D1Q3 ADE,
a supplementary material for
Lattice Boltzmann Method Analysis Tool (LBMAT)

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Contents

1 Global definitions

In $\mathbb{R}^1$, the position and velocity vectors are given by $\boldsymbol{x} = (x_1)$ and $\boldsymbol{v} = (v_1)$, respectively.

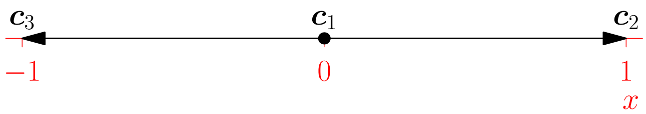
1.1 Discrete velocity vectors

Discrete velocity vectors and the lattice speed of sound are defined by

$$\{\boldsymbol{c}_i\}_{i=1}^{3} = \left((0)\,,(1)\,,(-1)\right),$$

$$c_s = \frac{1}{\sqrt{3}},$$

respectively [1].



1.2 Raw and central moments

The raw and central moments are defined by

$$m_{\boldsymbol{\alpha}} := \sum_{i=1}^{3} f_i \boldsymbol{c}_i^{\boldsymbol{\alpha}},$$

and

$$k_{\boldsymbol{\alpha}} := \sum_{i=1}^{3} f_i (\boldsymbol{c}_i - \boldsymbol{v})^{\boldsymbol{\alpha}},$$

respectively, where $\boldsymbol{\alpha} = (\alpha_1) \in \mathbb{Z}^1$ denotes a multi-index and $\boldsymbol{c}_i^{\boldsymbol{\alpha}} := [\boldsymbol{c}_i]_1^{\alpha_1}$.

1.3 Transformation matrix M

Matrix $\mathbf{M}$, that defines macroscopic quantities (moments) $\boldsymbol{\mu}$ by

$$\boldsymbol{\mu} = \mathbf{M}\boldsymbol{f},$$

with $\boldsymbol{f} = (f_1, f_2, f_3)^T$, is selected such that

$$\boldsymbol{\mu} = \left(m_{(0)}, m_{(1)}, m_{(2)}\right)^T,$$

i.e., **M** is given by

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}.$$

1.4 Equilibrium

The corresponding equilibrium raw moments are defined using the continuous Maxwell–Boltzmann distribution function [1]

$$f^{(eq)}(\xi) = \frac{\rho}{(2\pi c_s^2)^{\frac{1}{2}}} \exp\left(-\frac{(\xi - v_1)^2}{2c_s^2} \right)$$

as

$$m_{(\alpha)}^{(eq)} = \int_{\mathbb{R}} \xi^\alpha f^{(eq)}(\xi) \mathrm{d}\xi,$$

where $\alpha \in \{0, 1, 2\}$. Hence, the equilibrium moments $\boldsymbol{\mu}^{(eq)}$ satisfy

$$\boldsymbol{\mu}^{(eq)} = \Big(\rho, \rho v_1, \rho(v_1^2 + c_s^2) \Big)^T.$$

2 Spatial EPDEs

2.1 SRT

2.1.1 Definitions

Collision operator $\boldsymbol{C}$:

$$\boldsymbol{C}\left(\boldsymbol{f}\right) = \omega \left(\mathbf{M}^{-1} \boldsymbol{\mu}^{(eq)} - \boldsymbol{f} \right),$$

$\omega \in (0, 2)$.

2.1.2 Conservation of mass equation

$$\frac{\partial \rho}{\partial t} + \frac{\delta_l v_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + (-2+\omega) \frac{\delta_l}{2\omega} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial t} + (-2+\omega) \frac{\delta_l^2 v_1}{2\omega \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + (-2+\omega) \frac{\rho \delta_l^2}{2\omega \delta_t} \left(\frac{\partial v_1}{\partial x_1} \right)^2 +$$
$$(-2+\omega) \frac{\rho \delta_l}{2\omega} \frac{\partial^2 v_1}{\partial t \partial x_1} + (-2+\omega) \frac{c_s^2 \delta_l^2}{2\omega \delta_t} \frac{\partial^2 \rho}{\partial x_1^2} + (-2+\omega) \frac{\rho \delta_l^2 v_1}{2\omega \delta_t} \frac{\partial^2 v_1}{\partial x_1^2} + (12 - 12\omega + \omega^2) \frac{\rho \delta_l \delta_t}{12\omega^2} \frac{\partial^3 v_1}{\partial t^2 \partial x_1} +$$
$$(12 - 12\omega + \omega^2) \frac{\rho \delta_l^2 v_1}{6\omega^2} \frac{\partial^3 v_1}{\partial t \partial x_1^2} + (6 - 18c_s^2 - 6\omega + 6\omega v_1^2 - \omega^2 v_1^2 + 18c_s^2\omega - 6v_1^2 + \omega^2 - 3c_s^2\omega^2) \frac{\delta_l^3 v_1}{6\omega^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3} +$$
$$(12 - 24c_s^2 - 12\omega + 24\omega v_1^2 - 5\omega^2 v_1^2 + 24c_s^2\omega - 24v_1^2 + 2\omega^2 - 3c_s^2\omega^2) \frac{\rho \delta_l^3}{12\omega^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + (-2 + 3\omega - \omega^2) \frac{\rho \delta_l \delta_t^2}{2\omega^3} \frac{\partial^4 v_1}{\partial t^3 \partial x_1} +$$
$$(-2 + 3\omega - \omega^2) \frac{3\rho \delta_l^2 v_1 \delta_t}{2\omega^3} \frac{\partial^4 v_1}{\partial t^2 \partial x_1^2} + C_1 \frac{\rho \delta_l^3}{12\omega^3} \frac{\partial^4 v_1}{\partial t \partial x_1^3} + C_2 \frac{\delta_l^4}{24\omega^3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + C_3 \frac{\rho \delta_l^4 v_1}{12\omega^3 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} = 0,$$

where:

$$C_1 = -36 + 60c_s^2 + 54\omega - 108\omega v_1^2 + 42\omega^2 v_1^2 + \omega^3 - 90c_s^2\omega + 72v_1^2 - 20\omega^2 - 2c_s^2\omega^3 - 3\omega^3 v_1^2 + 34c_s^2\omega^2$$

$$C_2 = -84c_s^2\omega^2 v_1^2 - 24c_s^2 + 3\omega^3 v_1^4 - 72c_s^4\omega + 30c_s^4\omega^2 - 42\omega^2 v_1^4 + 6c_s^2\omega^3 v_1^2 - 3c_s^4\omega^3 - 108\omega v_1^2 - 72v_1^4 + 42\omega^2 v_1^2 + 36c_s^2\omega + 72v_1^2 + 108\omega v_1^4 + c_s^2\omega^3 + 48c_s^4 - 3\omega^3 v_1^2 + 216c_s^2\omega v_1^2 - 14c_s^2\omega^2 - 144c_s^2 v_1^2$$

$$C_3 = 24 - 48c_s^2 - 36\omega + 54\omega v_1^2 - 22\omega^2 v_1^2 - \omega^3 + 72c_s^2\omega - 36v_1^2 + 14\omega^2 + c_s^2\omega^3 + 2\omega^3 v_1^2 - 26c_s^2\omega^2$$

2.2 MRT

2.2.1 Definitions

Collision operator $\boldsymbol{C}$:

$$\boldsymbol{C}\left(\boldsymbol{f}\right) = \mathbf{M}^{-1}\mathbf{S}\left(\boldsymbol{\mu}^{(eq)} - \mathbf{M}\boldsymbol{f}\right),$$

where

$$\mathbf{S} = \mathrm{diag}(\omega_1, \omega_2, \omega_3),$$

$\omega_1, \omega_2, \omega_3 \in (0, 2)$.

2.2.2 Conservation of mass equation

$$\frac{\partial\rho}{\partial t} + \frac{v_1\delta_l}{\delta_t}\frac{\partial\rho}{\partial x_1} + \frac{\rho\delta_l}{\delta_t}\frac{\partial v_1}{\partial x_1} + (-2+\omega_2)\frac{\delta_l}{2\omega_2}\frac{\partial\rho}{\partial x_1}\frac{\partial v_1}{\partial t} + (-2+\omega_2)\frac{v_1\delta_l^2}{2\delta_t\omega_2}\frac{\partial\rho}{\partial x_1}\frac{\partial v_1}{\partial x_1} + (-2+\omega_2)\frac{\rho\delta_l^2}{2\delta_t\omega_2}\left(\frac{\partial v_1}{\partial x_1}\right)^2 + (-2+\omega_2)\frac{\rho\delta_l}{2\omega_2}\frac{\partial^2 v_1}{\partial t\partial x_1} + (-2+\omega_2)\frac{c_s^2\delta_l^2}{2\delta_t\omega_2}\frac{\partial^2\rho}{\partial x_1^2} + (-2+\omega_2)\frac{v_1\rho\delta_l^2}{2\delta_t\omega_2}\frac{\partial^2 v_1}{\partial x_1^2} + (12+\omega_2^2-12\omega_2)\frac{\delta_t\rho\delta_l}{12\omega_2^2}\frac{\partial^3 v_1}{\partial t^2\partial x_1} + (12-6\omega_3+\omega_3\omega_2-6\omega_2)\frac{v_1\rho\delta_l^2}{6\omega_3\omega_2}\frac{\partial^3 v_1}{\partial t\partial x_1^2} + C_1\frac{v_1\delta_l^3}{6\omega_3\delta_t\omega_2^2}\frac{\partial^3\rho}{\partial x_1^3} + C_2\frac{\rho\delta_l^3}{12\omega_3\delta_t\omega_2^2}\frac{\partial^3 v_1}{\partial x_1^3} + (-2-\omega_2^2+3\omega_2)\frac{\delta_t^2\rho\delta_l}{2\omega_2^3}\frac{\partial^4 v_1}{\partial t^3\partial x_1} + (-\omega_3^2\omega_2^2-\omega_3^2\omega_2-4\omega_2^2-4\omega_3\omega_2+2\omega_2^3-2\omega_3\omega_2^3+8\omega_3\omega_2^2+2\omega_3^2)\frac{\delta_t v_1\rho\delta_l^2}{2\omega_3^2\omega_2^3}\frac{\partial^4 v_1}{\partial t^2\partial x_1^2} + C_3\frac{\rho\delta_l^3}{12\omega_3^2\omega_2^3}\frac{\partial^4 v_1}{\partial t\partial x_1^3} + C_4\frac{\delta_l^4}{24\omega_3^2\delta_t\omega_2^3}\frac{\partial^4\rho}{\partial x_1^4} + C_5\frac{v_1\rho\delta_l^4}{12\omega_3^2\delta_t\omega_2^3}\frac{\partial^4 v_1}{\partial x_1^4} = 0,$$

where:

$$C_1 = 15\omega_3 c_s^2\omega_2 - 6c_s^2\omega_2 + 3v_1^2\omega_2^2 - \omega_3 v_1^2\omega_2^2 - 12\omega_3 c_s^2 - 6v_1^2\omega_2 + 3\omega_3 v_1^2\omega_2 - 3\omega_3 c_s^2\omega_2^2 + 3c_s^2\omega_2^2 - 3\omega_2^2 - 3\omega_3\omega_2 + 6\omega_2 + \omega_3\omega_2^2$$

$$C_2 = 18\omega_3 c_s^2\omega_2 - 12\omega_3 v_1^2 - 12c_s^2\omega_2 + 6v_1^2\omega_2^2 - 5\omega_3 v_1^2\omega_2^2 - 12\omega_3 c_s^2 - 12v_1^2\omega_2 + 18\omega_3 v_1^2\omega_2 - 3\omega_3 c_s^2\omega_2^2 + 6c_s^2\omega_2^2 - 6\omega_2^2 - 6\omega_3\omega_2 + 12\omega_2 + 2\omega_3\omega_2^2$$

$$C_3 = 24\omega_3 c_s^2\omega_2 + 15\omega_3 v_1^2\omega_2^3 - 6v_1^2\omega_2^3 - 11\omega_3^2\omega_2^2 + \omega_3^2\omega_2^3 + 12v_1^2\omega_2^2 - 60\omega_3 v_1^2\omega_2^2 - 6c_s^2\omega_2^3 + 48\omega_3 v_1^2\omega_2 + 9\omega_3 c_s^2\omega_2^3 - 36\omega_3 c_s^2\omega_2^2 + 12\omega_3^2\omega_2 + 12c_s^2\omega_2^2 - 12\omega_2^2 - 2\omega_3^2 c_s^2\omega_2^3 - 42\omega_3^2 v_1^2\omega_2 - 24\omega_3\omega_2 + 6\omega_2^3 + 25\omega_3^2 c_s^2\omega_2^2 - 9\omega_3\omega_2^3 - 3\omega_3^2 v_1^2\omega_2^3 + 24\omega_3^2 c_s^2 - 48\omega_3^2 c_s^2\omega_2 + 12\omega_3^2 v_1^2 + 27\omega_3^2 v_1^2\omega_2^2 + 36\omega_3\omega_2^2$$

$$C_4 = -24\omega_3 c_s^2\omega_2 + 18\omega_3 v_1^2\omega_2^3 - 12v_1^2\omega_2^3 + 24\omega_3^2 c_s^4 + 156\omega_3^2 v_1^2 c_s^2\omega_2 + 24\omega_3^2 c_s^4\omega_2^2 - 96\omega_3^2 v_1^2 c_s^2 + 24v_1^2\omega_2^2 - 72\omega_3 v_1^2\omega_2^2 - 3\omega_3^2 c_s^4\omega_2^3 + 24\omega_3^2 v_1^4\omega_2 + 6\omega_3^2 v_1^2 c_s^2\omega_2^3 - 24\omega_3^2 v_1^4\omega_2^2 + 48\omega_3 v_1^2\omega_2 - 6\omega_3 c_s^2\omega_2^3 - 72\omega_3^2 v_1^2 c_s^2\omega_2^2 + 3\omega_3^2 v_1^4\omega_2^3 - 48\omega_3^2 c_s^4\omega_2 + 24\omega_3 c_s^2\omega_2^2 - 24\omega_3 v_1^2 c_s^2\omega_2 - 24v_1^4\omega_2^2 + 72\omega_3 v_1^4\omega_2^2 + \omega_3^2 c_s^2\omega_2^3 - 24\omega_3^2 v_1^2\omega_2 + 24\omega_3 c_s^4\omega_2 - 18\omega_3 v_1^4\omega_2^3 + 12v_1^4\omega_2^3 - 8\omega_3^2 c_s^2\omega_2^2 + 12v_1^2 c_s^2\omega_2^3 - 3\omega_3^2 v_1^2\omega_2^3 + 12\omega_3^2 c_s^2\omega_2 - 24\omega_3 c_s^4\omega_2^2 - 12\omega_3 v_1^2 c_s^2\omega_2^3 + 24\omega_3^2 v_1^2\omega_2^2 - 24v_1^2 c_s^2\omega_2^2 + 48\omega_3 v_1^2 c_s^2\omega_2^2 - 48\omega_3 v_1^4\omega_2 + 6\omega_3 c_s^4\omega_2^3$$

$$C_5 = -12\omega_3 c_s^2\omega_2 - 6\omega_3 v_1^2\omega_2^3 + 6v_1^2\omega_2^3 + 8\omega_3^2\omega_2^2 - \omega_3^2\omega_2^3 - 12v_1^2\omega_2^2 + 24\omega_3 v_1^2\omega_2^2 + 6c_s^2\omega_2^3 - 12\omega_3 v_1^2\omega_2 - 6\omega_3 c_s^2\omega_2^3 + 24\omega_3 c_s^2\omega_2^2 - 6\omega_3^2\omega_2 - 12c_s^2\omega_2^2 + 12\omega_2^2 + \omega_3^2 c_s^2\omega_2^3 + 24\omega_3^2 v_1^2\omega_2 + 12\omega_3\omega_2 - 6\omega_2^3 - 20\omega_3^2 c_s^2\omega_2^2 + 6\omega_3\omega_2^3 + 2\omega_3^2 v_1^2\omega_2^3 - 24\omega_3^2 c_s^2 + 42\omega_3^2 c_s^2\omega_2 - 12\omega_3^2 v_1^2 - 16\omega_3^2 v_1^2\omega_2^2 - 24\omega_3\omega_2^2$$

2.3 CLBM

2.3.1 Definitions

Collision operator $\boldsymbol{C}$:

$$\boldsymbol{C}\left(\boldsymbol{f}\right) = \mathbf{K}^{-1}\mathbf{S}\left(\boldsymbol{\kappa}^{(eq)} - \mathbf{K}\boldsymbol{f}\right),$$

where

$$\mathbf{S} = \mathrm{diag}(\omega_1, \omega_2, \omega_3),$$

$\omega_1, \omega_2, \omega_3 \in (0, 2)$.
Matrix $\mathbf{K}$ corresponds to the transformation matrix to the central moment basis defined by

$$\boldsymbol{\kappa} = \left(k_{(0)}, k_{(1)}, k_{(2)}\right)^T$$

and is given by

$$\mathbf{K} = \begin{pmatrix} 1 & 1 & 1 \\ -v_1 & 1-v_1 & -v_1-1 \\ v_1^2 & (1-v_1)^2 & (v_1+1)^2 \end{pmatrix}.$$

The equilibrium central moments are defined by

$$\boldsymbol{\kappa}^{(eq)} = \mathbf{K}\mathbf{M}^{-1}\boldsymbol{\mu}^{(eq)},$$

i.e.,

$$\boldsymbol{\kappa}^{(eq)} = \left(\rho, 0, \rho c_s^2\right)^T.$$

2.3.2 Conservation of mass equation

$$\frac{\partial \rho}{\partial t} + \frac{\delta_l v_1}{\delta_t}\frac{\partial \rho}{\partial x_1} + \frac{\rho\delta_l}{\delta_t}\frac{\partial v_1}{\partial x_1} + (-2+\omega_2)\frac{\delta_l}{2\omega_2}\frac{\partial \rho}{\partial x_1}\frac{\partial v_1}{\partial t} + (-2+\omega_2)\frac{\delta_l^2 v_1}{2\omega_2\delta_t}\frac{\partial \rho}{\partial x_1}\frac{\partial v_1}{\partial x_1} + (-2+\omega_2)\frac{\rho\delta_l^2}{2\omega_2\delta_t}\left(\frac{\partial v_1}{\partial x_1}\right)^2 + (-2+\omega_2)\frac{\rho\delta_l}{2\omega_2}\frac{\partial^2 v_1}{\partial t\partial x_1} + (-2+\omega_2)\frac{\delta_l^2 c_s^2}{2\omega_2\delta_t}\frac{\partial^2 \rho}{\partial x_1^2} + (-2+\omega_2)\frac{\rho\delta_l^2 v_1}{2\omega_2\delta_t}\frac{\partial^2 v_1}{\partial x_1^2} + (12-12\omega_2+\omega_2^2)\frac{\rho\delta_l\delta_t}{12\omega_2^2}\frac{\partial^3 v_1}{\partial t^2\partial x_1} + (12-12\omega_2+\omega_2^2)\frac{\rho\delta_l^2 v_1}{6\omega_2^2}\frac{\partial^3 v_1}{\partial t\partial x_1^2} + C_1\frac{\delta_l^3 v_1}{6\omega_2\omega_3\delta_t}\frac{\partial^3 \rho}{\partial x_1^3} + C_2\frac{\rho\delta_l^3}{12\omega_2^2\omega_3\delta_t}\frac{\partial^3 v_1}{\partial x_1^3} + (-2+3\omega_2-\omega_2^2)\frac{\rho\delta_l\delta_t^2}{2\omega_2^3}\frac{\partial^4 v_1}{\partial t^3\partial x_1} + (-2+3\omega_2-\omega_2^2)\frac{3\rho\delta_l^2\delta_t v_1}{2\omega_2^3}\frac{\partial^4 v_1}{\partial t^2\partial x_1^2} + C_3\frac{\rho\delta_l^3}{12\omega_2^3\omega_3^2}\frac{\partial^4 v_1}{\partial t\partial x_1^3} + C_4\frac{\delta_l^4}{24\omega_2^3\omega_3^2\delta_t}\frac{\partial^4 \rho}{\partial x_1^4} + C_5\frac{\rho\delta_l^4 v_1}{12\omega_2^3\omega_3^2\delta_t}\frac{\partial^4 v_1}{\partial x_1^4} = 0,$$

where:

$C_1 = 6 - 6v_1^2 - 3\omega_2 - 18c_s^2 + 3\omega_3 v_1^2 + 9\omega_3 c_s^2 - 3\omega_3 + 3\omega_2 v_1^2 + \omega_2\omega_3 + 9\omega_2 c_s^2 - 3\omega_2\omega_3 c_s^2 - \omega_2\omega_3 v_1^2$

$C_2 = -3\omega_2^2\omega_3 c_s^2 + 12\omega_2 - 5\omega_2^2\omega_3 v_1^2 + 12\omega_3 v_1^2 + 6\omega_2^2 c_s^2 + 18\omega_2^2 v_1^2 - 12\omega_3 c_s^2 + 2\omega_2^2\omega_3 - 36\omega_2 v_1^2 - 6\omega_2\omega_3 - 6\omega_2^2 - 12\omega_2 c_s^2 + 18\omega_2\omega_3 c_s^2 + 6\omega_2\omega_3 v_1^2$

$C_3 = -36\omega_2^2\omega_3 c_s^2 + 18\omega_2\omega_3^2 v_1^2 - 6\omega_2^3 c_s^2 - 11\omega_2^2\omega_3^2 - 108\omega_2^2\omega_3 v_1^2 - 48\omega_2\omega_3^2 c_s^2 - 9\omega_2^3\omega_3 - 18\omega_2^3 v_1^2 + \omega_2^3\omega_3^2 + 12\omega_2^2 c_s^2 - 2\omega_2^3\omega_3^2 c_s^2 + 36\omega_2^2 v_1^2 + 36\omega_2^2\omega_3 - 3\omega_2^3\omega_3^2 v_1^2 + 24\omega_3^2 c_s^2 - 24\omega_2\omega_3 - 12\omega_2^2 + 27\omega_2^3\omega_3 v_1^2 - 36\omega_3^2 v_1^2 + 9\omega_2^3\omega_3 c_s^2 + 6\omega_2^3 + 15\omega_2^2\omega_3^2 v_1^2 + 24\omega_2\omega_3 c_s^2 + 25\omega_2^2\omega_3^2 c_s^2 + 72\omega_2\omega_3 v_1^2 + 12\omega_2\omega_3^2$

$C_4 = 24\omega_2^2\omega_3 c_s^2 - 12\omega_2^2\omega_3^2 v_1^4 + 24\omega_2\omega_3 c_s^4 - 72\omega_2^2\omega_3 v_1^2 + 12\omega_2\omega_3^2 c_s^2 + 24\omega_2^2\omega_3^2 c_s^4 - 36\omega_2^3 v_1^2 - 12\omega_2^2\omega_3^2 c_s^2 v_1^2 - 30\omega_2^3\omega_3 v_1^4 + 6\omega_2^3\omega_3^2 c_s^2 v_1^2 + 24\omega_3^2 c_s^4 + \omega_2^3\omega_3^2 c_s^2 + 72\omega_2^2 v_1^2 + 6\omega_2^3\omega_3 c_s^4 + 72\omega_2\omega_3 c_s^2 v_1^2 - 3\omega_2^3\omega_3^2 v_1^2 - 216\omega_2^2 c_s^2 v_1^2 - 3\omega_2^3\omega_3^2 c_s^4 + 30\omega_2^3\omega_3 v_1^2 - 72\omega_2^3\omega_3 c_s^2 v_1^2 + 3\omega_2^3\omega_3^2 v_1^4 - 36\omega_2\omega_3^2 c_s^2 v_1^2 - 6\omega_2^3\omega_3 c_s^2 - 72\omega_2^2 v_1^4 + 108\omega_2^3 c_s^2 v_1^2 + 12\omega_2^2\omega_3^2 v_1^2 - 24\omega_2\omega_3 c_s^2 - 24\omega_2^2\omega_3 c_s^4 - 8\omega_2^2\omega_3^2 c_s^2 + 36\omega_2^3 v_1^4 + 72\omega_2^2\omega_3 v_1^4 + 144\omega_2^2\omega_3 c_s^2 v_1^2 - 48\omega_2\omega_3^2 c_s^4$

$C_5 = 72\omega_2^2\omega_3 c_s^2 - 12\omega_2\omega_3^2 v_1^2 + 30\omega_2^3 c_s^2 + 2\omega_2^2\omega_3^2 + 24\omega_2^2\omega_3 v_1^2 - 30\omega_2\omega_3^2 c_s^2 + 12\omega_2^3\omega_3 + 42\omega_2^3 v_1^2 - \omega_2^3\omega_3^2 - 60\omega_2^2 c_s^2 + \omega_2^3\omega_3^2 c_s^2 - 84\omega_2^2 v_1^2 - 24\omega_2^2\omega_3 + 2\omega_2^3\omega_3^2 v_1^2 + 24\omega_3^2 c_s^2 - 12\omega_2\omega_3 + 36\omega_2^2 - 24\omega_2^3\omega_3 v_1^2 - 12\omega_3^2 v_1^2 - 24\omega_2^3\omega_3 c_s^2 - 18\omega_2^3 + 2\omega_2^2\omega_3^2 v_1^2 - 12\omega_2\omega_3 c_s^2 - 2\omega_2^2\omega_3^2 c_s^2 + 60\omega_2\omega_3 v_1^2 + 6\omega_2\omega_3^2$

3 Comparison of SRT, MRT, and CLBM

3.1 Conservation of mass equation

$$\frac{\partial \rho}{\partial t} + v_1\frac{\delta_l}{\delta_t}\frac{\partial \rho}{\partial x_1} + \rho\frac{\delta_l}{\delta_t}\frac{\partial v_1}{\partial x_1} + C^{(0)}_{\mathrm{D}_x\rho,\mathrm{D}_t v_1}\delta_l\frac{\partial \rho}{\partial x_1}\frac{\partial v_1}{\partial t} + C^{(0)}_{\mathrm{D}_x\rho,\mathrm{D}_x v_1}\frac{\delta_l^2}{\delta_t}\frac{\partial \rho}{\partial x_1}\frac{\partial v_1}{\partial x_1} + C^{(0)}_{\mathrm{D}_x v_1,\mathrm{D}_x v_1}\frac{\delta_l^2}{\delta_t}\left(\frac{\partial v_1}{\partial x_1}\right)^2 + C^{(0)}_{\mathrm{D}_t\mathrm{D}_x v_1}\delta_l\frac{\partial^2 v_1}{\partial t\partial x_1} + C^{(0)}_{\mathrm{D}_x^2\rho}\frac{\delta_l^2}{\delta_t}\frac{\partial^2 \rho}{\partial x_1^2} + C^{(0)}_{\mathrm{D}_x^2 v_1}\frac{\delta_l^2}{\delta_t}\frac{\partial^2 v_1}{\partial x_1^2} + C^{(0)}_{\mathrm{D}_t^2\mathrm{D}_x v_1}\delta_l\delta_t\frac{\partial^3 v_1}{\partial t^2\partial x_1} + C^{(0)}_{\mathrm{D}_t\mathrm{D}_x^2 v_1}\delta_l^2\frac{\partial^3 v_1}{\partial t\partial x_1^2} + C^{(0)}_{\mathrm{D}_x^3\rho}\frac{\delta_l^3}{\delta_t}\frac{\partial^3 \rho}{\partial x_1^3} + C^{(0)}_{\mathrm{D}_x^3 v_1}\frac{\delta_l^3}{\delta_t}\frac{\partial^3 v_1}{\partial x_1^3} + C^{(0)}_{\mathrm{D}_t^3\mathrm{D}_x v_1}\delta_l\delta_t^2\frac{\partial^4 v_1}{\partial t^3\partial x_1} + C^{(0)}_{\mathrm{D}_t^2\mathrm{D}_x^2 v_1}\delta_l^2\delta_t\frac{\partial^4 v_1}{\partial t^2\partial x_1^2} + C^{(0)}_{\mathrm{D}_t\mathrm{D}_x^3 v_1}\delta_l^3\frac{\partial^4 v_1}{\partial t\partial x_1^3} + C^{(0)}_{\mathrm{D}_x^4\rho}\frac{\delta_l^4}{\delta_t}\frac{\partial^4 \rho}{\partial x_1^4} + C^{(0)}_{\mathrm{D}_x^4 v_1}\frac{\delta_l^4}{\delta_t}\frac{\partial^4 v_1}{\partial x_1^4} = 0,$$

where:

coefficient $C^{(0)}_{\mathrm{D}_x\rho,\mathrm{D}_t v_1}$ at $\frac{\partial \rho}{\partial x_1}\frac{\partial v_1}{\partial t}$:

$C^{(0),\mathrm{SRT}}_{\mathrm{D}_x\rho,\mathrm{D}_t v_1} = (-2+\omega)\frac{1}{2\omega}$

$C^{(0),\mathrm{MRT1}}_{\mathrm{D}_x\rho,\mathrm{D}_t v_1} = (-2+\omega_2)\,\frac{1}{2\omega_2}$

$C^{(0),\mathrm{CLBM1}}_{\mathrm{D}_x\rho,\mathrm{D}_t v_1} = C^{(0),\mathrm{MRT1}}_{\mathrm{D}_x\rho,\mathrm{D}_t v_1}$

coefficient $C^{(0)}_{\mathrm{D}_x\rho,\mathrm{D}_x v_1}$ **at** $\frac{\partial\rho}{\partial x_1}\frac{\partial v_1}{\partial x_1}$**:**

$C^{(0),\mathrm{SRT}}_{\mathrm{D}_x\rho,\mathrm{D}_x v_1} = (-2+\omega)\,\frac{v_1}{2\omega}$

$C^{(0),\mathrm{MRT1}}_{\mathrm{D}_x\rho,\mathrm{D}_x v_1} = (-2+\omega_2)\,\frac{v_1}{2\omega_2}$

$C^{(0),\mathrm{CLBM1}}_{\mathrm{D}_x\rho,\mathrm{D}_x v_1} = C^{(0),\mathrm{MRT1}}_{\mathrm{D}_x\rho,\mathrm{D}_x v_1}$

coefficient $C^{(0)}_{\mathrm{D}_x v_1,\mathrm{D}_x v_1}$ **at** $\left(\frac{\partial v_1}{\partial x_1}\right)^2$**:**

$C^{(0),\mathrm{SRT}}_{\mathrm{D}_x v_1,\mathrm{D}_x v_1} = (-2+\omega)\,\frac{\rho}{2\omega}$

$C^{(0),\mathrm{MRT1}}_{\mathrm{D}_x v_1,\mathrm{D}_x v_1} = (-2+\omega_2)\,\frac{\rho}{2\omega_2}$

$C^{(0),\mathrm{CLBM1}}_{\mathrm{D}_x v_1,\mathrm{D}_x v_1} = C^{(0),\mathrm{MRT1}}_{\mathrm{D}_x v_1,\mathrm{D}_x v_1}$

coefficient $C^{(0)}_{\mathrm{D}_t\mathrm{D}_x v_1}$ **at** $\frac{\partial^2 v_1}{\partial t\partial x_1}$**:**

$C^{(0),\mathrm{SRT}}_{\mathrm{D}_t\mathrm{D}_x v_1} = (-2+\omega)\,\frac{\rho}{2\omega}$

$C^{(0),\mathrm{MRT1}}_{\mathrm{D}_t\mathrm{D}_x v_1} = (-2+\omega_2)\,\frac{\rho}{2\omega_2}$

$C^{(0),\mathrm{CLBM1}}_{\mathrm{D}_t\mathrm{D}_x v_1} = C^{(0),\mathrm{MRT1}}_{\mathrm{D}_t\mathrm{D}_x v_1}$

coefficient $C^{(0)}_{\mathrm{D}_x^2\rho}$ **at** $\frac{\partial^2\rho}{\partial x_1^2}$**:**

$C^{(0),\mathrm{SRT}}_{\mathrm{D}_x^2\rho} = (-2+\omega)\,\frac{c_s^2}{2\omega}$

$C^{(0),\mathrm{MRT1}}_{\mathrm{D}_x^2\rho} = (-2+\omega_2)\,\frac{c_s^2}{2\omega_2}$

$C^{(0),\mathrm{CLBM1}}_{\mathrm{D}_x^2\rho} = C^{(0),\mathrm{MRT1}}_{\mathrm{D}_x^2\rho}$

coefficient $C^{(0)}_{\mathrm{D}_x^2 v_1}$ **at** $\frac{\partial^2 v_1}{\partial x_1^2}$**:**

$C^{(0),\mathrm{SRT}}_{\mathrm{D}_x^2 v_1} = (-2+\omega)\,\frac{\rho v_1}{2\omega}$

$C^{(0),\mathrm{MRT1}}_{\mathrm{D}_x^2 v_1} = (-2+\omega_2)\,\frac{\rho v_1}{2\omega_2}$

$C^{(0),\mathrm{CLBM1}}_{\mathrm{D}_x^2 v_1} = C^{(0),\mathrm{MRT1}}_{\mathrm{D}_x^2 v_1}$

coefficient $C^{(0)}_{\mathrm{D}_t^2\mathrm{D}_x v_1}$ **at** $\frac{\partial^3 v_1}{\partial t^2\partial x_1}$**:**

$C^{(0),\mathrm{SRT}}_{\mathrm{D}_t^2\mathrm{D}_x v_1} = (12-12\omega+\omega^2)\,\frac{\rho}{12\omega^2}$

$C^{(0),\mathrm{MRT1}}_{\mathrm{D}_t^2\mathrm{D}_x v_1} = (12+\omega_2^2-12\omega_2)\,\frac{\rho}{12\omega_2^2}$

$C^{(0),\mathrm{CLBM1}}_{\mathrm{D}_t^2\mathrm{D}_x v_1} = C^{(0),\mathrm{MRT1}}_{\mathrm{D}_t^2\mathrm{D}_x v_1}$

coefficient $C^{(0)}_{\mathrm{D}_t\mathrm{D}_x^2 v_1}$ **at** $\frac{\partial^3 v_1}{\partial t\partial x_1^2}$**:**

$C^{(0),\mathrm{SRT}}_{\mathrm{D}_t\mathrm{D}_x^2 v_1} = (12-12\omega+\omega^2)\,\frac{\rho v_1}{6\omega^2}$

$C^{(0),\mathrm{MRT1}}_{\mathrm{D}_t\mathrm{D}_x^2 v_1} = (12 - 6\omega_2 - 6\omega_3 + \omega_2\omega_3)\frac{\rho v_1}{6\omega_2\omega_3}$

$C^{(0),\mathrm{CLBM1}}_{\mathrm{D}_t\mathrm{D}_x^2 v_1} = (12 + \omega_2^2 - 12\omega_2)\frac{\rho v_1}{6\omega_2^2}$

coefficient $C^{(0)}_{\mathrm{D}_x^3 \rho}$ **at** $\frac{\partial^3 \rho}{\partial x_1^3}$**:**

$C^{(0),\mathrm{SRT}}_{\mathrm{D}_x^3 \rho} = (6 - 6v_1^2 - 6\omega - 3\omega^2 c_s^2 + \omega^2 + 6\omega v_1^2 - \omega^2 v_1^2 - 18c_s^2 + 18\omega c_s^2)\frac{v_1}{6\omega^2}$

$C^{(0),\mathrm{MRT1}}_{\mathrm{D}_x^3 \rho} = (\omega_2^2\omega_3 + 3\omega_2^2 v_1^2 - 3\omega_2^2 - \omega_2^2\omega_3 v_1^2 + 6\omega_2 - 6\omega_2 c_s^2 + 15\omega_2\omega_3 c_s^2 - 12\omega_3 c_s^2 - 3\omega_2^2\omega_3 c_s^2 + 3\omega_2^2 c_s^2 + 3\omega_2\omega_3 v_1^2 - 6\omega_2 v_1^2 - 3\omega_2\omega_3)\frac{v_1}{6\omega_2^2\omega_3}$

$C^{(0),\mathrm{CLBM1}}_{\mathrm{D}_x^3 \rho} = (6 - 6v_1^2 + 3\omega_3 v_1^2 - 3\omega_2 + 9\omega_2 c_s^2 - 3\omega_2\omega_3 c_s^2 - 3\omega_3 + 9\omega_3 c_s^2 - 18c_s^2 - \omega_2\omega_3 v_1^2 + 3\omega_2 v_1^2 + \omega_2\omega_3)\frac{v_1}{6\omega_2\omega_3}$

coefficient $C^{(0)}_{\mathrm{D}_x^3 v_1}$ **at** $\frac{\partial^3 v_1}{\partial x_1^3}$**:**

$C^{(0),\mathrm{SRT}}_{\mathrm{D}_x^3 v_1} = (12 - 24v_1^2 - 12\omega - 3\omega^2 c_s^2 + 2\omega^2 + 24\omega v_1^2 - 5\omega^2 v_1^2 - 24c_s^2 + 24\omega c_s^2)\frac{\rho}{12\omega^2}$

$C^{(0),\mathrm{MRT1}}_{\mathrm{D}_x^3 v_1} =$
$(2\omega_2^2\omega_3 + 6\omega_2^2 v_1^2 - 6\omega_2^2 - 5\omega_2^2\omega_3 v_1^2 - 12\omega_3 v_1^2 + 12\omega_2 - 12\omega_2 c_s^2 + 18\omega_2\omega_3 c_s^2 - 12\omega_3 c_s^2 - 3\omega_2^2\omega_3 c_s^2 + 6\omega_2^2 c_s^2 + 18\omega_2\omega_3 v_1^2 - 12\omega_2 v_1^2 - 6\omega_2\omega_3)\frac{\rho}{12\omega_2^2\omega_3}$

$C^{(0),\mathrm{CLBM1}}_{\mathrm{D}_x^3 v_1} =$
$(2\omega_2^2\omega_3 + 18\omega_2^2 v_1^2 - 6\omega_2^2 - 5\omega_2^2\omega_3 v_1^2 + 12\omega_3 v_1^2 + 12\omega_2 - 12\omega_2 c_s^2 + 18\omega_2\omega_3 c_s^2 - 12\omega_3 c_s^2 - 3\omega_2^2\omega_3 c_s^2 + 6\omega_2^2 c_s^2 + 6\omega_2\omega_3 v_1^2 - 36\omega_2 v_1^2 - 6\omega_2\omega_3)\frac{\rho}{12\omega_2^2\omega_3}$

coefficient $C^{(0)}_{\mathrm{D}_t^3\mathrm{D}_x v_1}$ **at** $\frac{\partial^4 v_1}{\partial t^3 \partial x_1}$**:**

$C^{(0),\mathrm{SRT}}_{\mathrm{D}_t^3\mathrm{D}_x v_1} = (-2 + 3\omega - \omega^2)\frac{\rho}{2\omega^3}$

$C^{(0),\mathrm{MRT1}}_{\mathrm{D}_t^3\mathrm{D}_x v_1} = (-2 - \omega_2^2 + 3\omega_2)\frac{\rho}{2\omega_2^3}$

$C^{(0),\mathrm{CLBM1}}_{\mathrm{D}_t^3\mathrm{D}_x v_1} = C^{(0),\mathrm{MRT1}}_{\mathrm{D}_t^3\mathrm{D}_x v_1}$

coefficient $C^{(0)}_{\mathrm{D}_t^2\mathrm{D}_x^2 v_1}$ **at** $\frac{\partial^4 v_1}{\partial t^2 \partial x_1^2}$**:**

$C^{(0),\mathrm{SRT}}_{\mathrm{D}_t^2\mathrm{D}_x^2 v_1} = (-2 + 3\omega - \omega^2)\frac{3\rho v_1}{2\omega^3}$

$C^{(0),\mathrm{MRT1}}_{\mathrm{D}_t^2\mathrm{D}_x^2 v_1} = (8\omega_2^2\omega_3 - 4\omega_2^2 + 2\omega_2^3 - 2\omega_2^3\omega_3 - \omega_2^2\omega_3^2 + 2\omega_3^2 - \omega_2\omega_3^2 - 4\omega_2\omega_3)\frac{\rho v_1}{2\omega_2^3\omega_3^2}$

$C^{(0),\mathrm{CLBM1}}_{\mathrm{D}_t^2\mathrm{D}_x^2 v_1} = (-2 - \omega_2^2 + 3\omega_2)\frac{3\rho v_1}{2\omega_2^3}$

coefficient $C^{(0)}_{\mathrm{D}_t\mathrm{D}_x^3 v_1}$ **at** $\frac{\partial^4 v_1}{\partial t \partial x_1^3}$**:**

$C^{(0),\mathrm{SRT}}_{\mathrm{D}_t\mathrm{D}_x^3 v_1} = (-36 + 72v_1^2 + 54\omega + 34\omega^2 c_s^2 - 20\omega^2 - 2\omega^3 c_s^2 - 108\omega v_1^2 + \omega^3 + 42\omega^2 v_1^2 + 60c_s^2 - 3\omega^3 v_1^2 - 90\omega c_s^2)\frac{\rho}{12\omega^3}$

$C^{(0),\mathrm{MRT1}}_{\mathrm{D}_t\mathrm{D}_x^3 v_1} =$
$(9\omega_2^3\omega_3 c_s^2 + 36\omega_2^2\omega_3 + 12\omega_2^2 v_1^2 - 12\omega_2^2 - 60\omega_2^2\omega_3 v_1^2 + 6\omega_2^3 - 42\omega_2\omega_3^2 v_1^2 + \omega_2^3\omega_3^2 - 6\omega_2^3 v_1^2 - 9\omega_2^3\omega_3 - 3\omega_2^3\omega_3^2 v_1^2 + 25\omega_2^2\omega_3^2 c_s^2 - 11\omega_2^2\omega_3^2 + 24\omega_3^2 c_s^2 + 24\omega_2\omega_3 c_s^2 + 12\omega_2\omega_3^2 - 48\omega_2\omega_3^2 c_s^2 + 15\omega_2^3\omega_3 v_1^2 - 36\omega_2^2\omega_3 c_s^2 + 12\omega_2^2 c_s^2 + 48\omega_2\omega_3 v_1^2 + 12\omega_3^2 v_1^2 - 2\omega_2^3\omega_3^2 c_s^2 - 6\omega_2^3 c_s^2 - 24\omega_2\omega_3 + 27\omega_2^2\omega_3^2 v_1^2)\frac{\rho}{12\omega_2^3\omega_3^2}$

$C^{(0),\mathrm{CLBM1}}_{\mathrm{D}_t\mathrm{D}_x^3 v_1} =$
$(9\omega_2^3\omega_3 c_s^2 + 36\omega_2^2\omega_3 + 36\omega_2^2 v_1^2 - 12\omega_2^2 - 108\omega_2^2\omega_3 v_1^2 + 6\omega_2^3 + 18\omega_2\omega_3^2 v_1^2 + \omega_2^3\omega_3^2 - 18\omega_2^3 v_1^2 - 9\omega_2^3\omega_3 - 3\omega_2^3\omega_3^2 v_1^2 + 25\omega_2^2\omega_3^2 c_s^2 - 11\omega_2^2\omega_3^2 + 24\omega_3^2 c_s^2 + 24\omega_2\omega_3 c_s^2 + 12\omega_2\omega_3^2 - 48\omega_2\omega_3^2 c_s^2 + 27\omega_2^3\omega_3 v_1^2 - 36\omega_2^2\omega_3 c_s^2 + 12\omega_2^2 c_s^2 + 72\omega_2\omega_3 v_1^2 - 36\omega_3^2 v_1^2 - 2\omega_2^3\omega_3^2 c_s^2 - 6\omega_2^3 c_s^2 - 24\omega_2\omega_3 + 15\omega_2^2\omega_3^2 v_1^2)\frac{\rho}{12\omega_2^3\omega_3^2}$

coefficient $C^{(0)}_{\mathrm{D}_x^4 \rho}$ **at** $\frac{\partial^4 \rho}{\partial x_1^4}$**:**

$$C^{(0),\mathrm{SRT}}_{\mathrm{D}^4_x\rho} = (6\omega^3 v_1^2 c_s^2 - 72\omega c_s^4 + 72 v_1^2 + 3\omega^3 v_1^4 - 14\omega^2 c_s^2 - 42\omega^2 v_1^4 + \omega^3 c_s^2 - 108\omega v_1^2 + 48 c_s^4 + 216\omega v_1^2 c_s^2 - 144 v_1^2 c_s^2 + 42\omega^2 v_1^2 - 24 c_s^2 + 108\omega v_1^4 - 3\omega^3 c_s^4 - 84\omega^2 v_1^2 c_s^2 - 3\omega^3 v_1^2 - 72 v_1^4 + 36\omega c_s^2 + 30\omega^2 c_s^4)\frac{1}{24\omega^3}$$

$$C^{(0),\mathrm{MRT1}}_{\mathrm{D}^4_x\rho} = (12\omega_2^3 v_1^2 c_s^2 - 48\omega_2\omega_3 v_1^4 + 6\omega_2^3\omega_3^2 v_1^2 c_s^2 - 6\omega_2^3\omega_3 c_s^2 + 24\omega_2^2 v_1^2 - 72\omega_2^2\omega_3 v_1^2 - 24\omega_2^2\omega_3^2 v_1^4 - 96\omega_3^2 v_1^2 c_s^2 - 3\omega_2^3\omega_3^2 c_s^4 - 24\omega_2\omega_3^2 v_1^2 - 24\omega_2\omega_3 v_1^2 c_s^2 - 48\omega_2\omega_3^2 c_s^4 - 12\omega_2^3 v_1^2 - 3\omega_2^3\omega_3^2 v_1^2 - 8\omega_2^2\omega_3^2 c_s^2 - 24\omega_2^2\omega_3 c_s^4 - 72\omega_2^2\omega_3^2 v_1^2 c_s^2 - 18\omega_2^3\omega_3 v_1^4 - 24\omega_2\omega_3 c_s^2 + 24\omega_2^2\omega_3^2 c_s^4 + 12\omega_2\omega_3^2 c_s^2 + 12\omega_2^3 v_1^4 + 3\omega_2^3\omega_3^2 v_1^4 + 48\omega_2^2\omega_3 v_1^2 c_s^2 + 24\omega_3^2 c_s^4 + 18\omega_2^3\omega_3 v_1^2 + 24\omega_2\omega_3 c_s^4 + 24\omega_2^2\omega_3 c_s^2 - 24\omega_2^2 v_1^2 c_s^2 - 24\omega_2^2 v_1^4 - 12\omega_2^3\omega_3 v_1^2 c_s^2 + 72\omega_2^2\omega_3 v_1^4 + 48\omega_2\omega_3 v_1^2 + 6\omega_2^3\omega_3 c_s^4 + \omega_2^3\omega_3^2 c_s^2 + 24\omega_2\omega_3^2 v_1^4 + 156\omega_2\omega_3^2 v_1^2 c_s^2 + 24\omega_2^2\omega_3^2 v_1^2)\frac{1}{24\omega_2^3\omega_3^2}$$

$$C^{(0),\mathrm{CLBM1}}_{\mathrm{D}^4_x\rho} = (108\omega_2^3 v_1^2 c_s^2 + 6\omega_2^3\omega_3^2 v_1^2 c_s^2 - 6\omega_2^3\omega_3 c_s^2 + 72\omega_2^2 v_1^2 - 72\omega_2^2\omega_3 v_1^2 - 12\omega_2^2\omega_3^2 v_1^4 - 3\omega_2^3\omega_3^2 c_s^4 + 72\omega_2\omega_3 v_1^2 c_s^2 - 48\omega_2\omega_3^2 c_s^4 - 36\omega_2^3 v_1^2 - 3\omega_2^3\omega_3^2 v_1^2 - 8\omega_2^2\omega_3^2 c_s^2 - 24\omega_2^2\omega_3 c_s^4 - 12\omega_2^2\omega_3^2 v_1^2 c_s^2 - 30\omega_2^3\omega_3 v_1^4 - 24\omega_2\omega_3 c_s^2 + 24\omega_2^2\omega_3^2 c_s^4 + 12\omega_2\omega_3^2 c_s^2 + 36\omega_2^3 v_1^4 + 3\omega_2^3\omega_3^2 v_1^4 + 144\omega_2^2\omega_3 v_1^2 c_s^2 + 24\omega_3^2 c_s^4 + 30\omega_2^3\omega_3 v_1^2 + 24\omega_2\omega_3 c_s^4 + 24\omega_2^2\omega_3 c_s^2 - 216\omega_2^2 v_1^2 c_s^2 - 72\omega_2^2 v_1^4 - 72\omega_2^3\omega_3 v_1^2 c_s^2 + 72\omega_2^2\omega_3 v_1^4 + 6\omega_2^3\omega_3 c_s^4 + \omega_2^3\omega_3^2 c_s^2 - 36\omega_2\omega_3^2 v_1^2 c_s^2 + 12\omega_2^2\omega_3^2 v_1^2)\frac{1}{24\omega_2^3\omega_3^2}$$

coefficient $C^{(0)}_{\mathrm{D}^4_x v_1}$ **at** $\frac{\partial^4 v_1}{\partial x_1^4}$**:**

$$C^{(0),\mathrm{SRT}}_{\mathrm{D}^4_x v_1} = (24 - 36 v_1^2 - 36\omega - 26\omega^2 c_s^2 + 14\omega^2 + \omega^3 c_s^2 + 54\omega v_1^2 - \omega^3 - 22\omega^2 v_1^2 - 48 c_s^2 + 2\omega^3 v_1^2 + 72\omega c_s^2)\frac{\rho v_1}{12\omega^3}$$

$$C^{(0),\mathrm{MRT1}}_{\mathrm{D}^4_x v_1} = (-6\omega_2^3\omega_3 c_s^2 - 24\omega_2^2\omega_3 - 12\omega_2^2 v_1^2 + 12\omega_2^2 + 24\omega_2^2\omega_3 v_1^2 - 6\omega_2^3 + 24\omega_2\omega_3^2 v_1^2 - \omega_2^3\omega_3^2 + 6\omega_2^3 v_1^2 + 6\omega_2^3\omega_3 + 2\omega_2^3\omega_3^2 v_1^2 - 20\omega_2^2\omega_3^2 c_s^2 + 8\omega_2^2\omega_3^2 - 24\omega_3^2 c_s^2 - 12\omega_2\omega_3 c_s^2 - 6\omega_2\omega_3^2 + 42\omega_2\omega_3^2 c_s^2 - 6\omega_2^3\omega_3 v_1^2 + 24\omega_2^2\omega_3 c_s^2 - 12\omega_2^2 c_s^2 - 12\omega_2\omega_3 v_1^2 - 12\omega_3^2 v_1^2 + \omega_2^3\omega_3^2 c_s^2 + 6\omega_2^3 c_s^2 + 12\omega_2\omega_3 - 16\omega_2^2\omega_3^2 v_1^2)\frac{\rho v_1}{12\omega_2^3\omega_3^2}$$

$$C^{(0),\mathrm{CLBM1}}_{\mathrm{D}^4_x v_1} = (-24\omega_2^3\omega_3 c_s^2 - 24\omega_2^2\omega_3 - 84\omega_2^2 v_1^2 + 36\omega_2^2 + 24\omega_2^2\omega_3 v_1^2 - 18\omega_2^3 - 12\omega_2\omega_3^2 v_1^2 - \omega_2^3\omega_3^2 + 42\omega_2^3 v_1^2 + 12\omega_2^3\omega_3 + 2\omega_2^3\omega_3^2 v_1^2 - 2\omega_2^2\omega_3^2 c_s^2 + 2\omega_2^2\omega_3^2 + 24\omega_3^2 c_s^2 - 12\omega_2\omega_3 c_s^2 + 6\omega_2\omega_3^2 - 30\omega_2\omega_3^2 c_s^2 - 24\omega_2^3\omega_3 v_1^2 + 72\omega_2^2\omega_3 c_s^2 - 60\omega_2^2 c_s^2 + 60\omega_2\omega_3 v_1^2 - 12\omega_3^2 v_1^2 + \omega_2^3\omega_3^2 c_s^2 + 30\omega_2^3 c_s^2 - 12\omega_2\omega_3 + 2\omega_2^2\omega_3^2 v_1^2)\frac{\rho v_1}{12\omega_2^3\omega_3^2}$$

References

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