_4 \omega_1 \omega_2^2 + \\ & \omega_3 \omega_1^2 - \frac{7}{2} \omega_3 \omega_4^2 \omega_1 + \frac{39}{2} \omega_4 \omega_1 \omega_2 - 5\omega_3 \omega_1 + \omega_4^2 \omega_2 + \omega_3 \omega_4^2 - \frac{7}{2} \omega_3^2 \omega_4 \omega_1 - 5\omega_1 \omega_2 + 8\omega_3 \omega_4 \omega_1 \omega_2^2 + \omega_4 \omega_1^2 - \\ & \frac{7}{2} \omega_3 \omega_4 \omega_2^2 + \omega_1^2 \omega_2 - \frac{7}{2} \omega_4^2 \omega_1 \omega_2 + 8\omega_3^2 \omega_4 \omega_1 \omega_2 + \omega_3 \omega_2^2 + \omega_3^2 \omega_1 - 5\omega_4 \omega_2 + \frac{39}{2} \omega_3 \omega_4 \omega_1 - \frac{7}{2} \omega_4 \omega_1^2 \omega_2 - \frac{7}{2} \omega_3 \omega_1 \omega_2^2 - \\ & \frac{7}{2} \omega_3^2 \omega_4 \omega_2 + \frac{39}{2} \omega_3 \omega_1 \omega_2 + \omega_3^2 \omega_4 + 8\omega_3 \omega_4^2 \omega_1 \omega_2 + \omega_4 \omega_2^2 - \frac{7}{2} \omega_3 \omega_4 \omega_1^2 + 8\omega_3 \omega_4 \omega_1^2 \omega_2 - \frac{7}{2} \omega_3 \omega_4^2 \omega_2 - 5\omega_3 \omega_2 + \omega_4^2 \omega_1, \\ \gamma_{[tx]}^{[\mu_1]} &= \omega_3^2 \omega_4 c_s^2 - 5\omega_3 c_s^2 \omega_2 - \frac{1}{2} \omega_4^2 u^2 \omega_2 - 3\omega_3 \omega_1^2 c_s^2 \omega_2 + \frac{33}{2} \omega_4 \omega_1 u^2 \omega_2 + \frac{1}{2} \omega_3 \omega_4 \omega_2 + \omega_3 c_s^2 \omega_1 \omega_2 + \frac{33}{2} \omega_3 \omega_1 u^2 \omega_2 + \\ & 3\omega_3^2 \omega_4 u^2 \omega_2 + 2\omega_1 u^2 \omega_2^2 + \frac{1}{2} v^2 \omega_4 \omega_1 \omega_2 - 5\omega_4 \omega_1 u^2 + \omega_3 \omega_4^2 c_s^2 - \frac{1}{2} \omega_3 u^2 \omega_2^2 - \frac{3}{2} v^2 \omega_3^2 \omega_2 + \frac{5}{2} \omega_3 u^2 \omega_2 + \omega_3 \omega_1^2 u^2 + \end{aligned}$$

$$\begin{aligned}
& \frac{1}{2}\omega_4\omega_1\omega_2 + \frac{1}{2}\omega_4^2\omega_2 + 3\omega_3\omega_4^2u^2\omega_2 - 10\omega_1u^2\omega_2 - 3\omega_3^2\omega_4c_s^2\omega_2 + \omega_3\omega_4\omega_1u^2 + \omega_3\omega_4c_s^2\omega_1 - 3\omega_3\omega_1u^2\omega_2^2 + \\
& \omega_3^2\omega_1u^2 + 5\omega_3\omega_4u^2 + \omega_3c_s^2\omega_2^2 - 3\omega_4\omega_1u^2\omega_2^2 + \frac{3}{2}v^2\omega_4^2\omega_2 - 3\omega_3^2\omega_1u^2\omega_2 - 16\omega_3\omega_4u^2\omega_2 - \omega_3\omega_4^2u^2 - \frac{1}{2}\omega_3^2u^2\omega_2 - \\
& 3\omega_3\omega_4c_s^2\omega_1\omega_2 - 3\omega_4^2\omega_1u^2\omega_2 - \frac{5}{2}v^2\omega_4\omega_2 - \frac{1}{2}\omega_3\omega_2^2 - 3\omega_3\omega_4\omega_1u^2\omega_2 + 2\omega_1^2u^2\omega_2 - 5\omega_3\omega_1u^2 - \omega_3^2\omega_4u^2 - 2\omega_4\omega_2 + \\
& \frac{3}{2}v^2\omega_3\omega_2^2 - \frac{1}{2}v^2\omega_3\omega_1\omega_2 - 3\omega_3\omega_4c_s^2\omega_2^2 - \frac{1}{2}\omega_4u^2\omega_2^2 + \frac{1}{2}\omega_3\omega_1\omega_2 - 3\omega_4\omega_1^2u^2\omega_2 + 16\omega_3\omega_4c_s^2\omega_2 + \omega_3^2c_s^2\omega_2 + \\
& \frac{5}{2}\omega_4u^2\omega_2 - 5\omega_3\omega_4c_s^2 + \omega_4\omega_1^2u^2 - 3\omega_3\omega_1^2u^2\omega_2 + \frac{1}{2}\omega_4\omega_2^2 + \frac{5}{2}v^2\omega_3\omega_2 + \omega_4^2\omega_1u^2 - \frac{3}{2}v^2\omega_4\omega_2^2 + 3\omega_3\omega_4u^2\omega_2^2, \\
\gamma_{[tx]}^{[\mu_2]} &= -2\omega_3\omega_4\omega_1u - 6\omega_3\omega_4u\omega_2^2 - 10\omega_3\omega_4u - \omega_3^2\omega_1u + 3\omega_4\omega_1^2u\omega_2 - 5\omega_4u\omega_2 + 3\omega_3\omega_1^2u\omega_2 - \omega_3\omega_1^2u + 5\omega_4\omega_1u + \\
& \omega_4u\omega_2^2 + 34\omega_3\omega_4u\omega_2 + 3\omega_3^2\omega_1u\omega_2 + 2\omega_3^2u\omega_2 + 3\omega_4^2\omega_1u\omega_2 - 2\omega_1^2u\omega_2 + 6\omega_3\omega_4\omega_1u\omega_2 + 3\omega_3\omega_1u\omega_2^2 - \omega_4^2\omega_1u + \\
& 3\omega_4\omega_1u\omega_2^2 - 5\omega_3\omega_2 - 9\omega_3\omega_4^2u\omega_2 + 10\omega_1u\omega_2 - \omega_4\omega_1^2u - 2\omega_1u\omega_2^2 + 3\omega_3^2\omega_4u + 5\omega_3\omega_1u + \omega_3u\omega_2^2 + 2\omega_4^2u\omega_2 - \\
& 18\omega_4\omega_1u\omega_2 + 3\omega_3\omega_4^2u - 9\omega_3^2\omega_4u\omega_2 - 18\omega_3\omega_1u\omega_2, \\
\gamma_{[ty]}^{[\mu_1]} &= 17v\omega_4\omega_1u\omega_2 + 2v\omega_3\omega_4^2u + 17v\omega_3\omega_1u\omega_2 - 3v\omega_3^2\omega_4u\omega_2 + 6v\omega_3\omega_4\omega_1^2u + 5v\omega_3\omega_1u + 2v\omega_3^2\omega_4u + 2v\omega_1u\omega_2^2 + \\
& 9v\omega_3\omega_1^2\omega_4u - 10v\omega_1u\omega_2 - 3v\omega_3\omega_4^2u\omega_2 - v\omega_4\omega_1^2u - 2v\omega_4^2\omega_1u - 3v\omega_3\omega_1u\omega_2^2 + 9v\omega_3^2\omega_4\omega_1u - 3v\omega_4\omega_1u\omega_2^2 - \\
& 3v\omega_3^2\omega_1u\omega_2 + 2v\omega_1^2u\omega_2 - 3v\omega_4^2\omega_1u\omega_2 + 5v\omega_4\omega_1u - v\omega_3\omega_1^2u - 3v\omega_4\omega_1^2u\omega_2 - 3v\omega_3\omega_1^2u\omega_2 - 2v\omega_3^2\omega_1u - \\
& 36v\omega_3\omega_4\omega_1u, \\
\gamma_{[ty]}^{[\mu_2]} &= 5v\omega_3\omega_2 + 2v\omega_4^2\omega_1 - v\omega_4\omega_2^2 - 6v\omega_3\omega_4\omega_1^2 - 18v\omega_3\omega_1\omega_2 + 3v\omega_3^2\omega_4 + 3v\omega_3\omega_1\omega_2^2 + 2v\omega_3^2\omega_1 + 5v\omega_4\omega_2 + \\
& 34v\omega_3\omega_4\omega_1 + 3v\omega_4\omega_1^2\omega_2 - v\omega_3\omega_2^2 + 3v\omega_4^2\omega_1\omega_2 - 2v\omega_1^2\omega_2 + v\omega_4\omega_1^2 - 9v\omega_3^2\omega_4\omega_1 + 10v\omega_1\omega_2 + 3v\omega_3\omega_4^2 - \\
& 5v\omega_3\omega_1 - v\omega_4^2\omega_2 - 9v\omega_3\omega_4^2\omega_1 - 18v\omega_4\omega_1\omega_2 + v\omega_3\omega_1^2 + 3v\omega_4\omega_1\omega_2^2 - 10v\omega_3\omega_4 + 3v\omega_3\omega_1^2\omega_2 - 2v\omega_1\omega_2^2 + \\
& 6v\omega_3\omega_4\omega_1\omega_2 - v\omega_3^2\omega_2 - 5v\omega_4\omega_1 + 3v\omega_3^2\omega_1\omega_2 - 2v\omega_3\omega_4\omega_2, \\
\gamma_{[x^2]}^{[\mu_1]} &= -\frac{1}{2}\omega_3\omega_4^2u^3\omega_2 - \frac{1}{2}\omega_3^2\omega_4u^3\omega_2 - v^2\omega_4\omega_1u\omega_2^2 + v^2\omega_3\omega_1u\omega_2^2 - \omega_3\omega_4c_s^2\omega_1u\omega_2 - \frac{1}{4}\omega_3^2\omega_1u\omega_2 + \frac{1}{2}\omega_3\omega_4^2c_s^2u\omega_2 - \\
& \frac{1}{4}\omega_3\omega_4\omega_1u\omega_2 - \frac{1}{2}\omega_3\omega_1u\omega_2^2 - \omega_3\omega_1^2u^3\omega_2 - \omega_4\omega_1^2u^3\omega_2 + \frac{1}{2}v^2\omega_4^2\omega_1u\omega_2 - \frac{1}{2}v^2\omega_3^2\omega_1u\omega_2 + \frac{1}{2}\omega_3^2\omega_4c_s^2u\omega_2 + \\
& 2\omega_3\omega_4\omega_1u^3\omega_2 + \frac{1}{2}\omega_4^2\omega_1u^3\omega_2 + \frac{1}{2}\omega_3^2\omega_1u^3\omega_2 + \frac{1}{4}\omega_4\omega_1u\omega_2 + \frac{5}{4}\omega_3\omega_1u\omega_2, \\
\gamma_{[x^2]}^{[\mu_2]} &= \frac{3}{4}\omega_3\omega_4\omega_1\omega_2 - \frac{1}{2}\omega_3^2\omega_2 - 3\omega_3\omega_4\omega_2 + \frac{1}{4}\omega_3^2\omega_1\omega_2 + \omega_3^2\omega_4u^2\omega_2 - v^2\omega_3^2\omega_2 - \frac{1}{4}\omega_4\omega_1\omega_2 + \omega_3\omega_4^2u^2\omega_2 + \\
& v^2\omega_4\omega_1\omega_2^2 + \frac{1}{2}v^2\omega_3^2\omega_1\omega_2 + v^2\omega_4^2\omega_2 + \frac{1}{2}\omega_3\omega_4\omega_2^2 - \frac{1}{2}\omega_3^2\omega_1u^2\omega_2 - \frac{1}{2}\omega_4^2\omega_1u^2\omega_2 - \omega_3\omega_2^2 - 3\omega_3\omega_4\omega_1u^2\omega_2 + \\
& \frac{1}{2}\omega_4\omega_2 + 2v^2\omega_3\omega_2^2 + \frac{1}{2}\omega_3\omega_1\omega_2^2 + \frac{1}{2}\omega_3^2\omega_4\omega_2 - \frac{5}{4}\omega_3\omega_1\omega_2 + \omega_4\omega_1^2u^2\omega_2 + \omega_3\omega_1^2u^2\omega_2 - v^2\omega_3\omega_1\omega_2^2 - \\
& \frac{1}{2}v^2\omega_4^2\omega_1\omega_2 + \frac{1}{2}\omega_3\omega_4^2\omega_2 - 2v^2\omega_4\omega_2^2 + \frac{5}{2}\omega_3\omega_2, \\
\gamma_{[xy]}^{[\mu_1]} &= 3v\omega_4^2\omega_1u^2 - \frac{5}{2}v\omega_3\omega_2 + v^3\omega_3\omega_2^2 - 2v\omega_3\omega_4c_s^2\omega_2 - 5v^3\omega_4\omega_2 + v\omega_3\omega_1\omega_2 - 10v\omega_3\omega_4c_s^2 + 2v\omega_4\omega_1^2u^2 - \\
& 5v\omega_4u^2\omega_2 - 2v\omega_3^2c_s^2\omega_2 + v^3\omega_4\omega_1\omega_2 + v\omega_4u^2\omega_2^2 + \frac{1}{2}v\omega_4\omega_2 - 3v\omega_3^2\omega_4u^2 - 10v\omega_3\omega_1u^2 - v^3\omega_4\omega_2^2 - \\
& 4v\omega_1^2u^2\omega_2 + 2v\omega_3\omega_4u^2\omega_2 + 5v^3\omega_3\omega_2 + v\omega_3^2u^2\omega_2 - 3v\omega_3\omega_4^2u^2 - 2v\omega_3c_s^2\omega_2^2 - 2v^3\omega_3^2\omega_2 + 2v\omega_3\omega_4c_s^2\omega_1 + \\
& 4v\omega_3\omega_4\omega_1u^2 + 10v\omega_3\omega_4u^2 + 3v\omega_3^2\omega_1u^2 + 20v\omega_1u^2\omega_2 + 2v\omega_3\omega_1^2u^2 - 5v\omega_3u^2\omega_2 - v^3\omega_3\omega_1\omega_2 + v\omega_3u^2\omega_2^2 + \\
& 2v^3\omega_4^2\omega_2 - 4v\omega_1u^2\omega_2^2 - 10v\omega_4\omega_1u^2 + 3v\omega_3\omega_4^2c_s^2 - 3v\omega_4\omega_1u^2\omega_2 + \frac{1}{2}v\omega_3^2\omega_2 + v\omega_4^2u^2\omega_2 + 10v\omega_3c_s^2\omega_2 + \\
& 3v\omega_3^2\omega_4c_s^2 - 3v\omega_3\omega_1u^2\omega_2 + \frac{1}{2}v\omega_3\omega_4\omega_2 - 2v\omega_3c_s^2\omega_1\omega_2, \\
\gamma_{[xy]}^{[\mu_2]} &= 4v\omega_4\omega_1u\omega_2 + 8v\omega_3\omega_4^2u + 4v\omega_3\omega_1u\omega_2 - 3v\omega_4^2u\omega_2 - 2v\omega_3u\omega_2^2 + 10v\omega_3\omega_1u + 8v\omega_3^2\omega_4u + 4v\omega_1u\omega_2^2 - \\
& 20v\omega_1u\omega_2 - 2v\omega_4\omega_1^2u + 10v\omega_3u\omega_2 - 3v\omega_4^2\omega_1u - 6v\omega_3\omega_4u\omega_2 + 4v\omega_1^2u\omega_2 - 3v\omega_3^2u\omega_2 - 2v\omega_4u\omega_2^2 + \\
& 10v\omega_4\omega_1u - 2v\omega_3\omega_1^2u + 10v\omega_4u\omega_2 - 3v\omega_3^2\omega_1u - 20v\omega_3\omega_4u - 6v\omega_3\omega_4\omega_1u, \\
\gamma_{[y^2]}^{[\mu_1]} &= -v^2\omega_3\omega_1^2u - v^2\omega_3^2u\omega_2 + 3\omega_3\omega_4\omega_1u - \omega_3\omega_1^2c_s^2u + \frac{1}{2}\omega_3^2\omega_1u + \frac{1}{2}v^2\omega_3^2\omega_4u\omega_2 + 2v^2\omega_3\omega_1u\omega_2 + \\
& 2v^2\omega_4\omega_1u\omega_2 - \frac{1}{2}\omega_3\omega_1^2u\omega_2 + \omega_3\omega_1^2u + \omega_3^2\omega_1u^3 - v^2\omega_4\omega_1u\omega_2^2 - \omega_3^2\omega_4c_s^2u - v^2\omega_3\omega_1u\omega_2^2 - v^2\omega_3^2\omega_4\omega_1u - \\
& \frac{1}{2}\omega_4\omega_1u - \omega_3\omega_4c_s^2\omega_1u\omega_2 - \frac{1}{4}\omega_3^2\omega_1u\omega_2 - 2\omega_3\omega_1^2u^3 + \frac{1}{2}\omega_3\omega_4^2c_s^2u\omega_2 + \frac{1}{2}v^2\omega_3\omega_4^2u\omega_2 - \frac{3}{4}\omega_3\omega_4\omega_1u\omega_2 + \\
& 2\omega_3\omega_4c_s^2\omega_1u - \frac{1}{2}\omega_3^2\omega_4\omega_1u - v^2\omega_4^2u\omega_2 + \omega_3\omega_1^2u^3\omega_2 + v^2\omega_3\omega_4^2u - \omega_4\omega_1^2u^3\omega_2 + \frac{1}{2}v^2\omega_4^2\omega_1u\omega_2 - \\
& 2v^2\omega_3\omega_4u\omega_2 + \frac{1}{2}v^2\omega_3^2\omega_1u\omega_2 + v^2\omega_3^2\omega_4u + 2v^2\omega_3\omega_4\omega_1u\omega_2 + \frac{1}{2}\omega_3^2\omega_4c_s^2u\omega_2 - \omega_4^2\omega_1u^3 + \frac{1}{2}\omega_4^2\omega_1u^3\omega_2 - \\
& 2v^2\omega_3\omega_4\omega_1u - \frac{5}{2}\omega_3\omega_1u - \frac{1}{2}\omega_3^2\omega_1u^3\omega_2 - \frac{1}{2}\omega_3\omega_4^2\omega_1u + 2\omega_4\omega_1^2u^3 + \frac{1}{4}\omega_4\omega_1u\omega_2 - \frac{1}{2}\omega_3\omega_4\omega_1^2u + \frac{5}{4}\omega_3\omega_1u\omega_2. \\
\gamma_{[y^2]}^{[\mu_2]} &= \\
& \frac{3}{4}\omega_3\omega_4\omega_1\omega_2 + \frac{1}{2}\omega_4\omega_1 + v^2\omega_3^2\omega_4\omega_1 + \frac{1}{4}\omega_3^2\omega_1\omega_2 + v^2\omega_3\omega_4^2\omega_1 + \frac{1}{2}\omega_3\omega_1^2\omega_2 - \omega_3\omega_1^2 + 2\omega_3\omega_1^2u^2 + \frac{1}{2}\omega_3\omega_1^2\omega_1 - \frac{1}{4}\omega_4\omega_1\omega_2 + \\
& \frac{5}{2}\omega_3\omega_1 + v^2\omega_4\omega_1\omega_2^2 + \frac{1}{2}\omega_3^2\omega_4\omega_1 - \frac{1}{2}v^2\omega_3^2\omega_1\omega_2 - \omega_3^2\omega_1u^2 + \frac{1}{2}\omega_3^2\omega_1u^2\omega_2 - \frac{1}{2}\omega_4^2\omega_1u^2\omega_2 - \frac{1}{2}\omega_3^2\omega_1 - 3\omega_3\omega_4\omega_1 -
\end{aligned}$$

$$\frac{5}{4}\omega_3\omega_1\omega_2 + \omega_4\omega_1^2u^2\omega_2 - 2\omega_4\omega_1^2u^2 - \omega_3\omega_1^2u^2\omega_2 - 3v^2\omega_3\omega_4\omega_1\omega_2 + v^2\omega_3\omega_1\omega_2^2 + \frac{1}{2}\omega_3\omega_4\omega_1^2 + \omega_4^2\omega_1u^2 - \frac{1}{2}v^2\omega_4^2\omega_1\omega_2,$$

#### 8.4 EPDE for $\mu_3$

$$\begin{aligned} & \gamma_{[1]}^{[\mu_1]}\mu_1 + \gamma_{[1]}^{[\mu_3]}\mu_3 + \gamma_{[t]}^{[\mu_1]}\delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_3]}\delta_t \frac{\partial \mu_3}{\partial t} + \gamma_{[x]}^{[\mu_1]}\delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[x]}^{[\mu_3]}\delta_l \frac{\partial \mu_3}{\partial x} + \gamma_{[y]}^{[\mu_1]}\delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[y]}^{[\mu_3]}\delta_l \frac{\partial \mu_3}{\partial y} \\ & + \gamma_{[t^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_3]}\delta_l^2 \frac{\partial^2 \mu_3}{\partial t^2} + \gamma_{[tx]}^{[\mu_1]}\delta_l \delta_t \frac{\partial^2 \mu_1}{\partial t \partial x} + \gamma_{[tx]}^{[\mu_3]}\delta_l \delta_t \frac{\partial^2 \mu_3}{\partial t \partial x} + \gamma_{[ty]}^{[\mu_1]}\delta_l \delta_t \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[ty]}^{[\mu_3]}\delta_l \delta_t \frac{\partial^2 \mu_3}{\partial t \partial y} \\ & + \gamma_{[x^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_3]}\delta_l^2 \frac{\partial^2 \mu_3}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial x \partial y} + \gamma_{[xy]}^{[\mu_3]}\delta_l^2 \frac{\partial^2 \mu_3}{\partial x \partial y} + \gamma_{[y^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_3]}\delta_l^2 \frac{\partial^2 \mu_3}{\partial y^2} = 0, \end{aligned}$$

where

$$\gamma_{[1]}^{[\mu_1]} = -v\omega_3\omega_4\omega_1^2\omega_2 - v\omega_3\omega_4^2\omega_1\omega_2 - v\omega_3^2\omega_4\omega_1\omega_2 - v\omega_3\omega_4\omega_1\omega_2^2 + 5v\omega_3\omega_4\omega_1\omega_2,$$

$$\gamma_{[1]}^{[\mu_3]} = -5\omega_3\omega_4\omega_1\omega_2 + \omega_3\omega_4\omega_1\omega_2^2 + \omega_3^2\omega_4\omega_1\omega_2 + \omega_3\omega_4^2\omega_1\omega_2 + \omega_3\omega_4\omega_1^2\omega_2,$$

$$\begin{aligned} \gamma_{[t]}^{[\mu_1]} &= 4v\omega_3\omega_4\omega_1^2\omega_2 - v\omega_3\omega_4^2\omega_2 + 4v\omega_3\omega_4^2\omega_1\omega_2 + 5v\omega_3\omega_1\omega_2 - v\omega_3^2\omega_4\omega_2 - v\omega_3\omega_1\omega_2^2 - v\omega_4\omega_1^2\omega_2 - v\omega_4^2\omega_1\omega_2 + \\ & 4v\omega_3^2\omega_4\omega_1\omega_2 - v\omega_3\omega_4\omega_2^2 + v\omega_3^2\omega_4\omega_1 + 4v\omega_3\omega_4\omega_1\omega_2^2 + v\omega_3\omega_4^2\omega_1 + 5v\omega_4\omega_1\omega_2 - v\omega_4\omega_1\omega_2^2 - v\omega_3\omega_1^2\omega_2 - \\ & 24v\omega_3\omega_4\omega_1\omega_2 - v\omega_3^2\omega_1\omega_2 + 5v\omega_3\omega_4\omega_2, \end{aligned}$$

$$\begin{aligned} \gamma_{[t]}^{[\mu_3]} &= 23\omega_3\omega_4\omega_1\omega_2 - 5\omega_3\omega_4\omega_2 + \omega_3^2\omega_1\omega_2 + \omega_3\omega_1^2\omega_2 + \omega_4\omega_1\omega_2^2 + \omega_3\omega_4^2\omega_1 - 5\omega_4\omega_1\omega_2 + \omega_3^2\omega_4\omega_1 - 4\omega_3\omega_4\omega_1\omega_2^2 + \\ & \omega_3\omega_4\omega_2^2 + \omega_4^2\omega_1\omega_2 - 4\omega_3^2\omega_4\omega_1\omega_2 - 5\omega_3\omega_4\omega_1 + \omega_4\omega_1^2\omega_2 + \omega_3\omega_1\omega_2^2 + \omega_3^2\omega_4\omega_2 - 5\omega_3\omega_1\omega_2 - 4\omega_3\omega_4^2\omega_1\omega_2 + \\ & \omega_3\omega_4\omega_1^2 - 4\omega_3\omega_4\omega_1^2\omega_2 + \omega_3\omega_4^2\omega_2, \end{aligned}$$

$$\begin{aligned} \gamma_{[x]}^{[\mu_1]} &= -5v\omega_4\omega_1u\omega_2 - 5v\omega_3\omega_1u\omega_2 - 3v\omega_3^2\omega_4u\omega_2 + v\omega_3\omega_4^2u\omega_1 - 3v\omega_3\omega_4^2u\omega_2 + v\omega_3\omega_1u\omega_2^2 + v\omega_3^2\omega_4u\omega_1 + \\ & v\omega_4\omega_1u\omega_2^2 + v\omega_3^2\omega_1u\omega_2 + 10v\omega_3\omega_4u\omega_2 + v\omega_4^2u\omega_1u\omega_2 + v\omega_4\omega_1^2u\omega_2 + v\omega_3\omega_1^2u\omega_2 - 2v\omega_3\omega_4u\omega_2^2, \end{aligned}$$

$$\begin{aligned} \gamma_{[x]}^{[\mu_3]} &= 2\omega_3\omega_4u\omega_2^2 - \omega_4\omega_1^2u\omega_2 - \omega_3\omega_1^2u\omega_2 - 10\omega_3\omega_4u\omega_2 - \omega_3^2\omega_1u\omega_2 - \omega_4^2\omega_1u\omega_2 - 2\omega_3\omega_4\omega_1u\omega_2 - \omega_3\omega_1u\omega_2^2 - \\ & \omega_4\omega_1u\omega_2^2 + 3\omega_3\omega_4^2u\omega_2 + 5\omega_4\omega_1u\omega_2 + 3\omega_3^2\omega_4u\omega_2 + 5\omega_3\omega_1u\omega_2, \end{aligned}$$

$$\begin{aligned} \gamma_{[y]}^{[\mu_1]} &= -v^2\omega_3^2\omega_4\omega_1 + \omega_3\omega_4c_s^2\omega_1^2 - v^2\omega_3\omega_4^2\omega_1 - 5v^2\omega_4\omega_1\omega_2 + v^2\omega_3\omega_1^2\omega_2 + v^2\omega_4\omega_1\omega_2^2 + v^2\omega_3^2\omega_1\omega_2 - \\ & 5\omega_3\omega_4c_s^2\omega_1 + \omega_3\omega_4c_s^2\omega_1\omega_2 + \omega_3\omega_4^2c_s^2\omega_1 - 5v^2\omega_3\omega_1\omega_2 - v^2\omega_3\omega_4\omega_1^2 + 5v^2\omega_3\omega_4\omega_1 + v^2\omega_4\omega_1^2\omega_2 + \\ & v^2\omega_3\omega_4\omega_1\omega_2 + v^2\omega_3\omega_1\omega_2^2 + v^2\omega_4^2\omega_1\omega_2 + \omega_3^2\omega_4c_s^2\omega_1, \end{aligned}$$

$$\begin{aligned} \gamma_{[y]}^{[\mu_3]} &= 2v\omega_3\omega_4\omega_1^2 + 5v\omega_3\omega_1\omega_2 - v\omega_3\omega_1\omega_2^2 - 10v\omega_3\omega_4\omega_1 - v\omega_4\omega_1^2\omega_2 - v\omega_4^2\omega_1\omega_2 + 3v\omega_3^2\omega_4\omega_1 + 3v\omega_3\omega_4^2\omega_1 + \\ & 5v\omega_4\omega_1\omega_2 - v\omega_4\omega_1\omega_2^2 - v\omega_3\omega_1^2\omega_2 - 2v\omega_3\omega_4\omega_1\omega_2 - v\omega_3^2\omega_1\omega_2, \end{aligned}$$

$$\begin{aligned} \gamma_{[t^2]}^{[\mu_1]} &= -8v\omega_3\omega_4\omega_1^2\omega_2 + 5v\omega_3\omega_2 + v\omega_4^2\omega_1 + \frac{7}{2}v\omega_3\omega_4^2\omega_2 - 8v\omega_3\omega_4^2\omega_1\omega_2 - v\omega_4\omega_2^2 - \frac{41}{2}v\omega_3\omega_1\omega_2 + \frac{7}{2}v\omega_3^2\omega_4\omega_2 + \\ & v\omega_3^2\omega_4 + \frac{7}{2}v\omega_3\omega_1\omega_2^2 + v\omega_3^2\omega_1 + 5v\omega_4\omega_2 + 2v\omega_3\omega_4\omega_1 + \frac{7}{2}v\omega_4\omega_1^2\omega_2 - v\omega_3\omega_2^2 + \frac{7}{2}v\omega_4^2\omega_1\omega_2 - 8v\omega_3^2\omega_4\omega_1\omega_2 - \\ & v\omega_1^2\omega_2 + \frac{7}{2}v\omega_3\omega_4\omega_2^2 - \frac{7}{2}v\omega_3^2\omega_4\omega_1 - 8v\omega_3\omega_4\omega_1\omega_2^2 + 5v\omega_1\omega_2 + v\omega_3\omega_4^2 - v\omega_4^2\omega_2 - \frac{7}{2}v\omega_3\omega_4^2\omega_1 - \frac{41}{2}v\omega_4\omega_1\omega_2 + \\ & \frac{7}{2}v\omega_4\omega_1\omega_2^2 + \frac{7}{2}v\omega_3\omega_1^2\omega_2 - v\omega_1\omega_2^2 + 54v\omega_3\omega_4\omega_1\omega_2 - v\omega_3^2\omega_2 + \frac{7}{2}v\omega_3^2\omega_1\omega_2 - \frac{41}{2}v\omega_3\omega_4\omega_2, \end{aligned}$$

$$\begin{aligned} \gamma_{[t^2]}^{[\mu_3]} &= -\frac{101}{2}\omega_3\omega_4\omega_1\omega_2 + \omega_3^2\omega_2 - 5\omega_4\omega_1 + \frac{39}{2}\omega_3\omega_4\omega_2 - \frac{7}{2}\omega_3^2\omega_1\omega_2 + \omega_1\omega_2^2 - \frac{7}{2}\omega_3\omega_1^2\omega_2 - 5\omega_3\omega_4 - \frac{7}{2}\omega_4\omega_1\omega_2^2 + \\ & \omega_3\omega_1^2 - \frac{7}{2}\omega_3\omega_4^2\omega_1 + \frac{39}{2}\omega_4\omega_1\omega_2 - 5\omega_3\omega_1 + \omega_4^2\omega_2 + \omega_3\omega_4^2 - \frac{7}{2}\omega_3^2\omega_4\omega_1 - 5\omega_1\omega_2 + 8\omega_3\omega_4\omega_1\omega_2^2 + \omega_4\omega_1^2 - \\ & \frac{7}{2}\omega_3\omega_4\omega_2^2 + \omega_1^2\omega_2 - \frac{7}{2}\omega_4^2\omega_1\omega_2 + 8\omega_3^2\omega_4\omega_1\omega_2 + \omega_3\omega_2^2 + \omega_3^2\omega_1 - 5\omega_4\omega_2 + \frac{39}{2}\omega_3\omega_4\omega_1 - \frac{7}{2}\omega_4\omega_1^2\omega_2 - \frac{7}{2}\omega_3\omega_1\omega_2^2 - \\ & \frac{7}{2}\omega_3^2\omega_4\omega_2 + \frac{39}{2}\omega_3\omega_1\omega_2 + \omega_3^2\omega_4 + 8\omega_3\omega_4^2\omega_1\omega_2 + \omega_4\omega_2^2 - \frac{7}{2}\omega_3\omega_4\omega_1^2 + 8\omega_3\omega_4\omega_1^2\omega_2 - \frac{7}{2}\omega_3\omega_4^2\omega_2 - 5\omega_3\omega_2 + \omega_4^2\omega_1, \end{aligned}$$

$$\begin{aligned}
\gamma_{[tx]}^{[\mu_1]} &= 17v\omega_4\omega_1u\omega_2 + 2v\omega_3\omega_4^2u + 17v\omega_3\omega_1u\omega_2 + 9v\omega_3^2\omega_4u\omega_2 - 2v\omega_4^2u\omega_2 - v\omega_3u\omega_2^2 + 2v\omega_3^2\omega_4u + 2v\omega_1u\omega_2^2 - \\
&\quad 3v\omega_3\omega_4^2\omega_1u - 10v\omega_1u\omega_2 + 9v\omega_3\omega_4^2u\omega_2 + 5v\omega_3u\omega_2 - 3v\omega_3\omega_1u\omega_2^2 - 3v\omega_3^2\omega_4\omega_1u - 3v\omega_4\omega_1u\omega_2^2 - \\
&\quad 3v\omega_3^2\omega_1u\omega_2 - 36v\omega_3\omega_4u\omega_2 + 2v\omega_1^2u\omega_2 - 3v\omega_4^2\omega_1u\omega_2 - 2v\omega_3^2u\omega_2 - v\omega_4u\omega_2^2 - 3v\omega_4\omega_1^2u\omega_2 + 5v\omega_4u\omega_2 - \\
&\quad 3v\omega_3\omega_1^2u\omega_2 + 6v\omega_3\omega_4u\omega_2^2, \\
\gamma_{[tx]}^{[\mu_3]} &= -2\omega_3\omega_4\omega_1u - 6\omega_3\omega_4u\omega_2^2 - 10\omega_3\omega_4u - \omega_3^2\omega_1u + 3\omega_4\omega_1^2u\omega_2 - 5\omega_4u\omega_2 + 3\omega_3\omega_1^2u\omega_2 - \omega_3\omega_1^2u + 5\omega_4\omega_1u + \\
&\quad \omega_4u\omega_2^2 + 34\omega_3\omega_4u\omega_2 + 3\omega_3^2\omega_1u\omega_2 + 2\omega_3^2u\omega_2 + 3\omega_4^2\omega_1u\omega_2 - 2\omega_1^2u\omega_2 + 6\omega_3\omega_4\omega_1u\omega_2 + 3\omega_3\omega_1u\omega_2^2 - \omega_4^2\omega_1u + \\
&\quad 3\omega_4\omega_1u\omega_2^2 - 5\omega_3u\omega_2 - 9\omega_3\omega_4^2u\omega_2 + 10\omega_1u\omega_2 - \omega_4\omega_1^2u - 2\omega_1u\omega_2^2 + 3\omega_3^2\omega_4u + 5\omega_3\omega_1u + \omega_3u\omega_2^2 + 2\omega_4^2u\omega_2 - \\
&\quad 18\omega_4\omega_1u\omega_2 + 3\omega_3\omega_4^2u - 9\omega_3^2\omega_4u\omega_2 - 18\omega_3\omega_1u\omega_2, \\
\gamma_{[ty]}^{[\mu_1]} &= \omega_3^2\omega_4c_s^2 - \frac{1}{2}v^2\omega_3\omega_1^2 - 2\omega_4\omega_1 + \frac{1}{2}\omega_4\omega_1u^2\omega_2 + 3v^2\omega_3^2\omega_4\omega_1 + \omega_3c_s^2\omega_1\omega_2 - 3\omega_3\omega_4c_s^2\omega_1^2 - \frac{1}{2}\omega_3\omega_1u^2\omega_2 + \\
&\quad 5v^2\omega_3\omega_4 + 2v^2\omega_1\omega_2^2 + 3v^2\omega_3\omega_4^2\omega_1 + \frac{33}{2}v^2\omega_4\omega_1\omega_2 - \frac{5}{2}\omega_4\omega_1u^2 + \omega_3\omega_4^2c_s^2 - \frac{1}{2}\omega_3\omega_1^2 + \frac{5}{2}v^2\omega_4\omega_1 + v^2\omega_3^2\omega_2 + \\
&\quad \frac{3}{2}\omega_3\omega_1^2u^2 + \frac{1}{2}\omega_4\omega_1\omega_2 - 3v^2\omega_3\omega_1^2\omega_2 - \frac{1}{2}v^2\omega_4\omega_1^2 - 10v^2\omega_1\omega_2 - 3v^2\omega_4\omega_1\omega_2^2 + v^2\omega_3\omega_4\omega_2 - 3v^2\omega_3^2\omega_1\omega_2 + \\
&\quad 16\omega_3\omega_4c_s^2\omega_1 + \omega_3^2c_s^2\omega_1 - \frac{3}{2}\omega_3^2\omega_1u^2 - v^2\omega_3\omega_4^2 + v^2\omega_4^2\omega_2 + \frac{5}{2}v^2\omega_3\omega_1 + \frac{1}{2}\omega_4\omega_1^2 - 3\omega_3\omega_4c_s^2\omega_1\omega_2 - 5v^2\omega_4\omega_2 - \\
&\quad \frac{1}{2}v^2\omega_3^2\omega_1 + \frac{5}{2}\omega_3\omega_1u^2 - 5\omega_3c_s^2\omega_1 - 3\omega_3\omega_4^2c_s^2\omega_1 + v^2\omega_3\omega_2^2 + \frac{1}{2}\omega_3\omega_4\omega_1 + \frac{33}{2}v^2\omega_3\omega_1\omega_2 + 3v^2\omega_3\omega_4\omega_1^2 + \\
&\quad 2v^2\omega_1^2\omega_2 - 16v^2\omega_3\omega_4\omega_1 - 3v^2\omega_4\omega_1^2\omega_2 + \frac{1}{2}\omega_3\omega_1\omega_2 + \omega_3\omega_4c_s^2\omega_2 - 5\omega_3\omega_4c_s^2 - \frac{3}{2}\omega_4\omega_1^2u^2 + \omega_3c_s^2\omega_1^2 - \frac{1}{2}v^2\omega_4^2\omega_1 - \\
&\quad 5v^2\omega_3\omega_2 - 3v^2\omega_3\omega_4\omega_1\omega_2 - 3v^2\omega_3\omega_1\omega_2^2 + \frac{3}{2}\omega_4^2\omega_1u^2 - 3v^2\omega_4^2\omega_1\omega_2 + v^2\omega_4\omega_2^2 + \frac{1}{2}\omega_4^2\omega_1 - v^2\omega_3^2\omega_4 - 3\omega_3^2\omega_4c_s^2\omega_1, \\
\gamma_{[ty]}^{[\mu_3]} &= 5v\omega_3\omega_2 + 2v\omega_4^2\omega_1 - v\omega_4\omega_2^2 - 6v\omega_3\omega_4\omega_1^2 - 18v\omega_3\omega_1\omega_2 + 3v\omega_3^2\omega_4 + 3v\omega_3\omega_1\omega_2^2 + 2v\omega_3^2\omega_1 + 5v\omega_4\omega_2 + \\
&\quad 34v\omega_3\omega_4\omega_1 + 3v\omega_4\omega_1^2\omega_2 - v\omega_3\omega_2^2 + 3v\omega_4^2\omega_1\omega_2 - 2v\omega_1^2\omega_2 + v\omega_4\omega_1^2 - 9v\omega_3^2\omega_4\omega_1 + 10v\omega_1\omega_2 + 3v\omega_3\omega_4^2 - \\
&\quad 5v\omega_3\omega_1 - v\omega_4^2\omega_2 - 9v\omega_3\omega_4^2\omega_1 - 18v\omega_4\omega_1\omega_2 + v\omega_3\omega_1^2 + 3v\omega_4\omega_1\omega_2^2 - 10v\omega_3\omega_4 + 3v\omega_3\omega_1^2\omega_2 - 2v\omega_1\omega_2^2 + \\
&\quad 6v\omega_3\omega_4\omega_1\omega_2 - v\omega_3^2\omega_2 - 5v\omega_4\omega_1 + 3v\omega_3^2\omega_1\omega_2 - 2v\omega_3\omega_4\omega_2, \\
\gamma_{[x^2]}^{[\mu_1]} &= -\frac{1}{2}v^3\omega_3^2\omega_1\omega_2 - v\omega_4^2\omega_1u^2 + \frac{1}{2}v\omega_3^2\omega_4c_s^2\omega_1 - \frac{5}{2}v\omega_3\omega_2 + \frac{1}{2}v\omega_3^2\omega_4\omega_1u^2 - 2v^3\omega_3\omega_2^2 - \frac{1}{2}v\omega_3\omega_4^2\omega_2 - \\
&\quad v\omega_4\omega_1^2u^2\omega_2 + 2v\omega_3\omega_4c_s^2\omega_2 + \frac{5}{4}v\omega_3\omega_1\omega_2 - \frac{1}{2}v\omega_3^2\omega_4\omega_2 - v^3\omega_4\omega_1\omega_2^2 - v\omega_3\omega_1^2u^2\omega_2 + \frac{1}{2}v\omega_3\omega_4^2\omega_1u^2 + \\
&\quad 2v\omega_3\omega_4\omega_1u^2\omega_2 + \frac{1}{2}v\omega_3\omega_3^2c_s^2\omega_1 - \frac{1}{2}v\omega_3\omega_1\omega_2^2 - v\omega_3\omega_4c_s^2\omega_1\omega_2 - \frac{1}{2}v\omega_4\omega_2 + v\omega_3^2\omega_4u^2 + 2v^3\omega_4\omega_2^2 + v\omega_3\omega_2^2 - \\
&\quad 2v\omega_3\omega_4u^2\omega_2 + \frac{1}{2}v\omega_3^2\omega_1u^2\omega_2 + v\omega_3\omega_4^2u^2 + \frac{1}{2}v\omega_4^2\omega_1u^2\omega_2 + \frac{1}{2}v\omega_4^2\omega_1\omega_2 + v^3\omega_3^2\omega_2 - \frac{1}{2}v\omega_3\omega_4\omega_2^2 - \\
&\quad 2v\omega_3\omega_4\omega_1u^2 - v\omega_3^2\omega_1u^2 - v\omega_3\omega_4^2u^2\omega_2 + v^3\omega_3\omega_1\omega_2^2 + \frac{1}{4}v\omega_4\omega_1\omega_2 - v^3\omega_4^2\omega_2 - v\omega_3\omega_4^2c_s^2 + 2v\omega_4\omega_1u^2\omega_2 - \\
&\quad \frac{3}{4}v\omega_3\omega_4\omega_1\omega_2 + \frac{1}{2}v\omega_3^2\omega_2 - v\omega_3^2\omega_4c_s^2 - v\omega_3^2\omega_4u^2\omega_2 + 2v\omega_3\omega_1u^2\omega_2 - \frac{1}{4}v\omega_3^2\omega_1\omega_2 + 3v\omega_3\omega_4\omega_2, \\
\gamma_{[x^2]}^{[\mu_3]} &= \frac{3}{4}\omega_3\omega_4\omega_1\omega_2 - \frac{1}{2}\omega_3^2\omega_2 - 3\omega_3\omega_4\omega_2 + \frac{1}{4}\omega_3^2\omega_1\omega_2 + \omega_3^2\omega_4u^2\omega_2 - v^2\omega_3^2\omega_2 - \frac{1}{4}\omega_4\omega_1\omega_2 + \omega_3\omega_4^2u^2\omega_2 + \\
&\quad v^2\omega_4\omega_1\omega_2^2 + \frac{1}{2}v^2\omega_3^2\omega_1\omega_2 + v^2\omega_4^2\omega_2 + \frac{1}{2}\omega_3\omega_4\omega_2^2 - \frac{1}{2}\omega_3^2\omega_1u^2\omega_2 - \frac{1}{2}\omega_4^2\omega_1u^2\omega_2 - \omega_3\omega_2^2 - 3\omega_3\omega_4\omega_1u^2\omega_2 + \\
&\quad \frac{1}{2}\omega_4\omega_2 + 2v^2\omega_3\omega_2^2 + \frac{1}{2}\omega_3\omega_1\omega_2^2 + \frac{1}{2}\omega_3^2\omega_4\omega_2 - \frac{5}{4}\omega_3\omega_1\omega_2 + \omega_4\omega_1^2u^2\omega_2 + \omega_3\omega_1^2u^2\omega_2 - v^2\omega_3\omega_1\omega_2^2 - \\
&\quad \frac{1}{2}v^2\omega_4^2\omega_1\omega_2 + \frac{1}{2}\omega_3\omega_4^2\omega_2 - 2v^2\omega_4\omega_2^2 + \frac{5}{2}\omega_3\omega_2, \\
\gamma_{[xy]}^{[\mu_1]} &= -5\omega_4\omega_1u^3 + 3v^2\omega_3^2u\omega_2 + 10\omega_3c_s^2\omega_1u + \frac{1}{2}\omega_3\omega_4\omega_1u - 4v^2\omega_1^2u\omega_2 + 3\omega_3\omega_4^2c_s^2u + \frac{1}{2}\omega_3^2\omega_1u - 3v^2\omega_3\omega_1u\omega_2 - \\
&\quad 3v^2\omega_4\omega_1u\omega_2 + 2v^2\omega_4u\omega_2^2 - 5v^2\omega_4\omega_1u - 2\omega_3^2\omega_1u^3 - 10v^2\omega_4u\omega_2 - \omega_3\omega_1u^3\omega_2 + 3\omega_3^2\omega_4c_s^2u - 2\omega_3c_s^2\omega_1u\omega_2 + \\
&\quad \omega_4\omega_1u^3\omega_2 + \frac{1}{2}\omega_4\omega_1u + v^2\omega_3\omega_1^2u + \omega_3\omega_1^2u^3 + 10v^2\omega_3\omega_4u + v^2\omega_3^2\omega_1u - 2\omega_3\omega_4c_s^2\omega_1u + 5\omega_3\omega_1u^3 + \\
&\quad 3v^2\omega_4^2u\omega_2 - 3v^2\omega_3\omega_4^2u - 2\omega_3^2c_s^2\omega_1u - 4v^2\omega_1u\omega_2^2 + 4v^2\omega_3\omega_4u\omega_2 + 2v^2\omega_3u\omega_2^2 - 3v^2\omega_3^2\omega_4u - 5v^2\omega_3\omega_1u + \\
&\quad 2\omega_4^2\omega_1u^3 - 10v^2\omega_3u\omega_2 + 2v^2\omega_3\omega_4\omega_1u + v^2\omega_4\omega_1^2u - \frac{5}{2}\omega_3\omega_1u - 10\omega_3\omega_4c_s^2u + 20v^2\omega_1u\omega_2 - \omega_4\omega_1^2u^3 - \\
&\quad 2\omega_3c_s^2\omega_1^2u + 2\omega_3\omega_4c_s^2u\omega_2 + v^2\omega_4^2\omega_1u + \omega_3\omega_1u\omega_2, \\
\gamma_{[xy]}^{[\mu_3]} &= 4v\omega_4\omega_1u\omega_2 + 8v\omega_3\omega_4^2u + 4v\omega_3\omega_1u\omega_2 - 3v\omega_4^2u\omega_2 - 2v\omega_3u\omega_2^2 + 10v\omega_3\omega_1u + 8v\omega_3^2\omega_4u + 4v\omega_1u\omega_2^2 - \\
&\quad 20v\omega_1u\omega_2 - 2v\omega_4\omega_1^2u + 10v\omega_3u\omega_2 - 3v\omega_4^2\omega_1u - 6v\omega_3\omega_4u\omega_2 + 4v\omega_1^2u\omega_2 - 3v\omega_3^2u\omega_2 - 2v\omega_4u\omega_2^2 + \\
&\quad 10v\omega_4\omega_1u - 2v\omega_3\omega_1^2u + 10v\omega_4u\omega_2 - 3v\omega_3^2\omega_1u - 20v\omega_3\omega_4u - 6v\omega_3\omega_4\omega_1u, \\
\gamma_{[y^2]}^{[\mu_1]} &= 2v^3\omega_3\omega_4\omega_1\omega_2 + \frac{1}{2}v^3\omega_3^2\omega_1\omega_2 + \frac{1}{2}v\omega_3^2\omega_4c_s^2\omega_1 - v\omega_4\omega_1^2u^2\omega_2 + \frac{5}{4}v\omega_3\omega_1\omega_2 - v^3\omega_4\omega_1\omega_2^2 + v\omega_3\omega_1^2u^2\omega_2 + \\
&\quad \frac{1}{2}v\omega_3\omega_4^2c_s^2\omega_1 - \frac{1}{2}v^3\omega_3\omega_4^2\omega_1 - v\omega_3\omega_4c_s^2\omega_1\omega_2 - \frac{1}{2}v^3\omega_3^2\omega_4\omega_1 - \frac{1}{2}v\omega_3^2\omega_1u^2\omega_2 + \frac{1}{2}v\omega_4^2\omega_1u^2\omega_2 + \frac{1}{2}v^3\omega_4^2\omega_1\omega_2 - \\
&\quad v^3\omega_3\omega_1\omega_2^2 + \frac{1}{4}v\omega_4\omega_1\omega_2 - \frac{1}{2}v\omega_3\omega_1^2\omega_2 - \frac{1}{4}v\omega_3\omega_4\omega_1\omega_2 - \frac{1}{4}v\omega_3^2\omega_1\omega_2, \\
\gamma_{[y^2]}^{[\mu_3]} &=
\end{aligned}$$

$$\begin{aligned} & \frac{3}{4}\omega_3\omega_4\omega_1\omega_2 + \frac{1}{2}\omega_4\omega_1 + v^2\omega_3^2\omega_4\omega_1 + \frac{1}{4}\omega_3^2\omega_1\omega_2 + v^2\omega_3\omega_4^2\omega_1 + \frac{1}{2}\omega_3\omega_1^2\omega_2 - \omega_3\omega_1^2 + 2\omega_3\omega_1^2u^2 + \frac{1}{2}\omega_3\omega_4^2\omega_1 - \frac{1}{4}\omega_4\omega_1\omega_2 + \\ & \frac{5}{2}\omega_3\omega_1 + v^2\omega_4\omega_1\omega_2^2 + \frac{1}{2}\omega_3^2\omega_4\omega_1 - \frac{1}{2}v^2\omega_3^2\omega_1\omega_2 - \omega_3^2\omega_1u^2 + \frac{1}{2}\omega_3^2\omega_1u^2\omega_2 - \frac{1}{2}\omega_4^2\omega_1u^2\omega_2 - \frac{1}{2}\omega_3^2\omega_1 - 3\omega_3\omega_4\omega_1 - \\ & \frac{5}{4}\omega_3\omega_1\omega_2 + \omega_4\omega_1^2u^2\omega_2 - 2\omega_4\omega_1^2u^2 - \omega_3\omega_1^2u^2\omega_2 - 3v^2\omega_3\omega_4\omega_1\omega_2 + v^2\omega_3\omega_1\omega_2^2 + \frac{1}{2}\omega_3\omega_4\omega_1^2 + \omega_4^2\omega_1u^2 - \frac{1}{2}v^2\omega_4^2\omega_1\omega_2, \end{aligned}$$

## 8.5 EPDE for $\mu_4$

$$\begin{aligned} & \gamma_{[1]}^{[\mu_1]}\mu_1 + \gamma_{[1]}^{[\mu_4]}\mu_4 + \gamma_{[t]}^{[\mu_1]}\delta_t\frac{\partial\mu_1}{\partial t} + \gamma_{[t]}^{[\mu_4]}\delta_t\frac{\partial\mu_4}{\partial t} + \gamma_{[x]}^{[\mu_1]}\delta_l\frac{\partial\mu_1}{\partial x} + \gamma_{[x]}^{[\mu_4]}\delta_l\frac{\partial\mu_4}{\partial x} + \gamma_{[y]}^{[\mu_1]}\delta_l\frac{\partial\mu_1}{\partial y} + \gamma_{[y]}^{[\mu_4]}\delta_l\frac{\partial\mu_4}{\partial y} \\ & + \gamma_{[t^2]}^{[\mu_1]}\delta_t^2\frac{\partial^2\mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_4]}\delta_t^2\frac{\partial^2\mu_4}{\partial t^2} + \gamma_{[tx]}^{[\mu_1]}\delta_t\delta_l\frac{\partial^2\mu_1}{\partial t\partial x} + \gamma_{[tx]}^{[\mu_4]}\delta_t\delta_l\frac{\partial^2\mu_4}{\partial t\partial x} + \gamma_{[ty]}^{[\mu_1]}\delta_t\delta_l\frac{\partial^2\mu_1}{\partial t\partial y} + \gamma_{[ty]}^{[\mu_4]}\delta_t\delta_l\frac{\partial^2\mu_4}{\partial t\partial y} \\ & + \gamma_{[x^2]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_4]}\delta_l^2\frac{\partial^2\mu_4}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial x\partial y} + \gamma_{[xy]}^{[\mu_4]}\delta_l^2\frac{\partial^2\mu_4}{\partial x\partial y} + \gamma_{[y^2]}^{[\mu_1]}\delta_l^2\frac{\partial^2\mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_4]}\delta_l^2\frac{\partial^2\mu_4}{\partial y^2} = 0, \end{aligned}$$

where

$$\begin{aligned} \gamma_{[1]}^{[\mu_1]} &= -\omega_3\omega_4^2\omega_1u^2\omega_2 - \omega_3\omega_4^2c_s^2\omega_1\omega_2 - \omega_3\omega_4\omega_1^2u^2\omega_2 - \omega_3\omega_4c_s^2\omega_1^2\omega_2 + 5\omega_3\omega_4c_s^2\omega_1\omega_2 + 5\omega_3\omega_4\omega_1u^2\omega_2 - \\ & \omega_3^2\omega_4c_s^2\omega_1\omega_2 - \omega_3^2\omega_4\omega_1u^2\omega_2 - \omega_3\omega_4c_s^2\omega_1\omega_2^2 - \omega_3\omega_4\omega_1u^2\omega_2^2, \\ \gamma_{[1]}^{[\mu_4]} &= -5\omega_3\omega_4\omega_1\omega_2 + \omega_3\omega_4\omega_1\omega_2^2 + \omega_3^2\omega_4\omega_1\omega_2 + \omega_3\omega_4^2\omega_1\omega_2 + \omega_3\omega_4\omega_1^2\omega_2, \\ \gamma_{[t]}^{[\mu_1]} &= -\frac{1}{2}\omega_3\omega_4\omega_1\omega_2 + 4\omega_3\omega_4^2\omega_1u^2\omega_2 + 4\omega_3\omega_4^2c_s^2\omega_1\omega_2 - \omega_3\omega_4^2c_s^2\omega_2 + \frac{15}{2}\omega_4\omega_1u^2\omega_2 + 5\omega_3c_s^2\omega_1\omega_2 - \omega_3\omega_4c_s^2\omega_1^2 + \\ & \frac{15}{2}\omega_3\omega_1u^2\omega_2 + 3\omega_3^2\omega_4u^2\omega_2 + \frac{5}{2}v^2\omega_4\omega_1\omega_2 - \frac{1}{2}\omega_3\omega_1^2\omega_2 - \frac{1}{2}\omega_4\omega_1\omega_2^2 + 2\omega_4\omega_1\omega_2 + \frac{1}{2}v^2\omega_3\omega_1^2\omega_2 + 3\omega_3\omega_4^2u^2\omega_2 + \\ & 4\omega_3\omega_4\omega_1^2u^2\omega_2 + \frac{3}{2}v^2\omega_4\omega_1\omega_2^2 - \omega_3^2\omega_4c_s^2\omega_2 + \frac{3}{2}v^2\omega_3^2\omega_1\omega_2 + 5\omega_3\omega_4\omega_1u^2 + 5\omega_3\omega_4c_s^2\omega_1 - \omega_3c_s^2\omega_1\omega_2^2 - \\ & \frac{3}{2}\omega_3\omega_1u^2\omega_2^2 + 4\omega_3\omega_4c_s^2\omega_1^2\omega_2 - \frac{3}{2}\omega_4\omega_1u^2\omega_2^2 - \frac{5}{2}\omega_3^2\omega_1u^2\omega_2 - 5\omega_3\omega_4u^2\omega_2 - \omega_3\omega_4\omega_1^2u^2 - \frac{1}{2}\omega_4^2\omega_1\omega_2 - \omega_3^2c_s^2\omega_1\omega_2 - \\ & 22\omega_3\omega_4c_s^2\omega_1\omega_2 - \frac{5}{2}\omega_4^2\omega_1u^2\omega_2 - 28\omega_3\omega_4\omega_1u^2\omega_2 + 4\omega_3^2\omega_4c_s^2\omega_1\omega_2 - \omega_3\omega_4^2c_s^2\omega_1 - \frac{1}{2}\omega_4\omega_1^2\omega_2 - \frac{5}{2}v^2\omega_3\omega_1\omega_2 - \\ & \omega_3\omega_4^2\omega_1u^2 + 4\omega_3^2\omega_4\omega_1u^2\omega_2 + \frac{1}{2}\omega_3\omega_1\omega_2^2 - \omega_3\omega_4c_s^2\omega_2^2 - \frac{1}{2}v^2\omega_4\omega_1^2\omega_2 + \frac{1}{2}\omega_4\omega_1^2u^2\omega_2 + 5\omega_3\omega_4c_s^2\omega_2 + \frac{1}{2}\omega_3\omega_1^2u^2\omega_2 - \\ & \frac{3}{2}v^2\omega_3\omega_1\omega_2^2 + 4\omega_3\omega_4c_s^2\omega_1\omega_2^2 - \frac{3}{2}v^2\omega_4^2\omega_1\omega_2 + 4\omega_3\omega_4\omega_1u^2\omega_2^2 + \omega_3\omega_4u^2\omega_2^2 - \omega_3c_s^2\omega_1^2\omega_2 - \omega_3^2\omega_4\omega_1u^2 - \omega_3^2\omega_4c_s^2\omega_1, \\ \gamma_{[t]}^{[\mu_4]} &= 23\omega_3\omega_4\omega_1\omega_2 - 5\omega_3\omega_4\omega_2 + \omega_3^2\omega_1\omega_2 + \omega_3\omega_1^2\omega_2 + \omega_4\omega_1\omega_2^2 + \omega_3\omega_4^2\omega_1 - 5\omega_4\omega_1\omega_2 + \omega_3^2\omega_4\omega_1 - 4\omega_3\omega_4\omega_1\omega_2^2 + \\ & \omega_3\omega_4\omega_2^2 + \omega_4^2\omega_1\omega_2 - 4\omega_3^2\omega_4\omega_1\omega_2 - 5\omega_3\omega_4\omega_1 + \omega_4\omega_1^2\omega_2 + \omega_3\omega_1\omega_2^2 + \omega_3^2\omega_4\omega_2 - 5\omega_3\omega_1\omega_2 - 4\omega_3\omega_4^2\omega_1\omega_2 + \\ & \omega_3\omega_4\omega_1^2 - 4\omega_3\omega_4\omega_1^2\omega_2 + \omega_3\omega_4^2\omega_2, \\ \gamma_{[x]}^{[\mu_1]} &= \omega_3\omega_4^2u^3\omega_2 + \omega_3^2\omega_4u^3\omega_2 + 2v^2\omega_4\omega_1u\omega_2^2 - 2v^2\omega_3\omega_1u\omega_2^2 + 2\omega_3\omega_4c_s^2\omega_1u\omega_2 + \frac{1}{2}\omega_3^2\omega_1u\omega_2 - \omega_3\omega_4^2c_s^2u\omega_2 + \\ & \frac{1}{2}\omega_3\omega_4\omega_1u\omega_2 + \omega_3\omega_1u\omega_2^2 + 2\omega_3\omega_1^2u^3\omega_2 + 2\omega_4\omega_1^2u^3\omega_2 - v^2\omega_4^2\omega_1u\omega_2 + v^2\omega_3^2\omega_1u\omega_2 - \omega_3^2\omega_4c_s^2u\omega_2 - \\ & 4\omega_3\omega_4\omega_1u^3\omega_2 - \omega_4^2\omega_1u^3\omega_2 - \omega_3^2\omega_1u^3\omega_2 - \frac{1}{2}\omega_4\omega_1u\omega_2 - \frac{5}{2}\omega_3\omega_1u\omega_2, \\ \gamma_{[x]}^{[\mu_4]} &= 2\omega_3\omega_4u\omega_2^2 - \omega_4\omega_1^2u\omega_2 - \omega_3\omega_1^2u\omega_2 - 10\omega_3\omega_4u\omega_2 - \omega_3^2\omega_1u\omega_2 - \omega_4^2\omega_1u\omega_2 - 2\omega_3\omega_4\omega_1u\omega_2 - \omega_3\omega_1u\omega_2^2 - \\ & \omega_4\omega_1u\omega_2^2 + 3\omega_3\omega_4^2u\omega_2 + 5\omega_4\omega_1u\omega_2 + 3\omega_3^2\omega_4u\omega_2 + 5\omega_3\omega_1u\omega_2, \\ \gamma_{[y]}^{[\mu_1]} &= 2v^3\omega_3^2\omega_1\omega_2 - 3v\omega_3^2\omega_4c_s^2\omega_1 - 3v\omega_3^2\omega_4\omega_1u^2 + 2v\omega_3c_s^2\omega_1^2\omega_2 + v^3\omega_3\omega_1^2\omega_2 + 3v\omega_4\omega_1^2u^2\omega_2 + \frac{5}{2}v\omega_3\omega_1\omega_2 + \\ & v^3\omega_4\omega_1\omega_2^2 + 3v\omega_3\omega_1^2u^2\omega_2 - 3v\omega_3\omega_4^2\omega_1u^2 - 4v\omega_3\omega_4\omega_1u^2\omega_2 - 3v\omega_3\omega_4^2c_s^2\omega_1 + 5v^3\omega_4\omega_1\omega_2 + 2v\omega_3\omega_4c_s^2\omega_1\omega_2 + \\ & 2v\omega_3^2c_s^2\omega_1\omega_2 - 2v\omega_3\omega_4\omega_1^2u^2 - 2v^3\omega_4^2\omega_1\omega_2 + v\omega_3\omega_1u^2\omega_2^2 + 2v\omega_3c_s^2\omega_1\omega_2^2 + 10v\omega_3\omega_4c_s^2\omega_1 + 10v\omega_3\omega_4\omega_1u^2 + \\ & v\omega_4\omega_1u^2\omega_2^2 + 2v\omega_3\omega_4^2u^2\omega_2 - v^3\omega_4\omega_1^2\omega_2 - v^3\omega_3\omega_1\omega_2^2 - \frac{1}{2}v\omega_4\omega_1\omega_2 - 5v^3\omega_3\omega_1\omega_2 - v\omega_3\omega_1^2\omega_2 - \\ & 5v\omega_4\omega_1u^2\omega_2 - \frac{1}{2}v\omega_3\omega_4\omega_1\omega_2 + 2v\omega_3^2\omega_4u^2\omega_2 - 5v\omega_3\omega_1u^2\omega_2 - \frac{1}{2}v\omega_3^2\omega_1\omega_2 - 2v\omega_3\omega_4c_s^2\omega_1^2 - 10v\omega_3c_s^2\omega_1\omega_2, \\ \gamma_{[y]}^{[\mu_4]} &= 2v\omega_3\omega_4\omega_1^2 + 5v\omega_3\omega_1\omega_2 - v\omega_3\omega_1\omega_2^2 - 10v\omega_3\omega_4\omega_1 - v\omega_4\omega_1^2\omega_2 - v\omega_4^2\omega_1\omega_2 + 3v\omega_3^2\omega_4\omega_1 + 3v\omega_3\omega_4^2\omega_1 + \\ & 5v\omega_4\omega_1\omega_2 - v\omega_4\omega_1\omega_2^2 - v\omega_3\omega_1^2\omega_2 - 2v\omega_3\omega_4\omega_1\omega_2 - v\omega_3^2\omega_1\omega_2, \end{aligned}$$

$$\begin{aligned}
\gamma_{[t^2]}^{[\mu_1]} &= \frac{7}{4}\omega_3\omega_4\omega_1\omega_2 - \omega_3^2\omega_4c_s^2\omega_2 + 5\omega_3c_s^2\omega_2 - 8\omega_3\omega_4^2\omega_1u^2\omega_2 + \frac{3}{2}\omega_4^2u^2\omega_2 - 8\omega_3\omega_4^2c_s^2\omega_1\omega_2 + \frac{1}{2}v^2\omega_3\omega_1^2 + 2\omega_4\omega_1 + \\
&\quad \frac{7}{2}\omega_3\omega_4^2c_s^2\omega_2 - \frac{127}{4}\omega_4\omega_1u^2\omega_2 - \frac{1}{2}\omega_3\omega_4\omega_2 - \frac{37}{2}\omega_3c_s^2\omega_1\omega_2 + \frac{7}{2}\omega_3\omega_4c_s^2\omega_1^2 - \frac{127}{4}\omega_3\omega_1u^2\omega_2 - \frac{21}{2}\omega_3^2\omega_4u^2\omega_2 - \\
&\quad 2\omega_1u^2\omega_2^2 - \frac{29}{4}v^2\omega_4\omega_1\omega_2 + \frac{7}{4}\omega_3\omega_1^2\omega_2 + \frac{15}{2}\omega_4\omega_1u^2 - \omega_3\omega_4^2c_s^2 + \frac{1}{2}\omega_3u^2\omega_2^2 + \frac{7}{4}\omega_4\omega_1\omega_2^2 - \frac{1}{2}\omega_3\omega_1^2 + \frac{5}{2}v^2\omega_4\omega_1 + \\
&\quad \frac{3}{2}v^2\omega_3^2\omega_2 - \frac{5}{2}\omega_3u^2\omega_2 + \frac{1}{2}\omega_3\omega_1^2u^2 - \frac{17}{2}\omega_4\omega_1\omega_2 - \frac{7}{4}v^2\omega_3\omega_1^2\omega_2 - \frac{1}{2}\omega_4^2\omega_2 - \frac{1}{2}v^2\omega_4\omega_1^2 - \frac{21}{2}\omega_3\omega_4^2u^2\omega_2 - \\
&\quad 8\omega_3\omega_4\omega_1^2u^2\omega_2 + 10\omega_1u^2\omega_2 - \frac{21}{4}v^2\omega_4\omega_1\omega_2^2 + \frac{7}{2}\omega_3^2\omega_4c_s^2\omega_2 - \frac{21}{4}v^2\omega_3^2\omega_1\omega_2 - \frac{49}{2}\omega_3\omega_4\omega_1u^2 - \frac{37}{2}\omega_3\omega_4c_s^2\omega_1 + \\
&\quad 2\omega_1\omega_2 + \frac{7}{2}\omega_3c_s^2\omega_1\omega_2^2 - \omega_3^2c_s^2\omega_1 + \frac{21}{4}\omega_3\omega_1u^2\omega_2^2 - \frac{5}{2}\omega_3^2\omega_1u^2 - 5\omega_3\omega_4u^2 - \omega_3c_s^2\omega_2^2 - 8\omega_3\omega_4c_s^2\omega_1^2\omega_2 + \\
&\quad \frac{21}{4}\omega_4\omega_1u^2\omega_2^2 - \frac{3}{2}v^2\omega_4^2\omega_2 - \frac{5}{2}v^2\omega_3\omega_1 - \frac{1}{2}\omega_4\omega_1^2 - \omega_1^2\omega_2 + \frac{35}{4}\omega_3^2\omega_1u^2\omega_2 + \frac{41}{2}\omega_3\omega_4u^2\omega_2 + \frac{7}{2}\omega_3\omega_4\omega_1^2u^2 + \frac{7}{4}\omega_4^2\omega_1\omega_2 + \\
&\quad 3\omega_3\omega_4^2u^2 + \frac{3}{2}\omega_3^2u^2\omega_2 + \frac{7}{2}\omega_3^2c_s^2\omega_1\omega_2 + 47\omega_3\omega_4c_s^2\omega_1\omega_2 + \frac{35}{4}\omega_4^2\omega_1u^2\omega_2 + \frac{5}{2}v^2\omega_4\omega_2 + \frac{3}{2}v^2\omega_3^2\omega_1 + \frac{1}{2}\omega_3\omega_2^2 + \\
&\quad 68\omega_3\omega_4\omega_1u^2\omega_2 + 2\omega_1^2u^2\omega_2 + \frac{15}{2}\omega_3\omega_1u^2 + 3\omega_3^2\omega_4u^2 - 8\omega_3^2\omega_4c_s^2\omega_1\omega_2 + 5\omega_3c_s^2\omega_1 + \frac{7}{2}\omega_3\omega_4^2c_s^2\omega_1 + 2\omega_4\omega_2 - \\
&\quad \frac{3}{2}v^2\omega_3\omega_2^2 - \frac{1}{2}\omega_3\omega_4\omega_1 + \frac{7}{4}\omega_4\omega_1^2\omega_2 + \frac{29}{4}v^2\omega_3\omega_1\omega_2 + \frac{7}{2}\omega_3\omega_4^2\omega_1u^2 - 8\omega_3^2\omega_4\omega_1u^2\omega_2 - \frac{7}{4}\omega_3\omega_1\omega_2^2 + \frac{7}{2}\omega_3\omega_4c_s^2\omega_2^2 + \\
&\quad \frac{7}{4}\omega_4u^2\omega_2^2 + \frac{7}{4}v^2\omega_4\omega_1^2\omega_2 - \frac{1}{2}\omega_3\omega_1\omega_2 - \frac{7}{4}\omega_4\omega_1^2u^2\omega_2 - \frac{37}{2}\omega_3\omega_4c_s^2\omega_2 - \omega_3^2c_s^2\omega_2 - \frac{5}{2}\omega_4u^2\omega_2 + 5\omega_3\omega_4c_s^2 + \\
&\quad \frac{7}{4}\omega_4\omega_1^2u^2 - \frac{7}{4}\omega_3\omega_1^2u^2\omega_2 - \omega_3c_s^2\omega_1^2 - \frac{1}{2}\omega_4\omega_2^2 - \frac{3}{2}v^2\omega_4\omega_1 - \frac{5}{2}v^2\omega_3\omega_2 + \frac{21}{4}v^2\omega_3\omega_1\omega_2^2 - 8\omega_3\omega_4c_s^2\omega_1\omega_2^2 - \frac{5}{2}\omega_4^2\omega_1u^2 + \\
&\quad \frac{21}{4}v^2\omega_4^2\omega_1\omega_2 - 8\omega_3\omega_4\omega_1u^2\omega_2^2 + \frac{3}{2}v^2\omega_4\omega_2^2 - \frac{1}{2}\omega_4^2\omega_1 - \frac{7}{2}\omega_3\omega_4u^2\omega_2^2 + \frac{1}{2}\omega_3c_s^2\omega_1^2\omega_2 + \frac{7}{2}\omega_3^2\omega_4\omega_1u^2 + \frac{7}{2}\omega_3^2\omega_4c_s^2\omega_1, \\
\gamma_{[t^2]}^{[\mu_4]} &= -\frac{101}{2}\omega_3\omega_4\omega_1\omega_2 + \omega_3^2\omega_2 - 5\omega_4\omega_1 + \frac{39}{2}\omega_3\omega_4\omega_2 - \frac{7}{2}\omega_3^2\omega_1\omega_2 + \omega_1\omega_2^2 - \frac{7}{2}\omega_3\omega_1^2\omega_2 - 5\omega_3\omega_4 - \frac{7}{2}\omega_4\omega_1\omega_2^2 + \\
&\quad \omega_3\omega_1^2 - \frac{7}{2}\omega_3\omega_4^2\omega_1 + \frac{39}{2}\omega_4\omega_1\omega_2 - 5\omega_3\omega_1 + \omega_4^2\omega_2 + \omega_3\omega_1^2 - \frac{7}{2}\omega_3^2\omega_4\omega_1 - 5\omega_1\omega_2 + 8\omega_3\omega_4\omega_1\omega_2^2 + \omega_4\omega_1^2 - \\
&\quad \frac{7}{2}\omega_3\omega_4\omega_2^2 + \omega_1^2\omega_2 - \frac{7}{2}\omega_4^2\omega_1\omega_2 + 8\omega_3^2\omega_4\omega_1\omega_2 + \omega_3\omega_2^2 + \omega_3^2\omega_1 - 5\omega_4\omega_2 + \frac{39}{2}\omega_3\omega_4\omega_1 - \frac{7}{2}\omega_4\omega_1^2\omega_2 - \frac{7}{2}\omega_3\omega_1\omega_2^2 - \\
&\quad \frac{7}{2}\omega_3^2\omega_4\omega_2 + \frac{39}{2}\omega_3\omega_1\omega_2 + \omega_3^2\omega_4 + 8\omega_3\omega_4^2\omega_1\omega_2 + \omega_4\omega_2^2 - \frac{7}{2}\omega_3\omega_4\omega_1^2 + 8\omega_3\omega_4\omega_1^2\omega_2 - \frac{7}{2}\omega_3\omega_4^2\omega_2 - 5\omega_3\omega_2 + \omega_4^2\omega_1, \\
\gamma_{[tx]}^{[\mu_1]} &= \frac{1}{2}v^2\omega_3^2u\omega_2 + \frac{1}{2}\omega_3\omega_4\omega_1u - 3\omega_3\omega_4^2u^3\omega_2 - \omega_3\omega_4^2c_s^2u + \frac{1}{2}\omega_3^2\omega_1u - 3v^2\omega_3\omega_1u\omega_2 + 3v^2\omega_4\omega_1u\omega_2 - \\
&\quad 3\omega_3^2\omega_4u^3\omega_2 - \omega_3^2\omega_1u^3 - 6v^2\omega_4\omega_1u\omega_2^2 - 3\omega_3\omega_1u^3\omega_2 - 4\omega_3\omega_4\omega_1u^3 - \omega_3^2\omega_4c_s^2u + 2\omega_3c_s^2\omega_1u\omega_2 + \\
&\quad 6v^2\omega_3\omega_1u\omega_2^2 - 3\omega_4\omega_1u^3\omega_2 - \frac{1}{2}\omega_4\omega_1u - 6\omega_3\omega_4c_s^2\omega_1u\omega_2 + \frac{1}{2}\omega_4^2u^3\omega_2 - \omega_3\omega_4u\omega_2 - \frac{3}{2}\omega_3^2\omega_1u\omega_2 - \frac{1}{2}\omega_3^2u\omega_2 + \\
&\quad 2\omega_3\omega_1^2u^3 + 3\omega_3\omega_4^2c_s^2u\omega_2 + v^2\omega_3^2\omega_1u - \frac{3}{2}\omega_3\omega_4\omega_1u\omega_2 + 2\omega_3\omega_4c_s^2\omega_1u + \omega_3^2\omega_4u^3 - \frac{1}{2}v^2\omega_4^2u\omega_2 - 3\omega_3\omega_1u\omega_2^2 - \\
&\quad 6\omega_3\omega_1^2u^3\omega_2 - 6\omega_4\omega_1^2u^3\omega_2 + \omega_3\omega_4^2u^3 + 3v^2\omega_4^2\omega_1u\omega_2 - 3\omega_1u\omega_2 - 3v^2\omega_3^2\omega_1u\omega_2 + 4\omega_1^2u^3\omega_2 + \omega_1u\omega_2^2 + \\
&\quad 3\omega_3^2\omega_4c_s^2u\omega_2 + 12\omega_3\omega_4\omega_1u^3\omega_2 - \omega_4^2\omega_1u^3 + 3\omega_4^2\omega_1u^3\omega_2 - \frac{5}{2}\omega_3\omega_1u + \frac{1}{2}\omega_3^2u^3\omega_2 + \omega_3\omega_4u^3\omega_2 + 3\omega_3^2\omega_1u^3\omega_2 + \\
&\quad 2\omega_4\omega_1^2u^3 - \frac{1}{2}\omega_4^2u\omega_2 - \omega_3^2c_s^2u\omega_2 + \frac{5}{2}\omega_4\omega_1u\omega_2 - \omega_3\omega_4c_s^2u\omega_2 - v^2\omega_4^2\omega_1u + \frac{19}{2}\omega_3\omega_1u\omega_2, \\
\gamma_{[tx]}^{[\mu_4]} &= -2\omega_3\omega_4\omega_1u - 6\omega_3\omega_4u\omega_2^2 - 10\omega_3\omega_4u - \omega_3^2\omega_1u + 3\omega_4\omega_1^2u\omega_2 - 5\omega_4u\omega_2 + 3\omega_3\omega_1^2u\omega_2 - \omega_3\omega_1^2u + 5\omega_4\omega_1u + \\
&\quad \omega_4u\omega_2^2 + 34\omega_3\omega_4u\omega_2 + 3\omega_3^2\omega_1u\omega_2 + 2\omega_3^2u\omega_2 + 3\omega_4^2\omega_1u\omega_2 - 2\omega_1^2u\omega_2 + 6\omega_3\omega_4\omega_1u\omega_2 + 3\omega_3\omega_1u\omega_2^2 - \omega_4^2\omega_1u + \\
&\quad 3\omega_4\omega_1u\omega_2^2 - 5\omega_3u\omega_2 - 9\omega_3\omega_4^2u\omega_2 + 10\omega_1u\omega_2 - \omega_4\omega_1^2u - 2\omega_1u\omega_2^2 + 3\omega_3^2\omega_4u + 5\omega_3\omega_1u + \omega_3u\omega_2^2 + 2\omega_4^2u\omega_2 - \\
&\quad 18\omega_4\omega_1u\omega_2 + 3\omega_3\omega_4^2u - 9\omega_3^2\omega_4u\omega_2 - 18\omega_3\omega_1u\omega_2, \\
\gamma_{[ty]}^{[\mu_1]} &= -6v^3\omega_3^2\omega_1\omega_2 - \frac{11}{2}v\omega_4^2\omega_1u^2 + 9v\omega_3^2\omega_4c_s^2\omega_1 + \frac{5}{2}v\omega_3\omega_2 - \frac{3}{2}v\omega_4^2\omega_1 + 9v\omega_3^2\omega_4\omega_1u^2 - 6v\omega_3c_s^2\omega_1^2\omega_2 - \\
&\quad v^3\omega_3\omega_2^2 - 3v^3\omega_3\omega_1^2\omega_2 - 9v\omega_4\omega_1^2u^2\omega_2 + 2v\omega_3\omega_4c_s^2\omega_2 + \frac{1}{2}v^3\omega_3^2\omega_1 + 5v^3\omega_4\omega_2 - \frac{15}{2}v\omega_3\omega_1\omega_2 + 10v\omega_3\omega_4c_s^2 + \\
&\quad 2v\omega_4\omega_1^2u^2 - 3v^3\omega_4\omega_1\omega_2^2 - 9v\omega_3\omega_1^2u^2\omega_2 + 5v\omega_4u^2\omega_2 + 2v\omega_3^2c_s^2\omega_2 + 9v\omega_3\omega_4^2\omega_1u^2 + 12v\omega_3\omega_4\omega_1u^2\omega_2 + \\
&\quad 9v\omega_3\omega_4^2c_s^2\omega_1 - 14v^3\omega_4\omega_1\omega_2 - 6v\omega_3\omega_4c_s^2\omega_1\omega_2 - v\omega_4u^2\omega_2^2 + \frac{1}{2}v\omega_3^2\omega_1 - \frac{1}{2}v\omega_4\omega_2 + 7v\omega_3^2\omega_4u^2 + \\
&\quad 10v\omega_3\omega_1u^2 + v^3\omega_4\omega_2^2 - v\omega_3\omega_4\omega_1 - 6v\omega_3^2c_s^2\omega_1\omega_2 + 6v\omega_3\omega_4\omega_1^2u^2 - 4v\omega_3\omega_4u^2\omega_2 - 5v^3\omega_3\omega_2 - \frac{1}{2}v^3\omega_4^2\omega_1 - \\
&\quad 2v\omega_3^2u^2\omega_2 + v\omega_1^2\omega_2 + 7v\omega_3\omega_4^2u^2 + 2v\omega_3c_s^2\omega_2^2 - v\omega_4\omega_1^2 + 6v^3\omega_4^2\omega_1\omega_2 - 3v\omega_3\omega_1u^2\omega_2^2 + 2v^3\omega_3^2\omega_2 - \\
&\quad 6v\omega_3c_s^2\omega_1\omega_2^2 - 31v\omega_3\omega_4c_s^2\omega_1 - 45v\omega_3\omega_4\omega_1u^2 - 3v\omega_4\omega_1u^2\omega_2^2 - 10v\omega_3\omega_4u^2 - \frac{11}{2}v\omega_3^2\omega_1u^2 - v\omega_3^2c_s^2\omega_1 - \\
&\quad 2v\omega_1\omega_2 - 6v\omega_3\omega_4^2u^2\omega_2 - 20v\omega_1u^2\omega_2 + 3v^3\omega_4\omega_1^2\omega_2 + 2v\omega_3\omega_1^2u^2 + 3v^3\omega_3\omega_1\omega_2^2 + 5v\omega_3u^2\omega_2 + \frac{3}{2}v\omega_4\omega_1\omega_2 + \\
&\quad 14v^3\omega_3\omega_1\omega_2 - v\omega_3u^2\omega_2^2 - v\omega_3\omega_1^2 - 2v^3\omega_4^2\omega_2 + 3v\omega_3\omega_1^2\omega_2 + 4v\omega_1u^2\omega_2^2 + 10v\omega_4\omega_1u^2 - 3v\omega_3\omega_4^2c_s^2 + \\
&\quad 22v\omega_4\omega_1u^2\omega_2 + \frac{3}{2}v\omega_3\omega_4\omega_1\omega_2 - \frac{1}{2}v\omega_3^2\omega_2 + 4v\omega_4\omega_1 - 2v\omega_4^2u^2\omega_2 - 10v\omega_3c_s^2\omega_2 - 3v\omega_3^2\omega_4c_s^2 - \\
&\quad 6v\omega_3^2\omega_4u^2\omega_2 + 22v\omega_3\omega_1u^2\omega_2 + \frac{3}{2}v\omega_3^2\omega_1\omega_2 + 6v\omega_3\omega_4c_s^2\omega_1^2 - \frac{1}{2}v\omega_3\omega_4\omega_2 + 32v\omega_3c_s^2\omega_1\omega_2, \\
\gamma_{[ty]}^{[\mu_4]} &= 5v\omega_3\omega_2 + 2v\omega_4^2\omega_1 - v\omega_4\omega_2^2 - 6v\omega_3\omega_4\omega_1^2 - 18v\omega_3\omega_1\omega_2 + 3v\omega_3^2\omega_4 + 3v\omega_3\omega_1\omega_2^2 + 2v\omega_3^2\omega_1 + 5v\omega_4\omega_2 + \\
&\quad 34v\omega_3\omega_4\omega_1 + 3v\omega_4\omega_1^2\omega_2 - v\omega_3\omega_2^2 + 3v\omega_4^2\omega_1\omega_2 - 2v\omega_1^2\omega_2 + v\omega_4\omega_1^2 - 9v\omega_3^2\omega_4\omega_1 + 10v\omega_1\omega_2 + 3v\omega_3\omega_4^2 - \\
&\quad 5v\omega_3\omega_1 - v\omega_4^2\omega_2 - 9v\omega_3\omega_4^2\omega_1 - 18v\omega_4\omega_1\omega_2 + v\omega_3\omega_1^2 + 3v\omega_4\omega_1\omega_2^2 - 10v\omega_3\omega_4 + 3v\omega_3\omega_1^2\omega_2 - 2v\omega_1\omega_2^2 + \\
&\quad 6v\omega_3\omega_4\omega_1\omega_2 - v\omega_3^2\omega_2 - 5v\omega_4\omega_1 + 3v\omega_3^2\omega_1\omega_2 - 2v\omega_3\omega_4\omega_2, \\
\gamma_{[x^2]}^{[\mu_1]} &= -\frac{1}{2}\omega_3\omega_4^2c_s^2\omega_2 + \frac{5}{2}\omega_4\omega_1u^2\omega_2 + \frac{5}{2}\omega_3\omega_1u^2\omega_2 + \frac{1}{2}\omega_3^2\omega_4u^2\omega_2 + \frac{1}{2}\omega_3\omega_4^2u^2\omega_2 - \frac{1}{2}\omega_3^2\omega_4c_s^2\omega_2 - \frac{1}{2}\omega_3\omega_1u^2\omega_2^2 -
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{2}\omega_4\omega_1u^2\omega_2^2 - \frac{1}{2}\omega_3^2\omega_1u^2\omega_2 - \frac{5}{2}\omega_3\omega_4u^2\omega_2 - \frac{1}{2}\omega_3\omega_4c_s^2\omega_1\omega_2 - \frac{1}{2}\omega_4^2\omega_1u^2\omega_2 - \frac{1}{2}\omega_3\omega_4\omega_1u^2\omega_2 - \frac{1}{2}\omega_3\omega_4c_s^2\omega_2^2 - \\
& \frac{1}{2}\omega_4\omega_1^2u^2\omega_2 + \frac{5}{2}\omega_3\omega_4c_s^2\omega_2 - \frac{1}{2}\omega_3\omega_1^2u^2\omega_2 + \frac{1}{2}\omega_3\omega_4u^2\omega_2^2, \\
\gamma_{[x^2]}^{[\mu_4]} &= \frac{3}{4}\omega_3\omega_4\omega_1\omega_2 - \frac{1}{2}\omega_3^2\omega_2 - 3\omega_3\omega_4\omega_2 + \frac{1}{4}\omega_3^2\omega_1\omega_2 + \omega_3^2\omega_4u^2\omega_2 - v^2\omega_3^2\omega_2 - \frac{1}{4}\omega_4\omega_1\omega_2 + \omega_3\omega_4^2u^2\omega_2 + \\
& v^2\omega_4\omega_1\omega_2^2 + \frac{1}{2}v^2\omega_3^2\omega_1\omega_2 + v^2\omega_4^2\omega_2 + \frac{1}{2}\omega_3\omega_4\omega_2^2 - \frac{1}{2}\omega_3^2\omega_1u^2\omega_2 - \frac{1}{2}\omega_4^2\omega_1u^2\omega_2 - \omega_3\omega_2^2 - 3\omega_3\omega_4\omega_1u^2\omega_2 + \\
& \frac{1}{2}\omega_4\omega_2 + 2v^2\omega_3\omega_2^2 + \frac{1}{2}\omega_3\omega_1\omega_2^2 + \frac{1}{2}\omega_3^2\omega_4\omega_2 - \frac{5}{4}\omega_3\omega_1\omega_2 + \omega_4\omega_1^2u^2\omega_2 + \omega_3\omega_1^2u^2\omega_2 - v^2\omega_3\omega_1\omega_2^2 - \\
& \frac{1}{2}v^2\omega_4^2\omega_1\omega_2 + \frac{1}{2}\omega_3\omega_4^2\omega_2 - 2v^2\omega_4\omega_2^2 + \frac{5}{2}\omega_3\omega_2, \\
\gamma_{[xy]}^{[\mu_1]} &= -v\omega_4\omega_1u\omega_2 + 2v\omega_3^2c_s^2u\omega_2 + v\omega_3\omega_1u\omega_2 + 4v\omega_4\omega_1^2u^3 - v\omega_3^2u^3\omega_2 - 5v\omega_3\omega_1u + 2v\omega_3\omega_4c_s^2u\omega_2 - 2v\omega_1u\omega_2^2 - \\
& 8v\omega_1^2u^3\omega_2 - 2v\omega_3\omega_4u^3\omega_2 - 2v\omega_4^2\omega_1u^3 + 6v\omega_1u\omega_2 + 2v\omega_3\omega_4^2u^3 + 4v\omega_3\omega_4c_s^2\omega_1u + v^3\omega_3^2u\omega_2 + 2v\omega_3^2\omega_4u^3 - \\
& v\omega_3\omega_4u\omega_2 - 4v\omega_3c_s^2\omega_1u\omega_2 + 4v\omega_3\omega_1^2u^3 - v\omega_3^2u\omega_2 - 2v\omega_3^2\omega_4c_s^2u - 8v\omega_3\omega_4\omega_1u^3 - v\omega_4^2u^3\omega_2 + \\
& 6v\omega_3\omega_1u^3\omega_2 - v\omega_4\omega_1u - 2v\omega_3^2\omega_1u^3 + 6v\omega_4\omega_1u^3\omega_2 - v^3\omega_4^2u\omega_2 + 2v\omega_3^2\omega_1u + 2v^3\omega_4\omega_1u\omega_2 - 2v\omega_3\omega_4^2c_s^2u - \\
& 2v^3\omega_3\omega_1u\omega_2 + 2v\omega_3\omega_4\omega_1u, \\
\gamma_{[xy]}^{[\mu_4]} &= 4v\omega_4\omega_1u\omega_2 + 8v\omega_3\omega_4^2u + 4v\omega_3\omega_1u\omega_2 - 3v\omega_4^2u\omega_2 - 2v\omega_3u\omega_2^2 + 10v\omega_3\omega_1u + 8v\omega_3^2\omega_4u + 4v\omega_1u\omega_2^2 - \\
& 20v\omega_1u\omega_2 - 2v\omega_4\omega_1^2u + 10v\omega_3u\omega_2 - 3v\omega_4^2\omega_1u - 6v\omega_3\omega_4u\omega_2 + 4v\omega_1^2u\omega_2 - 3v\omega_3^2u\omega_2 - 2v\omega_4u\omega_2^2 + \\
& 10v\omega_4\omega_1u - 2v\omega_3\omega_1^2u + 10v\omega_4u\omega_2 - 3v\omega_3^2\omega_1u - 20v\omega_3\omega_4u - 6v\omega_3\omega_4\omega_1u, \\
\gamma_{[y^2]}^{[\mu_1]} &= -v^2\omega_3^2\omega_1u^2 + v^2\omega_3\omega_1^2u^2\omega_2 - v^2\omega_3\omega_1^2 + \frac{1}{4}\omega_4\omega_1u^2\omega_2 + \frac{5}{2}\omega_3c_s^2\omega_1\omega_2 - \frac{1}{2}\omega_3\omega_4c_s^2\omega_1^2 + \frac{5}{4}\omega_3\omega_1u^2\omega_2 + \\
& v^2\omega_4\omega_1^2u^2\omega_2 + 2\omega_4\omega_1^2u^4 + \omega_3^2\omega_4c_s^2u^2\omega_2 - v^2\omega_3^2\omega_4c_s^2\omega_1 - \frac{1}{2}\omega_3^2\omega_1u^4\omega_2 + \frac{13}{4}v^2\omega_4\omega_1\omega_2 - v^2\omega_3^2\omega_4\omega_1u^2 + \\
& 2v^2\omega_3\omega_1^2u^2 + 3v^2\omega_3\omega_4c_s^2\omega_1\omega_2 - \frac{1}{2}\omega_4\omega_1u^2 + \frac{1}{2}\omega_4^2\omega_1u^4\omega_2 - \omega_4^2\omega_1u^4 - 2\omega_3^2\omega_4c_s^2u^2 - \frac{1}{2}v^2\omega_4\omega_1 + \omega_3\omega_1^2u^2 - \\
& 4v^2\omega_1\omega_2 + v^2\omega_3^2c_s^2\omega_1\omega_2 - v^4\omega_3\omega_1\omega_2^2 - \frac{1}{2}v^2\omega_4\omega_1\omega_2^2 - 4v^2\omega_3\omega_4u^2\omega_2 + \frac{1}{4}v^2\omega_3^2\omega_1\omega_2 + 3\omega_3\omega_4\omega_1u^2 - \\
& \frac{1}{2}v^4\omega_4^2\omega_1\omega_2 + \frac{7}{2}\omega_3\omega_4c_s^2\omega_1 - \omega_4\omega_1^2u^4\omega_2 - 2\omega_3\omega_4^2c_s^2u^2 + \omega_3^2c_s^2\omega_1 - 2v^2\omega_4^2u^2\omega_2 + \frac{1}{2}\omega_3^2\omega_1u^2 - v^2\omega_3\omega_4^2\omega_1u^2 - \\
& v^2\omega_3\omega_4^2c_s^2\omega_1 + \omega_3\omega_1^2u^4\omega_2 + \frac{5}{2}v^2\omega_3\omega_1 + 2\omega_3c_s^2\omega_1^2u^2\omega_2 - \frac{1}{4}\omega_3^2\omega_1u^2\omega_2 - \frac{1}{2}\omega_3\omega_4\omega_1^2u^2 - \omega_3^2c_s^2\omega_1u^2\omega_2 - \\
& v^2\omega_4^2\omega_1u^2 + \omega_3\omega_4^2c_s^2u^2\omega_2 - \frac{5}{2}\omega_3^2c_s^2\omega_1\omega_2 - 4\omega_3c_s^2\omega_1^2u^2 - \omega_3\omega_4c_s^2\omega_1\omega_2 + v^2\omega_3\omega_4^2u^2\omega_2 - 2\omega_3\omega_1^2u^4 - \\
& \frac{1}{2}v^2\omega_3^2\omega_1 - \frac{3}{4}\omega_3\omega_4\omega_1u^2\omega_2 - \frac{5}{2}\omega_3\omega_1u^2 + 2v^2\omega_4\omega_1^2u^2 - v^2\omega_4\omega_1u^2\omega_2^2 - 5\omega_3c_s^2\omega_1 - \frac{1}{2}\omega_3\omega_4^2c_s^2\omega_1 - \frac{1}{4}v^2\omega_3\omega_1\omega_2 - \\
& \frac{1}{2}\omega_3\omega_4^2\omega_1u^2 - v^2\omega_3\omega_1u^2\omega_2^2 + 2v^2\omega_2^2\omega_2 - 2v^2\omega_3c_s^2\omega_1\omega_2^2 - 3\omega_3\omega_4c_s^2\omega_1u^2\omega_2 + \omega_3^2\omega_1u^4 - 6v^2\omega_3\omega_4\omega_1u^2 - \\
& \frac{1}{2}v^2\omega_3\omega_4\omega_1 - \frac{1}{2}v^2\omega_4\omega_1^2\omega_2 + 8v^2\omega_3\omega_1u^2\omega_2 + v^2\omega_3^2\omega_4u^2\omega_2 + 2v^2\omega_3^2\omega_4u^2 - \frac{1}{2}\omega_3\omega_1^2u^2\omega_2 + 2\omega_3c_s^2\omega_1^2 + \\
& 8v^2\omega_4\omega_1u^2\omega_2 + v^4\omega_4\omega_1\omega_2^2 - \frac{1}{4}v^2\omega_3\omega_4\omega_1\omega_2 + \frac{1}{2}v^2\omega_3\omega_1\omega_2^2 - 2v^2\omega_3^2u^2\omega_2 + 2v^2\omega_3\omega_4^2u^2 - \frac{1}{2}v^2\omega_4^2\omega_1\omega_2 + \\
& \frac{1}{2}v^4\omega_3^2\omega_1\omega_2 + 2\omega_3^2c_s^2\omega_1u^2 - 8v^2\omega_1^2u^2\omega_2 - \omega_3c_s^2\omega_1^2\omega_2 + 6\omega_3\omega_4c_s^2\omega_1u^2 - \frac{1}{2}\omega_3^2\omega_4\omega_1u^2 - \frac{1}{2}\omega_3^2\omega_4c_s^2\omega_1, \\
\gamma_{[y^2]}^{[\mu_4]} &= \\
& \frac{3}{4}\omega_3\omega_4\omega_1\omega_2 + \frac{1}{2}\omega_4\omega_1 + v^2\omega_3^2\omega_4\omega_1 + \frac{1}{4}\omega_3^2\omega_1\omega_2 + v^2\omega_3\omega_4^2\omega_1 + \frac{1}{2}\omega_3\omega_1^2\omega_2 - \omega_3\omega_1^2 + 2\omega_3\omega_1^2u^2 + \frac{1}{2}\omega_3\omega_4^2\omega_1 - \frac{1}{4}\omega_4\omega_1\omega_2 + \\
& \frac{5}{2}\omega_3\omega_1 + v^2\omega_4\omega_1\omega_2^2 + \frac{1}{2}\omega_3^2\omega_4\omega_1 - \frac{1}{2}v^2\omega_3^2\omega_1\omega_2 - \omega_3^2\omega_1u^2 + \frac{1}{2}\omega_3^2\omega_1u^2\omega_2 - \frac{1}{2}\omega_4^2\omega_1u^2\omega_2 - \frac{1}{2}\omega_3^2\omega_1 - 3\omega_3\omega_4\omega_1 - \\
& \frac{5}{4}\omega_3\omega_1\omega_2 + \omega_4\omega_1^2u^2\omega_2 - 2\omega_4\omega_1^2u^2 - \omega_3\omega_1^2u^2\omega_2 - 3v^2\omega_3\omega_4\omega_1\omega_2 + v^2\omega_3\omega_1\omega_2^2 + \frac{1}{2}\omega_3\omega_4\omega_1^2 + \omega_4^2\omega_1u^2 - \frac{1}{2}v^2\omega_4^2\omega_1\omega_2,
\end{aligned}$$

## 8.6 EPDE for $\mu_5$

$$\begin{aligned}
& \gamma_{[1]}^{[\mu_1]}\mu_1 + \gamma_{[1]}^{[\mu_5]}\mu_5 + \gamma_{[t]}^{[\mu_1]}\delta_t \frac{\partial \mu_1}{\partial t} + \gamma_{[t]}^{[\mu_5]}\delta_t \frac{\partial \mu_5}{\partial t} + \gamma_{[x]}^{[\mu_1]}\delta_l \frac{\partial \mu_1}{\partial x} + \gamma_{[x]}^{[\mu_5]}\delta_l \frac{\partial \mu_5}{\partial x} + \gamma_{[y]}^{[\mu_1]}\delta_l \frac{\partial \mu_1}{\partial y} + \gamma_{[y]}^{[\mu_5]}\delta_l \frac{\partial \mu_5}{\partial y} \\
& + \gamma_{[t^2]}^{[\mu_1]}\delta_t^2 \frac{\partial^2 \mu_1}{\partial t^2} + \gamma_{[t^2]}^{[\mu_5]}\delta_t^2 \frac{\partial^2 \mu_5}{\partial t^2} + \gamma_{[tx]}^{[\mu_1]}\delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial x} + \gamma_{[tx]}^{[\mu_5]}\delta_t \delta_l \frac{\partial^2 \mu_5}{\partial t \partial x} + \gamma_{[ty]}^{[\mu_1]}\delta_t \delta_l \frac{\partial^2 \mu_1}{\partial t \partial y} + \gamma_{[ty]}^{[\mu_5]}\delta_t \delta_l \frac{\partial^2 \mu_5}{\partial t \partial y} \\
& + \gamma_{[x^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial x^2} + \gamma_{[x^2]}^{[\mu_5]}\delta_l^2 \frac{\partial^2 \mu_5}{\partial x^2} + \gamma_{[xy]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial x \partial y} + \gamma_{[xy]}^{[\mu_5]}\delta_l^2 \frac{\partial^2 \mu_5}{\partial x \partial y} + \gamma_{[y^2]}^{[\mu_1]}\delta_l^2 \frac{\partial^2 \mu_1}{\partial y^2} + \gamma_{[y^2]}^{[\mu_5]}\delta_l^2 \frac{\partial^2 \mu_5}{\partial y^2} = 0,
\end{aligned}$$

where

$$\gamma_{[1]}^{[\mu_1]} = -\omega_3\omega_4^2c_s^2\omega_1\omega_2 - v^2\omega_3\omega_4^2\omega_1\omega_2 - v^2\omega_3\omega_4\omega_1^2\omega_2 - v^2\omega_3^2\omega_4\omega_1\omega_2 - \omega_3\omega_4c_s^2\omega_1^2\omega_2 + 5\omega_3\omega_4c_s^2\omega_1\omega_2 - \omega_3^2\omega_4c_s^2\omega_1\omega_2 - v^2\omega_3\omega_4\omega_1\omega_2^2 + 5v^2\omega_3\omega_4\omega_1\omega_2 - \omega_3\omega_4c_s^2\omega_1\omega_2^2,$$

$$\gamma_{[1]}^{[\mu_5]} = -5\omega_3\omega_4\omega_1\omega_2 + \omega_3\omega_4\omega_1\omega_2^2 + \omega_3^2\omega_4\omega_1\omega_2 + \omega_3\omega_4^2\omega_1\omega_2 + \omega_3\omega_4\omega_1^2\omega_2,$$

$$\gamma_{[t]}^{[\mu_1]} = -\frac{1}{2}\omega_3\omega_4\omega_1\omega_2 + 4\omega_3\omega_4^2c_s^2\omega_1\omega_2 - \omega_3\omega_1^2c_s^2\omega_2 + \frac{5}{2}\omega_4\omega_1u^2\omega_2 + 3v^2\omega_3^2\omega_4\omega_1 + 4v^2\omega_3\omega_4^2\omega_1\omega_2 - v^2\omega_3\omega_4\omega_2^2 + 5\omega_3c_s^2\omega_1\omega_2 - \omega_3\omega_4c_s^2\omega_1^2 - \frac{5}{2}\omega_3\omega_1u^2\omega_2 + 3v^2\omega_3\omega_4^2\omega_1 + \frac{15}{2}v^2\omega_4\omega_1\omega_2 + \frac{1}{2}\omega_3\omega_1^2\omega_2 - \frac{1}{2}\omega_4\omega_1\omega_2^2 + 4v^2\omega_3\omega_4\omega_1^2\omega_2 + 4v^2\omega_3^2\omega_4\omega_1\omega_2 + 2\omega_4\omega_1\omega_2 - \frac{3}{2}v^2\omega_3\omega_1^2\omega_2 + \frac{1}{2}v^2\omega_4\omega_1\omega_2^2 - \omega_3^2\omega_4c_s^2\omega_2 + 5v^2\omega_3\omega_4\omega_2 - \frac{5}{2}v^2\omega_3^2\omega_1\omega_2 + 5\omega_3\omega_4c_s^2\omega_1 - \omega_3c_s^2\omega_1\omega_2^2 + \frac{1}{2}\omega_3\omega_1u^2\omega_2^2 + 4\omega_3\omega_4c_s^2\omega_1^2\omega_2 - \frac{1}{2}\omega_4\omega_1u^2\omega_2^2 + \frac{3}{2}\omega_3^2\omega_1u^2\omega_2 - \frac{1}{2}\omega_4^2\omega_1\omega_2 - v^2\omega_3\omega_4^2\omega_2 - \omega_3^2c_s^2\omega_1\omega_2 - 22\omega_3\omega_4c_s^2\omega_1\omega_2 - \frac{3}{2}\omega_4^2\omega_1u^2\omega_2 + 4\omega_3^2\omega_4c_s^2\omega_1\omega_2 - \omega_3\omega_1^2c_s^2\omega_1 - v^2\omega_3^2\omega_4\omega_2 - \frac{1}{2}\omega_4\omega_1^2\omega_2 + \frac{15}{2}v^2\omega_3\omega_1\omega_2 + 4v^2\omega_3\omega_4\omega_1\omega_2^2 - \frac{1}{2}\omega_3\omega_1\omega_2^2 + v^2\omega_3\omega_4\omega_1^2 - \omega_3\omega_4c_s^2\omega_2^2 - 5v^2\omega_3\omega_4\omega_1 - \frac{3}{2}v^2\omega_4\omega_1^2\omega_2 + \frac{3}{2}\omega_4\omega_1^2u^2\omega_2 + 5\omega_3\omega_4c_s^2\omega_2 - \frac{3}{2}\omega_3\omega_1^2u^2\omega_2 - 28v^2\omega_3\omega_4\omega_1\omega_2 + \frac{1}{2}v^2\omega_3\omega_1\omega_2^2 + 4\omega_3\omega_4c_s^2\omega_1\omega_2^2 - \frac{5}{2}v^2\omega_4^2\omega_1\omega_2 - \omega_3c_s^2\omega_1^2\omega_2 - \omega_3^2\omega_4c_s^2\omega_1,$$

$$\gamma_{[t]}^{[\mu_5]} = 23\omega_3\omega_4\omega_1\omega_2 - 5\omega_3\omega_4\omega_2 + \omega_3^2\omega_1\omega_2 + \omega_3\omega_1^2\omega_2 + \omega_4\omega_1\omega_2^2 + \omega_3\omega_4^2\omega_1 - 5\omega_4\omega_1\omega_2 + \omega_3^2\omega_4\omega_1 - 4\omega_3\omega_4\omega_1\omega_2^2 + \omega_3\omega_4\omega_2^2 + \omega_1^2\omega_1\omega_2 - 4\omega_3^2\omega_4\omega_1\omega_2 - 5\omega_3\omega_4\omega_1 + \omega_4\omega_1^2\omega_2 + \omega_3\omega_1\omega_2^2 + \omega_3^2\omega_4\omega_2 - 5\omega_3\omega_1\omega_2 - 4\omega_3\omega_4^2\omega_1\omega_2 + \omega_3\omega_4\omega_1^2 - 4\omega_3\omega_4\omega_1^2\omega_2 + \omega_3\omega_1^2\omega_2,$$

$$\gamma_{[x]}^{[\mu_1]} = 2v^2\omega_3\omega_4^2\omega_1u - 3v^2\omega_3^2\omega_4u\omega_2 - 5v^2\omega_3\omega_1u\omega_2 - \omega_4\omega_1u^3\omega_2^2 + 2\omega_3c_s^2\omega_1u\omega_2^2 - 5v^2\omega_4\omega_1u\omega_2 + \omega_3\omega_1u^3\omega_2^2 + 3v^2\omega_4\omega_1u\omega_2^2 - 5\omega_3\omega_1u^3\omega_2 - 10\omega_3c_s^2\omega_1u\omega_2 + 3v^2\omega_3\omega_1u\omega_2^2 + 5\omega_4\omega_1u^3\omega_2 + 2v^2\omega_3^2\omega_4\omega_1u + 2\omega_3\omega_4c_s^2\omega_1u\omega_2 + 2\omega_3^2c_s^2\omega_1u\omega_2 - \frac{1}{2}\omega_3^2\omega_1u\omega_2 - 3\omega_3\omega_4^2c_s^2u\omega_2 - 3v^2\omega_3\omega_4^2u\omega_2 - \frac{1}{2}\omega_3\omega_4\omega_1u\omega_2 - 2\omega_3\omega_4c_s^2u\omega_2^2 - \omega_3\omega_1u\omega_2^2 - \omega_3\omega_1^2u^3\omega_2 + 2\omega_3c_s^2\omega_1^2u\omega_2 + \omega_4\omega_1^2u^3\omega_2 + 10v^2\omega_3\omega_4u\omega_2 - 4v^2\omega_3\omega_4\omega_1u\omega_2 - 3\omega_3^2\omega_4c_s^2u\omega_2 - 2\omega_4^2\omega_1u^3\omega_2 - 2v^2\omega_3\omega_4u\omega_2^2 + 2\omega_3^2\omega_1u^3\omega_2 - \frac{1}{2}\omega_4\omega_1u\omega_2 + v^2\omega_3\omega_1^2u\omega_2 + 10\omega_3\omega_4c_s^2u\omega_2 + v^2\omega_4\omega_1^2u\omega_2 + \frac{5}{2}\omega_3\omega_1u\omega_2,$$

$$\gamma_{[x]}^{[\mu_5]} = 2\omega_3\omega_4u\omega_2^2 - \omega_4\omega_1^2u\omega_2 - \omega_3\omega_1^2u\omega_2 - 10\omega_3\omega_4u\omega_2 - \omega_3^2\omega_1u\omega_2 - \omega_4^2\omega_1u\omega_2 - 2\omega_3\omega_4\omega_1u\omega_2 - \omega_3\omega_1u\omega_2^2 - \omega_4\omega_1u\omega_2^2 + 3\omega_3\omega_4^2u\omega_2 + 5\omega_4\omega_1u\omega_2 + 3\omega_3^2\omega_4u\omega_2 + 5\omega_3\omega_1u\omega_2,$$

$$\gamma_{[y]}^{[\mu_1]} = -4v^3\omega_3\omega_4\omega_1\omega_2 - v^3\omega_3^2\omega_1\omega_2 - v\omega_3^2\omega_4c_s^2\omega_1 + 2v\omega_4\omega_1^2u^2\omega_2 - \frac{5}{2}v\omega_3\omega_1\omega_2 + 2v^3\omega_4\omega_1\omega_2^2 - 2v\omega_3\omega_1^2u^2\omega_2 - v\omega_3\omega_4^2c_s^2\omega_1 + v^3\omega_3\omega_4^2\omega_1 + 2v\omega_3\omega_4c_s^2\omega_1\omega_2 + v^3\omega_3^2\omega_4\omega_1 + v\omega_3^2\omega_1u^2\omega_2 - v\omega_4^2\omega_1u^2\omega_2 - v^3\omega_4^2\omega_1\omega_2 + 2v^3\omega_3\omega_1\omega_2^2 - \frac{1}{2}v\omega_4\omega_1\omega_2 + v\omega_3\omega_1^2\omega_2 + \frac{1}{2}v\omega_3\omega_4\omega_1\omega_2 + \frac{1}{2}v\omega_3^2\omega_1\omega_2,$$

$$\gamma_{[y]}^{[\mu_5]} = 2v\omega_3\omega_4\omega_1^2 + 5v\omega_3\omega_1\omega_2 - v\omega_3\omega_1\omega_2^2 - 10v\omega_3\omega_4\omega_1 - v\omega_4\omega_1^2\omega_2 - v\omega_4^2\omega_1\omega_2 + 3v\omega_3^2\omega_4\omega_1 + 3v\omega_3\omega_4^2\omega_1 + 5v\omega_4\omega_1\omega_2 - v\omega_4\omega_1\omega_2^2 - v\omega_3\omega_1^2\omega_2 - 2v\omega_3\omega_4\omega_1\omega_2 - v\omega_3^2\omega_1\omega_2,$$

$$\gamma_{[t^2]}^{[\mu_1]} = \frac{7}{4}\omega_3\omega_4\omega_1\omega_2 - \omega_3^2\omega_4c_s^2 + 5\omega_3c_s^2\omega_2 - \frac{3}{2}\omega_4^2u^2\omega_2 - 8\omega_3\omega_4^2c_s^2\omega_1\omega_2 + \frac{1}{2}v^2\omega_3\omega_1^2 + 2\omega_4\omega_1 + \frac{7}{2}\omega_3\omega_4^2c_s^2\omega_2 - \frac{29}{4}\omega_4\omega_1u^2\omega_2 - \frac{21}{2}v^2\omega_3^2\omega_4\omega_1 - \frac{1}{2}\omega_3\omega_4\omega_2 - 8v^2\omega_3\omega_4^2\omega_1\omega_2 + \frac{7}{2}v^2\omega_3\omega_4\omega_2^2 - \frac{37}{2}\omega_3c_s^2\omega_1\omega_2 - \omega_1\omega_2^2 + \frac{7}{2}\omega_3\omega_4c_s^2\omega_1^2 + \frac{29}{4}\omega_3\omega_1u^2\omega_2 - 5v^2\omega_3\omega_4 + 2v^2\omega_1\omega_2^2 - \frac{21}{2}v^2\omega_3\omega_4^2\omega_1 - \frac{127}{4}v^2\omega_4\omega_1\omega_2 - \frac{7}{4}\omega_3\omega_1^2\omega_2 + \frac{5}{2}\omega_4\omega_1u^2 - \omega_3\omega_4^2c_s^2 + \frac{1}{2}\omega_3u^2\omega_2^2 + \frac{7}{4}\omega_4\omega_1\omega_2^2 - 8v^2\omega_3\omega_4\omega_1^2\omega_2 + \frac{1}{2}\omega_3\omega_1^2 - \frac{5}{2}v^2\omega_4\omega_1 - \frac{5}{2}v^2\omega_3^2\omega_2 - \frac{5}{2}\omega_3u^2\omega_2 - 8v^2\omega_3^2\omega_4\omega_1\omega_2 - \frac{3}{2}\omega_3\omega_1^2u^2 - \frac{17}{2}\omega_4\omega_1\omega_2 + \frac{21}{4}v^2\omega_3\omega_1^2\omega_2 - \frac{1}{2}\omega_4^2\omega_2 + \frac{1}{2}v^2\omega_4\omega_1^2 + 10v^2\omega_1\omega_2 - \frac{7}{4}v^2\omega_4\omega_1\omega_2^2 + \frac{7}{2}\omega_3^2\omega_4c_s^2\omega_2 - \frac{49}{2}v^2\omega_3\omega_4\omega_2 + \frac{35}{4}v^2\omega_3^2\omega_1\omega_2 - \frac{37}{2}\omega_3\omega_4c_s^2\omega_1 + 2\omega_1\omega_2 + \frac{7}{2}\omega_3c_s^2\omega_1\omega_2^2 - \omega_3^2c_s^2\omega_1 - \frac{7}{4}\omega_3\omega_1u^2\omega_2^2 + \frac{3}{2}\omega_3^2\omega_1u^2 + 3v^2\omega_3\omega_4^2 - \omega_3c_s^2\omega_2^2 - 8\omega_3\omega_4c_s^2\omega_1^2\omega_2 + \frac{7}{4}\omega_4\omega_1u^2\omega_2^2 - \frac{5}{2}v^2\omega_4^2\omega_2 - \frac{5}{2}v^2\omega_3\omega_1 - \frac{1}{2}\omega_4\omega_1^2 - \frac{21}{4}\omega_3^2\omega_1u^2\omega_2 + \frac{7}{4}\omega_4\omega_1\omega_2 + \frac{7}{2}v^2\omega_3\omega_4^2\omega_2 + \frac{3}{2}\omega_3^2u^2\omega_2 + \frac{7}{2}\omega_3^2c_s^2\omega_1\omega_2 + 47\omega_3\omega_4c_s^2\omega_1\omega_2 + \frac{21}{4}\omega_4^2\omega_1u^2\omega_2 + \frac{15}{2}v^2\omega_4\omega_2 + \frac{3}{2}v^2\omega_3^2\omega_1 - \frac{1}{2}\omega_3\omega_2^2 - \frac{5}{2}\omega_3\omega_1u^2 - 8\omega_3^2\omega_4c_s^2\omega_1\omega_2 + 5\omega_3c_s^2\omega_1 + \frac{7}{2}\omega_3\omega_4^2c_s^2\omega_1 + 2\omega_4\omega_2 + \frac{1}{2}v^2\omega_3\omega_2^2 + \frac{7}{2}v^2\omega_3^2\omega_4\omega_2 - \frac{1}{2}\omega_3\omega_4\omega_1 + \frac{7}{4}\omega_4\omega_1^2\omega_2 - \frac{127}{4}v^2\omega_3\omega_1\omega_2 - 8v^2\omega_3\omega_4\omega_1\omega_2^2 + \frac{7}{4}\omega_3\omega_1\omega_2^2 - \frac{7}{2}v^2\omega_3\omega_4\omega_1^2 + \frac{7}{2}\omega_3\omega_4c_s^2\omega_2^2 - 2v^2\omega_1^2\omega_2 - \frac{1}{2}\omega_4u^2\omega_2^2 + \frac{41}{2}v^2\omega_3\omega_4\omega_1 + \frac{21}{4}v^2\omega_4\omega_1^2\omega_2 - \frac{1}{2}\omega_3\omega_1\omega_2 - \frac{21}{4}\omega_4\omega_1^2u^2\omega_2 - \frac{37}{2}\omega_3\omega_4c_s^2\omega_2 - \omega_3^2c_s^2\omega_2 + \frac{5}{2}\omega_4u^2\omega_2 + 5\omega_3\omega_4c_s^2 + \frac{3}{2}\omega_4\omega_1^2u^2 + \frac{21}{4}\omega_3\omega_1^2u^2\omega_2 - \omega_3c_s^2\omega_1^2 - \frac{1}{2}\omega_4\omega_2^2 + \frac{3}{2}v^2\omega_4\omega_1 + \frac{15}{2}v^2\omega_3\omega_2 + 68v^2\omega_3\omega_4\omega_1\omega_2 - \frac{7}{4}v^2\omega_3\omega_1\omega_2^2 - 8\omega_3\omega_4c_s^2\omega_1\omega_2^2 - \frac{3}{2}\omega_4^2\omega_1u^2 + \frac{35}{4}v^2\omega_4^2\omega_1\omega_2 + \frac{1}{2}v^2\omega_4\omega_2^2 - \frac{1}{2}\omega_4^2\omega_1 + \frac{7}{2}\omega_3c_s^2\omega_1^2\omega_2 + 3v^2\omega_3^2\omega_4 + \frac{7}{2}\omega_3^2\omega_4c_s^2\omega_1,$$

$$\gamma_{[t^2]}^{[\mu_5]} = -\frac{101}{2}\omega_3\omega_4\omega_1\omega_2 + \omega_3^2\omega_2 - 5\omega_4\omega_1 + \frac{39}{2}\omega_3\omega_4\omega_2 - \frac{7}{2}\omega_3^2\omega_1\omega_2 + \omega_1\omega_2^2 - \frac{7}{2}\omega_3\omega_1^2\omega_2 - 5\omega_3\omega_4 - \frac{7}{2}\omega_4\omega_1\omega_2^2 + \omega_3\omega_1^2 - \frac{7}{2}\omega_3\omega_4^2\omega_1 + \frac{39}{2}\omega_4\omega_1\omega_2 - 5\omega_3\omega_1 + \omega_4^2\omega_2 + \omega_3\omega_4^2 - \frac{7}{2}\omega_3^2\omega_4\omega_1 - 5\omega_1\omega_2 + 8\omega_3\omega_4\omega_1\omega_2^2 + \omega_4\omega_1^2 -$$



$$\begin{aligned}
& \frac{7}{2}\omega_3\omega_4\omega_2^2 + \omega_2^2\omega_2 - \frac{7}{2}\omega_4^2\omega_1\omega_2 + 8\omega_3^2\omega_4\omega_1\omega_2 + \omega_3\omega_2^2 + \omega_3^2\omega_1 - 5\omega_4\omega_2 + \frac{39}{2}\omega_3\omega_4\omega_1 - \frac{7}{2}\omega_4\omega_2^2\omega_2 - \frac{7}{2}\omega_3\omega_1\omega_2^2 - \\
& \frac{7}{2}\omega_3^2\omega_4\omega_2 + \frac{39}{2}\omega_3\omega_1\omega_2 + \omega_3^2\omega_4 + 8\omega_3\omega_4^2\omega_1\omega_2 + \omega_4\omega_2^2 - \frac{7}{2}\omega_3\omega_4\omega_1^2 + 8\omega_3\omega_4\omega_1^2\omega_2 - \frac{7}{2}\omega_3\omega_4^2\omega_2 - 5\omega_3\omega_2 + \omega_4^2\omega_1, \\
\gamma_{[tx]}^{[\mu_1]} = & -6v^2\omega_3\omega_4^2\omega_1u + 5\omega_4\omega_1u^3 - \frac{11}{2}v^2\omega_3^2u\omega_2 - 10\omega_3c_s^2\omega_1u - \frac{1}{2}\omega_3\omega_4\omega_1u + 4v^2\omega_1^2u\omega_2 - 3\omega_3\omega_4^2c_s^2u - \\
& \frac{1}{2}\omega_3^2\omega_1u + 9v^2\omega_3^2\omega_4u\omega_2 + 22v^2\omega_3\omega_1u\omega_2 + 3\omega_4\omega_1u^3\omega_2^2 - 6\omega_3c_s^2\omega_1u\omega_2^2 + 4\omega_4u\omega_2 + 22v^2\omega_4\omega_1u\omega_2 - \\
& 3\omega_3\omega_1u^3\omega_2^2 + 2v^2\omega_4u\omega_2^2 + 5v^2\omega_4\omega_1u + 2\omega_3^2\omega_1u^3 + 10v^2\omega_4u\omega_2 - 9v^2\omega_4\omega_1u\omega_2^2 + 14\omega_3\omega_1u^3\omega_2 - \\
& 3\omega_3^2\omega_4c_s^2u + 32\omega_3c_s^2\omega_1u\omega_2 - 9v^2\omega_3\omega_1u\omega_2^2 - 14\omega_4\omega_1u^3\omega_2 - 6v^2\omega_3^2\omega_4\omega_1u - \frac{1}{2}\omega_4\omega_1u - 6\omega_3\omega_4c_s^2\omega_1u\omega_2 - \\
& v^2\omega_3\omega_1^2u - \omega_4u\omega_2^2 - \frac{1}{2}\omega_4^2u^3\omega_2 - 6\omega_3^2c_s^2\omega_1u\omega_2 - \omega_3\omega_4u\omega_2 + \frac{3}{2}\omega_3^2\omega_1u\omega_2 + \frac{1}{2}\omega_3^2u\omega_2 - \omega_3\omega_1^2u^3 + \\
& 9\omega_3\omega_4^2c_s^2u\omega_2 - 10v^2\omega_3\omega_4u - 2v^2\omega_3^2\omega_1u + 9v^2\omega_3\omega_4^2u\omega_2 + \frac{3}{2}\omega_3\omega_4\omega_1u\omega_2 + 2\omega_3\omega_4c_s^2\omega_1u + 6\omega_3\omega_4c_s^2u\omega_2^2 - \\
& 5\omega_3\omega_1u^3 - \frac{11}{2}v^2\omega_4^2u\omega_2 + 3\omega_3\omega_1u\omega_2^2 + 3\omega_3\omega_1^2u^3\omega_2 + 7v^2\omega_3\omega_4^2u - 6\omega_3c_s^2\omega_1^2u\omega_2 - 3\omega_4\omega_1^2u^3\omega_2 + \\
& 2\omega_3^2c_s^2\omega_1u - 2\omega_1u\omega_2 - 45v^2\omega_3\omega_4u\omega_2 + 2v^2\omega_3u\omega_2^2 + 7v^2\omega_3^2\omega_4u + 5v^2\omega_3\omega_1u + 12v^2\omega_3\omega_4\omega_1u\omega_2 + \omega_1u\omega_2^2 + \\
& 9\omega_3^2\omega_4c_s^2u\omega_2 - 2\omega_4^2\omega_1u^3 + 6\omega_4^2\omega_1u^3\omega_2 + 10v^2\omega_3u\omega_2 + 6v^2\omega_3\omega_4u\omega_2^2 - 4v^2\omega_3\omega_4\omega_1u - v^2\omega_4\omega_1^2u + \frac{5}{2}\omega_3\omega_1u - \\
& \omega_3u\omega_2^2 + 10\omega_3\omega_4c_s^2u + \frac{1}{2}\omega_3^2u^3\omega_2 - 6\omega_3^2\omega_1u^3\omega_2 - 20v^2\omega_1u\omega_2 + \omega_4\omega_1^2u^3 + 2\omega_3c_s^2\omega_1^2u - \frac{3}{2}\omega_4^2u\omega_2 - \\
& \omega_3^2c_s^2u\omega_2 + \frac{3}{2}\omega_4\omega_1u\omega_2 - 3v^2\omega_3\omega_1^2u\omega_2 - 31\omega_3\omega_4c_s^2u\omega_2 - 2v^2\omega_4^2\omega_1u - 3v^2\omega_4\omega_1^2u\omega_2 - \frac{15}{2}\omega_3\omega_1u\omega_2, \\
\gamma_{[tx]}^{[\mu_5]} = & -2\omega_3\omega_4\omega_1u - 6\omega_3\omega_4u\omega_2^2 - 10\omega_3\omega_4u - \omega_3^2\omega_1u + 3\omega_4\omega_1^2u\omega_2 - 5\omega_4u\omega_2 + 3\omega_3\omega_1^2u\omega_2 - \omega_3\omega_1^2u + 5\omega_4\omega_1u + \\
& \omega_4u\omega_2^2 + 34\omega_3\omega_4u\omega_2 + 3\omega_3^2\omega_1u\omega_2 + 2\omega_3^2u\omega_2 + 3\omega_4^2\omega_1u\omega_2 - 2\omega_1^2u\omega_2 + 6\omega_3\omega_4\omega_1u\omega_2 + 3\omega_3\omega_1u\omega_2^2 - \omega_4^2\omega_1u + \\
& 3\omega_4\omega_1u\omega_2^2 - 5\omega_3u\omega_2 - 9\omega_3\omega_4^2u\omega_2 + 10\omega_1u\omega_2 - \omega_4\omega_1^2u - 2\omega_1u\omega_2^2 + 3\omega_3^2\omega_4u + 5\omega_3\omega_1u + \omega_3u\omega_2^2 + 2\omega_4^2u\omega_2 - \\
& 18\omega_4\omega_1u\omega_2 + 3\omega_3\omega_4^2u - 9\omega_3^2\omega_4u\omega_2 - 18\omega_3\omega_1u\omega_2, \\
\gamma_{[ty]}^{[\mu_1]} = & 12v^3\omega_3\omega_4\omega_1\omega_2 + 3v^3\omega_3^2\omega_1\omega_2 - 4v^3\omega_3\omega_4\omega_2 - \frac{1}{2}v\omega_4^2\omega_1u^2 + 3v\omega_3^2\omega_4c_s^2\omega_1 - \frac{5}{2}v\omega_3\omega_2 - \frac{1}{2}v\omega_4^2\omega_1 + 2v^3\omega_3\omega_2^2 - \\
& 6v\omega_4\omega_1^2u^2\omega_2 + 2v\omega_3\omega_4c_s^2\omega_2 + \frac{1}{2}v^3\omega_3^2\omega_1 + \frac{19}{2}v\omega_3\omega_1\omega_2 - 6v^3\omega_4\omega_1\omega_2^2 + 6v\omega_3\omega_1^2u^2\omega_2 + 3v\omega_3\omega_4^2c_s^2\omega_1 - \\
& 3v^3\omega_4\omega_1\omega_2 - 3v^3\omega_3\omega_4^2\omega_1 + v^3\omega_3^2\omega_4 - 6v\omega_3\omega_4c_s^2\omega_1\omega_2 - \frac{1}{2}v\omega_3^2\omega_1 - \frac{1}{2}v\omega_4\omega_2 + 2v^3\omega_4\omega_2^2 - v\omega_3\omega_4\omega_1 - \\
& 3v^3\omega_3^2\omega_4\omega_1 + \frac{1}{2}v^3\omega_4^2\omega_1 - 3v\omega_3^2\omega_1u^2\omega_2 + v\omega_3^2u^2\omega_2 + v\omega_1^2\omega_2 + 3v\omega_4^2\omega_1u^2\omega_2 + 3v^3\omega_4^2\omega_1\omega_2 - v^3\omega_3^2\omega_2 - \\
& v\omega_3\omega_4c_s^2\omega_1 + 4v^3\omega_1\omega_2^2 + \frac{1}{2}v\omega_3^2\omega_1u^2 - v\omega_3^2c_s^2\omega_1 - 3v\omega_1\omega_2 + v^3\omega_3\omega_4\omega_1 - 6v^3\omega_3\omega_1\omega_2^2 + \frac{5}{2}v\omega_4\omega_1\omega_2 - \\
& 3v^3\omega_3\omega_1\omega_2 - v^3\omega_4^2\omega_2 - 3v\omega_3\omega_1^2\omega_2 + v^3\omega_3\omega_4^2 - v\omega_3\omega_4^2c_s^2 + 3v\omega_4\omega_1u^2\omega_2 - \frac{3}{2}v\omega_3\omega_4\omega_1\omega_2 + \frac{1}{2}v\omega_3^2\omega_2 - \\
& v\omega_4^2u^2\omega_2 - v\omega_3^2\omega_4c_s^2 - 3v\omega_3\omega_1u^2\omega_2 - \frac{3}{2}v\omega_3^2\omega_1\omega_2 + \frac{1}{2}v\omega_3\omega_4\omega_2 + 2v\omega_3c_s^2\omega_1\omega_2, \\
\gamma_{[ty]}^{[\mu_5]} = & 5v\omega_3\omega_2 + 2v\omega_4^2\omega_1 - v\omega_4\omega_2^2 - 6v\omega_3\omega_4\omega_1^2 - 18v\omega_3\omega_1\omega_2 + 3v\omega_3^2\omega_4 + 3v\omega_3\omega_1\omega_2^2 + 2v\omega_3^2\omega_1 + 5v\omega_4\omega_2 + \\
& 34v\omega_3\omega_4\omega_1 + 3v\omega_4\omega_1^2\omega_2 - v\omega_3\omega_2^2 + 3v\omega_4^2\omega_1\omega_2 - 2v\omega_1^2\omega_2 + v\omega_4\omega_1^2 - 9v\omega_3^2\omega_4\omega_1 + 10v\omega_1\omega_2 + 3v\omega_3\omega_4^2 - \\
& 5v\omega_3\omega_1 - v\omega_4^2\omega_2 - 9v\omega_3\omega_4^2\omega_1 - 18v\omega_4\omega_1\omega_2 + v\omega_3\omega_1^2 + 3v\omega_4\omega_1\omega_2^2 - 10v\omega_3\omega_4 + 3v\omega_3\omega_1^2\omega_2 - 2v\omega_1\omega_2^2 + \\
& 6v\omega_3\omega_4\omega_1\omega_2 - v\omega_3^2\omega_2 - 5v\omega_4\omega_1 + 3v\omega_3^2\omega_1\omega_2 - 2v\omega_3\omega_4\omega_2, \\
\gamma_{[x^2]}^{[\mu_1]} = & -2v^2\omega_3^2\omega_1u^2 - 5\omega_3c_s^2\omega_2 - v^2\omega_3\omega_1^2u^2\omega_2 - \frac{1}{2}\omega_3\omega_4^2c_s^2\omega_2 + \frac{13}{4}\omega_4\omega_1u^2\omega_2 - \frac{1}{2}v^2\omega_3\omega_4\omega_2^2 + \frac{5}{2}\omega_3c_s^2\omega_1\omega_2 - \\
& \frac{1}{4}\omega_3\omega_1u^2\omega_2 - 4v^2\omega_3c_s^2\omega_2^2 + 6v^2\omega_3\omega_4c_s^2\omega_2 - v^2\omega_4\omega_1^2u^2\omega_2 - \omega_3^2\omega_4c_s^2u^2\omega_2 + 2\omega_1u^2\omega_2^2 + v^2\omega_3^2\omega_4c_s^2\omega_1 + \\
& \frac{1}{2}\omega_3^2\omega_1u^4\omega_2 + \frac{1}{4}v^2\omega_4\omega_1\omega_2 + v^2\omega_3^2\omega_4\omega_1u^2 - 3v^2\omega_3\omega_4c_s^2\omega_1\omega_2 - \frac{1}{2}\omega_4^2\omega_1u^4\omega_2 - \omega_3u^2\omega_2^2 + \frac{1}{2}v^2\omega_3^2\omega_2 - v^4\omega_4^2\omega_2 + \\
& \frac{5}{2}\omega_3u^2\omega_2 - 2v^2\omega_3\omega_4^2c_s^2 - 8v^2\omega_1u^2\omega_2^2 - v^2\omega_3^2c_s^2\omega_1\omega_2 - 4\omega_1u^2\omega_2 + v^4\omega_3\omega_1\omega_2^2 + 2v^2\omega_3u^2\omega_2^2 - 6v^2\omega_3\omega_4u^2\omega_2 - \\
& \frac{1}{2}\omega_3^2\omega_4c_s^2\omega_2 + 3v^2\omega_3\omega_4\omega_2 - \frac{1}{4}v^2\omega_3^2\omega_1\omega_2 + \frac{1}{2}v^4\omega_4^2\omega_1\omega_2 + \omega_4\omega_1^2u^4\omega_2 - \omega_3c_s^2\omega_1\omega_2^2 - v^2\omega_4^2u^2\omega_2 - \\
& 2v^2\omega_3^2\omega_4c_s^2 + v^2\omega_3\omega_4^2\omega_1u^2 + v^2\omega_3\omega_4^2c_s^2\omega_1 - \omega_3\omega_1^2u^4\omega_2 + 2\omega_3c_s^2\omega_2^2 - \frac{1}{2}\omega_4\omega_1u^2\omega_2^2 - 2\omega_3c_s^2\omega_1^2u^2\omega_2 + \\
& v^4\omega_3^2\omega_2 + \frac{1}{4}\omega_3^2\omega_1u^2\omega_2 - \frac{1}{2}\omega_3\omega_4u^2\omega_2 + \omega_3^2c_s^2\omega_1u^2\omega_2 - \frac{1}{2}v^2\omega_3\omega_4^2\omega_2 - 2v^2\omega_4^2\omega_1u^2 - \omega_3\omega_4^2c_s^2u^2\omega_2 - \\
& \frac{1}{2}\omega_3^2u^2\omega_2 - \frac{1}{2}\omega_3^2c_s^2\omega_1\omega_2 - \omega_3\omega_4c_s^2\omega_1\omega_2 - v^2\omega_3\omega_4^2u^2\omega_2 - \frac{1}{2}\omega_4^2\omega_1u^2\omega_2 - \frac{1}{2}v^2\omega_4\omega_2 - \frac{1}{4}\omega_3\omega_4\omega_1u^2\omega_2 + \\
& v^2\omega_4\omega_1u^2\omega_2^2 + 2v^2\omega_3^2c_s^2\omega_2 + 2v^4\omega_4\omega_2^2 + v^2\omega_3\omega_2^2 - \frac{1}{2}v^2\omega_3^2\omega_4\omega_2 + \frac{5}{4}v^2\omega_3\omega_1\omega_2 - \frac{1}{2}\omega_3\omega_4c_s^2\omega_2^2 + v^2\omega_3\omega_1u^2\omega_2^2 + \\
& 2v^2\omega_3c_s^2\omega_1\omega_2^2 + 3\omega_3\omega_4c_s^2\omega_1u^2\omega_2 - 4v^2\omega_3\omega_4\omega_1u^2 + 8v^2\omega_3\omega_1u^2\omega_2 - \frac{1}{2}\omega_4\omega_1^2u^2\omega_2 + \frac{7}{2}\omega_3\omega_4c_s^2\omega_2 - \\
& v^2\omega_3^2\omega_4u^2\omega_2 + \omega_3^2c_s^2\omega_2 - \frac{1}{2}\omega_4u^2\omega_2 + 2v^2\omega_3^2\omega_4u^2 + \frac{1}{2}\omega_3\omega_1^2u^2\omega_2 + 2v^2\omega_4u^2\omega_2^2 + 8v^2\omega_4\omega_1u^2\omega_2 - \frac{5}{2}v^2\omega_3\omega_2 - \\
& v^4\omega_4\omega_1\omega_2^2 - \frac{3}{4}v^2\omega_3\omega_4\omega_1\omega_2 - \frac{1}{2}v^2\omega_3\omega_1\omega_2^2 - v^2\omega_3^2u^2\omega_2 + 2v^2\omega_3\omega_4^2u^2 - \frac{1}{2}v^4\omega_3^2\omega_1\omega_2 - 2v^4\omega_3\omega_2^2, \\
\gamma_{[x^2]}^{[\mu_5]} = & \frac{3}{4}\omega_3\omega_4\omega_1\omega_2 - \frac{1}{2}\omega_3^2\omega_2 - 3\omega_3\omega_4\omega_2 + \frac{1}{4}\omega_3^2\omega_1\omega_2 + \omega_3^2\omega_4u^2\omega_2 - v^2\omega_3^2\omega_2 - \frac{1}{4}\omega_4\omega_1\omega_2 + \omega_3\omega_1^2u^2\omega_2 + \\
& v^2\omega_4\omega_1\omega_2^2 + \frac{1}{2}v^2\omega_3^2\omega_1\omega_2 + v^2\omega_4^2\omega_2 + \frac{1}{2}\omega_3\omega_4\omega_2^2 - \frac{1}{2}\omega_3^2\omega_1u^2\omega_2 - \frac{1}{2}\omega_4^2\omega_1u^2\omega_2 - \omega_3\omega_2^2 - 3\omega_3\omega_4\omega_1u^2\omega_2 + \\
& \frac{1}{5}\omega_4\omega_2 + 2v^2\omega_3\omega_2^2 + \frac{1}{2}\omega_3\omega_1\omega_2^2 + \frac{1}{2}\omega_3^2\omega_4\omega_2 - \frac{5}{4}\omega_3\omega_1\omega_2 + \omega_4\omega_1^2u^2\omega_2 + \omega_3\omega_1^2u^2\omega_2 - v^2\omega_3\omega_1\omega_2^2 - \\
& \frac{7}{2}v^2\omega_4^2\omega_1\omega_2 + \frac{1}{2}\omega_3\omega_4^2\omega_2 - 2v^2\omega_4\omega_2^2 + \frac{5}{2}\omega_3\omega_2,
\end{aligned}$$

$$\begin{aligned}
\gamma_{[xy]}^{[\mu_1]} &= -v\omega_4\omega_1u\omega_2 + v\omega_3\omega_1u\omega_2 - 2v^3\omega_3\omega_4\omega_1u - v^3\omega_3^2\omega_1u + 4v\omega_3\omega_4c_s^2u\omega_2 - v\omega_4^2\omega_1u^3 + 6v\omega_1u\omega_2 - 5v\omega_3u\omega_2 + \\
&\quad 2v\omega_3\omega_4c_s^2\omega_1u + 4v^3\omega_4u\omega_2^2 - 8v^3\omega_3\omega_4u\omega_2 - 2v^3\omega_3^2u\omega_2 + 2v\omega_3^2c_s^2\omega_1u + 2v\omega_3\omega_4u\omega_2 - 4v\omega_3c_s^2\omega_1u\omega_2 - \\
&\quad 2v\omega_1^2u\omega_2 - v^3\omega_4^2\omega_1u + 2v\omega_3^2u\omega_2 - 2v\omega_3^2\omega_4c_s^2u - 2v\omega_3\omega_1u^3\omega_2 + v\omega_3^2\omega_1u^3 + 2v\omega_4\omega_1u^3\omega_2 - 8v^3\omega_1u\omega_2^2 + \\
&\quad 2v^3\omega_3^2\omega_4u + 4v^3\omega_3u\omega_2^2 - v\omega_4u\omega_2 - 2v^3\omega_4^2u\omega_2 - v\omega_3^2\omega_1u + 6v^3\omega_4\omega_1u\omega_2 - 2v\omega_3\omega_4^2c_s^2u + 6v^3\omega_3\omega_1u\omega_2 + \\
&\quad 2v^3\omega_3\omega_4^2u - v\omega_3\omega_4\omega_1u, \\
\gamma_{[xy]}^{[\mu_5]} &= 4v\omega_4\omega_1u\omega_2 + 8v\omega_3\omega_4^2u + 4v\omega_3\omega_1u\omega_2 - 3v\omega_4^2u\omega_2 - 2v\omega_3u\omega_2^2 + 10v\omega_3\omega_1u + 8v\omega_3^2\omega_4u + 4v\omega_1u\omega_2^2 - \\
&\quad 20v\omega_1u\omega_2 - 2v\omega_4\omega_1^2u + 10v\omega_3u\omega_2 - 3v\omega_4^2\omega_1u - 6v\omega_3\omega_4u\omega_2 + 4v\omega_1^2u\omega_2 - 3v\omega_3^2u\omega_2 - 2v\omega_4u\omega_2^2 + \\
&\quad 10v\omega_4\omega_1u - 2v\omega_3\omega_1^2u + 10v\omega_4u\omega_2 - 3v\omega_3^2\omega_1u - 20v\omega_3\omega_4u - 6v\omega_3\omega_4\omega_1u, \\
\gamma_{[y^2]}^{[\mu_1]} &= \frac{1}{2}v^2\omega_3^2\omega_4\omega_1 - \frac{1}{2}\omega_3\omega_4c_s^2\omega_1^2 + \frac{1}{2}v^2\omega_3\omega_4^2\omega_1 + \frac{5}{2}v^2\omega_4\omega_1\omega_2 - \frac{1}{2}v^2\omega_3\omega_1^2\omega_2 - \frac{1}{2}v^2\omega_4\omega_1\omega_2^2 - \frac{1}{2}v^2\omega_3^2\omega_1\omega_2 + \\
&\quad \frac{5}{2}\omega_3\omega_4c_s^2\omega_1 - \frac{1}{2}\omega_3\omega_4c_s^2\omega_1\omega_2 - \frac{1}{2}\omega_3\omega_4^2c_s^2\omega_1 + \frac{5}{2}v^2\omega_3\omega_1\omega_2 + \frac{1}{2}v^2\omega_3\omega_4\omega_1^2 - \frac{5}{2}v^2\omega_3\omega_4\omega_1 - \frac{1}{2}v^2\omega_4\omega_1^2\omega_2 - \\
&\quad \frac{1}{2}v^2\omega_3\omega_4\omega_1\omega_2 - \frac{1}{2}v^2\omega_3\omega_1\omega_2^2 - \frac{1}{2}v^2\omega_4^2\omega_1\omega_2 - \frac{1}{2}\omega_3^2\omega_4c_s^2\omega_1. \\
\gamma_{[y^2]}^{[\mu_5]} &= \\
&\quad \frac{3}{4}\omega_3\omega_4\omega_1\omega_2 + \frac{1}{2}\omega_4\omega_1 + v^2\omega_3^2\omega_4\omega_1 + \frac{1}{4}\omega_3^2\omega_1\omega_2 + v^2\omega_3\omega_4^2\omega_1 + \frac{1}{2}\omega_3\omega_1^2\omega_2 - \omega_3\omega_1^2 + 2\omega_3\omega_1^2u^2 + \frac{1}{2}\omega_3\omega_1^2\omega_1 - \frac{1}{4}\omega_4\omega_1\omega_2 + \\
&\quad \frac{5}{2}\omega_3\omega_1 + v^2\omega_4\omega_1\omega_2^2 + \frac{1}{2}\omega_3^2\omega_4\omega_1 - \frac{1}{2}v^2\omega_3^2\omega_1\omega_2 - \omega_3^2\omega_1u^2 + \frac{1}{2}\omega_3^2\omega_1u^2\omega_2 - \frac{1}{2}\omega_4^2\omega_1u^2\omega_2 - \frac{1}{2}\omega_3^2\omega_1 - 3\omega_3\omega_4\omega_1 - \\
&\quad \frac{5}{4}\omega_3\omega_1\omega_2 + \omega_4\omega_1^2u^2\omega_2 - 2\omega_4\omega_1^2u^2 - \omega_3\omega_1^2u^2\omega_2 - 3v^2\omega_3\omega_4\omega_1\omega_2 + v^2\omega_3\omega_1\omega_2^2 + \frac{1}{2}\omega_3\omega_4\omega_1^2 + \omega_4^2\omega_1u^2 - \frac{1}{2}v^2\omega_4^2\omega_1\omega_2,
\end{aligned}$$