

**D2Q9 NSE,**  
a supplementary material for  
**Lattice Boltzmann Method Analysis Tool (LBMAT)**

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## Contents

<b>1 Global definitions</b>	<b>2</b>
1.1 Discrete velocity vectors . . . . .	2
1.2 Raw and central moments . . . . .	3
1.3 Transformation matrix $\mathbf{M}$ . . . . .	3
1.4 Equilibrium . . . . .	4
<b>2 Spatial EPDEs</b>	<b>4</b>
2.1 SRT . . . . .	4
2.1.1 Definitions . . . . .	4
2.1.2 Conservation of mass: $\rho$ . . . . .	4
2.1.3 Conservation of momentum: $\rho v_1$ . . . . .	5
2.1.4 Conservation of momentum: $\rho v_2$ . . . . .	6
2.2 MRT . . . . .	6
2.2.1 Definitions . . . . .	6
2.2.2 Conservation of mass: $\rho$ . . . . .	7
2.2.3 Conservation of momentum: $\rho v_1$ . . . . .	7
2.2.4 Conservation of momentum: $\rho v_2$ . . . . .	11
2.3 MRT2 . . . . .	14
2.3.1 Definitions . . . . .	14
2.3.2 Conservation of mass: $\rho$ . . . . .	15
2.3.3 Conservation of momentum: $\rho v_1$ . . . . .	16
2.3.4 Conservation of momentum: $\rho v_2$ . . . . .	19
2.4 CLBM1 . . . . .	23
2.4.1 Definitions . . . . .	23
2.4.2 Conservation of mass: $\rho$ . . . . .	24
2.4.3 Conservation of momentum: $\rho v_1$ . . . . .	24
2.4.4 Conservation of momentum: $\rho v_2$ . . . . .	26
2.5 CLBM2 . . . . .	27
2.5.1 Definitions . . . . .	27

2.5.2	Conservation of mass: $\rho$	28
2.5.3	Conservation of momentum: $\rho v_1$	28
2.5.4	Conservation of momentum: $\rho v_2$	30
2.6	CuLBM1	32
2.6.1	Definitions	32
2.6.2	Conservation of mass: $\rho$	32
2.6.3	Conservation of momentum: $\rho v_1$	33
2.6.4	Conservation of momentum: $\rho v_2$	34
2.7	CuLBM2	35
2.7.1	Definitions	35
2.7.2	Conservation of mass: $\rho$	35
2.7.3	Conservation of momentum: $\rho v_1$	36
2.7.4	Conservation of momentum: $\rho v_2$	39
<b>3</b>	<b>Comparison of SRT, MRT, CLBM, and CuLBM</b>	<b>41</b>
3.1	Conservation of mass: $\rho$	41
3.2	Conservation of momentum: $\rho v_1$	46
3.3	Conservation of momentum: $\rho v_2$	63

## 1 Global definitions

In  $\mathbb{R}^2$ , the position and velocity vectors are given by  $x = (x_1, x_2)^T$  and  $v = (v_1, v_2)^T$ , respectively.

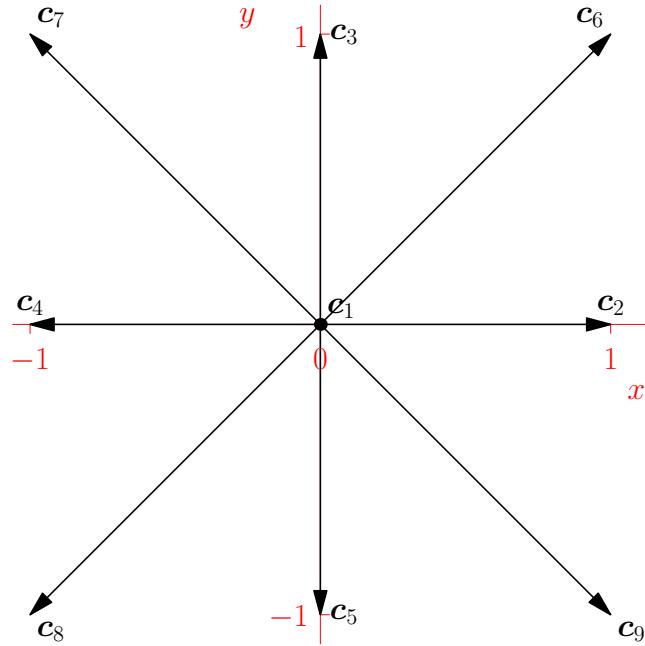
### 1.1 Discrete velocity vectors

Discrete velocity vectors and the lattice speed of sound are defined by

$$\{\mathbf{c}_i\}_{i=1}^9 = \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right),$$

$$c_s = \frac{1}{\sqrt{3}},$$

respectively [1].



## 1.2 Raw and central moments

The raw and central moments are defined by

$$m_{\alpha} := \sum_{i=1}^9 f_i \mathbf{c}_i^{\alpha},$$

and

$$k_{\alpha} := \sum_{i=1}^9 f_i (\mathbf{c}_i - \mathbf{v})^{\alpha},$$

respectively, where  $\alpha = (\alpha_1, \alpha_2) \in \mathbb{Z}^2$  denotes a multi-index (as a row vector) and  $\mathbf{c}_i^{\alpha} := \prod_{j=1}^2 [\mathbf{c}_i]_j^{\alpha_j}$ .

## 1.3 Transformation matrix M

Matrix  $\mathbf{M}$ , that defines macroscopic quantities (moments)  $\boldsymbol{\mu}$  by

$$\boldsymbol{\mu} = \mathbf{M} \mathbf{f},$$

with  $\mathbf{f} = (f_1, f_2, \dots, f_9)^T$ , is selected such that

$$\boldsymbol{\mu} = \left( m_{(0,0)}, m_{(1,0)}, m_{(0,1)}, m_{(1,1)}, m_{(2,0)}, m_{(0,2)}, m_{(2,1)}, m_{(1,2)}, m_{(2,2)} \right)^T,$$

i.e.,  $\mathbf{M}$  is given by

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

## 1.4 Equilibrium

The corresponding equilibrium raw moments are defined using the continuous Maxwell–Boltzmann distribution function [1]

$$f^{(eq)}(\boldsymbol{\xi}) = \frac{\rho}{2\pi c_s^2} \exp\left(-\frac{\|\boldsymbol{\xi} - \mathbf{v}\|^2}{2c_s^2}\right)$$

as

$$m_{\boldsymbol{\alpha}}^{(eq)} = \int_{\mathbb{R}^2} \boldsymbol{\xi}^{\boldsymbol{\alpha}} f^{(eq)}(\boldsymbol{\xi}) d\boldsymbol{\xi},$$

where  $\alpha_i \in \{0, 1, 2\}$ ,  $i = 1, 2$ . Hence, the equilibrium moments  $\boldsymbol{\mu}^{(eq)}$  satisfy

$$\boldsymbol{\mu}^{(eq)} = \begin{pmatrix} \rho \\ \rho v_1 \\ \rho v_2 \\ \rho v_1 v_2 \\ \rho(v_1^2 + c_s^2) \\ \rho(v_2^2 + c_s^2) \\ \rho v_2(c_s^2 + v_1^2) \\ \rho v_1(c_s^2 + v_2^2) \\ \rho(c_s^4 + c_s^2(v_1^2 + v_2^2) + v_1^2 v_2^2) \end{pmatrix}.$$

## 2 Spatial EPDEs

### 2.1 SRT

#### 2.1.1 Definitions

Collision operator  $\mathbf{C}$ :

$$\mathbf{C}(\mathbf{f}) = \omega \left( \mathbf{M}^{-1} \boldsymbol{\mu}^{(eq)} - \mathbf{f} \right),$$

$\omega \in (0, 2)$ .

#### 2.1.2 Conservation of mass: $\rho$

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$$\begin{aligned}
& \frac{\partial \rho}{\partial t} + \frac{\delta_l v_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\delta_l \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_l v_2}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\delta_l \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + 3c_s^2 + v_1^2) \frac{\delta_l^3 v_1}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3v_1^2) \frac{\delta_l^3 \rho}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\
& \frac{\delta_l^3 c_s^2 \rho}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{\delta_l^3 c_s^2 \rho}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{\delta_l^3 v_2}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + c_s^2 + 3v_2^2) \frac{\delta_l^3 \rho}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& (6v_1^4 + \omega c_s^2 + 2c_s^4 + 3\omega v_1^2 - 2c_s^2 - 3\omega v_1^4 + 24c_s^2 v_1^2 - 6v_1^2 - \omega c_s^4 - 12\omega c_s^2 v_1^2) \frac{\delta_l^4}{24\omega \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& (-4 + 2\omega - 3\omega c_s^2 - 5\omega v_1^2 + 6c_s^2 + 10v_1^2) \frac{\delta_l^4 \rho v_1}{12\omega \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + (2 - \omega + 3\omega c_s^2 + \omega v_1^2 - 6c_s^2 - 2v_1^2) \frac{\delta_l^4 \rho v_1}{12\omega \delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& (-2 + \omega) \frac{\delta_l^4 c_s^4}{6\omega \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + (2 - \omega + 3\omega c_s^2 + \omega v_2^2 - 6c_s^2 - 2v_2^2) \frac{\delta_l^4 v_2 \rho}{12\omega \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& (\omega c_s^2 + 6v_2^4 + 3\omega v_2^2 + 2c_s^4 - 2c_s^2 + 24c_s^2 v_2^2 - 3\omega v_2^4 - 12\omega c_s^2 v_2^2 - 6v_2^2 - \omega c_s^4) \frac{\delta_l^4}{24\omega \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\
& (-4 + 2\omega - 3\omega c_s^2 - 5\omega v_2^2 + 6c_s^2 + 10v_2^2) \frac{\delta_l^4 v_2 \rho}{12\omega \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0.
\end{aligned}$$

### 2.1.3 Conservation of momentum: $\rho v_1$

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$$\begin{aligned}
& v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (c_s^2 + v_1^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2\delta_l \rho v_1}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_l v_2 v_1}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\delta_l v_2 \rho}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{\delta_l \rho v_1}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\
& (-2 + \omega - 2\omega c_s^2 - 3\omega v_1^2 + 4c_s^2 + 6v_1^2) \frac{\delta_l^2}{\omega \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + (2 - \omega) \frac{3\delta_l^2 \rho v_1}{\omega \delta_t} \left( \frac{\partial v_1}{\partial x_1} \right)^2 + (-2 + \omega) \frac{\delta_l^2 c_s^2}{2\omega \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\
& (-2 + \omega) \frac{\delta_l^2 c_s^2}{2\omega \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (-2 + \omega - 3\omega c_s^2 - \omega v_1^2 + 6c_s^2 + 2v_1^2) \frac{\delta_l^2 v_1}{2\omega \delta_t} \frac{\partial^2 \rho}{\partial x_1^2} + \\
& (-2 + \omega - \omega c_s^2 - 3\omega v_1^2 + 2c_s^2 + 6v_1^2) \frac{\delta_l^2 \rho}{2\omega \delta_t} \frac{\partial^2 v_1}{\partial x_2^2} + (-2 + \omega) \frac{\delta_l^2 c_s^2 \rho}{2\omega \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + (-2 + \omega) \frac{\delta_l^2 c_s^2 \rho}{2\omega \delta_t} \frac{\partial^2 v_1}{\partial x_2^2} + C_1 \frac{\delta_l^3}{12\omega^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + \\
& (-24 + 24\omega - 4\omega^2 - 36\omega c_s^2 - 60\omega v_1^2 + 36c_s^2 + 11\omega^2 v_1^2 + 60v_1^2 + 5\omega^2 c_s^2) \frac{\delta_l^3 \rho v_1}{6\omega^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + \\
& (12 - 12\omega + 3\omega^2 + 36\omega c_s^2 + 12\omega v_1^2 - 36c_s^2 - 3\omega^2 v_1^2 - 12v_1^2 - 11\omega^2 c_s^2) \frac{\delta_l^3 \rho v_1}{12\omega^2 \delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + \\
& (-12 + 12\omega - \omega^2) \frac{\delta_l^3 c_s^4}{6\omega^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} - \frac{\delta_l^3 c_s^2 \rho v_1}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{\delta_l^3 v_2 v_1}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\
& (6 - 6\omega + \omega^2 + 18\omega c_s^2 + 6\omega v_2^2 - 18c_s^2 - \omega^2 v_2^2 - 3\omega^2 c_s^2 - 6v_2^2) \frac{\delta_l^3 v_2 \rho}{6\omega^2 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + (-1 + c_s^2 + 3v_2^2) \frac{\delta_l^3 \rho v_1}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& C_2 \frac{\delta_l^4 v_1}{12\omega^3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + C_3 \frac{\delta_l^4 \rho}{12\omega^3 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_4 \frac{\delta_l^4 \rho}{12\omega^3 \delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_5 \frac{\delta_l^4 c_s^2 v_1}{12\omega^3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\
& (-24 + 36\omega - 12\omega^2 - 18\omega c_s^2 - 108\omega v_1^2 + 12c_s^2 + 36\omega^2 v_1^2 - \omega^3 c_s^2 + 72v_1^2 + 8\omega^2 c_s^2) \frac{\delta_l^4 c_s^2 \rho}{12\omega^3 \delta_t} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_6 \frac{\delta_l^4 c_s^2 v_2}{12\omega^3 \delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + \\
& +(2 - \omega + 3\omega c_s^2 + \omega v_2^2 - 6c_s^2 - 2v_2^2) \frac{\delta_l^4 v_2 \rho v_1}{12\omega \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& (-12 + 18\omega - 6\omega^2 - 54\omega v_2^2 - \omega^3 c_s^2 + 18\omega^2 v_2^2 + 2\omega^2 c_s^2 + 36v_2^2) \frac{\delta_l^4 c_s^2 \rho}{12\omega^3 \delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\
& (\omega c_s^2 + 6v_2^4 + 3\omega v_2^2 + 2c_s^4 - 2c_s^2 + 24c_s^2 v_2^2 - 3\omega v_2^4 - 12\omega c_s^2 v_2^2 - 6v_2^2 - \omega c_s^4) \frac{\delta_l^4 v_1}{24\omega \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_7 \frac{\delta_l^4 \rho}{24\omega^3 \delta_t} \frac{\partial^4 v_1}{\partial x_2^4} + \\
& (-4 + 2\omega - 3\omega c_s^2 - 5\omega v_2^2 + 6c_s^2 + 10v_2^2) \frac{\delta_l^4 v_2 \rho v_1}{12\omega \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= 36v_1^4 + \omega^2 c_s^4 + 12\omega c_s^2 + 24\omega^2 c_s^2 v_1^2 + 12c_s^4 + 7\omega^2 v_1^4 + 36\omega v_1^2 - 12c_s^2 - 7\omega^2 v_1^2 - 36\omega v_1^4 + 144c_s^2 v_1^2 - 36v_1^2 - \omega^2 c_s^2 - 12\omega c_s^4 - 144\omega c_s^2 v_1^2 \\
C_2 &= 12 - 9\omega^3 v_1^4 - 18\omega + 144v_1^4 + 8\omega^2 + 82\omega^2 c_s^4 + 198\omega c_s^2 - \omega^3 + 404\omega^2 c_s^2 v_1^2 - 5\omega^3 c_s^4 + 144c_s^4 + 90\omega^2 v_1^4 + 234\omega v_1^2 - 34\omega^3 c_s^2 v_1^2 - 132c_s^2 - 98\omega^2 v_1^2 + 6\omega^3 c_s^2 - 216\omega v_1^4 + 672c_s^2 v_1^2 - 156v_1^2 - 78\omega^2 c_s^2 + 10\omega^3 v_1^2 - 216\omega c_s^4 - 1008\omega c_s^2 v_1^2 \\
C_3 &= 12 - 29\omega^3 v_1^4 - 18\omega + 504v_1^4 + 8\omega^2 + 14\omega^2 c_s^4 + 54\omega c_s^2 - \omega^3 + 252\omega^2 c_s^2 v_1^2 - \omega^3 c_s^4 + 24c_s^4 + 310\omega^2 v_1^4 + 378\omega v_1^2 - 18\omega^3 c_s^2 v_1^2 - 36c_s^2 - 154\omega^2 v_1^2 + 2\omega^3 c_s^2 - 756\omega v_1^4 + 432c_s^2 v_1^2 - 252v_1^2 - 22\omega^2 c_s^2 + 14\omega^3 v_1^2 - 36\omega c_s^4 - 648\omega c_s^2 v_1^2 \\
C_4 &= 4\omega^3 v_1^4 - 36v_1^4 + 20\omega^2 c_s^4 + 36\omega c_s^2 - 42\omega^2 c_s^2 v_1^2 - \omega^3 c_s^4 + 36c_s^4 - 26\omega^2 v_1^4 - 54\omega v_1^2 + 12\omega^3 c_s^2 v_1^2 - 24c_s^2 + 26\omega^2 v_1^2 + 54\omega v_1^4 - 36c_s^2 v_1^2 + 36v_1^2 - 12\omega^2 c_s^2 - 4\omega^3 v_1^2 - 54\omega c_s^4 + 54\omega c_s^2 v_1^2 \\
C_5 &= 24 - 36\omega + 14\omega^2 + 108\omega c_s^2 - \omega^3 + 36\omega v_1^2 - 72c_s^2 - 14\omega^2 v_1^2 + 5\omega^3 c_s^2 - 24v_1^2 - 46\omega^2 c_s^2 + \omega^3 v_1^2 \\
C_6 &= 24 - 36\omega + 14\omega^2 + 108\omega c_s^2 - \omega^3 + 36\omega v_1^2 - 72c_s^2 + 3\omega^3 c_s^2 - 14\omega^2 v_1^2 - 42\omega^2 c_s^2 - 24v_1^2 + \omega^3 v_1^2
\end{aligned}$$

$$C_7 = -84\omega^2 c_s^2 v_2^2 + 30\omega^2 c_s^4 + 3\omega^3 v_2^4 + 36\omega c_s^2 - 72v_2^4 - 3\omega^3 c_s^4 - 108\omega v_2^2 + 48c_s^4 - 42\omega^2 v_2^4 - 24c_s^2 + \omega^3 c_s^2 - 144c_s^2 v_2^2 + 108\omega v_2^4 + 6\omega^3 c_s^2 v_2^2 + 42\omega^2 v_2^2 - 14\omega^2 c_s^2 + 216\omega c_s^2 v_2^2 + 72v_2^2 - 3\omega^3 v_2^2 - 72\omega c_s^4$$

### 2.1.4 Conservation of momentum: $\rho v_2$

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$$\begin{aligned}
& v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{\delta_l v_2 v_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\delta_l v_2 \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_l \rho v_1}{\delta_t} \frac{\partial v_2}{\partial x_1} + (c_s^2 + v_2^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{2\delta_l v_2 \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega) \frac{\delta_l^2 c_s^2}{2\omega \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\
& (-2 + \omega) \frac{\delta_l^2 c_s^2}{2\omega \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (-2 + \omega - 2\omega c_s^2 - 3\omega v_2^2 + 4c_s^2 + 6v_2^2) \frac{\delta_l^2}{\omega \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (2 - \omega) \frac{3\delta_l^2 v_2 \rho}{\omega \delta_t} \left( \frac{\partial v_2}{\partial x_2} \right)^2 + \\
& (-2 + \omega) \frac{\delta_l^2 c_s^2 \rho}{2\omega \delta_t} \frac{\partial^2 v_2}{\partial x_1^2} + (-2 + \omega) \frac{\delta_l^2 c_s^2 \rho}{2\omega \delta_t} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + (-2 + \omega - 3\omega c_s^2 - \omega v_2^2 + 6c_s^2 + 2v_2^2) \frac{\delta_l^2 v_2}{2\omega \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + \\
& (-2 + \omega - \omega c_s^2 - 3\omega v_2^2 + 2c_s^2 + 6v_2^2) \frac{\delta_l^2 \rho}{2\omega \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + (-1 + 3c_s^2 + v_1^2) \frac{\delta_l^3 v_2 v_1}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3v_1^2) \frac{\delta_l^3 v_2 v_1}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + \\
& (6 - 6\omega + \omega^2 + 18\omega c_s^2 + 6\omega v_1^2 - 18c_s^2 - \omega^2 v_1^2 - 6v_1^2 - 3\omega^2 c_s^2) \frac{\delta_l^3 \rho v_1}{6\omega^2 \delta_t} \frac{\partial^3 v_2}{\partial x_1^3} + (-12 + 12\omega - \omega^2) \frac{\delta_l^3 c_s^4}{6\omega^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} - \\
& \frac{\delta_l^3 c_s^2 v_2 \rho}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + (12 - 12\omega + 3\omega^2 + 36\omega c_s^2 + 12\omega v_2^2 - 36c_s^2 - 3\omega^2 v_2^2 - 11\omega^2 c_s^2 - 12v_2^2) \frac{\delta_l^3 v_2 \rho}{12\omega^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + \\
& C_1 \frac{\delta_l^3}{12\omega^2 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-24 + 24\omega - 4\omega^2 - 36\omega c_s^2 - 60\omega v_2^2 + 36c_s^2 + 11\omega^2 v_2^2 + 5\omega^2 c_s^2 + 60v_2^2) \frac{\delta_l^3 v_2 \rho}{6\omega^2 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& (6v_1^4 + \omega c_s^2 + 2c_s^4 + 3\omega v_1^2 - 2c_s^2 - 3\omega v_1^4 + 24c_s^2 v_1^2 - 6v_1^2 - \omega c_s^4 - 12\omega c_s^2 v_1^2) \frac{\delta_l^4 v_2}{24\omega \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& (-4 + 2\omega - 3\omega c_s^2 - 5\omega v_1^2 + 6c_s^2 + 10v_1^2) \frac{\delta_l^4 v_2 \rho v_1}{12\omega \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_2 \frac{\delta_l^4 \rho}{24\omega^3 \delta_t} \frac{\partial^4 v_2}{\partial x_1^4} + C_3 \frac{\delta_l^4 c_s^2 v_1}{12\omega^3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + \\
& (-12 + 18\omega - 6\omega^2 - 54\omega v_1^2 + 18\omega^2 v_1^2 - \omega^3 c_s^2 + 36v_1^2 + 2\omega^2 c_s^2) \frac{\delta_l^4 c_s^2 \rho}{12\omega^3 \delta_t} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + \\
& (2 - \omega + 3\omega c_s^2 + \omega v_1^2 - 6c_s^2 - 2v_1^2) \frac{\delta_l^4 v_2 \rho v_1}{12\omega \delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_4 \frac{\delta_l^4 c_s^2 v_2}{12\omega^3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\
& (-24 + 36\omega - 12\omega^2 - 18\omega c_s^2 - 108\omega v_2^2 + 12c_s^2 - \omega^3 c_s^2 + 36\omega^2 v_2^2 + 8\omega^2 c_s^2 + 72v_2^2) \frac{\delta_l^4 c_s^2 \rho}{12\omega^3 \delta_t} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_5 \frac{\delta_l^4 \rho}{12\omega^3 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& + C_6 \frac{\delta_l^4 v_2}{12\omega^3 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_7 \frac{\delta_l^4 \rho}{12\omega^3 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= 24\omega^2 c_s^2 v_2^2 + \omega^2 c_s^4 + 12\omega c_s^2 + 36v_2^4 + 36\omega v_2^2 + 12c_s^4 + 7\omega^2 v_2^4 - 12c_s^2 + 144c_s^2 v_2^2 - 36\omega v_2^4 - 7\omega^2 v_2^2 - \omega^2 c_s^2 - 144\omega c_s^2 v_2^2 - 36v_2^2 - 12\omega c_s^4 \\
C_2 &= 3\omega^3 v_1^4 - 72v_1^4 + 30\omega^2 c_s^4 + 36\omega c_s^2 - 84\omega^2 c_s^2 v_1^2 - 3\omega^3 c_s^4 + 48c_s^4 - 42\omega^2 v_1^4 - 108\omega v_1^2 + 6\omega^3 c_s^2 v_1^2 - 24c_s^2 + 42\omega^2 v_1^2 + \omega^3 c_s^2 + 108\omega v_1^4 - \\
& 144c_s^2 v_1^2 + 72v_1^2 - 14\omega^2 c_s^2 - 3\omega^3 v_1^2 - 72\omega c_s^4 + 216\omega c_s^2 v_1^2 \\
C_3 &= 24 - 36\omega + 14\omega^2 + 108\omega c_s^2 - \omega^3 + 36\omega v_1^2 - 72c_s^2 - 14\omega^2 v_1^2 + 3\omega^3 c_s^2 - 24v_1^2 - 42\omega^2 c_s^2 + \omega^3 v_1^2 \\
C_4 &= 24 - 36\omega + 14\omega^2 + 108\omega c_s^2 - \omega^3 + 36\omega v_2^2 - 72c_s^2 + 5\omega^3 c_s^2 - 14\omega^2 v_2^2 - 46\omega^2 c_s^2 - 24v_2^2 + \omega^3 v_2^2 \\
C_5 &= -42\omega^2 c_s^2 v_2^2 + 20\omega^2 c_s^4 + 4\omega^3 v_2^4 + 36\omega c_s^2 - 36v_2^4 - \omega^3 c_s^4 - 54\omega v_2^2 + 36c_s^4 - 26\omega^2 v_2^4 - 24c_s^2 - 36c_s^2 v_2^2 + 54\omega v_2^4 + 12\omega^3 c_s^2 v_2^2 + 26\omega^2 v_2^4 - \\
& 12\omega^2 c_s^2 + 54\omega c_s^2 v_2^2 + 36v_2^2 - 4\omega^3 v_2^2 - 54\omega c_s^4 \\
C_6 &= 12 + 404\omega^2 c_s^2 v_2^2 - 18\omega + 8\omega^2 + 82\omega^2 c_s^4 - 9\omega^3 v_2^4 + 198\omega c_s^2 + 144v_2^4 - \omega^3 - 5\omega^3 c_s^4 + 234\omega v_2^2 + 144c_s^4 + 90\omega^2 v_2^4 - 132c_s^2 + 6\omega^3 c_s^2 + \\
& 672c_s^2 v_2^2 - 216\omega v_2^4 - 34\omega^3 c_s^2 v_2^2 - 98\omega^2 v_2^2 - 78\omega^2 c_s^2 - 1008\omega c_s^2 v_2^2 - 156v_2^2 + 10\omega^3 v_2^2 - 216\omega c_s^4 \\
C_7 &= 12 + 252\omega^2 c_s^2 v_2^2 - 18\omega + 8\omega^2 + 14\omega^2 c_s^4 - 29\omega^3 v_2^4 + 54\omega c_s^2 + 504v_2^4 - \omega^3 - \omega^3 c_s^4 + 378\omega v_2^2 + 24c_s^4 + 310\omega^2 v_2^4 - 36c_s^2 + 2\omega^3 c_s^2 + \\
& 432c_s^2 v_2^2 - 756\omega v_2^4 - 18\omega^3 c_s^2 v_2^2 - 154\omega^2 v_2^2 - 22\omega^2 c_s^2 - 648\omega c_s^2 v_2^2 - 252v_2^2 + 14\omega^3 v_2^2 - 36\omega c_s^4
\end{aligned}$$

## 2.2 MRT

### 2.2.1 Definitions

Collision operator  $C$ :

$$C(f) = M^{-1}S(\mu^{(eq)} - Mf),$$

where

$$\mathbf{S} = \text{diag}(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9),$$

$$\omega_1, \omega_2, \dots, \omega_9 \in (0, 2).$$

### 2.2.2 Conservation of mass: $\rho$

attached text file: `output_d2q9_nse_mrt1_symbolic_pde_00.txt`

$$\begin{aligned} & \frac{\partial \rho}{\partial t} + \frac{\delta_t v_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\delta_l \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_l v_2}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\delta_l \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + v_1^2 + 3c_s^2) \frac{\delta_l^3 v_1}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + 3v_1^2 + c_s^2) \frac{\delta_l^3 \rho}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\ & \frac{\delta_l^3 c_s^2 \rho}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{\delta_l^3 c_s^2 \rho}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + v_2^2 + 3c_s^2) \frac{\delta_l^3 v_2}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + 3v_2^2 + c_s^2) \frac{\delta_l^3 \rho}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & (2c_s^4 + 24c_s^2 v_1^2 + c_s^2 \omega_5 + 6v_1^4 - 3v_1^4 \omega_5 - c_s^4 \omega_5 - 12c_s^2 v_1^2 \omega_5 - 6v_1^2 + 3v_1^2 \omega_5 - 2c_s^2) \frac{\delta_l^4}{24\delta_t \omega_5} \frac{\partial^4 \rho}{\partial x_1^4} + \\ & (-4 - 3c_s^2 \omega_5 + 10v_1^2 - 5v_1^2 \omega_5 + 6c_s^2 + 2\omega_5) \frac{\delta_l^4 v_1 \rho}{12\delta_t \omega_5} \frac{\partial^4 v_1}{\partial x_1^4} + (-3c_s^2 \omega_5 + \omega_7 v_1^2 - \omega_7 - v_1^2 \omega_5 + \omega_5 + 3c_s^2 \omega_7) \frac{\delta_l^4 v_2 \omega_1}{4\delta_t \omega_7 \omega_5} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + \\ & + (-c_s^2 \omega_5 + 3\omega_7 v_1^2 - \omega_7 - 3v_1^2 \omega_5 + \omega_5 + c_s^2 \omega_7) \frac{\delta_l^4 v_2 \rho}{4\delta_t \omega_7 \omega_5} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + (3\omega_4 \omega_5 - 6c_s^2 \omega_7 \omega_5 + 3c_s^2 \omega_7 \omega_4 \omega_5 - 3c_s^2 \omega_4 \omega_5 + \\ & \omega_7 \omega_4 v_1^2 \omega_5 - \omega_7 \omega_4 + \omega_7 \omega_4 v_1^2 - \omega_7 \omega_4 \omega_5 - 3\omega_4 v_1^2 \omega_5 + 3c_s^2 \omega_7 \omega_4) \frac{\delta_l^4 v_1 \rho}{12\delta_t \omega_7 \omega_4 \omega_5} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + (-2 + \omega_4) \frac{\delta_l^4 c_s^4}{6\delta_t \omega_4} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\ & (-\omega_7 + \omega_4) \frac{\delta_l^4 c_s^2 v_1 \rho}{2\delta_t \omega_7 \omega_4} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + (\omega_4 - \omega_8) \frac{\delta_l^4 c_s^2 v_2 \rho}{2\delta_t \omega_4 \omega_8} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + (\omega_6 + 3c_s^2 \omega_8 - \omega_6 v_2^2 - \omega_8 + v_2^2 \omega_8 - 3\omega_6 c_s^2) \frac{\delta_l^4 v_2 v_1}{4\omega_6 \delta_t \omega_8} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + \\ & + (3c_s^2 \omega_4 \omega_8 - 3\omega_6 \omega_4 v_2^2 - \omega_6 \omega_4 \omega_8 + 3\omega_6 c_s^2 \omega_4 \omega_8 - 6\omega_6 c_s^2 \omega_8 + 3\omega_6 \omega_4 - \omega_4 \omega_8 + \omega_6 \omega_4 v_2^2 \omega_8 - 3\omega_6 c_s^2 \omega_4 + \\ & \omega_4 v_2^2 \omega_8) \frac{\delta_l^4 v_2 \rho}{12\omega_6 \delta_t \omega_4 \omega_8} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + (\omega_6 + c_s^2 \omega_8 - 3\omega_6 v_2^2 - \omega_8 + 3v_2^2 \omega_8 - \omega_6 c_s^2) \frac{\delta_l^4 v_1 \rho}{4\omega_6 \delta_t \omega_8} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\ & (24c_s^2 v_2^2 + 2c_s^4 - 12\omega_6 c_s^2 v_2^2 - \omega_6 c_s^4 + 6v_2^4 - 3\omega_6 v_2^4 - 6v_2^2 + 3\omega_6 v_2^2 - 2c_s^2 + \omega_6 c_s^2) \frac{\delta_l^4}{24\omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\ & (-4 + 2\omega_6 + 10v_2^2 - 5\omega_6 v_2^2 + 6c_s^2 - 3\omega_6 c_s^2) \frac{\delta_l^4 v_2 \rho}{12\omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0. \end{aligned}$$

### 2.2.3 Conservation of momentum: $\rho v_1$

attached text file: `output_d2q9_nse_mrt1_symbolic_pde_01.txt`

$$\begin{aligned} & v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (v_1^2 + c_s^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2\delta_l v_1 \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_l v_2 v_1}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\delta_l v_1 \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\ & (-2 - 2c_s^2 \omega_5 + 6v_1^2 - 3v_1^2 \omega_5 + 4c_s^2 + \omega_5) \frac{\delta_l^2}{\delta_t \omega_5} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + (2 - \omega_5) \frac{3\delta_l^2 v_1 \rho}{\delta_t \omega_5} \left( \frac{\partial v_1}{\partial x_1} \right)^2 + (-2 + \omega_4) \frac{\delta_l^2 c_s^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\ & (-2 + \omega_4) \frac{\delta_l^2 c_s^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_1} + (-2 - 3c_s^2 \omega_5 + 2v_1^2 - v_1^2 \omega_5 + 6c_s^2 + \omega_5) \frac{\delta_l^2 v_1}{2\delta_t \omega_5} \frac{\partial^2 \rho}{\partial x_1^2} + \\ & (-2 - c_s^2 \omega_5 + 6v_1^2 - 3v_1^2 \omega_5 + 2c_s^2 + \omega_5) \frac{\delta_l^2 \rho}{2\delta_t \omega_5} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega_4) \frac{\delta_l^2 c_s^2 \rho}{2\delta_t \omega_4} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_4) \frac{\delta_l^2 c_s^2 \rho}{2\delta_t \omega_4} \frac{\partial^2 v_2}{\partial x_2^2} + C_1 \frac{\delta_l^3}{12\delta_t \omega_5^2} \frac{\partial^3 \rho}{\partial x_1^3} + \\ & + (-24 - 36c_s^2 \omega_5 + 5c_s^2 \omega_5^2 + 60v_1^2 + 11v_1^2 \omega_5^2 - 4\omega_5^2 - 60v_1^2 \omega_5 + 36c_s^2 + 24\omega_5) \frac{\delta_l^3 v_1 \rho}{6\delta_t \omega_5^2} \frac{\partial^3 v_1}{\partial x_1^3} + C_2 \frac{\delta_l^3 v_2 v_1}{\delta_t \omega_7 \omega_4 \omega_5^2} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + \\ & C_3 \frac{\delta_l^3 v_2 \rho}{\delta_t \omega_7 \omega_4 \omega_5^2} \frac{\partial^3 v_1}{\partial x_2^3 \partial x_2} + C_4 \frac{\delta_l^3 v_1 \rho}{12\delta_t \omega_7 \omega_4 \omega_5^2} \frac{\partial^3 v_2}{\partial x_1^3 \partial x_2} + (-12 - \omega_4^2 + 12\omega_4) \frac{\delta_l^3 c_s^4}{6\delta_t \omega_4^2} \frac{\partial^3 \rho}{\partial x_2^3} + \\ & (12\omega_4 \omega_5 + 12\omega_4^2 - 12\omega_7 \omega_5 - \omega_7 \omega_4^2 \omega_5 - 12\omega_7 \omega_4 + 12\omega_7 \omega_4 \omega_5 - 12\omega_4^2 \omega_5) \frac{\delta_l^3 c_s^2 v_1 \rho}{6\delta_t \omega_7 \omega_4^2 \omega_5} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + \\ & (-\omega_4^2 + \omega_4 \omega_8 + 2\omega_4 - 2\omega_8) \frac{\delta_l^3 c_s^2 v_2 \rho}{\delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_5 \frac{\delta_l^3 v_2 v_1}{12\omega_6 \delta_t \omega_4 \omega_8} \frac{\partial^3 \rho}{\partial x_3^3} + C_6 \frac{\delta_l^3 v_2 \rho}{6\delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_1}{\partial x_3^2} + C_7 \frac{\delta_l^3 v_1 \rho}{12\omega_6 \delta_t \omega_4 \omega_8} \frac{\partial^3 v_2}{\partial x_2^2} + \\ & C_8 \frac{\delta_l^4 v_1}{12\delta_t \omega_5^3} \frac{\partial^4 \rho}{\partial x_1^4} + C_9 \frac{\delta_l^4 \rho}{12\delta_t \omega_5^3} \frac{\partial^4 v_1}{\partial x_1^4} + C_{10} \frac{\delta_l^4 v_2}{4\delta_t \omega_7^2 \omega_4^2 \omega_5^3} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{11} \frac{\delta_l^4 v_2 v_1 \rho}{4\delta_t \omega_7^2 \omega_4^2 \omega_5^3} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + C_{12} \frac{\delta_l^4 \rho}{12\delta_t \omega_7^2 \omega_4^2 \omega_5^3} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\ & C_{13} \frac{\delta_l^4 v_1}{12\omega_9 \delta_t \omega_7^2 \omega_4^2 \omega_8 \omega_5^3} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_{14} \frac{\delta_l^4 \rho}{12\omega_9 \delta_t \omega_7^2 \omega_4^2 \omega_8 \omega_5^3} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{15} \frac{\delta_l^4 v_2 v_1 \rho}{2\omega_9 \omega_6 \delta_t \omega_7^2 \omega_4^2 \omega_8 \omega_5^3} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + \\ & C_{16} \frac{\delta_l^4 v_2}{12\omega_9 \omega_6^2 \delta_t \omega_7 \omega_4^2 \omega_8 \omega_5^3} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{17} \frac{\delta_l^4 v_2 v_1 \rho}{12\omega_9 \omega_6^2 \delta_t \omega_7 \omega_4^2 \omega_8 \omega_5^3} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{18} \frac{\delta_l^4 \rho}{12\omega_9 \omega_6^2 \delta_t \omega_7 \omega_4^2 \omega_8 \omega_5^3} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + C_{19} \frac{\delta_l^4 v_1}{24\omega_6^2 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 \rho}{\partial x_2^4} + \\ & + C_{20} \frac{\delta_l^4 \rho}{24\delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 v_1}{\partial x_2^4} + C_{21} \frac{\delta_l^4 v_2 v_1 \rho}{12\omega_6^2 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:



$$\begin{aligned}
& 36c_4^4w_7^2w_4^2w_8w_5^3 - 12w_9w_7w_4^2v_2^2w_8v_2^2w_5^2 + 72w_9c_8^2w_7w_3^2v_2^2w_8w_5^2 - 12w_9c_8^2w_7w_4w_8v_1^2w_5^3 - 6w_9c_8^2w_7w_3^2v_1^2w_5^2 - 12w_9w_7w_2^3v_2^2w_8s - 24w_9w_7w_3^3v_2^2w_8w_5^2 - \\
& 18c_8^2w_7w_3^2v_2^2w_8w_5^2 + 18w_9c_8^2w_7w_3^2v_2^2w_8w_5^2 + 12w_9c_8^2w_7w_4w_8v_1^2w_5^3 + 12w_9c_8^2w_7w_4w_8w_5^3 + 36c_4^2w_7w_4^2w_5^3 - w_9c_8^2w_7w_3^2v_1^2w_8w_5^2 - 6w_9w_7w_2^3v_2^2w_8w_5^2 + \\
& 36c_4^2w_7w_3^2v_2^2w_8w_5^2 + 12c_8^2w_7w_3^2v_2^2w_8w_5^3 + 18c_7^2w_7w_3^2v_2^2w_8w_5^3 - 12w_9w_7w_2^2v_2^2w_8w_5 + 12w_9w_7w_3^2v_2^2w_8w_5^3 + 12w_7^2w_4^2v_2^2v_1^1w_5^3 + 12w_9c_4^2w_8w_5^2 - \\
& 18c_8^2w_7w_4^2v_2^2w_5^3 - 36c_4^2w_7w_4^2w_8w_5^3 - 36w_9c_8^2w_7w_3^2v_2^2w_8w_5^3 + 36w_9w_7w_4^2v_2^2w_8v_2^2w_5^3 + 36w_9c_2^2w_7w_4^2v_2^2w_8w_5^3 + 5w_9c_4^2w_7w_3^2v_1^2w_8w_5^3 - 36w_9c_8^2w_7w_4^2v_2^2w_8w_5^2 + \\
& 24w_9c_7w_4^2v_2^2w_8v_2^2w_5^2 + 24w_9w_7c_4^2w_4^2v_2^2w_8w_5^2 + 12w_9c_8^2w_7w_4^2v_2^2w_8w_5^2 + 180w_9c_4^2w_7w_4w_8w_5^3 - 72w_9c_8^2w_7w_4^2v_2^2w_8w_5^2 + 12w_9w_7w_3^2v_2^2w_8v_1^2w_5^3 - \\
& 36w_9c_8^2w_7w_3^2v_2^2w_8w_5^2 - 12w_9c_8^2w_7w_4^2v_2^2w_8w_5^3 - 54w_9c_8^2w_7w_4^2v_2^2w_8w_5 + 108w_9c_8^2w_7w_4^2v_2^2w_8w_5^3 + 12w_9w_7w_4^2w_8v_2^2w_8w_5^2 - 6w_9c_7w_4^2v_2^2w_8v_1^2w_5^3 - 12w_7^2w_4^2v_2^2w_8v_1^2w_5^2 - \\
& 36w_9w_7w_4^2v_2^2w_8w_5^3 + 18w_9c_8^2w_7w_3^2v_2^2w_8w_5^3 - 36c_4^2w_7w_4^2v_2^2w_8w_5^3 + 36w_9c_8^2w_7w_3^2v_2^2w_8w_5^3 - 36w_9c_4^2w_7w_3^2v_1^2w_8w_5 - 12w_9w_7w_3^2v_2^2w_8v_1^2w_5^2 - \\
& 36w_9c_8^2w_7w_3^2v_2^2w_8w_5^3 - 12w_7^2w_4^2v_2^2w_8v_1^2w_5^3 - 84w_9c_8^2w_7w_4w_8w_5^2 + 36c_4^2w_7w_4^2v_2^2w_8w_5^3 + 12w_9w_7w_3^2v_2^2w_8w_5^3 - 18c_4^2w_7w_3^2v_1^2w_5^3 - 24w_9w_7w_4^2v_2^2w_8v_1^2w_5^2 + \\
& 12w_9c_8^2w_7w_3^2v_1^2w_5^3 + 12w_9w_7w_4^2v_2^2w_8w_5^2 - 12w_9c_8^2w_7w_4^2v_1^2w_5^2 + 12w_9c_8^2w_7w_4^2v_2^2w_8w_5^2 - 36w_9c_8^2w_7w_4^2v_2^2w_8w_5^2 + 12w_9c_8^2w_7w_4^2w_8w_5^2
\end{aligned}$$

$$\begin{aligned}
C_{14} = & -12w_7w_3^2v_2^2w_8w_5^2 - 36w_9w_7w_2^4v_2^2v_1^2w_5^2 - 36c_8^2w_7^2w_4^2w_8v_2^2w_5^3 + 6w_9c_8^2w_7^2w_4^2v_2^2w_5^2 - 12w_9c_8^4w_7w_3^4w_8w_5^2 + 12w_9w_4^2v_2^2w_8w_5^3 - \\
& 4w_9c_8^4w_7^2w_4^2w_8w_5^3 - 6c_8^4w_7^2w_3^4w_8w_5^2 + 12c_8^2w_7^2w_4^2v_2^2w_5^3 - 132w_9c_8^2w_7w_4^2w_8v_1^2w_5^3 + 18w_9c_8^2w_7^2w_4^2w_8v_1^2w_5^2 + 12c_8^4w_7w_4^2w_8w_5^3 + \\
& 180w_9c_8^2w_7^2w_4^2w_8v_2^2w_5^2 + 6w_9c_8^4w_7^2w_3^4w_8w_5^2 - 36w_9w_4^2v_2^2w_8v_1^2w_5^3 - 18w_7w_4^2v_2^2w_8v_1^2w_5^2 + 24w_9c_8^2w_3^4w_8v_1^2w_5^2 - 12c_8^2w_7w_4^2w_8w_5^3 + 6c_8^2w_7^2w_4^3w_8w_5^2 - \\
& 6w_7w_3^2v_2^2w_8w_5^3 + 12w_9c_8^2w_7^2w_4^2v_2^2w_8 + 12w_9c_8^2w_7w_4^3w_8w_5^2 + 6c_8^4w_7^2w_4^3w_8w_5^3 - 12c_8^2w_7w_4^2w_8w_5^3 + 60w_9c_8^2w_7w_4^2w_8v_1^2w_5^2 + 12w_9c_8^4w_7w_4^3w_8w_5^3 + \\
& 24w_9c_8^4w_7^2w_4^2w_8w_5^2 + 18w_9w_7w_3^2v_2^2w_8v_1^2w_5^2 + 12w_7w_4^2v_2^2w_8w_5^3 - 24w_9c_8^2w_7^2w_4^2w_8w_5^2 + 6w_7^2w_4^2v_2^2w_8w_5^2 - 12w_9c_8^2w_7w_4^3w_8w_5^3 + 36c_8^2w_7w_4^2w_8v_1^2w_5^3 - \\
& 6c_8^2w_7^2w_4^3w_8w_5^3 - 48w_9c_8^2w_7w_3^2v_1^2w_5^3 - 24w_9c_8^2w_3^4w_8v_2^2w_5^2 + 12w_9w_7w_4^2v_2^2w_8v_1^2w_5^3 - 42w_9c_8^2w_7^2w_4^2w_8v_2^2w_5^3 + 72w_9c_8^2w_7w_4^3w_8v_2^2w_5^3 + 18w_7^2w_3^2v_2^2w_8v_1^2w_5^3 - \\
& 36w_9w_7w_4^2v_2^2w_8v_1^2w_5^3 + 12w_9c_8^2w_7^2w_4^2w_8w_5^2 - 12w_7w_4^2v_2^2w_8w_5^3 + 18w_9c_8^2w_7^2w_4^3v_2^2w_5^3 + 36c_8^2w_7w_4^2v_2^2w_5^3 - 18c_8^2w_7w_4^3v_2^2w_5^3 + 24w_9w_7w_4v_2^2w_8w_5^3 - \\
& 12w_9c_8^2w_7w_4w_8w_5^3 - 12w_9c_8^4w_7^2w_4^2w_5^2 + 12w_9w_4^2v_2^2w_8w_5^2 + 72w_9w_7w_4^3v_2^2w_8v_1^2w_5^2 - 144w_9c_8^2w_7w_4^3w_8v_1^2w_5^2 - 24w_9c_8^2w_7w_4^2v_2^2w_8w_5^3 + \\
& 24w_9c_8^2w_7w_4v_2^2w_8w_5^2 - 12w_9c_8^4w_7^2w_4^2w_8w_5^2 + 24w_9c_8^2w_7w_4^3v_2^2w_8v_1^2w_5^3 - 24w_9w_7w_4^2v_2^2w_8w_5^2 - 36c_8^2w_7w_4^3v_2^2w_8v_1^2w_5^3 - 54w_9w_7w_4^3v_2^2w_8v_1^2w_5^3 - \\
& 36w_7w_3^2v_2^2w_8v_1^2w_5^3 + 12w_9c_8^4w_7w_4w_8w_5^3 - 36w_9w_7w_4^3v_2^2w_8v_1^2w_5^3 - 108w_9c_8^2w_7w_4^2w_8v_1^2w_5^3 + 78w_9c_8^2w_7w_4^3w_8v_1^2w_5^3 - 12w_9w_7w_4^3v_2^2w_8w_5^3 + \\
& 18w_9w_7w_4^2v_2^2v_1^2w_5^2 + 18c_8^2w_7w_3^2v_8w_1^2w_5^3 + 6w_7w_4^2v_2^2w_5^3 + 12w_7w_4^2v_2^2w_8w_5^3 + 36c_8^2w_7w_4^3w_8v_1^2w_5^2 + 36w_7w_4^2v_2^2w_8v_1^2w_5^2 + 6c_8^2w_7^2w_4^3w_8v_1^2w_5^3 - \\
& 18w_9c_8^2w_7w_3^2v_8w_1^2w_5^2 - 12c_8^2w_7w_4^3v_2^2w_8w_5^3 + 12c_8^2w_7^2w_4^2w_8w_5^3 + 36w_7w_4^2v_2^2w_8v_1^2w_5^3 + 36w_9w_7w_4^2v_2^2w_8v_1^2w_5^2 - 12c_8^2w_7w_4^3w_8w_5^3 + \\
& 24w_9c_8^2w_7w_3^2v_8w_1^2w_5^3 + 6w_9c_8^2w_7^2w_4^3w_8w_5^2 - 84w_9c_8^2w_7^2w_4w_8v_1^2w_5^2 - 72w_9w_7w_4v_2^2w_8v_1^2w_5^3 - 6w_9w_7w_4^2v_2^2w_5^2 - 12w_9c_8^2w_4^2v_2^2w_8w_5^3 - \\
& 6w_9c_8^4w_7^2w_4^2w_8w_5^2 - 24w_9c_8^4w_7w_4w_8w_5^3 - 12w_9c_8^2w_7^2w_8w_5^3 + 12c_8^2w_7w_3^2w_8w_5^2 - 12c_8^2w_7^2w_4^2w_8w_5^3 - 36w_9w_7w_4^2v_2^2w_8v_1^2w_5^2 + 24w_9c_8^2w_7w_4^3v_2^2w_8w_5^2 + \\
& 60w_9c_8^2w_7w_4w_8v_1^2w_5^3 - 6w_9c_8^2w_7w_3^2w_5^3 - 12w_9c_8^2w_7^2w_4^3v_2^2w_8 - 24w_9w_7w_4^2v_2^2w_8w_5^2 - 6c_8^2w_7w_3^2v_2^2w_8w_5^2 + 6w_9c_8^2w_7^2w_3^2v_2^2w_8w_5^2 + 84w_9c_8^2w_7^2w_4w_8v_1^2w_5^3 - \\
& 12w_9c_8^2w_7w_3^2w_8w_5^2 + 12c_8^2w_7w_4^2v_2^2w_5^3 - 6w_9w_7w_4^2v_2^2w_8w_5^2 + 12c_8^2w_7w_4^3v_2^2w_8w_5^2 + 12c_8^2w_7w_4^2w_8v_1^2w_5^3 - 12w_9w_7w_4^2v_2^2w_8w_5^2 + \\
& 12w_9w_7w_4^3v_2^2w_8w_5^3 + 36w_7w_4^2v_2^2v_1^2w_5^3 - 6c_8^2w_7w_3^2v_2^2w_5^3 - 12c_8^2w_7w_4^3v_2^2w_8w_5^3 - 12w_9c_8^2w_7w_4^2v_2^2w_8w_5^3 + 108w_9w_7w_4^2v_2^2w_8v_1^2w_5^3 + 12w_9c_8^2w_7w_4^2v_2^2w_8w_5^2 - \\
& w_9c_8^2w_7w_4^3w_8w_5^3 - 12w_9c_8^2w_7w_4v_2^2w_8w_5^2 - 12w_9c_8^2w_7w_4^2v_2^2w_8w_5^3 + 12w_9c_8^2w_7w_4^3v_2^2w_8w_5^2 + 18w_9c_8^2w_7^2w_4w_8w_5^3 - \\
& 24w_9c_8^2w_7w_2^2v_2^2w_8w_5^2 + 36w_9w_7c_8^2v_2^2w_8v_1^2w_5^3 - 12w_9c_8^2w_7^2w_4^3v_2^2w_8w_5^2 - 18w_9c_8^2w_7^2w_4^2v_2^2w_8w_5^2 + 36w_9c_8^2w_7w_4^3v_2^2w_8w_5^3 + 36w_9c_8^2w_7w_4^2v_2^2w_8v_1^2w_5^3 - \\
& 18w_7w_3^2v_2^2v_1^2w_5^3 - 12w_9w_7w_4^2v_2^2w_5^3 - 36w_9w_7w_4v_2^2w_8w_5^3 + 18w_9w_7w_4^2v_2^2w_8w_5^2 - 12c_8^2w_7w_4^2v_2^2w_8w_5^3 + 12w_9c_8^2w_7^2w_4^3v_2^2w_8w_5^3 + 6w_9c_8^2w_7^2w_4^3w_8w_5^3 - \\
& 36w_9w_7c_8^2v_2^2w_8v_1^2w_5^3 - 12w_9c_8^2w_7w_4^3v_2^2w_8w_5^2 - 36w_7w_4^2v_2^2w_8v_1^2w_5^3 - 12w_9c_8^4w_7^2w_4w_8w_5^2 + 12c_8^2w_7w_4^2v_2^2w_8w_5^3 + 12w_9w_7w_4^3v_2^2w_8w_5^2 - 6c_8^4w_7w_4^3w_8v_1^2w_5^3 - \\
& 72w_9w_7w_4^2v_2^2w_8v_1^2w_5^2 - 18c_8^2w_7w_4^3v_2^2w_8v_1^2w_5^3 + 12w_9w_7w_4^2v_2^2w_8w_5^2 - 36w_9c_8^2w_7w_4^2v_2^2w_8v_1^2w_5^2 - 6w_9c_8^2w_7w_4^2v_2^2w_8w_5^2 + 12w_9c_8^2w_7w_4w_8w_5^2
\end{aligned}$$

$$\begin{aligned}
C_{15} = & -2w_9w_6w_7w_4^2w_8^2v_1^2w_5^2 + 8w_9w_6c_8^2w_4^2w_8^2w_5^3 - 16w_9w_6c_8^2w_7w_8^2w_5^3 + 12w_9w_6c_8^2w_7w_4^2w_8^3w_5^2 - 4w_6c_8^2w_7w_4^2w_8^2w_5^3 + 4w_6w_7w_8^2v_1^2w_5^3 - \\
& 4w_6w_7w_4^2w_8^2w_5^2 + 4w_6w_7^2w_4^2w_8^2w_5^3 - w_9w_6w_7^2w_4^2w_8^2w_5^2 - 9w_9w_6w_7w_4^2w_8^2w_5^3 - w_9w_6w_7w_4^2w_8^2v_1^2w_5^3 + 13w_9w_6c_8^2w_7w_4^2w_8^2w_5^2 - 4w_9w_6w_7w_4^2w_8^2v_1^2w_5^3 - \\
& 8w_9w_6c_8^2w_7w_4^2w_8^2w_5^3 + 4w_9c_8^2w_7w_4^2w_8w_5^3 + 4w_9w_6c_8^2w_7w_4^2w_8^2w_5^3 - 4w_6c_8^2w_7w_4^2w_8^2w_5^3 - 4w_6w_8^2w_4^2w_8^2v_1^2w_5^3 + 4w_6c_8^2w_7w_4^2w_8^2w_5^2 - 8w_9w_6c_8^2w_7w_4^2w_8^2w_5^2 + \\
& 9w_9w_6w_7w_4^2w_8^2w_5^3 - 5w_9w_6w_7w_4^2w_8^2v_1^2w_5 + 12w_9w_6c_8^2w_7w_4w_8w_5^3 - 5w_9w_6c_8^2w_7w_4^2w_8^2w_5^3 - 3w_9w_6w_7^2w_4^2w_8^2v_1^2w_5^2 - 4w_9c_8^2w_7w_4^2w_8^2w_5^2 - \\
& 9w_9w_6c_8^2w_7w_4^2w_8^2w_5^2 + 2w_9w_6w_7w_4^2w_8^2w_5^3 + 4w_6w_7w_4^2w_8^2w_5^3 + 2w_6w_7^2w_4^2w_8^2w_5^3 - 4w_9w_6w_7^2w_4^2w_8^2w_5^3 - 2w_9w_6w_7^2w_4^2w_8^2w_5^2 + 2w_9w_6c_8^2w_7w_4^2w_8^2w_5^3 + \\
& 2w_9w_6c_8^2w_7w_4^2w_8^2w_5^3 - 4w_9w_6w_7^2w_4^2w_8^2w_5^2 + 2w_9w_6w_7^2w_4^2w_8^2v_1^2w_5^3 - 3w_9w_6w_7^2w_4^2w_8^2v_1^2w_8^3 + 8w_9w_6c_8^2w_7w_4^2w_8^2w_5^3 - 6w_9w_6c_8^2w_7w_4^2w_8^2w_5^3 + \\
& 2w_6w_7w_4^2w_8^2v_1^2w_5^3 + 4w_6w_7w_4^2w_8^2v_1^2w_5^2 + w_9w_6w_7^2w_4^2w_8^2v_1^2w_5^2 + 4w_9w_6w_7^2w_4^2w_8^2v_1^2w_5^3 - 2w_6w_7^2w_4^2w_8^2v_1^2w_5^2 - 4w_9w_6c_8^2w_7w_4^2w_8^2v_1^2w_5^3 + 5w_9w_6w_7^2w_4^2w_8^2w_5^3 + \\
& 2w_9w_6w_7^2w_4^2w_8w_5^3 + 7w_9w_6w_7w_4^2w_8^2v_1^2w_5^2 - 8w_9w_6c_8^2w_7w_4^2w_8^2w_5^2 - 2w_9w_6w_7^2w_4w_8w_5^2 + 4w_9w_6w_7^2w_4^2w_8^2w_5^2 - 2w_6w_7^2w_4^2w_8^2w_5^3 - 4w_6w_7w_4^2w_8^2v_1^2w_5^3 + \\
& 2w_9w_6w_7^2w_4^2w_8^2v_1^2w_5^3 + 3w_9w_6c_8^2w_7w_4^2w_8^2w_5^2 + 4w_9w_6w_7^2w_4^2w_8^2v_1^2w_5^3 - 2w_6w_7^2w_4^2w_8^2v_1^2w_5^3 + 11w_9w_6c_8^2w_7w_4^2w_8^2w_5^3 + 2w_6w_7^2w_4^2w_8^2w_5^2 - 4w_6w_7w_4^2w_8^2w_5^3 - \\
& 7w_9w_6w_7w_4^2w_8^2w_5^2 + w_9w_6w_7^2w_4^2w_8^2w_5^3 + 2w_6c_8^2w_7w_4^2w_8^2w_5^3 + 3w_9w_6w_7w_4^2w_8^2w_5^3 + 3w_9w_6w_7w_4^2w_8^2w_5^2 - 2w_6w_7^2w_4^2w_8^2w_5^3 - 2w_9c_8^2w_7w_4^2w_8^2w_5^3 + \\
& 2w_9w_6w_7^2w_4^2w_8^2v_1^2w_5^2 + 2w_9w_6c_8^2w_7w_4^2w_8^2w_5^2 - 4w_9w_6w_7^2w_4^2w_8^2v_1^2w_5^2 - 6w_9w_6w_7w_4^2w_8^2v_1^2w_5^3 + w_9w_6w_7w_4^2w_8^2v_1^2w_5^2 - \\
& 2w_6c_8^2w_7w_4^2w_8^2w_5^2 - 8w_9w_6c_8^2w_7w_4^2w_8^2w_5^3 + 4w_6c_8^2w_7w_4^2w_8w_5^3 - 2w_9w_6w_7w_4^2w_8^2w_5^3 + 8w_9w_6c_8^2w_7w_4^2w_8^2w_5^2 - 2w_9w_6w_7^2w_4^2w_8^2v_1^2w_5^3 + \\
& 26w_9w_6c_8^2w_7w_4^2w_8^2w_5^3 + 6w_9w_6w_7w_4w_8w_5^3 - w_9w_6w_7^2w_4^2w_8^2w_5^3 + 4w_6w_7^2w_4^2w_8^2v_1^2w_5^3 - 4w_9w_6w_7^2w_4^2w_8^2v_1^2w_5^3 - 15w_9w_6c_8^2w_7w_4^2w_8^2w_5^3 + 4w_9w_6w_7^2w_4^2w_8^2w_5^3 - \\
& 4w_6w_7^2w_4^2w_8w_5^3 + 2w_9c_8^2w_7w_4^2w_8^2w_5^3 - 6w_9w_6c_8^2w_7w_4^2w_8^2w_5^2 - 24w_9w_6c_8^2w_7w_4^2w_8^2w_5^3 + 4w_9w_6w_7^2w_4^2w_8^2w_5^3
\end{aligned}$$

$$\begin{aligned}
C_{16} = & -24w_9w_6^2w_7w_4^2v_2^2w_8v_1^2 - 36w_9w_6c_4^4w_7w_4w_8^2w_5 - 12w_6^2c_2^2w_4^3w_8^2 + w_9w_6^2c_2^2w_7w_3^4v_2^2w_8w_5 - w_9w_6^2c_2^2w_7w_3w_4^3w_8w_5 + 12w_9w_6c_2^2w_7w_4w_8^2w_5 - \\
& 12w_6^2c_2^2w_7w_3^2v_2^2w_8w_5 + 6w_9w_6c_2^2w_7w_3^4v_2^2w_8^2 - 36w_6^2c_2^2w_4^3w_8^2v_1^2w_5 + 3w_9w_6c_2^4w_7w_4w_8^2w_5 - 9w_6w_6w_7w_4^3w_8^2v_1^2w_5 - 12w_9w_6^2c_2^2w_7w_4v_2^2w_8w_5 - \\
& 36w_9w_6c_4^4w_8^2w_5 + 12w_9w_6w_3^2w_8^2v_1^2 - 12w_6^2w_2^2w_8^2v_1^2w_5 - 96w_9w_6^2c_4^2w_7w_4w_8^2w_5 + 18w_6^2c_4^2w_7w_4^3w_8^2w_5 - 36w_9w_6c_2^2w_4^2w_8^2v_1^2w_5 - \\
& 12w_9w_6w_7w_4^2v_2^2w_8v_1^2w_5 + 12w_9w_7w_2^2v_2^2w_8^2v_1^2w_5 - 12w_9w_6w_3^2v_2^2w_8^2v_1^2 - 12w_9w_6c_2^2w_3^4v_2^2w_8^2v_1^2 + 12w_6^2w_7w_4^2w_2^2v_1^2w_5 + 36w_6^2c_2^2w_7w_4^2w_8v_1^2w_5 + \\
& 18w_9w_6^2c_2^2w_7w_3^4v_2^2w_5 - 12w_9w_6^2w_7w_3^2w_8v_1^2 + 9w_9w_6w_7w_3^4v_2^2w_8^2v_1^2w_5 - 18w_6^2s_w7w_3^2w_8^2v_1^2 + 27w_9w_6c_2^2w_7w_4^2w_8^2v_1^2w_5 - 45w_9w_6^2c_2^2w_7w_3^4w_8v_1^2w_5 - \\
& 12w_9w_6c_2^2w_7w_4v_2^2w_8w_5 + 12w_9w_6w_4^2s_w7w_2^2w_5 - 6w_2^2s_w7w_4^3w_8^2w_5 - 15w_9w_6^2w_7w_3^4v_2^2w_8v_1^2w_5 + 12w_6^2w_4^2v_2^2w_8^2v_1^2w_5 - 18w_6^2s_w7w_3^4w_8v_1^2w_5 - \\
& 6w_6^2w_7w_4^3v_2^2w_8v_1^2w_5 + 24w_9w_6^2w_7w_4w_8v_1^2w_5 - 6w_6^2w_7w_4^2w_8^2v_1^2w_5 - 42w_9w_6^2c_4^2w_7w_4w_8^2w_5 + 36w_9w_6c_2^2w_3^4w_8^2v_1^2w_5 - 6w_6^2s_w7w_4^2v_2^2w_8^2 + \\
& 18w_9w_6c_4^4w_7w_3^2w_8^2 - 12w_6^2w_3^2w_8^2v_1^2 + 36w_6^2c_4^2w_3^2w_8^2 - 24w_9w_6w_7w_4w_8^2v_1^2 - 12w_6^2c_2^2w_4^2w_8^2w_5 - 36w_9w_6c_2^2w_3^2w_8^2v_1^2 - 12w_6^2w_7w_4^2v_2^2w_8^2v_1^2w_5 + \\
& 6w_6^2w_7w_4^2w_8v_1^2 - 12w_9w_6c_2^2w_4^2w_8^2w_5 - 72w_9w_6^2s_w7w_2^2w_8v_1^2 + 12w_9w_6^2w_7w_4^2v_2^2w_5 + 144w_9w_6^2s_w7w_4^2w_8v_1^2w_5 - 108w_9w_6c_2^2w_7w_4^2w_8^2v_1^2w_5 + \\
& 12w_9w_6c_2^2w_3^2w_8^2 - 18w_9w_6^2c_2^2w_7w_4w_8w_5 - 12w_9w_6w_2^2v_2^2w_8^2v_1^2w_5 + 156w_9w_6^2c_4^2w_7w_4w_8^2w_5 - 12w_6^2c_2^2w_7w_4^2w_8^2v_1^2w_5 + 6w_9w_6^2c_2^2w_7w_3^4w_8^2v_1^2w_5 - \\
& 12w_9w_6^2c_2^2w_7w_4^2v_2^2w_8w_5 - 36w_9w_6w_4^2s_w7w_4^2w_8^2w_5 + 5w_9w_6c_2^2w_7w_4w_8^2w_5 - 18w_9w_6c_2^2w_7w_4^3w_8^2v_1^2w_5 + 12w_6^2s_w7w_4^2w_8^2v_1^2w_5 + 6w_6^2s_w7w_3^4w_8^2v_1^2w_5 - \\
& 6w_9w_6^2c_4^2w_7w_4^2w_8w_5 + 36w_6^2c_4^2w_7w_4^2w_8w_5 - 36w_6^2c_4^2w_7w_4^2w_8^2w_5 + 12w_6^2w_3^2w_8^2v_1^2w_5 - 12w_9w_6^2c_2^2w_7w_4w_8^2w_5 + 36w_9w_6w_7w_4^2w_8^2v_1^2w_5 + \\
& 36w_6^2c_2^2w_7w_4^2w_8^2v_1^2w_5 - 15w_9w_6c_4^4w_7w_3^2w_8^2w_5 + 6w_6^2c_4^2w_7w_4^2v_2^2w_8^2w_5 + 24w_9w_6w_7w_4^2v_2^2w_8^2w_5 + 18w_9w_6^2c_2^2w_7w_4^2v_2^2w_8w_5 + 12w_6^2c_2^2w_7w_4^2w_8^2w_5 + \\
& 12w_9w_6^2c_4^4w_7w_4w_8w_5 + 12w_6^2w_3^2v_2^2w_8^2v_1^2 + 12w_6^2c_3^2w_4^2v_2^2w_8^2 + 12w_6^2s_w2^2w_4^2v_2^2w_8w_5 + 6w_6^2w_7w_3^2w_8v_1^2w_5 + 18w_6^2c_2^2w_7w_4^2w_8^2v_1^2w_5 + \\
& 6w_6^2w_7w_4^2v_2^2w_8^2v_1^2w_5 + 12w_9w_6^2c_2^2w_7w_4w_8w_5 - 36w_9w_6c_2^2w_7w_4^2w_8^2v_1^2w_5 - 36w_6^2c_4^2w_7w_4^2w_8^2w_5 - 12w_9w_7w_4^2w_8^2v_1^2w_5 + 12w_6^2w_7w_4^2v_2^2w_8v_1^2w_5 - \\
& 6w_6^2s_w7w_4^2w_8w_5 + 24w_9w_6^2w_7w_4^2w_8v_1^2 - 36w_9w_6^2s_w7w_4^2v_1^2w_5 + 36w_9w_6^2s_w7w_4^2w_8^2v_1^2w_5 + 6w_9w_6^2w_7w_4^3v_2^2w_8v_1^2w_5 + 18w_9w_6c_2^2w_7w_4^2v_2^2w_8^2w_5 -
\end{aligned}$$

$$\begin{aligned}
& 60w_9w_6^2c_8^4w_7w_4^2w_8^2w_5 + 15w_9w_6^2w_7w_3^4w_8w_8v_1^2w_5 + 12w_9w_6w_4^2w_8^2v_1^2w_5 - 6w_6^2w_7w_3^4v_2^2w_8v_1^2 + 12w_9w_6w_7w_4^2w_8^2v_1^2w_5 + 12w_9w_6^2w_8^2w_7w_4^2w_8^2w_5 + \\
& 12w_9w_6c_8^2v_2^2w_8^2w_5 + 12w_9w_6w_4^3v_2^2w_8^2v_1^2w_5 + 12w_9w_6^2c_8^2w_7w_4^2w_8^2w_5 - 6w_9w_6c_8^2w_7w_3^4v_2^2w_8^2w_5 - 5w_9w_6c_8^2w_7w_4^3v_2^2w_8^2w_5 - 72w_9w_6^2w_8^2w_7w_4w_8v_1^2w_5 - \\
& 12w_9w_6^2w_7w_4^2v_2^2w_8^2w_5 + 36w_6^2c_8^4w_4^2w_8^2w_5 - 18w_6^2c_8^4w_3^2w_8^2w_5 + 72w_9w_6c_8^2w_7w_4w_8v_1^2w_5 + 54w_9w_6c_8^4w_7w_4^2w_8^2w_5 - 12w_9w_6^2c_8^2w_7w_4^2w_8^2w_5 + \\
& 6w_6^2c_8^2w_7w_3^4w_8w_5 - 6w_9w_6w_7w_3^4w_8v_1^2w_5 - 12w_9w_6c_8^2w_4^2v_2^2w_8^2w_5 + 24w_9w_6w_7w_4w_8v_2^2w_8^2v_1^2 + 72w_9w_6c_8^2w_7w_4^2w_8^2w_5 - 12w_9w_6w_4^3w_8^2w_5 - \\
& 18w_9w_6c_8^2w_7w_4^2w_8^2w_5 - 48w_9w_6w_7w_4^2w_8v_1^2w_5 + 36w_9w_6c_8^4w_3^2w_8^2w_5 - 24w_9w_6^2w_7w_4v_2^2w_8^2v_1^2w_5 - 36w_6^2c_8^2w_7w_4^2v_2^2w_8^2v_1^2w_5 + 12w_9w_6c_8^2w_4^2w_8^2w_5 - \\
& 18w_9w_6^2c_8^2w_7w_4^3w_8w_5 - 18w_6^2c_8^4w_7w_3^4w_8w_5 - 12w_6^2c_8^2w_3^2w_8^2w_5 - 12w_6^2w_3^2v_2^2w_8^2v_1^2w_5 + 3w_9w_6^2w_7w_4^2w_8^2v_1^2w_5 - \\
& 36w_9w_6w_7w_4^2v_2^2w_8^2v_1^2w_5 - 6w_9w_6w_7w_3^2v_2^2w_8^2v_1^2 + 48w_9w_6^2w_7w_4v_2^2w_8v_1^2w_5 - 3w_9w_6^2c_8^2w_7w_4^3v_2^2w_8^2v_1^2w_5 - 24w_9w_6w_7w_4w_8v_1^2w_5 - 6w_9w_6^2w_7w_3^4v_1^2w_5 + \\
& 36w_9w_6^2c_8^2w_7w_4^3w_8w_1^2v_5 - 18w_9c_8^2w_7w_4^3w_8v_1^2w_5 - 6w_9w_7w_4^3v_2^2w_8v_1^2w_5 + 12w_6^2c_8^2w_7w_3^2v_2^2w_8w_5 + 6w_9w_7w_4^3w_8v_1^2w_5 + 6w_9w_6w_7w_3^4w_8v_1^2 + \\
& 12w_6^2c_8^2w_4^3w_8^2w_5 + 6w_9w_6w_7w_4^2v_2^2w_8v_1^2w_5 + 15w_9w_6^2c_8^4w_7w_4^2w_8w_5 + 36w_6^2c_8^2w_4^3w_8^2v_1^2 + 18w_9w_6c_8^2w_7w_4^3w_8w_1^2w_5 + 12w_9w_6^2w_8^2w_7w_4^2w_8^2w_5 \\
\\
C_{17} = & -48w_9w_6^2c_8^2w_7w_4^2w_8^2w_5 - 24w_9w_6^2c_8^2w_7^2w_4^3w_8w_5 + 24w_9w_6^2w_7^2w_4^2w_8w_5 + 72w_9w_6^2c_8^2w_7w_4w_8w_5^2 + 12w_9w_6^2w_7w_4^2w_8^2w_5^2 + 24w_6^2w_7^2w_4^2w_8^2w_5^2 - \\
& 84w_9w_6^2c_8^2w_7w_4^3w_8w_5^2 - 12w_6^2w_7^2w_4^3v_2^2w_8w_5^2 - 4w_9w_6w_7w_4^2w_8^3w_5^2 - 96w_9w_6^2c_8^2w_7w_8^2w_5^2 - 24w_9w_6^2c_8^2w_3^2w_8^2w_5^2 - 12w_9w_6w_7w_4^2v_2^2w_8w_5^2 + \\
& 84w_9w_6^2c_8^2w_7^2w_4^2w_8^2w_5^2 + 24w_6^2w_7w_4^3w_8w_5^2 - w_9w_6^2w_7w_4^2w_8^2w_5^2 + 12w_9w_6^2w_4^2v_2^2w_8w_5^2 - 18w_9w_6^2w_7w_4^2w_8^2w_5^2 + 24w_6^2c_8^2w_7w_4^3w_8w_5^2 - \\
& 24w_9w_6^2w_7^2w_4^2v_2^2w_8w_5^2 - 42w_9w_6c_8^2w_7^2w_4^2w_8^2w_5^2 - 12w_6^2c_8^2w_7^2w_4^2w_8^3w_5^2 + 12w_9w_6^2w_7w_4v_2^2w_8w_5^2 + 12w_9w_6w_7w_4^2w_8^2w_5^2 + 66w_9w_6^2w_7w_4^2v_2^2w_8w_5^2 - \\
& 24w_6^2c_8^2w_7w_3^2w_8w_5^2 - 6w_9w_7w_4^3v_2^2w_8w_5^2 + 6w_9w_6w_7w_4^2v_2^2w_8w_5^2 + 12w_9w_6^2w_7w_4^2v_2^2w_8w_5^2 - 24w_6^2w_7w_4^3w_8w_5^2 - 12w_9w_6^2w_4^2w_8w_5^2 - \\
& 66w_9w_6^2c_8^2w_7^2w_4^2w_8w_5^2 + 24w_9w_6^2c_8^2w_7^2w_4^3w_5^2 + 12w_9w_6^2w_7w_4^2v_2^2w_8w_5^2 - 12w_9w_6^2w_7w_4w_8w_5^2 + 12w_6^2w_7w_3^2w_8w_5^2 + 12w_9w_6w_7w_4^2w_8w_5^2 - \\
& 18w_9c_8^2w_7w_4^3w_8w_5^2 + 24w_9w_6^2c_8^2w_7^2w_4^2w_8w_5^2 - 24w_9w_6^2c_8^2w_7^2w_4^2w_8w^2 + 6w_9w_6c_8^2w_7w_4^2w_8w_5^2 - 24w_6^2c_8^2w_7w_4^2w_8^2w_5^2 - 12w_9w_6w_7w_4w_8w_5^2 + \\
& 12w_9w_6^2c_8^2w_7w_4^3w_8w_5^2 - 66w_9w_6^2w_7w_4^2w_8w_5^2 - 36w_9w_6^2c_8^2w_7^2w_4^2w_8w_5^2 + 60w_9w_6^2c_8^2w_7w_3^2w_8w_5^2 + 24w_6^2w_7w_4^2w_8^2w_5^2 + 12w_9w_6c_8^2w_7w_4^2w_8^2w_5^2 - \\
& 12w_6^2c_8^2w_7^2w_4^2w_8^2w_5^2 + 24w_6^2c_8^2w_7w_4^2w_8w_5^2 - 12w_9w_6^2w_7w_4^2v_2^2w_8w_5^2 + 12w_9w_6^2w_7w_4^2w_8^2w_5^2 - 12w_9w_6^2w_7w_4^2v_2^2w_8w_5^2 - 24w_6^2w_7w_4^2w_8^2w_5^2 - \\
& 24w_6^2c_8^2w_7^2w_4^2w_8^2w_5^2 + 156w_9w_6^2c_8^2w_7w_4^2w_8w_5^2 + 24w_9w_6^2c_8^2w_8w_5^2 + 18w_9w_6w_7w_2^2w_8^2w_5^2 - 12w_9w_6^2w_7w_2^2w_8^2w_5^2 + 24w_6^2w_7w_4^2v_2^2w_8w_5^2 + \\
& 4w_9w_6w_7w_4^2v_2^2w_8w_5^2 - 12w_6^2c_8^2w_7^2w_4^2w_8w_5^2 - 12w_9w_6^2w_7w_4^2w_8w_5^2 + 90w_9w_6^2w_7w_4^2w_8w_5^2 + 24w_6^2w_7w_4^2v_2^2w_8w_5^2 + 48w_9w_6^2c_8^2w_7w_4^2w_8w_5^2 - \\
& 12w_9w_6^2w_7w_4^2w_8^3w_5^2 + 36w_9w_6^2w_7w_4^2w_8w_5^2 - 12w_6^2w_7w_4^2v_2^2w_8w_5^2 - 12w_9w_6^2w_7w_4^2w_8w_5^2 + 18w_9w_6^2w_7w_4^2w_8w_5^2 - 24w_6^2w_7w_4^2v_2^2w_8w_5^2 - \\
& 18w_9w_6w_7w_4^2v_2^2w_8w_5^2 - 24w_9w_6^2c_8^2w_7w_4^2w_8w_5^2 + 12w_6^2w_7w_4^2w_8^3w_5^2 - 132w_9w_6^2c_8^2w_7w_4^2w_8w_5^2 + 12w_9w_6^2w_7w_4^2w_8^3w_5^2 + w_9w_6^2w_7w_4^2v_2^2w_8w_5^2 + \\
& 12w_9w_6^2w_7w_4^2v_2^2w_8w_5^2 + 36w_9w_6^2c_8^2w_7w_4^2w_8w_5^2 + 36w_9w_6^2c_8^2w_7w_4^2w_8w_5^2 - 36w_9w_6^2w_7w_4^2v_2^2w_8w_5^2 + 12w_6^2c_8^2w_7w_4^2v_2^2w_8w_5^2 + 24w_9w_6^2w_7w_4^2w_8^3w_5^2 - \\
& 12w_9w_6^2w_7w_4^2w_8^2w_5^2 + 3w_9w_6^2c_8^2w_7^2w_4^2w_8w_5^2 + 6w_9w_6^2w_7w_4^2w_8w_5^2 - 24w_6^2w_7w_4^2w_8w_5^2 - 72w_9w_6^2w_7w_4w_8w_5^2 + 24w_6^2c_8^2w_7w_4^2w_8^2w_5^2 - \\
& 6w_9w_6w_7w_4^2w_8w_5^2 + 24w_9w_6^2c_8^2w_7w_4^2w_8w_5^2 + 12w_6^2w_7w_4^2v_2^2w_8w_5^2 - 24w_9w_6^2w_7w_4^2v_2^2w_8w_5^2 + 12w_9w_6c_8^2w_7w_4w_8w_5^2 + 12w_9w_6^2w_7w_4^2v_2^2w_8w_5^2
\end{aligned}$$

$$\begin{aligned}
C_{18} = & -72w_9w_6^2w_7w_4^2v_2w_8v_1^2 - 12w_9w_6c_s^4w_7w_4w_8w_5 - 12w_6^2c_s^2w_3^4w_8^2 + 12w_9w_6c_s^2w_7w_4w_8^2w_5 - 36w_6^2s_w^2w_7w_4^2w_8^2w_5 + 18w_9w_6c_s^2w_7w_3v_2^2w_8^2 - \\
& 12w_6^2c_s^3w_3^2w_8v_1w_5 - w_9w_6^2c_s^5w_7w_3^4w_8^2w_5 - 9w_9w_6w_7w_3^2w_8^2v_1^2w_5 + 60w_9w_6c_s^2w_7w_4v_2^2w_8w_5 - 12w_9w_6c_s^4w_3^4w_8^2v_1^2 - \\
& 12w_6w_8^2c_s^4w_8^2v_1^2w_5 - 12w_9w_6^2c_s^4w_7w_3^2w_8^2w_5 - 12w_9w_6c_s^2w_4^2w_8^2v_1^2w_5 - 36w_9w_6w_7w_2^2v_2^2w_8v_1^2w_5 + 36w_9w_7w_4v_2^2w_8^2v_1^2w_5 - \\
& 36w_9w_6w_3^2v_2^2w_8v_1^2 - 36w_9w_6c_s^2w_3^2v_2^2w_8 + 12w_6^2w_7w_4^2w_8^2v_1^2w_5 + 12w_6^2c_s^2w_7w_4w_8^2v_1^2w_5 - 12w_9w_6^2c_s^2w_7w_4^2w_8^2v_1^2 + \\
& 27w_9w_6w_7w_3^2w_8^2v_1^2w_5 - 12w_6^2c_s^2w_7w_4w_8^2v_1^2 - 9w_9w_6c_s^2w_7w_4w_8^2v_1^2w_5 - 15w_9w_6c_s^2w_7w_4w_8^2v_1^2w_5 - 36w_9w_6c_s^2w_7w_4w_8^2v_1^2w_5 - 6w_6^2c_s^2w_7w_4w_8^2v_1^2w_5 - \\
& 45w_9w_6^2w_7w_3^2w_8v_1^2w_5 + 36w_6^2w_4^2v_2^2w_8^2v_1^2 - 6w_6^2s_w^2w_7w_4w_8^2v_1^2 - 18w_6^2w_7w_3^2v_2^2w_8v_1^2w_5 - 48w_9w_6c_s^2w_7w_2^2w_8^2w_5 + 24w_9w_6^2w_7w_4w_8v_1^2w_5 - \\
& 6w_6^2w_7w_4^2w_8^2v_1^2w_5 - 18w_9w_6c_s^2w_7w_4w_8^2w_5 + 12w_9w_6c_s^3w_3^2w_8^2v_1^2w_5 - 18w_6^2c_s^2w_7w_4^2v_2^2w_8 + 6w_9w_6c_s^4w_7w_4w_8^2w_5 - 12w_6^2w_7w_3^2w_8^2v_1^2 + 12w_6^2c_s^4w_8^2w_5 - \\
& 24w_9w_6w_7w_4^2s_w^2v_1^2 - 12w_6^2c_s^2w_4^2w_8^2w_5 - 12w_9w_6c_s^2w_3^4w_8^2v_1^2 - 36w_6^2w_7w_4v_2^2w_8^2v_1^2w_5 + 6w_6^2w_7w_3^4w_8^2v_1^2 - 12w_9w_6c_s^2w_3^2w_8^2w_5 - \\
& 24w_9w_6^2c_s^2w_7w_4^2w_8v_1^2 + 12w_9w_6^2w_7w_4^2v_1^2w_5 + 48w_9w_6^2c_s^2w_7w_4w_8^2v_1^2w_5 - 36w_9w_6c_s^2w_7w_4^2w_8^2v_1^2w_5 + 12w_9w_6c_s^2w_3^4w_8^2v_1^2 + 18w_9w_6^2c_s^2w_7w_4^2w_8^2w_5 - \\
& 36w_9w_6w_4^2v_2^2w_8^2v_1^2w_5 + 18w_9w_6^2c_s^2w_7w_4w_8^2w_5 - 12w_6^2c_s^2w_7w_4^2w_8w_5 - 12w_9w_6c_s^2w_7w_4^2v_2^2w_8^2v_1^2w_5 - 15w_9w_6^2c_s^2w_7w_4^2w_8^2v_1^2w_5 - 12w_9w_6c_s^4w_7w_4^2w_8^2v_1^2w_5 + \\
& 5w_9w_6c_s^2w_7w_4^2w_8^2v_1^2w_5 - 6w_9w_6c_s^2w_7w_3^2w_8^2v_1^2 + 36w_9w_6c_s^2w_7w_4v_2^2w_8v_1^2 + 6w_6^2c_s^2w_7w_4^2w_8w_5 - 12w_6^2c_s^4w_4^2w_8^2w_5 + 12w_6^2w_4^2w_8^2v_1^2w_5 + \\
& 36w_9w_6w_7w_4^2s_w^2v_1^2w_5 + 12w_6^2w_4^2w_8^2v_1^2w_5 - 5w_9w_6c_s^4w_7w_3^2w_8^2w_5 + 18w_6^2c_s^2w_7w_4^2v_2^2w_8^2w_5 + 72w_9w_6w_7w_4^2v_2^2w_8^2v_1^2 - 102w_9w_6^2c_s^2w_7w_4^2v_2^2w_8w_5 + \\
& 12w_6^2s_w^2w_7w_4^2w_8^2w_5 + 12w_9w_6c_s^4w_7w_4w_8w_5 + 36w_6^2w_4^2v_2^2w_8^2v_1^2 + 36w_6^2c_s^2w_4^2v_2^2w_8^2v_1^2 + 36w_6^2w_4^2v_2^2w_8^2v_1^2w_5 + 6w_6^2w_7w_4^2w_8v_1^2w_5 + 6w_6^2s_w^2w_7w_4^2w_8v_1^2w_5 + \\
& 18w_6^2w_7w_4^2v_2^2w_8^2v_1^2w_5 - 12w_6^2w_8c_s^2w_7w_4w_8w_5 - 12w_9w_6c_s^2w_7w_4w_8^2w_5 - 12w_6^2c_s^2w_7w_4^2w_8^2v_1^2w_5 - 36w_6^2w_7w_4^2v_2^2w_8v_1^2w_5 - \\
& 18w_6^2s_w^2w_7w_4^2v_2^2w_8w_5 + 24w_9w_6^2w_7w_4^2w_8v_1^2 - 12w_9w_6c_s^2w_7w_4w_8^2v_1^2w_5 + 12w_9c_s^2w_7w_4^2w_8^2v_1^2w_5 + 18w_9w_6^2w_7w_4^2v_2^2w_8^2v_1^2w_5 + 54w_9w_6c_s^2w_7w_4^2v_2^2w_8^2w_5 - \\
& 5w_9w_6c_s^4w_7w_4^2w_8^2w_5 + 15w_9w_6^2c_s^2w_7w_4^3w_8^2v_1^2w_5 + 12w_9w_6w_4^2w_8^2v_1^2w_5 - 18w_6^2w_7w_4^2v_2^2w_8^2v_1^2 + 12w_9w_6w_7w_4^2w_8^2v_1^2w_5 + w_9w_6c_s^2w_7w_4^2w_8^2w_5 + \\
& 36w_9w_6c_s^2w_4^2v_2^2w_8^2w_5 + 36w_9w_6w_4^2c_s^2w_8^2v_1^2w_5 - 6w_9w_6c_s^2w_7w_4^3w_8^2v_1^2 - 15w_9w_6c_s^2w_7w_4^2v_2^2w_8^2w_5 - 24w_9w_6^2c_s^2w_7w_4w_8v_1^2w_5 - 36w_9w_6^2w_7w_4^2v_2^2v_1w_5 + \\
& 12w_6^2c_s^4w_4^2w_8^2w_5 - 6w_6^2c_s^4w_7w_4^3w_8^2w_5 + 24w_9w_6c_s^2w_7w_4w_8^2v_1^2w_5 + 18w_9w_6c_s^4w_7w_4^2w_8^2w_5 + 24w_9w_6^2c_s^2w_7w_4^2v_2^2w_5 + 6w_6^2c_s^2w_7w_4^2w_8w_5 - \\
& 6w_9w_6w_7w_4^2w_8v_1^2w_5 - 36w_9w_6c_s^2w_4^2v_2^2w_8^2w_5 + 72w_9w_6w_7w_4^2v_2^2w_8v_1^2w_5 + 24w_9w_6c_s^2w_7w_4^2v_2^2w_8^2v_1^2 - 12w_9w_6c_s^2w_7w_4^2w_8^2v_1^2w_5 - 18w_9w_6c_s^2w_7w_4^2w_8^2w_5 - \\
& 48w_9w_6^2w_7w_4^2v_2^2w_8v_1^2w_5 + 12w_9w_6c_s^4w_4^3w_8^2w_5 - 72w_9w_6w_7w_4^2v_2^2w_8v_1^2w_5 - 12w_6^2c_s^2w_7w_4^2w_8^2v_1^2w_5 + 12w_9w_6c_s^2w_7w_4^2w_8^2w_5 - 6w_6^2c_s^4w_7w_4^2w_8^2v_1^2w_5 - \\
& 36w_6^2c_s^2w_4^2v_2^2w_8^2w_5 - 36w_6^2w_4^2v_2^2w_8^2v_1^2w_5 - 12w_6^2w_7w_4^2w_8v_1^2w_5 - 6w_9w_6^2c_s^2w_7w_4^2w_8^2v_1^2w_5 - 108w_9w_6w_7w_4^2v_2^2w_8^2v_1^2w_5 - 18w_9w_6w_7w_4^2v_2^2w_8^2v_1^2w_5 + \\
& 144w_9w_6^2w_7w_4^2v_2^2w_8v_1^2w_5 + 30w_9w_6c_s^2w_7w_4^3w_8^2v_1^2w_5 - 24w_9w_6w_7w_4w_8v_1^2w_5 - 6w_9w_6^2c_s^2w_7w_4^3v_1^2w_5 + 12w_9w_6c_s^2w_7w_4^2w_8^2v_1^2w_5 - 6w_9c_s^2w_7w_4^2w_8^2v_1^2w_5 - \\
& 18w_9w_7w_4^2v_2^2w_8^2v_1^2w_5 + 36w_6^2c_s^2w_7w_4^2v_2^2w_8w_5 + 6w_9w_7w_4^2w_8^2v_1^2w_5 + 6w_9w_6w_7w_4^2w_8^2v_1^2w_5 + 18w_9w_6w_7w_4^2v_2^2w_8v_1^2w_5 + \\
& 6w_9w_6^2c_s^4w_7w_4^3w_8w_5 + 12w_6^2w_4^2w_8^2w_8^2v_1^2 + 6w_9w_6c_s^2w_7w_4^3w_8^2v_1^2w_5 + 60w_9w_6^2c_s^2w_7w_4w_8v_1^2w_5
\end{aligned}$$

$$\begin{aligned}
C_{19} = & 150w_6c^2s^2w_4^2v_2^2w_8^2 + 48w_6^2v_2^2w_8 - 48w_6w_4v_4^2w_8 - 24w_6^2w_4v_4^2 - 24w_6^2c_4^4w_8 - 24w_6c_2^2w_8^2 - w_6^2c_4^4w_2^2w_8^2 - 36w_6w_2^2v_2^2w_8^2 + 48w_6v_4^2w_8^2 + 12w_6^2s^2w_4^2s_8 - 3w_6^2w_2^2v_4^2w_8^2 + 24w_6^2s^2w_4 + 72w_4v_4^2w_8^2 + 48w_6c_2^2w_4w_8^2 - 12w_6^2c_2^2s_2^2v_2^2w_8^2 + 24c_4^4w_4w_8^2 - 96w_6^2w_4v_4^2w_8^2 - 12w_6^2w_4^2v_2^2 - 12w_6^2c_2^2s_2^2v_2^2w_8^2 + 24w_6^2s^2w_4^2s_8 - 30w_6^2w_4v_4^2w_8^2 - 12w_6^2c_2^2s_2^2w_4^2 - 216w_6^2s_2^2v_2^2w_8^2 - 12w_6^2c_2^4s_4^2w_8 + 72w_6^2c_2^2s_2^2w_2^2v_2^2 - 48w_6c_4^4w_4w_8^2 - 24w_6c_2^2v_2^2w_8 + 28w_6c_2^2s_2^2w_4^2s_8 - 72w_6c_2^2v_2^2w_8^2 - 96w_6w_4v_4^2w_8^2 - 24c_2^2w_4w_8^2 - 144w_2^2c_2^2w_4v_2^2 + 48w_2^2c_4^4w_4w_8^2 + 3w_2^2w_4^2v_2^2w_8^2 - 24w_6^2c_4^4w_4 - 72w_4v_4^2w_8^2 + 432w_6^2w_2^2w_4^2s_8 + 14w_6c_4^4s_4^2w_8^2 + 12c_2^2w_4^2s_8^2 + 96w_6^2w_4v_4^2w_8^2 - 144w_2^2c_2^2v_2^2w_8^2 - 48w_2^2v_2^2w_8 + 48w_6w_4v_4^2w_8^2 + 24w_6^2c_2^2s_2^2w_8 + 24w_6c_2^2s_2^2w_4^2s_8 + 216w_6^2s_2^2v_2^2w_8^2 + 36w_6w_4^2v_4^2w_8^2 - 48w_6v_2^2w_8^2 - 144w_6c_2^2s_2^2w_4v_2^2w_8^2 - 48w_6^2c_2^2s_2^2w_4w_8^2 - 14w_6c_2^2s_2^2w_4^2w_8^2 + 24w_6w_4^2v_2^2w_8^2 + 24w_6c_2^2v_2^2w_8^2 - 12c_4^4w_4^2w_8^2 - 36w_4^2v_2^2w_8^2 + 12w_6^2c_2^2v_2^2v_4^2 + 30w_6^2w_2^2v_2^2w_8^2 + 12w_6^2c_4^4w_4
\end{aligned}$$

$$\begin{aligned} C_{20} = & -24w_4^2v_4^4 - 24c_s^2w_4w_8 + 6c_s^3w_3^3v_2^2w_8^2 - 24c_s^2w_4v_2^2w_8 + 24w_4v_4^2w_8^2 - 18w_4^3v_4^2w_8 - 72w_4^2v_2^2w_8 - 48c_s^4w_4w_8^2 + 24c_s^4w_4w_8 + 24w_4^2v_2^2w_8^2 + \\ & 12w_3^2v_4^2 - 48w_4v_2^2w_8 + 3w_4^3v_4^2w_8^2 - 12c_s^2w_3^3v_2^2w_8 + 156c_s^2w_4v_2^2w_8^2 + 12c_s^2w_4w_8^2 - 12w_3^2v_2^2 - 3c_s^3w_3^3w_8^2 - 24w_4v_2^2w_8^2 + 18w_4^3v_2^2w_8 + 24c_s^4w_8^2 + \\ & 72w_2^2v_4^2w_8 - 8c_s^2w_4^2w_8^2 - 72c_s^2w_4v_2^2w_8^2 - 24c_s^4w_4w_8 - 24c_s^2w_4^2v_2^2 - 6c_s^2w_3^3w_8 + c_s^2w_3^4w_8^2 + 48c_s^2w_4^2v_2^2w_8 + 24w_4^2v_2^2 + 24c_s^4w_4^2w_8^2 - 96c_s^2v_2^2w_8^2 - \\ & 24w_4v_2^2w_8^2 + 24c_s^2w_4^2w_8 + 48w_4v_2^2w_8 + 12c_s^2w_3^4v_2^2 + 6c_s^4w_3^3w_8 - 3w_3^3v_2^2w_8^2 \end{aligned}$$

$$\begin{aligned}
C_{21} = & 36\omega_6^2\omega_8 - 84\omega_6^2v_2^2\omega_8 - 48\omega_4\omega_8^2 + 2\omega_6^2\omega_4^2\omega_8^2 + 60\omega_6c_s^2\omega_8^2 + 61\omega_6\omega_4^2v_2^2\omega_8^2 - 33\omega_6^2c_s^2\omega_8^2\omega_8 + 72\omega_6\omega_4\omega_8^2 - 48\omega_6^2c_s^2\omega_4\omega_8^2 - 120\omega_6c_s^2\omega_4\omega_8^2 + \\
& 168\omega_6^2\omega_4v_2^2\omega_8 + 24\omega_6^2\omega_4^2v_2^2 - 60\omega_4^2v_2^2\omega_8^2 - 3\omega_6^2c_s^2\omega_4^2\omega_8^2 + 24\omega_6\omega_4\omega_8 + 24\omega_6^2c_s^2\omega_4^2\omega_8^2 - 24\omega_6c_s^2\omega_4\omega_8 + 36\omega_6\omega_4^2v_2^2\omega_8 + 21\omega_6^2\omega_4^2\omega_8 + 72c_s^2\omega_4\omega_8^2 - \\
& 5\omega_6^2\omega_4^2v_2^2\omega_8^2 + 120\omega_4v_2^2\omega_8^2 + 24\omega_6^2\omega_4^2 - 36\omega_6\omega_8^2 - 36c_s^2\omega_4^2\omega_8^2 - 72\omega_6^2\omega_4\omega_8 - 72\omega_6\omega_4^2v_2^2\omega_8 - 60\omega_6^2c_s^2\omega_8 - 12\omega_6\omega_4^2\omega_8 + 12\omega_6c_s^2\omega_4^2\omega_8 + \\
& 84\omega_6v_2^2\omega_8^2 + 120\omega_6^2c_s^2\omega_4\omega_8 - 25\omega_6\omega_4^2\omega_8^2 + 39\omega_6c_s^2\omega_4^2\omega_8^2 - 168\omega_6\omega_4v_2^2\omega_8^2 - 48\omega_6^2\omega_4v_2^2 + 24\omega_4^2\omega_8^2 - 51\omega_6^2\omega_4^2v_2^2\omega_8 - 12\omega_6^2\omega_4^2
\end{aligned}$$

#### 2.2.4 Conservation of momentum: $\rho v_2$

 attached text file: output\_d2q9\_nse\_mrt1\_symbolic\_pde\_02.txt

$$\begin{aligned}
v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{\delta_l v_2 v_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\delta_l v_2 \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + (v_2^2 + c_s^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{2\delta_l v_2 \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_4) \frac{\delta_l^2 c_s^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\
(-2 + \omega_4) \frac{\delta_l^2 c_s^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (-2 + \omega_6 + 6v_2^2 - 3\omega_6 v_2^2 + 4c_s^2 - 2\omega_6 c_s^2) \frac{\delta_l^2}{\omega_6 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (2 - \omega_6) \frac{3\delta_l^2 v_2 \rho}{\omega_6 \delta_t} \left( \frac{\partial v_2}{\partial x_2} \right)^2 + \\
(-2 + \omega_4) \frac{\delta_l^2 c_s^2 \rho}{2\delta_t \omega_4} \frac{\partial^2 v_2}{\partial x_1^2} + (-2 + \omega_4) \frac{\delta_l^2 c_s^2 \rho}{2\delta_t \omega_4} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + (-2 + \omega_6 + 2v_2^2 - \omega_6 v_2^2 + 6c_s^2 - 3\omega_6 c_s^2) \frac{\delta_l^2 v_2}{2\omega_6 \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + \\
(-2 + \omega_6 + 6v_2^2 - 3\omega_6 v_2^2 + 2c_s^2 - \omega_6 c_s^2) \frac{\delta_l^2 \rho}{2\omega_6 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + C_1 \frac{\delta_l^3 v_2 v_1}{12\delta_t \omega_7 \omega_4 \omega_5} \frac{\partial^3 v_1}{\partial x_1^3} + C_2 \frac{\delta_l^3 v_2 \rho}{12\delta_t \omega_7 \omega_4 \omega_5} \frac{\partial^3 v_1}{\partial x_1^3} + C_3 \frac{\delta_l^3 v_1 \rho}{6\delta_t \omega_7 \omega_4^2} \frac{\partial^3 v_2}{\partial x_1^3} + \\
(-12 - \omega_4^2 + 12\omega_4) \frac{\delta_l^3 c_s^4}{6\delta_t \omega_4^2} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + (-\omega_4^2 - 2\omega_7 + 2\omega_4 + \omega_7 \omega_4) \frac{\delta_l^3 c_s^2 v_1 \rho}{\delta_t \omega_7 \omega_4^2} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} + \\
(-12\omega_6\omega_4^2 + 12\omega_4^2 + 12\omega_6\omega_4\omega_8 + 12\omega_6\omega_4 - 12\omega_4\omega_8 - \omega_6\omega_4^2\omega_8 - 12\omega_6\omega_8) \frac{\delta_l^3 c_s^2 v_2 \rho}{6\omega_6 \delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + C_4 \frac{\delta_l^3 v_2 v_1}{\omega_6^2 \delta_t \omega_4 \omega_8} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + \\
C_5 \frac{\delta_l^3 v_2 \rho}{12\omega_6^2 \delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_6 \frac{\delta_l^3 v_1 \rho}{\omega_6^2 \delta_t \omega_4 \omega_8} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_7 \frac{\delta_l^3}{12\omega_6^2 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\
(-24 + 5\omega_6^2 c_s^2 + 24\omega_6 + 11\omega_6^2 v_2^2 - 4\omega_6^2 + 60v_2^2 - 60\omega_6 v_2^2 + 36c_s^2 - 36\omega_6 c_s^2) \frac{\delta_l^3 v_2 \rho}{6\omega_6^2 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_8 \frac{\delta_l^4 v_2}{24\delta_t \omega_7^2 \omega_4^2 \omega_5^2} \frac{\partial^4 \rho}{\partial x_1^4} + \\
C_9 \frac{\delta_l^4 v_2 v_1 \rho}{12\delta_t \omega_7^2 \omega_4^2 \omega_5^2} \frac{\partial^4 v_1}{\partial x_1^4} + C_{10} \frac{\delta_l^4 \rho}{24\delta_t \omega_7^2 \omega_4^3} \frac{\partial^4 v_2}{\partial x_1^4} + C_{11} \frac{\delta_l^4 v_1}{12\omega_9 \omega_6 \delta_t \omega_7^2 \omega_4^3 \omega_8 \omega_5^2} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{12} \frac{\delta_l^4 \rho}{12\omega_9 \omega_6 \delta_t \omega_7^2 \omega_4^3 \omega_8 \omega_5^2} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + \\
C_{13} \frac{\delta_l^4 v_2 v_1 \rho}{12\omega_9 \omega_6^2 \delta_t \omega_7^2 \omega_4^3 \omega_8 \omega_5^2} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_{14} \frac{\delta_l^4 v_2}{12\omega_9 \omega_6^3 \delta_t \omega_7 \omega_4^3 \omega_8^2} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2} + C_{15} \frac{\delta_l^4 v_2 v_1 \rho}{2\omega_9 \omega_6^3 \delta_t \omega_7^2 \omega_4^3 \omega_8^2 \omega_5} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2} + \\
C_{16} \frac{\delta_l^4 \rho}{12\omega_9 \omega_6^3 \delta_t \omega_7 \omega_4^3 \omega_8^2} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2} + C_{17} \frac{\delta_l^4 v_1}{4\omega_6^3 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{18} \frac{\delta_l^4 \rho}{12\omega_6^3 \delta_t \omega_4^3 \omega_8^2} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{19} \frac{\delta_l^4 v_2 v_1 \rho}{4\omega_6^3 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\
C_{20} \frac{\delta_l^4 v_2}{12\omega_6^3 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{21} \frac{\delta_l^4 \rho}{12\omega_6^3 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$C_1 = -6\omega_4\omega_5 - 36c_s^2\omega_5 + 12\omega_7v_1^2 + 3c_s^2\omega_7\omega_4\omega_5 + 18c_s^2\omega_4\omega_5 - 12\omega_7 + \omega_7\omega_4v_1^2\omega_5 + 6\omega_7\omega_4 - 6\omega_7\omega_4v_1^2 - \omega_7\omega_4\omega_5 - 12v_1^2\omega_5 + 6\omega_4v_1^2\omega_5 - 18c_s^2\omega_7\omega_4 + 12\omega_5 + 36c_s^2\omega_7$$

$$C_2 = -6\omega_4\omega_5 - 12c_s^2\omega_5 + 36\omega_7v_1^2 + c_s^2\omega_7\omega_4\omega_5 + 6c_s^2\omega_4\omega_5 - 12\omega_7 + 3\omega_7\omega_4v_1^2\omega_5 + 6\omega_7\omega_4 - 18\omega_7\omega_4v_1^2 - \omega_7\omega_4\omega_5 - 36v_1^2\omega_5 + 18\omega_4v_1^2\omega_5 - 6c_s^2\omega_7\omega_4 + 12\omega_5 + 12c_s^2\omega_7$$

$$C_3 = -6\omega_4v_1^2 - 3\omega_4^2 - \omega_7\omega_4v_1^2 + 6\omega_4 - 3\omega_7\omega_4 - 3c_s^2\omega_7\omega_4^2 + 3c_s^2\omega_4^2 + 3\omega_7\omega_4v_1^2 - 6c_s^2\omega_4 + \omega_7\omega_4^2 + 15c_s^2\omega_7\omega_4 - 12c_s^2\omega_7 + 3\omega_4^2v_1^2$$

$$C_4 = 3c_s^2\omega_4\omega_8 - 3\omega_6^2c_s^2 - \omega_6\omega_4v_2^2 + 3\omega_6^2c_s^2\omega_4 - \omega_6^2v_2^2 + \omega_6\omega_4\omega_8 - 3\omega_6c_s^2\omega_4\omega_8 + 3\omega_6c_s^2\omega_8 + \omega_6^2 + \omega_6\omega_4 - \omega_4\omega_8 - \omega_6^2\omega_4 - \omega_6\omega_4v_2^2\omega_8 + \omega_6v_2^2\omega_8 + \omega_6^2\omega_4v_2^2 - \omega_6\omega_8 - 3\omega_6c_s^2\omega_4 + \omega_4v_2^2\omega_8$$

$$C_5 = 12\omega_6\omega_4^2 - 11\omega_6^2c_s^2\omega_4\omega_8 - 12\omega_6^2c_s^2\omega_4 + 12\omega_4^2v_2^2\omega_8 + 6\omega_6^2\omega_4v_2^2\omega_8 + 12\omega_6^2c_s^2v_2^2 + 12\omega_6^2c_s^2\omega_4^2 - 24\omega_6c_s^2\omega_4\omega_8 - 6\omega_6\omega_4^2v_2^2\omega_8 + 3\omega_6^2\omega_4^2\omega_8 + 12\omega_6^2\omega_4 - 12\omega_6c_s^2\omega_4^2 - 12\omega_4^2\omega_8 - 6\omega_6^2\omega_4\omega_8 - 24\omega_6^2c_s^2\omega_8 + 6\omega_6\omega_4^2\omega_8 - 18\omega_6c_s^2\omega_4\omega_8 + 42\omega_6^2c_s^2\omega_4\omega_8 - 12\omega_6^2\omega_4v_2^2 + 36c_s^2\omega_4^2\omega_8 - 3\omega_6^2\omega_4^2v_2^2\omega_8 - 12\omega_6\omega_4^2v_2^2 - 12\omega_6^2\omega_4^2$$

$$C_6 = c_s^2\omega_4\omega_8 - \omega_6^2c_s^2 - 3\omega_6\omega_4v_2^2 + \omega_6^2c_s^2\omega_4 - 3\omega_6^2v_2^2 + \omega_6\omega_4\omega_8 - \omega_6c_s^2\omega_8 + \omega_6^2 + \omega_6\omega_4 - \omega_4\omega_8 - \omega_6^2\omega_4 - 3\omega_6\omega_4v_2^2\omega_8 + 3\omega_6v_2^2\omega_8 + 3\omega_6^2\omega_4v_2^2 - \omega_6\omega_8 - \omega_6c_s^2\omega_4 + 3\omega_4v_2^2\omega_8$$

$$C_7 = 144c_s^2v_2^2 - \omega_6^2c_s^2 + 12c_s^4 - 144\omega_6c_s^2v_2^2 - 12\omega_6c_s^4 - 7\omega_6^2v_2^2 + 36v_4^2 - 36\omega_6v_2^4 + 7\omega_6^2v_2^4 - 36v_2^2 + 36\omega_6v_2^2 + \omega_6^2c_s^4 - 12c_s^2 + 24\omega_6^2c_s^2v_2^2 + 12\omega_6c_s^2$$

$$\begin{aligned}
C_8 = & 96\omega_7\omega_4v_1^2\omega_5 + 48\omega_7^2v_1^4\omega_5 + 24\omega_4v_1^4\omega_5^2 - 24\omega_4v_1^4\omega_5^2 + c_s^2\omega_7^2\omega_4^2\omega_5^2 - 12c_s^4\omega_7^2\omega_4^2 - 48c_s^4\omega_7\omega_4\omega_5^2 + 24c_s^2\omega_4\omega_5^2 + 150c_s^2\omega_7^2\omega_4^2v_1^2\omega_5 - \\
& 3\omega_7^2\omega_4^2v_1^2\omega_5^2 + 48\omega_7v_1^2\omega_5^2 - 48c_s^2\omega_7\omega_4\omega_5^2 + 30\omega_7\omega_4v_1^2\omega_5^2 - 24c_s^2\omega_7^2\omega_4^2\omega_5^2 - 48\omega_7\omega_4v_1^4\omega_5 + 432c_s^2\omega_7\omega_4v_1^2\omega_5^2 + 24c_s^2\omega_7\omega_5^2 - 36\omega_7^2\omega_4^2v_1^4 - \\
& 144c_s^2\omega_4v_1^2\omega_5^2 + 96\omega_7\omega_4v_1^4\omega_5^2 + 12c_s^2\omega_7^2\omega_4^2\omega_5^2 + 216c_s^2\omega_7^2v_1^2\omega_5^2 - 12c_s^4\omega_7\omega_4^2\omega_5^2 - 144c_s^2\omega_7\omega_4v_1^2\omega_5^2 - 24\omega_7\omega_4v_1^2\omega_5^2 - 24c_s^4\omega_4v_1^2\omega_5^2 - 12c_s^2v_1^2\omega_5^2 - \\
& 12c_s^2\omega_7^2v_1^2\omega_5^2 + 36\omega_7^2\omega_4^2v_1^2\omega_5^2 - 72\omega_7^2\omega_4v_1^2\omega_5^2 - 144c_s^2\omega_7\omega_4^2v_1^2\omega_5^2 + 24c_s^4\omega_7\omega_4^2\omega_5^2 - 14c_s^2\omega_7^2\omega_4^2\omega_5^2 + 72c_s^2\omega_7\omega_4^2v_1^2\omega_5^2 - 30\omega_7\omega_4v_1^4\omega_5^2 - c_s^4\omega_7^2\omega_4^2\omega_5^2 + \\
& 72c_s^2\omega_7^2v_1^2\omega_5^2 + 24c_s^4\omega_7^2\omega_4^2\omega_5^2 + 36\omega_7^2\omega_4^2v_1^2\omega_5^2 + 48\omega_7\omega_4v_1^2\omega_5^2 + 48c_s^2\omega_7^2\omega_4\omega_5^2 + 288c_s^2\omega_7^2\omega_4^2v_1^2\omega_5^2 - 48\omega_7^2\omega_4v_1^2\omega_5^2 - 96\omega_7^2\omega_4v_1^4\omega_5^2 + 72\omega_7^2\omega_4v_1^2\omega_5^2 + 24\omega_4v_1^2\omega_5^2 + \\
& 48c_s^4\omega_7\omega_4\omega_5^2 + 12c_s^4\omega_7^2\omega_4^2\omega_5^2 - 48\omega_7v_1^4\omega_5^2 + 3\omega_7^2\omega_4^2v_1^2\omega_5^2 + 12c_s^2\omega_7\omega_4^2\omega_5^2 + 12\omega_4^2v_1^2\omega_5^2 - 36\omega_7^2\omega_4^2v_1^2\omega_5^2 - 24c_s^2\omega_7^2\omega_4\omega_5^2 - 432c_s^2\omega_7^2\omega_4v_1^2\omega_5^2 - \\
& 96\omega_7\omega_4v_1^2\omega_5^2 - 126c_s^2\omega_7\omega_4^2v_1^2\omega_5^2 + 14c_s^4\omega_7^2\omega_4^2\omega_5^2 - 216c_s^2\omega_7v_1^2\omega_5^2 + 24\omega_7\omega_4^2v_1^4\omega_5^2 - 12c_s^2\omega_4^2\omega_5^2
\end{aligned}$$

$$\begin{aligned}
C_9 = & -168w_2^2w_4v_1^2w_5 - 3c_s^2w_7^2w_4^2w_5^2 - 48c_s^2w_4w_5^2 - 84w_7v_1^2w_5^2 + 120c_s^2w_7w_4w_5^2 - 51w_7w_4^2v_1^2w_5^2 + 21w_7w_4^2w_5^2 + 36w_7w_5^2 + 72c_s^2w_7w_4 + 24w_7^2w_4^2 - \\
& 60c_s^2w_7w_5^2 - 36c_s^2w_7^2w_4^2 - 48w_7^2w_4 - 24c_s^2w_7w_4w_5 + 36w_7w_4^2v_1^2w_5 - 12w_7w_4^2w_5 + 24w_4^2v_1^2w_5^2 + 24w_4w_5^2 + 72w_7^2w_4w_5 + 120w_7^2w_4v_1^2 + \\
& 39c_s^2w_7^2w_4^2w_5 - 12w_4^2w_5^2 - 60w_7^2w_4^2v_1^2 - 25w_7^2w_4^2w_5 - 72w_7w_4v_1^2w_5 - 120c_s^2w_7^2w_4w_5 + 84w_7^2v_1^2w_5^2 - 36w_7^2w_5^2 - 48w_4v_1^2w_5^2 + 12c_s^2w_7w_4^2w_5 + \\
& 24w_7w_4w_5 - 5w_7^2w_4^2v_1^2w_5^2 - 33c_s^2w_7w_4^2w_5^2 + 61w_7^2w_4^2v_1^2w_5 - 72w_7w_4w_5^2 + 60c_s^2w_7^2w_5 + 2w_7^2w_4^2w_5^2 + 168w_7w_4v_1^2w_5^2 + 24c_s^2w_4^2w_5^2
\end{aligned}$$

$$\begin{aligned} C_{10} = & 18\omega_7 w_4^3 v_1^2 + 24c_s^4 w_7^2 w_4^2 + 3w_7 w_4^3 v_1^4 - 3c_s^4 w_7^2 w_4^3 + 6c_s^2 w_7^2 w_3^3 v_1^2 - 24w_4^2 v_1^4 + 12c_s^2 w_7^2 w_4 + 24c_s^4 w_7^2 - 24w_7^2 w_4^2 v_1^4 - 8c_s^2 w_7^2 w_4^2 - 24c_s^2 w_7 w_4 v_1^2 - \\ & 72w_7 w_4^2 v_1^2 + c_s^2 w_7^2 w_3^4 - 48w_7 w_4 v_1^4 + 12w_3^2 v_1^4 - 24w_7^2 w_4 v_1^2 - 72c_s^2 w_7 w_2^2 v_1^2 - 48c_s^4 w_7^2 w_4 - 6c_s^2 w_7 w_4^3 - 24c_s^2 w_4^2 v_1^2 + 72w_7 w_4^2 v_1^4 + 24w_7^2 w_2^2 v_1^2 + \\ & 24c_s^4 w_7 w_4 + 156c_s^2 w_7^2 w_4 v_1^2 + 24w_2^2 w_4 v_1^4 - 12w_3^2 v_1^2 + 24c_s^2 w_7 w_4^2 + 48c_s^2 w_7 w_4^2 v_1^2 + 48w_7 w_4 v_1^2 + 6c_s^4 w_7 w_4^3 - 3w_7^2 w_4^3 v_1^2 + 12c_s^2 w_4^2 v_1^2 - 96c_s^2 w_7^2 v_1^2 - \\ & 18w_7 w_4^3 v_1^4 - 24c_s^2 w_7 w_4 - 24c_s^4 w_7 w_4^2 - 12c_s^2 w_7 w_4^3 v_1^2 + 24w_4^2 v_1^2 \end{aligned}$$

$$\begin{aligned}
C_{11} = & -15w_9w_6c_s^4w_7^2w_4^3w_8w_5 + 6w_9w_6w_3^4v_2^2w_8v_1^2w_5 + 156w_9w_6c_s^4w_7^2w_4w_8w_5^2 - 6w_9w_6w_7w_3^4v_2^2w_8w_5 - 18w_4^4c_s^2w_7^3w_3^4w_8w_5^2 + 6w_9c_s^2w_7^2w_4^3w_8v_1^2w_5 - \\
& 12w_9w_6w_7^2w_3^4v_2^2w_5 - 48w_9w_6w_7w_4^2v_2^2w_8w_5^2 + 18w_6c_s^2w_7^2w_3^4v_2^2w_8w_5^2 - 6w_7w_3^2v_2^2w_8v_1^2w_5 + 27w_9w_6c_s^2w_7^2w_3^4v_2^2w_8w_5 + 5w_9w_6c_s^2w_7^2w_4^3w_8w_5 - \\
& 12w_9w_7^2w_3^4v_2^2w_1^2w_5 + 12w_6w_7w_4^2v_2^2w_8w_5^2 - 12w_9w_6c_s^2w_7^2w_4^3w_5 + 6c_s^2w_7^2w_3^4w_8w_5^2 - 12w_9w_6c_s^2w_7^2w_4w_8w_5^2 - 18w_6c_s^2w_7w_3^4v_2^2w_8w_5^2 + \\
& 12w_6w_7w_4^2v_2^2w_8v_1^2w_5 + 36w_6c_s^4w_7^2w_4^3w_5 - 12w_9w_6w_7w_4^2v_2^2w_8v_1^2w_5 + 18w_9w_6c_s^2w_7w_4^3w_8w_5 - 12w_9w_7w_4^2v_2^2w_8w_5^2 - 36w_9w_6c_s^2w_7^2w_4^3w_2w_5 + \\
& 12w_6c_s^2w_7^2w_4^3w_1^2w_5 + 18w_9w_6c_s^2w_7^2w_4^3w_8w_5^2 - 12w_9c_s^2w_7^2w_4^3v_2^2w_5 + 6w_2^2v_4^2w_2^2w_8w_5^2 + 144w_9w_6c_s^2w_7w_4^2v_2^2w_8w_5^2 - 12w_9w_6c_s^2w_7^2w_4w_8v_1^2w_5 - \\
& 6w_9w_6w_7^2w_3^4v_2^2w_8w_5^2 - 9w_9w_6w_7w_4^2v_2^2w_8w_5 + 12w_9w_6c_s^2w_7^2w_4^2w_5 - 36w_9w_6c_s^2w_7^2v_2^2w_8w_5^2 - 12w_6w_7w_4^2v_2^2w_8w_5^2 - 36w_9w_6w_7w_2^2w_8v_1^2w_5 + \\
& 6w_6w_7w_4^2v_2^2w_8w_5^2 - 12w_6w_7w_2^2v_2^2w_8w_1^2w_5^2 - 6c_s^2w_7^2w_3^4w_8v_1^2w_5^2 - 18w_9w_6c_s^2w_7w_4^2w_8w_5^2 - 6w_9w_6c_s^4w_3^2w_8w_5^2 + 36w_9w_6c_s^2w_7^2w_4^2v_2^2w_8w_8 - \\
& 96w_9w_6c_s^4w_7^2w_3^4w_8w_5^2 + 12w_9w_6c_s^2w_7^2w_4w_8v_1^2w_5^2 + 18w_6c_s^4w_7^2w_3^4w_8w_5^2 - 36w_9w_6c_s^2w_7w_4^2v_2^2w_8w_5 + 12w_9w_7w_4^2v_2^2w_8v_1^2w_5^2 + 36w_6c_s^2w_7w_4^2v_2^2w_8w_5^2 + \\
& 36w_9w_6w_7^2w_4^2v_2^2w_8w_5 + 12w_9w_6w_7^2w_4^2v_2^2w_8v_1^2w_5^2 - 45w_9w_6c_s^2w_7w_4^3v_2^2w_8w_5^2 + 48w_9w_6w_7w_4^2v_2^2w_8v_1^2w_5^2 - 12w_6w_7w_4^2v_2^2w_8w_5^2 - 12c_s^2w_7^2w_4^3w_5^2 - \\
& 6w_9w_7^2w_4^3v_2^2w_8v_1^2w_5^2 - 6w_6c_s^2w_7^2w_4^3w_8w_5^2 - 42w_9w_6c_s^4w_7w_4^2w_8w_5^2 + 12w_9w_6w_7w_4^2v_2^2w_8w_5^2 - 36w_9c_s^4w_7^2w_3^4w_5^2 + 36w_6c_s^4w_7w_4^2w_8w_5^2 + \\
& 36w_6c_s^2w_7^2w_4^2v_2^2w_5^2 - 12w_9w_6c_s^2w_7^2w_4^2v_2^2w_5^2 - 24w_9w_6w_7w_4v_2^2w_8v_1^2w_5^2 - 9w_9w_6c_s^2w_7^2w_4^3w_8w_5^2 + 6w_9w_6w_6c_s^2w_7^2w_4^2v_2^2w_8 + 12w_9w_6c_s^2w_7^2w_4w_8w_5^2 + \\
& 24w_9w_6w_7w_4^2v_2^2w_8v_1^2w_5^2 + 3w_9w_6c_s^2w_7^2w_4^3w_8w_5^2 - 12w_6c_s^2w_7w_4^2w_8w_5^2 + 12w_9w_6c_s^2w_7^2w_4^2w_8w_5^2 - 6w_6w_7w_3^2v_2^2w_8v_1^2w_5^2 - 12w_9w_6c_s^2w_7w_4^3w_8v_1^2w_5^2 - \\
& 36w_9w_6c_s^4w_7^2w_4^2w_8w_5^2 - 108w_9w_6c_s^2w_7^2w_4^2v_2^2w_8w_5^2 - 36w_9c_s^2w_7^2w_4^3v_2^2w_8w_5^2 - 36w_6c_s^4w_7^2w_3^4w_5^2 + 15w_9w_6w_7w_3^2v_2^2w_8w_5^2 + \\
& 24w_7w_4^2v_2^2w_8v_1^2w_5^2 + 12w_9w_6c_s^2w_7w_4w_8w_5^2 + 24w_9w_6w_7w_4^2v_2^2w_8w_5^2 - 3w_9w_6c_s^2w_7^2w_3^4w_8v_1^2w_5^2 - 15w_9w_6w_7w_4^3v_2^2w_8v_1^2w_5^2 + 36w_9w_6c_s^4w_7^2w_3^4w_5^2 + \\
& 12w_9w_6c_s^4w_7w_4w_8w_5^2 + 12w_6c_s^2w_7^2w_4^2w_8v_1^2w_5^2 + 36c_s^2w_7^2w_3^4v_2^2w_5^2 - 12w_9w_6w_7w_4^2v_2^2w_1^2v_5^2 - 24w_9w_7w_4^2v_2^2w_8v_1^2w_5^2 + 36w_9c_s^2w_7w_3^2w_8w_5^2 - \\
& 12w_6c_s^2w_7^2w_3^4v_2^2w_5^2 + 9w_9w_6w_7c_s^2w_7^2w_4^2v_2^2w_8v_1^2w_5^2 + 36w_9w_6c_s^2w_7^2w_4^2v_2^2w_5^2 - 12w_9w_6c_s^2w_7^2w_4^2w_8v_1^2w_5^2 + 6w_6w_7w_4^2v_2^2w_8v_1^2w_5^2 - 12w_9w_7w_4^2v_2^2w_8w_5^2 - \\
& 18c_s^2w_7^2w_3^4v_2^2w_8w_5^2 + 18w_9w_6c_s^2w_7^2w_4^2w_8v_1^2w_5^2 - 72w_9w_6c_s^2w_7w_4v_2^2w_8w_5^2 - 18w_9w_6c_s^2w_7w_4^2w_8w_5^2 - 12w_6c_s^2w_7^2w_4^2w_8v_1^2w_5^2 - 12w_6c_s^2w_7^2w_4^2w_5^2 - \\
& 24w_9w_6c_s^2w_7^2w_4^2w_8w_5^2 - 12w_6w_7w_4^2v_2^2v_1^2w_5^2 + 54w_9w_6c_s^4w_7^2w_4^2w_8w_5^2 + 12w_9w_6c_s^2w_7^2w_4^2v_2^2w_5^2 - 6w_9w_6w_3^4v_2^2w_8w_5^2 + 72w_9c_s^2w_7^2w_4^2v_2^2w_8w_5^2 + \\
& 9w_9w_6c_s^2w_7^2w_4^2w_8v_1^2w_5^2 - 6w_9w_6c_s^2w_7^2w_4^3w_8w_5^2 + 6w_6c_s^2w_7w_4^2w_8w_5^2 - 60w_9w_6c_s^4w_7w_4^2w_8w_5^2 - 36w_9w_6c_s^4w_7^2w_4^2w_5^2 - 24w_9w_6w_7w_4^2v_2^2w_8w_5^2 + \\
& 6w_6c_s^2w_7^2w_4^3w_8v_1^2w_5^2 - 5w_9w_6c_s^2w_7^2w_4^2w_8v_1^2w_5^2 + 12w_9w_6c_s^2w_7^2w_4^2v_2^2w_8w_5^2 - 12w_9w_6c_s^2w_7^2w_4^2v_1^2w_5^2 - 18w_6c_s^4w_7^2w_4^2w_8w_5^2 - 12w_9w_6c_s^2w_7^2w_4^2w_8v_1^2w_5^2 - \\
& 36w_6c_s^2w_7^2w_4^3v_2^2w_5^2 - 18w_9c_s^2w_7^2w_4^2w_8w_5^2 + 12w_9w_6w_7^2w_4^2v_2^2w_8w_5^2 - 12w_6c_s^4w_7^2w_4^2v_2^2w_8v_1^2w_5^2 + 36c_s^2w_7^2w_4^2v_2^2w_8v_1^2w_5^2 + 12c_s^2w_7^2w_4^2v_1^2w_5^2 + \\
& 12w_6w_7w_4^2v_2^2v_1^2w_5^2 + 6w_9w_7w_4^2v_2^2w_8w_5^2 + 72w_9w_6c_s^2w_7w_4^2v_2^2w_8w_5^2 + 18w_9w_6c_s^2w_7w_4^2w_8v_1^2w_5^2 + 12w_9w_7w_4^2v_2^2w_8w_5^2 + 18w_9c_s^4w_7^2w_4^2w_8w_5^2 + \\
& 15w_9w_6c_s^4w_7^2w_4^3w_8w_5^2 + 12w_9c_s^2w_7^2w_4^3w_5^2 + 12w_6c_s^2w_7^2w_4^2w_8w_5^2 + 24w_9w_7w_4^2v_2^2w_8w_5^2 + 12w_6w_7w_4^2v_2^2v_1^2w_5^2 + 12w_6c_s^2w_7^2w_4^2v_3^2w_5^2 - 6w_9c_s^2w_7^2w_4^3w_8w_5^2 + \\
& 6w_9w_6c_s^2w_4^2w_8v_1^2w_5^2 - 6w_6c_s^2w_7w_4^3w_8w_5^2 - 72w_9c_s^2w_7w_4^2v_2^2w_8w_5^2 + 12w_9w_6w_7w_4^2v_2^2v_1^2w_5^2 - 6w_6w_7w_4^3v_2^2w_8v_1^2w_5^2 + 6w_9w_6w_7w_4^2v_2^2w_8v_1^2w_5^2 - \\
& 36w_6c_s^4w_7^2w_4^2w_8w_5^2 + 12w_9w_6c_s^4w_7^2w_4^2w_8w_5^2 + 3w_9w_6c_s^2w_7w_4^3w_8w_5^2 + 12w_9w_7w_4^3v_2^2w_8v_1^2w_5^2 - 18w_9w_6c_s^2w_7w_4^2w_8v_1^2w_5^2 + 6w_9w_6w_7w_4^2v_2^2w_8v_1^2w_5^2
\end{aligned}$$

$$\begin{aligned}
C_{12} = & -5w_9w_6c_4^4w_7^2w_4^3w_8w_5 + 18w_9w_6w_3^4v_2^2w_8v_1^2w_5 + 18w_9w_6e_4^4s^2w_7^2w_4w_8w_5^2 - 6w_9w_6w_7w_3^3v_2^2w_8w_5 - 6c_4^4s^2w_7^2w_3^4w_8w_5^2 + 18w_9s^2w_7w_3^2w_8v_1^2w_5 - \\
& 12w_9w_6w_7w_3^2v_2^2w_5 - 48w_9w_6w_7w_4^2v_2^2w_8w_5^2 + 6w_6c_5^2s^2w_7w_3^4v_2^2w_8w_5^2 - 18w_7^2w_3^4v_2^2w_8v_1^2w_5^2 + 9w_9w_6c_5^2s^2w_7^2w_4^3w_8w_5^2 + 5w_9w_6c_5^2s^2w_7^2w_4^3w_8w_5^2 - \\
& 36w_9w_7w_3^4v_2^2w_1^2w_5 + 12w_6w_7w_2^4v_2^2w_8w_5^2 - 12w_9w_6c_5^2s^2w_7w_4^3w_5 + 6c_6^2s^2w_7^2w_4^3w_8w_5^2 - 6w_6c_5^2s^2w_7w_4^3v_2^2w_8w_5^2 + 36w_6w_7w_2^4v_2^2w_8v_1^2w_5^2 + 12w_6c_4^4s^2w_7^2w_4^2w_5^2 - \\
& 36w_9w_6w_7w_4^2v_2^2w_8v_1^2w_5 + 6w_9w_6c_5^2s^2w_7w_4^3v_2^2w_8w_5^2 - 12w_6w_7w_4^2v_2^2w_8w_5^2 - 12w_9w_6c_5^2s^2w_7w_4^2v_2^2w_5 + 36w_6c_5^2s^2w_7^2w_4^2v_1^2w_5^2 + 6w_9w_6c_5^2s^2v_2^2w_8w_5^2 - \\
& 36w_6c_5^2s^2w_7^2w_3^1v_1^2w_5 + 6w_7^2w_4^3v_2^2w_8w_5^2 + 48w_9w_6c_5^2s^2w_7w_4^2v_2^2w_8w_5^2 - 36w_9w_6c_5^2s^2w_7w_4^3w_8v_1^2w_5 - 18w_9w_6w_7w_3^4v_2^2w_8w_5^2 - 9w_9w_6w_7w_3^4v_2^2w_8w_5^2 + \\
& 12w_9w_6c_5^2s^2w_7w_4^2w_5 - 12w_9w_6c_5^2s^2w_7w_4^3w_8w_5^2 - 12w_7^2w_3^4v_2^2w_5^2 - 108w_9w_6w_7w_2^4v_2^2w_8v_1^2w_5 + 6w_6w_7w_3^4v_2^2w_8w_5^2 - 36w_6w_7w_2^4v_2^2w_8v_1^2w_5^2 - \\
& 18w_9w_6c_5^2s^2w_7w_3^2w_8w_5^2 + 12w_9w_6c_5^2s^2w_7w_4^2w_5^2 + 12w_9w_6c_5^2s^2w_7w_4^3w_8w_5^2 - 12w_9w_6c_5^2s^2w_7w_4^2w_8v_1^2w_5^2 + 60w_9w_6c_5^2s^2w_7w_4^2w_8v_1^2w_5^2 + 6w_6c_4^4s^2w_7w_4^3w_8w_5^2 - \\
& 12w_9w_6c_5^2s^2w_7w_4^2v_2^2w_8w_5^2 + 36w_9w_6w_7w_3^4v_2^2w_8v_1^2w_5^2 + 12w_6c_5^2s^2w_7w_4^2v_2^2w_8w_5^2 + 36w_9w_6w_7w_4^2v_2^2w_8w_5^2 - 12w_6c_5^2s^2w_7w_4^2v_2^2w_5^2 - 18w_9w_7w_3^4v_2^2w_8v_1^2w_5^2 - 6w_6c_5^2s^2w_7w_4^3w_8w_5^2 - \\
& 15w_9w_6c_5^2s^2w_7w_4^3v_2^2w_8w_5^2 + 144w_9w_6w_7w_4^2v_2^2w_8v_1^2w_5^2 - 12w_6w_7w_2^4v_2^2w_5^2 - 12c_2^2w_7w_3^4v_2^2w_5^2 - 18w_9w_7w_3^4v_2^2w_8v_1^2w_5^2 - 6w_6c_5^2s^2w_7w_4^3w_8w_5^2 - \\
& 18w_9w_6c_5^2s^2w_7w_4^2w_8w_5^2 + 12w_9w_6w_7w_4^2v_2^2w_8w_5^2 - 12w_9c_4^4s^2w_7w_3^4w_5^2 + 12w_6c_5^2s^2w_7w_4^2w_8w_5^2 + 12w_6c_5^2s^2w_7w_4^2v_2^2w_5^2 - 36w_9w_6c_5^2s^2w_7w_4^2v_2^2w_1^2w_5 - \\
& 72w_9w_6w_7w_4^2v_2^2w_8v_1^2w_5^2 + 6w_9w_6w_7w_4^3v_2^2w_8w_5^2 + 12w_9w_6c_5^2s^2w_7w_4w_8w_5^2 + 72w_9w_6w_7w_4v_2^2w_8v_1^2w_5 - w_9w_6c_5^2s^2w_7w_4^3w_8w_5^2 - 12w_6c_5^2s^2w_7w_4^2w_8w_5^2 - \\
& 6w_6w_7w_3^4v_2^2w_8w_5^2 + 60w_9w_6c_5^2s^2w_7w_4w_8v_1^2w_5^2 - 12w_9w_6c_5^2s^2w_7w_4w_8w_5^2 - 36w_9w_6c_5^2s^2w_7w_4^2v_2^2w_8w_5^2 - 12w_9c_5^2s^2w_7w_4^3v_2^2w_5^2 - 12w_6c_5^2s^2w_7w_4^2v_2^2w_8w_5^2 - \\
& 12w_6c_5^2s^2w_7w_3^5w_5^2 + 15w_9w_6w_7w_3^2v_2^2w_8w_5^2 + 72w_9w_6c_5^2s^2w_7w_4w_8v_1^2w_5^2 - 12w_9w_6c_5^2s^2w_7w_4w_8w_5^2 + 24w_9w_6w_7w_4v_2^2w_8w_5^2 + 30w_9w_6c_5^2s^2w_7w_4^3w_8v_1^2w_5^2 - \\
& 45w_9w_6w_7w_4^3v_2^2w_8v_1^2w_5^2 + 12w_9w_6c_5^2s^2w_7w_4^3w_5^2 + 12w_9w_6c_5^2s^2w_7w_4w_8w_5^2 + 36w_6c_5^2s^2w_7w_4^2w_8v_1^2w_5^2 + 12c_2^2w_7w_3^4v_2^2w_5^2 - 36w_9w_6w_7w_4^2v_2^2w_1^2w_5 - \\
& 72w_9w_6c_5^2s^2w_7w_4^2w_8v_1^2w_5^2 + 12w_9w_6c_5^2s^2w_7w_4^3v_2^2w_8w_5^2 - 36w_6c_5^2s^2w_7w_4^3v_2^2w_5^2 + 27w_9w_6c_5^2s^2w_7w_4^2v_2^2w_8v_1^2w_5 + 12w_9w_6c_5^2s^2w_7w_4^3v_2^2w_5^2 + 24w_9w_6c_5^2s^2w_7w_4^2w_8v_1^2w_5^2 + \\
& 18w_6c_5^2s^2w_7w_4^2v_2^2w_8v_1^2w_5^2 - 12w_9w_6c_5^2s^2w_7w_4^3v_2^2w_8w_5^2 - 6c_3^2w_7w_3^4v_2^2w_8w_5^2 + 54w_9w_6c_5^2s^2w_7w_4^2w_8v_1^2w_5^2 - 24w_9w_6c_5^2s^2w_7w_4v_2^2w_8w_5^2 - 18w_9w_6c_5^2s^2w_7w_4^2w_8w_5^2 - \\
& 36w_6c_5^2s^2w_7w_4^2w_8v_1^2w_5^2 - 12w_6c_5^2s^2w_7w_4^2v_2^2w_5^2 - 24w_9w_6c_5^2s^2w_7w_4^2v_2^2w_8w_5^2 - 36w_6w_7w_3^4v_2^2w_1^2w_5^2 + 18w_9w_6c_5^2s^2w_7w_4^2w_8w_5^2 + 12w_9w_6w_7w_4^2v_2^2w_5^2 - \\
& 6w_9w_6c_5^4v_2^2w_8w_5^2 + 24w_9c_5^2s^2w_7w_4^2v_2^2w_8w_5^2 + 6w_6c_5^2s^2w_7w_3^4w_8w_5^2 - 5w_9w_6c_5^4s^2w_7w_4^2w_8w_5^2 - 12w_9w_6c_5^4s^2w_7w_4^2v_2^2w_5^2 - 24w_9w_6w_7w_4v_2^2w_8w_5^2 + \\
& 18w_6c_5^2w_7w_3^4w_8v_1^2w_5^2 - 15w_9w_6c_5^2s^2w_7w_4^2w_8v_1^2w_5 + 9w_6c_5^2s^2w_7w_4^2w_8v_1^2w_5^2 + 36w_9w_6c_5^2s^2w_7w_4^3v_1^2w_5^2 - 6w_6c_5^4s^2w_7w_4^3w_8w_8v_1^2w_5^2 - 15w_9w_6c_5^2s^2w_7w_4^2w_8v_1^2w_5^2 - \\
& 12w_6c_5^2w_7w_3^2v_2^2w_5^2 - 6w_9c_5^2w_7w_4^2v_2^2w_8w_5^2 + 12w_9w_6w_7w_4v_2^2w_8w_5^2 - 12w_9w_6w_7w_4^2v_2^2w_8w_5^2 + 12c_4^4s^2w_7w_4^2w_5^2 - 36w_9w_6w_7w_4^2v_2^2w_8v_1^2w_5^2 + 36c_5^2s^2w_7w_4^2v_1^2w_5^2 + \\
& 36w_6w_7w_4^2v_2^2w_1^2w_5^2 + 6w_9w_7w_4^3v_2^2w_8w_5^2 + 24w_9w_6c_5^2s^2w_7w_4^2v_2^2w_8w_5^2 - 102w_9w_6c_5^2s^2w_7w_4w_8v_1^2w_5^2 + 12w_6w_7w_4^2v_2^2w_5^2 + 6w_9c_5^4s^2w_7w_4^3w_8w_5^2 + \\
& 6w_9w_6c_5^4s^2w_7w_4^3w_8w_5^2 + 12w_9w_6c_5^2s^2w_7w_4^3w_5^2 + 12w_6c_5^2s^2w_7w_4^2w_8w_5^2 + 24w_9w_7w_4v_2^2w_8w_5^2 - 48w_9w_6c_5^2s^2w_7w_8v_1^2w_5^2 + 36w_7^2w_3^4v_2^2v_1^2w_5^2 + 12w_6c_5^2s^2w_7w_4^3w_5^2 - \\
& 6w_9c_5^2s^2w_7w_4^3w_8w_5^2 - 12w_9w_6c_5^2s^2w_7w_4^3w_8v_1^2w_5^2 - 18w_6c_5^2s^2w_7w_4^3w_8v_2^2w_5^2 - 24w_9c_5^2s^2w_7w_4^2v_2^2w_8w_5^2 + 36w_9w_6w_7w_4^2v_2^2w_1^2w_5^2 - 18w_6w_7w_4^3v_2^2w_8v_1^2w_5^2 + \\
& 18w_9w_6w_7w_4^3v_2^2w_8v_1^2w_5^2 - 12w_9c_5^4s^2w_7w_4^2w_8w_5^2 - 6w_9w_6c_5^2s^2w_7w_4^3w_8w_5^2 + 12w_9w_6w_7w_4^2v_2^2w_5^2 - 6w_9w_6c_5^2s^2w_7w_4^3w_8v_1^2w_5^2
\end{aligned}$$

$$\begin{aligned} C_{13} = & 24w_9w_6^2c_s^2w_7^2w_4w_5^2 + 60w_9w_6^2c_s^2w_7^2w_4w_8w_5^2 - 36w_9w_6^2c_s^2w_7w_4w_8w_5^2 + 12w_6^2w_7^2w_4^2w_8^2v_1w_5^2 - 24w_9w_6w_7w_4w_8^2v_1^2w_5^2 - 66w_9w_6^2w_7w_4w_8w_5^2 + \\ & 24w_6^2w_7^2w_4^2w_8w_5^2 + 6w_9w_6^2c_s^2w_7w_3^2w_8w_5 - 18w_9w_6^2w_7w_3^2w_8^2v_1^2w_5^2 - 12w_9w_6^2w_7w_2^2w_8^2v_1^2w_5 - 96w_9w_6^2c_s^2w_7w_2^2w_8^2 + 24w_9w_6^2c_s^2w_4^3w_5^2 + \\ & 12w_9w_6c_s^2w_7w_4^2w_8w_5^2 - 42w_9w_6^2c_s^2w_7w_4w_8^2w_5 + 12w_6^2w_7w_3^2w_8w_5^2 - 18w_9w_6^2w_7w_2^2w_8^2v_1^2w_5 - w_9w_6^2w_7^2w_3^2w_8w_5^2 - 12w_9w_6^2w_7^2w_4w_8w_5 - \end{aligned}$$

$$\begin{aligned}
C_{15} = & -2w_3^6w_7w_3^3v_2^2w_8w_5 + 2w_9w_6^2w_7^2w_4^2w_8w_5 + 4w_9w_6^3c_2^2w_4^2w_8w_5 - 4w_9w_6^2c_2^2w_7w_3^2w_8w_5 - 24w_9w_6^3c_2^2w_7w_3^2w_8w_5 + 6w_9w_6^3w_7^2w_4w_8w_5 + 12w_9c_2^2w_7^2w_3^2w_8w_5 - \\
& -2w_3^2w_7^2c_3^4w_8w_5 - 8w_9w_6c_2^2w_7^2w_4^3w_8w_5 - w_9w_6^3w_7w_3^2c_2^2w_8w_5 - 6w_9w_6^3w_2^2w_4w_8w_5 + 2w_6^3c_2^2w_7^2w_3^2w_8w_5 - w_6w_6^2c_3^2w_4^2w_8w_5 - \\
& -2w_9w_6^3c_2^2w_7w_3^2w_8^2 - 4w_6^2w_2^2w_3^2w_8w_5 - 2w_9w_6^2w_7^2w_4c_2^2w_8w_5 + 2w_6^2w_2^2w_3^2c_2^2w_8w_5 + 4w_6^3c_2^2w_7w_4^2w_8w_5 - 2w_9w_6^2w_2^2w_4^2v_2^2w_8w_5 + w_9w_6^3w_7w_3^2v_2^2w_8w_5 + \\
& w_9w_6^3w_2^2w_4^2w_8w_5 + 4w_9w_7c_2^3v_2^2w_8w_5 - 4w_6^3w_7w_4^2w_8w_5 - 4w_9w_6^2w_7w_3^2v_2^2w_8w_5 - 4w_6^3w_7w_4^2w_8w_5 + 2w_9w_6c_2^2w_7w_3^2w_8w_5 + \\
& 2w_9w_6^2w_7w_4^2v_2^2w_8w_5 + 4w_6^3c_2^2w_7w_4^2w_8w_5 + 4w_6^2w_2^2w_7w_3^2w_8w_5 - 4w_9w_6^3w_7w_4^2v_2^2w_8w_5 - w_9w_6^2w_7w_4^2w_8w_5 - 2w_9w_6^3c_2^2w_7w_4w_8w_5 + \\
& 26w_9w_6^3c_2^2w_7w_4w_8w_5 + 7w_9w_6^2w_7w_4^2v_2^2w_8w_5 - 4w_6^3w_2^2w_4^2w_8w_5 + 4w_6^2w_7w_3^2v_2^2w_8w_5 - w_9w_6^3w_2^2w_4^2v_2^2w_8w_5 + 5w_9w_6w_7w_3^2w_8w_5 + 3w_9w_6^3w_7w_3^2w_8w_5 + \\
& 4w_6^3w_7w_4^2v_2^2w_8w_5 + 11w_9w_6^3c_2^2w_7w_4^2w_8w_5 + 13w_9w_6^2c_2^2w_7w_3^2w_8w_5 + 4w_9w_6^3c_2^2w_7w_4^2w_8^2 + 9w_9w_6^3w_7w_4^2v_2^2w_8w_5 + 4w_6^2w_7w_3^2w_8w_5 + 2w_9w_6^3c_2^2w_7w_4^2w_8w_5 - \\
& 2w_6^2w_7w_4^2v_2^2w_8w_5 + 2w_3^3w_7w_3^2w_8w_5 - 8w_9w_6^3c_2^2w_7w_3^2w_8w_5 + 3w_9w_6^2w_7w_3^2w_8w_5 - 4w_9w_6^2w_7w_3^2w_8w_5 + 2w_9w_6^3w_7w_3^2w_8w_5 - 4w_9w_6^3w_2^2w_4^2w_5 + \\
& 4w_6^3w_7w_4^2w_8w_5 - 8w_9w_6^3w_7w_3^2w_8w_5 - 4w_6^3w_2^2w_7w_4^2w_8w_5 - 2w_6^2c_2^2w_7w_4^2w_8w_5 - 7w_9w_6^2w_7w_3^2w_8w_5 + 2w_9w_6^2w_7w_4^2w_8w_5 + 8w_9w_6^3w_7w_4^2w_8w_5 + \\
& 8w_9w_6^3w_7w_4^2w_8w_5 + 4w_6^3w_7w_4^2v_2^2w_8w_5 - 6w_9w_6^3c_2^2w_7w_4w_8w_5 + w_9w_6^2w_7w_3^2v_2^2w_8w_5 - 16w_9w_6^3c_2^2w_7w_4^2w_8w_5 + 2w_9w_6^3c_2^2w_7w_4^2w_8w_5 + \\
& 12w_9w_6^3c_2^2w_7w_4w_8w_5 - 5w_9w_6^3c_2^2w_7w_4^2w_8w_5 + 2w_9w_6^2c_2^2w_7w_4^2w_8w_5 + 2w_6^2w_7w_3^2v_2^2w_8w_5 + 4w_9w_6^2w_7w_3^2v_2^2w_8w_5 - 3w_9w_6^2w_7w_4^2v_2^2w_8w_5 - 2w_9w_6w_7w_3^2w_4^2w_8w_5 - \\
& 4w_6^3w_2^2w_4^2v_2^2w_8w_5 - 5w_9w_6w_7w_3^2w_4^2v_2^2w_8w_5 - 9w_9w_6^3w_2^2w_4^2w_8w_5 + 3w_9w_6^2w_7w_3^2w_4^2w_8w_5 + 4w_9w_6^3w_2^2w_4^2v_2^2w_8w_5 - 3w_9w_6^3w_2^2w_4^2v_2^2w_8w_5 - 6w_9w_6^2w_7w_3^2w_4^2w_8w_5 - \\
& 15w_9w_6c_2^2w_7^2w_3^2w_4^2w_8w_5 + 8w_9w_6^3c_2^2w_7w_3^2w_4^2w_8w_5 - 2w_9w_6^3w_7w_4w_8w_5 + 4w_6^3w_7w_3^2c_2^2w_4^2w_8w_5 + 2w_9w_6^3w_7w_4^2v_2^2w_8w_5 + 4w_9w_6^3w_2^2w_4^2w_5
\end{aligned}$$

$$\begin{aligned}
& 12w_9w_3^6c_4^4w_7w_8^2 + 6w_9w_6^2c_8^2w_7w_3^3w_8^2v_1 - 12w_9w_7w_4^3w_8^2v_1^2 - 36w_9w_6c_2^2w_7w_4w_2^2w_8^2 - 36w_9w_6w_7w_3^2v_2^2w_8s^2v_1^2 - 48w_9w_3^2c_8w_7w_2^2w_8^2 + \\
& 36w_9w_7w_3^4v_2^2w_8^2v_1^2 - 12w_9w_6c_4^8w_7w_2^4w_8^2 + 18w_9w_6c_8^2w_7w_3^3v_2^2w_8^2 + 12w_6^3c_4^4w_7w_4w_2^4w_8 + 18w_9w_6^2c_8^2w_3^4v_2^2w_8^2 + 18w_9w_6^2w_4^3v_2^2w_8^2v_1^2 - 12w_6^2c_8^2w_7w_3^4w_8^2 - \\
& 24w_9w_6^3c_8^2w_7w_4^3v_2^2 + 12w_9w_6w_7w_3^4w_8s^2v_1^2 + 18w_9w_6^3c_8^4w_7w_4w_2^4w_8^2 - 6w_6^3c_4^4w_3^3w_8^2 + 6w_6^3c_8^2w_4^3v_2^2 + 24w_9w_6^2c_8^2w_7w_4^2w_8^2 - 24w_9w_6^3c_8^2w_7w_4w_2w_8s^2v_1^2 - \\
& 6w_6^3c_8^2w_3^3w_8^2v_1^2 - 72w_9w_6^3w_7w_4w_2^2w_8^2v_1^2 - 24w_9w_6^2w_7w_4^3w_8s^2v_1^2 + 36w_3^3c_4^2w_2^2v_2^2v_1^2 - 12w_6^3c_8^2w_7w_4^2w_8^2v_1^2 - 6w_2^2c_8^2w_7w_4^3w_8^2v_1^2 + 12w_9w_6^2w_4^2w_8^2v_1^2 + \\
& 12w_9w_6^3w_7w_4^2v_1^2 + 36w_6^3c_8^2w_7w_4^2w_2^2s^2 + 12w_9w_6^3c_8^2w_7w_4^2w_2^3v_1^2 + 36w_6^3c_8^2w_7w_4^2v_2^2w_8^2 + 12w_9w_7w_4^3w_8^2v_1^2 + 6w_6^3c_8^4w_7w_4^2v_2^2w_8^2 - 12w_6^3c_8^2w_4^2w_8^2 - \\
& 36w_9w_6^3w_7w_4^2w_8s^2v_1^2 + 24w_9w_6^3w_7w_4^2w_8^2v_2^2 - 18w_6^3c_8^2w_7w_4^3v_2^2w_8^2 - 36w_6^3c_8^2w_7w_4^2w_2^2s^2 + 6w_6w_9c_4^2w_7w_4^3w_8^2 - 12w_6^3c_8^2w_7w_4^3w_8^2s^2 - 12w_9w_6w_7w_4^2w_8^2v_1^2 + \\
& 12w_9w_6^2c_2^2w_7w_4w_2^4w_8 + 12w_9w_6^2c_4^8w_7w_4w_2^4w_8^2 + 36w_6^2w_7w_3^3v_2^2w_8^2 + 6w_2^2w_7w_3^4w_2^2v_1^2 - 12w_9w_6^3c_2^2w_7w_4^3w_8^2v_1^2 - 12w_9w_6^2c_2^2w_7w_4^2w_8^2v_1^2 - 12w_9w_6^3c_8^2w_7w_4^3w_8^2 - \\
& 18w_3^3c_4^2v_2^2w_8^2v_1^2 - 18w_3^3c_2^4w_4^2v_2^2w_8^2 - 12w_6^3c_8^2w_7w_3^3w_8s^2v_1^2 - 12w_9w_6^3c_8^2w_7w_4^2v_1^2 + 6w_9w_6^2c_8^2w_3^4v_2^2v_1^2 - 12w_6^3c_8^4w_7w_4^2w_8^2 + 6w_6^3c_8^2w_3^4w_8^2 + \\
& 72w_9w_6c_8^2w_7w_4^3v_2^2w_8^2 - 36w_6^3w_7w_4^2v_2^2w_8^2v_1^2 + 12w_6^3w_7w_4^3w_8s^2v_1^2 - 18w_9w_6c_8^2w_7w_4^3w_8^2v_1^2 + 12w_9w_6^3c_8^4w_7w_4w_2^4w_8 + 72w_9w_6^2w_7w_3^2w_8s^2v_1^2 + 6w_6^2c_8^2w_7w_4^3w_8^2 - \\
& 12w_9w_6^3w_7w_4^3v_1^2 - 18w_9w_6^2c_8^2w_7w_4^3v_2^2w_8^2 - 42w_9w_6^3c_8^2w_7w_4^2v_2^2w_8^2 + 24w_9w_6^2c_8^2w_7w_4w_2^4w_8^2v_1^2 - 36w_9w_6^2w_4^2v_2^2w_8^2v_1^2 + 36w_9w_6^3w_7w_4^3v_2^2v_1^2 - 6w_9w_6^2c_8^4w_7w_4^2w_8^2 +
\end{aligned}$$

$$\begin{aligned}
& 12\omega_6^3 c_s^4 \omega_8^2 \omega_8^2 + 72\omega_9 \omega_6^2 \omega_7 \omega_4 v_2^2 \omega_8^2 v_1^2 + 24\omega_9 \omega_6^2 \omega_7 \omega_4 v_2^2 \omega_8^2 v_1^2 + 36\omega_9 \omega_6 \omega_7 \omega_4 v_2^2 \omega_8^2 v_1^2 + 12\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 \omega_8^2 - 6\omega_9 \omega_6^2 \omega_3^4 \omega_8^2 v_1^2 + \\
& 6\omega_6^3 c_s^2 \omega_7 \omega_4^3 \omega_8^2 v_1^2 + 18\omega_9 \omega_6^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 - 24\omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4^2 \omega_8 - 12\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 \omega_8 v_1^2 + 24\omega_9 \omega_6^3 \omega_7 \omega_4^3 \omega_8^2 v_1^2 + 12\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 \omega_8^2 v_1^2 + \\
& 36\omega_6^3 \omega_7 \omega_4^2 \omega_8 v_1^2 - 36\omega_6^3 c_s^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 + 12\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 v_1^2 - 12\omega_6^3 \omega_7 \omega_4^2 \omega_8 v_1^2 + 12\omega_6^2 c_s^2 \omega_7 \omega_4^2 \omega_8^2 + 12\omega_9 \omega_6^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 + 12\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 \omega_8^2 - \\
& 12\omega_6^3 c_s^2 \omega_7 \omega_4^2 \omega_8 + 12\omega_9 \omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 9\omega_6^3 c_s^4 \omega_7 \omega_4^2 \omega_8^2 - 108\omega_9 \omega_6 c_s^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 - 36\omega_9 \omega_6^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 - 24\omega_9 \omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 36\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 \omega_8^2 - \\
& 12\omega_6^2 \omega_7 \omega_4^3 \omega_8 v_1^2 - 24\omega_9 \omega_6^2 \omega_7 \omega_4^3 \omega_8^2 v_1^2 - 18\omega_6^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 - 6\omega_6^3 c_s^2 \omega_7 \omega_4^2 \omega_8^2 + 108\omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 + 78\omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 + 60\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 \omega_8^2 + \\
& 84\omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^2 \omega_8^2 - 24\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 - 6\omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^2 \omega_8^2 - 12\omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4^2 \omega_8^2 + 180\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 + 24\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 + 12\omega_6^3 c_s^2 \omega_7 \omega_4^2 \omega_8^2 - \\
& 6\omega_9 \omega_6^2 c_s^2 \omega_4^3 \omega_8^2 - 6\omega_9 \omega_6^2 \omega_7 \omega_4^3 \omega_8^2 v_1^2 + 60\omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 - 36\omega_9 \omega_6^3 \omega_7 \omega_4^2 \omega_8^2 v_1^2 + 12\omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4^2 \omega_8^2 + 12\omega_6^3 c_s^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 - 12\omega_9 \omega_6^2 c_s^2 \omega_4^2 \omega_8^2 v_1^2 + \\
& 12\omega_6^2 c_s^2 \omega_7 \omega_4^3 \omega_8^2 v_1^2 - 12\omega_9 \omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 36\omega_9 \omega_6^3 \omega_7 \omega_4^2 \omega_8^2 v_1^2 - 12\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 + 12\omega_9 \omega_6 c_s^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 - 12\omega_6^3 \omega_7 \omega_4^2 \omega_8^2 v_1^2 - 36\omega_6^3 \omega_7 \omega_4^2 \omega_8^2 v_1^2 - \\
& 12\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 \omega_8^2 v_1^2 - 6\omega_6^3 c_s^4 \omega_7 \omega_4^3 \omega_8^2 - 72\omega_9 \omega_6^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 - 12\omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^2 \omega_8^2 + 36\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 54\omega_9 \omega_6 \omega_7 \omega_4^2 \omega_8^2 v_1^2 + 36\omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 + 12\omega_6^3 c_s^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 + 18\omega_6^2 c_s^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 + 18\omega_9 \omega_6 \omega_7 \omega_4^2 \omega_8^2 v_1^2 + 18\omega_6^2 c_s^2 \omega_7 \omega_4^2 \omega_8^2
\end{aligned}$$

$$\begin{aligned}
C_{17} = & -144\omega_6 c_s^2 \omega_4^2 \omega_8^2 - 4\omega_6^3 c_s^4 \omega_4^2 \omega_8 + 24\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 8\omega_6^3 c_s^4 \omega_4^2 \omega_8 + 4\omega_6^2 c_s^4 \omega_4^2 \omega_8^2 + 4\omega_6^3 c_s^2 \omega_8 + 36\omega_6 \omega_6^2 \omega_4^2 \omega_8^2 - 8\omega_6^2 c_s^2 \omega_4^2 \omega_8 + 13\omega_6^2 \omega_4^2 \omega_8^2 + \\
& 36\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 4\omega_6^2 c_s^4 \omega_4^2 \omega_8 - 4\omega_6^2 c_s^4 \omega_4^2 \omega_8 + 51\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 36\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + 16\omega_6^2 \omega_4^2 \omega_8^2 - 13\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + 4\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 120\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + \\
& 4\omega_6^3 c_s^4 \omega_4^2 \omega_8^2 - 24\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 4\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + 20\omega_6^2 \omega_4^2 \omega_8^2 - 20\omega_6^3 \omega_4^2 \omega_8^2 - 4\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 32\omega_6^2 \omega_4^2 \omega_8^2 + 84\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + 4\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + \\
& 8\omega_6^2 c_s^4 \omega_4^2 \omega_8^2 - 24\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 4\omega_6 c_s^4 \omega_4^2 \omega_8^2 + 20\omega_6 \omega_4^2 \omega_8^2 + 4\omega_6^3 \omega_4^2 \omega_8^2 - 72\omega_6 c_s^2 \omega_4^2 \omega_8^2 + 20\omega_6 \omega_4^2 \omega_8^2 - 8\omega_6^2 \omega_4^2 \omega_8^2 + 4\omega_6^2 c_s^4 \omega_8^2 - 13\omega_6^2 \omega_4^2 \omega_8^2 + \\
& 4\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 48\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 12\omega_6 c_s^4 \omega_4^2 \omega_8^2 + 4\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 51\omega_3^3 c_s^2 \omega_4^2 \omega_8^2 - 8\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + 13\omega_6^3 \omega_4^2 \omega_8^2 - 16\omega_6^2 \omega_4^2 \omega_8^2 + 96\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - \\
& 8\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + 8\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 8\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 4\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 36\omega_6 \omega_6^2 \omega_4^2 \omega_8^2 - 4\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 12\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 20\omega_6 \omega_4^2 \omega_8^2 + 72\omega_6 c_s^2 \omega_4^2 \omega_8^2 - \\
& 20\omega_6 \omega_4^2 \omega_8^2 + 8\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 4\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 8\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 4\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 24\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 4\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 8\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - \\
& 32\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 8\omega_6^2 c_s^4 \omega_4^2 \omega_8^2 + 20\omega_6^3 \omega_4^2 \omega_8^2 - 4\omega_6^2 c_s^4 \omega_4^2 \omega_8^2 - 84\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 4\omega_6 c_s^4 \omega_4^2 \omega_8^2
\end{aligned}$$

$$\begin{aligned}
C_{18} = & -108\omega_6 c_s^2 \omega_4^2 \omega_8^2 + 12\omega_6^3 \omega_4^2 \omega_8^2 - \omega_6^2 c_s^2 \omega_4^3 \omega_8^2 - 18\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 12\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 90\omega_6 \omega_6^3 \omega_4^2 \omega_8^2 + 252\omega_6^2 c_s^2 \omega_4^3 \omega_8^2 + 6\omega_6^2 c_s^4 \omega_7 \omega_4^3 \omega_8^2 - 6\omega_6^3 c_s^2 \omega_4^3 \omega_8^2 - \omega_6^3 c_s^4 \omega_7 \omega_4^3 \omega_8^2 - \\
& 12\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 12\omega_6^2 \omega_6^2 \omega_4^2 \omega_8^2 - 12\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + 102\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + 27\omega_6^3 \omega_4^2 \omega_8^2 - 21\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 19\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 12\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + \\
& 48\omega_6^3 \omega_4^2 \omega_8^2 - 5\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 6\omega_6^2 c_s^4 \omega_4^2 \omega_8^2 - 48\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 12\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 60\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + 18\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + 12\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + \\
& 24\omega_6^3 \omega_4^2 \omega_8^2 - 6\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 6\omega_6^3 c_s^2 \omega_4^3 \omega_8^2 - 24\omega_6^2 \omega_4^2 \omega_8^2 - 12\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 4\omega_6^3 \omega_4^2 \omega_8^2 + 12\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + 72\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 12\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + \\
& 12\omega_6^2 c_s^4 \omega_4^2 \omega_8^2 + 13\omega_6^3 c_s^4 \omega_4^2 \omega_8^2 + 6\omega_6^2 c_s^2 \omega_4^3 \omega_8^2 - 36\omega_6 \omega_6^3 \omega_4^2 \omega_8^2 + 12\omega_6^3 \omega_4^2 \omega_8^2 - 12\omega_6^2 \omega_4^2 \omega_8^2 + 12\omega_6^4 c_s^2 \omega_4^2 \omega_8^2 + 60\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 12\omega_6^3 \omega_4^2 \omega_8^2 - 27\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - \\
& 12\omega_6 c_s^4 \omega_4^2 \omega_8^2 + 12\omega_6 c_s^2 \omega_4^3 \omega_8^2 + 19\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 30\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 24\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 48\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 12\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + 90\omega_6 \omega_6^3 \omega_4^2 \omega_8^2 - 12\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - \\
& 306\omega_6 c_s^2 \omega_4^3 \omega_8^2 - 12\omega_6 c_s^2 \omega_4^2 \omega_8^2 - 36\omega_6 c_s^2 \omega_4^2 \omega_8^2 + 12\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 12\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + 6\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + 36\omega_6 \omega_6^3 \omega_4^2 \omega_8^2 + 60\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - \\
& 81\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + 18\omega_6^3 \omega_4^2 \omega_8^2 + 12\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 24\omega_6^3 \omega_4^2 \omega_8^2 + 54\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 4\omega_6^3 \omega_4^2 \omega_8^2 - 48\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 72\omega_6^2 c_s^2 \omega_4^2 \omega_8^2
\end{aligned}$$

$$\begin{aligned}
C_{19} = & -17\omega_6^2 \omega_4^2 \omega_8^2 - 20\omega_6^3 c_s^2 \omega_8 - 120\omega_6 \omega_4^2 \omega_8^2 + 56\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 24\omega_6 \omega_4^2 \omega_8^2 - 28\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 48\omega_6^2 \omega_4^2 \omega_8^2 - 16\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + \\
& 80\omega_6^2 \omega_4^2 \omega_8^2 - 25\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 68\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 68\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + 25\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 16\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 12\omega_6^3 \omega_4^2 \omega_8^2 - 64\omega_6 \omega_4^2 \omega_8^2 - 16\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 12\omega_6^2 \omega_4^2 \omega_8^2 + \\
& 28\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 40\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 43\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 16\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 16\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 48\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 43\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + 16\omega_6^2 \omega_4^2 \omega_8^2 - 8\omega_6^3 \omega_4^2 \omega_8^2 + 44\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - \\
& 28\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 44\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 24\omega_6 \omega_4^2 \omega_8^2 - 32\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 16\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 48\omega_6 \omega_4^2 \omega_8^2 - 72\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 17\omega_6^3 \omega_4^2 \omega_8^2 + 8\omega_6^3 \omega_4^2 \omega_8^2 + 64\omega_6 \omega_4^2 \omega_8^2 + \\
& 20\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 16\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 + 28\omega_6^2 \omega_4^2 \omega_8^2 - 32\omega_6^2 \omega_4^2 \omega_8^2 + 104\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 8\omega_6^2 \omega_4^2 \omega_8^2
\end{aligned}$$

$$C_{20} = 12 + 672 c_s^2 v_2^2 - 78 \omega_6^2 c_s^2 + 144 c_s^4 + 10 \omega_6^3 v_2^2 - 1008 \omega_6 c_s^2 v_2^2 - 216 \omega_6 c_s^4 - 18 \omega_6 - 98 \omega_6^2 v_2^2 - \omega_6^3 + 144 v_2^4 + 6 \omega_6^3 c_s^2 v_2^2 + 8 \omega_6^2 - \\
216 \omega_6 v_2^4 + 90 \omega_6^2 v_2^4 - 156 v_2^4 - 5 \omega_6^3 c_s^4 + 234 \omega_6 v_2^2 + 82 \omega_6^2 c_s^4 - 132 c_s^2 - 9 \omega_6^3 v_2^4 + 404 \omega_6^2 c_s^2 v_2^2 + 198 \omega_6 c_s^2$$

$$C_{21} = 12 + 432 c_s^2 v_2^2 - 22 \omega_6^2 c_s^2 + 24 c_s^4 + 14 \omega_6^3 v_2^2 - 648 \omega_6 c_s^2 v_2^2 - 36 \omega_6 c_s^4 - 18 \omega_6 - 154 \omega_6^2 v_2^2 - \omega_6^3 + 504 v_2^4 + 2 \omega_6^3 c_s^2 v_2^2 + 18 \omega_6^3 c_s^2 v_2^2 + 8 \omega_6^2 - \\
756 \omega_6 v_2^4 + 310 \omega_6^2 v_2^4 - 252 v_2^2 - \omega_6^3 c_s^4 + 378 \omega_6 v_2^2 + 14 \omega_6^2 c_s^4 - 36 c_s^2 - 29 \omega_6^3 v_2^4 + 252 \omega_6^2 c_s^2 v_2^2 + 54 \omega_6 c_s^2$$

## 2.3 MRT2

### 2.3.1 Definitions

Collision operator  $\mathbf{C}$ :

$$\mathbf{C}(\mathbf{f}) = \mathbf{M}_2^{-1} \mathbf{S} \left( \boldsymbol{\mu}_2^{(eq)} - \mathbf{M}_2 \mathbf{f} \right),$$

where

$$\mathbf{S} = \text{diag}(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9),$$

$$\omega_1, \omega_2, \dots, \omega_9 \in (0, 2).$$

Matrix  $\mathbf{M}_2$  corresponds to the transformation matrix to the raw moment basis defined by

$$\boldsymbol{\mu}_2 = \begin{pmatrix} m_{(0,0)} \\ m_{(1,0)} \\ m_{(0,1)} \\ m_{(1,1)} \\ m_{(2,0)} + m_{(0,2)} \\ m_{(2,0)} - m_{(0,2)} \\ m_{(2,1)} \\ m_{(1,2)} \\ m_{(2,2)} \end{pmatrix},$$

and is given by

$$\mathbf{M}_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

The equilibrium moments  $\boldsymbol{\mu}_2^{(eq)}$  are defined by

$$\boldsymbol{\mu}_2^{(eq)} = \mathbf{M}_2 \mathbf{M}^{-1} \boldsymbol{\mu}^{(eq)},$$

i.e.,

$$\boldsymbol{\mu}_2^{(eq)} = \begin{pmatrix} \rho \\ \rho v_1 \\ \rho v_2 \\ \rho v_1 v_2 \\ \rho(v_2^2 + v_1^2 + 2c_s^2) \\ \rho(v_1^2 - v_2^2) \\ \rho v_2(v_1^2 + c_s^2) \\ \rho v_1(v_2^2 + c_s^2) \\ \rho(v_1^2 v_2^2 + c_s^2 v_2^2 + c_s^2 v_1^2 + c_s^4) \end{pmatrix}.$$

### 2.3.2 Conservation of mass: $\rho$

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$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{v_1 \delta_t}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho \delta_t}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 \delta_t}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho \delta_t}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + 3c_s^2 + v_1^2) \frac{v_1 \delta_t^3}{12 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3v_1^2) \frac{\rho \delta_t^3}{12 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\ \frac{\rho c_s^2 \delta_t^3}{6 \delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{\rho c_s^2 \delta_t^3}{6 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{v_2 \delta_t^3}{12 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + c_s^2 + 3v_2^2) \frac{\rho \delta_t^3}{12 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\ (-c_s^4 \omega_5 + 24v_1^2 c_s^2 - 2c_s^2 + 3v_1^2 \omega_5 + 6v_1^4 + 2c_s^4 + c_s^2 \omega_5 - 6v_1^2 - 12v_1^2 c_s^2 \omega_5 - 3v_1^4 \omega_5) \frac{\delta_t^4}{24 \delta_t \omega_5} \frac{\partial^4 \rho}{\partial x_1^4} + \\ (-4 + 6c_s^2 - 5v_1^2 \omega_5 - 3c_s^2 \omega_5 + 10v_1^2 + 2\omega_5) \frac{\rho v_1 \delta_t^4}{128 \delta_t \omega_5} \frac{\partial^4 v_1}{\partial x_1^4} + (v_1^2 \omega_7 - \omega_7 - v_1^2 \omega_5 - 3c_s^2 \omega_5 + \omega_5 + 3c_s^2 \omega_7) \frac{v_2 v_1 \delta_t^4}{48 \delta_t \omega_7 \omega_5} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + \\ (3v_1^2 \omega_7 - \omega_7 - 3v_1^2 \omega_5 - c_s^2 \omega_5 + \omega_5 + c_s^2 \omega_7) \frac{\rho v_2 \delta_t^4}{48 \delta_t \omega_7 \omega_5} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + (3c_s^2 \omega_7 \omega_4 - 3c_s^2 \omega_4 \omega_5 - 6c_s^2 \omega_7 \omega_5 - \omega_7 \omega_4 - 3v_1^2 \omega_4 \omega_5 + \end{aligned}$$

$$\begin{aligned}
& v_1^2 \omega_7 \omega_4 + 3 \omega_4 \omega_5 + v_1^2 \omega_7 \omega_4 \omega_5 - \omega_7 \omega_4 \omega_5 + 3 c_s^2 \omega_7 \omega_4 \omega_5 \frac{\rho v_1 \delta_l^4}{12 \delta_t \omega_7 \omega_4 \omega_5} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + (-2 + \omega_4) \frac{c_s^4 \delta_l^4}{6 \delta_t \omega_4} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\
& (-\omega_7 + \omega_4) \frac{\rho v_1 c_s^2 \delta_l^4}{2 \delta_t \omega_7 \omega_4} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + (\omega_4 - \omega_8) \frac{\rho v_2 c_s^2 \delta_l^4}{2 \delta_t \omega_4 \omega_8} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + (\omega_6 + 3 c_s^2 \omega_8 - 3 \omega_6 c_s^2 - \omega_6 v_2^2 - \omega_8 + v_2^2 \omega_8) \frac{v_2 v_1 \delta_l^4}{4 \omega_6 \delta_t \omega_8} \frac{\partial^4 \rho}{\partial x_1 \partial x_3^3} \\
& + (-3 \omega_6 c_s^2 \omega_4 - \omega_4 \omega_8 + 3 \omega_6 c_s^2 \omega_4 \omega_8 + \omega_6 v_2^2 \omega_4 \omega_8 + 3 \omega_6 \omega_4 - 6 \omega_6 c_s^2 \omega_8 - 3 \omega_6 v_2^2 \omega_4 - \omega_6 \omega_4 \omega_8 + v_2^2 \omega_4 \omega_8 + \\
& 3 c_s^2 \omega_4 \omega_8) \frac{\rho v_2 \delta_l^4}{12 \omega_6 \delta_t \omega_4 \omega_8} \frac{\partial^4 v_1}{\partial x_1 \partial x_3^2} + (\omega_6 + c_s^2 \omega_8 - \omega_6 c_s^2 - 3 \omega_6 v_2^2 - \omega_8 + 3 v_2^2 \omega_8) \frac{\rho v_1 \delta_l^4}{4 \omega_6 \delta_t \omega_8} \frac{\partial^4 v_2}{\partial x_1 \partial x_3^2} + \\
& (-2 c_s^2 + 24 v_2^2 c_s^2 + 6 v_2^4 + \omega_6 c_s^2 - 3 \omega_6 v_2^4 - 6 v_2^2 + 2 c_s^4 - 12 \omega_6 v_2^2 c_s^2 + 3 \omega_6 v_2^2 - \omega_6 c_s^4) \frac{\delta_l^4}{24 \omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\
& (-4 + 2 \omega_6 + 6 c_s^2 - 3 \omega_6 c_s^2 + 10 v_2^2 - 5 \omega_6 v_2^2) \frac{\rho v_2 \delta_l^4}{12 \omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0.
\end{aligned}$$

### 2.3.3 Conservation of momentum: $\rho v_1$



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$$\begin{aligned}
& v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (c_s^2 + v_1^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2 \rho v_1 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho v_2 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{\rho v_1 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\
& (-2 + 4 c_s^2 - 3 v_1^2 \omega_5 - 2 c_s^2 \omega_5 + 6 v_1^2 + \omega_5) \frac{\delta_l^2}{\delta_t \omega_5} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + (2 - \omega_5) \frac{3 \rho v_1 \delta_l^2}{\delta_t \omega_5} \left( \frac{\partial v_1}{\partial x_1} \right)^2 + (-2 + \omega_4) \frac{c_s^2 \delta_l^2}{2 \delta_t \omega_4} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\
& (-2 + \omega_4) \frac{c_s^2 \delta_l^2}{2 \delta_t \omega_4} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (-2 + 6 c_s^2 - v_1^2 \omega_5 - 3 c_s^2 \omega_5 + 2 v_1^2 + \omega_5) \frac{v_1 \delta_l^2}{2 \delta_t \omega_5} \frac{\partial^2 \rho}{\partial x_1^2} + \\
& (-2 + 2 c_s^2 - 3 v_1^2 \omega_5 - c_s^2 \omega_5 + 6 v_1^2 + \omega_5) \frac{\rho \delta_l^2}{2 \delta_t \omega_5} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega_4) \frac{\rho c_s^2 \delta_l^2}{2 \delta_t \omega_4} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_4) \frac{\rho c_s^2 \delta_l^2}{2 \delta_t \omega_4} \frac{\partial^2 v_1}{\partial x_2^2} + C_1 \frac{\delta^3}{12 \delta_t \omega_5^2} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + \\
& + (-24 + 11 v_1^2 \omega_5^2 + 36 c_s^2 - 4 \omega_5^2 - 60 v_1^2 \omega_5 - 36 c_s^2 \omega_5 + 60 v_1^2 + 24 \omega_5 + 5 c_s^2 \omega_5^2) \frac{\rho v_1 \delta_l^3}{6 \delta_t \omega_5^2} \frac{\partial^3 v_1}{\partial x_1^3} + C_2 \frac{v_2 v_1 \delta_l^3}{\delta_t \omega_7 \omega_4 \omega_5^2} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + \\
& C_3 \frac{\rho v_2 \delta_l^3}{\delta_t \omega_7 \omega_4 \omega_5^2} \frac{\partial^3 v_1}{\partial x_2^2 \partial x_2} + C_4 \frac{\rho v_1 \delta_l^3}{12 \delta_t \omega_7 \omega_4 \omega_5^2} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + (-12 - \omega_4^2 + 12 \omega_4) \frac{c_s^4 \delta_l^3}{6 \delta_t \omega_4^2} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + \\
& (-\omega_7 \omega_4^2 \omega_5 + 12 \omega_4^2 - 12 \omega_4^2 \omega_5 - 12 \omega_7 \omega_4 + 12 \omega_4 \omega_5 - 12 \omega_7 \omega_5 + 12 \omega_7 \omega_4 \omega_5) \frac{\rho v_1 c_s^2 \delta_l^3}{6 \delta_t \omega_7 \omega_4^2 \omega_5} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + \\
& (-\omega_4^2 + \omega_4 \omega_8 + 2 \omega_4 - 2 \omega_8) \frac{\rho v_2 c_s^2 \delta_l^3}{\delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_5 \frac{v_2 v_1 \delta_l^3}{12 \omega_6 \delta_t \omega_4 \omega_8} \frac{\partial^3 \rho}{\partial x_2^3} + C_6 \frac{\rho v_2 \delta_l^3}{6 \delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_1}{\partial x_2^3} + C_7 \frac{\rho v_1 \delta_l^3}{12 \omega_6 \delta_t \omega_4 \omega_8} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& C_8 \frac{v_1 \delta_l^4}{12 \delta_t \omega_5^3} \frac{\partial^4 \rho}{\partial x_1^4} + C_9 \frac{\rho \delta_l^4}{12 \delta_t \omega_5^3} \frac{\partial^4 v_1}{\partial x_1^4} + C_{10} \frac{v_2 \delta_l^4}{4 \delta_t \omega_5^2 \omega_4^2 \omega_5^3} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2} + C_{11} \frac{\rho v_2 v_1 \delta_l^4}{4 \delta_t \omega_5^2 \omega_4^2 \omega_5^3} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2} + C_{12} \frac{\rho \delta_l^4}{12 \delta_t \omega_5^2 \omega_4^2 \omega_5^3} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2} + \\
& C_{13} \frac{v_1 \delta_l^4}{12 \omega_9 \delta_t \omega_5^2 \omega_4^2 \omega_8 \omega_5^3} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_{14} \frac{\rho \delta_l^4}{12 \omega_9 \delta_t \omega_5^2 \omega_4^2 \omega_8 \omega_5^3} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{15} \frac{\rho v_2 v_1 \delta_l^4}{2 \omega_9 \omega_6 \delta_t \omega_5^2 \omega_4^2 \omega_8 \omega_5^3} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + \\
& C_{16} \frac{v_2 \delta_l^4}{12 \omega_9 \omega_6^2 \delta_t \omega_7 \omega_4^2 \omega_5^2} \frac{\partial^4 \rho}{\partial x_1 \partial x_3^2} + C_{17} \frac{\rho v_2 v_1 \delta_l^4}{12 \omega_9 \omega_6^2 \delta_t \omega_7 \omega_4^2 \omega_5^2} \frac{\partial^4 v_1}{\partial x_1 \partial x_3^2} + C_{18} \frac{\rho \delta_l^4}{12 \omega_9 \omega_6^2 \delta_t \omega_7 \omega_4^2 \omega_5^2} \frac{\partial^4 v_2}{\partial x_1 \partial x_3^2} + C_{19} \frac{v_1 \delta_l^4}{24 \omega_6^2 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 \rho}{\partial x_2^4} \\
& + C_{20} \frac{\rho \delta_l^4}{24 \delta_t \omega_4^3 \omega_8^2} \frac{\partial^4 v_1}{\partial x_2^4} + C_{21} \frac{\rho v_2 v_1 \delta_l^4}{12 \omega_6^2 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= -12 c_s^4 \omega_5 + 144 v_1^2 c_s^2 - 7 v_1^2 \omega_5^2 - 12 c_s^2 + 36 v_1^2 \omega_5 + c_s^4 \omega_5^2 + 36 v_1^4 + 12 c_s^4 + 12 c_s^2 \omega_5 - 36 v_1^2 - 144 v_1^2 c_s^2 \omega_5 + 7 v_1^4 \omega_5^2 - 36 v_1^4 \omega_5 + 24 v_1^2 c_s^2 \omega_5^2 - c_s^2 \omega_5^2 \\
C_2 &= -v_1^2 \omega_5^2 + 3 c_s^2 \omega_7 \omega_4 - 3 c_s^2 \omega_4 \omega_5 + 3 c_s^2 \omega_7 \omega_5 + 3 c_s^2 \omega_4 \omega_5^2 + \omega_5^2 - \omega_7 \omega_4 - v_1^2 \omega_4 \omega_5 + v_1^2 \omega_7 \omega_4 + \omega_4 \omega_5 - \omega_4 \omega_5^2 - \omega_7 \omega_5 - 3 c_s^2 \omega_5^2 + v_1^2 \omega_7 \omega_5 - v_1^2 \omega_7 \omega_4 \omega_5 + v_1^2 \omega_4 \omega_5^2 + \omega_7 \omega_4 \omega_5 - 3 c_s^2 \omega_7 \omega_4 \omega_5
\end{aligned}$$

$$C_3 = -3 v_1^2 \omega_5^2 + c_s^2 \omega_7 \omega_4 - c_s^2 \omega_4 \omega_5 + c_s^2 \omega_7 \omega_5 + c_s^2 \omega_4 \omega_5^2 + \omega_5^2 - \omega_7 \omega_4 - 3 v_1^2 \omega_4 \omega_5 + 3 v_1^2 \omega_7 \omega_4 + \omega_4 \omega_5 - \omega_4 \omega_5^2 - \omega_7 \omega_5 - c_s^2 \omega_5^2 + 3 v_1^2 \omega_7 \omega_5 - 3 v_1^2 \omega_7 \omega_4 \omega_5 + 3 v_1^2 \omega_4 \omega_5^2 + \omega_7 \omega_4 \omega_5 - c_s^2 \omega_7 \omega_4 \omega_5$$

$$C_4 = 6 \omega_7 \omega_4^2 \omega_5 - 18 c_s^2 \omega_7 \omega_4 \omega_5 - 12 \omega_4^2 \omega_5^2 - 24 c_s^2 \omega_7 \omega_4 \omega_5 - 6 v_1^2 \omega_7 \omega_4 \omega_5 + 12 v_1^2 \omega_4 \omega_5^2 - 3 v_1^2 \omega_7 \omega_4 \omega_5^2 - 12 v_1^2 \omega_4 \omega_5^2 - 12 c_s^2 \omega_4 \omega_5^2 + 36 c_s^2 \omega_7 \omega_4^2 + 12 \omega_4^2 \omega_5 - 11 c_s^2 \omega_7 \omega_4 \omega_5^2 + 3 \omega_7 \omega_4^2 \omega_5^2 + 42 c_s^2 \omega_7 \omega_4 \omega_5^2 - 6 \omega_7 \omega_4 \omega_5^2 + 12 c_s^2 \omega_4 \omega_5^2 + 6 v_1^2 \omega_7 \omega_4 \omega_5^2 + 12 v_1^2 \omega_7 \omega_4^2 + 12 \omega_4 \omega_5^2 - 12 c_s^2 \omega_4 \omega_5^2 - 12 \omega_7 \omega_4^2 - 12 v_1^2 \omega_4 \omega_5^2 - 24 c_s^2 \omega_7 \omega_4 \omega_5$$

$$C_5 = 18 \omega_6 c_s^2 \omega_4 + 12 \omega_6 + 6 \omega_4 \omega_8 + 36 c_s^2 \omega_8 - 36 \omega_6 c_s^2 + 3 \omega_6 c_s^2 \omega_4 \omega_8 + \omega_6 v_2^2 \omega_4 \omega_8 - 6 \omega_6 \omega_4 + 6 \omega_6 v_2^2 \omega_4 - \omega_6 \omega_4 \omega_8 - 12 \omega_6 v_2^2 - 12 \omega_8 - 6 v_2^2 \omega_4 \omega_8 - 18 c_s^2 \omega_4 \omega_8 + 12 v_2^2 \omega_8$$

$$C_6 = -3 c_s^2 \omega_4^2 \omega_8 - v_2^2 \omega_4^2 \omega_8 + 3 v_2^2 \omega_4^2 - 3 \omega_4^2 - 6 v_2^2 \omega_4 - 3 \omega_4 \omega_8 - 12 c_s^2 \omega_8 + 6 \omega_4 + \omega_4^2 \omega_8 + 3 c_s^2 \omega_4^2 - 6 c_s^2 \omega_4 + 3 v_2^2 \omega_4 \omega_8 + 15 c_s^2 \omega_4 \omega_8 \\
C_7 = 6 \omega_6 c_s^2 \omega_4 + 12 \omega_6 + 6 \omega_4 \omega_8 + 12 c_s^2 \omega_8 - 12 \omega_6 c_s^2 + \omega_6 c_s^2 \omega_4 \omega_8 + 3 \omega_6 v_2^2 \omega_4 \omega_8 - 6 \omega_6 \omega_4 + 18 \omega_6 v_2^2 \omega_4 - \omega_6 \omega_4 \omega_8 - 36 \omega_6 v_2^2 - 12 \omega_8 - 18 v_2^2 \omega_4 \omega_8 - 6 c_s^2 \omega_4 \omega_8 + 36 v_2^2 \omega_8$$

$$C_8 = 12 - 216c_s^4\omega_5 + 672v_1^2\omega_s^2 - 98v_1^2\omega_5^2 + 10v_1^2\omega_5^3 - 132c_s^2 + 8\omega_5^2 - 5c_s^4\omega_5^3 + 234v_1^2\omega_5 + 82c_s^4\omega_5^2 + 144v_1^4 - \omega_5^3 + 144c_s^4 - 9v_1^4\omega_5^3 + 198c_s^2\omega_5 - 156v_1^2 - 1008v_1^2c_s^2\omega_5 + 90v_1^4\omega_5^2 - 18\omega_5 - 216v_1^4\omega_5 + 404v_1^2c_s^2\omega_5^2 - 78c_s^2\omega_5^2 - 34v_1^2c_s^2\omega_5^3 + 6c_s^2\omega_5^3$$

$$\textcolor{red}{C_9} = 12 - 36c_s^4\omega_5 + 432v_1^2c_s^2 - 154v_1^2\omega_5^2 + 14v_1^2\omega_5^3 - 36c_s^2 + 8\omega_5^2 - c_s^4\omega_5^3 + 378v_1^2\omega_5 + 14c_s^4\omega_5^2 + 504v_1^4 - \omega_5^3 + 24c_s^4 - 29v_1^4\omega_5^3 + 54c_s^2\omega_5 - 252v_1^2 - 648v_1^2c_s^2\omega_5 + 310v_1^4\omega_5^2 - 18\omega_5 - 756v_1^4\omega_5 + 252v_1^2c_s^2\omega_5^2 - 22c_s^2\omega_5^2 - 18v_1^2c_s^2\omega_5^3 + 2c_s^2\omega_5^3$$

$$\begin{aligned}
C_{10} = & 8v_4^4w_7^2w_5^2 + 120v_1^2c_s^2w_7w_4w_5^2 + 20v_4^1w_7w_4w_5^3 + 13v_4^1w_2^2w_4^2w_5^2 - 20v_1^2w_2^2w_4w_5 - 4v_1^2w_4^2w_5^3 + 4c_s^2w_7w_4w_5^3 - 16v_4^1w_7w_4w_5^2 - \\
& 24v_2^2w_7^2w_4^2 - 4c_s^2w_4w_5^3 + 8c_s^2w_7w_4w_5^3 - 36v_1^2c_s^2w_7w_4w_5^3 - 51v_1^2c_s^2w_7w_2w_4w_5^2 - 84v_1^2c_s^2w_7w_2w_4w_5^3 + 4c_s^2w_7w_2w_4w_5^2 - 4c_s^2w_7w_4w_5 + 20v_1^2w_7w_2w_5 + \\
& 4v_2^2w_4w_5^2 + 8c_s^2w_7w_4w_5^2 - 8v_2^2w_7^2w_5^2 + 4c_s^2w_7w_2w_5^3 - 32v_1^2w_7w_2w_5^2 + 4v_4^4w_4w_5^3 - 24v_1^2c_s^2w_4w_5^3 - 4c_4^4w_7w_5^3 - 12c_4^4w_5^2w_4w_5 + 72v_1^2c_s^2w_7w_4w_5 + \\
& 24v_4^4w_7^2w_4^2 + 4c_s^2w_4w_5^3 + 13v_1^2w_7w_4w_5^2 - 8c_s^2w_7w_4w_5^2 - 72v_1^2c_s^2w_7w_2w_5^2 - 36v_1^2w_7w_2w_5^3 - 4v_1^4w_4w_5^2 - 20v_1^2w_7w_4w_5^3 + \\
& 96v_1^2c_s^2w_7w_2w_4^2 - 13v_1^2w_7w_2w_4w_5^2 - 4c_s^2w_7w_2w_4w_5^3 + 8v_1^2w_7w_4w_5^3 - 4c_s^2w_7w_4w_5^2 + 4c_s^4w_4w_5^2 + 20v_4^4w_2^2w_4w_5^2 - 24v_1^2c_s^2w_4w_5^2 - 8c_s^2w_7w_4w_5^3 - 4c_s^2w_7w_2w_4w_5^2 + \\
& 4c_s^2w_4w_5^2 + 16v_2^1w_7w_4w_5^2 + 24v_1^2c_s^2w_4w_5^3 - 20v_4^1w_7w_4w_5^2 - 4v_4^4w_4w_5^3 - 8c_2^2w_7w_4^2 + 4c_s^2w_7w_4w_5 - 144v_1^2c_s^2w_7w_2w_5^2 + 32v_1^4w_7w_4w_5^2 + 4c_4^4w_4w_5^3 - \\
& 8v_1^4w_7w_5^3 + 51v_1^2c_s^2w_7w_4w_5^2 - 8c_s^4w_7w_4w_5^2 - 4c_s^4w_7w_4w_5^3 + 84v_1^2c_s^2w_7w_4w_5^3 + 12c_2^2w_7w_4w_5^2 - 4c_s^2w_7w_5^2 - 48v_1^2c_s^2w_7w_4w_5^2 + 36v_1^2c_s^2w_7w_5^2 + \\
& 8c_s^4w_7w_4w_5^2 - 20v_4^1w_7^2w_4w_5^2 - 4c_s^4w_4w_5^2 - 13v_4^1w_7w_4w_5^3 + 4v_1^2w_4w_5^3 + 8c_4^4w_7w_4^2 + 36v_1^2w_7^2w_4w_5
\end{aligned}$$

$$\begin{aligned} C_{11} = & 24w_7w_4^2w_5 - 8w_4^2w_5^3 + 64v_1^2w_7^2w_4w_5 + 16v_1^2w_4^2w_5^3 - 32c_s^2w_7w_4^2w_5 - 20c_s^2w_7w_3^3 + 8w_4^2w_5^2 + 80v_1^2w_7^2w_4^2 - 24w_7^2w_4w_5 + 32c_s^2w_7^2w_4w_5 - \\ & 64v_1^2w_7w_4^2w_5 - 16v_1^2w_4^2w_5^2 - 32w_7^2w_4^2 - 44c_s^2w_7^2w_4w_5^2 + 28v_1^2w_7^2w_5^2 - 25c_s^2w_7w_4^2w_5^3 + 104v_1^2w_7w_4^2w_5^2 + 17w_7w_4^2w_5^3 + 28w_7^2w_4w_5^2 - 12w_7^2w_5^2 - \\ & 16c_s^2w_4w_5^3 - 68v_1^2w_7^2w_4w_5^2 - 43v_1^2w_7w_4^2w_5^3 + 56c_s^2w_7w_4^2w_5^2 - 40w_7w_4^2w_5^2 + 68v_1^2w_7w_4w_5^3 + 43v_1^2w_7^2w_4^2w_5^2 + 16c_s^2w_4w_5^3 - 16c_s^2w_7w_4w_5^2 - \\ & 28v_1^2w_7w_3^3 + 12w_7w_5^3 + 16w_7w_4w_5^2 + 44c_s^2w_7w_4w_5^3 + 25c_s^2w_7^2w_4^2w_5^2 - 16c_s^2w_7^2w_5^2 - 48v_1^2w_7w_4w_5^2 - 17w_7^2w_4^2w_5^2 + 48c_s^2w_7^2w_4^2 - 28w_7w_4w_5^3 + \\ & 48w_7^2w_4^2w_5^2 - 72c_s^2w_7^2w_4^2w_5 + 20c_s^2w_7^2w_5^2 + 8w_4w_5^3 - 16v_1^2w_4w_5^3 - 120v_1^2w_7^2w_4^2w_5 \end{aligned}$$

$$\begin{aligned}
C_{12} = & 13c_s^4w_7^2w_4^2w_5^3 + 102v_1^2c_s^2w_7^2w_4w_5^3 - 12c_s^4w_7^2w_4^3w_5 - 12v_1^2c_s^2w_7w_4^2w_5^2 - 72v_1^2w_7^2w_4^3w_5^3 - 24v_1^4w_7w_4w_5^3 + 12v_1^4w_7^3w_5^3 + 12v_1^4w_7^2w_4^2w_5^2 - \\
& 60v_1^2w_7w_3^3w_5^2 + 12v_1^2w_7^2w_4^2w_5^3 - 6c_s^2w_7w_4^3w_5^3 - 18v_1^4w_7^2w_4^2w_5^3 - 12v_1^4w_7^3w_5^2 - 90v_1^4w_7^2w_3^3w_5 + 12c_s^4w_7w_4w_5^3 + 30v_1^2c_s^2w_7w_4^2w_5^3 - 48v_1^2c_s^2w_7w_4w_5^2 + \\
& 6c_s^4w_7^2w_4^2w_5^2 - 36v_1^2c_s^2w_7w_4^3w_5^3 + 6c_s^2w_7w_4^3w_5^2 + 27v_1^2w_7w_4w_5^3 + 18c_s^2w_7w_4^2w_5^3 + 24v_1^2w_7w_4^2w_5^2 - 12v_1^2w_7^3w_5^3 + 72v_1^4w_7^2w_4^3w_5^3 - 12v_1^2w_7w_4w_5^3 - \\
& 12v_1^4w_7^2w_4^3w_5^3 + 19v_1^4w_7w_4^3w_5^3 - c_s^4w_7w_4^3w_5^2 + 54v_1^2c_s^2w_7w_4^3w_5^2 - 12c_s^4w_7w_4^2w_5^4 + 36v_1^2w_7w_4w_5^3 + 12v_1^2w_7^3w_5^3 - 48v_1^2w_7w_4w_5^3 + 12c_s^2w_7w_4w_5^2 + \\
& 6c_s^2w_7^2w_4w_5^3 - 21v_1^2c_s^2w_7w_4^3w_5^3 + c_s^4w_7w_4^3w_5^2 + 4v_1^4w_7w_4^3w_5^3 + 24v_1^2w_7w_4w_5^3 - 12v_1^2w_7^2w_4^2w_5^2 - 5c_s^2w_7w_4^2w_5^3 + 12c_s^2w_7w_4^2w_5^4 + 60v_1^2c_s^2w_7w_4^3w_5^2 + \\
& 6c_s^2w_7w_4^3w_5^3 - 12c_s^2w_7^2w_4^3w_5^3 + 60v_1^2w_7w_4^3w_5^2 - 12c_s^2w_7w_4w_5^3 - 6c_s^2w_7^2w_4^2w_5^2 + 18v_1^2w_7w_4^2w_5^3 + 90v_1^2w_7w_4w_5^3 + 252v_1^2c_s^2w_7w_4^2w_5^3 - 12v_1^2c_s^2w_7^2w_4^3w_5^3 - \\
& 27v_1^4w_7w_4^3w_5^3 - 108v_1^2c_s^2w_7^2w_4^2w_5^5 - 6c_s^4w_7w_4^3w_5^2 + 12v_1^2c_s^2w_7w_4^3w_5^3 + 12c_s^4w_7w_4^2w_5^3 - 24v_1^4w_7w_4^2w_5^2 + 12v_1^4w_7^2w_4w_5^3 - 12v_1^2c_s^2w_7w_4^2w_5^2 + \\
& 162v_1^2c_s^2w_7^2w_4^2w_5^2 - 18c_s^4w_7w_4^2w_5^3 - 48v_1^2c_s^2w_7w_4^2w_5^3 - 12v_1^2c_s^2w_7w_4w_5^3 + 12c_s^2w_7w_4^2w_5^2 + 12c_s^4w_7w_4^2w_5^3 - 19v_1^2w_7w_4^3w_5^2 + 12c_s^4w_7w_4^2w_5^2 - \\
& 306v_1^2c_s^2w_7^2w_4^3w_5^3 - 24c_s^4w_7w_4w_5^3 - 81v_1^2c_s^2w_7w_4^2w_5^3 - 36v_1^4w_7w_4w_5^3 + 48v_1^4w_7w_4^2w_5^3 - 4v_1^2w_7w_4^3w_5^2 - c_s^2w_7^2w_4^3w_5^2
\end{aligned}$$

$$\begin{aligned}
C_{13} = & 36c_s^4w_2^2w_4^2w_5^3 + 36w_9v_2^2c_2^2w_7^2w_4^2w_8w_5 - 12w_9v_1^2c_2^2w_7^2w_4^2w_8w_5 - 36w_9c_4^4w_2^2w_4^2w_8w_5 - 24w_9v_2^2w_7^2w_4w_8w_5^2 + 36w_9v_2^2c_2^2w_7^2w_4^2w_8w_8 - \\
& 6c_2^2w_7^2w_4^2w_8w_5^3 - 6v_2^2c_1^2w_7^2w_3^2w_8w_5^2 - 84w_9c_4^4w_2^2w_4w_8w_5^2 - 12w_9v_1^2c_2^2w_7^2w_4w_8w_5^2 + 12w_9c_4^4w_2^2w_4w_8w_5^3 + 12v_2^2v_1^2w_7w_2^2w_8w_5^3 + 18w_9v_2^2w_7^2w_4^2w_8w_5 - \\
& 72w_9v_2^2c_2^2w_7w_4w_8w_5^3 - 36w_9c_4^4w_2^2w_7^2w_4^2w_5^2 + 12w_9c_2^2w_2^2w_4^2w_8w_5^2 + 12w_9v_2^2w_4^2w_8w_5^2 - 6v_2^2w_7^2w_3^2w_8w_5^3 - 12w_9v_1^2c_2^2w_4^2w_8w_5^2 + 12w_9c_2^2w_7^2w_4w_8w_5^2 + \\
& 12w_9c_2^2w_3^2w_8w_5^2 + 18c_4^4w_2^2w_3^2w_8w_5^2 + 24w_9v_2^2c_1^2w_7w_3^2w_8w_5^2 + 180w_9c_4^4w_2^2w_7^2w_4w_8w_5^2 + 6v_2^2v_1^2w_7w_2^2w_8w_5^3 + 12w_9v_1^2c_2^2w_7^2w_4w_8w_5^2 + 12w_9v_2^2v_1^2w_7w_2^2w_3^2w_8w_5^2 + \\
& 6c_2^2w_7^2w_4^2w_8w_5^2 - 12c_2^2w_7w_4w_8w_5^2 - 36w_9v_2^2c_2^2w_7w_4^2w_8w_5^2 - 12w_9c_2^2w_3^2w_8w_5^2 - 24w_9v_2^2v_1^2w_7w_4^2w_8w_5^2 + 36c_4^4w_2^2w_7w_4w_8w_5^2 - 12w_9v_2^2v_1^2w_7w_4^2w_8w_5^2 - \\
& 18c_4^4w_2^2w_3^2w_8w_5^2 - 12w_9c_2^2w_2^2w_4w_8w_5^2 + 6v_2^2w_7^2w_3^2w_8w_5^2 + 12w_9v_2^2c_2^2w_4^2w_8w_5^3 - 12v_2^2w_7w_2^2w_8w_5^3 - 12w_9v_2^2w_3^2w_8w_5^2 + 12w_9v_2^2v_1^2w_7w_4^2w_8w_5^2 + \\
& 12w_9v_2^2v_1^2w_3^2w_8w_5^2 - 24w_9v_2^2v_1^2w_7w_4w_8w_5^2 - 12v_2^2c_2^2w_7w_3^2w_8w_5^3 - 36v_2^2c_2^2w_7w_2^2w_8w_5^3 - 18w_9v_2^2c_2^2w_7w_3^2w_8w_5^2 - 36w_9v_2^2c_2^2w_3^2w_8w_5^2 - \\
& 12w_9v_2^2w_7w_2^2w_8w_5^2 + 18w_9c_2^2w_5^2w_3^2w_8w_5^2 - 12w_9c_2^2w_7^2w_4^2w_8w_5^2 + 72w_9v_2^2c_2^2w_7w_4^2w_8w_5^2 - 12w_9v_2^2w_7w_2^2w_3^2w_8w_5^2 + w_9v_2^2c_2^2w_7w_4^2w_8w_5^3 + 5w_9c_2^2w_7^2w_4^2w_8w_5^3 - \\
& 18c_4^4w_2^2w_3^2w_5^3 + 12w_9v_1^2c_2^2w_3^2w_4^2w_8 + 2w_9c_2^2w_2^2w_3^2w_8w_5^2 + 36w_9v_2^2c_2^2w_3^2w_8w_5^2 - 18w_9c_2^2w_7w_4^2w_8w_5^2 + 12v_2^2c_2^2w_7w_4^2w_8w_5^2 + 18v_2^2c_2^2w_7w_3^2w_8w_5^2 - \\
& 12v_1^2c_2^2w_7w_2^2w_8w_5^2 - 12w_9v_2^2v_1^2w_7w_3^2w_8w_5^2 - 12w_9v_2^2v_1^2w_4^2w_8w_5^2 + 18w_9v_1^2c_2^2w_7w_4^2w_8w_5^2 - 42w_9c_4^4w_7w_2^2w_8w_5^3 - 36w_9v_2^2c_2^2w_7w_1^2w_8w_5^2 - \\
& 2w_9v_1^2c_2^2w_2^2w_3^2w_8w_5^2 - 6w_9c_4^4w_2^2w_3^2w_8w_5^2 - 72w_9v_2^2c_2^2w_2^2w_4^2w_8w_5^2 - 6w_9v_2^2w_7^2w_3^2w_8w_5^2 - 36w_9v_2^2w_7w_2^2w_8w_5^3 + 12w_9c_2^2w_2^2w_4^2w_5^2 + \\
& 36w_9v_2^2v_1^2w_7w_4^2w_8w_5^2 - 12v_2^2c_2^2w_4^2w_5^3 + 6w_9v_2^2v_1^2w_7^2w_3^2w_8w_5^2 + 12w_9c_2^2w_4^2w_8w_5^3 - 36c_4^4w_7w_4^2w_8w_5^3 + 12w_9v_2^2w_4^2w_8w_5^3 + 12w_9c_2^2w_7w_3^2w_8w_5^2 + \\
& 12w_9v_2^2w_7w_2^2w_5^2 - 12c_2^2w_7^2w_2^2w_4^3 + 12v_2^2w_7w_3^2w_8w_5^3 - 12w_9v_1^2c_2^2w_4^2w_8w_5^3 + 18w_9v_2^2c_2^2w_7w_4^2w_5^2 - 12w_9v_1^2c_2^2w_7w_2^2w_5^2 + 12c_2^2w_7w_3^2w_8w_5^3 + \\
& 12v_2^2v_1^2w_7w_4^2w_8w_5^2 + 12w_9v_2^2w_7w_3^2w_8w_5^2 + 12w_9c_4^4w_3^2w_8w_5^2 - 12w_9v_2^2v_1^2w_7w_2^2w_4^2w_8w_5^2 + 12w_9c_2^2w_7w_3^2w_8w_5^2 + 12w_9v_1^2c_2^2w_7w_4^2w_8w_5^2 + 12w_9v_2^2v_1^2w_7w_2^2w_4^2w_8w_5^3 - \\
& 12v_2^2w_7w_3^2w_8w_5^2 + 12v_2^2w_7^2w_4^2w_8w_5^2 - 36c_4^4w_2^2w_7w_4^2w_8w_5^2 - 12w_9v_2^2v_1^2w_7w_4^2w_8w_5^2 + 36c_4^4w_7w_3^2w_8w_5^2 + 6w_9v_2^2v_1^2w_7w_2^2w_4^2w_8w_5^2 + 12w_9c_2^2w_7w_4^2w_8w_5^3 - \\
& 54w_9v_2^2c_2^2w_7^2w_4^2w_8w_5^2 + 48w_9v_2^2w_7w_4w_8w_5^2 - 48w_9c_4^4w_2^2w_4^2w_8w_5^2 - 12w_9v_2^2c_2^2w_7w_4w_8w_5^2 - 12w_9v_4^2c_4^2w_7w_4w_8w_5^3 - 12v_2^2v_1^2w_7w_3^2w_8w_5^2 + 12w_9c_4^4w_7w_4^2w_8w_5^3 - \\
& 12w_9v_2^2w_7w_2^2w_8w_5^2 - 12w_9c_4^4w_3^2w_8w_5^2 + 72w_9v_2^2c_2^2w_7w_2^2w_8w_5^2 - 6v_2^2c_2^2w_7w_4^2w_8w_5^3 - 12c_2^2w_7w_3^2w_8w_5^2 + 36v_2^2c_2^2w_7w_2^2w_4^2w_5^3 + 12c_2^2w_7w_4^2w_8w_5^3 + \\
& 150w_9c_4^4w_2^2w_4^2w_8w_5^2 + 18w_9v_1^2c_2^2w_2^2w_4^2w_8w_5^2 + 18w_9v_2^2c_2^2w_7w_3^2w_8w_5^2 + 30w_9c_4^4w_7w_3^2w_8w_5^3 - 6w_9v_1^2c_2^2w_7w_4^2w_8w_5^3 + 108w_9v_2^2c_2^2w_7w_2^2w_8w_5^3 - \\
& 6v_2^2v_1^2w_7w_4^2w_8w_5^2 + 12w_9v_2^2w_7w_3^2w_8w_5^2 + 24w_9v_2^2w_2^2w_4^2w_8w_5^2 - 36v_2^2c_2^2w_7w_2^2w_4^2w_8w_5^2 + 6e_2^2w_7w_4^2w_8w_5^3 + 6v_2^2c_2^2w_7w_3^2w_8w_5^3 - 6w_9v_2^2w_7w_2^2w_4^2w_8w_5^3 + \\
& 36v_2^2c_2^2w_7w_3^2w_8w_5^2 - 36w_9v_2^2c_2^2w_7w_2^2w_5^2 + 6w_9v_2^2w_7w_3^2w_8w_5^3 + 6w_9v_1^2c_2^2w_7w_4^2w_8w_5^2 - 36w_9v_2^2c_2^2w_4^2w_8w_5^3 - 18w_9c_2^2w_7w_2^2w_4^2w_8w_5^2 + 36w_9c_4^4w_2^2w_7w_4^2w_8w_5^2 + \\
& 6v_2^2w_7w_2^2w_5^2 - 6w_9c_2^2w_5^2w_3^2w_8w_5^2 - 24w_9v_2^2w_7w_3^2w_8w_5^2 + 12v_2^2c_2^2w_7w_2^2w_5^2 - 18v_2^2c_2^2w_7w_4^2w_8w_5^3 - 88w_9c_4^4w_2^2w_4^2w_8w_5^2 - 12w_9v_1^2c_2^2w_7w_2^2w_4^2w_8w_5^3 - \\
& 36w_9v_2^2c_2^2w_7w_4^2w_8w_5^2 - 42w_9c_4^4w_7w_3^2w_8w_5^2 + 18w_9v_2^2c_2^2w_7w_4^2w_8w_5^2 - 12w_9v_2^2v_1^2w_7w_4^2w_8w_5^2 - 18w_9v_2^2v_1^2w_7w_3^2w_8w_5^2 - 96w_9c_4^4w_7w_3^2w_8w_5^2 - \\
& 12w_9v_2^2v_1^2w_7w_2^2w_4^2w_5^2 - 18w_9c_2^2w_7w_3^2w_8w_5^2 + 12w_9c_2^2w_7w_4^2w_8w_5^2 - 6v_2^2c_2^2w_7w_3^2w_8w_5^2 + 24w_9v_2^2v_1^2w_7w_4^2w_8w_5^2 + 12v_2^2c_2^2w_7w_2^2w_8w_5^3 - 36v_2^2c_2^2w_7w_4^2w_8w_5^3
\end{aligned}$$

$$\begin{aligned}
C_{14} = & 12c_s^4w_2^2w_4^2w_5^3 + 12w_9v_2^2s_2^2w_7^2w_4^2w_8w_5 + 18w_9v_1^2s_2^2w_7^2w_3^2w_8w_5 + 6w_9c_s^4w_2^2w_3^2w_8w_5 - 24w_9v_2^2w_7^2w_4w_8w_5^2 + 12w_9v_2^2c_2^2w_7^2w_4w_8w_5 - \\
& 6c_s^2w_7^2w_4w_8w_5^3 - 18v_2^2v_1^2w_7^2w_4^2w_8w_5^2 - 12w_9c_s^4w_2^2w_4w_8w_5^2 - 84w_9v_1^2c_2^2w_7^2w_4w_8w_5^2 + 36v_2^2v_1^2w_7w_4^2w_8w_5^3 + 18w_9v_2^2w_7^2w_4w_8w_5 - \\
& 24w_9v_2^2c_2^2w_7w_4w_8w_5^3 - 12w_9c_s^4w_2^2w_4w_8w_5^2 - 6w_9c_s^2w_7^2w_4w_8w_5^2 + 12w_9v_2^2s_2^2w_7^2w_4w_8w_5^2 - 6v_2^2w_7^2w_4w_8w_5^3 + 24w_9v_1^2c_2^2w_7^2w_4w_8w_5^2 + 12w_9c_s^2w_7^2w_4w_8w_5^2 + \\
& 6c_s^4w_2^2w_7w_4w_8w_5^3 + 72w_9v_2^2v_1^2w_7w_4^2w_8w_5^2 + 18w_9c_s^4w_2^2w_7w_4^2w_8w_5^3 + 18v_2^2v_1^2c_2^2w_7^2w_4w_8w_5^3 + 84w_9v_1^2c_2^2w_7^2w_4w_8w_5^3 + 36w_9v_2^2v_1^2w_7^2w_4w_8w_5^2 + 6c_s^2w_7^2w_4^2w_8w_5^2 - \\
& 12c_s^2w_7^2w_4w_8w_5^3 - 12w_9v_2^2s_2^2w_7w_4^2w_8w_5^2 - 72w_9v_2^2v_1^2w_7^2w_4w_8w_5^2 - 12c_4^2w_7w_4w_8w_5^3 - 36w_9v_2^2v_1^2w_7^2w_4w_8w_5^3 - 6c_s^4w_2^2w_7^2w_4w_8w_5^2 + 6v_2^2w_7^2w_4^2w_8w_5^2 - \\
& 24w_9v_2^2c_2^2w_7^2w_4^2w_8w_5^3 - 12v_2^2w_7^2w_4^2w_8w_5^3 - 12w_9v_2^2w_7^2w_4^2w_8w_5^3 + 36w_9v_2^2v_1^2w_7^2w_4w_8w_5^3 - 72w_9v_2^2v_1^2w_7w_4w_8w_5^3 - \\
& 36v_1^2c_2^2w_7w_4^2w_8w_5^3 + 12v_2^2c_2^2w_7w_4^2w_8w_5^3 - 6v_2^2c_2^2w_7^2w_4^2w_8w_5^2 - 12w_9v_2^2c_2^2w_7^2w_4^2w_8w_5^2 - 12w_9c_s^2w_7^2w_4w_8w_5^2 + 12w_9v_2^2w_7w_4^2w_8w_5^2 + 6w_9c_s^4w_2^2w_7^2w_4^2w_8w_5^2 + \\
& 24w_9v_2^2c_2^2w_7w_4^2w_8w_5^2 + 60w_9v_1^2c_2^2w_7w_4^2w_8w_5^2 - 12w_9v_2^2w_7^2w_4^2w_8w_5^2 + 12w_9c_s^4w_7w_4^2w_8w_5^2 - w_9c_s^4w_7^2w_4^2w_8w_5^3 - 6c_s^4w_7^2w_4^2w_8w_5^3 + 6w_9c_s^2w_7^2w_4^2w_8w_5^2 + \\
& 12w_9v_2^2c_2^2w_7^2w_4^2w_8w_5^3 + 24w_9v_2^2w_7w_4^2w_8w_5^3 + 36v_1^2c_2^2w_7w_4^2w_8w_5^2 + 6v_2^2s_2^2w_7^2w_4^2w_8w_5^3 - 36w_9v_2^2v_1^2w_7w_4^2w_8w_5^2 -
\end{aligned}$$

$$\begin{aligned}
& 36w_9v_2^2v_1^2w_4^3w_8w_5^2 - 132w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 - 24w_9c_4^2w_7w_4^2w_8w_5^3 - 12w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 - 18w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 - 48w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 - \\
& 6w_9c_4^2w_7w_4^2w_8w_5^2 - 24w_9v_2^2c_s^2w_7w_4^2w_8w_5^2 - 6w_9v_2^2c_s^2w_7w_4^2w_8w_5^2 - 36w_9v_2^2c_s^2w_7w_4^2w_8w_5^2 - 12w_9c_2^2w_7w_4^2w_8w_5^2 + 108w_9v_2^2v_1^2w_7w_4^2w_8w_5^2 - 12v_2^2w_7w_4^2w_8w_5^2 + \\
& 18w_9v_2^2c_1^2w_7w_4^2w_8w_5^2 - 12c_4^2w_7w_4^2w_8w_5^2 + 12w_9v_2^2c_4^2w_8w_5^3 + 12w_9v_2^2c_4^2w_8w_5^3 - 12c_4^2w_7w_4^2w_8w_5^3 + 12v_2^2w_7w_4^2w_8w_5^3 + 24w_9v_1^2c_s^2w_7w_4^2w_8w_5^3 + \\
& 6w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 - 36w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 + 12c_2^2w_7w_4^2w_8w_5^3 + 36v_2^2v_1^2w_7w_4^2w_8w_5^3 + 36v_2^2v_1^2w_7w_4^2w_8w_5^3 + 72w_9v_1^2c_s^2w_7w_4^2w_8w_5^3 + \\
& 36v_2^2v_1^2w_7w_4^2w_8w_5^3 - 12v_2^2w_7w_4^2w_8w_5^3 + 12v_2^2w_7w_4^2w_8w_5^3 + 12w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 - 12c_4^2w_7w_4^2w_8w_5^3 - 36w_9v_2^2v_1^2w_7w_4^2w_8w_5^3 + 12c_4^2w_7w_4^2w_8w_5^3 + \\
& 18w_9v_2^2v_1^2w_7w_4^2w_8w_5^3 - 12w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 - 18w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 + 24w_9v_2^2w_7w_4^2w_8w_5^3 - 108w_9v_1^2c_s^2w_7w_4^2w_8w_5^3 + \\
& 60w_9v_1^2c_s^2w_7w_4^2w_8w_5^3 - 36v_2^2v_1^2w_7w_4^2w_8w_5^3 + 12w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 - 12w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 + 24w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 - 18v_1^2c_s^2w_7w_4^2w_8w_5^3 - \\
& 12c_s^2w_7w_4^3w_8w_5^2 + 12v_2^2c_s^2w_7w_4^2w_8w_5^3 + 12c_s^2w_7w_4^2w_8w_5^3 + 24w_9c_4^2w_7w_4^2w_8w_5^3 + 180w_9v_1^2c_s^2w_7w_4^2w_8w_5^3 + 6w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 + 12w_9c_4^2w_7w_4^2w_8w_5^3 + \\
& 78w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 + 36w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 - 18v_2^2v_1^2w_7w_4^2w_8w_5^3 + 12w_9v_2^2w_7w_4^2w_8w_5^3 + 24w_9v_2^2w_7w_4^2w_8w_5^3 + 6c_4^2w_7w_4^2w_8w_5^3 + \\
& 18v_1^2c_s^2w_7w_4^2w_8w_5^3 - 6w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 + 12v_2^2c_s^2w_7w_4^2w_8w_5^3 - 12w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 - 12w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 - 12w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 - \\
& 24w_9c_2^2w_7w_4^2w_8w_5^3 + 6v_2^2w_7w_4^2w_8w_5^3 - 6w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 - 24w_9v_2^2w_7w_4^2w_8w_5^3 + 36v_2^2c_s^2w_7w_4^2w_8w_5^3 - 6v_2^2c_s^2w_7w_4^2w_8w_5^3 - 4w_9c_4^2w_7w_4^2w_8w_5^3 - \\
& 42w_9v_2^2v_1^2w_7w_4^2w_8w_5^3 - 12w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 - 12w_9v_2^2c_s^2w_7w_4^2w_8w_5^3 - 144w_9v_1^2c_s^2w_7w_4^2w_8w_5^3 - 36w_9v_2^2v_1^2w_7w_4^2w_8w_5^3 - 54w_9v_2^2v_1^2w_7w_4^2w_8w_5^3 - \\
& 12w_9c_4^2w_7w_4^2w_8w_5^3 - 36w_9v_2^2v_1^2w_7w_4^2w_8w_5^3 + 12w_9c_s^2w_7w_4^2w_8w_5^3 - 18v_1^2c_s^2w_7w_4^2w_8w_5^3 + 72w_9v_2^2v_1^2w_7w_4^2w_8w_5^3 + 36v_1^2c_s^2w_7w_4^2w_8w_5^3 - 12v_2^2c_s^2w_7w_4^2w_8w_5^3
\end{aligned}$$

$$\begin{aligned}
C_{15} = & 4w_6c_2^2w_7w_4^2w_8w_5^3 + 4w_6w_7w_4^2w_8w_5^3 - 3w_9w_6v_1^2w_7w_4^2w_8w_5^3 - 3w_9w_6v_1^2w_7w_4^2w_8w_5^3 - 4w_9w_6c_2^2w_7w_4^2w_8w_5^3 + 2w_6v_1^2w_7w_4^2w_8w_5^3 + 4w_9w_6w_7w_4^2w_8w_5^3 + \\
& 12w_9w_6c_2^2w_7w_4^2w_8w_5^2 + 2w_9v_2^2w_7w_4^2w_8w_5^3 - 8w_9w_6c_2^2w_7w_4^2w_8w_5^3 + 7w_9w_6v_1^2w_7w_4^2w_8w_5^3 - w_9w_6v_1^2w_7w_4^2w_8w_5^3 + 4w_6w_7w_4^2w_8w_5^3 - 4w_6w_7w_4^2w_8w_5^3 + \\
& 2w_9w_6c_2^2w_7w_4^2w_8w_5^2 - 2w_9w_6w_7w_4^2w_8w_5^3 + 4w_6v_1^2w_7w_4^2w_8w_5^3 - 6w_9w_6c_2^2w_7w_4^2w_8w_5^3 - w_9w_6w_7w_4^2w_8w_5^3 - 2w_6v_1^2w_7w_4^2w_8w_5^2 + 6w_9w_6w_7w_4^2w_8w_5^3 + \\
& 8w_9w_6c_2^2w_7w_4^2w_8w_5^3 + 4w_9w_6v_1^2w_7w_4^2w_8w_5^3 + 4w_9w_6w_7w_4^2w_8w_5^3 - w_9w_6c_2^2w_7w_4^2w_8w_5^2 - 5w_9w_6c_2^2w_7w_4^2w_8w_5^3 + 3w_9w_6w_7w_4^2w_8w_5^3 + \\
& 12w_9w_6c_2^2w_7w_4^2w_8w_5^2 + 4w_6v_1^2w_7w_4^2w_8w_5^3 - 2w_9c_2^2w_7w_4^2w_8w_5^3 + 2w_6c_2^2w_7w_4^2w_8w_5^3 - 8w_9w_6c_2^2w_7w_4^2w_8w_5^3 - 4w_9w_6v_1^2w_7w_4^2w_8w_5^3 + w_9w_6w_7w_4^2w_8w_5^3 - \\
& 7w_9w_6w_7w_4^2w_8w_5^2 - 8w_9w_6c_2^2w_7w_4^2w_8w_5^3 - 4w_9w_6w_7w_4^2w_8w_5^3 + 13w_9w_6c_2^2w_7w_4^2w_8w_5^3 + 2w_9w_6v_1^2w_7w_4^2w_8w_5^3 + w_9w_6v_1^2w_7w_4^2w_8w_5^3 - 4w_9w_6v_1^2w_7w_4^2w_8w_5^3 + \\
& 2w_6w_7w_4^3w_8w_5^3 - 2w_6c_2^2w_7w_4^2w_8w_5^3 - 6w_9w_6v_1^2w_7w_4^2w_8w_5^3 + 4w_6c_2^2w_7w_4^2w_8w_5^3 + 5w_9w_6w_7w_4^2w_8w_5^3 - 15w_9w_6c_2^2w_7w_4^2w_8w_5^3 - 2w_9w_6w_7w_4^2w_8w_5^3 - \\
& 24w_9w_6c_2^2w_7w_4^2w_8w_5^2 - 6w_9w_6c_2^2w_7w_4^2w_8w_5^3 - 4w_6v_1^2w_7w_4^2w_8w_5^3 + 2w_9w_6w_7w_4^2w_8w_5^3 + 4w_9w_6c_2^2w_7w_4^2w_8w_5^3 - 2w_6w_7w_4^2w_8w_5^3 - 4w_9w_6w_7w_4^2w_8w_5^3 + \\
& 2w_9w_6v_1^2w_7w_4^2w_8w_5^2 + 26w_9w_6c_2^2w_7w_4^2w_8w_5^3 - 16w_9w_6c_2^2w_7w_4^2w_8w_5^3 - 16w_9w_6w_7w_4^2w_8w_5^3 + 2w_9w_6w_7w_4^2w_8w_5^3 + 8w_9w_6c_2^2w_7w_4^2w_8w_5^3 + \\
& w_9w_6v_1^2w_7w_4^2w_8w_5^2 - 9w_9w_6c_2^2w_7w_4^2w_8w_5^3 - 4w_6w_7w_4^2w_8w_5^3 - 2w_6c_2^2w_7w_4^2w_8w_5^3 + 2w_9w_6v_1^2w_7w_4^2w_8w_5^3 + 4w_6c_2^2w_7w_4^2w_8w_5^3 - 4w_9c_2^2w_7w_4^2w_8w_5^3 + \\
& 2w_9w_6c_2^2w_7w_4^2w_8w_5^2 + 4w_9w_6w_7w_4^2w_8w_5^3 + 2w_9w_6w_7w_4^2w_8w_5^3 - 4w_9w_6v_1^2w_7w_4^2w_8w_5^3 + 4w_6c_2^2w_7w_4^2w_8w_5^2 - 4w_6c_2^2w_7w_4^2w_8w_5^3 + 2w_9w_6v_1^2w_7w_4^2w_8w_5^3 + \\
& 8w_9w_6c_2^2w_7w_4^2w_8w_5^3 - 9w_9w_6w_7w_4^2w_8w_5^3 - 2w_6v_1^2w_7w_4^2w_8w_5^3 - w_9w_6w_7w_4^2w_8w_5^3 + 11w_9w_6c_2^2w_7w_4^2w_8w_5^3 + 3w_9w_6c_2^2w_7w_4^2w_8w_5^3 + 4w_9c_2^2w_7w_4^2w_8w_5^3
\end{aligned}$$

$$\begin{aligned}
C_{16} = & -12w_6^2v_2^2v_1^2w_7w_4^2w_8w_5 - 18w_6^2c_4^2w_7w_4^2w_8w_5 - 12w_9w_6^2c_2^2w_7w_4^2w_8w_5 - 3w_9w_6^2v_2^2c_2w_7w_4^2w_8w_5 + 12w_6^2v_1^2w_7w_4^2w_8w_5 + \\
& 144w_9w_6c_2^2c_1^2w_7w_4^2w_8w_5 - 6w_6^2v_2^2v_1^2w_7w_4^2w_8w_5 + 12w_9w_6c_2^2c_4^2w_8w_5 + 6w_9w_6v_1^2w_7w_4^2w_8w_5 + 12w_9w_6v_1^2w_7w_4^2w_8w_5 - 12w_9w_6v_2^2c_2^2w_4^2w_8 - \\
& 15w_9w_6c_2^4w_7w_4^2w_8w_5^2 + 24w_9w_6v_1^2c_2^2w_7w_4^2w_8w_5^2 + 21w_9w_6v_2^2w_7w_4^2w_8w_5^2 + 6w_6^2w_7w_4^2w_8w_5^2 + 156w_9w_6c_2^4w_7w_4^2w_8w_5^2 + 36w_6^2v_1^2w_7w_4^2w_8w_5^2 - \\
& 6w_6^2v_2^2c_2^2w_7w_4^2w_8w_5^2 + 12w_9w_6v_1^2w_7w_4^2w_8w_5^2 + 5w_9w_6c_2^2w_7w_4^2w_8w_5^2 - 36w_6^2v_2^2c_2^2w_7w_4^2w_8w_5^2 - 12w_9w_6v_1^2w_7w_4^2w_8w_5^2 - 24w_9w_6v_1^2w_7w_4^2w_8w_5^2 + \\
& 6w_9w_6v_2^2c_2^2w_7w_4^2w_8w_5^2 - 36w_9w_6v_1^2w_7w_4^2w_8w_5^2 + 12w_9w_6v_1^2w_7w_4^2w_8w_5^2 - 6w_6^2v_2^2c_2^2w_7w_4^2w_8w_5^2 - 36w_9w_6v_1^2c_2^2w_7w_4^2w_8w_5^2 - 18w_9w_6c_2^2w_7w_4^2w_8w_5^2 + \\
& 6w_9v_1^2w_7w_4^2w_8w_5^2 - 18w_6^2c_4^2w_7w_4^2w_8w_5^2 - 108w_9w_6v_1^2c_2^2w_7w_4^2w_8w_5^2 - 5w_9w_6v_2^2c_2^2w_7w_4^2w_8w_5^2 + 12w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 + \\
& 36w_9w_6c_4^2w_7w_4^2w_8w_5^2 - 12w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 + 12w_9v_2^2v_1^2w_7w_4^2w_8w_5^2 - 12w_6^2w_7w_4^2w_8w_5^2 - 42w_9w_6v_2^2w_7w_4^2w_8w_5^2 - 24w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 - \\
& 36w_9w_6v_1^2c_2^2w_7w_4^2w_8w_5^2 + 12w_6^2c_2^2w_7w_4^2w_8w_5^2 - 15w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 - 12w_6^2v_2^2c_2^2w_7w_4^2w_8w_5^2 + 18w_6^2v_1^2c_2^2w_7w_4^2w_8w_5^2 - 12w_6^2v_2^2c_2^2w_7w_4^2w_8w_5^2 - \\
& 6w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 - 36w_6^2c_2^2w_7w_4^2w_8w_5^2 - 6w_6^2v_2^2v_1^2w_7w_4^2w_8w_5^2 + 36w_6^2v_1^2c_2^2w_7w_4^2w_8w_5^2 - 36w_6^2c_4^2w_7w_4^2w_8w_5^2 - 12w_6^2v_1^2w_7w_4^2w_8w_5^2 + \\
& 6w_6^2v_1^2w_7w_4^2w_8w_5^2 - 12w_9w_6v_2^2c_2^2w_7w_4^2w_8w_5^2 - 9w_9w_6v_1^2w_7w_4^2w_8w_5^2 + 18w_9w_6c_2^2w_7w_4^2w_8w_5^2 - 12w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 - 12w_6^2s_2^2w_4^2w_8w_5^2 - \\
& 12w_9w_6c_2^2w_7w_4^2w_8w_5^2 + 9w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 - 6w_9w_6c_2^2w_7w_4^2w_8w_5^2 + 3w_9w_6v_2^2c_2^2w_7w_4^2w_8w_5^2 - 18w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 - 12w_6^2s_2^2w_4^2w_8w_5^2 + \\
& 36w_9w_6c_2^4w_7w_4^2w_8w_5^2 + 36w_6^2c_2^2w_7w_4^2w_8w_5^2 - 9w_9w_6c_2^2w_7w_4^2w_8w_5^2 + 18w_9w_6v_2^2c_2^2w_7w_4^2w_8w_5^2 + 12w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 + 12w_9w_6v_2^2c_2^2w_7w_4^2w_8w_5^2 + \\
& 18w_9w_6v_2^2c_2^2w_7w_4^2w_8w_5^2 - 12w_9w_6c_2^2w_7w_4^2w_8w_5^2 + 15w_9w_6c_2^2w_7w_4^2w_8w_5^2 - 6w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 - 12w_6^2s_2^2w_4^2w_8w_5^2 - 36w_6^2c_4^2w_7w_4^2w_8w_5^2 - 12w_6^2v_1^2w_7w_4^2w_8w_5^2 + \\
& 6w_6^2v_1^2w_7w_4^2w_8w_5^2 - 12w_9w_6v_2^2c_2^2w_7w_4^2w_8w_5^2 + 27w_9w_6v_1^2c_2^2w_7w_4^2w_8w_5^2 + 18w_9w_6v_2^2c_2^2w_7w_4^2w_8w_5^2 - 12w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 - 12w_9v_1^2w_7w_4^2w_8w_5^2 + \\
& 3w_9w_6c_2^2c_2^2w_7w_4^2w_8w_5^2 + 12w_9w_6v_2^2c_2^2w_7w_4^2w_8w_5^2 - 18w_6^2v_2^2c_2^2w_7w_4^2w_8w_5^2 + 12w_6^2v_2^2c_2^2w_7w_4^2w_8w_5^2 - 12w_6^2c_2^2w_7w_4^2w_8w_5^2 - 18w_9w_6c_2^2c_2^2w_7w_4^2w_8w_5^2 + \\
& 6w_6^2c_2^2w_7w_4^2w_8w_5^2 - 24w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 + 12w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 - 12w_9w_6v_2^2c_2^2w_7w_4^2w_8w_5^2 + 18w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 + 36w_9v_1^2c_2^2w_7w_4^2w_8w_5^2 + \\
& 12w_9w_6v_2^2c_2^2w_7w_4^2w_8w_5^2 - 72w_9w_6v_2^2c_2^2w_7w_4^2w_8w_5^2 - 6w_9w_6c_2^2c_4^2w_7w_4^2w_8w_5^2 + 12w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 + 15w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 - \\
& 36w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 - 72w_9w_6v_2^2c_2^2w_7w_4^2w_8w_5^2 - 6w_9w_6c_2^2c_4^2w_7w_4^2w_8w_5^2 + 72w_9w_6v_2^2c_2^2w_7w_4^2w_8w_5^2 + 18w_6^2c_4^2w_7w_4^2w_8w_5^2 + 12w_6^2v_1^2w_7w_4^2w_8w_5^2 + \\
& 12w_9w_6v_2^2w_7w_4^2w_8w_5^2 + w_9w_6v_2^2c_2^2w_7w_4^2w_8w_5^2 - 12w_6^2v_1^2w_7w_4^2w_8w_5^2 + 12w_6^2v_2^2c_2^2w_7w_4^2w_8w_5^2 + 24w_9w_6v_1^2w_7w_4^2w_8w_5^2 - 6w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 + \\
& 36w_9w_6v_1^2w_7w_4^2w_8w_5^2 + 12w_6^2v_2^2c_2^2w_7w_4^2w_8w_5^2 - 36w_6^2v_1^2w_7w_4^2w_8w_5^2 + 12w_6^2v_2^2c_2^2w_7w_4^2w_8w_5^2 + 6w_6^2v_2^2c_2^2w_7w_4^2w_8w_5^2 - 6w_6^2c_2^2w_7w_4^2w_8w_5^2 + \\
& 36w_9w_6v_1^2c_2^2w_7w_4^2w_8w_5^2 + 12w_9w_6v_1^2w_7w_4^2w_8w_5^2 + 48w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 + 36w_6^2c_2^2w_7w_4^2w_8w_5^2 + 12w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2
\end{aligned}$$

$$\begin{aligned}
C_{17} = & 12w_6^2c_2^2w_7w_4^2w_8w_5^2 - 12w_9w_6^2w_7w_4^2w_8w_5^2 + 12w_9w_6^2c_2^2w_7w_4^2w_8w_5^2 + 60w_9w_6^2c_2^2w_7w_4^2w_8w_5^2 + 6w_9w_6v_2^2w_7w_4^2w_8w_5^2 - 36w_9w_6^2c_2^2w_7w_4^2w_8w_5^2 + \\
& 6w_9w_6c_2^2w_7w_4^2w_8w_5^2 - 36w_9w_6^2v_2^2w_7w_4^2w_8w_5^2 - 6w_9w_6w_7w_4^2w_8w_5^2 + 12w_9w_6v_2^2w_7w_4^2w_8w_5^2 + 12w_9w_6v_2^2w_7w_4^2w_8w_5^2 + \\
& 24w_6^2w_7w_4^2w_8w_5^2 - 66w_9w_6^2c_2^2w_7w_4^2w_8w_5^2 + 18w_9w_6^2w_7w_4^2w_8w_5^2 + 12w_9w_6^2w_7w_4^2w_8w_5^2 + 24w_6^2v_2^2w_7w_4^2w_8w_5^2 + 12w_9w_6^2v_2^2w_7w_4^2w_8w_5^2 + \\
& 36w_9c_2^2w_7w_4^2w_8w_5^2 + 24w_6^2c_2^2w_7w_4^2w_8w_5^2 + 24w_6^2v_2^2w_7w_4^2w_8w_5^2 - 12w_9w_6^2v_2^2w_7w_4^2w_8w_5^2 - 42w_9w_6c_2^2w_7w_4^2w_8w_5^2 + 18w_9w_6v_2^2w_7w_4^2w_8w_5^2 + \\
& 24w_9w_2^2c_2^2w_7w_4^2w_8w_5^2 + 24w_6^2c_2^2w_7w_4^2w_8w_5^2 + 24w_6^2v_2^2w_7w_4^2w_8w_5^2 + 12w_9w_6^2v_2^2w_7w_4^2w_8w_5^2 - 12w_9w_6^2v_2^2w_7w_4^2w_8w_5^2 + 84w_9w_6v_2^2w_7w_4^2w_8w_5^2 - \\
& 12w_9w_6v_2^2w_7w_4^2w_8w_5^2 + 36w_9w_6^2v_2^2w_7w_4^2w_8w_5^2 + 12w_6^2v_2^2w_7w_4^2w_8w_5^2 + 24w_9w_6v_2^2w_7w_4^2w_8w_5^2 - 18w_9w_6v_2^2w_7w_4^2w_8w_5^2 - \\
& 12w_9w_6v_2^2w_7w_4^2w_8w_5^2 - 24w_6^2w_7w_4^2w_8w_5^2 + 72w_9w_6v_2^2w_7w_4^2w_8w_5^2 - 12w_9w_6v_2^2w_7w_4^2w_8w_5^2 - 24w_9w_6^2v_2^2w_7w_4^2w_8w_5^2 + 12w_9w_6^2c_2^2w_7w_4^2w_8w_5^2 + \\
& 12w_6^2c_2^2w_7w_4^2w_8w_5^2 - 24w_6^2w_7w_4^2w_8w_5^2 - 84w_9w_6^2c_2^2w_7w_4^2w_8w_5^2 - 12w_9w_6v_2^2w_7w_4^2w_8w_5^2 - 24w_9w_6^2v_2^2w_7w_4^2w_8w_5^2 + 24w_9w_6^2c_2^2w_7w_4^2w_8w_5^2 - \\
& 12w_6^2w_7w_4^2w_8w_5^2 - 18w_9w_6^2c_2^2w_7w_4^2w_8w_5^2 - 12w_9w_6w_7w_4^2w_8w_5^2 + 12w_9w_6c_2^2w_7w_4^2w_8w_5^2 + w_9w_6v_2^2w_7w_4^2w_8w_5^2 - 48w_9w_6^2c_2^2w_7w_4^2w_8w_5^2
\end{aligned}$$

$$\begin{aligned}
& 66w_9w_6^2w_7^2w_4^2w_8w_5 + 12w_9w_6v_2^2w_7^2w_4w_8^2w_5^2 + 3w_9w_6^2c_s^2w_7^2w_4^3w_8^2w_5^2 - 6w_9v_2^2w_7^2w_4^2w_8^3w_5^2 - 72w_9w_6^2c_s^2w_7^2w_4w_8^2w_5 - 24w_6^2v_2^2w_7w_4^3w_8^2w_5^2 - \\
& 12w_9w_6^2v_2^2w_7w_4^2w_8^2w_5^2 + 12w_9w_6w_7^2w_4^2w_8w_5^2 - 12w_9w_6c_s^2w_7^2w_4^2w_8w_5^2 + 24w_9w_6^2v_2^2w_7^2w_4^2w_8^2w_5^2 - 24w_9w_6^2v_2^2w_7^2w_4^2w_8w_5 - \\
& 24w_6^2c_s^2w_7^2w_4^2w_8w_5^2 - 24w_9w_6^2c_s^2w_7^2w_4^2w_8w_5 - 24w_6^2v_2^2w_7^2w_4^2w_8w_5^2 - 12w_9w_6^2w_7^2w_4w_8w_5^2 - 12w_9w_6^2v_2^2w_7^2w_4^2w_8w_5^2 - 96w_9w_6^2c_s^2w_7^2w_8^2w_5^2 - \\
& 132w_9w_6^2c_s^2w_7w_4^2w_8^2w_5^2 - w_9w_6^2w_7^2w_4^2w_8w_5^2 + 90w_9w_6^2c_s^2w_7^2w_4^2w_8w_5^2 + 6w_9w_7^2w_4^2w_8w_5^2 + 48w_9w_6^2w_7^2w_4^2w_8w_5^2 - 24w_6^2w_7^2w_4^2w_8w_5^2 + \\
& 24w_9w_6^2c_s^2w_7^2w_4^3w_8^2w_5^2 + 12w_9w_6^2v_2^2w_7^2w_4^2w_8w_5^2 + 66w_9w_6^2v_2^2w_7^2w_4^2w_8w_5^2 - 12w_6^2v_2^2w_7^2w_4^2w_8w_5^2 + 24w_9w_6^2c_s^2w_7^2w_4^2w_8w_5^2 + 24w_9w_6^2c_s^2w_7^2w_4^2w_8w_5^2 + \\
& 156w_9w_6^2c_s^2w_7^2w_4^2w_8w_5^2 + 4w_9w_6v_2^2w_7^2w_4^2w_8w_5^2 + 24w_6^2v_2^2w_7^2w_4^2w_8w_5^2 + 12w_9w_6c_s^2w_7^2w_4^2w_8w_5^2 - 24w_9w_6^2c_s^2w_7^2w_4^2w_8w_5^2 - 4w_9w_6^2w_7^2w_4^2w_8w_5^2 + \\
& 24w_6^2c_s^2w_7^2w_4^3w_8w_5^2 + 12w_9w_6^2v_2^2w_7^2w_4^2w_8w_5^2 + 12w_6^2w_7^2w_4^2w_8w_5^2 + 12w_9w_6c_s^2w_7^2w_4^2w_8w_5^2 - 12w_6^2c_s^2w_7^2w_4^2w_8w_5^2
\end{aligned}$$

$$\begin{aligned}
C_{18} = & -36w_6^2v_2^2v_1^2w_7w_4^2w_8w_5 - 6w_6^2c_s^4w_7w_4^3w_8w_5 + 30w_9w_6^2v_2^2c_s^2w_7w_4^3w_8w_5 + 12w_6^2v_2^2w_7w_4^2w_8w_5 + 48w_9w_6^2v_1^2c_s^2w_7w_4^2w_8w_5 - \\
& 18w_6^2v_2^2v_1^2w_7w_4^3w_8w_5 + 12w_9w_6c_s^2w_7w_4^2w_8w_5 + 18w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5 + 12w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 - 36w_9w_6c_s^4w_7w_4^3w_8w_5 + \\
& 72w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5 + 12w_9w_6^2c_s^2w_7w_4^2w_8w_5 + 6w_6^2v_2^2w_7w_4^3w_8w_5 + 18w_9w_6^2c_s^4w_7w_4^2w_8w_5 + 12w_6^2v_2^2c_s^2w_7w_4^2w_8w_5 - 18w_6^2v_2^2w_7w_4^3w_8w_5 + \\
& 12w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5 - 12w_6^2v_2^2c_s^2w_7w_4^2w_8w_5 - 36w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 - 24w_9w_6v_2^2w_7w_4^2w_8w_5 - 12w_9w_6^2v_2^2c_s^2w_7w_4^3w_8w_5 - \\
& 12w_9w_6v_2^2v_1^2c_s^2w_7w_4^2w_8w_5 - 12w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5 - 12w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 + 18w_9w_6^2c_s^2w_7w_4^2w_8w_5 + 6w_9w_6v_2^2w_7w_4^3w_8w_5 - 6w_6^2c_s^4w_7w_4^3w_8w_5 - \\
& 36w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 - 15w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5 - 36w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5 + 12w_9w_6c_s^4w_7w_4^3w_8w_5 - 36w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5 + 36w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5 - \\
& 12w_6^2c_s^2v_1^2w_7w_4^2w_8w_5 - 18w_9w_6^2c_s^4w_7w_4^2w_8w_5 - 72w_9w_6^2v_2^2v_1^2w_7w_4^2w_8w_5 - 12w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 + 12w_6^2c_s^2w_7w_4^2w_8w_5 - 45w_9w_6^2v_2^2v_1^2w_7w_4^3w_8w_5 - \\
& 36w_9w_6^2v_2^2c_s^2w_7w_4^2w_8w_5 + 6w_6^2v_1^2c_s^2w_7w_4^2w_8w_5 - 36w_6^2v_2^2c_s^2w_7w_4^2w_8w_5 - 6w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5 - 12w_6^2v_2^2c_s^2w_7w_4^2w_8w_5 - 48w_9w_6^2v_2^2c_s^2w_7w_8^2w_5 - \\
& 18w_6^2v_2^2v_1^2w_7w_4^3w_8w_5 + 12w_6^2v_1^2c_s^2w_7w_4^2w_8w_5 - 12w_9w_6^2v_2^2c_s^2w_7w_4^2w_8w_5 - 12w_6^2v_1^2w_7w_4^2w_8w_5 + 6w_6^2v_1^2w_7w_4^3w_8w_5 - \\
& 36w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 + 9w_9w_6v_1^2w_7w_4^2w_8w_5 + 6w_9w_6c_s^4w_7w_4^2w_8w_5 - 36w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5 + 12w_9w_6c_s^2w_7w_4^2w_8w_5 + \\
& 27w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5 - w_9w_6^2c_s^4w_7w_4^2w_8w_5 - 48w_9w_6^2v_1^2w_7w_4^2w_8w_5 - 12w_6^2v_2^2c_s^2w_7w_4^2w_8w_5 + 12w_6^2v_2^2c_s^4w_8w_5 + \\
& 60w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 - 6w_9v_2^2c_s^2w_7w_4^2w_8w_5 + 36w_6^2v_2^2c_s^2w_7w_4^2w_8w_5 - 6w_6^2v_2^2c_s^2w_7w_4^2w_8w_5 + 6w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 + 24w_9w_6^2v_2^2c_s^2w_7w_4^2w_8w_5 + \\
& 6w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5 + 6w_6^2v_2^2w_7w_4^2w_8w_5 + 12w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5 - 12w_9w_6v_2^2c_s^4w_7w_4^2w_8w_5 + 6w_9w_6c_s^4w_7w_4^2w_8w_5 - 18w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5 + \\
& 36w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5 - 12w_9w_6c_s^4w_7w_4^2w_8w_5 + 18w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5 - 6w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 - 12w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5 + 9w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5 + \\
& 54w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 - 36w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5 - 12w_9v_2^2v_1^2w_7w_4^2w_8w_5 - 6w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 + 60w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 - 6w_6^2v_1^2s^2w_7w_4^3w_8w_5 + \\
& 36w_6^2v_2^2c_s^2w_7w_4^2w_8w_5 - 12w_6^2c_s^2w_7w_4^2w_8w_5 + 24w_9w_6^2v_2^2w_7w_4^2w_8w_5 + 6w_6^2c_s^2w_7w_4^2w_8w_5 - 72w_9w_6^2v_2^2v_1^2w_7w_4^2w_8w_5 - 24w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5 + \\
& 36w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 + 12w_6^2v_1^2c_s^2w_7w_4^2w_8w_5 - 18w_9w_6c_s^2w_7w_4^2w_8w_5 - 12w_6^2v_1^2c_s^2w_7w_4^2w_8w_5 + 72w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5 - \\
& 12w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5 - 102w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5 + 12w_6^2c_s^2v_1^2w_7w_4^2w_8w_5 + 12w_6^2v_2^2c_s^2v_1^2w_7w_4^2w_8w_5 + 18w_6^2v_2^2v_1^2w_7w_4^3w_8w_5 - \\
& 6w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5 + 18w_9w_6c_s^4w_7w_4^2w_8w_5 + 12w_9w_6v_2^2c_s^4w_7w_4^2w_8w_5 + 12w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5 + 24w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 + \\
& 18w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 - 12w_9v_2^2c_s^2w_7w_4^2w_8w_5 - 6w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 + 60w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 - 6w_6^2v_1^2s^2w_7w_4^3w_8w_5 + \\
& 36w_6^2v_2^2c_s^2w_7w_4^2w_8w_5 - 12w_6^2c_s^2w_7w_4^2w_8w_5 + 24w_9w_6^2v_2^2w_7w_4^2w_8w_5 + 6w_6^2c_s^2w_7w_4^2w_8w_5 - 72w_9w_6^2v_2^2v_1^2w_7w_4^2w_8w_5 - 24w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5 + \\
& 36w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 + 12w_6^2v_1^2c_s^2w_7w_4^2w_8w_5 - 18w_9w_6c_s^2w_7w_4^2w_8w_5 - 12w_6^2v_1^2c_s^2w_7w_4^2w_8w_5 + 72w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5 - \\
& 12w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5 - 102w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5 + 12w_6^2c_s^2v_1^2w_7w_4^2w_8w_5 + 12w_6^2v_2^2c_s^2v_1^2w_7w_4^2w_8w_5 + 18w_6^2v_2^2v_1^2w_7w_4^3w_8w_5 - \\
& 6w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5 + 18w_9w_6c_s^4w_7w_4^2w_8w_5 + 12w_9w_6v_2^2c_s^4w_7w_4^2w_8w_5 + 12w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5 + 24w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 + \\
& 18w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 + 12w_9v_2^2c_s^2w_7w_4^2w_8w_5 + w_9w_6^2c_s^2w_7w_4^2w_8w_5 - 24w_9w_6v_2^2v_1^2c_s^2w_7w_4^2w_8w_5 - 6w_9w_6c_s^2w_7w_4^2w_8w_5 + 36w_6^2v_2^2v_1^2w_7w_4^2w_8w_5 - \\
& 5w_9w_6^2c_s^2w_7w_4^2w_8w_5 + 15w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5 - 108w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5 - 24w_9w_6v_2^2v_1^2c_s^2w_7w_4^2w_8w_5 + 24w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 + 6w_6^2c_s^4w_7w_4^2w_8w_5 + \\
& 12w_6^2v_1^2w_7w_4^3w_8w_5 - 12w_9w_6^2c_s^2w_7w_4^2w_8w_5 - 12w_6^2v_1^2w_7w_4^2w_8w_5 + 36w_6^2v_2^2v_1^2w_7w_4^2w_8w_5 + 24w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5 - 18w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5 + \\
& 36w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5 + 36w_6^2v_2^2v_1^2w_7w_4^3w_8w_5 - 12w_6^2v_1^2s^2w_7w_4^2w_8w_5 + 36w_6^2v_2^2c_s^2w_7w_4^2w_8w_5 + 18w_6^2v_2^2c_s^2w_7w_4^2w_8w_5 - 6w_6^2c_s^4w_7w_4^2w_8w_5 + \\
& 12w_9w_6v_2^2c_s^2w_7w_4^2w_8w_5 + 12w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5 + 6w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5 + 12w_6^2c_s^4w_7w_4^2w_8w_5 + 36w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5
\end{aligned}$$

$$\begin{aligned}
C_{19} = & 72w_6^2v_2^2c_s^2w_4^2 + 216w_6v_2^2c_s^2w_8^2 + 12w_6^2c_s^4w_4^2 + 30w_6^2v_2^2w_4^2w_8 - 36w_6^4w_4^2w_8^2 + 150w_6v_2^2c_s^2w_4^2w_8^2 + 12w_6^2c_s^2w_4^2w_8^2 + 48w_6c_s^4w_4^2w_8^2 + \\
& 96w_6v_2^2w_4^2w_8^2 - w_2^2c_s^4w_4^2w_8^2 - 3w_6^2v_4^2w_4^2w_8^2 - 12w_6^2v_2^2c_s^2w_4^2w_8^2 - 48w_6v_4^2w_4^2w_8^2 - 12w_6^2v_2^2w_4^2w_8^2 - 144w_6^2c_s^2w_4^2w_8^2 - 12w_6^2c_s^4w_4^2w_8^2 + \\
& 12c_s^2w_4^2w_8^2 - 12w_6^2v_2^2c_s^2w_4^2w_8^2 - 30w_6^2v_2^2c_s^2w_4^2w_8^2 + 36w_6^2v_2^2c_s^2w_4^2w_8^2 + 24w_6^2v_2^2w_4^2w_8^2 - 96w_6v_2^2c_s^4w_4^2w_8^2 + 24w_6c_s^2w_4^2w_8^2 + 3w_6^2v_2^2w_4^2w_8^2 - \\
& 48w_6^2v_2^2w_4^2w_8^2 + w_6^2c_s^2w_4^2w_8^2 + 72w_6^2v_2^2c_s^2w_4^2w_8^2 + 48w_6v_2^2w_4^2w_8^2 - 24w_6^2v_2^2c_s^4w_4^2w_8^2 - 144w_6^2v_2^2c_s^2w_4^2w_8^2 + 12w_6^2v_2^4w_4^2w_8^2 - \\
& 24w_6v_2^2w_4^2w_8^2 - 12w_6^2c_s^2w_4^2w_8^2 + 288w_6^2c_s^2w_4^2w_8^2 - 72w_6^2w_4^2w_8^2 + 96w_6^2v_2^2c_s^2w_4^2w_8^2 + 432w_6^2v_2^2c_s^2w_4^2w_8^2 - 24c_s^2w_4^2w_8^2 + 48w_6^2c_s^4w_4^2w_8^2 + 14w_6c_s^4w_4^2w_8^2 + \\
& 36w_6v_2^4w_4^2w_8^2 + 48w_6^2v_2^2w_8 + 24w_6v_2^4w_4^2w_8^2 + 24w_6^2v_2^2w_4^2w_8^2 - 24w_6^2c_s^4w_8^2 - 216w_6^2v_2^2c_s^2w_8^2 - 432w_6v_2^2c_s^2w_4^2w_8^2 + 24c_s^2w_4^2w_8^2 - \\
& 48w_6^2c_s^2w_4^2w_8^2 + 72w_6^2w_4^2w_8^2 - 96w_6^2v_2^2w_4^2w_8^2 - 36w_6^2v_2^2w_4^2w_8^2 - 24w_6^2v_2^2w_4^2w_8^2 - 24w_6c_s^2w_4^2w_8^2
\end{aligned}$$

$$\begin{aligned}
C_{20} = & c_s^2w_4^2w_8^2 - 3v_2^2c_s^2w_4^2w_8^2 + 6w_6^2c_s^2w_4^2w_8^2 + 12w_2^2c_s^2w_4^2w_8^2 - 24v_4^2w_4^2w_8^2 + 24c_4^2w_4^2w_8^2 - 12v_2^2w_4^2w_8^2 - 72v_2^2w_4^2w_8^2 + 24v_2^2w_4^2w_8^2 + 48v_2^2c_s^2w_4^2w_8^2 - \\
& 18v_4^2w_4^2w_8^2 + 6c_s^4w_4^2w_8^2 - 24v_2^2c_s^2w_4^2w_8^2 + 3v_2^2w_4^2w_8^2 - 72v_2^2c_s^2w_4^2w_8^2 - 3c_4^2w_4^2w_8^2 - 8c_2^2w_4^2w_8^2 + 24v_2^2w_4^2w_8^2 + 72v_2^4w_4^2w_8^2 - 12v_2^2c_s^2w_4^2w_8^2 - 24c_2^2w_4^2w_8^2 - \\
& 6c_s^2w_4^2w_8^2 + 18v_2^2c_s^2w_4^2w_8^2 + 24w_6v_2^2c_s^2w_4^2w_8^2 + 12w_6v_2^2w_4^2w_8^2 + 24w_6c_s^2w_4^2w_8^2 + 120w_6^2w_4^2w_8^2 + 21w_6^2w_4^2w_8^2 + 72c_s^2w_4^2w_8^2 - 84w_6^2v_2^2w_8^2 + 2w_6^2w_4^2w_8^2 - \\
& 48w_6^2c_s^2w_4^2w_8^2 + 24w_4^2w_8^2 + 72w_6w_4^2w_8^2 + 120w_6^2c_s^2w_4^2w_8^2 + 168w_6^2v_2^2w_4^2w_8^2 + 61w_6v_2^2w_4^2w_8^2 - 12w_6^2w_4^2w_8^2 + 60w_6c_s^2w_4^2w_8^2 + 39w_6c_s^2w_4^2w_8^2
\end{aligned}$$

### 2.3.4 Conservation of momentum: $\rho v_2$

attached text file: output\_d2q9\_nse\_mrt2\_symbolic\_pde\_02.txt

$$\begin{aligned}
v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{v_2 v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho v_2 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_1} + \frac{\rho v_1 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + \frac{2 \rho v_2 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_4) \frac{c_s^2 \delta_l^2}{2 \delta_t \omega_4} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\
(-2 + \omega_4) \frac{c_s^2 \delta_l^2}{2 \delta_t \omega_4} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (-2 + \omega_6 + 4c_s^2 - 2w_6c_s^2 + 6v_2^2 - 3\omega_6v_2^2) \frac{\delta_l^2}{\omega_6 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (2 - \omega_6) \frac{3\rho v_2 \delta_l^2}{\omega_6 \delta_t} \left( \frac{\partial v_2}{\partial x_2} \right)^2 + \\
(-2 + \omega_4) \frac{\rho c_s^2 \delta_l^2}{2 \delta_t \omega_4} \frac{\partial^2 v_2}{\partial x_1^2} + (-2 + \omega_4) \frac{\rho c_s^2 \delta_l^2}{2 \delta_t \omega_4} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + (-2 + \omega_6 + 6c_s^2 - 3w_6c_s^2 + 2v_2^2 - \omega_6v_2^2) \frac{v_2 \delta_l^2}{2 \omega_6 \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + \\
(-2 + \omega_6 + 2c_s^2 - \omega_6c_s^2 + 6v_2^2 - 3\omega_6v_2^2) \frac{\rho \delta_l^2}{2 \omega_6 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + C_1 \frac{v_2 v_1 \delta_l^3}{12 \delta_t \omega_7 w_4 \omega_5} \frac{\partial^3 \rho}{\partial x_1^3} + C_2 \frac{\rho v_2 \delta_l^3}{12 \delta_t \omega_7 w_4 \omega_5} \frac{\partial^3 v_1}{\partial x_1^3} + C_3 \frac{\rho v_1 \delta_l^3}{6 \delta_t \omega_7 w_4^2} \frac{\partial^3 v_2}{\partial x_1^3} +
\end{aligned}$$

$$\begin{aligned}
& (-12 - \omega_4^2 + 12\omega_4) \frac{c_s^4 \delta_l^3}{6\delta_t \omega_4^2} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + (-\omega_4^2 - 2\omega_7 + 2\omega_4 + \omega_7 \omega_8) \frac{\rho v_1 c_s^2 \delta_l^3}{\delta_t \omega_7 \omega_4^2} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} + \\
& (-12\omega_6 \omega_4^2 + 12\omega_4^2 - 12\omega_4 \omega_8 + 12\omega_6 \omega_4 - \omega_6 \omega_4^2 \omega_8 + 12\omega_6 \omega_4 \omega_8 - 12\omega_6 \omega_8) \frac{\rho v_2 c_s^2 \delta_l^3}{6\omega_6 \delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + C_4 \frac{v_2 v_1 \delta_l^3}{\omega_6^2 \delta_t \omega_4 \omega_8} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + \\
& C_5 \frac{\rho v_2 \delta_l^3}{12\omega_6^2 \delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_6 \frac{\rho v_1 \delta_l^3}{\omega_6^2 \delta_t \omega_4 \omega_8} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_7 \frac{\delta_l^3}{12\omega_6^2 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\
& (-24 + 24\omega_6 + 36c_s^2 + 11\omega_6^2 v_2^2 - 36\omega_6 c_s^2 + 60v_2^2 + 5\omega_6^2 c_s^2 - 60\omega_6 v_2^2 - 4\omega_6^2) \frac{\rho v_2 \delta_l^3}{6\omega_6^2 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_8 \frac{v_2 \delta_l^4}{24\delta_t \omega_7^2 \omega_4^2 \omega_5^2} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& C_9 \frac{\rho v_2 v_1 \delta_l^4}{12\delta_t \omega_7^2 \omega_4^2 \omega_5^2} \frac{\partial^4 v_1}{\partial x_1^4} + C_{10} \frac{\rho \delta_l^4}{24\delta_t \omega_7^2 \omega_4^3} \frac{\partial^4 v_2}{\partial x_1^4} + C_{11} \frac{v_1 \delta_l^4}{12\omega_9 \omega_6 \delta_t \omega_7^2 \omega_4^3 \omega_8 \omega_5^2} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{12} \frac{\rho \delta_l^4}{12\omega_9 \omega_6 \delta_t \omega_7^2 \omega_4^3 \omega_8 \omega_5^2} \frac{\partial^4 v_1}{\partial x_3 \partial x_2} + \\
& C_{13} \frac{\rho v_2 v_1 \delta_l^4}{12\omega_9 \omega_5^2 \delta_t \omega_7^2 \omega_4^3 \omega_8 \omega_5^2} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_{14} \frac{v_2 \delta_l^4}{12\omega_9 \omega_6^3 \delta_t \omega_7 \omega_4^3 \omega_8^2} \frac{\partial^4 \rho}{\partial x_2^3 \partial x_2} + C_{15} \frac{\rho v_2 v_1 \delta_l^4}{2\omega_9 \omega_6^3 \delta_t \omega_7^2 \omega_4^3 \omega_8^2 \omega_5} \frac{\partial^4 v_1}{\partial x_2^2 \partial x_2^2} + \\
& C_{16} \frac{\rho \delta_l^4}{12\omega_9 \omega_5^2 \delta_t \omega_7 \omega_4^3 \omega_8^2} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_{17} \frac{v_1 \delta_l^4}{4\omega_6^3 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{18} \frac{\rho \delta_l^4}{12\omega_6^3 \delta_t \omega_4^3 \omega_8^2} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{19} \frac{\rho v_2 v_1 \delta_l^4}{4\omega_6^3 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\
& C_{20} \frac{v_2 \delta_l^4}{12\omega_6^3 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{21} \frac{\rho \delta_l^4}{12\omega_6^3 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\text{C}_1 = -18c_s^2\omega_7\omega_4 + 12v_1^2\omega_7 + 18c_s^2\omega_4\omega_5 - 12\omega_7 - 12v_1^2\omega_5 - 36c_s^2\omega_5 + 6\omega_7\omega_4 + 6v_1^2\omega_4\omega_5 - 6v_1^2\omega_7\omega_4 - 6\omega_4\omega_5 + 12\omega_5 + v_1^2\omega_7\omega_4\omega_5 - \omega_7\omega_4\omega_5 + 3c_s^2\omega_7\omega_4\omega_5 + 36c_s^2\omega_7$$

$$C_2 = -6c_s^2\omega_7\omega_4 + 36v_1^2\omega_7 + 6c_s^2\omega_4\omega_5 - 12\omega_7 - 36v_1^2\omega_5 - 12c_s^2\omega_5 + 6\omega_7\omega_4 + 18v_1^2\omega_4\omega_5 - 18v_1^2\omega_7\omega_4 - 6\omega_4\omega_5 + 12\omega_5 + 3v_1^2\omega_7\omega_4\omega_5 - \omega_7\omega_4\omega_5 + c_s^2\omega_7\omega_4\omega_5 + 12c_s^2\omega_7$$

$$C_3 = -6v_1^2\omega_4 + 15c_s^2\omega_7\omega_4 - 3\omega_4^2 - 3c_s^2\omega_7\omega_4^2 + 6\omega_4 + 3v_1^2\omega_4^2 - 3\omega_7\omega_4 + 3v_1^2\omega_7\omega_4 + 3c_s^2\omega_4^2 - 6c_s^2\omega_4 - v_1^2\omega_7\omega_4^2 + \omega_7\omega_4^2 - 12c_s^2\omega_7$$

$$C_4 = w_6 v_2^2 w_8 - 3 w_6 c_s^2 w_4 - w_6^2 v_2^2 - w_4 w_8 + w_6^2 v_2^2 w_4 - 3 w_6 c_s^2 w_4 w_8 - w_6 v_2^2 w_4 w_8 + w_6 w_4 - 3 w_6^2 c_s^2 - w_6^2 w_4 + 3 w_6 c_s^2 w_8 - w_6 v_2^2 w_4 + w_6 w_4 w_8 + v_2^2 w_4 w_8 + 3 c_s^2 w_4 w_8 + 3 w_6^2 c_s^2 w_4 + w_6^2 - w_6 w_8$$

$$\begin{aligned} \textcolor{blue}{C_5} = & -3w_6^2v_2^2w_4^2w_8 - 11w_6^2c_2^2w_4^2w_8 + 12w_6w_4^2 + 36c_2^2w_4^2w_8 + 12v_2^2w_4^2w_8 + 12w_6^2c_2^2w_4^2 - 24w_6^2c_2^2w_8 - 6w_6^2w_4w_8 - 12w_6^2v_2^2w_4 - 12w_6c_2^2w_4^2 - 24w_6c_2^2w_4w_8 + \\ & 6w_6w_4^2w_8 + 12w_6^2w_4 - 6w_6v_2^2w_4^2w_8 - 18w_6c_2^2w_4^2w_8 + 12w_6^2c_2^2w_4 - 12w_4^2w_8 + 3w_6^2w_4^2w_8 - 12w_6^2c_2^2w_4 - 12w_6v_2^2w_4^2 + 42w_6^2c_2^2w_4w_8 + 6w_6^2v_2^2w_4w_8 - 12w_6^2w_4^2 \end{aligned}$$

$$\text{C}_6 = 3w_6v_2^2w_8 - w_6c_s^2w_4 - 3w_6^2v_2^2 - w_4w_8 + 3w_6^2v_2^2w_4 - w_6c_s^2w_4w_8 - 3w_6v_2^2w_4w_8 + w_6w_4 - w_6^2c_s^2 - w_6^2w_4 + w_6c_s^2w_8 - 3w_6v_2^2w_4 + w_6w_4w_8 + 3v_2^2w_4w_8 + c_s^2w_4w_8 + w_6^2c_s^2w_4 + w_6^2 - w_6w_8$$

$$C_7 = -12c_s^2 - 7\omega_6^2 v_2^2 + 144v_2^2 c_s^2 + 36v_4^2 + 24\omega_6^2 v_2^2 c_s^2 + \omega_6^2 c_4^2 + 12\omega_6 c_s^2 - 36\omega_6 v_2^4 - 36v_2^2 - \omega_6^2 c_4^2 + 12c_4^2 + 7\omega_6^2 v_2^4 - 144\omega_6 v_2^2 c_s^2 + 36\omega_6 v_2^2 - 12\omega_6 c_4^2$$

$$\begin{aligned}
C_8 = & -126v_1^4c_s^2w_7w_4w_5^2 - 216v_1^2c_s^2w_7w_5^2 + 48c_s^4w_7w_4w_5^2 - 24c_s^4w_5w_4w_5^2 - 3v_1^4w_2^2w_4w_5 + 96v_1^2w_7w_4w_5^2 + 36v_1^2w_7w_4w_4^2 - \\
& c_s^4w_2^2w_4w_5^2 + 24c_s^2w_7w_5^2 + 48c_s^2w_7w_4w_5^2 - 48v_1^2w_2^2w_5^2 - 24v_1^2w_4w_5^2 + 30v_1^2w_7w_4w_5^2 + 24c_s^2w_4w_5^2 - 72v_1^2w_7^3w_4w_5 - 48v_1^4w_7w_4w_5 + \\
& 14c_s^4w_7w_4w_5^2 - 432v_1^2c_s^2w_7w_4w_5^2 - 36v_1^4w_7w_4w_5^2 + 12c_s^2w_7w_4w_5^2 + 48v_1^4w_7w_5^2 - 24c_s^4w_7w_5^2 + 72v_1^2c_s^2w_7w_4w_5^2 - 144v_1^2c_s^2w_4w_5^2 + 36v_1^4w_2^2w_4w_5^2 + \\
& 12v_1^2w_2^2w_5^2 + 24c_s^4w_5w_4w_4 - 144v_1^2c_s^2w_7w_4w_5^2 + 3v_2^2w_2^2w_5^2 - 48c_s^2w_7w_4w_5^2 - 144v_1^2c_s^2w_7w_4w_5 + 216v_1^2c_s^2w_7w_5^2 - 24v_1^4w_4w_5^2 - 96v_1^2w_7w_4w_5 + \\
& 72v_1^2c_s^2w_4w_5^2 + 48v_1^2w_7w_5^2 - 24c_s^2w_7w_5^2 + c_s^2w_7w_4w_5^2 - 12c_s^2w_4w_5^2 - 96v_1^2w_7w_4w_5^2 + 24v_1^4w_7w_4w_5^2 + 12c_s^2w_7w_4w_5^2 - 48c_s^4w_7w_4w_5 + \\
& 150v_1^2c_s^2w_7w_4w_5^2 - 30v_1^4w_7w_4w_5^2 - 24c_s^2w_7w_4w_4 - 12v_1^2c_s^2w_7w_2w_5^2 - 14c_s^2w_7w_2w_5^2 + 48v_1^2w_7w_4w_5 + 24v_1^2w_4w_5^2 + 432v_1^2c_s^2w_7w_4w_5^2 + 24c_s^4w_7w_5^2 - \\
& 12c_s^4w_7w_4w_5^2 - 48v_1^2w_7w_5^2 + 12c_s^4w_5w_4w_5^2 - 12c_s^4w_7w_4w_4 + 288v_1^2c_s^2w_7w_4w_4 - 36v_1^2w_7w_4w_5^2
\end{aligned}$$

$$\begin{aligned} \textcolor{red}{C_9} = & -12w_7w_4^2w_5 - 168v_1^2w_7^2w_4w_5 + 12c_s^2w_7w_4^2w_5 - 12w_4^2w_5^2 - 60v_1^2w_7^2w_4^2 + 72w_7^2w_4w_5 - 36w_7^2w_5 - 60c_s^2w_7w_4w_5^2 - 120c_s^2w_7^2w_4w_5 + 84v_1^2w_7^2w_5 + 36v_1^2w_7w_4^2w_5 + 24v_1^2w_4^2w_5^2 + 24w_7^2w_4^2 - 51v_1^2w_7w_4^2w_5^2 - 48w_7^2w_4 - 48c_s^2w_4w_5^2 + 120v_1^2w_7^2w_4 - 33c_s^2w_7w_4^2w_5^2 + 21w_7w_4^2w_5^2 - 5v_1^2w_7^2w_4^2w_5^2 + 120c_s^2w_7w_4w_5^2 - 72w_7w_4w_5^2 - 84v_1^2w_7w_5^2 + 60c_s^2w_7^2w_5 - 3c_s^2w_7w_4^2w_5^2 + 24c_s^2w_4^2w_5^2 + 168v_1^2w_7w_4w_5^2 + 2w_7^2w_4^2w_5^2 - 36c_s^2w_7^2w_4^2 + 36w_7w_5^2 - 25w_7^2w_4^2w_5 + 24w_4w_5^2 + 72c_s^2w_7^2w_4 + 39c_s^2w_7^2w_4^2w_5 - 72v_1^2w_7w_4w_5 - 48v_1^2w_4w_5^2 + 24w_7w_4w_5 + 61v_1^2w_7^2w_4^2w_5 - 24c_s^2w_7w_4w_5 \end{aligned}$$

$$\begin{aligned}
C_{10} = & -3v_1^2 w_7^2 w_3^3 + 24 v_1^4 w_7^2 w_4 - 24 v_1^4 w_7 w_4^2 - 24 v_1^2 c_s^2 w_7 w_4 - 24 c_s^2 w_7 w_4 + 24 v_1^2 w_7^2 w_4^2 + 24 c_s^4 w_7^2 + 6 c_s^4 w_7 w_3^3 - 12 v_1^2 c_s^2 w_7 w_4^3 + 3 v_1^4 w_7^2 w_3^3 + 24 c_s^2 w_7 w_4 - 24 v_1^2 w_7^2 w_4 - 12 v_1^2 w_3^4 - 24 v_1^4 w_7^2 w_4^2 + 24 c_s^4 w_7 w_4 + 48 v_1^2 c_s^2 w_7 w_4^2 + 24 v_1^2 w_4^2 - 6 c_s^2 w_7 w_3^4 - 48 c_s^4 w_7^2 w_4 - 72 v_1^2 c_s^2 w_7^2 w_4^2 + 12 v_1^2 c_s^4 w_3^4 + 72 v_1^4 w_7 w_4^2 - 96 v_1^2 c_s^2 w_7^2 + c_s^2 w_7^2 w_3^3 - 24 v_1^2 c_s^2 w_4^2 - 18 v_1^4 w_7 w_3^4 + 6 v_1^2 c_s^2 w_7^2 w_4^3 + 48 v_1^2 w_7 w_4 - 8 c_s^2 w_7^2 w_4^2 - 72 v_1^2 w_7 w_4^2 + 12 c_s^2 w_7^2 w_4 - 24 v_1^4 w_4^2 - 3 c_s^4 w_7^2 w_3^4 + 18 v_1^2 w_7 w_3^4 + 24 c_s^4 w_7^2 w_4^2 + 156 v_1^2 c_s^2 w_7 w_4 + 12 v_1^4 w_3^4 - 48 v_1^4 w_7 w_4
\end{aligned}$$

$$\begin{aligned}
C_{11} = & 6w_9c_2^2w_7w_4^3w_8w_5^2 - 24w_9w_6v_2^2w_7^2w_4w_8w_5 + 72w_9v_2^2c_2^2w_7^2w_4w_8w_5 + 6w_9v_2^2c_3^2w_7^2w_4^3w_8w_5 + 18w_9c_4^2w_7^2w_4^3w_8w_5 + 3w_9w_6c_4^2w_7^2w_4^3w_8w_5^2 - \\
& 6v_2^2v_1^2w_7^2w_4^3w_8w_5^2 + 12w_6v_2^2v_1^2w_7^2w_4^2w_5^2 + 144w_9w_6v_2^2c_3^2w_7w_4^2w_8w_5^2 - 3w_9w_6v_1^2c_2^2w_7w_4^3w_8w_5^2 - 36w_9w_6c_4^2w_7^2w_4^2w_8w_5 + 12w_9w_6v_2^2v_1^2w_7^2w_4^2w_8 + \\
& + 6w_9v_1^2w_7^2w_4^3w_8w_5 - w_9w_6c_2^2w_7^2w_4^3w_8w_5^2 - 6w_9c_2^2w_7^2w_4^3w_8w_5 - 18w_6c_4^2w_7w_4^3w_8w_5^2 + 9w_9w_6v_2^2v_1^2w_7^2w_4^3w_8w_5 + 12w_9w_6c_2^2w_7^2w_4^4w_8w_5 - \\
& 6w_6v_1^2c_3^2w_7w_4^3w_8w_5^2 + 12w_9w_6c_4^2w_7^2w_4^3w_8w_5^2 + 36w_6v_2^2c_3^2w_7w_4^3w_8w_5^2 + 6w_6v_2^2w_7w_4^3w_8w_5^2 + 36w_9w_6c_4^2c_5^2w_7^2w_4^3w_5^2 + 6w_9w_6v_2^2v_1^2w_7^2w_4^3w_8w_5^2 + \\
& 12w_9v_2^2v_1^2w_7w_4^3w_8w_5^2 - 12w_9w_6v_1^2c_3^2w_7w_4^2w_8w_5^2 - 48w_9w_6v_2^2w_7w_4^2w_8w_5^2 - 6w_9w_6c_3^2w_8w_5^2 + 6c_5^2w_7^2w_4^3w_8w_5^2 - 36w_9c_4^2w_7^2w_4^3w_5^2 - \\
& 42w_9w_6c_4^2w_7w_4^2w_8w_5^2 - 36w_6c_4^2w_7w_4^2w_8w_5^2 + 12w_9w_6v_2^2c_3^2w_7w_4^2w_8w_5^2 - 6w_9w_6c_2^2w_7^2w_4^3w_8w_5^2 + 12w_6c_3^2w_8w_5^2 + 36w_9w_6v_2^2c_3^2w_7w_4^2w_8 - \\
& 12w_9w_6v_1^2c_3^2w_7w_4^3w_8w_5^2 + 36w_6v_2^2c_3^2w_7w_4^2w_8w_5^2 + 12w_6v_2^2w_7w_4^2w_8w_5^2 - 18c_4^2w_7w_4^3w_8w_5^2 + 18w_6v_2^2c_3^2w_7w_4^3w_8w_5^2 - 12w_6v_2^2c_3^2w_7w_4^2w_8w_5^2 - \\
& 12w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 + 6w_2^2w_7^2w_4^3w_8w_5^2 - 12w_9w_6v_2^2w_7w_4^2w_8w_5^2 - 36w_6c_4^2w_7w_4^2w_8w_5^2 + 18w_9w_6v_2^2c_3^2w_7w_4^3w_8w_5^2 - 18w_9w_6c_2^2w_7w_4^2w_8w_5^2 + \\
& 24w_9v_2^2v_1^2w_7^2w_4^2w_8w_5^2 - 36w_9w_6c_4^2w_7w_4^2w_8w_5^2 - 18v_2^2s_2^2w_7w_4^3w_8w_5^2 + 48w_9w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 + 27w_9w_6v_2^2c_2^2w_7w_4^3w_8w_5^2 + 18w_9w_6v_2^2c_3^2w_7w_4^2w_8w_5^2 + \\
& + 12w_9w_6v_2^2w_7w_4^2w_8w_5^2 + 24w_9v_2^2w_7w_4^2w_8w_5^2 - 6w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - 12w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 + 36w_9v_2^2c_3^2w_7w_4^3w_8w_5^2 + 12w_6v_2^2v_1^2w_7w_4^2w_8w_5^2 +
\end{aligned}$$

$$\begin{aligned}
& 6w_9w_6v_7^2w_7^2w_4^3w_8 + 5w_9w_6c_s^2w_7^2w_4^3w_8w_5 + 6w_9w_6v_1^2c_s^2w_7^2w_4^3w_8w_5^2 - 12w_9w_6c_2^2w_7^2w_4^3w_8w_5^2 + 24w_9w_6v_2^2v_1^2w_7^2w_4^3w_8w_5 + 6w_6v_2^2v_1^2w_7^2w_4^3w_8w_5^2 + \\
& 36w_6c_s^4w_7^2w_4^2w_5^2 - 15w_9w_6c_4^2w_7^2w_4^3w_8w_5 + 12w_6v_1^2c_s^2w_7^2w_4^3w_8w_5^2 - 36w_6v_2^2c_s^2w_7^2w_4^3w_8w_5^2 - 18w_9w_6v_2^2c_s^2w_7^2w_4^3w_8w_5 + 36c_4^2w_7^2w_4^3w_8w_5^2 - \\
& 12w_9w_6v_1^2c_s^2w_7^2w_4^3w_8w_5^2 + 156w_9w_6c_4^2w_7^2w_4^3w_8w_5^2 - 9w_9w_6v_2^2c_s^2w_7^2w_4^3w_8w_5^2 - 36w_9w_6v_2^2c_s^2w_7^2w_4^3w_8w_5^2 - 36w_9w_6v_2^2c_s^2w_7^2w_4^3w_8w_5^2 + 6w_6v_1^2c_s^2w_7^2w_4^3w_8w_5^2 - \\
& 24w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - 6w_6v_2^2c_s^2w_7^2w_4^3w_8w_5^2 + 3w_9w_6c_8^2w_7w_4^3w_8w_5^2 - 36w_9w_6v_2^2c_s^2w_7^2w_4^3w_8w_5^2 - 36w_9w_6v_2^2c_s^2w_7^2w_4^3w_8w_5^2 + 18w_6c_4^2w_7^2w_4^3w_8w_5^2 - \\
& 12w_9w_6v_1^2c_s^2w_7^2w_4^3w_8w_5^2 + 6w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 + 12v_2^2c_s^2w_7^2w_4^3w_8w_5^2 + 12w_9w_6c_8^2w_7w_4^3w_8w_5^2 + 15w_9w_6v_2^2w_7w_4^3w_8w_5^2 - 12w_9w_6c_8^2w_7w_4^3w_8w_5^2 + \\
& w_9w_6v_1^2c_s^2w_7^2w_4^3w_8w_5^2 - 12w_9w_6v_2^2w_7w_4^3w_8w_5^2 - 6w_6c_8^2w_7w_4^3w_8w_5^2 - 12w_9w_6v_1^2c_s^2w_7^2w_4^3w_8w_5^2 + 12w_9w_6v_2^2w_7w_4^3w_8w_5^2 + 15w_9w_6c_8^2w_7w_4^3w_8w_5^2 - \\
& 36w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 + 36w_6c_4^2w_7w_4^3w_8w_5^2 + 12w_6v_2^2c_s^2w_7^2w_4^3w_8w_5^2 + 12w_9w_6v_2^2w_7w_4^3w_8w_5^2 - 12w_9v_2^2v_1^2w_7w_4^3w_8w_5^2 - \\
& 12w_6v_2^2w_7w_4^3w_8w_5^2 - 24w_9v_2^2v_1^2w_7w_4^3w_8w_5^2 - 12w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 + 12w_6v_2^2w_7w_4^3w_8w_5^2 + 12w_9w_6c_4^2w_7w_4^3w_8w_5^2 + \\
& 12w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 12w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - 18w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 60w_9w_6c_8^2w_7w_4^3w_8w_5^2 - 18w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 12w_6c_8^2w_7w_4^3w_8w_5^2 - \\
& 24w_9v_2^2c_s^2w_7w_4^3w_8w_5^2 + 45w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 18w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 72w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 12w_9w_6c_8^2w_7w_4^3w_8w_5^2 + 54w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - \\
& 12w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 + 36w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 18w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 36v_2^2c_s^2w_7w_4^3w_8w_5^2 + 12w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 15w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - \\
& 18w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 36w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 12w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 12w_9v_2^2v_1^2w_7w_4^3w_8w_5^2 - 96w_9w_6c_8^2w_7w_4^3w_8w_5^2 + 12w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - \\
& 12w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 + 36w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 6w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 5w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 6w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 + 12v_2^2v_1^2w_7w_4^3w_8w_5^2 + \\
& 24w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 72w_9v_2^2c_s^2w_7w_4^3w_8w_5^2 + 12w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 12w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 12w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - \\
& 6w_9v_2^2v_1^2w_7w_4^3w_8w_5^2 - 12w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 12w_6c_8^2w_7w_4^3w_8w_5^2 - 6w_9v_2^2c_s^2w_7w_4^3w_8w_5^2 - 6v_1^2c_s^2w_7w_4^3w_8w_5^2 - 15w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - 12c_8^2w_7w_4^3w_8w_5^2
\end{aligned}$$

$$\begin{aligned}
C_{12} = & 6w_6c_8^2w_7w_4^3w_8w_5^2 - 24w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5 + 24w_9v_2^2c_s^2w_7w_4^3w_8w_5 + 18w_9v_1^2c_s^2w_7w_4^3w_8w_5 + 6w_9c_8^4w_7w_4^3w_8w_5 - w_9w_6c_8^4w_7w_4^3w_8w_5^2 - \\
& 18v_2^2v_1^2c_s^2w_7w_4^3w_8w_5 + 36w_6v_2^2v_1^2c_s^2w_7w_4^3w_8w_5 + 48w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 12w_9w_6v_1^2c_s^2w_7w_4^3w_8w_5^2 - 12w_9w_6c_4^2w_7w_4^3w_8w_5 + 36w_9w_6v_2^2v_1^2c_s^2w_7w_4^3w_8w_5^2 + \\
& 6w_9v_2^2w_7w_4^3w_8w_5^2 - 6w_9c_8^2w_7w_4^3w_8w_5^2 - 6w_6c_8^4w_7w_4^3w_8w_5^2 + 27w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 + 12w_9w_6c_8^2w_7w_4^3w_8w_5^2 - 18w_6v_1^2c_s^2w_7w_4^3w_8w_5^2 + \\
& 12w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 6w_6v_2^2w_7w_4^3w_8w_5^2 + 12w_9w_6c_8^4w_7w_4^3w_8w_5^2 + 18w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 + 36w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - 15w_9w_6v_1^2c_s^2w_7w_4^3w_8w_5^2 - \\
& 48w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 6c_8^2w_7w_4^3w_8w_5^2 - 12w_9c_8^4w_7w_4^3w_8w_5^2 - 18w_9w_6c_4^2w_7w_4^3w_8w_5^2 - 12w_6c_8^4w_7w_4^3w_8w_5^2 + 24w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - \\
& 48w_9w_6v_1^2c_s^2w_7w_4^3w_8w_5^2 - 6w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 12w_6c_8^2w_7w_4^3w_8w_5^2 + 12w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 36w_6v_1^2c_s^2w_7w_4^3w_8w_5^2 + 12w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + \\
& 12w_6v_2^2w_7w_4^3w_8w_5^2 - 6c_8^2w_7w_4^3w_8w_5^2 + 6w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 36w_6v_1^2c_s^2w_7w_4^3w_8w_5^2 - 36w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - 6v_2^2w_7w_4^3w_8w_5^2 - \\
& 12w_9w_6v_2^2w_7w_4^3w_8w_5^2 - 12w_6c_8^4w_7w_4^3w_8w_5^2 + 6w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 18w_9w_6c_8^2w_7w_4^3w_8w_5^2 + 72w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - 12w_9w_6c_8^4w_7w_4^3w_8w_5^2 - \\
& 6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 144w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 + 9w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 54w_9w_6v_1^2c_s^2w_7w_4^3w_8w_5^2 + 12w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 12w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - \\
& 18w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - 36w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 + 12w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - 36w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 + 6w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 + 5w_9w_6c_8^2w_7w_4^3w_8w_5^2 - \\
& 12w_9w_6v_1^2c_s^2w_7w_4^3w_8w_5^2 + 72w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 + 18w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 + 12w_6c_8^4w_7w_4^3w_8w_5^2 - 5w_9w_6c_8^2w_7w_4^3w_8w_5^2 + 36w_6v_1^2c_s^2w_7w_4^3w_8w_5^2 - \\
& 12w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 6w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 12c_8^4w_7w_4^3w_8w_5^2 + 60w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 18w_9w_6c_8^4w_7w_4^3w_8w_5^2 - 9w_9w_6v_2^2w_7w_4^3w_8w_5^2 - \\
& 12w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 12w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 18w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 72w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - 6w_6v_2^2w_7w_4^3w_8w_5^2 - 6w_9w_6c_8^2w_7w_4^3w_8w_5^2 - \\
& 12w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 12w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 6w_6c_8^4w_7w_4^3w_8w_5^2 - 36w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 18w_9w_6v_1^2c_s^2w_7w_4^3w_8w_5^2 + 36v_2^2c_s^2w_7w_4^3w_8w_5^2 + \\
& 15w_9w_6v_2^2w_7w_4^3w_8w_5^2 - 12w_9w_6c_8^2w_7w_4^3w_8w_5^2 - 12w_9w_6v_2^2w_7w_4^3w_8w_5^2 - 6w_6c_8^2w_7w_4^3w_8w_5^2 - 36w_9w_6v_1^2c_s^2w_7w_4^3w_8w_5^2 + 12w_9w_6v_2^2w_7w_4^3w_8w_5^2 + \\
& 6w_9w_6c_8^4w_7w_4^3w_8w_5^2 - 108w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 + 12w_6c_8^4w_7w_4^3w_8w_5^2 + 12w_6c_8^2w_7w_4^3w_8w_5^2 + 12w_9v_2^2v_1^2w_7w_4^3w_8w_5^2 + 12w_6c_8^4w_7w_4^3w_8w_5^2 - \\
& 36w_9v_2^2c_s^2w_7w_4^3w_8w_5^2 - 12w_9v_2^2v_1^2w_7w_4^3w_8w_5^2 - 72w_9v_2^2v_1^2w_7w_4^3w_8w_5^2 - 36w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 + 12w_9v_2^2w_7w_4^3w_8w_5^2 + 12w_9c_8^2w_7w_4^3w_8w_5^2 + \\
& 36w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 36w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - 5w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 6w_9v_2^2c_s^2w_7w_4^3w_8w_5^2 - 6w_9v_2^2w_7w_4^3w_8w_5^2 - 12w_6c_8^2w_7w_4^3w_8w_5^2 - \\
& 24w_9v_2^2c_s^2w_7w_4^3w_8w_5^2 - 15w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - 102w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 24w_9w_6v_2^2v_1^2c_s^2w_7w_4^3w_8w_5^2 + 12w_9w_6v_2^2v_1^2c_s^2w_7w_4^3w_8w_5^2 + \\
& 36w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 36w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 + 6w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 12w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 24w_9w_6v_2^2v_1^2c_s^2w_7w_4^3w_8w_5^2 + \\
& 18w_9w_6c_8^2w_7w_4^3w_8w_5^2 - 12w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 + 36w_9w_6v_2^2v_1^2c_s^2w_7w_4^3w_8w_5^2 - 36w_9w_6c_8^4w_7w_4^3w_8w_5^2 + 12w_9w_6c_8^4w_7w_4^3w_8w_5^2 - \\
& 12w_9w_6v_2^2w_7w_4^3w_8w_5^2 - 36w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 6w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - 15w_9w_6v_1^2c_s^2w_7w_4^3w_8w_5^2 - 18w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 + 36v_2^2v_1^2w_7w_4^3w_8w_5^2 + \\
& 24w_9w_6v_2^2w_7w_4^3w_8w_5^2 - 24w_9v_2^2c_s^2w_7w_4^3w_8w_5^2 + 60w_9w_6v_2^2v_1^2c_s^2w_7w_4^3w_8w_5^2 - 12w_9w_6v_2^2w_7w_4^3w_8w_5^2 - 12w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - 36w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - \\
& 18w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - 12w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 12w_6c_8^2w_7w_4^3w_8w_5^2 - 18w_9v_2^2c_s^2w_7w_4^3w_8w_5^2 - 12w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - 12w_6v_2^2w_7w_4^3w_8w_5^2 - 36w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - \\
& 156w_9w_6v_2^2c_s^2w_7w_4^3w_8w_5^2 - 24w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - 66w_9w_6v_2^2w_7w_4^3w_8w_5^2 + 24w_9w_6c_8^2c_s^2w_7w_4^3w_8w_5^2 + 66w_9w_6v_2^2v_1^2w_7w_4^3w_8w_5^2 - 24w_6c_8^2c_s^2w_7w_4^3w_8w_5^2
\end{aligned}$$

$$\begin{aligned}
C_{14} = & -48w_9w_6c_8^4w_7w_4^3w_8^2 - 36w_9w_6c_8^4w_7w_4^3w_8^2 + 12w_6v_2^2v_1^2w_7w_4^3w_8^2 - 6w_6v_2^2v_1^2w_7w_4^3w_8^2 + 12w_9w_6c_8^2v_1^2w_7w_4^3w_8^2 - 12w_9w_6c_8^2v_1^2w_7w_4^3w_8^2 - \\
& 36w_6v_1^2c_s^2w_7w_4^3w_8^2 + 12w_6c_8^2c_s^2w_7w_4^3w_8^2 + 180w_9w_6c_8^2c_s^2w_7w_4^3w_8^2 + 36w_9w_6c_8^2v_1^2w_7w_4^3w_8^2 - 12w_9w_6v_2^2c_s^2w_7w_4^3w_8^2 - 18w_9w_6c_8^2c_s^2w_7w_4^3w_8^2 + \\
& 18w_9w_6v_1^2c_s^2w_7w_4^3w_8^2 - 42w_9w_6c_8^2c_s^2w_7w_4^3w_8^2 + w_9w_6v_3^2v_2^2c_s^2w_7w_4^3w_8^2 - 6w_9w_6c_8^2c_s^2w_7w_4^3w_8^2 + 12w_6v_2^2c_s^2w_7w_4^3w_8^2 + 12w_9w_6c_8^2c_s^2w_7w_4^3w_8^2 + \\
& 12w_9w_6c_8^2c_s^2w_7w_4^3w_8^2 - 12w_9w_6c_8^2v_1^2w_7w_4^3w_8^2 - 18w_6c_8^2c_s^2w_7w_4^3w_8^2 + 12w_3v_1^2w_7w_4^3w_8^2 + 12w_9w_6c_8^2c_s^2w_7w_4^3w_8^2 + 150w_9w_6c_8^2c_s^2w_7w_4^3w_8^2 - 36w_9w_6v_1^2c_s^2w_7w_4^3w_8^2 + \\
& 24w_9w_6v_2^2v_1^2w_7w_4^3w_8^2 + 18w_9w_6v_2^2c_s^2c_s^2w_7w_4^3w_8^2 - 36w_9w_6v_2^2v_1^2c_s^2w_7w_4^3w_8^2 - 6w_6v_2^2v_1^2w_7w_4^3w_8^2 + 6w_9w_6v_2^2c_s^2c_s^2w_7w_4^3w_8^2 - w_9w_6c_8^2c_s^2w_7w_4^3w_8^2 - \\
& 12w_9w_6v_2^2c_s^2w_7w_4^3w_8^2 - 12w_9w_6c_8^2c_s^2w_7w_4^3w_8^2 + 12w_9w_6c_8^2c_s^2w_7w_4^3w_8^2 + 72w_9w_6v_2^2c_s^2w_7w_4^3w_8^2 + 6w_9w_6c_8^2c_s^2w_7w_4^3w_8^2 - 12w_9w_6v_2^2v_1^2w_7w_4^3w_8^2 -
\end{aligned}$$

$$\begin{aligned}
& 12w_9w_6^2v_2^2c_s^2w_7w_4^3 - 36w_9w_6^2v_1^2c_s^2w_4^2w_8^2 + 36w_9^3c_s^4w_2^3w_8^2 + 6w_6^3c_s^2w_4^3w_8^2 - 12w_6^3c_s^2w_7w_4^2w_8 + 12w_6^2v_2^2c_s^2w_7w_3^4w_8 + 18w_9w_6^2c_s^4w_3^4w_8^2 - \\
& 36w_9w_6c_s^4w_7w_4^2w_8^2 + 36w_6^2c_s^4w_7w_4^3w_8^2 - 6w_6^3v_1^2w_7w_4^3w_8^2 + 12w_9w_6^2c_s^2w_7w_4w_8^2 - 36w_9w_6^2v_1^2c_s^2w_7w_4^3w_8^2 + 12w_9w_6^3c_s^2w_7w_4^2w_8^2 - 12w_9w_6^2v_2^2c_s^2w_4^2w_8^2 + \\
& 36w_9c_s^4w_7w_3^2w_8^2 - 6w_9w_3^2v_2^2c_s^2w_7w_3^4w_8 + 108w_9w_6^2v_1^2c_s^2w_7w_4^2w_8 + 12w_9w_6^3v_1^2w_7w_4^3w_8^2 - 6w_9w_6^2c_s^4w_7w_4^3w_8^2 + 12w_9w_6^2c_s^2w_7w_3^4w_8 + 12w_9w_6^3v_1^2w_7w_4^2w_8^2 + \\
& 12w_9w_6^2c_s^4w_7w_4^2w_8^2 + 12w_6^2v_2^2v_1^2w_7w_3^4w_8 - 12w_6^3v_1^2w_7w_4^2w_8^2 - 6w_6^3c_s^2w_7w_3^4w_8^2 - 24w_9w_6^2v_1^2w_7w_4w_8^2 + 12w_9w_6^3c_s^4w_7w_4w_8^2 - 18w_6^2v_1^2c_s^2w_7w_3^4w_8^2 + \\
& 12w_9w_6v_1^2w_7w_4^3w_8 - 36w_6^3c_s^4w_7w_4^3w_8 + 6w_6^2v_1^2w_7w_4^3w_8^2 + 72w_9w_6^2v_1^2c_s^2w_7w_3^4w_8^2 - 12w_9w_6^3c_s^2w_7w_4w_8^2 - 18w_9w_6^2v_1^2c_s^2w_7w_3^4w_8^2 - \\
& 72w_9w_6^3v_2^2c_s^2w_7w_4w_8 + 6w_9w_6^2v_2^2v_1^2w_7w_3^4w_8^2 - 36w_6^3v_1^2c_s^2w_7w_4^2w_8^2 - 18w_9w_6^2c_s^2w_7w_3^4w_8^2 + 6w_3^2v_2^2c_s^2w_7w_3^4w_8^2 - 54w_9w_6^2v_1^2c_s^2w_7w_3^4w_8^2 - \\
& 12w_9w_6^3v_1^2w_7w_4^3 - 12w_9w_6^3v_2^2c_s^2w_7w_4^3 - 36w_9w_6^3v_1^2c_s^2w_7w_4^2 - 12w_9w_6^2v_2^2v_1^2w_7w_3^4w_8 + 12w_3^2v_2^2c_s^2w_7w_4^2w_8^2 - 12w_6^3v_1^2w_7w_4^3w_8^2 - 42w_9w_6^3c_s^4w_7w_4^2w_8 + \\
& 24w_9w_6^2v_1^2w_7w_4^2w_8^2 + 12w_9w_6v_1^2v_2^2w_7w_4^3w_8^2 + 12w_9w_6^2c_s^2w_7w_4^3w_8 + 12w_9w_6^3v_2^2c_s^2w_7w_4w_8^2 - 6w_6^3c_s^2c_2^2w_4^3w_8^2 + 6w_6^2c_s^2w_7w_3^4w_8^2 - 12w_9w_6^2v_2^2v_1^2w_7w_4^2w_8 - \\
& 12w_9w_6v_1^2w_7w_4^2w_8^2 + 6w_9w_6^2v_2^2v_1^2w_4^3w_8^2 - 72w_9w_6^2v_1^2c_s^2w_7w_3^4w_8^2 - 36w_6^3c_s^4w_7w_4^2w_8^2 + 12w_9w_6v_2^2v_1^2w_7w_4^2w_8^2 + \\
& 6w_6^3v_2^2v_1^2w_7w_4^3w_8^2 - 24w_9w_6^2v_2^2w_7w_4^3w_8^2 + 5w_9w_6^3c_s^4w_7w_4^3w_8^2 + 12w_9w_6^2c_s^2w_7w_4^3w_8^2 - 18w_9w_6^2c_s^2w_7w_4^2w_8^2 - 36w_6^3v_1^2c_s^2w_7w_4^3w_8^2 + 12w_6^3v_2^2c_s^2w_7w_4^2w_8^2 - \\
& 12w_9w_6^3v_2^2c_s^2w_7w_4^2 - 36w_9w_6^3v_1^2c_s^2w_7w_4^3 + 12w_9w_6^3v_1^2w_7w_4^2 - 12w_9w_6^2v_2^2c_s^2w_7w_4^3w_8^2 - 12w_3^2v_2^2w_7w_4^3w_8^2 - 6w_9w_6^2v_1^2w_7w_4^3w_8^2 + 30w_9w_6^3c_s^4w_7w_4^3w_8^2 + \\
& 6w_3^2v_1^2w_4^3w_8^2 + 36w_9w_6v_1^2c_s^2w_7w_4^2w_8^2 - 12w_9w_6^2v_2^2c_s^2w_7w_4^3w_8^2 + 18w_3^2v_1^2c_s^2w_7w_4^3w_8^2 - 12w_6^3v_2^2c_s^2w_7w_4^2w_8^2 - 12w_9w_1^2w_7w_4^3w_8^2 + 36w_6^3v_1^2c_s^2w_4^3w_8^2 + \\
& 12w_9w_6c_s^2w_7w_4^3w_8^2 + 24w_9w_6^3v_1^2w_7w_4w_8 - 12w_6^2c_s^2w_7w_4^3w_8^2 - 84w_9w_6^2c_s^4w_7w_4w_8^2 - 24w_9w_6^2v_1^2v_2^2w_7w_4^2w_8^2 - 12w_9w_6^3v_2^2c_s^2w_7w_4w_8 + 12w_9w_6^2v_1^2w_4^2w_8^2 - \\
& 12w_9w_6^2v_2^2v_1^2w_7w_4^3 + 18w_9w_6^2v_2^2c_s^2w_7w_4^3w_8^2 - 36w_9w_6^2v_1^2c_s^2w_7w_4^2w_8^2 + 36w_6^3c_s^4w_7w_4^2w_8^2 - 36w_9w_6^1v_2^1c_s^2w_7w_4^3w_8^2 + 36w_6^3v_1^2c_s^2w_7w_4^2w_8^2 + \\
& 2w_9w_6^2c_s^2w_7w_4^3w_8^2 - 12w_6^3v_2^2c_s^2w_7w_4^3w_8^2 + 36w_9w_6^1v_2^1c_s^2w_7w_4^3w_8^2 - 12w_9c_s^2w_7w_4^3w_8^2 - 12w_6^3v_2^2v_1^2w_7w_4^2w_8^2 - 88w_9w_6^3c_s^4w_7w_4^2w_8^2 + 12w_9w_6^2v_1^2w_7w_4^2w_8^2 - \\
& 18w_9w_6v_2^2v_1^2w_7w_4^3w_8^2 - 12w_9w_6^2v_2^2v_1^2w_4^2w_8^2 + 18w_9w_6v_2^2v_1^2w_7w_4^3w_8^2 - 24w_9w_6^2v_2^2v_1^2w_7w_4w_8^2 + 18w_6^3v_2^2c_s^2w_7w_4^2w_8^2 + 18w_6^3c_s^4w_7w_4^3w_8^2 + \\
& 18w_9w_6^2v_1^2c_s^2w_7w_4^3w_8^2 + 12w_9w_6^3c_s^4w_7w_4w_8^2 - 96w_9w_6^3c_s^4w_7w_4w_8^2 - 12w_6^2v_1^2w_7w_4^3w_8^2 + 12w_9w_6^2v_2^2v_1^2w_7w_4^3w_8^2 + 24w_9w_6^2v_2^2v_1^2w_7w_4^2w_8^2 + 12w_6^3v_2^2c_s^2w_4^2w_8^2
\end{aligned}$$

$$\begin{aligned}
C_{15} = & -4w_6^3w_7w_2^2w_8^2w_5 + 3w_9w_6^2w_7^2w_2^2w_8^2w_5 + 11w_9w_3^2c_s^2w_7^2w_4^2w_8w_5 + 13w_9w_6^2c_s^2w_7^2w_4^3w_8w_5 - 4w_6^2w_7^2w_4^3w_8w_5 - 2w_9w_3^2w_7^2w_4w_8^2w_5 - 16w_9w_3^2c_s^2w_7^2w_8^2w_5 + \\
& 9w_9w_3^2w_7^2w_4^2w_8w_5 + 7w_9w_6^2v_2^2w_7^2w_4^3w_8w_5 - 8w_9w_6^2c_s^2w_7^2w_4^4w_5 - 4w_6^3v_2^2w_7^2w_4^3w_8w_5 - 4w_9w_6^2c_s^2w_7^2w_4^2w_5 + 26w_9w_3^2c_s^2w_7^2w_4w_8^2w_5 - 2w_6^3w_7^2w_4^3w_8^2w_5 + \\
& 2w_9w_6^2v_2^2w_7^2w_4^2w_8^2w_5 + 2w_9w_6^2v_2^2w_7^2w_4w_8^2w_5 + 2w_9w_6^2c_s^2w_7^2w_4^2w_8^2w_5 - 2w_9w_6^2v_2^2w_7^2w_4^2w_8^2w_5 - 9w_9w_6^2v_2^2w_7^2w_4^3w_8w_5 - 2w_9w_6^2c_s^2w_7^2w_4^3w_8w_5 - 4w_6^3c_s^2w_7^2w_4^3w_8w_5 - \\
& 2w_6^3c_s^2w_7w_4^3w_8^2w_5 + 2w_9w_3^2w_7w_4^2w_8^2w_5 - 4w_9w_6^2c_s^2w_7^2w_4w_8^2w_5 - 3w_9w_6^2v_2^2w_7^2w_4^2w_8^2w_5 - 7w_9w_6^2v_2^2w_7^2w_4^3w_8w_5 - w_9w_6^2c_s^2w_7^2w_4^2w_8^2w_5 - 2w_9w_3^2c_s^2w_7^2w_4^3w_8^2w_5 - \\
& 2w_6^3v_2^2w_7w_4^3w_8^2w_5 - 8w_9w_6^2c_s^2w_7^2w_4^3w_8w_5 + 4w_9w_6^2c_s^2w_7^2w_4^2w_8^2w_5 - 2w_9w_3^2v_2^2w_7w_4^2w_8^2w_5 + 4w_9w_6^2w_7^2w_4^3w_8w_5 - 4w_6^3c_s^2w_7^2w_4^2w_8^2w_5 - 2w_6^3c_s^2w_7^2w_4^3w_8^2w_5 + \\
& 4w_9w_6^2c_s^2w_7^2w_4^3w_8w_5 - 4w_6^3v_2^2w_7^2w_4^2w_8^2w_5 - 2w_6^3v_2^2w_7^2w_4^3w_8^2w_5 - 4w_9w_6^2c_s^2w_7w_4^2w_8^2w_5 - 24w_9w_3^2c_s^2w_7w_4^2w_8^2w_5 + 8w_9w_6^2c_s^2w_7^2w_4^3w_8w_5 - 4w_6^3w_7^2w_4^2w_8w_5 - \\
& 4w_9w_6^2v_2^2w_7^2w_4^3w_8w_5 + 5w_9w_6^2w_7^2w_4^2w_8^2w_5 - 15w_9w_6^2c_s^2w_7^2w_4^3w_8^2w_5 + 2w_9w_6^2v_2^2w_7^2w_4^2w_8^2w_5 + 4w_6^3c_s^2w_7^2w_4^3w_8w_5 + 4w_6^2c_s^2w_7^2w_4^3w_8^2w_5 + 3w_9w_3^2w_7^2w_4^3w_8w_5 + \\
& 4w_9w_6^2c_s^2w_7^2w_4^3w_8^2w_5 + 4w_6^3v_2^2w_7^2w_4^2w_8^2w_5 + 4w_6^2v_2^2w_7^2w_4^3w_8^2w_5 - 5w_9w_6^2v_2^2w_7^2w_4^2w_8^2w_5 + 4w_6^3w_7^2w_4^2w_8^2w_5 - 6w_9w_6^2c_s^2w_7^2w_4^2w_8^2w_5 - 8w_9w_6^2c_s^2w_7^2w_4^3w_5 - \\
& 4w_9w_6^2c_s^2w_7^2w_4^3w_8^2w_5 - 3w_9w_6^2v_2^2w_7^2w_4^3w_8w_5 - 2w_9w_6^2v_2^2w_7^2w_4^2w_8w_5 + 2w_9w_6^2c_s^2w_7^2w_4^2w_8^2w_5 - 5w_9w_6^2v_2^2w_7^2w_4^3w_8^2w_5 + 2w_9w_6^2c_s^2w_7^2w_4^2w_8^2w_5 - \\
& 2w_9w_6^2c_s^2w_7^2w_4^3w_8^2w_5 + 12w_9w_3^2c_s^2w_7w_4^2w_8^2w_5 + 4w_9w_6^2c_s^2w_7^2w_4^2w_5 - 4w_9w_7^2v_2^2w_7^2w_4^2w_8^2w_5 + 2w_6^3v_2^2w_7^2w_4^3w_8^2w_5 - 2w_9w_6^2w_7^2w_4w_8^2w_5 - 6w_9w_6^2c_s^2w_7^2w_4w_8w_5 + \\
& 4w_6^3w_7^2w_4^3w_8w_5 + 8w_9w_6^2c_s^2w_7w_4^2w_5 + 8w_9w_6^2c_s^2w_7^2w_4^2w_5 + 8w_9w_6^2v_2^2w_7^2w_4^3w_8^2w_5 + w_9w_6^2v_2^2w_7w_4^2w_8^2w_5 - 6w_9w_6^2v_2^2w_7^2w_4w_8w_5 + 2w_3^2c_s^2w_7^2w_4^3w_8^2w_5 + w_9w_6^2w_7^2w_4^2w_8^2w_5 - \\
& 8w_9w_6^2c_s^2w_7^2w_4^2w_8^2w_5 + 3w_9w_6^2c_s^2w_7^2w_4^3w_8^2w_5 - 4w_9w_6^2w_7^2w_4^2w_8^2w_5 + 2w_9w_6^2w_7^2w_4^2w_8w_5 + 4w_9w_6^2v_2^2w_7^2w_4^3w_8^2w_5 + 4w_6^3v_2^2w_7w_4^2w_8^2w_5 + 4w_9w_6^2c_s^2w_7^2w_4^2w_8^2w_5 - \\
& w_9w_6^2c_s^2w_7w_4^3w_8^2w_5 + 4w_6^3c_s^2w_7w_4^2w_8^2w_5 + 6w_9w_6^2v_2^2w_7w_4w_8w_5 + 12w_9w_6^2c_s^2w_7^2w_4^2w_8^2w_5 + 2w_6^2w_7^2w_4^3w_8^2w_5 - w_9w_6^2v_2^2w_7^2w_4w_8^2w_5 + w_9w_6^2v_2^2w_7^2w_4^3w_8^2w_5
\end{aligned}$$

$$\begin{aligned}
C_{16} = & -12w_9w_6c_4^4w_7w_4^2w_8^2 - 12w_9w_6c_2^4s_8w_4^2w_8^2 + 12w_3^3c_1^2w_7w_4^2w_8^2 - 18w_2^2v_2^2v_1^2w_7w_4^3w_8^2 + 36w_9w_3^2v_2^2v_1^2w_7w_4^3 - 12w_3^3c_2^2w_4^2s_8^2 + 12w_6^2v_1^2c_8^2w_7w_4^3w_8 + \\
& 12w_6^3c_3^2w_7w_4^3w_8 + 18w_9w_3^2c_3^4w_7w_4w_8^2 + 108w_9w_3^2v_2^2v_1^2w_7w_4^2w_8 - 84w_9w_6^2c_2^2c_8^2w_7w_4w_8^2 + 24w_9w_3^2c_3^2w_7w_4^2w_8 - 6w_3^4s_4^2w_4^2w_8 + 6w_9w_6^2v_2^2c_8^2w_4^3w_8^2 - \\
& 12w_9w_2^2c_5^4w_7w_4^3w_8 - 6w_9w_6c_2^2w_3^4w_8^2 - 48w_9w_3^2v_2^2s_8^2w_7w_8^3 - 18w_8^2v_2^2c_2^2w_7w_4^2w_8^2 + 12w_6^3c_2^2w_7w_4^2w_8^2 - 36w_9w_3^2v_2^2v_1^2w_7w_4^2w_8^2 - 6w_6^2c_4^2w_7w_4^3w_8^2 + \\
& 12w_6^3c_1^2w_7w_4^3w_8 + 24w_9w_6c_2^4s_8w_4^2w_8^2 - 36w_9w_3^2v_1^2w_7w_4^2w_8 + 72w_9w_2^2v_2^2v_1^2w_7w_4w_8^2 - 132w_9w_3^2v_2^2c_2^2w_7w_4^2w_8^2 - 12w_9w_3^2v_1^2c_8^2w_7w_4^3w_8^2 - \\
& 18w_8^2v_2^2v_1^2w_7w_4^2w_8^2 + 18w_9w_4^2v_2^2s_8^2w_4^2 - 42w_9w_6c_2^2c_8^2w_7w_4^2w_8^2 + 12w_9w_6s_4^2w_4^2w_7w_4^2w_8^2 + 12w_9w_6^2c_2^2s_8^2w_4^2w_8^2 + 24w_9w_6^2v_2^2s_8^2w_7w_4w_8^2 - \\
& 12w_9w_3^2c_2^2w_7w_4^3w_8 - 36w_9w_3^2v_1^2w_7w_4^3w_8 + 24w_9w_2^2v_2^2s_8^2w_7w_4^3 - 12w_9w_6^2v_1^2c_8^2w_7w_4^2w_8^2 + 12w_3^3c_2^2w_4^2w_8^2 + 6w_6^2c_2^2s_8^2w_4^2w_8^2 - 12w_9w_3^2s_8^2w_7w_4^2w_8^2 + \\
& 36w_6^2v_2^2c_2^2w_7w_4^3w_8 + 6w_9w_6c_2^4c_4^3w_8^2 + 6w_9w_6c_4^2w_7w_4^3w_8^2 + 12w_6^2c_4^2w_7w_4^3w_8^2 - 6w_3^2v_2^2w_7w_4^3w_8^2 + 12w_9w_2^2c_2^2w_7w_4w_8^2 - 12w_9w_6^2v_1^2c_8^2w_7w_4^3 - \\
& 36w_9w_6^2v_2^2c_2^2w_4^2w_8^2 + 78w_9w_3^2v_2^2c_8^2w_7w_4^3w_8 + 36w_9w_3^2v_2^2c_8^2w_7w_4^2w_8 + 12w_9w_3^2v_2^2w_7w_4^3w_8^2 - 6w_9w_6^2s_4^2w_7w_4^3w_8^2 + 36w_6^3c_2^2v_1^2w_2^2w_8^2 + \\
& 36w_6^2v_2^2v_1^2w_7w_4^3w_8 - 12w_6^3v_2^2w_7w_4^2w_8^2 - 6w_6^3c_2^2w_7w_4^3w_8^2 - 24w_9w_6^2v_2^2w_7w_4w_8^2 + 12w_9w_6^2s_4^2w_7w_4^3w_8^2 - 6w_6^2v_1^2c_8^2w_7w_4^3w_8^2 + 12w_9w_61^2v_1^2w_7w_4^3w_8 + \\
& 60w_9w_6^2v_2^2c_2^2w_7w_4^2w_8 - 12w_6^3c_2^2w_7w_4^3w_8 + 6w_6^2v_1^2c_8^2w_7w_4^3w_8 + 24w_9w_6^2v_1^2c_8^2w_7w_4^2w_8 - 6w_6^2v_1^2c_8^2w_4^2w_8 + 12w_9w_6^2c_2^2w_7w_4^2w_8^2 - 6w_9w_6^2v_1^2w_4^2w_8^2 - \\
& 24w_9w_6^2v_1^2c_8^2w_7w_4w_8 + 18w_9w_6^2v_2^2v_1^2w_7w_4^3w_8^2 - 12w_6^3v_2^2c_2^2s_8^2w_7w_4^2w_8 + 12w_9w_6^2c_2^2w_7w_4^2w_8^2 + 18w_9w_6^2v_2^2c_8^2w_7w_4^3w_8^2 - 18w_9w_6^2v_2^2s_8^2w_7w_4^3w_8^2 - \\
& 108w_9w_6v_2^2s_8^2w_7w_4^2w_8^2 - 12w_9w_6^3v_1^2w_7w_4^3 - 24w_9w_6^3v_2^2s_8^2w_7w_4^3 - 12w_9w_6^3v_2^2c_8^2w_7w_4^3 - 36w_9w_6v_2^2v_1^2w_7w_4^3w_8^2 + 36w_3^2v_2^2w_7w_4^2w_8^2 - 12w_3^3v_1^2w_2^2w_8^2 - \\
& 24w_9w_6^3c_4^2w_7w_4^2w_8^2 + 24w_9w_6^2v_1^2w_7w_4^2w_8^2 + 36w_9w_2^2v_1^2w_7w_4^3w_8^2 + 84w_9w_6^3v_2^2c_2^2w_7w_4^2w_8^2 - 18w_3^2v_2^2c_2^2w_4^2w_8^2 + 6w_6^2c_2^2w_7w_4^3w_8^2 - 36w_9w_6^2v_2^2v_1^2w_7w_4^2w_8^2 - \\
& 12w_9w_6v_1^2w_7w_4^3w_8^2 + 18w_9w_6^2v_2^2v_1^2w_4^2w_8^2 - 18w_9w_6^2v_2^2c_2^2w_7w_4^3w_8^2 - 24w_9w_6^2v_1^2c_8^2w_7w_4^2w_8^2 - 12w_3^2c_4^2w_7w_4^2w_8^2 + 36w_9w_6^2v_2^2v_1^2w_7w_4^2w_8^2 + \\
& 18w_8^2v_2^2v_1^2w_7w_4^3w_8^2 - 24w_9w_6^2v_1^2w_7w_4^2w_8 - w_9w_6^3c_4^2w_7w_4^3w_8^2 - 24w_9w_6^2s_8^2w_7w_4^2w_8^2 - 12w_3^3v_1^2c_2^2w_7w_4^3w_8^2 + 36w_3^2v_2^2c_2^2s_8^2w_7w_4^2w_8^2 + 24w_9w_6^3v_2^2c_2^2w_7w_4^2w_8^2 + \\
& 12w_9w_6^2v_2^2c_5^2w_7w_4^3 + 12w_9w_6^3s_1^2w_7w_4^2w_8^2 + 72w_9w_6v_2^2c_8^2w_7w_4^3w_8^2 - 36w_3^2v_2^2v_1^2w_7w_4^3w_8^2 - 6w_9w_6^2v_1^2w_7w_4^3w_8^2 + 12w_9w_6^3c_4^2w_7w_4^3w_8^2 + 6w_3^2v_2^2w_4^2w_8^2 + \\
& 12w_9w_6v_1^2c_8^2w_7w_4^2w_8^2 + 18w_9w_6^2v_2^2c_8^2w_7w_4^3w_8^2 - 12w_9w_6^2c_2^2w_7w_4^2w_8^2 + 6w_6^2v_1^2c_8^2w_7w_4^3w_8^2 - 36w_6^2v_2^2c_2^2w_7w_4^2w_8^2 - 12w_9w_6^2v_1^2w_7w_4^3w_8^2 + 12w_6^3v_1^2c_8^2w_4^2w_8^2 - \\
& 6w_9w_6^2s_8^2w_7w_4^2w_8^2 + 24w_9w_6^3v_1^2w_7w_4w_8^2 - 12w_6^2c_2^2w_7w_4^3w_8^2 - 12w_9w_6^2c_4^2w_7w_4^3w_8^2 - 72w_9w_6^2v_2^2w_7w_4^2w_8^2 + 60w_9w_6^2v_2^2c_2^2w_7w_4w_8^2 + \\
& 12w_9w_6^2v_2^2w_4^2w_8^2 - 36w_9w_6^2v_2^2v_1^2w_7w_4^3 - 144w_9w_6^2v_2^2c_2^2w_7w_4^3w_8^2 + 12w_9w_6^2v_2^2c_2^2w_7w_4^2w_8^2 + 12w_9w_6^2v_1^2c_8^2w_7w_4^3w_8^2 + \\
& 12w_9w_6^3v_1^2c_2^2w_7w_4^2w_8^2 + 6w_9w_6^2c_2^2w_7w_4^3w_8^2 - 36w_6^2v_2^2c_2^2w_7w_4^3w_8^2 + 12w_9w_6^2v_2^2s_8^2w_7w_4^2w_8^2 - 36w_3^2v_2^2v_1^2w_7w_4^2w_8^2 - 4w_9w_6^3c_4^2w_7w_4^2w_8^2 + 12w_9w_6^2v_1^2w_7w_4^2w_8^2 - \\
& 54w_9w_6v_2^2v_1^2w_7w_4^3w_8^2 - 36w_9w_6^2v_2^2v_1^2w_2^2w_8^2 + 18w_9w_6^2v_1^2w_7w_4^3w_8^2 - 72w_9w_6^2v_2^2v_1^2w_7w_4w_8^2 + 180w_9w_6^2v_2^2c_2^2w_7w_4^2w_8^2 + 6w_6^3c_4^2w_7w_4^3w_8^2 + \\
& 6w_9w_6^2v_1^2c_8^2w_7w_4^3w_8^2 - 12w_9w_6^3s_8^2w_7w_4w_8^2 - 12w_9w_6^3c_4^2w_7w_4^2w_8^2 - 12w_6^2v_1^2w_7w_4^3w_8^2 + 12w_9w_6^2v_2^2w_7w_4^3w_8^2 + 72w_9w_6^2v_2^2v_1^2w_7w_4^3w_8^2 + 36w_6^2v_2^2c_2^2w_4^2w_8^2
\end{aligned}$$

$$\begin{aligned}
C_{17} = & -24w_0^2v_2^2c_2^2w_4^2 - 4w_0^2c_s^4w_4^2 + 8w_0^2v_2^2w_8^2 - 32w_0^2v_2^2w_4^2w_8 + 24v_4^2w_2^2w_8^2 - 144w_6v_2^2c_s^2w_4^2w_8^2 - 8w_6^2c_2^2w_4^2w_8 + 8c_s^4w_4^2w_2^2 - 24w_0^3v_2^2c_s^2w_0^2 - \\
& 4w_0c_s^2w_4^2w_8^2 - 20w_6v_2^2w_4^2w_8^2 - 4w_6^3c_s^2w_4^2 + 4w_6^2c_s^4w_4^2w_8^2 + 13w_6^2v_2^4w_4^2w_8^2 + 51w_6^2v_2^2c_s^2w_4^2w_8^2 + 4w_6^2v_2^2w_2^2 - 4w_6^2c_s^2w_8^2 - 13w_6^3v_2^4w_4^2w_8 - 4w_6^3c_s^4w_4^2w_8 + \\
& 4w_6^3v_2^2w_4^2 + 96w_7^2c_2^2w_4^2w_8^2 - 4w_8^3v_2^4w_4^2 + 8w_8^2c_s^4w_4^2w_8^2 - 8c_8^2w_4^2w_8^2 + 120w_6^2v_2^2c_s^2w_4^2w_8^2 + 32w_6^2v_2^4w_4^2w_8^2 - 24w_7^2c_2^2w_4^2w_8^2 - 4w_6^3c_s^4w_4^2w_8 + 20w_6^2c_4^2w_4^2w_8 + \\
& 4w_6c_4^2w_4^2w_8^2 - 13w_6^2v_2^2w_4^2w_8^2 + 24w_6^3v_2^2c_s^2w_4^2 - 4w_6^2c_s^2w_4^2w_8^2 - 72w_6v_2^2c_2^2w_4^2w_8 + 84w_6^3v_2^2w_4^2w_8 + 4w_6^3c_2^2w_4^2w_8 + 8w_6^3v_2^2w_4^2w_8 + 4w_6^3c_2^2w_4^2w_8 + \\
& 13w_6^3v_2^2w_4^2w_8^2 + 8w_6^2c_s^2w_4^2w_8^2 + 20w_6^2v_2^2w_4^2w_8^2 - 4w_6^2c_2^2w_4^2w_8^2 + 20w_6v_2^2c_2^2w_4^2w_8 + 4w_6c_2^2w_4^2w_8^2 - 51w_6^2v_2^2c_s^2w_4^2w_8^2 - 36w_6^3v_2^2c_2^2w_4^2w_8 + 36w_6^2v_2^2c_s^2w_8^2 + \\
& 4w_2^2c_s^2w_4^2w_8^2 - 20w_6^3v_2^2w_4^2w_8^2 - 4w_6^3v_2^2w_4^2w_8^2 - 8w_6^3c_2^2w_4^2w_8^2 + 4w_6^2c_2^2w_4^2w_8^2 - 16w_6^2v_2^4w_4^2w_8^2 - 48w_6^2v_2^2c_s^2w_4^2w_8^2 - 8w_6^2v_2^2w_2^2 - 12w_6c_4^2w_4^2w_8^2 - 36w_6^2c_2^2w_4^2w_8^2 + \\
& 4w_6^3c_4^2w_4^2 - 8w_6^4v_2^2c_2^2w_4^2w_8^2 - 20w_6^2v_4^4w_4^2w_8^2 - 8w_6^2c_4^2w_4^2w_8^2 - 4w_6c_4^2w_4^2w_8^2 - 4w_6^3c_4^2w_4^2 - 20w_6v_2^4w_4^2w_8^2 - 8w_6^3v_2^2w_4^2w_8^2 + 8w_6^3c_4^2w_4^2w_8^2 + 20w_6^3v_2^4w_4^2w_8^2 + \\
& 4w_6^3v_2^2w_4^2 + 72w_6v_2^2c_s^2w_4^2w_8^2 + 16w_6^2v_2^2w_4^2w_8^2 + 4w_6^3c_2^2w_8^2 + 36w_6v_2^2w_4^2w_8^2 + 12w_6c_4^2w_4^2w_8^2
\end{aligned}$$

$$\begin{aligned} C_{18} = & 12\omega_6^2 v_2^2 \omega_4^3 - 12c_5^2 \omega_4^3 \omega_8^2 - 5\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 6\omega_6^2 c_4^4 \omega_4^3 \omega_8 - 72v_2^2 \omega_4^3 \omega_8^2 + 18\omega_6^3 v_2^2 \omega_4^2 \omega_8^2 + 60\omega_6^2 v_2^4 \omega_4^3 \omega_8 + 54\omega_6^2 v_2^2 c_5^2 \omega_4^3 \omega_8 + 252v_2^2 c_5^2 \omega_4^3 \omega_8^2 + \\ & 4\omega_6^3 v_2^4 \omega_4^3 \omega_8^2 + 24\omega_6^2 v_2^2 \omega_4^2 \omega_8 + 12\omega_6^3 v_2^4 \omega_4^3 - 108\omega_6 v_2^2 c_s^2 \omega_4^2 \omega_8^2 - \omega_6^3 c_4^4 \omega_4^3 \omega_8^2 - 12\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 102\omega_6^3 v_2^2 c_s^2 \omega_4 \omega_8^2 + 6\omega_6^2 c_s^4 \omega_4^2 \omega_8^2 - 6\omega_6^3 c_5^2 \omega_4^3 \omega_8 + \\ & 12\omega_6^2 v_2^2 \omega_4^2 \omega_8^2 + 27\omega_6^3 v_2^2 \omega_4^3 \omega_8 + 12\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 12\omega_6^2 v_2^2 c_s^2 \omega_4^3 + 162\omega_6^3 v_2^2 c_s^2 \omega_4^2 \omega_8^2 - 19\omega_6^2 v_2^2 \omega_4^3 \omega_8^2 + 48\omega_6^3 v_2^2 c_s^2 \omega_4^3 \omega_8 - 36\omega_6^2 v_2^2 c_s^2 \omega_4^3 \omega_8 - \omega_6^2 c_s^2 \omega_4^3 \omega_8^2 - \\ & 18\omega_6^3 c_s^4 \omega_4^2 \omega_8 - 12\omega_6^2 v_2^2 \omega_4^4 + 12\omega_6^2 v_2^2 c_s^2 \omega_4^3 + 72v_2^2 \omega_4^3 \omega_8^2 - 18\omega_6^3 v_2^4 \omega_4^2 \omega_8^2 - 60\omega_6^2 v_2^2 \omega_4^3 \omega_8 + 12c_s^4 \omega_4^3 \omega_8^2 + 13\omega_6^3 c_s^4 \omega_4^2 \omega_8^2 + 6\omega_6^2 c_s^2 \omega_4^3 \omega_8 - \\ & 306\omega_6 v_2^2 c_s^2 \omega_4^3 \omega_8^2 + 12\omega_6^3 c_s^4 \omega_4^2 \omega_8 - 12\omega_6^2 v_2^2 c_s^2 \omega_4^2 \omega_8 - 4\omega_6^3 v_2^2 \omega_4^2 \omega_8^2 - 24\omega_6^2 v_2^4 \omega_4^2 \omega_8 - 24\omega_6^2 v_2^2 \omega_4^2 \omega_8^2 - 27\omega_6^3 v_2^4 \omega_4^3 \omega_8 - 12\omega_6^2 v_2^2 c_s^2 \omega_4^2 - 6\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + \\ & 6\omega_6^3 c_5^2 \omega_4^3 \omega_8 - 12\omega_6^3 v_2^2 c_s^2 \omega_4 \omega_8 + \omega_6^2 c_s^4 \omega_4^3 \omega_8^2 + 18\omega_6^3 c_s^2 \omega_4^3 \omega_8 + 60\omega_6^2 v_2^2 c_s^2 \omega_4^3 \omega_8^2 + 19\omega_6^2 v_2^4 \omega_4^3 \omega_8^2 - 48\omega_6^3 v_2^2 \omega_4^3 \omega_8 + 30\omega_6^2 v_2^2 c_s^2 \omega_4^2 \omega_8 - 36\omega_6 v_2^2 \omega_4^3 \omega_8 + 24\omega_6^3 v_2^2 \omega_4 \omega_8 + 12\omega_6^3 v_2^2 \omega_4^2 - 12\omega_6^2 v_2^4 \omega_4^3 + 12\omega_6^3 v_2^2 c_s^2 \omega_4^3 \omega_8^2 - 24\omega_6^3 c_4^4 \omega_4 \omega_8^2 + 12\omega_6^3 v_2^4 \omega_4 \omega_8^2 - \\ & 12\omega_6^3 v_2^2 \omega_4^3 - 12\omega_6 c_s^4 \omega_4^2 \omega_8^2 - 48\omega_6^2 v_2^2 c_s^2 \omega_4 \omega_8 + 36\omega_6 v_2^2 \omega_4^3 \omega_8 - 21\omega_6^3 v_2^2 c_s^2 \omega_4^3 \omega_8 + 12\omega_6^3 c_s^4 \omega_4 \omega_8 - 24\omega_6^3 v_2^4 \omega_4 \omega_8 - 12\omega_6 c_s^4 \omega_4^3 \omega_8^2 - 90\omega_6 v_2^4 \omega_4^3 \omega_8^2 - \\ & 12\omega_6^3 v_2^2 \omega_4 \omega_8 + 6\omega_6^3 c_s^2 \omega_4 \omega_8^2 - 81\omega_6^3 v_2^2 c_s^2 \omega_4^2 \omega_8^2 - 48\omega_6^3 v_2^2 c_s^2 \omega_4^2 \omega_8^2 + 12\omega_6 c_s^2 \omega_4^3 \omega_8^2 \end{aligned}$$

$$C_{19} = 17\omega_6^3 \omega_4^2 \omega_8 + 48\omega_6 \omega_4^2 \omega_8^2 + 104\omega_6^2 v_2^2 \omega_4^2 \omega_8 + 56\omega_6^2 c_s^2 \omega_4^2 \omega_8 + 32\omega_6 c_s^2 \omega_4 \omega_8^2 + 64\omega_6 v_2^2 \omega_4 \omega_8^2 + 16\omega_6^3 c_s^2 \omega_4^2 + 28\omega_6^2 \omega_4 \omega_8^2 - 16\omega_6^2 v_2^2 \omega_4^2 +$$

$$20\omega_6^2 c_s^2 \omega_4^2 + 12\omega_6^3 \omega_8 + 48c_s^2 \omega_4^2 \omega_8^2 + 16\omega_6^2 \omega_4 \omega_8 + 80v_2^2 \omega_4^2 \omega_8^2 + 43\omega_6^2 v_2^2 \omega_4^2 \omega_8^2 - 12\omega_6^2 \omega_4^2 + 25\omega_6^2 c_s^2 \omega_4^2 - 16\omega_6^3 v_2^2 \omega_8 - \\ 25\omega_6^2 c_s^2 \omega_4^2 \omega_8 + 24\omega_6 \omega_4^2 \omega_8 - 43\omega_6^3 v_2^2 \omega_4^2 \omega_8 - 44\omega_6^2 c_s^2 \omega_4 \omega_8^2 - 68\omega_6^2 v_2^2 \omega_4 \omega_8^2 - 64\omega_6 v_2^2 \omega_4 \omega_8 - 32\omega_6 c_s^2 \omega_4^2 \omega_8 + 68\omega_6^2 v_2^2 \omega_4 \omega_8 + 16\omega_6^3 v_2^2 \omega_8 + \\ 44\omega_6^3 c_2^2 \omega_4 \omega_8 - 8\omega_6^3 \omega_4^2 - 16\omega_6^2 c_s^2 \omega_4^2 - 40\omega_6^2 \omega_4^2 \omega_8 + 28\omega_6^2 v_2^2 \omega_8^2 - 17\omega_6^2 \omega_4^2 \omega_8^2 + 8\omega_6^3 \omega_4^2 - 32\omega_4^2 \omega_8^2 - 28\omega_6^3 \omega_4 \omega_8 - 24\omega_6 \omega_4 \omega_8^2 - 16\omega_6^3 v_2^2 \omega_4 - \\ 16\omega_6^2 c_s^2 \omega_4 \omega_8 - 48\omega_6^2 v_2^2 \omega_4 \omega_8 - 20\omega_6^3 c_s^2 \omega_8 + 120\omega_6 v_2^2 \omega_4^2 \omega_8^2 + 8\omega_6^2 \omega_4^2 - 72\omega_6 c_s^2 \omega_4^2 \omega_8^2$$

$$C_{20} = 12 - 5\omega_6^3 c_s^4 - 18\omega_6 + 10\omega_6^3 v_2^2 - 132c_s^2 - 98\omega_6^2 v_2^2 + 672v_2^2 c_s^2 + 144v_2^4 + 404\omega_6^2 v_2^2 \omega_8^2 + 82\omega_6^2 c_s^4 + 198\omega_6 c_s^2 - 216\omega_6 v_2^4 - 156v_2^2 - 78\omega_6^2 c_s^2 + 144c_s^4 + 90\omega_6^2 v_2^4 - 1008\omega_6 v_2^2 c_s^2 + 234\omega_6 v_2^2 - 216\omega_6 c_s^4 - \omega_6^3 - 9\omega_6^3 v_2^4 + 6\omega_6^3 c_s^2 + 8\omega_6^2 - 34\omega_6^3 v_2^2 c_s^2$$

$$C_{21} = 12 - \omega_6^3 c_s^4 - 18\omega_6 + 14\omega_6^3 v_2^2 - 36c_s^2 - 154\omega_6^2 v_2^2 + 432v_2^2 c_s^2 + 504v_2^4 + 252\omega_6^2 v_2^2 c_s^2 + 14\omega_6^2 c_s^4 + 54\omega_6 c_s^2 - 756\omega_6 v_2^4 - 252v_2^2 - 22\omega_6^2 c_s^2 + 24c_s^4 + 310\omega_6^2 v_2^4 - 648\omega_6 v_2^2 c_s^2 + 378\omega_6 v_2^2 - 36\omega_6 c_s^4 - \omega_6^3 - 29\omega_6^3 v_2^4 + 2\omega_6^3 c_s^2 + 8\omega_6^2 - 18\omega_6^3 v_2^2 c_s^2$$

## 2.4 CLBM1

### 2.4.1 Definitions

Collision operator  $\mathbf{C}$ :

$$\mathbf{C}(\mathbf{f}) = \mathbf{K}^{-1} \mathbf{S} (\boldsymbol{\kappa}^{(eq)} - \mathbf{K}\mathbf{f}) ,$$

where

$$\mathbf{S} = \text{diag}(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9),$$

$\omega_1, \omega_2, \dots, \omega_9 \in (0, 2)$ .

Matrix  $\mathbf{K}$  corresponds to the transformation matrix to the central moment basis defined by

$$\boldsymbol{\kappa} = \left( k_{(0,0)}, k_{(1,0)}, k_{(0,1)}, k_{(1,1)}, k_{(2,0)}, k_{(0,2)}, k_{(2,1)}, k_{(1,2)}, k_{(2,2)} \right)^T,$$

and is given by

$$\begin{aligned} \mathbf{K}_{1,i} &= (\mathbf{c}_i - \mathbf{v})^{(0,0)} \\ \mathbf{K}_{2,i} &= (\mathbf{c}_i - \mathbf{v})^{(1,0)} \\ \mathbf{K}_{3,i} &= (\mathbf{c}_i - \mathbf{v})^{(0,1)} \\ \mathbf{K}_{4,i} &= (\mathbf{c}_i - \mathbf{v})^{(1,1)} \\ \mathbf{K}_{5,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,0)} \\ \mathbf{K}_{6,i} &= (\mathbf{c}_i - \mathbf{v})^{(0,2)} \\ \mathbf{K}_{7,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,1)} \\ \mathbf{K}_{8,i} &= (\mathbf{c}_i - \mathbf{v})^{(1,2)} \\ \mathbf{K}_{9,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,2)}, \end{aligned}$$

$$\forall i \in \{1, 2, \dots, 9\}.$$

The equilibrium central moments are defined by

$$\boldsymbol{\kappa}^{(eq)} = \mathbf{KM}^{-1} \boldsymbol{\mu}^{(eq)},$$

i.e.,

$$\boldsymbol{\kappa}^{(eq)} = \left( \rho, 0, 0, 0, \rho c_s^2, \rho c_s^2, 0, 0, \rho c_s^4 \right)^T.$$

#### 2.4.2 Conservation of mass: $\rho$

attached text file: `output_d2q9_nse_clbm1_symbolic_pde_00.txt`

$$\begin{aligned} & \frac{\partial \rho}{\partial t} + \frac{v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + 3c_s^2 + v_1^2) \frac{v_1 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3v_1^2) \frac{\rho \delta_l^3}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\ & \frac{\rho c_s^2 \delta_l^3}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{\rho c_s^2 \delta_l^3}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{v_2 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + c_s^2 + 3v_2^2) \frac{\rho \delta_l^3}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & (-c_s^4 \omega_5 - 2c_s^2 - 6v_1^2 - 3v_1^4 \omega_5 + c_s^2 \omega_5 - 12c_s^2 v_1^2 \omega_5 + 2c_s^4 + 6v_1^4 + 24c_s^2 v_1^2 + 3v_1^2 \omega_5) \frac{\delta_l^4}{24\omega_5 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\ & (-4 + 6c_s^2 + 10v_1^2 + 2\omega_5 - 3c_s^2 \omega_5 - 5v_1^2 \omega_5) \frac{\rho v_1 \delta_l^4}{12\omega_5 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + \\ & (v_1^2 \omega_7 \omega_5 - \omega_7 + 3\omega_5 - 9c_s^2 \omega_5 + v_1^2 \omega_7 + 3c_s^2 \omega_7 \omega_5 + 3c_s^2 \omega_7 - 3v_1^2 \omega_5 - \omega_7 \omega_5) \frac{\rho v_1 \delta_l^4}{12\omega_7 \omega_5 \delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + (-2 + \omega_4) \frac{c_s^4 \delta_l^4}{6\omega_4 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\ & + (v_2^2 \omega_8 \omega_6 - \omega_8 \omega_6 - 9c_s^2 \omega_6 + 3c_s^2 \omega_8 \omega_6 + 3c_s^2 \omega_8 - \omega_8 - 3v_2^2 \omega_6 + v_2^2 \omega_8 + 3\omega_6) \frac{\rho v_2 \delta_l^4}{12\omega_8 \omega_6 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\ & (-3v_2^4 \omega_6 + c_s^2 \omega_6 + 24c_s^2 v_2^2 + 6v_2^4 - 2c_s^2 + 3v_2^2 \omega_6 - c_s^4 \omega_6 - 6v_2^2 + 2c_s^4 - 12c_s^2 v_2^2 \omega_6) \frac{\delta_l^4}{24\omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\ & (-4 - 3c_s^2 \omega_6 + 6c_s^2 - 5v_2^2 \omega_6 + 10v_2^2 + 2\omega_6) \frac{\rho v_2 \delta_l^4}{12\omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0. \end{aligned}$$

#### 2.4.3 Conservation of momentum: $\rho v_1$

attached text file: `output_d2q9_nse_clbm1_symbolic_pde_01.txt`

$$\begin{aligned} & v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (c_s^2 + v_1^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2\rho v_1 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_1 v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho v_2 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{\rho v_1 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\ & (-2 + 4c_s^2 + 6v_1^2 + \omega_5 - 2c_s^2 \omega_5 - 3v_1^2 \omega_5) \frac{\delta_l^2}{\omega_5 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + (2 - \omega_5) \frac{3\rho v_1 \delta_l^2}{\omega_5 \delta_t} \left( \frac{\partial v_1}{\partial x_1} \right)^2 + (-2 + \omega_4) \frac{c_s^2 \delta_l^2}{2\omega_4 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\ & (-2 + \omega_4) \frac{c_s^2 \delta_l^2}{2\omega_4 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (-2 + 6c_s^2 + 2v_1^2 + \omega_5 - 3c_s^2 \omega_5 - v_1^2 \omega_5) \frac{v_1 \delta_l^2}{2\omega_5 \delta_t} \frac{\partial^2 \rho}{\partial x_1^2} + \\ & (-2 + 2c_s^2 + 6v_1^2 + \omega_5 - c_s^2 \omega_5 - 3v_1^2 \omega_5) \frac{\rho \delta_l^2}{2\omega_5 \delta_t} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega_4) \frac{\rho c_s^2 \delta_l^2}{2\omega_4 \delta_t} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_4) \frac{\rho c_s^2 \delta_l^2}{2\omega_4 \delta_t} \frac{\partial^2 v_1}{\partial x_2^2} + C_1 \frac{\delta_l^3}{12\omega_5^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + \\ & + (-24 - 4\omega_5^2 + 36c_s^2 + 60v_1^2 + 24\omega_5 - 36c_s^2 \omega_5 + 11v_1^2 \omega_5^2 - 60v_1^2 \omega_5 + 5c_s^2 \omega_5^2) \frac{\rho v_1 \delta_l^3}{6\omega_5^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + C_2 \frac{\rho v_1 \delta_l^3}{12\omega_7 \omega_4 \omega_5^2 \delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + \\ & (-12 - \omega_4^2 + 12\omega_4) \frac{c_s^4 \delta_l^3}{6\omega_4^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} - \frac{\rho c_s^2 v_1 \delta_l^3}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{v_1 v_2 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_3^3} + C_3 \frac{\rho v_2 \delta_l^3}{6\omega_4 \omega_8 \delta_t} \frac{\partial^3 v_1}{\partial x_3^3} + \\ & (-1 + c_s^2 + 3v_2^2) \frac{\rho v_1 \delta_l^3}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_4 \frac{v_1 \delta_l^4}{12\omega_3^2 \delta_t} \frac{\partial^4 \rho}{\partial x_4^4} + C_5 \frac{\rho \delta_l^4}{12\omega_3^2 \delta_t} \frac{\partial^4 v_1}{\partial x_4^4} + C_6 \frac{\rho \delta_l^4}{12\omega_7^2 \omega_4^2 \omega_5^2 \delta_t} \frac{\partial^4 v_2}{\partial x_3^3 \partial x_2} + C_7 \frac{c_s^2 v_1 \delta_l^4}{12\omega_7^2 \omega_4^2 \omega_8 \omega_5^2 \omega_9 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\ & + C_8 \frac{\rho c_s^2 \delta_l^4}{12\omega_7 \omega_4^2 \omega_8 \omega_5^2 \omega_9 \delta_t} \frac{\partial^4 v_1}{\partial x_2^2 \partial x_3^2} + C_9 \frac{c_s^2 v_2 \delta_l^4}{12\omega_7 \omega_4^2 \omega_8^2 \omega_5 \omega_9 \omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_3^3} + \\ & (v_2^2 \omega_8 \omega_6 - \omega_8 \omega_6 - 9c_s^2 \omega_6 + 3c_s^2 \omega_8 \omega_6 + 3c_s^2 \omega_8 - \omega_8 - 3v_2^2 \omega_6 + v_2^2 \omega_8 + 3\omega_6) \frac{\rho v_1 v_2 \delta_l^4}{12\omega_8 \omega_6 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_3^3} + \\ & C_{10} \frac{\rho c_s^2 \delta_l^4}{12\omega_7 \omega_4^3 \omega_8 \omega_5 \omega_9 \omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_3^3} + \\ & (-3v_2^4 \omega_6 + c_s^2 \omega_6 + 24c_s^2 v_2^2 + 6v_2^4 - 2c_s^2 + 3v_2^2 \omega_6 - c_s^4 \omega_6 - 6v_2^2 + 2c_s^4 - 12c_s^2 v_2^2 \omega_6) \frac{v_1 \delta_l^4}{24\omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{11} \frac{\rho \delta_l^4}{24\omega_4^3 \omega_8^2 \delta_t} \frac{\partial^4 v_1}{\partial x_2^4} + \\ & (-4 - 3c_s^2 \omega_6 + 6c_s^2 - 5v_2^2 \omega_6 + 10v_2^2 + 2\omega_6) \frac{\rho v_1 v_2 \delta_l^4}{12\omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:

$$C_1 = 7v_4^4\omega_5^2 - 12c_s^4\omega_5 - 12c_s^2 + c_s^4\omega_5^2 - 36v_4^2 - 36v_4^4\omega_5 + 12c_s^2\omega_5 - 144c_s^2v_1^2\omega_5 + 12c_s^4 - 7v_1^2\omega_5^2 + 36v_4^4 + 144c_s^2v_1^2 + 36v_2^2\omega_5 - c_s^2\omega_5^2 + 24c_s^2v_1^2\omega_5^2$$

$$\begin{aligned} \textcolor{red}{C}_2 = & 12v_1^2\omega_4w_5^2 + 6\omega_7w_4w_5 + 3\omega_7w_4w_5^2 + 12w_5^2 - 12v_1^2\omega_4w_5 + 6v_1^2\omega_7w_5^2 + 12v_1^2\omega_7w_4 + 36c_s^2\omega_4w_5^2 - 12\omega_7w_4 - 6\omega_7w_5^2 - 18c_s^2\omega_7w_4w_5 - \\ & 3v_1^2\omega_7w_4w_5^2 + 12w_4w_5 - 12v_1^2\omega_5^2 + 18c_s^2\omega_7w_5^2 - 12w_4w_5^2 - 6v_1^2\omega_7w_4w_5 + 36c_s^2\omega_7w_4 - 11c_s^2\omega_7w_4w_5^2 - 36c_s^2\omega_4w_5 - 36c_s^2\omega_5^2 \end{aligned}$$

$$C_3 = 6 + 3\omega_4 v_2^2 - 3c_s^2 \omega_4 \omega_8 - 3\omega_4 + \omega_4 \omega_8 - 18c_s^2 + 9c_s^2 \omega_8 - 3\omega_8 - 6v_2^2 - \omega_4 v_2^2 \omega_8 + 3v_2^2 \omega_8 + 9c_s^2 \omega_4$$

$$\begin{aligned} C_4 = & 12 + 90v_4^4\omega_5^2 - 9v_4^4\omega_5^3 - 216c_s^4\omega_5 + 8\omega_5^2 - 132c_s^2 + 82c_s^4\omega_5^2 - 156a_1^2 - \omega_5^3 - 5c_s^4\omega_5^3 - 216v_4^4\omega_5 - 18\omega_5 + 10v_1^2\omega_5^3 + 198c_s^2\omega_5 - \\ & 1008c_s^2v_1^2\omega_5 + 144c_s^4 - 98v_2^2\omega_5^2 + 144v_4^4 - 34c_s^2v_1^2\omega_5^3 + 6c_s^2\omega_5^3 + 672c_s^2v_1^2 + 234v_2^2\omega_5 - 78c_s^2\omega_5^2 + 404c_s^2v_1^2\omega_5^2 \end{aligned}$$

$$\begin{aligned} C_5 = & 12 + 310v_4^4\omega_5^2 - 29v_4^4\omega_5^3 - 36c_s^4\omega_5 + 8\omega_5^2 - 36c_s^4 + 14c_s^4\omega_5^2 - 252v_1^2 - \omega_5^3 - c_s^4\omega_5^3 - 756v_1^4\omega_5 - 18\omega_5 + 14v_1^2\omega_5^3 + 54c_s^2\omega_5 - 648c_s^2v_1^2\omega_5 + \\ & 24c_s^4 - 154v_1^2\omega_5^2 + 504v_4^4 - 18c_s^2v_1^2\omega_5^3 + 2c_s^2\omega_5^3 + 432c_s^2v_1^4 + 378v_1^2\omega_5 - 22c_s^2\omega_5^2 + 252c_s^2v_1^2\omega_5^2 \end{aligned}$$

$$\begin{aligned}
& \text{C}_6 = 6c_s^2 w_7^2 w_4 w_5 - 12c_s^2 w_7 w_4^2 w_5 + c_s^4 w_2^2 w_3^2 w_5^2 - 18c_s^2 v_1^2 w_7^2 w_4 w_5^3 + 36c_s^2 v_1^2 w_7 w_4^2 w_5^2 + 108c_s^2 v_2^2 w_4^2 w_5^3 - 72v_1^2 w_7 w_4^3 w_5^2 + 12c_s^2 w_5^2 w_4^3 + \\
& 18c_s^2 w_7 w_4 w_5^3 - 90v_1^4 w_7^2 w_4^2 w_5^3 - 6v_1^4 w_7^2 w_4^2 w_5^3 + 39v_1^2 w_7 w_4^3 w_5^3 - 108c_s^2 v_2^2 w_4^3 w_5^3 + 54c_s^2 v_2^2 w_7 w_4^2 w_5^3 - 12c_s^4 w_7^2 w_4^2 w_5^3 - 108c_s^2 v_1^2 w_7 w_4^3 w_5^3 - 12c_s^2 v_1^2 w_7 w_4^2 w_5^3 + \\
& 12c_s^4 w_7 w_5^3 - 108c_s^2 v_1^2 w_2^2 w_5^3 + 6c_s^4 w_7^2 w_4^2 w_5^2 + 12c_s^4 w_7 w_4^3 w_5^3 + 198c_s^2 v_1^2 w_7 w_4^3 w_5^2 + 6c_s^2 w_7 w_4^3 w_5^2 + 19v_1^4 w_7^2 w_4^3 w_5^2 - 12c_s^4 w_7^2 w_4^3 w_5^3 - 99c_s^2 v_1^2 w_7 w_4^3 w_5^3 - \\
& 12c_s^2 w_2^2 w_3^3 + 13c_s^4 w_7^2 w_5^3 - 36v_1^2 w_7 w_4^3 w_5^3 + 252c_s^2 v_2^2 w_2^2 w_5^3 + 36v_1^2 w_7 w_4^3 w_5 + 44c_s^2 w_7^2 w_4^3 w_5^3 - 6c_s^2 w_7 w_4^3 w_5^3 - 24c_s^4 w_7 w_4^3 w_5^3 + 60c_s^2 v_1^2 w_7 w_4^3 w_5^3 + \\
& 72v_1^2 w_7 w_4^2 + 12c_s^2 w_7 w_4^2 w_5^2 + 72v_1^2 w_7 w_4^3 w_5^2 - c_s^2 w_7^2 w_4^3 w_5^2 + 36v_1^2 w_7 w_4^3 w_5^3 + 90v_1^2 w_7^2 w_4^3 w_5^3 + 6v_1^2 w_7^2 w_4^2 w_5^3 - 18c_s^4 w_7 w_4^2 w_5^3 - 36c_s^2 v_2^2 w_7^2 w_4^2 w_5^3 + \\
& 12c_s^2 v_1^2 w_7 w_3^3 w_5^3 - 36v_1^4 w_4^2 w_5^3 + 12c_s^2 w_7^2 w_4^2 w_5^3 - 36v_1^2 w_7^2 w_4^3 w_5^3 - 39v_1^4 w_7 w_4^3 w_5^3 - 72v_1^2 w_7^2 w_4^3 w_5^3 - 6c_s^2 w_7^2 w_4^2 w_5^3 - 12c_s^2 w_7 w_4 w_5^3 - 19c_s^2 w_7^2 w_4^3 w_5^3 + \\
& 18c_s^2 v_1^2 w_2^2 w_5^3 - 36v_1^4 w_4^3 w_5^2 - 6c_s^4 w_7 w_4^3 w_5^2 + 36c_s^2 v_2^2 w_7 w_4 w_5^3 + 36v_1^4 w_7 w_4^2 w_5^3 - 36v_1^4 w_7 w_4^3 w_5^3 + 36v_1^2 w_4^2 w_5^3 + 12c_s^2 w_7^2 w_4^3 w_5^3 - 5c_s^2 w_7^2 w_4^2 w_5^3 - \\
& 306c_s^2 v_1^2 w_7 w_4^3 w_5 + 6c_s^4 w_7 w_4^3 w_5^3 + 36v_1^4 w_3^3 w_5 - 3c_s^2 v_1^2 w_7 w_4^2 w_5^3 - 4v_1^2 w_7 w_4^3 w_5^3
\end{aligned}$$

$$\begin{aligned}
C_7 = & 36c_s^2w_7^2w_4^2w_8w_9 - 18w_7w_4w_8w_5^3w_9 + 36c_s^2w_7^2w_4w_5^3 + 5c_s^2w_7^2w_4^2w_8w_5^3w_9 + 12v_1^2w_7w_4w_8w_5^3 + 12w_7^2w_4^2w_8w_5w_9 - 6v_1^2w_7w_4^2w_8w_5^3w_9 - \\
& 36c_s^2w_4w_8w_5^2w_9 + 12v_1^2w_4^2w_8w_5^3w_9 + 12w_7w_4^2w_8w_5^3 - 36c_s^2w_7^2w_4w_8w_5^3 + v_1^2w_7^2w_4^2w_8w_5^3w_9 + 12w_7w_4^2w_8w_5w_9 + 18c_s^2w_7^2w_4^2w_5^2w_9 + 12v_1^2w_7^2w_4^2w_8w_5^3w_9 + \\
& 12w_7w_4w_8w_5^3w_9 - 12w_7w_4^2w_8w_5^2 - 18c_s^2w_7w_4^2w_8w_5^3w_9 - 36c_s^2w_7w_4w_8w_5^3w_9 - 12v_1^2w_4^2w_8w_5^2w_9 + 12v_1^2w_7^2w_4w_4^3w_5 + 6v_1^2w_7^2w_4^2w_8w_5^3 + 12w_4w_8w_5^3w_9 + \\
& 6w_7^2w_4^2w_5^3 + 18v_1^2w_7w_4^2w_8w_5^2w_9 - 36c_s^2w_7w_4^2w_8w_5^3 + 12w_7^2w_4w_8w_5^3 - 12w_7^2w_4^2w_8w_5^2 - 6c_s^2w_7^2w_4^2w_8w_5^2w_9 + 36c_s^2w_7w_4^2w_8w_5^2 + 54c_s^2w_7w_4^2w_8w_5^2w_9 - \\
& 6v_1^2w_7^2w_4^2w_8w_5^2 - 12v_1^2w_7^2w_4w_8w_5^3w_9 + 12w_7w_4w_8w_5^3w_9 - 18w_7^2w_4w_8w_5^3w_9 - 12v_1^2w_7^2w_4w_8w_5^2w_9 - 2v_1^2w_7^2w_4^2w_8w_5^2w_9 - 6w_7^2w_4^2w_8w_5^3w_9 - 12v_1^2w_4w_8w_5^3w_9 - \\
& 36c_s^2w_7w_4^2w_8w_5w_9 - 6w_7^2w_4^2w_8w_5^3 + 12w_4^2w_8w_5^3w_9 + 36c_s^2w_7w_4w_8w_5^3 - 12v_1^2w_7^2w_4w_8w_5^3 + 18v_1^2w_7w_4w_8w_5^3w_9 + 36c_s^2w_4^2w_8w_5^3w_9 + \\
& 12v_1^2w_7^2w_4^2w_8w_9 - 40c_s^2w_7^2w_4w_8w_5^3w_9 - 12v_1^2w_7^2w_4^2w_8w_5w_9 + 6w_7w_4^2w_8w_5^3w_9 - 12w_7w_4w_5^3 - 6v_1^2w_7^2w_4^2w_5^3 + 54c_s^2w_7w_4w_8w_5^3w_9 + 36c_s^2w_7^2w_8w_5^3w_9 - \\
& 12v_1^2w_7w_8w_5^3w_9 - w_7^2w_4^2w_8w_5^3w_9 + 6w_7^2w_4^2w_8w_5^2 - 12v_1^2w_7w_4^2w_8w_5w_9 + 12w_7^2w_8w_5^2w_9 + 12w_7^2w_4w_5^2w_9 - 36c_s^2w_7^2w_4^2w_8w_5w_9 - 12v_1^2w_7^2w_4w_8w_5^3w_9 + \\
& 6v_1^2w_7^2w_4^2w_5^2w_9 - 18w_7w_7^2w_8w_5^2w_9 + 54c_s^2w_7^2w_4w_8w_5^2w_9 + 18c_s^2w_7^2w_4^2w_8w_5^3 - 36c_s^2w_7^2w_4^2w_5^2w_9 - 12w_7w_4w_8w_5^3 - 12w_7^2w_4^2w_8w_5^3 - 12w_7^2w_4^2w_8w_5^3w_9 - \\
& 36c_s^2w_7^2w_4w_5^3w_9 + 18v_1^2w_7^2w_4w_8w_5^2w_9 - 18c_s^2w_7^2w_4^2w_5^3 + 12v_1^2w_7w_4^2w_8w_5^3 - 12w_7^2w_8w_5^3w_9 + 2w_7^2w_4^2w_8w_5^2w_9 - 36c_s^2w_7^2w_8w_5^2w_9 - 18c_s^2w_7^2w_4^2w_8w_5^2
\end{aligned}$$

$$\begin{aligned}
C_8 = & 12w_7w_4^2w_5w_9 - 12c_s^2w_3^3w_8w_5^2 - c_s^2w_7w_4^3w_8w_5^2w_9 - 36v_1^2w_7w_4^2w_8w_9 + 12c_s^2w_7w_4^2w_5^2 + 12w_3^2w_8w_5w_9 - 12w_4^2w_8w_5^2 - 12c_s^2w_7w_4w_8w_5w_9 - 12w_4^2w_8w_5^2w_9 - 18v_1^2w_7w_4^2w_5^2 - 24w_7w_4^2w_8w_5w_9 + 6c_2^2w_7w_4^3w_8w_5w_9 + 36v_1^2w_4^3w_8w_5^2w_9 + 36v_1^2w_4^3w_8w_5w_9 + 12c_s^2w_4^2w_8w_5w_9 - 18v_1^2w_7w_4^3w_8w_5w_9 + 12w_7w_4^2w_8w_5^2 - 36v_1^2w_7w_4w_8w_5w_9 - 36v_1^2w_8w_5w_9 + 36v_1^2w_7w_4^2w_5^2 - 72v_1^2w_2^2w_8w_5^2w_9 + 36v_1^2w_7w_4^2w_5^2 + 18v_1^2w_7w_4^3w_8w_5^2 + 12c_s^2w_4^2w_8w_5^2w_9 + 6w_7w_4^3w_8w_5w_9 - 12c_s^2w_7w_4^2w_5^2w_9 - 6c_s^2w_7w_4^3w_5^2 - 12c_s^2w_3^3w_8w_5w_9 - 12w_4^2w_8w_5w_9 - 12c_s^2w_7w_4^3w_8w_5^2w_9 - 4c_2^2w_7w_4^2w_8w_5^2w_9 - 12c_s^2w_7w_4w_8w_5^2w_9 + 18v_1^2w_7w_4^3w_5^2w_9 + 12c_s^2w_4^2w_8w_5w_9 - 6w_7w_3^3w_8w_9 + 24c_2^2w_7w_4^2w_8w_5w_9 - 6w_7w_3^2w_8w_5^2 + 24w_7w_4^2w_8w_5^2w_9 - 12c_2^2w_7w_4^2w_8w_9 + 12c_2^2w_7w_4^2w_8w_5^2 - 6c_2^2w_7w_4^3w_8w_5w_9 + 36v_1^2w_4^2w_8w_5w_9 - 12w_7w_4^2w_5^2 + 72v_1^2w_7w_4^2w_8w_5w_9 - 6w_7w_4^3w_5w_9 + 12c_2^2w_3^3w_8w_5w_9 + 12w_7w_4w_8w_5w_9 + 18v_1^2w_7w_4^3w_8w_9 + 12c_2^2w_4^2w_8w_5^2 - 18v_1^2w_7w_4^3w_8w_5w_9 - 36v_1^2w_7w_4^2w_5w_9 - 36v_1^2w_4^3w_8w_5w_9 + 12w_7w_4^2w_8w_9 + 6c_2^2w_7w_4^3w_8w_5^2 - 12w_4^3w_8w_5 - 24c_2^2w_4^2w_8w_5^2w_9 + 36v_1^2w_4w_8w_5^2w_9 + 6c_2^2w_7w_4^2w_5w_9 - 12w_4^2w_8w_5^2w_9 - 6c_2^2w_7w_4^3w_8w_5w_9 - 36v_1^2w_7w_4^2w_8w_5^2 + 6w_7w_4^3w_8w_5 + 36v_1^2w_4^2w_8w_5^2 + 6w_7w_4^3w_8w_5^2 + 18c_2^2w_7w_4w_8w_5^2w_9
\end{aligned}$$

$$\begin{aligned}
C_9 = & 6w_7w_2^2w_8^2w_6 + 6w_7w_4^2w_8w_5w_6 + 12w_4^2w_8^2w_9 + 12w_4v_2^2w_8^2w_5w_6 + 6w_7w_4^2v_2^2w_8^2w_9 + 12w_2^2v_2^2w_8^2w_5w_9 + 12w_7w_4w_5w_9w_6 + 36c_s^2w_7w_8^2w_5w_9w_6 + 3c_s^2w_7w_8^2w_5w_9w_6 + 12w_7w_8^2w_5w_9 - 12w_4w_8^2w_5w_6 - 12w_4^2w_8^2w_6 + 12w_7w_8w_5w_9w_6 + 18c_s^2w_7w_8^2w_5w_9w_6 - 12w_4v_2^2w_8^2w_5w_9 - 36c_s^2w_7w_4w_5w_9w_6 - 6w_7w_4^2w_8^2w_5w_9 + 36c_s^2w_7w_4w_5w_6 - 3w_7w_4^2v_2^2w_8w_5w_6 + 12w_4w_8^2w_5w_6 - 6w_7w_4^2w_8^2w_5w_9 - 15c_s^2w_7w_4w_5w_9w_6 + 36c_s^2w_7w_4w_5w_9w_6 + 12w_4w_8^2w_5w_6 - 6w_7w_4^2v_2^2w_8^2w_5w_6 - 18c_s^2w_7w_4^2w_8^2w_6 + 18w_7w_4v_2^2w_8w_5w_9w_6 - 12w_4^2v_2^2w_8^2w_5w_9 + 12w_7w_4w_8^2w_5w_6 - w_7w_4^2w_8^2w_5w_9 - 36c_s^2w_7w_4w_8^2w_5w_9 + 18c_s^2w_7w_4w_8^2w_5w_6 + 12w_7w_4v_2^2w_8^2w_5w_9w_6 + 12w_7w_4w_8^2w_5w_9w_6 - 6w_7w_4^2v_2^2w_8w_5w_6 - 12w_7w_4v_2^2w_8^2w_5w_9 - 18w_7w_4w_8^2w_5w_9 + 18c_s^2w_7w_4w_8^2w_5w_9 + 12w_4^2v_2^2w_8^2w_5w_6 - 6w_7w_4^2w_5w_9w_6 - 9c_s^2w_7w_4^2w_8w_5w_9w_6 - 36c_s^2w_7w_8w_5w_9w_6 + 12w_7w_8^2w_5w_6 + 36c_s^2w_4^2w_8^2w_6 - 12w_7w_4v_2^2w_8^2w_5w_6 - 12w_7w_8w_5w_6 - 5w_7w_4^2v_2^2w_8^2w_5w_9 - 36c_s^2w_4^2w_8^2w_5w_9 + 18w_7w_4v_2^2w_8^2w_5w_9 - 18c_s^2w_7w_4^2w_8w_5w_6 + w_7w_4^2v_2^2w_8^2w_5w_9w_6 - 12w_4^2w_8^2w_5w_9 + 54c_s^2w_7w_4w_8w_5w_9w_6 - 12w_7w_8^2w_5w_9w_6 - 36c_s^2w_7w_4w_5w_9w_6 + 6w_7w_4^2v_2^2w_8^2w_5w_6 - 6w_7w_4^2w_8^2w_5w_6 + 3w_7w_4^2w_8w_5w_9w_6 - 12w_7w_4v_2^2w_8^2w_5w_9w_6 - 12w_7w_4v_2^2w_8^2w_5w_9w_6 - 36c_s^2w_4^2w_8^2w_5w_6 + 54c_s^2w_7w_4w_5w_9w_6 + 5w_7w_4^2w_8^2w_5w_9 - 18w_7w_4w_8w_5w_9w_6 + 6w_7w_4^2v_2^2w_5w_9w_6 - 12w_7w_4^2w_8w_5w_9w_6 - 36c_s^2w_7w_4w_8^2w_5w_6 + 36c_s^2w_4^2w_8^2w_5w_9
\end{aligned}$$

$$\begin{aligned}
C_{10} = & -6d_s^2 w_7 w_4^3 w_8 w_6 + 36w_4^3 v_2^2 w_8 w_5 w_9 + 12w_7 w_4^2 w_8 w_5 w_6 - 12w_4^3 w_8 w_5 w_9 + 36w_7 w_4^2 v_2^2 w_5 w_6 - 12w_7 w_4 w_5 w_9 w_6 + 12c_s^2 w_4^2 w_8 w_5 w_6 - \\
& 12c_s^2 w_7 w_4 w_8 w_5 w_9 + 18w_7 w_4^3 v_2^2 w_5 w_9 w_6 - 18c_s^2 w_7 w_4^2 w_5 w_9 w_6 - 18w_7 w_4^2 w_8 w_5 w_9 + 12w_4^3 w_8 w_5 w_6 - 36w_4^2 v_2^2 w_8 w_5 w_6 + 6c_s^2 w_7 w_3^2 w_8 w_9 - \\
& 12w_7 w_4^2 w_5 w_6 - 12c_s^2 w_4^2 w_8 w_5 w_9 - 3w_7 w_4^2 v_2^2 w_8 w_5 w_9 w_6 - 12w_4^2 w_8 w_5 w_6 + 5w_7 w_4^2 w_8 w_5 w_9 + 6c_s^2 w_7 w_3^2 w_5 w_9 w_6 + 6w_7 w_4^2 w_8 w_5 w_6 - c_6^2 w_7 w_3^2 w_8 w_5 w_9 w_6 + \\
& 5w_7 w_4^2 v_2^2 w_8 w_5 w_9 + 12w_4^2 w_8 w_5 w_9 + 12c_s^2 w_4^2 w_8 w_5 w_9 + 12w_7 w_4^2 w_8 w_5 w_9 - 6w_7 w_4^2 w_8 w_5 w_9 - 36w_7 w_4^2 v_2^2 w_8 w_5 w_9 - 36w_7 w_4^2 v_2^2 w_8 w_5 w_6 - \\
& 12c_s^2 w_4^2 w_8 w_5 w_6 + 12c_s^2 w_7 w_4^2 w_5 w_6 - 18w_7 w_4^3 v_2^2 w_5 w_6 - 6w_7 w_4^2 w_8 w_5 w_9 + 18c_s^2 w_7 w_4^2 w_8 w_5 w_9 + 18w_7 w_4^2 w_8 w_5 w_6 - 5c_s^2 w_7 w_4^2 w_8 w_5 w_9 w_6 - \\
& 15w_7 w_4^2 v_2^2 w_8 w_5 w_9 - 12c_s^2 w_7 w_8 w_5 w_9 w_6 + 6w_7 w_4^2 w_8 w_5 w_6 + 18w_7 w_4^3 v_2^2 w_8 w_9 + 18w_7 w_4^2 v_2^2 w_8 w_5 w_6 - 12c_s^2 w_7 w_4^2 w_8 w_5 w_6 + 18c_s^2 w_7 w_4 w_8 w_5 w_9 w_6 - \\
& 18w_7 w_4^2 v_2^2 w_8 w_6 + 12w_7 w_4 w_8 w_5 w_9 + 12c_s^2 w_7 w_4 w_5 w_9 w_6 + 6c_s^2 w_7 w_4^2 w_8 w_5 w_6 + w_7 w_4^2 w_8 w_5 w_9 w_6 + 36w_7 w_4 v_2^2 w_5 w_9 w_6 - 6c_s^2 w_7 w_3^2 w_5 w_6 + 12c_s^2 w_4^2 w_8 w_6 - \\
& 36w_4^2 v_2^2 w_8 w_5 w_9 - 36w_4^2 v_2^2 w_8 w_9 - 6w_7 w_4^2 w_5 w_9 w_6 - 5c_s^2 w_7 w_4^2 w_8 w_5 w_9 + 36w_4^2 v_2^2 w_8 w_5 w_6 - 12c_s^2 w_4^2 w_8 w_9 + 36w_4^2 v_2^2 w_8 w_6 - 54w_7 w_4^2 v_2^2 w_5 w_9 w_6
\end{aligned}$$

$$\begin{aligned} C_{11} = & -72c_2^4v_2^2w_8 - 24c_2^2w_4w_8 - 72c_2^3w_3^2v_2w_8 - 30w_3^4v_2^4w_8 - 36c_2^5w_4v_2^2w_8^2 - 48c_4^4w_4w_8^2 - 72w_2^4v_2^4 + 24c_4^4w_4w_8 + 6c_2^2w_3^4v_2^2w_8^2 + 12c_2^2w_4w_8^2 + 72c_2^2w_4v_2^2w_8 + 3w_3^4v_2^4w_8^2 + 36w_3^4v_2^4 + 12w_4^2v_2^2w_8^2 - 8c_2^2w_4^2w_8^2 - 3c_4^4w_3^2w_8^2 + 144c_2^3w_4v_2^2w_8 - 6c_2^3w_3^4w_8 + 72w_2^4v_2^4w_8 - 216c_2^3w_2^2v_2^2 - 24c_4^4w_4^2w_8 + 30w_3^4v_2^2w_8 + 24c_4^4w_8^2 - 36w_3^4v_2^2 + 24c_4^4w_4^2w_8^2 - 3w_3^4v_2^2w_8^2 - 12c_2^2w_4v_2^2w_8^2 - 12w_4^2v_2^4w_8 + c_2^2w_3^4w_8^2 + 72w_2^4v_2^2 + 6c_4^4w_3^4w_8 + 108c_2^3w_3^4v_2^2 + 24c_2^3w_4^2w_8 \end{aligned}$$

#### 2.4.4 Conservation of momentum: $\rho v_2$

 attached text file: output\_d2q9\_nse\_clbm1\_symbolic\_pde\_02.txt

$$\begin{aligned}
& v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{v_1 v_2 \delta_t}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho v_2 \delta_t}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\rho v_1 \delta_t}{\delta_t} \frac{\partial v_2}{\partial x_1} + (c_s^2 + v_2^2) \frac{\delta_t}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{2 \rho v_2 \delta_t}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_4) \frac{c_s^2 \delta_t^2}{2 \omega_4 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + \\
& (-2 + \omega_4) \frac{c_s^2 \delta_t^2}{2 \omega_4 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (-2 - 2 c_s^2 \omega_6 + 4 c_s^2 - 3 v_2^2 \omega_6 + 6 v_2^2 + \omega_6) \frac{\delta_t^2}{\omega_6 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (2 - \omega_6) \frac{3 \rho v_2 \delta_t^2}{\omega_6 \delta_t} \left( \frac{\partial v_2}{\partial x_2} \right)^2 + \\
& (-2 + \omega_4) \frac{\rho c_s^2 \delta_t^2}{2 \omega_4 \delta_t} \frac{\partial^2 v_2}{\partial x_1^2} + (-2 + \omega_4) \frac{\rho c_s^2 \delta_t^2}{2 \omega_4 \delta_t} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + (-2 - 3 c_s^2 \omega_6 + 6 c_s^2 - v_2^2 \omega_6 + 2 v_2^2 + \omega_6) \frac{v_2 \delta_t^2}{2 \omega_6 \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + \\
& (-2 - c_s^2 \omega_6 + 2 c_s^2 - 3 v_2^2 \omega_6 + 6 v_2^2 + \omega_6) \frac{\rho \delta_t^2}{2 \omega_6 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + (-1 + 3 c_s^2 + v_1^2) \frac{v_1 v_2 \delta_t^3}{12 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3 v_1^2) \frac{\rho v_2 \delta_t^3}{12 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + \\
& C_1 \frac{\rho v_1 \delta_t^3}{6 \omega_7 \omega_4 \delta_t} \frac{\partial^3 v_2}{\partial x_1^3} + (-12 - \omega_4^2 + 12 \omega_4) \frac{c_s^4 \delta_t^3}{6 \omega_4 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3 \partial x_2} - \frac{\rho c_s^2 v_2 \delta_t^3}{6 \delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + C_2 \frac{\rho v_2 \delta_t^3}{12 \omega_4 \omega_8 \omega_6 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_3 \frac{\delta_t^3}{12 \omega_6^2 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\
& (-24 - 36 c_s^2 \omega_6 + 36 c_s^2 + 5 c_s^2 \omega_6^2 - 60 v_2^2 \omega_6 + 60 v_2^2 + 24 \omega_6 - 4 \omega_6^2 + 11 v_2^2 \omega_6^2) \frac{\rho v_2 \delta_t^3}{6 \omega_6^2 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& (-c_s^4 \omega_5 - 2 c_s^2 - 6 v_1^2 - 3 v_1^4 \omega_5 + c_s^2 \omega_5 - 12 c_s^2 v_1^2 \omega_5 + 2 c_s^4 + 6 v_1^4 + 24 c_s^2 v_1^2 + 3 v_1^2 \omega_5) \frac{v_2 \delta_t^4}{24 \omega_5 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& (-4 + 6 c_s^2 + 10 v_1^2 + 2 \omega_5 - 3 c_s^2 \omega_5 - 5 v_1^2 \omega_5) \frac{\rho v_1 v_2 \delta_t^4}{12 \omega_5 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_4 \frac{\rho \delta_t^4}{24 \omega_7^2 \omega_4^3 \delta_t} \frac{\partial^4 v_2}{\partial x_1^4} + C_5 \frac{c_s^2 v_1 \delta_t^4}{12 \omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_9 \omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + \\
& C_6 \frac{\rho c_s^2 \delta_t^4}{12 \omega_7 \omega_4^3 \omega_8 \omega_5 \omega_9 \omega_6 \delta_t} \frac{\partial^4 v_1}{\partial x_3^3 \partial x_2} + \\
& (v_1^2 \omega_7 \omega_5 - \omega_7 + 3 \omega_5 - 9 c_s^2 \omega_5 + v_1^2 \omega_7 + 3 c_s^2 \omega_7 \omega_5 + 3 c_s^2 \omega_7 - 3 v_1^2 \omega_5 - \omega_7 \omega_5) \frac{\rho v_1 v_2 \delta_t^4}{12 \omega_7 \omega_5 \delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& C_7 \frac{c_s^2 v_2 \delta_t^4}{12 \omega_7 \omega_4^2 \omega_8^2 \omega_9 \omega_6^3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_8 \frac{\rho c_s^2 \delta_t^4}{12 \omega_7 \omega_4^3 \omega_8 \omega_9 \omega_6^2 \delta_t} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_9 \frac{\rho \delta_t^4}{12 \omega_4^3 \omega_8^2 \omega_6^3 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{10} \frac{v_2 \delta_t^4}{12 \omega_6^3 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{11} \frac{\rho \delta_t^4}{12 \omega_6^3 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= 6 - 3\omega_7 - 3\omega_4 - 18c_s^2 - 6v_1^2 - v_1^2\omega_7w_4 + \omega_7w_4 + 3v_1^2\omega_7 + 3v_1^2\omega_4 + 9c_s^2\omega_7 - 3c_s^2\omega_7w_4 + 9c_s^2w_4 \\
C_2 &= 12\omega_4\omega_6 - 6\omega_4v_2^2\omega_8w_6 + 6\omega_4\omega_8w_6 + 36c_s^2\omega_4\omega_8 + 36c_s^2\omega_4\omega_6^2 + 18c_s^2\omega_8\omega_6^2 - 12\omega_4\omega_8 - 36c_s^2\omega_4\omega_6 - 36c_s^2\omega_6^2 - 3\omega_4v_2^2\omega_8w_6^2 - 3\omega_4w_8\omega_6^2 + 6v_2^2\omega_8w_6^2 - 6\omega_8w_6^2 - 12\omega_4v_2^2w_6 - 18c_s^2\omega_4w_8w_6 + 12\omega_4v_2^2w_8 - 11c_s^2\omega_4w_8w_6^2 + 12\omega_6^2 - 12v_2^2w_6^2 + 12\omega_4v_2^2w_6^2 \\
C_3 &= -36v_2^4\omega_6 + 12c_s^2\omega_6 + 144c_s^2v_2^2 + 36v_2^4 - 12c_s^2 + 7v_2^4\omega_6^2 - c_s^2\omega_6^2 + 36v_2^2\omega_6 - 12c_s^4\omega_6 - 36v_2^2 + 12c_s^4 + 24c_s^2v_2^2\omega_6^2 - 144c_s^2v_2^2\omega_6 - 7v_2^2\omega_6^2 + c_s^4\omega_6^2 \\
C_4 &= -72v_2^2\omega_7\omega_4^2 + 24c_s^4\omega_7^2\omega_4^2 + 24c_s^4\omega_7^2 - 3c_s^4\omega_7^2\omega_4^3 + 30v_1^2\omega_7\omega_4^3 - 36c_s^2v_2^2\omega_7^2\omega_4 + 12c_s^2\omega_7^2\omega_4^2 + 36v_4^4\omega_4^3 - 12c_s^2v_1^2\omega_7^2\omega_4^2 - 8c_s^2\omega_7^2\omega_4^2 + 72v_4^4\omega_7\omega_4^2 - 30v_4^4\omega_7\omega_4^3 + c_s^2\omega_7^2\omega_4^3 + 6c_s^2v_1^2\omega_7^2\omega_4^3 - 72v_4^4\omega_4^2 - 48c_s^4\omega_7^2\omega_4 - 6c_s^2\omega_7\omega_4^3 + 3v_1^4\omega_7^2\omega_4^3 - 72c_s^2v_1^2\omega_7\omega_4^3 + 24c_s^4\omega_7\omega_4 - 216c_s^2v_1^2\omega_4^2 + 144c_s^2v_1^2\omega_7\omega_4^2 - 12v_4^4\omega_7^2\omega_4^2 + 24c_s^2\omega_7\omega_4^2 + 108c_s^2v_1^2\omega_4^3 - 3v_1^2\omega_7\omega_4^3 + 6c_s^4\omega_7\omega_4^3 + 72v_2^2\omega_4^2 + 72c_s^2v_1^2\omega_7\omega_4 - 24c_s^2\omega_7\omega_4 - 36v_1^2\omega_4^3 - 24c_s^4\omega_7\omega_4^2 + 12v_1^2\omega_7\omega_4^2 \\
C_5 &= -36c_s^2\omega_7^2\omega_4w_8w_5w_6 + 18c_s^2\omega_7^2\omega_4^2w_8w_9 + 6\omega_7w_4^2\omega_8w_5w_6 + 54c_s^2\omega_7^2\omega_4w_8w_9w_6 - 12w_7^2\omega_8w_5w_9w_6 - 12w_7^2\omega_7^2\omega_4^2w_5 - 6v_1^2\omega_7^2\omega_4^2\omega_8w_5 + 12v_1^2\omega_7w_4w_8w_5w_6 + 12w_7w_4w_8w_5w_6 - 36c_s^2\omega_7^2w_8w_5w_6 + 5\omega_7^2\omega_4^2w_8w_9w_6 - 12v_1^2\omega_7^2w_4w_8w_5w_6 + 36c_s^2\omega_7w_4w_8w_5w_6 - 36c_s^2\omega_7^2w_4^2w_9 - 6\omega_7^2\omega_4^2w_8w_5w_6 + 18v_1^2\omega_7^2\omega_4w_8w_9w_6 - 36c_s^2\omega_7^2\omega_4w_9w_6 - 6\omega_7^2\omega_8w_5w_9w_6 - 6\omega_7^2\omega_4^2w_8w_9w_6 + 12v_1^2\omega_7^2\omega_4^2w_9w_6 + 12w_7^2\omega_8w_5w_9w_6 - 12w_7^2\omega_7^2\omega_4w_5w_6 - 9c_s^2\omega_7w_4^2\omega_8w_5w_9w_6 - 36c_s^2\omega_7w_8w_5w_9w_6 - 6v_1^2\omega_7^2\omega_4w_8w_5w_6 - 15c_s^2\omega_7^2\omega_4^2w_8w_9w_6 - 12v_1^2\omega_7^2w_4w_8w_5w_6 - 18c_s^2\omega_7^2\omega_4^2w_5w_6 + 12w_7^2\omega_7^2\omega_4^2w_9 + 12w_7^2\omega_7^2w_4w_8w_5w_6 - 12w_7^2\omega_7w_4w_8w_5w_6 + 6v_1^2\omega_7^2\omega_4^2w_8w_9w_6 + 18c_s^2\omega_7^2\omega_4^2w_8w_5w_6 - 12v_1^2\omega_7^2\omega_8w_5w_9w_6 - 12w_7^2\omega_7^2\omega_4^2w_9w_6 - 5v_1^2\omega_7^2\omega_4^2w_8w_9w_6 - 18c_s^2\omega_7^2\omega_4^2w_8w_5w_6 + 36c_s^2\omega_7^2\omega_4^2w_5w_6 + 12w_7^2\omega_7^2\omega_4^2w_5w_6 - 12v_1^2\omega_7^2\omega_4^2w_9w_6 + 6v_1^2\omega_7^2\omega_4^2w_8w_5w_6 + 6v_1^2\omega_7^2\omega_4^2w_8w_5w_6 + 12v_1^2\omega_7^2\omega_4^2w_5w_6 + v_1^2\omega_7^2\omega_4^2w_8w_5w_9w_6 + 3w_7\omega_7^2\omega_4w_8w_5w_9w_6 + 6v_1^2\omega_7^2\omega_4^2w_8w_5w_9w_6 - 36c_s^2\omega_7^2\omega_4w_8w_5w_9w_6 - 36c_s^2\omega_7^2\omega_4^2w_8w_5w_9w_6 - 3v_1^2\omega_7^2\omega_4^2w_8w_5w_9w_6 + 6\omega_7^2\omega_4^2w_8w_5w_6 - 36c_s^2\omega_7^2\omega_4^2w_5w_6 + 18c_s^2\omega_7^2\omega_4^2w_8w_5w_9w_6 + 3c_s^2\omega_7^2\omega_4^2w_8w_5w_9w_6 + 18v_1^2\omega_7w_4w_8w_5w_9w_6 + 36c_s^2\omega_7^2\omega_4^2w_9w_6 - 18w_7w_4w_8w_5w_9w_6 + 12v_1^2\omega_7^2\omega_4^2w_5 - 12v_1^2\omega_7^2\omega_4^2w_8w_5w_9w_6 \\
C_6 &= 12\omega_7w_4^3\omega_9 + 12\omega_7w_4^2\omega_8w_5w_6 - 36v_1^2\omega_7w_4w_8w_9w_6 + 18v_1^2\omega_7^2\omega_8w_5w_9w_6 + 12c_s^2\omega_7^2\omega_8w_5w_6 - 18\omega_7w_4^2\omega_8w_9w_6 - 12c_s^2\omega_7w_4w_8w_9w_6 + 12c_s^2\omega_7w_4^2\omega_5w_6 - 18v_1^2\omega_7w_4^3\omega_8w_5 + 12w_7\omega_4^2\omega_9w_6 + 6c_s^2\omega_7w_4^2\omega_8w_5w_6 + 18w_4^2\omega_8w_5w_9w_6 - 12w_4^2\omega_8w_5w_6 + 12c_s^2\omega_7w_4^2\omega_3w_9w_6 + 36v_1^2\omega_7w_4^2\omega_9w_6 - 12c_s^2\omega_7w_4^2\omega_8w_5w_6 - 6w_7\omega_4^2\omega_8w_5w_6 - 6w_7\omega_4^2\omega_8w_5w_6 + 36v_1^2\omega_7w_4^2\omega_9w_6 - 12c_s^2\omega_7w_4^2\omega_8w_5w_6 + 12w_4w_8w_5w_9w_6 + 12w_4w_8w_5w_9w_6 + 5\omega_7w_4^2\omega_8w_9w_6 + 12c_s^2\omega_7w_4^2\omega_5w_6 - 6w_7\omega_4^2\omega_8w_9w_6 - 5c_s^2\omega_7w_4^2\omega_8w_5w_9w_6 + 12w_7w_4w_8w_9w_6 - 12c_s^2\omega_7w_4w_8w_9w_6 - 36v_1^2\omega_7w_4^2\omega_8w_5w_6 - 12w_7w_4^2\omega_5w_6 - 12w_7w_4^2\omega_3w_9w_6 + 36v_1^2\omega_7w_4^2\omega_8w_5w_6 + 54v_1^2\omega_7w_4^2\omega_8w_9w_6 + 12w_7w_4^2\omega_5w_6 - 6w_4^2\omega_8w_5w_9w_6 - 6c_s^2\omega_7w_4^2\omega_8w_5w_6 - 12c_s^2\omega_7w_4^2\omega_8w_5w_6 - 12c_s^2\omega_7w_4^2\omega_3w_9w_6 + 18c_s^2\omega_7w_4w_8w_5w_9w_6 + 18c_s^2\omega_7w_4^2\omega_8w_9w_6 + 6c_s^2\omega_7w_4^2\omega_8w_5w_6 + w_7w_4^2\omega_8w_5w_9w_6 - 54v_1^2\omega_7w_4^2\omega_8w_5w_9w_6 - 12c_s^2\omega_7w_4^2\omega_5w_6 + 12c_s^2\omega_7w_4\omega_8w_5w_9w_6 - 5c_s^2\omega_7w_4^2\omega_8w_9w_6 - 3v_1^2\omega_7w_4^2\omega_8w_5w_9w_6 + 12c_s^2\omega_7w_4^2\omega_9w_6 - 18v_1^2\omega_7w_4^2\omega_8w_5w_6 - 36v_1^2\omega_7w_4w_8w_5w_9w_6 - 15v_1^2\omega_7w_4^2\omega_8w_5w_9w_6 + 36v_1^2\omega_7w_4w_8w_5w_9w_6 - 36v_1^2\omega_7w_4^2\omega_9w_6
\end{aligned}$$

$$\begin{aligned}
& 5c_s^2 \omega_7 \omega_4^2 \omega_8^2 \omega_9 \omega_6^3 + 36c_s^2 \omega_7 \omega_8^2 \omega_9 \omega_6^3 - 12 \omega_7 \omega_4^2 \omega_8^2 \omega_9 - 36c_s^2 \omega_4 \omega_8^2 \omega_9 \omega_6^2 + 36c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^3 - 12 \omega_7 v_2^2 \omega_8^2 \omega_9 \omega_6^2 - 18 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 6 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^2 + 12 \omega_7 \omega_8^2 \omega_9 \omega_6^2 + 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 + 6 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^2 - 6 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^3 - 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^2 + 6 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 - 12 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 - 18 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 - 12 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^2 + 6 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 - 12 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^3 - 36c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 - 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 + 12 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 36c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^3 + 12 \omega_7 \omega_8^2 \omega_9 \omega_6^2 + 12 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 + 54c_s^2 \omega_7 \omega_4 \omega_8 \omega_9 \omega_6^3 - 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 - 12 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 - 12 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 - 40c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^3 + 12 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 54c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 2 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^2 + 12 \omega_7 \omega_4 \omega_8 \omega_9 \omega_6^3 - 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^2 - 12 \omega_4^2 \omega_8 \omega_9 \omega_6^3 + 18c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 + 12 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 - 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 - 36c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^3 - 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^2 - 36c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 54c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 - 12 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 - 18c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 - 2 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^2 - 6 \omega_2^2 \omega_8 \omega_9 \omega_6^2 - 36c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^3 - 18c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 - 36c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 + 18 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^2 + \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 - 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 - 36c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^3
\end{aligned}$$

$$\begin{aligned}
C_8 = & -6c_s^2 \omega_7 \omega_4^3 \omega_8 \omega_6 + 6 \omega_4^3 \omega_8 \omega_6^2 + 36 \omega_7 \omega_4^2 v_2^2 \omega_9 \omega_6 - 12 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 36 \omega_7 \omega_4^2 v_2^2 \omega_9 \omega_6^2 + 36 \omega_7 \omega_4^3 v_2^2 \omega_9 \omega_6^2 - 24 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6 - 24c_s^2 \omega_7 \omega_4^2 \omega_9 \omega_6^2 + 12c_s^2 \omega_2^2 \omega_8 \omega_6^2 - 12c_s^2 \omega_7 \omega_4 \omega_8 \omega_9 \omega_6 - 6 \omega_7 \omega_4^3 \omega_8 \omega_6^2 + 6c_s^2 \omega_7 \omega_4^3 \omega_8 \omega_9 - 12 \omega_7 \omega_4^3 \omega_6 - 6 \omega_4^3 \omega_8 \omega_9 \omega_6 + 18 \omega_4^3 v_2^2 \omega_8 \omega_9 \omega_6 - 12 \omega_7 \omega_4^2 \omega_9 \omega_6^2 + 24 \omega_7 \omega_4^2 \omega_9 \omega_6^2 + 6 \omega_7 \omega_4^2 \omega_8 \omega_6 - 36 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6 + 18c_s^2 \omega_7 \omega_4 \omega_8 \omega_9 \omega_6^2 + 12 \omega_2^2 \omega_8 \omega_9 \omega_6 + 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_6^2 + 12 \omega_7 \omega_4 \omega_8 \omega_9 \omega_6^3 - 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^2 - 12 \omega_4^2 \omega_8 \omega_9 \omega_6^3 + 40c_s^2 \omega_7 \omega_4 \omega_8 \omega_9 \omega_6^3 + 18c_s^2 \omega_7 \omega_4^2 \omega_9 \omega_6^2 + 12 \omega_7 \omega_8 \omega_9 \omega_6^3 - 36c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6 + 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 + 12 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 - 36c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 - 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 - 12 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 - 18c_s^2 \omega_7 \omega_4 \omega_8 \omega_9 \omega_6^3 - 2 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^2 - 6 \omega_2^2 \omega_8 \omega_9 \omega_6^2 - 36c_s^2 \omega_7 \omega_4 \omega_8 \omega_9 \omega_6^3 - 18c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 - 36c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 + 12 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 54c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 - 12 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 - 18c_s^2 \omega_7 \omega_4 \omega_8 \omega_9 \omega_6^3 - 2 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^2 - 6 \omega_2^2 \omega_8 \omega_9 \omega_6^2 - 36c_s^2 \omega_7 \omega_4 \omega_8 \omega_9 \omega_6^3 - 18c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 - 36c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 + 18 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^2 + \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 - 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^3 - 36c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^3
\end{aligned}$$

$$\begin{aligned}
C_9 = & -12c_s^4 \omega_4^2 \omega_8 \omega_6 + 36 \omega_3^3 v_2^4 \omega_6^3 + 108c_s^2 \omega_3^2 v_2^2 \omega_6^3 + 4 \omega_4^3 v_2^4 \omega_8 \omega_6^3 + 36c_s^4 \omega_3^2 \omega_8 \omega_6^3 + 90 \omega_3^3 v_2^2 \omega_8 \omega_6^2 + 36 \omega_2^2 v_2^2 \omega_6^3 + 6 \omega_2^2 \omega_4 \omega_8 \omega_6^3 + 12c_s^2 \omega_3^2 v_2^2 \omega_8 \omega_6^3 - 6c_s^2 \omega_3^2 \omega_8 \omega_6^2 + 6 \omega_4^2 v_2^2 \omega_8 \omega_6^3 - 108c_s^2 \omega_3^2 v_2^2 \omega_6^2 + 19 \omega_4^3 v_2^4 \omega_8 \omega_6^2 - 36c_s^2 \omega_4^2 v_2^2 \omega_8 \omega_6^2 - 36 \omega_4^3 v_2^4 \omega_6^2 + 18c_s^2 \omega_4^2 \omega_8 \omega_6^3 + 12c_s^2 \omega_7 \omega_4^3 \omega_6 - 12 \omega_7 \omega_4 \omega_8 \omega_6^2 + 12 \omega_7 \omega_4^2 \omega_8 \omega_6^2 - 12c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_6^2 + 18 \omega_7 \omega_4^3 v_2^2 \omega_8 \omega_6 - c_s^2 \omega_7 \omega_4^3 \omega_8 \omega_9 \omega_6^2 - 36 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_6^2 - 6 \omega_2^2 \omega_8 \omega_6^2 + 12c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_6^2 - 36 \omega_7 \omega_4^3 v_2^2 \omega_6^2 + 12c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_6^2 - 12 \omega_4^2 \omega_8 \omega_6^2 - 18 \omega_7 \omega_4^3 v_2^2 \omega_8 \omega_6 + 24c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 - 18 \omega_7 \omega_4^3 v_2^2 \omega_8 \omega_9 \omega_6^2 - 12c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 12c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 - 36c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^2 - 12c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 36c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 12 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^2 - 12c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 252c_s^2 \omega_4^2 v_2^2 \omega_8^2 - 19 \omega_4^3 v_2^2 \omega_8 \omega_6^2 - 6 \omega_4^2 v_2^4 \omega_2 \omega_6^3 + 72 \omega_4^3 v_2^4 \omega_8 \omega_6^2 + 6c_s^4 \omega_2^2 \omega_8 \omega_6^2 - 3c_s^2 \omega_2^2 \omega_8 \omega_6^3 + 12c_s^4 \omega_4^2 v_2^2 \omega_8 \omega_6^3 + 36 \omega_4^2 v_2^4 \omega_8 \omega_6^3 - 72 \omega_4^3 v_2^2 \omega_8 \omega_6^2 - 6c_s^2 \omega_4^3 \omega_6 + 12c_s^2 \omega_4^2 \omega_8 \omega_6^2 + 36 \omega_4^3 v_2^2 \omega_6^2 - 24c_s^2 \omega_4 \omega_8 \omega_6^2 + 54c_s^2 \omega_4^2 v_2^2 \omega_8 \omega_6^3 + 12c_s^4 \omega_4^2 \omega_8 \omega_6^2 - 36 \omega_4^3 v_2^4 \omega_8 \omega_6^2 - 12c_s^4 \omega_4^2 \omega_8 \omega_6^2 + 39 \omega_3^3 v_2^2 \omega_8 \omega_6^3 + 6c_s^2 \omega_4^3 \omega_8 \omega_6^2 - 36 \omega_4^3 v_2^4 \omega_8 \omega_6^2 - 36c_s^2 \omega_4^2 v_2^2 \omega_8 \omega_6^3 + c_s^2 \omega_4^3 \omega_8 \omega_6^2 - 72 \omega_4^3 v_2^2 \omega_8 \omega_6^2 + 198c_s^2 \omega_4^2 v_2^2 \omega_8 \omega_6^3 - 5c_s^2 \omega_4^2 \omega_8 \omega_6^2 - 12c_s^2 \omega_4^2 v_2^2 \omega_8 \omega_6^3 - 18c_s^2 \omega_4^2 v_2^2 \omega_8 \omega_6^3 - 12c_s^2 \omega_4^2 v_2^2 \omega_8 \omega_6^3 - 72 \omega_4^3 v_2^2 \omega_8 \omega_6^2 - 5c_s^2 \omega_4^2 v_2^2 \omega_8 \omega_6^3 - 18c_s^2 \omega_4^2 v_2^2 \omega_8 \omega_6^3 - c_s^2 \omega_4^3 \omega_8 \omega_6^2 - 39 \omega_3^3 v_2^4 \omega_8 \omega_6^3 + 36 \omega_4^3 v_2^2 \omega_8 \omega_6^2 - 99c_s^2 \omega_4^2 v_2^2 \omega_8 \omega_6^3 - 6c_s^2 \omega_4^2 \omega_8 \omega_6^2 - 12c_s^2 \omega_4 \omega_8 \omega_6^3
\end{aligned}$$

$$C_{10} = 12 - 216v_2^4 \omega_6 + 198c_s^2 \omega_6 + 672c_s^2 v_2^2 + 144v_2^4 - 132c_s^2 + 90v_2^4 \omega_6^2 - 78c_s^2 \omega_6^2 + 6c_s^2 \omega_6^3 - 9v_2^4 \omega_6^3 + 234v_2^2 \omega_6 - 34c_s^2 v_2^2 \omega_6^3 - 216c_s^4 \omega_6 - 156v_2^2 + 144c_s^4 + 404c_s^2 v_2^2 \omega_6^2 - \omega_6^3 - 1008c_s^2 v_2^2 \omega_6 - 5c_s^4 \omega_6^3 - 18 \omega_6 + 10v_2^2 \omega_6^3 + 8 \omega_6^2 - 98v_2^2 \omega_6^2 + 82c_s^4 \omega_6^2$$

$$C_{11} = 12 - 756v_2^4 \omega_6 + 54c_s^2 \omega_6 + 432c_s^2 v_2^2 + 504v_2^4 - 36c_s^2 + 310v_2^4 \omega_6^2 - 22c_s^2 \omega_6^2 + 2c_s^2 \omega_6^3 - 29v_2^4 \omega_6^3 + 378v_2^2 \omega_6 - 18c_s^2 v_2^2 \omega_6^3 - 36c_s^4 \omega_6 - 252v_2^2 + 24c_s^4 + 252c_s^2 v_2^2 \omega_6^2 - \omega_6^3 - 648c_s^2 v_2^2 \omega_6 - c_s^4 \omega_6^3 - 18 \omega_6 + 14v_2^2 \omega_6^3 + 8 \omega_6^2 - 154v_2^2 \omega_6^2 + 14c_s^4 \omega_6^2$$

## 2.5 CLBM2

### 2.5.1 Definitions

Collision operator  $\mathbf{C}$ :

$$\mathbf{C}(\mathbf{f}) = \mathbf{K}^{-1} \mathbf{S} (\boldsymbol{\kappa}^{(eq)} - \mathbf{K} \mathbf{f}),$$

where

$$\mathbf{S} = \text{diag}(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9),$$

$\omega_1, \omega_2, \dots, \omega_9 \in (0, 2)$ .

Matrix  $\mathbf{K}$  corresponds to the transformation matrix to the central moment basis defined by

$$\boldsymbol{\kappa} = \begin{pmatrix} k_{(0,0)} \\ k_{(1,0)} \\ k_{(0,1)} \\ k_{(1,1)} \\ k_{(2,0)} + k_{(0,2)} \\ k_{(2,0)} - k_{(0,2)} \\ k_{(2,1)} \\ k_{(1,2)} \\ k_{(2,2)} \end{pmatrix},$$

thus, the transformation matrix  $\mathbf{K}$  satisfies

$$\begin{aligned}
\mathbf{K}_{1,i} &= (\mathbf{c}_i - \mathbf{v})^{(0,0)} \\
\mathbf{K}_{2,i} &= (\mathbf{c}_i - \mathbf{v})^{(1,0)} \\
\mathbf{K}_{3,i} &= (\mathbf{c}_i - \mathbf{v})^{(0,1)} \\
\mathbf{K}_{4,i} &= (\mathbf{c}_i - \mathbf{v})^{(1,1)} \\
\mathbf{K}_{5,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,0)} + (\mathbf{c}_i - \mathbf{v})^{(0,2)} \\
\mathbf{K}_{6,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,0)} - (\mathbf{c}_i - \mathbf{v})^{(0,2)} \\
\mathbf{K}_{7,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,1)} \\
\mathbf{K}_{8,i} &= (\mathbf{c}_i - \mathbf{v})^{(1,2)} \\
\mathbf{K}_{9,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,2)},
\end{aligned}$$

$$\forall i \in \{1, 2, \dots, 9\}.$$

The equilibrium central moments are defined by

$$\boldsymbol{\kappa}^{(eq)} = \mathbf{KM}^{-1} \boldsymbol{\mu}^{(eq)},$$

i.e.,

$$\boldsymbol{\kappa}^{(eq)} = \left( \rho, 0, 0, 0, 2\rho c_s^2, 0, 0, 0, \rho c_s^4 \right)^T.$$

### 2.5.2 Conservation of mass: $\rho$



attached text file: `output_d2q9_nse_clbm2_symbolic_pde_00.txt`

$$\begin{aligned}
&\frac{\partial \rho}{\partial t} + \frac{v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + 3c_s^2 + v_1^2) \frac{v_1 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3v_1^2) \frac{\rho \delta_l^3}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\
&\frac{\rho \delta_l^3 c_s^2}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{\rho \delta_l^3 c_s^2}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2} + (-1 + 3c_s^2 + v_2^2) \frac{v_2 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + c_s^2 + 3v_2^2) \frac{\rho \delta_l^3}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
&(\omega_5 c_s^2 + 3\omega_5 v_1^2 + 6v_1^4 + 2c_s^4 - 2c_s^2 - 6v_1^2 - 12\omega_5 v_1^2 c_s^2 + 24v_1^2 c_s^2 - 3\omega_5 v_1^4 - \omega_5 c_s^4) \frac{\delta_l^4}{24\omega_5 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\
&(-4 - 3\omega_5 c_s^2 - 5\omega_5 v_1^2 + 2\omega_5 + 6c_s^2 + 10v_1^2) \frac{\rho v_1 \delta_l^4}{12\omega_5 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + \\
&(3\omega_7 \omega_5 c_s^2 + \omega_7 \omega_5 v_1^2 - \omega_7 - 9\omega_5 c_s^2 - 3\omega_5 v_1^2 + 3\omega_5 + 3\omega_7 c_s^2 + \omega_7 v_1^2 - \omega_7 \omega_5) \frac{\rho v_1 \delta_l^4}{12\omega_7 \omega_5 \delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + (-2 + \omega_4) \frac{\delta_l^4 c_s^4}{6\omega_4 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} \\
&+ (v_2^2 \omega_8 \omega_6 - \omega_8 \omega_6 - \omega_8 + 3\omega_8 \omega_6 c_s^2 + 3\omega_8 c_s^2 - 3v_2^2 \omega_6 + 3\omega_6 + v_2^2 \omega_8 - 9\omega_6 c_s^2) \frac{\rho v_2 \delta_l^4}{12\omega_8 \omega_6 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
&(-3v_2^4 \omega_6 - \omega_6 c_s^4 + 2c_s^4 + 6v_2^4 + 3v_2^2 \omega_6 - 2c_s^2 + 24v_2^2 c_s^2 - 6v_2^2 - 12v_2^2 \omega_6 c_s^2 + \omega_6 c_s^2) \frac{\delta_l^4}{24\omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\
&(-4 - 5v_2^2 \omega_6 + 6c_s^2 + 10v_2^2 + 2\omega_6 - 3\omega_6 c_s^2) \frac{\rho v_2 \delta_l^4}{12\omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0.
\end{aligned}$$

### 2.5.3 Conservation of momentum: $\rho v_1$



attached text file: `output_d2q9_nse_clbm2_symbolic_pde_01.txt`

$$\begin{aligned}
&v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (c_s^2 + v_1^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2\rho v_1 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho v_2 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{\rho v_1 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\
&(-2 - 2\omega_5 c_s^2 - 3\omega_5 v_1^2 + \omega_5 + 4c_s^2 + 6v_1^2) \frac{\delta_l^2}{\omega_5 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + (2 - \omega_5) \frac{3\rho v_1 \delta_l^2}{\omega_5 \delta_t} \left( \frac{\partial v_1}{\partial x_1} \right)^2 + (-2 + \omega_4) \frac{\delta_l^2 c_s^2}{2\omega_4 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} +
\end{aligned}$$

$$\begin{aligned}
& (-2 + \omega_4) \frac{\delta_l^2 c_s^2}{2\omega_4 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (-2 - 3\omega_5 c_s^2 - \omega_5 v_1^2 + \omega_5 + 6c_s^2 + 2v_1^2) \frac{v_1 \delta_l^2}{2\omega_5 \delta_t} \frac{\partial^2 \rho}{\partial x_1^2} + \\
& (-2 - \omega_5 c_s^2 - 3\omega_5 v_1^2 + \omega_5 + 2c_s^2 + 6v_1^2) \frac{\rho \delta_l^2}{2\omega_5 \delta_t} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega_4) \frac{\rho \delta_l^2 c_s^2}{2\omega_4 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + (-2 + \omega_4) \frac{\rho \delta_l^2 c_s^2}{2\omega_4 \delta_t} \frac{\partial^2 v_1}{\partial x_2^2} + C_1 \frac{\delta_l^3}{12\omega_5^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + \\
& + (-24 - 36\omega_5 c_s^2 - 60\omega_5 v_1^2 + 24\omega_5 - 4\omega_5^2 + 36c_s^2 + 60v_1^2 + 5\omega_5^2 c_s^2 + 11\omega_5^2 v_1^2) \frac{\rho v_1 \delta_l^3}{6\omega_5^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + C_2 \frac{\rho v_1 \delta_l^3}{12\omega_7 \omega_4 \omega_5^2 \delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + \\
& (-12 - \omega_4^2 + 12\omega_4) \frac{\delta_l^3 c_s^4}{6\omega_4^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} - \frac{\rho v_1 \delta_l^3 c_s^2}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{v_2 v_1 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + C_3 \frac{\rho v_2 \delta_l^3}{6\omega_4 \omega_8 \delta_t} \frac{\partial^3 v_1}{\partial x_2^3} + \\
& (-1 + c_s^2 + 3v_2^2) \frac{\rho v_1 \delta_l^3}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_4 \frac{v_1 \delta_l^4}{12\omega_5^3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + C_5 \frac{\rho \delta_l^4}{12\omega_8^3 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_6 \frac{\rho \delta_l^4}{12\omega_7^2 \omega_4^2 \omega_5^3 \delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_7 \frac{v_1 \delta_l^4 c_s^2}{12\omega_7^2 \omega_4^2 \omega_8 \omega_5^3 \omega_9 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} \\
& + C_8 \frac{\rho \delta_l^4 c_s^2}{12\omega_7 \omega_4^3 \omega_8 \omega_5^2 \omega_9 \delta_t} \frac{\partial^4 v_1}{\partial x_2^2 \partial x_2} + C_9 \frac{v_2 \delta_l^4 c_s^2}{12\omega_7 \omega_4^2 \omega_8^2 \omega_5 \omega_9 \omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_3^2} + \\
& (v_2^2 \omega_8 \omega_6 - \omega_8 w_6 - \omega_8 + 3\omega_8 \omega_6 c_s^2 + 3\omega_8 c_s^2 - 3v_2^2 \omega_6 + 3\omega_6 + v_2^2 \omega_8 - 9\omega_6 c_s^2) \frac{\rho v_2 v_1 \delta_l^4}{12\omega_8 \omega_6 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& C_{10} \frac{\rho \delta_l^4 c_s^2}{12\omega_7 \omega_4^3 \omega_8 \omega_5 \omega_9 \omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\
& (-3v_2^4 \omega_6 - \omega_6 c_s^4 + 2c_s^4 + 6v_2^4 + 3v_2^2 \omega_6 - 2c_s^2 + 24v_2^2 c_s^2 - 6v_2^2 - 12v_2^2 \omega_6 c_s^2 + \omega_6 c_s^2) \frac{v_1 \delta_l^4}{24\omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{11} \frac{\rho \delta_l^4}{24\omega_4^3 \omega_8^2 \delta_t} \frac{\partial^4 v_1}{\partial x_2^4} + \\
& (-4 - 5v_2^2 \omega_6 + 6c_s^2 + 10v_2^2 + 2\omega_6 - 3\omega_6 c_s^2) \frac{\rho v_2 v_1 \delta_l^4}{12\omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= 7\omega_5^2 v_1^4 + 24\omega_5^2 v_1^2 c_s^2 + \omega_5^2 c_s^4 + 12\omega_5 v_1^2 + 36v_1^4 + 12c_s^4 - 12c_s^2 - 36v_1^2 - \omega_5^2 c_s^2 - 7\omega_5^2 v_1^2 - 144\omega_5 v_1^2 c_s^2 + 144v_1^2 c_s^2 - 36\omega_5 v_1^4 - 12\omega_5 c_s^4 \\
C_2 &= 12\omega_4 \omega_5 - 3\omega_7 \omega_4 \omega_5^2 v_1^2 - 11\omega_7 \omega_4 \omega_5^2 c_s^2 + 6\omega_7 \omega_4 \omega_5 + 3\omega_7 \omega_4 \omega_5^2 + 36\omega_7 \omega_4 c_s^2 + 12\omega_7 \omega_4 v_1^2 + 12\omega_4 \omega_5^2 v_1^2 - 12\omega_4 \omega_5^2 - 12\omega_7 \omega_4 + 36\omega_4 \omega_5^2 c_s^2 + \\
& 12\omega_5^2 - 12\omega_4 \omega_5 v_1^2 - 36\omega_4 \omega_5 c_s^2 - 6\omega_7 \omega_5^2 - 36\omega_5^2 c_s^2 - 6\omega_7 \omega_4 \omega_5 v_1^2 - 12\omega_5^2 v_1^2 - 18\omega_7 \omega_4 \omega_5 c_s^2 + 18\omega_7 \omega_5^2 c_s^2 + 6\omega_7 \omega_5^2 v_1^2 \\
C_3 &= 6 - 3\omega_4 + \omega_4 \omega_8 + 3\omega_4 v_2^2 - 3\omega_8 + 9\omega_4 c_s^2 - \omega_4 v_2^2 \omega_8 + 9\omega_8 c_s^2 - 18c_s^2 - 6v_2^2 + 3v_2^2 \omega_8 - 3\omega_4 \omega_8 c_s^2 \\
C_4 &= 12 + 90\omega_5^2 v_1^4 + 404\omega_5^2 v_1^2 c_s^2 + 82\omega_5^2 c_s^4 + 198\omega_5 c_s^2 + 234\omega_5 v_1^2 + 144v_1^4 - 5\omega_5^3 c_s^4 - 9\omega_5^3 v_1^4 + 144c_s^4 - 18\omega_5 + 8\omega_5^2 + 10\omega_5^3 v_1^2 - 132c_s^2 - \\
& 156v_1^2 + 6\omega_5^3 c_s^2 - \omega_5^3 - 34\omega_5^3 v_1^2 c_s^2 - 78\omega_5^3 c_s^2 - 98\omega_5^2 v_1^2 - 1008\omega_5 v_1^2 c_s^2 + 672v_1^2 c_s^2 - 216\omega_5 v_1^4 - 216\omega_5 c_s^4 \\
C_5 &= 12 + 310\omega_5^2 v_1^4 + 252\omega_5^2 v_1^2 c_s^2 + 14\omega_5^2 c_s^4 + 54\omega_5 c_s^2 + 378\omega_5 v_1^2 + 504v_1^4 - \omega_5^3 c_s^4 - 29\omega_5^3 v_1^4 + 24c_s^4 - 18\omega_5 + 8\omega_5^2 + 14\omega_5^3 v_1^2 - 36c_s^2 - 252v_1^2 + \\
& 2\omega_5^3 c_s^2 - \omega_5^3 - 18\omega_5^3 v_1^2 c_s^2 - 22\omega_5^2 c_s^2 - 154\omega_5^2 v_1^2 - 648\omega_5 v_1^2 c_s^2 + 432v_1^2 c_s^2 - 756\omega_5 v_1^4 - 36\omega_5 c_s^4 \\
C_6 &= \omega_7^2 \omega_4^3 \omega_5^2 c_s^4 + 39\omega_7 \omega_4^3 \omega_5^3 v_1^2 - 6\omega_7 \omega_4^3 \omega_5^3 c_s^2 + 36\omega_4^3 \omega_5^2 v_1^2 + 19\omega_7^2 \omega_4^3 \omega_5^2 v_1^2 - 72\omega_7^2 \omega_4^3 v_1^2 + 6\omega_7^2 \omega_4^3 \omega_5^3 v_1^2 - 24\omega_7^2 \omega_4 \omega_5^3 c_s^4 - \\
& 36\omega_4^2 \omega_5^3 v_1^4 + 12\omega_7 \omega_4^2 \omega_5^2 c_s^4 + 60\omega_7 \omega_4^2 \omega_5^2 v_1^2 + 90\omega_7 \omega_4^2 \omega_5 v_1^2 + 12\omega_7 \omega_4^2 \omega_5 c_s^2 + 108\omega_7 \omega_4^3 \omega_5^3 v_1^2 c_s^2 - 5\omega_7^2 \omega_4^2 \omega_5^3 c_s^2 - 12\omega_7^2 \omega_4^2 \omega_5 c_s^4 - 36\omega_4^3 \omega_5^3 v_1^2 + \\
& 4\omega_7^2 \omega_4^3 \omega_5^3 v_1^4 + 18\omega_7^2 \omega_4^2 \omega_5^2 v_1^2 c_s^2 + 6\omega_7 \omega_4^3 \omega_5^2 v_1^2 - 72\omega_7 \omega_4^3 \omega_5^2 v_1^2 - 108\omega_7 \omega_4^3 \omega_5^2 v_1^2 - 12\omega_7^2 \omega_4^2 \omega_5 c_s^4 + 36\omega_7 \omega_4 \omega_5^3 v_1^2 c_s^2 - \omega_7^2 \omega_4^3 \omega_5^3 c_s^4 - 6\omega_7^2 \omega_4^2 \omega_5^2 c_s^2 - \\
& 36\omega_7 \omega_4^3 \omega_5 v_1^4 - 18\omega_7 \omega_4 \omega_5^3 v_1^2 c_s^2 + 36\omega_7 \omega_4 \omega_5^3 v_1^2 - 12\omega_7 \omega_4 \omega_5^3 c_s^4 + 36\omega_7 \omega_4^2 \omega_5^2 v_1^2 c_s^2 + 72\omega_7 \omega_4^2 \omega_5^2 v_1^2 + 12\omega_7^2 \omega_4^2 \omega_5^2 c_s^2 - 36\omega_7 \omega_4^2 \omega_5^2 v_1^2 c_s^2 + 36\omega_7 \omega_4^2 \omega_5^2 v_1^2 - 6\omega_7 \omega_4^2 \omega_5^2 c_s^2 - \\
& 36\omega_7 \omega_4^2 \omega_5^2 v_1^2 c_s^2 + 36\omega_7 \omega_4^2 \omega_5^2 v_1^2 - 4\omega_7 \omega_4^2 \omega_5^2 v_1^2 + 12\omega_7 \omega_4^2 \omega_5^2 v_1^2 c_s^2 - 6\omega_7 \omega_4^2 \omega_5^2 c_s^2 + 18\omega_7 \omega_4^2 \omega_5^2 v_1^2 - 99\omega_7 \omega_4^2 \omega_5^2 v_1^2 c_s^2 + 12\omega_7^2 \omega_5^3 c_s^4 + \\
& 6\omega_7^2 \omega_4^2 \omega_5^2 c_s^2 + 36\omega_7 \omega_4^3 \omega_5 v_1^2 - 36\omega_7 \omega_4^2 \omega_5^2 v_1^2 + 12\omega_7 \omega_4 \omega_5^3 c_s^4 + 6\omega_7 \omega_4^3 \omega_5^3 c_s^4 + 54\omega_7 \omega_4^2 \omega_5^3 v_1^2 c_s^2 - 36\omega_7 \omega_4^2 \omega_5^3 v_1^2 - 19\omega_7 \omega_4^2 \omega_5^2 v_1^2 - 108\omega_7 \omega_4 \omega_5 v_1^2 c_s^2 - \\
& \omega_7^2 \omega_4^3 \omega_5^2 c_s^2 - 39\omega_7 \omega_4^3 \omega_5^3 v_1^2 - 306\omega_7 \omega_4^3 \omega_5 v_1^2 c_s^2 - 12\omega_7^2 \omega_4^3 \omega_5 c_s^4 + 13\omega_7^2 \omega_4^2 \omega_5^3 c_s^4 + 12\omega_7^2 \omega_4^2 \omega_5^3 c_s^4 + 72\omega_7 \omega_4^2 \omega_5^3 v_1^2 - 6\omega_7^2 \omega_4^2 \omega_5^3 v_1^2 + 36\omega_4^2 \omega_5^3 v_1^2 \\
& 6\omega_7^2 \omega_4 \omega_5^3 c_s^2 - 3\omega_7 \omega_4^2 \omega_5^3 v_1^2 c_s^2 + 252\omega_7 \omega_4^2 \omega_5^2 c_s^2 - 12\omega_7 \omega_4 \omega_5^2 c_s^2 - 90\omega_7 \omega_4^2 \omega_5 v_1^4
\end{aligned}$$

$$\begin{aligned}
C_7 &= -12\omega_4^2 \omega_8 \omega_5^3 \omega_9 - 36\omega_4^2 \omega_4 \omega_5^2 \omega_9 c_s^2 + 18\omega_7 \omega_4^2 \omega_8 \omega_5^2 v_1^2 \omega_9 - 18\omega_7 \omega_4 \omega_8 \omega_5^3 \omega_9 - 18\omega_7^2 \omega_4^2 \omega_8 \omega_5^2 c_s^2 - 36\omega_4^2 \omega_8 \omega_5^2 \omega_9^2 - 6\omega_7^2 \omega_4^2 \omega_8 \omega_5^2 v_1^2 + \\
& 12\omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_9 - 12\omega_7^2 \omega_4^2 \omega_8 \omega_9 - 12\omega_4^2 \omega_8 \omega_5^2 \omega_9^2 + 12\omega_7 \omega_4^2 \omega_8 \omega_5^2 v_1^2 - 6\omega_7^2 \omega_4^2 \omega_5^3 v_1^2 - 12\omega_7^2 \omega_4 \omega_5^3 \omega_9 + 12\omega_7 \omega_4^2 \omega_8 \omega_5 \omega_9 + \\
& 54\omega_7 \omega_4^2 \omega_8 \omega_5^2 \omega_9 c_s^2 + 12\omega_7 \omega_4^2 \omega_8 \omega_5^3 - 18\omega_7 \omega_4^2 \omega_5^3 c_s^2 + 36\omega_7 \omega_4^2 \omega_8 \omega_5^2 c_s^2 + 12\omega_7 \omega_4^2 \omega_8 \omega_5^3 \omega_9 - 12\omega_7^2 \omega_4^2 \omega_8 \omega_5^2 + 6\omega_7^2 \omega_4^2 \omega_8 \omega_5^2 \omega_9 c_s^2 + \\
& 6\omega_7^2 \omega_4^2 \omega_8 \omega_5^3 v_1^2 + 18\omega_7^2 \omega_4 \omega_8 \omega_5^3 v_1^2 + 12\omega_4^2 \omega_8 \omega_5^2 \omega_9^2 - 36\omega_7^2 \omega_4 \omega_8 \omega_5^2 \omega_9 + 12\omega_7^2 \omega_4 \omega_8 \omega_5^2 \omega_9 - 2\omega_7^2 \omega_4^2 \omega_8 \omega_5^2 v_1^2 \omega_9 - \\
& 36\omega_7 \omega_4^2 \omega_8 \omega_5^3 c_s^2 + 12\omega_7 \omega_4^2 \omega_8 \omega_5^3 \omega_9 - 12\omega_7 \omega_4^2 \omega_8 \omega_5^3 v_1^2 \omega_9 + 54\omega_7 \omega_4^2 \omega_8 \omega_5^3 v_1^2 + 18\omega_7 \omega_4 \omega_8 \omega_5^3 \omega_9 - 99\omega_7 \omega_4^2 \omega_8 \omega_5^3 v_1^2 c_s^2 + 12\omega_7^2 \omega_5^3 c_s^4 + \\
& 6\omega_7^2 \omega_4^2 \omega_5^2 c_s^2 + 36\omega_7 \omega_4^3 \omega_5 v_1^2 - 36\omega_7 \omega_4^2 \omega_5^2 v_1^2 + 12\omega_7 \omega_4 \omega_5^3 c_s^4 + 6\omega_7 \omega_4^3 \omega_5^3 c_s^4 + 54\omega_7 \omega_4^2 \omega_5^3 v_1^2 c_s^2 - 36\omega_7 \omega_4^2 \omega_5^3 v_1^2 - 19\omega_7 \omega_4^2 \omega_5^2 v_1^2 - 108\omega_7 \omega_4 \omega_5 v_1^2 c_s^2 - \\
& \omega_7^2 \omega_4^3 \omega_5^2 c_s^2 - 39\omega_7 \omega_4^3 \omega_5^3 v_1^2 - 306\omega_7 \omega_4^3 \omega_5 v_1^2 c_s^2 - 12\omega_7^2 \omega_4^3 \omega_5 c_s^4 + 13\omega_7^2 \omega_4^2 \omega_5^3 c_s^4 + 12\omega_7^2 \omega_4^2 \omega_5^3 c_s^4 + 72\omega_7 \omega_4^2 \omega_5^3 v_1^2 - 6\omega_7^2 \omega_4^2 \omega_5^3 v_1^2 + 36\omega_4^2 \omega_5^3 v_1^2 \\
& 6\omega_7^2 \omega_4 \omega_5^3 c_s^2 - 3\omega_7 \omega_4^2 \omega_5^3 v_1^2 c_s^2 + 252\omega_7 \omega_4^2 \omega_5^2 c_s^2 - 12\omega_7 \omega_4 \omega_5^2 c_s^2 - 90\omega_7 \omega_4^2 \omega_5 v_1^4
\end{aligned}$$

$$\begin{aligned}
C_9 = & 18\omega_7\omega_4^2\omega_8^2\omega_9c_s^2 + 12\omega_4v_2^2\omega_8^2\omega_5\omega_6 - 12\omega_4\omega_8^2\omega_5\omega_6 - 6\omega_7\omega_4^2\omega_8^2\omega_5\omega_6 + 36\omega_7\omega_4\omega_8\omega_5\omega_6c_s^2 + 6\omega_7\omega_4^2v_2^2\omega_8^2\omega_5\omega_6 - 36\omega_7\omega_4\omega_8^2\omega_5\omega_6c_s^2 + \\
& 12\omega_4^2\omega_8^2\omega_9 - 12\omega_7\omega_4^2\omega_5\omega_9\omega_6 - 12\omega_7\omega_4v_2^2\omega_8^2\omega_5\omega_6 + 54\omega_7\omega_4\omega_8\omega_5\omega_9\omega_6c_s^2 + 12\omega_4^2v_2^2\omega_8^2\omega_5\omega_9\omega_6 - 12\omega_4^2v_2^2\omega_8^2\omega_5\omega_6 - \\
& 18\omega_7\omega_4^2\omega_8\omega_5\omega_6c_s^2 - 36\omega_7\omega_4^2\omega_5\omega_9c_s^2 - 9\omega_7\omega_4^2\omega_8\omega_5\omega_9\omega_6c_s^2 - 12\omega_7\omega_4^2\omega_8\omega_5\omega_9\omega_6 + 54\omega_7\omega_4\omega_8^2\omega_5\omega_9c_s^2 + 36\omega_7\omega_4^2\omega_5\omega_9\omega_6c_s^2 + \\
& 6\omega_7\omega_4^2v_2^2\omega_8^2\omega_9 + 6\omega_7\omega_4^2v_2^2\omega_8^2\omega_5\omega_9\omega_6 - 12\omega_7\omega_4^2\omega_5\omega_9\omega_6 + 12\omega_7\omega_4\omega_8^2\omega_5\omega_9\omega_6 + 36\omega_4^2\omega_8^2\omega_5\omega_9c_s^2 - 12\omega_7\omega_4\omega_8^2\omega_5\omega_9\omega_6 - 6\omega_7\omega_4^2\omega_8^2\omega_9 + \\
& 6\omega_7\omega_4^2v_2^2\omega_5\omega_9\omega_6 - \omega_7\omega_4^2\omega_8^2\omega_5\omega_9\omega_6 - 36\omega_4\omega_8^2\omega_5\omega_9c_s^2 - 12\omega_4^2\omega_8^2\omega_5\omega_9 + 3\omega_7\omega_4^2\omega_8^2\omega_5\omega_9\omega_6c_s^2 - 18\omega_7\omega_4\omega_8^2\omega_5\omega_9 - 15\omega_7\omega_4^2\omega_8^2\omega_5\omega_9c_s^2 - \\
& 6\omega_7\omega_4^2\omega_5\omega_9\omega_6 - 36\omega_7\omega_4\omega_5\omega_9\omega_6c_s^2 - 36\omega_4^2\omega_8^2\omega_5\omega_6c_s^2 - 36\omega_7\omega_4^2\omega_8^2\omega_5\omega_9c_s^2 - 36\omega_7\omega_4\omega_8^2\omega_5\omega_6c_s^2 + 12\omega_7\omega_4\omega_8^2\omega_5\omega_6 + 12\omega_4^2\omega_8^2\omega_5\omega_6 + \\
& 18\omega_7\omega_4^2\omega_8^2\omega_5\omega_6c_s^2 + 18\omega_7\omega_4v_2^2\omega_8\omega_5\omega_9\omega_6 + 6\omega_7\omega_4^2\omega_8^2\omega_6 + 36\omega_4\omega_8^2\omega_5\omega_6c_s^2 + 12\omega_7\omega_8\omega_5\omega_9\omega_6 - 12\omega_7\omega_4^2\omega_8^2\omega_5\omega_9 - 6\omega_7\omega_4^2\omega_8^2\omega_6 - \\
& 12\omega_4^2v_2^2\omega_8^2\omega_9 + 12\omega_7\omega_4^2\omega_8\omega_5\omega_9 + 6\omega_7\omega_4^2\omega_8\omega_5\omega_6 + 18\omega_7\omega_4v_2^2\omega_8^2\omega_5\omega_9 - 3\omega_7\omega_4^2v_2^2\omega_8\omega_5\omega_9\omega_6 - 18\omega_7\omega_4\omega_8\omega_5\omega_9\omega_6 - 36\omega_7\omega_8\omega_5\omega_9\omega_6c_s^2 - \\
& 6\omega_7\omega_4^2v_2^2\omega_8\omega_5\omega_6 + 12\omega_4^2v_2^2\omega_8\omega_5\omega_9 - 18\omega_7\omega_4^2\omega_8^2\omega_6c_s^2 + 12\omega_7\omega_4^2\omega_8^2\omega_5\omega_9\omega_6 + 5\omega_7\omega_4^2\omega_8^2\omega_5\omega_9 + 12\omega_7\omega_4\omega_8\omega_5\omega_9\omega_6 + 12\omega_4\omega_8^2\omega_5\omega_9 - \\
& 12\omega_4v_2^2\omega_8\omega_5\omega_9 + 12\omega_7\omega_4v_2^2\omega_8\omega_5\omega_6 - 12\omega_4^2\omega_8^2\omega_6 - 5\omega_7\omega_4^2v_2^2\omega_8^2\omega_5\omega_9 + 36\omega_7\omega_8\omega_5\omega_9\omega_6c_s^2 + 3\omega_7\omega_4^2\omega_8\omega_5\omega_9\omega_6
\end{aligned}$$

$$\begin{aligned}
C_{10} = & -12\omega_7\omega_4^2\omega_5\omega_6 + 18\omega_7\omega_4^3v_2^2\omega_8\omega_5\omega_6 - 36\omega_7\omega_4v_2^2\omega_8\omega_5\omega_9 + 36\omega_7\omega_4^2v_2^2\omega_5\omega_6 - 6\omega_7\omega_4^3\omega_8\omega_5\omega_9 - 6\omega_7\omega_4^3\omega_5\omega_9\omega_6 + \\
& 12\omega_4^3\omega_8\omega_5\omega_6c_s^2 + 12\omega_4^2\omega_8\omega_5\omega_6c_s^2 - 36\omega_4^3v_2^2\omega_8\omega_5\omega_6 + 18\omega_7\omega_4\omega_8\omega_5\omega_9\omega_6c_s^2 - 18\omega_7\omega_4^2\omega_8\omega_5\omega_9 - 12\omega_4^3\omega_8\omega_9c_s^2 - 18\omega_7\omega_4^3v_2^2\omega_8\omega_6 + \\
& 54\omega_7\omega_4^2v_2^2\omega_8\omega_5\omega_9 - 5\omega_7\omega_4^3\omega_8\omega_5\omega_9c_s^2 - 12\omega_7\omega_4^2\omega_8\omega_5\omega_6c_s^2 + 12\omega_4^3\omega_8\omega_5\omega_6 + 5\omega_7\omega_4^2\omega_8\omega_5\omega_9 - 5\omega_7\omega_4^2\omega_8\omega_5\omega_9\omega_6c_s^2 + 12\omega_4^3\omega_8\omega_9 - 36\omega_4^3v_2^2\omega_8\omega_9 - \\
& 18\omega_7\omega_4^2\omega_5\omega_9\omega_6c_s^2 - 12\omega_4^2\omega_8\omega_5\omega_6 + 36\omega_7\omega_4v_2^2\omega_5\omega_9\omega_6 - 18\omega_7\omega_4^3v_2^2\omega_5\omega_6 - 54\omega_7\omega_4^2v_2^2\omega_5\omega_9\omega_6 + 18\omega_7\omega_4^2\omega_5\omega_9\omega_6 - 6\omega_7\omega_4^3\omega_8\omega_6c_s^2 + \\
& 12\omega_7\omega_4\omega_5\omega_9\omega_6c_s^2 + 6\omega_7\omega_4^3\omega_5\omega_9\omega_6c_s^2 + 6\omega_7\omega_4^3\omega_8\omega_9\omega_6c_s^2 + 36\omega_4^3v_2^2\omega_8\omega_6 - 12\omega_4^3\omega_8\omega_6 - 6\omega_7\omega_4^3\omega_8\omega_5\omega_6 + 12\omega_7\omega_4\omega_8\omega_5\omega_9 + 6\omega_7\omega_4^3\omega_8\omega_6 + \\
& 12\omega_7\omega_4^2\omega_5\omega_6c_s^2 + 18\omega_7\omega_4^3v_2^2\omega_5\omega_9\omega_6 + 12\omega_4^2\omega_8\omega_5\omega_9c_s^2 - 12\omega_7\omega_4^2\omega_8\omega_5\omega_9c_s^2 + 12\omega_7\omega_4^2\omega_8\omega_5\omega_6c_s^2 - 3\omega_7\omega_4^2v_2^2\omega_8\omega_5\omega_9\omega_6 + 36\omega_4^3v_2^2\omega_8\omega_5\omega_9 - \\
& 12\omega_7\omega_8\omega_5\omega_9\omega_6c_s^2 - 12\omega_7\omega_4\omega_8\omega_5\omega_9c_s^2 - 12\omega_4^3\omega_8\omega_5\omega_9 - 6\omega_7\omega_4^2\omega_5\omega_6c_s^2 - 36\omega_7\omega_4^2v_2^2\omega_8\omega_5\omega_6 + 18\omega_7\omega_4^3v_2^2\omega_8\omega_9 + 18\omega_7\omega_4^2\omega_8\omega_5\omega_9c_s^2 - \\
& 12\omega_7\omega_4\omega_5\omega_9\omega_6 + 6\omega_7\omega_4^3\omega_8\omega_8\omega_6 + 6\omega_7\omega_4^3\omega_8\omega_5\omega_6c_s^2 - 15\omega_7\omega_4^3v_2^2\omega_8\omega_5\omega_9 + 12\omega_4^3\omega_8\omega_6c_s^2 - \omega_7\omega_4^3\omega_8\omega_5\omega_9\omega_6c_s^2 + 36\omega_4^2v_2^2\omega_8\omega_5\omega_6 + \omega_7\omega_4^2\omega_8\omega_5\omega_9\omega_6
\end{aligned}$$

$$\begin{aligned}
C_{11} = & 30\omega_4^3v_2^2\omega_8 + 24\omega_4^2\omega_8c_s^2 + 72\omega_4^2v_2^4\omega_8 + 24\omega_4\omega_8c_s^4 + 36\omega_4^3v_4^2 + 12\omega_4\omega_8^2c_s^2 - 12\omega_4^2v_2^2\omega_8^2c_s^2 + 24\omega_4^2\omega_8^2c_s^4 + 24\omega_4^2c_4^4 + 72\omega_4^2\omega_8c_s^2 + \\
& 108\omega_4^3v_2^2c_s^2 + \omega_4^3\omega_8^2c_s^2 + 6\omega_4^3v_2^2\omega_8^2c_s^2 + 6\omega_4^3\omega_8c_s^4 - 12\omega_4^2v_2^4\omega_8^2 - 3\omega_4^3v_2^2\omega_8^2 - 72\omega_4^2v_2^4 - 3\omega_4^3\omega_8^2c_s^4 - 30\omega_4^3v_4^2\omega_8 - 36\omega_4v_2^2\omega_8^2c_s^2 + 72\omega_4^2v_2^2 - 6\omega_4^3\omega_8c_s^2 - \\
& 72\omega_4^2v_2^2\omega_8 - 72\omega_4^3v_2^2\omega_8c_s^2 + 12\omega_4^2v_2^2\omega_8^2 - 24\omega_4\omega_8c_s^2 + 3\omega_4^3v_2^4\omega_8^2 - 24\omega_4^2\omega_8c_s^4 + 144\omega_4^2v_2^2\omega_8c_s^2 - 216\omega_4^2v_2^2c_s^2 - 8\omega_4^2\omega_8^2c_s^2 - 36\omega_4^3v_2^2 - 48\omega_4\omega_8c_s^4
\end{aligned}$$

#### 2.5.4 Conservation of momentum: $\rho v_2$

attached text file: output\_d2q9\_nse\_clbm2\_symbolic\_pde\_02.txt

$$\begin{aligned}
v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{v_2 v_1 \delta_t}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho v_2 \delta_t}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\rho v_1 \delta_t}{\delta_t} \frac{\partial v_2}{\partial x_1} + (c_s^2 + v_2^2) \frac{\delta_t}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{2\rho v_2 \delta_t}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_4) \frac{\delta_t^2 c_s^2}{2\omega_4 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\
(-2 + \omega_4) \frac{\delta_t^2 c_s^2}{2\omega_4 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (-2 - 3v_2^2 \omega_6 + 4c_s^2 + 6v_2^2 + \omega_6 - 2\omega_6 c_s^2) \frac{\delta_t^2}{\omega_6 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (2 - \omega_6) \frac{3\rho v_2 \delta_t^2}{\omega_6 \delta_t} \left( \frac{\partial v_2}{\partial x_2} \right)^2 + \\
(-2 + \omega_4) \frac{\rho \delta_t^2 c_s^2}{2\omega_4 \delta_t} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_4) \frac{\rho \delta_t^2 c_s^2}{2\omega_4 \delta_t} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + (-2 - v_2^2 \omega_6 + 6c_s^2 + 2v_2^2 + \omega_6 - 3\omega_6 c_s^2) \frac{v_2 \delta_t^2}{2\omega_6 \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + \\
(-2 - 3v_2^2 \omega_6 + 2c_s^2 + 6v_2^2 + \omega_6 - \omega_6 c_s^2) \frac{\rho \delta_t^2}{2\omega_6 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + (-1 + 3c_s^2 + v_1^2) \frac{v_2 v_1 \delta_t^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3v_1^2) \frac{\rho v_2 \delta_t^3}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + \\
C_1 \frac{\rho v_1 \delta_t^3}{6\omega_7 \omega_4 \delta_t} \frac{\partial^3 v_2}{\partial x_1^3} + (-12 - \omega_4^2 + 12\omega_4) \frac{\delta_t^3 c_s^4}{6\omega_4^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} - \frac{\rho v_2 \delta_t^3 c_s^2}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + C_2 \frac{\rho v_2 \delta_t^3}{12\omega_4 \omega_5 \omega_6^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_3 \frac{\delta_t^3}{12\omega_6^2 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\
(-24 + 5\omega_6^2 c_s^2 - 60v_2^2 \omega_6 + 36c_s^2 + 60v_2^2 + 24\omega_6 - 4\omega_6^2 - 36\omega_6 c_s^2 + 11v_2^2 \omega_6^2) \frac{\rho v_2 \delta_t^3}{6\omega_6^2 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
(\omega_5 c_s^2 + 3\omega_5 v_1^2 + 6v_1^4 + 2c_s^4 - 2c_s^2 - 6v_1^2 - 12\omega_5 v_1^2 c_s^2 + 24v_1^2 c_s^2 - 3\omega_5 v_1^4 - \omega_5 c_s^4) \frac{v_2 \delta_t^4}{24\omega_5 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\
(-4 - 3\omega_5 c_s^2 - 5\omega_5 v_1^2 + 2\omega_5 + 6c_s^2 + 10v_1^2) \frac{\rho v_2 v_1 \delta_t^4}{12\omega_5 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_4 \frac{\rho \delta_t^4}{24\omega_7^2 \omega_4^3 \delta_t} \frac{\partial^4 v_2}{\partial x_1^4} + C_5 \frac{v_1 \delta_t^4 c_s^2}{12\omega_7^2 \omega_4^2 \omega_5 \omega_9 \omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + \\
C_6 \frac{\rho \delta_t^4 c_s^2}{12\omega_7 \omega_4^3 \omega_8 \omega_5 \omega_9 \omega_6 \delta_t} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + \\
(3\omega_7 \omega_5 c_s^2 + \omega_7 \omega_5 v_1^2 - \omega_7 - 9\omega_5 c_s^2 - 3\omega_5 v_1^2 + 3\omega_5 + 3\omega_7 c_s^2 + \omega_7 v_1^2 - \omega_7 \omega_5) \frac{\rho v_2 v_1 \delta_t^4}{12\omega_7 \omega_5 \delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
C_7 \frac{v_2 \delta_t^4 c_s^2}{12\omega_7 \omega_4^2 \omega_8^2 \omega_9 \omega_6^2 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_8 \frac{\rho \delta_t^4 c_s^2}{12\omega_7 \omega_4^3 \omega_8 \omega_9 \omega_6^2 \delta_t} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_9 \frac{\rho \delta_t^4}{12\omega_4^3 \omega_8^2 \omega_6^3 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{10} \frac{v_2 \delta_t^4}{12\omega_6^3 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{11} \frac{\rho \delta_t^4}{12\omega_6^3 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} \\
= 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 = & 6 - 3\omega_7 - 3\omega_4 - 3\omega_7 \omega_4 c_s^2 - \omega_7 \omega_4 v_1^2 + 9\omega_4 c_s^2 + 3\omega_4 v_1^2 + \omega_7 \omega_4 - 18c_s^2 - 6v_1^2 + 9\omega_7 c_s^2 + 3\omega_7 v_1^2 \\
C_2 = & -36\omega_6^2 c_s^2 - 12\omega_4 \omega_6^2 - 6\omega_8 \omega_6^2 - 18\omega_4 \omega_8 \omega_6 c_s^2 - 3\omega_4 v_2^2 \omega_8 \omega_6^2 - 12\omega_4 v_2^2 \omega_6 - 12\omega_4 \omega_8 + 6v_2^2 \omega_8 \omega_6^2 + 12\omega_4 v_2^2 \omega_6^2 - 6\omega_4 v_2^2 \omega_8 \omega_6 - 36\omega_4 \omega_6 c_s^2 + \\
& 12\omega_4 \omega_6 + 12\omega_4 v_2^2 \omega_8 + 18\omega_8 \omega_6^2 c_s^2 + 3\omega_4 \omega_8 \omega_6^2 + 36\omega_4 \omega_6^2 c_s^2 + 6\omega_4 \omega_8 \omega_6 + 12\omega_6^2 - 12v_2^2 \omega_6^2 - 11\omega_4 \omega_8 \omega_6^2 c_s^2 \\
C_3 = & -\omega_6^2 c_s^2 - 36v_2^4 \omega_6 - 12\omega_6 c_s^4 + 24v_2^2 \omega_6^2 c_s^2 + 7v_2^4 \omega_6^2 + 12c_s^4 + 36v_4^2 + 36v_2^2 \omega_6 - 12c_s^2 + 144v_2^2 \omega_6 c_s^2 - 36v_2^2 - 144v_2^2 \omega_6 c_s^2 + \omega_6^2 c_s^4 + 12\omega_6 c_s^2 - 7v_2^2 \omega_6^2
\end{aligned}$$

$$\begin{aligned}
C_4 = & 36w_7^3v_1^4 + 72w_7w_4v_2^2c_s^2 + 6w_7w_4^3c_s^4 - 3w_7w_3^3v_1^2 + 108w_4^3v_2^2c_s^2 - 30w_7w_4^3v_1^4 + w_7^2w_4^3c_s^2 - 72w_2^2v_1^4 - 24w_7w_4c_s^2 - 12w_7^2w_4^2v_2^2c_s^2 - 48w_7^2w_4c_s^4 - \\
& 72w_7w_4v_1^2c_s^2 + 72w_7w_2^4v_1^4 - 8w_7^2w_4^2c_s^2 - 24w_7w_2^4c_s^4 + 12w_7^2w_4^2c_s^2 + 12w_7^2w_4c_s^2 - 36w_7^2w_4v_1^2c_s^2 + 72w_2^4v_1^2 + 24w_7^2c_s^4 + 24w_7w_4c_s^4 - 216w_2^4v_1^2c_s^2 + \\
& 24w_7w_4^2c_s^2 - 12w_7^2w_4^2v_1^4 - 72w_7w_4^2v_1^2 + 24w_7^2w_4^2c_s^4 - 36w_3^3v_1^2 + 144w_7w_4^2v_1^2c_s^2 + 30w_7w_4^3v_1^2 - 3w_7^2w_4^3c_s^4 - 6w_7w_4^3c_s^2 + 3w_7^2w_4^3v_1^4 + 6w_7^2w_4^3v_1^2c_s^2
\end{aligned}$$

$$\begin{aligned}
C_5 = & 18w_7^2w_8w_5w_9w_6c_s^2 + 12w_7^2w_4w_5v_1^2w_6 - 5w_7^2w_4^2w_8v_1^2w_9w_6 + 36w_7w_4w_8w_5w_6c_s^2 + 36w_7^2w_4w_5w_6c_s^2 + 12w_7w_4w_8w_5v_1^2w_6 - \\
& 12w_4w_8w_5v_1^2w_9w_6 - 12w_7^2w_8v_1^2w_9w_6 - 12w_7w_2^2w_5 - 6w_7^2w_4^2w_8w_9 - 6w_7w_4^2w_8w_5v_1^2w_6 + 54w_7w_4w_8w_5w_9w_6c_s^2 - 3w_7w_4^2w_8w_5v_2^2w_9w_6 - \\
& 36w_7^2w_4w_9w_6c_s^2 + 18w_7^2w_4w_8v_1^2w_9w_6 - 18w_7w_7^2w_8w_5w_6c_s^2 - 9w_7w_4^2w_8w_5w_9w_6c_s^2 + 6w_7^2w_4^2w_8w_5v_1^2w_6 - 12w_7^2w_4v_1^2w_9w_6 + 12w_7^2w_4w_9w_6 + \\
& 54w_7^2w_4w_8w_9w_6c_s^2 + 18w_7w_4w_8w_5v_1^2w_9w_6 - 12w_7^2w_4w_5w_6 + 18w_7^2w_4w_8w_5w_6c_s^2 - 12w_7w_4w_8w_5w_6 - 36w_7w_4w_8w_5w_9w_6c_s^2 + 12w_7^2w_4w_8w_5w_6 + \\
& 12w_7^2w_8w_9w_6 - 36w_7^2w_4w_8w_5w_6c_s^2 - 36w_7^2w_8w_9w_6c_s^2 - 12w_7^2w_4w_8w_5v_1^2w_6 + 6w_7^2w_8w_5v_1^2w_9w_6 - 18w_7^2w_4w_8w_9w_6 - 15w_7^2w_4^2w_8w_9w_6c_s^2 + \\
& 36w_7^2w_4^2w_5c_s^2 - 36w_7^2w_4^2w_5w_6c_s^2 + 3w_7^2w_4^2w_8w_5w_9w_6c_s^2 - 12w_7^2w_8w_5w_9w_6 - 12w_7^2w_4^2w_9w_6 - 12w_7^2w_4^2w_5v_1^2w_6 - 36w_7^2w_4^2w_9c_s^2 + 12w_7^2w_4^2w_9 - \\
& w_7^2w_4^2w_8w_5w_6w_6 + 12w_7^2w_4^2w_5w_6 + 12w_7^2w_4^2w_5v_1^2 - 12w_7^2w_4w_8w_5v_1^2w_9w_6 + 36w_7^2w_8w_5w_9w_6c_s^2 + 12w_7^2w_4w_8w_5w_9w_6 + 6w_7^2w_4^2w_8w_5 + \\
& 36w_7^2w_4^2w_9w_6c_s^2 - 12w_7w_4w_8w_5v_1^2w_9w_6 + 12w_7w_4w_8w_5w_9w_6 - 12w_7^2w_4^2v_1^2w_9 + 6w_7w_4^2w_8w_5w_6 - 18w_7w_4w_8w_5w_9w_6 - 36w_7w_4w_8w_5w_9w_6c_s^2 + \\
& 6w_7^2w_4^2w_8v_1^2w_9 + 12w_7^2w_4^2v_1^2w_9w_6 + 12w_7^2w_4w_8w_5v_1^2w_9w_6 - 6w_7^2w_4w_8w_5w_9w_6 + 5w_7^2w_4^2w_8w_9w_6 - 36w_7^2w_4w_8w_5w_9w_6c_s^2 + 12w_4w_8w_5w_9w_6 - \\
& 18w_7^2w_4^2w_8w_5v_1^2w_9 - 6w_7^2w_4^2w_8w_5w_6 - 6w_7^2w_4^2w_8w_5v_1^2 + 3w_7w_4^2w_8w_5w_9w_6 + 18w_7^2w_4^2w_8w_9c_s^2 + w_7^2w_4^2w_8w_5v_1^2w_9w_6
\end{aligned}$$

$$\begin{aligned}
C_6 = & 12w_7w_4^3w_9 - 12w_7w_4^2w_5w_6 - 18w_4^2w_8w_5w_9w_6c_s^2 + 36w_4^2w_8w_5v_1^2w_6 - 6w_3^4w_8w_5w_9w_6 - 6w_7w_4^3w_8w_9 + 12w_7w_4^2w_9w_6 + 36w_4w_8w_5v_1^2w_9w_6 + 18w_7w_4^2w_8w_9w_6c_s^2 + 12w_4^2w_8w_5w_6c_s^2 - 36w_7w_4^2w_8w_5v_1^2w_6 + 18w_7w_4w_8w_5w_9w_6c_s^2 - 12w_7w_4w_8w_9w_6c_s^2 - 3w_7w_4^2w_8w_5v_1^2w_9w_6 - 12w_7w_4^2w_8w_5w_6c_s^2 + 6w_4^3w_8w_5w_6 - 5w_7w_4^2w_8w_9w_6c_s^2 + 12w_7w_4w_8w_9w_6 - 12w_7w_4w_8w_5w_6 - 36w_7w_4w_8v_1^2w_6 + 12w_4w_8w_5w_9w_6c_s^2 + 54w_7w_4^2w_8v_1^2w_9w_6 - 54w_4^2w_8w_5v_1^2w_9w_6 + 6w_7w_4^2w_8w_5 + 36w_7w_4^2w_9w_6 - 12w_7w_4^2w_8v_1^2w_9 - 36w_7w_4^2w_1^2w_9 - 12w_7w_4^2w_9w_6c_s^2 + 36w_7w_4^2w_5v_1^2w_6 + 6w_7w_4^2w_8w_9c_s^2 - 12w_7w_4^2w_9w_6 + 12w_7w_4^2w_5c_s^2 - 18w_7w_4^2w_8w_5v_1^2 - 6w_7w_4^2w_8w_5w_6 - 6w_7w_4^2w_8w_5w_6c_s^2 + 6w_4^3w_8w_5w_9w_6c_s^2 - 15w_7w_4^2w_8v_1^2w_9w_6 + 36w_7w_4^2w_5v_1^2 + 12w_7w_4^2w_5w_6 + 12w_7w_4^2w_5w_6c_s^2 - 12w_7w_4^2w_9w_6c_s^2 + 5w_7w_4^2w_8w_9w_6 + 12w_7w_4^2w_8w_5w_6 - 5w_7w_4^2w_8w_9w_6c_s^2 - 6w_3^4w_8w_5w_6c_s^2 + 18w_4^3w_8w_5v_1^2w_9w_6 - 36w_7w_4^2w_5v_1^2w_6 - 12w_7w_4w_8w_5w_9w_6c_s^2 - 18w_7w_4^2w_8w_9w_6 - 12w_7w_4^2w_5w_6c_s^2 - 18w_4^3w_8w_5v_1^2w_6 + 18w_4^2w_8w_5w_9w_6 + 6w_7w_4^2w_8w_5w_6c_s^2 - 36w_7w_4^2w_9w_6 - 12w_4w_8w_5w_9w_6 + 12w_7w_4^2w_9w_6c_s^2 - w_7w_4^2w_8w_5w_9w_6c_s^2 + 18w_7w_4^2w_8w_5v_1^2w_6 + w_7w_4^2w_8w_5w_9w_6
\end{aligned}$$

$$\begin{aligned}
C_7 = & 36w_7w_4^2w_8w_9w_6^2 - 12w_7w_4^2v_2^2w_8w_9w_6 - 12w_7w_4^2v_2^2w_8w_6^3 + 12w_7w_4^2w_8w_6^3 - 40w_7w_4w_2^2w_9w_6c_s^2 - 36w_7w_4^2w_8w_9w_6c_s^2 + \\
& 12w_7w_4^2w_8w_9w_6 + 12w_7w_2^2w_8w_9w_6^3 + 12w_7w_4w_2^2w_8w_6^3 - 36w_7w_8w_9w_6c_s^2 + 36w_7w_4w_9w_6c_s^2 - 12w_7w_4^2w_8w_8w_6^2 + 12w_7w_4^2v_2^2w_8w_8w_6^2 - 12w_7v_2^2w_8w_9w_6^2 + \\
& 36w_7w_8w_8w_6c_s^2 - 18w_7w_2w_8w_6c_s^2 + 5w_7w_4w_2^2w_8w_9w_6c_s^2 + 12w_7w_4^2v_2^2w_9w_6^3 - w_7w_2^2w_8w_9w_6^3 + 18w_4^2w_8w_9w_6c_s^2 + 18w_7w_2w_2^2w_8w_9w_6^2 + 12w_7w_2w_9w_6^2 - \\
& 12w_7w_2^2w_9w_6^3 + 12w_7w_4^2v_2^2w_8w_9w_6 + 54w_7w_4w_2^2w_8w_9w_6c_s^2 + 6w_2^2v_2^2w_8w_9w_6c_s^2 + w_7w_4^2w_8w_9w_6^3 - 12w_7w_4^2w_8w_9 + 18w_7w_4w_2^2w_8w_6c_s^2 + 12w_7w_2^2w_9w_6^2 - \\
& 12w_7w_8w_9w_6^3 - 12w_4^2v_2^2w_8w_9w_6^2 - 6w_7w_4^2w_8w_9w_6c_s^2 + 12w_4w_2^2w_8w_9w_6^2 + 36w_4w_8w_6c_s^2 - 12w_4w_2^2w_8w_6^3 - 12w_7w_4v_2^2w_8w_9w_6^3 - 12w_7w_4^2v_2^2w_9w_6^2 + \\
& 2w_7w_4^2w_8w_9w_6^2 - 6w_4^2v_2^2w_8w_6^3 - 2w_7w_4^2v_2^2w_8w_9w_6^2 - 36w_4w_2^2w_9w_6c_s^2 - 12w_7w_4v_2^2w_8w_6^3 - 36w_7w_4w_2^2w_9w_6c_s^2 + 36w_7w_2^2w_8w_9w_6c_s^2 - 36w_7w_2w_8w_9w_6c_s^2 + \\
& 12w_7w_4w_9w_6^3 + 6w_7w_2w_8w_9w_6^3 - 6w_7w_4^2v_2^2w_8w_9w_6^3 + 54w_7w_4w_2^2w_9w_6c_s^2 + 12w_7w_4v_2^2w_8w_9w_6^3 - 18w_7w_2w_8w_9w_6^2 + 6w_2^2w_8w_6^3 - 18w_7w_7w_2w_8w_9w_6c_s^2 + \\
& 18w_7w_4v_2^2w_8w_9w_6^3 - 36w_7w_2w_8w_9w_6c_s^2 + 12w_7w_8w_9w_6^3 + 18w_7w_4^2v_2^2w_8w_9w_6^2 - 36w_7w_4w_2^2w_9w_6c_s^2 + 6w_7w_2^2w_8w_6^2 - 12w_7w_4w_8w_6^3 - 12w_7w_4^2v_2^2w_8w_9w_6^2 - \\
& 6w_7w_4^2v_2^2w_8w_6^2 + 12w_7w_2^2w_8w_9w_6 - 12w_7w_4^2w_8w_9w_6^3 - 12w_7w_4v_2^2w_9w_6^3 + 12w_7w_4w_2^2w_8w_9w_6^3 + 6w_7w_4^2v_2^2w_8w_8w_6^3 - 36w_7w_8w_9w_6c_s^2 + 12w_4v_2^2w_8w_8w_6^3 - \\
& 6w_7w_4w_8w_6^3 - 18w_4^2w_8w_6c_s^2 - 18w_7w_4w_2^2w_8w_9w_6^2 + 36w_7w_4w_2^2w_8w_6c_s^2 - 36w_7w_4w_8w_6c_s^2 + 54w_7w_4^2w_8w_9w_6c_s^2 - 6w_4^2w_8w_9w_6^2
\end{aligned}$$

$$\begin{aligned}
& \text{C}_8 = 36w_7^2w_4^3v_2^2w_9w_6^2 + 36w_7w_4^3v_2^2w_6 - 6w_3^4w_8w_6^2c_s^2 - 6w_7w_3^4w_8w_9 - 18w_7w_3^3v_2^2w_8w_9w_6 - 12w_7w_2^4w_9w_6 + 12w_7w_4^3w_9w_6^2c_s^2 + 24w_7w_2^4w_8w_9w_6c_s^2 + \\
& 36w_7w_2^3v_2^2w_9w_6 - w_7w_3^3w_8w_9w_6c_s^2 + 12w_7w_4^2w_8w_6^2 - 36w_7w_4^2v_2^2w_8w_6^2 - 12w_7w_4w_8w_9w_6c_s^2 - 6w_4^3w_8w_9w_6 + 18w_7w_4^2w_8w_9w_6 + 12w_7w_4^2w_6c_s^2 - \\
& 18w_7w_3^2v_2^2w_8w_6 - 12w_3^2w_8w_9w_6c_s^2 + 12w_4^2w_8w_9w_6 + 18w_7w_4^3v_2^2w_8w_6^2 + 12w_7w_4w_8w_9w_6 + 24w_7w_4^2w_9w_6^2 + 12w_7w_4^3w_6c_s^2 + 12w_4^2w_8w_6^2c_s^2 - \\
& 72w_7w_4^2v_2^2w_9w_6^2 - 12w_3^2w_8w_9w_6^2 - 36w_7w_4^3v_2^2w_9w_6 - 36w_7w_4^3v_2^2w_6^2 - 24w_7w_4^2w_9w_6^2c_s^2 - 6w_7w_4^3w_8w_6c_s^2 - 12w_7w_4^3v_2^2c_s^2 - 6w_7w_4^3w_8w_6^2 + \\
& 6w_7w_3^2w_8w_6^2c_s^2 + 12w_7w_4^2w_6^2 + 12w_7w_4^2w_9w_6c_s^2 + 6w_7w_4^3w_8w_9w_6c_s^2 + 12w_7w_4^2w_9w_6^2 - 12w_7w_4w_8w_9w_6^2 - 36w_7w_4^2v_2^2w_8w_9 - 12w_7w_8w_9w_6^2c_s^2 + \\
& 12w_7w_2^2w_8w_9 + 36w_4^2v_2^2w_8w_6^2 + 6w_7w_4^3w_8w_9w_6 + 36w_7w_4^2v_2^2w_6^2 - 6w_7w_3^2w_8w_9w_6c_s^2 - 12w_7w_4^2w_6^2 + 72w_7w_4^2v_2^2w_8w_9w_6 + 18w_7w_4w_8w_9w_6^2c_s^2 - \\
& 12w_7w_4^2w_8w_9w_6^2 - 24w_7w_4^2w_8w_9w_6 - 18w_4^3v_2^2w_8w_6^2 - 12w_7w_4^3w_9w_6^2 + 6w_4^3w_8w_6^2 + 18w_7w_4^3v_2^2w_8w_9 + 6w_4^3w_8w_9w_6c_s^2 + 12w_7w_4w_9w_6^2c_s^2 + \\
& 6w_7w_3^2w_8w_6 - 36w_4^2v_2^2w_8w_9w_6 - 12w_7w_4^3w_9w_6c_s^2 + 36w_7w_4v_2^2w_9w_6^2 - 12w_7w_4^2w_8w_6^2c_s^2 - 36w_7w_4v_2^2w_8w_9w_6 - 4w_7w_4^2w_8w_9w_6^2c_s^2
\end{aligned}$$

$$\begin{aligned}
C_9 = & 18w^2v^2w^2w^2s^2c_s^2 - 90w^3v^4w^2w^2w_6 - 36w^4v^2w^2w_6^3 + 6w^3v^2w^2w_6^2c_s^2 - 36w^3v^2w^2w_6^3 + 198w^3v^2w^2w^2s^2c_s^2 + w^3v^2w^2w_6^2c_s^4 - 4w^3v^2w^2w^2w_6^3 + 13w^2v^2w^2w_6^3c_s^4 + 12w^3v^2w^2w_6^2c_s^2 - \\
& 19w^3v^2w^2w^2w_6^2 - 6w^2v^4w^2w^2w_6^3 + 18w^2v^2w^2w_6^3c_s^2 + 36w^3v^2w^2w_6^2 - 306w^3v^2w^2w^2w_6^2s^2 + 90w^3v^2w^2w^2w_6^3c_s^2 - w^3v^2w^2w^2w_6^3c_s^4 - 99w^3v^2w^2w^2w_6^3c_s^2 - \\
& 6w^3v^2w^2w_6^3c_s^2 - 12w^3v^2w^2w_6^2c_s^2 + 4w^3v^2w^2w_6^2w^2s^2 - 3w^4v^2w^2w^2w_6^3s^2 + 252w^3v^2w^2w^2s^2 - 12w^4v^2w^2w^2w_6^2s^2 + 6w^4v^2w^2w^2w_6^3c_s^2 - 18w^4v^2w^2w^2w_6^3c_s^2 + 6w^4v^2w^2w^2w_6^3 - 72w^3v^2w^2w_6^2 + \\
& 12w^4w^2w^2w^2s^2c_s^4 + 19w^3v^4w^2w^2w_6^2 + 6w^2v^2w^2w^2w_6^2c_s^4 + 6w^3v^2w^2w^2w_6^3c_s^2 + 12w^3v^2w^2w^2w_6^3c_s^2 + 12w^3v^2w^2w^2w_6^3c_s^4 + 72w^3v^4w^2w^2w_6^3 + 54w^2v^2w^2w^2w_6^3c_s^2 - 36w^2v^2w^2w^2w_6^3 - 36w^3v^4w^2w^2w_6^3 - \\
& 12w^2v^2w^2w_6^3c_s^4 + 108w^3v^2w^2w_6^3c_s^2 - 12w^4w^2w^2w_6^3c_s^2 + 36w^3v^2w^2w^2w_6^3 + 36w^4v^2w^2w^2w_6^3c_s^2 - 6w^2v^2w^2w^2w_6^3c_s^2 + 36w^3v^2w^2w^2w_6^3c_s^4 + 12w^4v^2w^2w^2w_6^3c_s^2 - 24w^4w^2w^2w_6^3c_s^4 - \\
& 39w^3v^2w^2w_6^3 - 108w^3v^2w^2w_6^2c_s^2 - w^3v^2w^2w_6^2s^2 + 36w^4v^2w^2w^2w_6^2c_s^2 + 72w^3v^2w^2w^2w_6^2s^2 + 36w^4v^2w^2w^2w_6^3 - 72w^3v^2w^2w^2w_6^3s^2 + 60w^3v^2w^2w^2w_6^2s^2 - 6w^4v^2w^2w^2w_6^3c_s^2 + \\
& 12w^2v^2w^2w_6^3c_s^4 - 36w^3v^2w^2w^2w_6^2 - 36w^4v^2w^2w^2w_6^2c_s^2 - 18w^4v^2w^2w^2w_6^3c_s^4 - 12w^3v^2w^2w^2w_6^3c_s^4 - 5w^2v^2w^2w^2w_6^3s^2 - 108w^4v^2w^2w^2w_6^3c_s^2 - 108w^3v^2w^2w^2w_6^2s^2 + 39w^3v^2w^2w^2w_6^3
\end{aligned}$$

$$\begin{aligned} C_{10} = & 12 - 78\omega_6^2 c_s^2 - 34v_2^2 \omega_6^3 c_s^2 - 216v_4^2 \omega_6 - 216\omega_6 c_s^4 + 404v_2^2 \omega_6^2 c_s^2 + 90v_4^2 \omega_6^2 + 6\omega_6^3 c_s^2 + 144c_s^4 - 9v_4^2 \omega_6^3 + 144v_4^2 + 234v_2^2 \omega_6 - 5\omega_6^3 c_s^4 - \\ & 132c_s^2 + 672v_2^2 c_s^2 - 156v_6 - 1008v_2^2 \omega_6 c_s^2 - 18\omega_6 - \omega_6^3 + 82\omega_6^2 c_s^4 + 10v_2^2 \omega_6^3 + 8\omega_6^2 + 198\omega_6 c_s^2 - 98v_2^2 \omega_6^2 \end{aligned}$$

$$C_{11} = 12 - 22w_6^2c_s^2 - 18v_2^2w_6^3c_s^2 - 756v_2^4w_6 - 36w_6c_s^4 + 252v_2^2w_6^2c_s^2 + 310v_4^3w_6^2 + 2w_6^3c_s^2 + 24c_s^4 - 29v_2^4\omega_6^3 + 504v_2^4 + 378v_2^2w_6 - \omega_6^3c_s^4 - 36c_s^2 + 432v_2^2c_s^2 - 252v_2^2 - 648v_2^2w_6c_s^2 - 18w_6 - \omega_6^3 + 14w_6^2c_s^4 + 14v_2^2w_6^3 + 8w_6^2 + 54w_6c_s^2 - 154v_2^2w_6$$

## 2.6 CuLBM1

### 2.6.1 Definitions

Based on [2], collision operator  $\mathbf{C}$ :

$$\mathbf{C}(\mathbf{f}) = \mathbf{M}^{-1} \mathbf{G}^{-1} \left( \mathbf{S} \left( \boldsymbol{\gamma}^{(eq)} - \mathbf{G}(\mathbf{M}\mathbf{f}) \right) \right),$$

where

$$\mathbf{S} = \text{diag}(0, 0, 0, \omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6),$$

$$\omega_1, \omega_2, \dots, \omega_6 \in (0, 2).$$

The nonlinear operator  $\mathbf{G}$  (with its inverse  $\mathbf{G}^{-1}$ ) transforms the raw moment vector  $\boldsymbol{\mu}$  defined by matrix  $\mathbf{M}$  to the cumulant vector

$$\boldsymbol{\gamma} = \mathbf{G}(\boldsymbol{\mu}) = \left( \gamma_{(0,0)}, \gamma_{(1,0)}, \gamma_{(0,1)}, \gamma_{(2,0)}, \gamma_{(0,2)}, \gamma_{(1,1)}, \gamma_{(2,1)}, \gamma_{(1,2)}, \gamma_{(2,2)} \right)^T$$

as

$$\begin{aligned} \gamma_{(0,0)} &= m_{(0,0)}, \\ \gamma_{(1,0)} &= \frac{m_{(1,0)}}{m_{(0,0)}}, \\ \gamma_{(0,1)} &= \frac{m_{(0,1)}}{m_{(0,0)}}, \\ \gamma_{(2,0)} &= -\frac{m_{(1,0)}^2}{m_{(0,0)}^2} + \frac{m_{(2,0)}}{m_{(0,0)}}, \\ \gamma_{(0,2)} &= -\frac{m_{(0,1)}^2}{m_{(0,0)}^2} + \frac{m_{(0,2)}}{m_{(0,0)}}, \\ \gamma_{(1,1)} &= -\frac{m_{(1,0)}m_{(0,1)}}{m_{(0,0)}^2} + \frac{m_{(1,1)}}{m_{(0,0)}}, \\ \gamma_{(2,1)} &= \frac{m_{(2,1)}}{m_{(0,0)}} - \frac{m_{(0,1)}m_{(2,0)}}{m_{(0,0)}^2} - 2\frac{m_{(1,0)}m_{(1,1)}}{m_{(0,0)}^2} + 2\frac{m_{(1,0)}^2m_{(0,1)}}{m_{(0,0)}^3}, \\ \gamma_{(1,2)} &= \frac{m_{(1,2)}}{m_{(0,0)}} - \frac{m_{(1,0)}m_{(0,2)}}{m_{(0,0)}^2} - 2\frac{m_{(0,1)}m_{(1,1)}}{m_{(0,0)}^2} + 2\frac{m_{(0,1)}^2m_{(1,0)}}{m_{(0,0)}^3}, \\ \gamma_{(2,2)} &= -6\frac{m_{(1,0)}^2m_{(0,1)}}{m_{(0,0)}^4} + 2\frac{m_{(0,1)}^2m_{(2,0)} + m_{(1,0)}m_{(0,2)}}{m_{(0,0)}^2} + 8\frac{m_{(1,0)}m_{(0,1)}m_{(1,1)}}{m_{(0,0)}^3} - 2\frac{m_{(1,0)}m_{(1,2)} + m_{(0,1)}m_{(2,1)}}{m_{(0,0)}^2} + \frac{m_{(2,2)}}{m_{(0,0)}} - \\ &\quad \frac{m_{(2,0)}m_{(0,2)} + 2m_{(1,1)}^2}{m_{(0,0)}^2}. \end{aligned}$$

The equilibrium cumulant vector  $\boldsymbol{\gamma}^{(eq)}$  is defined by

$$\boldsymbol{\gamma}^{(eq)} = \left( \rho, v_1, v_2, c_s^2, c_s^2, 0, 0, 0, 0 \right)^T.$$

### 2.6.2 Conservation of mass: $\rho$

 attached text file: `output_d2q9_nse_culbm1_symbolic_pde_00.txt`

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\delta_l v_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\delta_l \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\delta_l \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + 3c_s^2 + v_1^2) \frac{\delta_l^3 v_1}{12 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3v_1^2) \frac{\delta_l^3 \rho}{12 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\ \frac{\delta_l^3 \rho c_s^2}{6 \delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{\delta_l^3 \rho c_s^2}{6 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{v_2 \delta_l^3}{12 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + c_s^2 + 3v_2^2) \frac{\delta_l^3 \rho}{12 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\ (-2c_s^2 - \omega_1 c_s^4 - 3\omega_1 v_1^4 - 6v_1^2 + 3\omega_1 v_1^2 + 6v_1^4 - 12\omega_1 v_1^2 c_s^2 + 24v_1^2 c_s^2 + 2c_s^4 + \omega_1 c_s^2) \frac{\delta_l^4}{24 \omega_1 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\ (-4 + 6c_s^2 + 2\omega_1 + 10v_1^2 - 5\omega_1 v_1^2 - 3\omega_1 c_s^2) \frac{\delta_l^4 v_1 \rho}{12 \omega_1 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + \\ (3\omega_1 + 3\omega_1 c_s^2 \omega_4 + 3c_s^2 \omega_4 - 3\omega_1 v_1^2 + \omega_1 v_1^2 \omega_4 + v_1^2 \omega_4 - \omega_1 \omega_4 - 9\omega_1 c_s^2 - \omega_4) \frac{\delta_l^4 v_1 \rho}{12 \omega_1 \delta_t \omega_4} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + (-2 + \omega_3) \frac{\delta_l^4 c_s^4}{6 \omega_3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} \end{aligned}$$

$$\begin{aligned}
& + (3\omega_2\omega_6c_s^2 + v_2^2\omega_6 + 3\omega_2 - \omega_2\omega_6 - \omega_6 + 3\omega_6c_s^2 - 9\omega_2c_s^2 + \omega_2v_2^2\omega_6 - 3\omega_2v_2^2) \frac{v_2\delta_l^4\rho}{12\omega_2\omega_6\delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& (-12\omega_2v_2^2c_s^2 - 2c_s^2 - 3\omega_2v_2^4 - 6v_2^2 - \omega_2c_s^4 + 6v_2^4 + 24v_2^2c_s^2 + \omega_2c_s^2 + 2c_s^4 + 3\omega_2v_2^2) \frac{\delta_l^4}{24\omega_2\delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\
& (-4 + 6c_s^2 + 2\omega_2 + 10v_2^2 - 3\omega_2c_s^2 - 5\omega_2v_2^2) \frac{v_2\delta_l^4\rho}{12\omega_2\delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0.
\end{aligned}$$

### 2.6.3 Conservation of momentum: $\rho v_1$

attached text file: output\_d2q9\_nse\_culbm1\_symbolic\_pde\_01.txt

$$\begin{aligned}
& v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (c_s^2 + v_1^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2\delta_l v_1 \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 \delta_l v_1}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{v_2 \delta_l \rho}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{\delta_l v_1 \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\
& (-2 + 4c_s^2 + \omega_1 + 6v_1^2 - 3\omega_1v_1^2 - 2\omega_1c_s^2) \frac{\delta_l^2}{\omega_1\delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + (2 - \omega_1) \frac{3\delta_l^2 v_1 \rho}{\omega_1\delta_t} \left( \frac{\partial v_1}{\partial x_1} \right)^2 + (-2 + \omega_3) \frac{\delta_l^2 c_s^2}{2\omega_3\delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\
& (-2 + \omega_3) \frac{\delta_l^2 c_s^2}{2\omega_3\delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (-2 + 6c_s^2 + \omega_1 + 2v_1^2 - \omega_1v_1^2 - 3\omega_1c_s^2) \frac{\delta_l^2 v_1}{2\omega_1\delta_t} \frac{\partial^2 \rho}{\partial x_1^2} + \\
& (-2 + 2c_s^2 + \omega_1 + 6v_1^2 - 3\omega_1v_1^2 - \omega_1c_s^2) \frac{\delta_l^2 \rho}{2\omega_1\delta_t} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega_3) \frac{\delta_l^2 \rho c_s^2}{2\omega_3\delta_t} \frac{\partial^2 v_2}{\partial x_1^2} + (-2 + \omega_3) \frac{\delta_l^2 \rho c_s^2}{2\omega_3\delta_t} \frac{\partial^2 v_1}{\partial x_2^2} + C_1 \frac{\delta_l^3}{12\omega_1^2\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + \\
& + (-24 + 36c_s^2 + 24\omega_1 + 5\omega_2c_s^2 + 11\omega_2^2v_1^2 + 60v_1^2 - 60\omega_1v_1^2 - 4\omega_1^2 - 36\omega_1c_s^2) \frac{\delta_l^3 v_1 \rho}{6\omega_1^2\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + C_2 \frac{\delta_l^3 v_1 \rho}{12\omega_1^2\omega_3\delta_t\omega_4} \frac{\partial^3 v_2}{\partial x_1^2\partial x_2} + \\
& (-12 + 12\omega_3 - \omega_3^2) \frac{\delta_l^3 c_s^4}{6\omega_2^2\delta_t} \frac{\partial^3 \rho}{\partial x_1\partial x_2^2} - \frac{\delta_l^3 v_1 \rho c_s^2}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1\partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{v_2 \delta_l^3 v_1}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + C_3 \frac{v_2 \delta_l^3 \rho}{6\omega_6\omega_3\delta_t} \frac{\partial^3 v_1}{\partial x_2^2} + \\
& (-1 + c_s^2 + 3v_2^2) \frac{\delta_l^3 v_1 \rho}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_4 \frac{\delta_l^4 v_1}{12\omega_1^3\delta_t} \frac{\partial^4 \rho}{\partial x_1^2} + C_5 \frac{\delta_l^4 \rho}{12\omega_1^3\delta_t} \frac{\partial^4 v_1}{\partial x_1^2} + C_6 \frac{\delta_l^4 \rho}{12\omega_1^3\omega_3^2\delta_t\omega_4^2} \frac{\partial^4 v_2}{\partial x_1^3\partial x_2} + C_7 \frac{\delta_l^4 v_1 c_s^2}{12\omega_1^3\omega_3^2\delta_t\omega_4^2} \frac{\partial^4 \rho}{\partial x_1^2\partial x_2^2} + \\
& C_8 \frac{\delta_l^4 \rho c_s^2}{12\omega_1^3\omega_3^2\delta_t\omega_4} \frac{\partial^4 v_1}{\partial x_1^2\partial x_2^2} + C_9 \frac{v_2 \delta_l^4 c_s^2}{12\omega_2\omega_6^2\omega_3^2\delta_t} \frac{\partial^4 \rho}{\partial x_1\partial x_2^3} + \\
& (3\omega_2\omega_6c_s^2 + v_2^2\omega_6 + 3\omega_2 - \omega_2\omega_6 - \omega_6 + 3\omega_6c_s^2 - 9\omega_2c_s^2 + \omega_2v_2^2\omega_6 - 3\omega_2v_2^2) \frac{v_2\delta_l^4 v_1 \rho}{12\omega_2\omega_6\delta_t} \frac{\partial^4 v_1}{\partial x_1\partial x_2^3} + C_{10} \frac{\delta_l^4 \rho c_s^2}{12\omega_2\omega_6\omega_3^2\delta_t} \frac{\partial^4 v_2}{\partial x_1\partial x_2^3} + \\
& + (-12\omega_2v_2^2c_s^2 - 2c_s^2 - 3\omega_2v_2^4 - 6v_2^2 - \omega_2c_s^4 + 6v_2^4 + 24v_2^2c_s^2 + \omega_2c_s^2 + 2c_s^4 + 3\omega_2v_2^2) \frac{\delta_l^4 v_1}{24\omega_2\delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{11} \frac{\delta_l^4 \rho}{24\omega_6^2\omega_3^2\delta_t} \frac{\partial^4 v_1}{\partial x_2^4} + \\
& + (-4 + 6c_s^2 + 2\omega_2 + 10v_2^2 - 3\omega_2c_s^2 - 5\omega_2v_2^2) \frac{v_2\delta_l^4 v_1 \rho}{12\omega_2\delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= -12c_s^2 + 24\omega_1^2v_1^2c_s^2 - \omega_1^2c_s^2 - 12\omega_1c_s^4 - 36\omega_1v_1^4 - 7\omega_1^2v_1^2 - 36v_1^2 + 36\omega_1v_1^2 + 36v_1^4 + 7\omega_1^2v_1^4 - 144\omega_1v_1^2c_s^2 + \omega_1^2c_s^4 + 144v_1^2c_s^2 + 12c_s^4 + 12\omega_1c_s^2 \\
C_2 &= 6\omega_1^2v_1^2\omega_4 - 6\omega_1v_1^2\omega_3\omega_4 - 6\omega_1^2\omega_4 + 36\omega_3c_s^2\omega_4 - 36\omega_1^2c_s^2 + 12v_1^2\omega_3\omega_4 - 18\omega_1\omega_3c_s^2\omega_4 + 12\omega_1\omega_3 - 12\omega_1v_1^2\omega_3 - 12\omega_3\omega_4 - 12\omega_1^2v_1^2 - \\
36\omega_1\omega_3c_s^2 &+ 6\omega_1\omega_3\omega_4 - 3\omega_1^2v_1^2\omega_3\omega_4 + 12\omega_1^2 + 12\omega_1^2v_1^2\omega_3\omega_4 - 11\omega_1^2\omega_3c_s^2\omega_4 + 3\omega_1^2\omega_3\omega_4 - 12\omega_1^2\omega_3 + 36\omega_1^2\omega_3c_s^2 + 18\omega_1^2c_s^2\omega_4 \\
C_3 &= 6 + \omega_6\omega_3 - 18c_s^2 + 3v_2^2\omega_3 + 3v_2^2\omega_6 - 6v_2^2 + 9\omega_3c_s^2 - 3\omega_3 - 3\omega_6\omega_3c_s^2 - v_2^2\omega_6\omega_3 \\
C_4 &= 12 - 132c_s^2 - 18\omega_1 + 404\omega_1^2v_1^2c_s^2 - 78\omega_1^2c_s^2 - 216\omega_1c_s^4 + 10\omega_1^3v_1^2 + 6\omega_1^3c_s^2 - 216\omega_1v_1^4 - 98\omega_2^2v_1^2 - 156v_1^2 + 234\omega_1v_1^2 - 5\omega_1^3c_s^4 + 8\omega_1^2 + 144v_1^4 - \omega_1^3 - 34\omega_1^3v_1^2c_s^2 + 90\omega_1^2v_1^4 - 1008\omega_1v_1^2c_s^2 + 82\omega_1^2c_s^4 + 672v_1^2c_s^2 + 144c_s^4 - 9\omega_1^3v_1^4 + 198\omega_1c_s^2 \\
C_5 &= 12 - 36c_s^2 - 18\omega_1 + 252\omega_1^2v_1^2c_s^2 - 22\omega_1^2c_s^2 - 36\omega_1c_s^4 + 14\omega_1^3v_1^2 + 2\omega_1^3c_s^2 - 756\omega_1v_1^4 - 154\omega_1^2v_1^2 - 252v_1^2 + 378\omega_1v_1^2 - \omega_1^3c_s^4 + 8\omega_1^2 + 504v_1^4 - \omega_1^3 - 18\omega_1^2v_1^2c_s^2 + 310\omega_1^2v_1^4 - 648\omega_1v_1^2c_s^2 + 14\omega_1^2c_s^4 + 432v_1^2c_s^2 + 24c_s^4 - 29\omega_1^3v_1^4 + 54\omega_1c_s^2 \\
C_6 &= \omega_1^2\omega_3^3c_s^4\omega_4^2 - 39\omega_1^3v_1^4\omega_3^2\omega_4 + 18\omega_1^3\omega_3^2c_s^2\omega_4 - 12\omega_3^2c_s^2\omega_4^2 - 108\omega_1^2v_1^2\omega_3^2c_s^2 - 24\omega_1^3\omega_3c_s^4\omega_4^2 - 6\omega_1^2\omega_3^2c_s^2\omega_4^2 - 36\omega_1^3v_1^2\omega_3^2\omega_4 + 19\omega_1^2v_1^4\omega_3^2\omega_4^2 + 6\omega_1^3\omega_3^3c_s^4\omega_4 + 36\omega_1^2v_1^2\omega_3^2c_s^2\omega_4 - 108\omega_1^2v_1^2\omega_3^3c_s^2 - 72\omega_1^2v_1^2\omega_3^2\omega_4 - 12\omega_1\omega_3c_s^4\omega_4^2 - 6\omega_1^3v_1^4\omega_3^2\omega_4 + 36\omega_1^2v_1^2\omega_3^2c_s^2\omega_4 - 4\omega_1^3v_1^2\omega_3^2\omega_4^2 + 12\omega_1\omega_3^3c_s^2\omega_4^2 + 12\omega_1^2\omega_3^2c_s^3c_s^2\omega_4^2 + 39\omega_1^2v_1^2\omega_3^2\omega_4 - 18\omega_1^2v_1^2\omega_3c_s^2\omega_4 - 99\omega_1^3v_1^2\omega_3^2c_s^2\omega_4 + 108\omega_1^2v_1^2\omega_3^2c_s^2 - 36\omega_1^2v_1^2\omega_3^2\omega_4 + 18\omega_1^2v_1^2\omega_3^2c_s^2\omega_4 - 36\omega_1^2v_1^2\omega_3^2c_s^2\omega_4 - 36\omega_1^2v_1^2\omega_3^2\omega_4 - 19\omega_1^2v_1^2\omega_3^2c_s^2\omega_4 - 36\omega_1^2v_1^2\omega_3^2\omega_4 - 36\omega_1^2v_1^2\omega_3^2c_s^2\omega_4 - 36\omega_1^2v_1^2\omega_3^2\omega_4 - 12\omega_1^2v_1^2\omega_3^2c_s^2\omega_4 + 12\omega_1^2v_1^2\omega_3^2c_s^2\omega_4 + 6\omega_1^2v_1^2\omega_3^2\omega_4^2 - 6\omega_1^2v_1^2\omega_3^2c_s^2\omega_4 + 4\omega_1^2v_1^2\omega_3^2\omega_4 - 5\omega_1^2v_1^2\omega_3^2c_s^2\omega_4 + 252v_1^2\omega_3^2c_s^2\omega_4 - 90\omega_1^2v_1^2\omega_3^2\omega_4 + 12\omega_1^2c_s^4\omega_4 - 3\omega_1^2v_1^2\omega_3^2\omega_4^2 + 12\omega_1\omega_3^2\omega_4^2 + 198\omega_1^2v_1^2\omega_3^2c_s^2\omega_4 - 12\omega_1\omega_3^3c_s^4\omega_4^2 - \omega_1^2\omega_3^2c_s^2\omega_4^2 - 108\omega_1v_1^2\omega_3^2\omega_4^2 - 18\omega_1^2\omega_3^2\omega_4^2 + 72v_1^2\omega_3^2\omega_4^2 + 12\omega_3^2\omega_4^2 + 36\omega_1v_1^2\omega_3^2\omega_4 + 6\omega_1^2\omega_3^2c_s^2\omega_4^2 + 6\omega_1^2\omega_3c_s^2\omega_4^2 + 90\omega_1v_1^2\omega_3^2\omega_4^2 - 306\omega_1v_1^2\omega_3^2c_s^2\omega_4 + 6\omega_1^2\omega_3^2c_s^2\omega_4^2 + 13\omega_1\omega_3^2c_s^2\omega_4^2 + 60\omega_1^2v_1^2\omega_3^2c_s^2\omega_4^2 + 36\omega_1^2v_1^2\omega_3^2\omega_4^2 - 72v_1^2\omega_3^2\omega_4^2 - 36\omega_1v_1^2\omega_3^2\omega_4 + 36\omega_1^2v_1^2\omega_3^2\omega_4 + 54\omega_1^2v_1^2\omega_3^2c_s^2\omega_4 \\
C_7 &= 4\omega_1^2v_1^2\omega_3^2\omega_4^2 - 18\omega_1^3\omega_3^2c_s^2\omega_4 - 18\omega_1^3\omega_3^2c_s^2\omega_4 - 36\omega_1^3\omega_3c_s^2\omega_4 - 12\omega_1^2v_1^2\omega_3^2c_s^2\omega_4 - 6\omega_1^3v_1^2\omega_3^2\omega_4 + 36\omega_1^3c_s^2\omega_4^2 - 4\omega_1^2\omega_3^2\omega_4^2 - 12\omega_1^3v_1^2\omega_3^2\omega_4^2 + 12\omega_1^3\omega_3^2c_s^2\omega_4^2 - 36\omega_1^2\omega_3^2\omega_4^2 - 6\omega_1^2\omega_3^2c_s^2\omega_4^2 + 12\omega_1^2v_1^2\omega_3^2\omega_4^2 + 18\omega_1^2\omega_3^2c_s^2\omega_4^2 + 6\omega_1^2\omega_3^2\omega_4^2 + 6\omega_1^2\omega_3^2c_s^2\omega_4^2 - 12\omega_1^2\omega_3^2\omega_4^2 - 12\omega_1^2v_1^2\omega_3^2\omega_4^2 + 12\omega_1^2\omega_3^2c_s^2\omega_4^2 - 40\omega_1^2\omega_3^2c_s^2\omega_4^2 + 12\omega_1^2v_1^2\omega_3^2\omega_4^2 - 6\omega_1^2\omega_3^2\omega_4^2 + 54\omega_1^2\omega_3^2c_s^2\omega_4^2 + 12\omega_1^2v_1^2\omega_3^2\omega_4^2 + 18\omega_1^2\omega_3^2c_s^2\omega_4^2 + 6\omega_1^2\omega_3^2\omega_4^2 + 6\omega_1^2\omega_3^2c_s^2\omega_4^2 + 36\omega_1^2v_1^2\omega_3^2\omega_4^2 + 18\omega_1^2\omega_3^2c_s^2\omega_4^2 + 18\omega_1^2\omega_3^2\omega_4^2 - 12\omega_1^2\omega_3^2\omega_4^2
\end{aligned}$$

$$\begin{aligned} \text{C}_8 = & 12w_1^2w_3^3c_s^2 - 36w_1v_1^2w_3w_4 + 24w_1w_3^3 + 72w_1v_1^2w_3^2 - 24w_1w_3^2 + 36v_1^2w_3^3 - 12w_1w_3c_s^2w_4 - 72w_1v_1^2w_3^3 - 24w_1^2w_3^2c_s^2 - 4w_1^2w_3^2c_s^2w_4 + \\ & 12w_1w_3w_4 + 12w_3^2w_4 + 24w_1w_3^2c_s^2 - 12w_1w_3^2w_4 + 36w_1^2v_1^2w_3 + 18w_1^2w_3c_s^2w_4 - 12w_1^2w_3 + 12w_1^2w_3c_s^2 - 36v_1^2w_3^2w_4 + 24w_1^2w_3^2 - 24w_1w_3^3c_s^2 - \\ & w_1^2w_3^3c_s^2w_4 - 12w_3^2c_s^2w_4 - 72w_1^2v_1^2w_3^2 + 12w_3^2c_s^2 - 12w_1^2c_s^2w_4 - 12w_1^2w_3^3 + 36w_1v_1^2w_3^2w_4 + 36w_1^2v_1^2w_3^3 + 12w_1w_3c_s^2w_4 - 12w_3^2 \end{aligned}$$

$$\begin{aligned}
C_9 = & -12w_2w_6^2 - 12w_6w_3 - 12w_2v_2^2w_6^2w_3 - w_2w_6^2c_s^2 - 9w_2w_6w_3^2c_s^2 + 12w_2w_3 + 18w_2w_3^2c_s^2 + 18w_6^2w_3c_s^2 - 12v_2^2w_6^2 - 36w_2w_6^2w_3c_s^2 - 36w_2w_6c_s^2 - \\
& 18w_6w_3^2c_s^2 + 6v_2^2w_6^2w_3 + v_2^2w_6^2w_3^2 - 36w_2^2c_s^2 - 6w_2w_3^2 + w_2v_2^2w_6^2w_3^2 + 12w_2w_6 + 6w_6w_3^2 + 12w_2w_6^2w_3 - 3w_2v_2^2w_6w_3^2 - w_6^2w_3^2 - 18w_2w_6w_3 - \\
& 12w_2v_2^2w_3 - 6v_2^2w_6w_3^2 + 12w_6^2 + 12w_2v_2^2w_6^2 + 3w_2w_6^2w_3^2c_s^2 - 12w_2v_2^2w_6 + 36w_2w_6^2c_s^2 + 36w_6w_3c_s^2 + 6w_2v_2^2w_3^2 + 12v_2^2w_6w_3 - 6w_6^2w_3 + \\
& 18w_2v_2^2w_6w_3 + 54w_2w_6w_3c_s^2 + 3w_2w_6w_3^2 + 3w_6^2w_3^2c_s^2 - 36w_2w_3c_s^2
\end{aligned}$$

$$\begin{aligned} C_{10} = & 12w_6\omega_3 - 5w_2w_6\omega_3^2c_s^2 - 12w_2\omega_3 - 18w_2\omega_3^2c_s^2 - 12w_2w_6c_s^2 + 6w_6\omega_3^2c_s^2 + 6w_2\omega_3^2c_s^2 + 36v_2^2\omega_3^2 - 6w_2\omega_3^3 - \omega_2w_6\omega_3^2c_s^2 - w_6\omega_3^3 + 18w_2\omega_3^2 - 18v_2^2\omega_3^3 + w_6\omega_3^2c_s^2 - 6w_6\omega_3^2 + 3v_2^2w_6\omega_3^3 + 12w_2^2\omega_3^2 - 3w_2v_2^2w_6\omega_3^2 + 36w_2v_2^2\omega_3 + 18v_2^2w_6\omega_3^2 - 12\omega_3^2 - 12w_6\omega_3c_s^2 - 54w_2v_2^2\omega_3^2 - 6w_3^3c_s^2 - 36v_2^2w_6\omega_3 + 18w_2w_6\omega_3c_s^2 + w_2w_6\omega_3^2 + 18w_2v_2^2\omega_3 + 12w_2w_3c_s^2 + 6\omega_3^2 \end{aligned}$$

$$\begin{aligned} C_{11} = & 24w_6w_3c_s^4 + 108v_2^2w_3^2c_s^2 - 12v_2^4w_6^2w_3^2 + 12w_6^2w_3c_s^4 + 6v_2^2w_6^2w_3^3 + 3v_2^4w_6^2w_3^3 + 24w_6^2w_3^4c_s^4 + 24w_6w_3^2c_s^2 + 72v_2^2w_3^2 - 12v_2^2w_6^2w_3^2 + \\ & 12v_2^2w_6^2w_3^2 - 216v_2^2w_3^2c_s^2 - 3v_2^2w_6^2w_3^3 - 36v_2^2w_6^2w_3^3 + 72v_2^2w_6w_3c_s^2 - 6w_6w_3^2c_s^2 - 3w_6^2w_3^4c_s^4 + 30v_2^2w_6w_3^3 + 144v_2^2w_6w_3^2c_s^2 + 24w_6^2c_s^4 + w_6^2w_3^2c_s^2 - \\ & 36v_2^2w_6^2w_3c_s^2 - 72v_2^2w_6w_3^3 + 6w_6w_3^2c_s^4 - 30v_2^4w_6w_3^3 + 36v_2^4w_3^3 - 48w_6^2w_3c_s^4 - 72v_2^2w_6w_3^2c_s^2 - 24w_6w_3c_s^2 - 72v_2^4w_3^2 - 24w_6w_3^2c_s^4 + 72v_2^4w_6w_3^2 - 8w_6^2w_3^2c_s^2 \end{aligned}$$

#### 2.6.4 Conservation of momentum: $\rho v_2$

 attached text file: output\_d2q9\_nse\_culbm1\_symbolic\_pde\_02.txt

$$\begin{aligned}
& v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{v_2 \delta_1 v_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{v_2 \delta_1 \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_1 v_1 \rho}{\delta_t} \frac{\partial v_2}{\partial x_1} + (c_s^2 + v_2^2) \frac{\delta_1}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{2 v_2 \delta_1 \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_3) \frac{\delta_1^2 c_s^2}{2 \omega_3 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\
& (-2 + \omega_3) \frac{\delta_1^2 c_s^2}{2 \omega_3 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (-2 + 4c_s^2 + \omega_2 + 6v_2^2 - 2\omega_2 c_s^2 - 3\omega_2 v_2^2) \frac{\delta_1^2}{\omega_2 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (2 - \omega_2) \frac{3 v_2 \delta_1^2 \rho}{\omega_2 \delta_t} \left( \frac{\partial v_2}{\partial x_2} \right)^2 + \\
& (-2 + \omega_3) \frac{\delta_1^2 \rho c_s^2}{2 \omega_3 \delta_t} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_3) \frac{\delta_1^2 \rho c_s^2}{2 \omega_3 \delta_t} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + (-2 + 6c_s^2 + \omega_2 + 2v_2^2 - 3\omega_2 c_s^2 - \omega_2 v_2^2) \frac{v_2 \delta_1^2}{2 \omega_2 \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + \\
& (-2 + 2c_s^2 + \omega_2 + 6v_2^2 - \omega_2 c_s^2 - 3\omega_2 v_2^2) \frac{\delta_1^2 \rho}{2 \omega_2 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + (-1 + 3c_s^2 + v_1^2) \frac{v_2 \delta_1^3 v_1}{12 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3v_1^2) \frac{v_2 \delta_1^3 \rho}{12 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + \\
& C_1 \frac{\delta_1^3 v_1 \rho}{6 \omega_3 \delta_t \omega_4} \frac{\partial^3 v_2}{\partial x_1^3} + (-12 + 12\omega_3 - \omega_3^2) \frac{\delta_1^3 c_s^4}{6 \omega_3^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} - \frac{v_2 \delta_1^3 \rho c_s^2}{6 \delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + C_2 \frac{v_2 \delta_1^3 \rho}{12 \omega_2^2 \omega_6 \omega_3 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_3 \frac{\delta_1^3}{12 \omega_2^2 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\
& (-24 + 36c_s^2 + 11\omega_2^2 v_2^2 - 4\omega_2^2 + 5\omega_2^2 c_s^2 + 24\omega_2 + 60v_2^2 - 36\omega_2 c_s^2 - 60\omega_2 v_2^2) \frac{v_2 \delta_1^3 \rho}{6 \omega_2^2 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& (-2c_s^2 - \omega_1 c_s^4 - 3\omega_1 v_1^4 - 6v_1^2 + 3\omega_1 v_1^2 + 6v_1^4 - 12\omega_1 v_1^2 c_s^2 + 24v_1^2 c_s^2 + 2c_s^4 + \omega_1 c_s^2) \frac{v_2 \delta_1^4}{24 \omega_1 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& (-4 + 6c_s^2 + 2\omega_1 + 10v_1^2 - 5\omega_1 v_1^2 - 3\omega_1 c_s^2) \frac{v_2 \delta_1^4 v_1 \rho}{12 \omega_1 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_4 \frac{\delta_1^4 \rho}{24 \omega_3^3 \delta_t \omega_4} \frac{\partial^4 v_2}{\partial x_1^4} + C_5 \frac{\delta_1^4 v_1 c_s^2}{12 \omega_1 \omega_2^2 \delta_t \omega_4^2} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + \\
& C_6 \frac{\delta_1^4 \rho c_s^2}{12 \omega_1 \omega_3^3 \delta_t \omega_4} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + (3\omega_1 + 3\omega_1 c_s^2 \omega_4 + 3c_s^2 \omega_4 - 3\omega_1 v_1^2 + \omega_1 v_1^2 \omega_4 + v_1^2 \omega_4 - \omega_1 \omega_4 - 9\omega_1 c_s^2 - \omega_4) \frac{v_2 \delta_1^4 v_1 \rho}{12 \omega_1 \delta_t \omega_4} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& C_7 \frac{v_2 \delta_1^4 c_s^2}{12 \omega_3^2 \omega_6^2 \omega_3^2 \delta_t} \frac{\partial^4 \rho}{\partial x_2^2 \partial x_2^2} + C_8 \frac{\delta_1^4 \rho c_s^2}{12 \omega_2^2 \omega_6 \omega_3^2 \delta_t} \frac{\partial^4 v_2}{\partial x_2^2 \partial x_2^2} + C_9 \frac{\delta_1^4 \rho}{12 \omega_3^2 \omega_6^2 \omega_3^2 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{10} \frac{v_2 \delta_1^4}{12 \omega_3^2 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{11} \frac{\delta_1^4 \rho}{12 \omega_2^2 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$C_1 = 6 - 18c_s^2 - 3\omega_3 c_s^2 \omega_4 - v_1^2 \omega_3 \omega_4 + 9c_s^2 \omega_4 + 9\omega_3 c_s^2 + \omega_3 \omega_4 - 6v_1^2 + 3v_1^2 \omega_3 - 3\omega_3 + 3v_1^2 \omega_4 - 3\omega_4$$

$$\textcolor{red}{C_2} = -12w_6w_3 + 6w_2^2v_2^2w_6 - 12w_2^2v_2^2 + 12w_2w_3 + 36w_2^2w_3c_s^2 + 12w_2^2 + 3w_2^2w_6w_3 - 36w_2^2c_s^2 + 12w_2^2v_2^2w_3 - 11w_2^2w_6w_3c_s^2 - 3w_2^2v_2^2w_6w_3 - 6w_2^2w_6 + 6w_2w_6w_3 - 12w_2v_2^2w_3 + 36w_6w_3c_s^2 + 12v_2^2w_6w_3 - 6w_2v_2^2w_6w_3 - 18w_2w_6w_3c_s^2 - 12w_2^2w_3 + 18w_2^2w_6c_s^2 - 36w_2w_3c_s^2$$

$$C_3 = -144\omega_2 v_2^2 c_s^2 - 12c_s^2 - 7\omega_2 v_2^2 - 36\omega_2 v_4^2 - \omega_2^2 c_s^2 - 36v_2^2 - 12\omega_2 c_s^4 + \omega_2^2 c_s^4 + 36v_4^2 + 144v_2^2 c_s^2 + 12\omega_2 c_s^2 + 24\omega_2 v_2^2 c_s^2 + 12c_s^4 + 7\omega_2^2 v_4^2 + 36\omega_2 v_2^2$$

$$\begin{aligned} C_4 = & 24w_2^2c_s^4w_4^2 + 72v_1^2w_3^2 + w_3^3c_s^2w_4^2 - 24w_3c_s^2w_4 - 36v_1^2w_3^3 - 12v_1^2w_2^2c_s^2w_4^2 + 144v_1^2w_2^3c_s^2w_4 - 6w_3^3c_s^2w_4 + 12w_3c_s^2w_4^2 - 24w_3^2c_s^4w_4 + 36v_1^4w_3^3 + \\ & 6v_2^2w_3^2c_s^2w_4^2 + 72v_1^4w_3^2w_4 + 72v_1^2w_3c_s^2w_4 + 108v_1^2w_3^2c_s^3 + 30v_1^2w_3^3w_4 - 8w_3^2c_s^2w_4^2 + 3v_1^4w_3^3w_4^2 - 72v_1^4w_3^2 - 3w_3^3c_s^4w_4^2 + 12v_1^2w_3^2w_4^2 + 24w_3c_s^4w_4 + \\ & 6w_3^3c_s^4w_4 - 72v_1^2w_3^2w_4 - 48w_3c_s^4w_4^2 - 216v_1^2w_3^2c_s^2 + 24w_3^2c_s^2w_4 - 30v_1^4w_3^3w_4 + 24c_s^4w_4^2 - 3v_1^2w_3^3w_4^2 - 12v_1^4w_3^2w_4^2 - 72v_1^2w_3^2c_s^4w_4 - 36v_1^2w_3c_s^2w_4^2 \end{aligned}$$

$$C_5 = 18w_1v_1^2w_3w_4 + 12w_1w_3s_8^2 + 36w_3c_8^2w_4 + 6w_1v_1^2w_3^2 - 6w_3w_4^2 + 12v_1^2w_3w_4 - 6w_1w_3^2 - 36w_1c_8^2w_4 + 54w_1w_3c_8^2w_4 - 36w_1w_3c_8^2w_4^2 - 36c_8^2w_4^2 + 12w_1w_3 + 6v_1^2w_3w_4^2 + 36w_1c_8^2w_4^2 - 12w_1v_1^2w_3 - 12w_3w_4 + 18w_3c_8^2w_4^2 + 12w_4^2 - 36w_1w_3s_8^2 - 12w_1v_1^2w_3w_4^2 - 18w_1w_3w_4 - 12w_1w_4^2 + 6w_3^2w_4 + 3w_1w_3s_8^2w_4^2 + 18w_1w_3s_8^2 + 3w_1w_3^2w_4 + w_1v_1^2w_3s_8^2w_4^2 + 3w_3^2c_8^2w_4^2 - 12v_1^2w_4^2 + v_1^2w_3^2w_4 + 12w_1v_1^2w_4^2 - 6v_1^2w_3^2w_4 - 12w_1v_1^2w_4 - 18w_3c_8^2w_4 - w_1w_3^2w_4^2 - 3w_1v_1^2w_3^2w_4 + 12w_1w_4 - 9w_1w_3c_8^2w_4 - w_3^2w_4$$

$$\begin{aligned} C_6 = & -6\omega_1 w_3^3 + 36v_1^2 w_3^2 - 12w_3 c_s^2 w_4 - 54w_1 v_1^2 w_3^2 - 36v_1^2 w_3 w_4 + 18w_1 w_3^2 - 18v_1^2 w_3^3 - 12w_1 c_s^2 w_4 + 18w_1 w_3 c_s^2 w_4 + 18w_1 v_1^2 w_3^2 - \omega_1 w_3^3 c_s^2 w_4 - \\ & 12w_1 w_3 + w_3^3 c_s^2 w_4 + 36w_1 v_1^2 w_3 + 12w_3 w_4 + 12w_1 w_3 c_s^2 + 12w_3^2 c_s^2 - 6w_3^2 w_4 - 18w_1 w_3^2 c_s^2 + \omega_1 w_3^2 w_4 + 3v_1^2 w_3^3 w_4 + 18v_1^2 w_3^2 w_4 + 6w_1 w_3^3 c_s^2 - 12w_3^2 + \\ & 6w_3^2 c_s^2 w_4 - \omega_3^3 w_4 - 6w_3^3 c_s^2 - 3w_1 v_1^2 w_3^2 w_4 - 5w_1 w_3^2 c_s^2 w_4 + 6w_3^3 \end{aligned}$$

$$\mathbf{C}_7 = 12\omega_2^3\omega_6^2\omega_3 + 18\omega_2\omega_6^2\omega_3^2 + 6\omega_2^2v_2^2\omega_6\omega_3^2 + 18\omega_2^3v_2^2\omega_6\omega_3 - 12\omega_2^2v_2^2\omega_3^2 - 36\omega_2^2\omega_6^2c_s^2 + 54\omega_2^3\omega_6\omega_3c_s^2 - 6\omega_2^2\omega_6\omega_3^2 + 5\omega_2^3\omega_6^2\omega_3^2c_s^2 + 12v_2^2\omega_6^2\omega_3^2 - 36\omega_2^3\omega_6c_s^2 - 12\omega_2^2\omega_6\omega_3 + 12\omega_2^2\omega_6^2\omega_3^2c_s^2 + 36\omega_2^3\omega_3^2c_s^2 - 6\omega_2^3v_2^2\omega_6\omega_3^2 + 36\omega_2^2\omega_6\omega_3c_s^2 - 12\omega_2^2v_2^2\omega_6^2 - \omega_2^3\omega_6^2\omega_3^2 - 18\omega_2v_2^2\omega_6^2\omega_3^2 + 12\omega_2^2v_2^2\omega_6^2\omega_3 + 12\omega_2^2v_2^2\omega_6^2\omega_3^2 + 36\omega_2^3\omega_6^2c_s^2 + 12\omega_2^3\omega_3 - 12\omega_2^3\omega_6^2 - 6\omega_2^2\omega_6^2\omega_3 + 18\omega_2^2\omega_6\omega_3^2 + 6\omega_2^2\omega_6\omega_3^2c_s^2 + 12\omega_2^2v_2^2\omega_6^2 + 12\omega_2^3\omega_6 - 40\omega_2^3\omega_6^2\omega_3c_s^2 - 12\omega_2^3v_2^2\omega_3 - 36\omega_2^2\omega_3^2c_s^2 - 12\omega_2^3\omega_3^2 + 12\omega_2^2\omega_6^2 - 18\omega_2^3\omega_6\omega_3 + 4\omega_2^2v_2^2\omega_6^2 - 12\omega_2^3v_2^2\omega_6^2\omega_3 + 36\omega_2^2\omega_6^2\omega_3^2 - 18\omega_2^3\omega_6\omega_3^2$$

$$\mathbf{C}_8 = 12\omega_2\omega_6\omega_3^2c_s^2 - 72\omega_2^2v_2^2\omega_3^2 + 24\omega_2\omega_3^2c_s^2 + 12\omega_2^2\omega_3c_s^2 - 12\omega_6\omega_3^2c_s^2 + 36\omega_2^2v_2^2\omega_3^2 - 24\omega_2\omega_3^2c_s^2 + 24\omega_2\omega_3^2 - 24\omega_2\omega_3^2 + 36\omega_2^2v_2^2\omega_3 + 36v_2^2\omega_3^2 + 18\omega_2^2\omega_6\omega_3c_s^2 + 12\omega_6\omega_3^2 + 24\omega_2^2\omega_3^2 + 12\omega_2^2\omega_3^2c_s^2 + 36\omega_2v_2^2\omega_6\omega_3^2 - 4\omega_2^2\omega_6\omega_3^2c_s^2 + 12\omega_2\omega_6\omega_3 - 36\omega_2^2\omega_6\omega_3^2 - 12\omega_2^2\omega_3^2 - 12\omega_2^2\omega_6\omega_3^2 - 72\omega_2v_2^2\omega_3^2 - 12\omega_2^2\omega_6\omega_3^2 - 12\omega_2^3$$

$$\mathbf{C}_9 = -36\omega_2^3v_2^4\omega_3^2 - \omega_2^2\omega_6^2\omega_3^2c_s^2 - 24\omega_2^3\omega_6^2\omega_3^4 + 18\omega_2^2v_2^2\omega_6^2\omega_3^2c_s^2 - 39\omega_2^3v_2^4\omega_6\omega_3^2 - 54\omega_2^3v_2^2\omega_6\omega_3^2c_s^2 - 12\omega_2\omega_6^2\omega_3^2c_s^4 - 90\omega_2v_2^4\omega_6^2\omega_3^2 + 252\omega_2^2\omega_6^2\omega_3^2c_s^2 - 12\omega_2^3\omega_6\omega_3c_s^2 - 6\omega_2^2\omega_6\omega_3^2c_s^4 - 72\omega_2^2v_2^2\omega_6\omega_3^2 + 72v_2^4\omega_6^2\omega_3^2 + 36\omega_2^3v_2^4\omega_6\omega_3^2 - 18\omega_2^3\omega_6\omega_3^2c_s^4 + 36\omega_2^3v_2^4\omega_3^2 - 5\omega_2^3\omega_6^2\omega_3^2c_s^2 - 36\omega_2v_2^2\omega_6^2\omega_3^2c_s^2 + 36\omega_2^2\omega_6^2\omega_3^2 - 18\omega_2^3v_2^2\omega_6\omega_3c_s^2 + 12\omega_2^2\omega_6\omega_3^2c_s^4 - 99\omega_2^3v_2^2\omega_6\omega_3^2c_s^2 - 12\omega_2\omega_6^2\omega_3^2c_s^4 + 39\omega_2^3v_2^2\omega_6\omega_3^2 - 6\omega_2^2\omega_6^2\omega_3^2c_s^2 + 90\omega_2v_2^2\omega_6^2\omega_3^2 + 60\omega_2^2v_2^2\omega_6^2\omega_3^2c_s^2 - 36\omega_2^3v_2^2\omega_6\omega_3^2 + 72\omega_2^2v_2^4\omega_6\omega_3^2 - 72v_2^2\omega_6^2\omega_3^2 - 306\omega_2v_2^2\omega_6^2\omega_3^2c_s^2 + 12\omega_2^2\omega_6^2\omega_3^4 + 6\omega_2^3\omega_6\omega_3^2c_s^4 + 12\omega_2^3\omega_6^2\omega_3^2c_s^4 + 19\omega_2^2v_2^4\omega_6^2\omega_3^2 + 12\omega_2^3v_2^2\omega_6^2\omega_3^2c_s^2 + 6\omega_2^3v_2^2\omega_6^2\omega_3^2 + 6\omega_2^2\omega_6^2\omega_3^2c_s^4 - 36\omega_2^3v_2^2\omega_6\omega_3^2 + 198\omega_2^2v_2^2\omega_6\omega_3^2c_s^2 - 108\omega_2^3v_2^2\omega_6\omega_3^2c_s^2 - 12\omega_2\omega_6\omega_3^2c_s^2 + 12\omega_2\omega_6\omega_3^2c_s^4 - 36\omega_2^2v_2^4\omega_6\omega_3^2 - 108\omega_2^3v_2^2\omega_6\omega_3^2c_s^2 + 36\omega_2v_2^2\omega_6\omega_3^2 - \omega_2^3\omega_6^2\omega_3^2c_s^4 + 12\omega_2\omega_6\omega_3^2c_s^2 - 6\omega_2^3v_2^2\omega_6^2\omega_3^2 - 19\omega_2^2v_2^2\omega_6^2\omega_3^2 + 36\omega_2^2v_2^2\omega_6\omega_3^2c_s^2 + 6\omega_2^2\omega_6\omega_3^2c_s^4 + 12\omega_2^3\omega_6\omega_3^2c_s^2 + 108\omega_2^3v_2^2\omega_6\omega_3^2c_s^2 + 6\omega_2^3\omega_6^2\omega_3^2c_s^4 + \omega_2^2\omega_6^2\omega_3^2c_s^4 - 3\omega_2^3v_2^2\omega_6^2\omega_3^2c_s^2 + 13\omega_2^3\omega_6^2\omega_3^2c_s^4 + 4\omega_2^3v_2^4\omega_6\omega_3^2 + 36\omega_2^2v_2^2\omega_6\omega_3^2c_s^2 + 18\omega_2^3\omega_6\omega_3^2c_s^2 - 36\omega_2v_2^2\omega_6\omega_3^2$$

$$\mathbf{C}_{10} = 12 + 6\omega_2^3c_s^2 - 1008\omega_2v_2^2c_s^2 - 132c_s^2 - 98\omega_2^2v_2^2 - \omega_2^3 - 216\omega_2v_2^4 + 8\omega_2^2 + 10\omega_2^3v_2^2 - 78\omega_2^2c_s^2 - 18\omega_2 - 156v_2^2 - 216\omega_2c_s^4 - 34\omega_2^3v_2^2c_s^2 - 9\omega_2^3v_2^4 + 82\omega_2^2c_s^4 + 144v_2^4 + 672v_2^2c_s^2 + 198\omega_2c_s^2 + 404\omega_2^2v_2^2c_s^2 - 5\omega_2^3c_s^4 + 144c_s^4 + 90\omega_2^2v_2^4 + 234\omega_2v_2^2$$

$$\mathbf{C}_{11} = 12 + 2\omega_2^3c_s^2 - 648\omega_2v_2^2c_s^2 - 36c_s^2 - 154\omega_2^2v_2^2 - \omega_2^3 - 756\omega_2v_2^4 + 8\omega_2^2 + 14\omega_2^3v_2^2 - 22\omega_2^2c_s^2 - 18\omega_2 - 252v_2^2 - 36\omega_2c_s^4 - 18\omega_2^3v_2^2c_s^2 - 29\omega_2^3v_2^4 + 14\omega_2^2c_s^4 + 504v_2^4 + 432v_2^2c_s^2 + 54\omega_2c_s^2 + 252\omega_2^2v_2^2c_s^2 - \omega_2^3c_s^4 + 24c_s^4 + 310\omega_2^2v_2^4 + 378\omega_2v_2^2$$

## 2.7 CuLBM2

### 2.7.1 Definitions

Collision operator  $\mathbf{C}$ :

$$\mathbf{C}(\mathbf{f}) = \mathbf{M}^{-1}\mathbf{G}^{-1}\left(\mathbf{N}^{-1}\mathbf{S}\mathbf{N}\left(\boldsymbol{\gamma}^{(eq)} - \mathbf{G}(\mathbf{M}\mathbf{f})\right)\right),$$

where

$$\mathbf{S} = \text{diag}(0, 0, 0, \omega_1, \omega_2, \omega_1, \omega_3, \omega_3, \omega_4),$$

$\omega_1, \omega_2, \omega_3, \omega_4 \in (0, 2)$ .

Matrix  $\mathbf{N}$  defines the combination of cumulants for the collision as

$$\mathbf{N} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

The nonlinear operator  $\mathbf{G}$  is the same as in CuLBM1 in Section ?? and, again, the equilibrium cumulant vector  $\boldsymbol{\gamma}^{(eq)}$  is defined by

$$\boldsymbol{\gamma}^{(eq)} = (\rho, v_1, v_2, 2c_s^2, 0, 0, 0, 0, 0)^T.$$

### 2.7.2 Conservation of mass: $\rho$

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$$\begin{aligned}
& \frac{\partial \rho}{\partial t} + \frac{v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + 3c_s^2 + v_1^2) \frac{v_1 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3v_1^2) \frac{\rho \delta_l^3}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\
& \frac{c_s^2 \rho \delta_l^3}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{c_s^2 \rho \delta_l^3}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{v_2 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + c_s^2 + 3v_2^2) \frac{\rho \delta_l^3}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_1 \frac{\delta_l^4}{24\delta_t \omega_1 \omega_2} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& (5v_1^2 \omega_2 + 2\omega_1 \omega_2 - 5\omega_1 v_1^2 \omega_2 + 5\omega_1 v_1^2 - 2\omega_1 + 3c_s^2 \omega_1 - 3c_s^2 \omega_1 \omega_2 - 2\omega_2 + 3c_s^2 \omega_2) \frac{\rho v_1 \delta_l^4}{12\delta_t \omega_1 \omega_2} \frac{\partial^4 v_1}{\partial x_1^4} + \\
& (\omega_1 v_2^2 - \omega_1 - \omega_2 v_2^2 + 3c_s^2 \omega_1 + \omega_2 - 3c_s^2 \omega_2) \frac{v_1 v_2 \delta_l^4}{8\delta_t \omega_1 \omega_2} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + \\
& (\omega_1 v_2^2 - \omega_1 - \omega_2 v_2^2 + 3c_s^2 \omega_1 + \omega_2 - 3c_s^2 \omega_2) \frac{\rho v_2 \delta_l^4}{24\delta_t \omega_1 \omega_2} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + C_2 \frac{\rho v_1 \delta_l^4}{24\omega_3 \delta_t \omega_1 \omega_2} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& (-3v_1^2 \omega_2 + 3\omega_1 v_1^2 + 3\omega_1 v_1^2 - 2\omega_1 - 3\omega_2 v_2^2 + 2c_s^2 \omega_1 + 4c_s^2 \omega_1 \omega_2 + 2\omega_2 - 10c_s^2 \omega_2) \frac{c_s^2 \delta_l^4}{24\delta_t \omega_1 \omega_2} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\
& (-v_1^2 \omega_2 + \omega_1 v_1^2 - \omega_1 + 3c_s^2 \omega_1 + \omega_2 - 3c_s^2 \omega_2) \frac{\rho v_1 \delta_l^4}{24\delta_t \omega_1 \omega_2} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + \\
& (\omega_1 v_2^2 - \omega_1 - \omega_2 v_2^2 + 3c_s^2 \omega_1 + \omega_2 - 3c_s^2 \omega_2) \frac{\rho v_2 \delta_l^4}{24\delta_t \omega_1 \omega_2} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + \\
& (-v_1^2 \omega_2 + \omega_1 v_1^2 - \omega_1 + 3c_s^2 \omega_1 + \omega_2 - 3c_s^2 \omega_2) \frac{v_1 v_2 \delta_l^4}{8\delta_t \omega_1 \omega_2} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_3 \frac{\rho v_2 \delta_l^4}{24\omega_3 \delta_t \omega_1 \omega_2} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& (-v_1^2 \omega_2 + \omega_1 v_1^2 - \omega_1 + 3c_s^2 \omega_1 + \omega_2 - 3c_s^2 \omega_2) \frac{\rho v_1 \delta_l^4}{24\delta_t \omega_1 \omega_2} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + C_4 \frac{\delta_l^4}{24\delta_t \omega_1 \omega_2} \frac{\partial^4 \rho}{\partial x_2^4} + \\
& (2\omega_1 \omega_2 + 5\omega_1 v_1^2 - 2\omega_1 + 5\omega_2 v_2^2 + 3c_s^2 \omega_1 - 3c_s^2 \omega_1 \omega_2 - 2\omega_2 + 3c_s^2 \omega_2 - 5\omega_1 \omega_2 v_2^2) \frac{\rho v_2 \delta_l^4}{12\delta_t \omega_1 \omega_2} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= -3v_1^2 \omega_2 + c_s^4 \omega_1 + 3\omega_1 v_1^2 \omega_2 - 3\omega_1 v_1^2 + c_s^4 \omega_2 - c_s^4 \omega_1 \omega_2 + 12c_s^2 \omega_1 v_1^2 + 3\omega_1 v_1^4 + 3v_1^4 \omega_2 - c_s^2 \omega_1 + 12c_s^2 v_1^2 \omega_2 + c_s^2 \omega_1 \omega_2 - c_s^2 \omega_2 - 3\omega_1 v_1^4 \omega_2 - 12c_s^2 \omega_1 v_1^2 \omega_2 \\
C_2 &= \omega_3 \omega_1 v_1^2 + 6\omega_1 \omega_2 + 9\omega_3 \omega_1 v_2^2 + 6\omega_3 c_s^2 \omega_1 \omega_2 - 6\omega_1 v_1^2 \omega_2 + \omega_3 v_1^2 \omega_2 - 2\omega_3 \omega_1 \omega_2 + 2\omega_3 \omega_2 + 2\omega_3 \omega_1 v_1^2 \omega_2 + 6\omega_3 c_s^2 \omega_1 - 18c_s^2 \omega_1 \omega_2 - 4\omega_3 \omega_1 - 9\omega_3 \omega_2 v_2^2 \\
C_3 &= 9\omega_3 \omega_1 v_1^2 + 6\omega_1 \omega_2 + 2\omega_3 \omega_1 \omega_2 v_2^2 + \omega_3 \omega_1 v_2^2 + 6\omega_3 c_s^2 \omega_1 \omega_2 - 9\omega_3 v_1^2 \omega_2 - 2\omega_3 \omega_1 \omega_2 + 2\omega_3 \omega_2 + 6\omega_3 c_s^2 \omega_1 - 18c_s^2 \omega_1 \omega_2 - 6\omega_1 \omega_2 v_2^2 - 4\omega_3 \omega_1 + \omega_3 \omega_2 v_2^2 \\
C_4 &= -3\omega_1 \omega_2 v_2^4 + c_s^4 \omega_1 + 12c_s^2 \omega_2 v_2^2 + 3\omega_2 v_2^4 - 3\omega_1 v_2^2 + c_s^4 \omega_2 - 12c_s^2 \omega_1 \omega_2 v_2^2 + 12c_s^2 \omega_1 v_2^2 - c_s^4 \omega_1 \omega_2 + 3\omega_1 v_2^4 - 3\omega_2 v_2^2 - c_s^2 \omega_1 + c_s^2 \omega_1 \omega_2 - c_s^2 \omega_2 + 3\omega_1 \omega_2 v_2^2
\end{aligned}$$

### 2.7.3 Conservation of momentum: $\rho v_1$

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$$\begin{aligned}
& v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (c_s^2 + v_1^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2\rho v_1 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_1 v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho v_2 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{\rho v_1 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\
& (3v_1^2 \omega_2 + \omega_1 \omega_2 - 3\omega_1 v_1^2 \omega_2 + 3\omega_1 v_1^2 - \omega_1 + 2c_s^2 \omega_1 - 2c_s^2 \omega_1 \omega_2 - \omega_2 + 2c_s^2 \omega_2) \frac{\delta_l^2}{\delta_t \omega_1 \omega_2} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + \\
& (-\omega_1 \omega_2 + \omega_1 + \omega_2) \frac{3\rho v_1 \delta_l^2}{\delta_t \omega_1 \omega_2} \left( \frac{\partial v_1}{\partial x_1} \right)^2 + (3\omega_1 v_2^2 - \omega_1 - 3\omega_2 v_2^2 + c_s^2 \omega_1 + \omega_2 - c_s^2 \omega_2) \frac{\delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_2} + \\
& (\omega_1 - \omega_2) \frac{3\rho v_2 \delta_l^2}{\delta_t \omega_1 \omega_2} \frac{\partial v_2}{\partial x_1} \frac{\partial v_2}{\partial x_2} + (3\omega_1 v_2^2 - \omega_1 - 3\omega_2 v_2^2 + 3c_s^2 \omega_1 + c_s^2 \omega_1 \omega_2 + \omega_2 - 5c_s^2 \omega_2) \frac{\delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_1} + \\
& (-2 + \omega_1) \frac{c_s^2 \delta_l^2}{2\delta_t \omega_1} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (v_1^2 \omega_2 + \omega_1 \omega_2 - \omega_1 v_1^2 \omega_2 + \omega_1 v_1^2 - \omega_1 + 3c_s^2 \omega_1 - 3c_s^2 \omega_1 \omega_2 - \omega_2 + 3c_s^2 \omega_2) \frac{v_1 \delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial^2 \rho}{\partial x_1^2} + \\
& (3v_1^2 \omega_2 + \omega_1 \omega_2 - 3\omega_1 v_1^2 \omega_2 + 3\omega_1 v_1^2 - \omega_1 + c_s^2 \omega_1 - c_s^2 \omega_1 \omega_2 - \omega_2 + c_s^2 \omega_2) \frac{\rho \delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial^2 v_1}{\partial x_1^2} + \\
& (\omega_1 v_2^2 - \omega_1 - \omega_2 v_2^2 + 3c_s^2 \omega_1 + \omega_2 - 3c_s^2 \omega_2) \frac{v_2 \delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial^2 \rho}{\partial x_1 \partial x_2} + \\
& (3\omega_1 v_2^2 - \omega_1 - 3\omega_2 v_2^2 + c_s^2 \omega_1 + c_s^2 \omega_1 \omega_2 + \omega_2 - 3c_s^2 \omega_2) \frac{\rho \delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_1) \frac{c_s^2 \rho \delta_l^2}{2\delta_t \omega_1} \frac{\partial^2 v_1}{\partial x_2^2} + C_1 \frac{\delta_l^3}{12\delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 \rho}{\partial x_1^3} + \\
& C_2 \frac{\rho v_1 \delta_l^3}{12\delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 v_1}{\partial x_1^3} + C_3 \frac{3v_1 v_2 \delta_l^3}{4\delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + C_4 \frac{\rho v_2 \delta_l^3}{4\omega_3 \delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 v_1}{\partial x_2^2 \partial x_2} + C_5 \frac{\rho v_1 \delta_l^3}{12\omega_3 \delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + C_6 \frac{\delta_l^3}{12\delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^3} - \\
& \frac{c_s^2 \rho v_1 \delta_l^3}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_7 \frac{\rho v_2 \delta_l^3}{4\delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{v_1 v_2 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + C_8 \frac{\rho v_2 \delta_l^3}{6\omega_3 \delta_t \omega_1} \frac{\partial^3 v_1}{\partial x_2^3} + (-1 + c_s^2 + 3v_2^2) \frac{\rho v_1 \delta_l^3}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& C_9 \frac{v_1 \delta_l^4}{24\omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 \rho}{\partial x_1^4} + C_{10} \frac{\rho \delta_l^4}{24\omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_1}{\partial x_1^4} + C_{11} \frac{v_2 \delta_l^4}{24\omega_3^2 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{12} \frac{\rho v_1 v_2 \delta_l^4}{24\omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + \\
& C_{13} \frac{\rho \delta_l^4}{24\omega_3^2 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_{14} \frac{v_1 \delta_l^4}{24\omega_3^2 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_{15} \frac{\rho \delta_l^4}{24\omega_3^2 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{16} \frac{\rho v_1 v_2 \delta_l^4}{24\omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} +
\end{aligned}$$

$$C_{17} \frac{v_2 \delta_t^4}{24 \omega_3^2 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{18} \frac{\rho v_1 v_2 \delta_t^4}{24 \omega_3 \delta_t \omega_1 \omega_2} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{19} \frac{\rho \delta_t^4}{24 \omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + C_{20} \frac{v_1 \delta_t^4}{24 \delta_t \omega_1 \omega_2} \frac{\partial^4 \rho}{\partial x_2^4} + C_{21} \frac{\rho \delta_t^4}{24 \omega_3^2 \delta_t \omega_1^3} \frac{\partial^4 v_1}{\partial x_2^4} + (2\omega_1 \omega_2 + 5\omega_1 v_2^2 - 2\omega_1 + 5\omega_2 v_2^2 + 3c_s^2 \omega_1 - 3c_s^2 \omega_1 \omega_2 - 2\omega_2 + 3c_s^2 \omega_2 - 5\omega_1 \omega_2 v_2^2) \frac{\rho v_1 v_2 \delta_t^4}{12 \delta_t \omega_1 \omega_2} \frac{\partial^4 v_2}{\partial x_2^4} = 0,$$

where:

$$\begin{aligned} C_1 = & 6c_s^4\omega_2^2 + c_s^4\omega_1^2\omega_2^2 + 6c_s^2\omega_1^2\omega_2 + 18\omega_1v_1^2\omega_2^2 - 72c_s^2\omega_1^2v_1^2\omega_2 - 18\omega_1^2v_1^4\omega_2 + 9\omega_1^2v_1^4 + 45c_s^2\omega_2^2v_1^2 + 24c_s^2\omega_2^2v_1^2\omega_2^2 + 7\omega_1^2v_1^4\omega_2^2 - c_s^2\omega_1^2\omega_2^2 - \\ & 18\omega_1v_1^2\omega_2^2 - 9v_1^2\omega_2^2 - 6c_s^4\omega_2^2\omega_2 + 6c_s^2\omega_1^2 - 18\omega_1v_1^4\omega_2^2 - 72c_s^2\omega_1v_1^2\omega_2^2 - 9\omega_1^2v_1^2 + 18\omega_1^2v_1^2\omega_2^2 - 6c_s^2\omega_2^2 + 45c_s^2v_1^2\omega_2^2 + 6c_s^2\omega_1\omega_2^2 + 9v_1^4\omega_2^2 - 6c_s^2\omega_1^2 - \\ & 7\omega_1^2v_1^2\omega_2^2 - 6c_s^4\omega_1\omega_2^2 + 18\omega_1v_1^4\omega_2^2 + 54c_s^2\omega_1v_1^2\omega_2 \end{aligned}$$

$$\textcolor{red}{C_2} = -18\omega_1\omega_2 - 36c_s^2\omega_1^2\omega_2 - 60\omega_1v_1^2\omega_2^2 + 10c_s^2v_1^2\omega_2^2 + 54\omega_1v_1^2\omega_2 + 33v_1^2\omega_2^2 + 24\omega_1\omega_2^2 - 8\omega_1^2\omega_2^2 - 15\omega_1^2 + 33\omega_1^2v_1^2 - 60\omega_1^2v_1^2\omega_2 + 27c_s^2\omega_2^2 - 36c_s^2\omega_1\omega_2^2 + 18c_s^2\omega_1\omega_2 + 27c_s^2\omega_1^2 + 22\omega_1^2v_1^2\omega_2^2 - 15\omega_2^2 + 24\omega_1^2\omega_2$$

$$C_3 = -\omega_1^2 \omega_2 v_2^2 + 2\omega_1 \omega_2 - 3c_s^2 \omega_1^2 \omega_2 + \omega_1 \omega_2^2 v_2^2 - 2\omega_1 v_1^2 \omega_2 + v_1^2 \omega_2^2 - \omega_1 \omega_2^2 - \omega_2^2 v_2^2 - 2\omega_1^2 + \omega_1^2 v_2^2 + \omega_1^2 v_2^2 + 3c_s^2 \omega_1 \omega_2^2 - 6c_s^2 \omega_1 \omega_2 + 6c_s^2 \omega_1^2 + \omega_1^2 \omega_2$$

$$\begin{aligned} \textcolor{red}{C_4} = & -2w_3w_2^2v_2^2 - 2w_1^2w_2v_2^2 + 9w_3v_2^2w_2^2 - w_3w_1w_2^2 - 6c_s^2w_1w_2 - 6w_3c_s^2w_1w_2 + 2w_1w_2^2v_2^2 + 3w_3c_s^2w_1w_2^2 - 2w_1w_2^2 + 6w_3w_1w_2 + w_3w_1^2w_2 - 5w_3w_1^2 - 18w_3w_1v_1^2w_2 + 9w_3c_s^2w_1^2 + 6c_s^2w_1w_2^2 + w_3w_1w_2^2v_2^2 + 2w_3w_1^2v_2^2 - 3w_3c_s^2w_2^2 - 3w_3c_s^2w_1^2w_2 - w_3w_1^2w_2v_2^2 + 9w_3w_1^2v_1^2 + 2w_1^2w_2 - w_3w_2^2 \end{aligned}$$

$$C_5 = -27w_3w_2^3v_2^2 - 3w_3w_1^2v_1w_2^2 + 6w_3v_1^2w_2^2 - 12w_3w_1w_2^2 - 18c_s^2w_1^2w_2 - 18w_1v_1^2w_2^2 + 18w_3c_s^2w_1w_2^2 + 36c_s^2w_1^2w_2^2 - 3w_3w_1^2v_1^2w_2 + 18w_1w_2^2 - 12w_1^2w_2^2 + 12w_3w_1^2w_2 - 6w_2^2v_1^2w_2 - 15w_3w_1^2 - 11w_3c_s^2w_1w_2^2 + 27w_3c_s^2w_1^2 - 54c_s^2w_1w_2^2 + 27w_3w_1w_2^2v_2^2 + 27w_3w_1^2v_2^2 + 9w_3c_s^2w_2^2 - 18w_3c_s^2w_1w_2 + 3w_3w_1v_1^2w_2^2 - 27w_3w_1^2w_2v_2^2 + 12w_2^2v_1^2w_2^2 + 6w_3w_1v_1^2 + 6w_1^2w_2 + 3w_3w_1^2w_2^2 + 3w_3w_2^2$$

$$\text{C}_6 = -30c_s^4w_2^2 + 9w_1^2w_2v_2^2 - 2c_s^4w_1^2w_2^2 + 6c_s^2w_1^2w_2 - 9w_1w_2^2v_2^2 + 9w_1^2v_2^4 + 45c_s^2w_1^2v_2^2 - 6c_s^4w_1^2w_2 + 6c_s^4w_1^2 + 9w_2^2v_2^2 + 45c_s^2w_1w_2^2v_2^2 - 9w_2^2v_2^4 - 45c_s^2w_2^2v_2^2 + 6c_s^2w_2^2 - 45c_s^2w_1^2w_2v_2^2 - 9w_1^2v_2^2 - 6c_s^2w_1w_2^2 - 9w_1^2w_2v_2^4 - 6c_s^2w_1^2 + 9w_1w_2^2v_2^2 + 30c_s^4w_1w_2^2$$

$$C_7 = -11\omega_1^2\omega_2v_2^2 - 9c_s^2\omega_1^2\omega_2 + 11\omega_1\omega_2^2v_2^2 - 5\omega_1\omega_2^2 - 11\omega_2^2v_2^2 - 5\omega_1^2 - 9c_s^2\omega_2^2 + 11\omega_1^2v_2^2 + 9c_s^2\omega_1\omega_2^2 + 9c_s^2\omega_1^2 + 5\omega_2^2 + 5\omega_1^2\omega_2$$

$$C_8 = 6 + 9\omega_3 c_s^2 - 3\omega_3 - 18c_s^2 - \omega_3 \omega_1 v_2^2 + 3\omega_1 v_2^2 - 3\omega_1 + 9c_s^2 \omega_1 - 3\omega_3 c_s^2 \omega_1 + 3\omega_3 v_2^2 - 6v_2^2 + \omega_3 \omega_1$$

$$\begin{aligned}
C_9 = & 90w_3^3\omega_1^3v_1^4w_2^2 + 54w_3c_s^4w_1\omega_2^2 + 210w_3\omega_1^2v_1^2w_2^2 - 36c_s^4\omega_1^2w_2^2 + 404w_3c_s^2\omega_1^3v_1^2w_2^2 + 6w_3\omega_1\omega_2^2 - 171w_3c_s^4\omega_1\omega_2^3 - 18w_3\omega_1^3v_1^4w_2^3 - \\
& 68w_3c_s^2\omega_1^3v_1^2\omega_2^3 - 12w_3\omega_1\omega_2^3 - 51w_3v_1^4\omega_2^3 + 18c_s^4\omega_1^3\omega_2 + 90w_3c_s^4\omega_1^3 - 98w_3\omega_1^2v_1^2\omega_2^3 - 60w_3c_s^2\omega_1\omega_2^2 - 12c_s^2\omega_1^2v_1^2\omega_2^2 - 600w_3c_s^2\omega_1v_1^2\omega_2^3 - \\
& 117w_3\omega_1v_1^4\omega_2^3 + 12c_s^2\omega_1^2w_2^2 + 90w_3c_s^2\omega_1^3 + 45w_3v_1^3v_4^4 - 105w_2\omega_1^2v_1^2\omega_2 + 411w_3c_s^2\omega_1v_1^2\omega_2^2 + 261w_3c_s^2v_1^2\omega_2^3 + 141w_3c_s^2\omega_1\omega_2^3 - 600w_3c_s^2\omega_1^3v_1^2\omega_2 - \\
& 117w_3\omega_1^3v_1^2\omega_2 - 6c_s^2\omega_1^3\omega_2 + 99w_3\omega_1v_1^4\omega_2^2 + 6c_s^2\omega_1^3v_1^2\omega_2 + 404w_3c_s^2\omega_1^2v_1^2\omega_2^2 - 78w_3c_s^2\omega_1^2\omega_2^3 + 20w_3\omega_1^3v_1^2\omega_2^3 + 6w_3\omega_1^2\omega_2 + 141w_3c_s^2\omega_1^3\omega_2 - \\
& 2w_3\omega_1^3\omega_2^3 + 54w_3c_s^4\omega_1^2\omega_2^2 + 45w_3v_1^4\omega_2^3 - 10w_3c_s^4\omega_1^3\omega_2^3 - 6c_s^2\omega_1^3\omega_2^3 - 72w_3\omega_1^3v_1^2\omega_2^3 + 90w_3\omega_1^2v_1^2\omega_2^3 + 8w_3\omega_1^3\omega_2^3 + 6c_s^2\omega_1^2v_1^2\omega_2^3 - 98w_3\omega_1^3v_1^2\omega_2^3 + \\
& 114w_3c_s^2\omega_1^2\omega_2^2 - 816w_3c_s^2\omega_1^3v_1^2\omega_2^2 - 198w_3\omega_1^2v_1^4\omega_2^2 - 51w_3\omega_1^3v_1^2 + 6w_3\omega_1^3 + 82w_3c_s^4\omega_1^3\omega_2^2 + 99w_3\omega_1^2v_1^4\omega_2 - 171w_3c_s^4\omega_1^3\omega_2^3 - 82w_3c_s^2\omega_1^2\omega_2^3 + \\
& 6w_3\omega_1^2 - 12w_3\omega_1^3\omega_2^3 + 12w_3c_s^2\omega_1^3v_1^3 + 8w_3\omega_1^2v_1^3 - 60w_3c_s^2\omega_1^2\omega_2^2 + 129w_3\omega_1^3v_1^2\omega_2^2 + 411w_3c_s^2\omega_1^2v_1^2\omega_2 - 105w_3\omega_1v_1^2\omega_2^2 + 18c_s^4\omega_1\omega_2^3 - \\
& 90w_3c_s^4\omega_1^2\omega_2^2 + 261w_3c_s^2\omega_1^3v_1^2 + 129w_3\omega_1v_1^2\omega_2^3 - 12w_3\omega_1^2\omega_2^2 - 78w_3c_s^2\omega_1^3\omega_2^2 - 72w_3c_s^2\omega_1^3
\end{aligned}$$

$$\begin{aligned} C_{10} = & 310w_3w_1^3v_1^4w_2^2 + 6w_3c_4^4w_1w_2^2 + 306w_3w_1^2v_1^2w_2^2 - 12c_4^4w_1^2w_2^2 + 252w_3c_2^3w_1^3v_1^2w_2^2 + 6w_3w_1w_2^2 - 33w_3c_4^4w_1w_2^3 - 58w_3w_1^3v_4^4w_2^3 - 36w_3c_2^3w_1^3v_2w_3^2 - \\ & 12w_3w_1w_2^3 - 99w_3v_1^2w_2^3 + 6c_4^4w_1^3w_2 + 18w_3c_4^4w_1^3 - 154w_3w_1^2v_2w_3^2 - 12w_3c_2^3w_1w_2^2 - 36c_2^4w_2^3v_1^2w_2^2 - 432w_3c_2^3w_1v_1^2w_2^3 - 423w_3w_1v_4^4w_2^3 + \\ & 18w_3c_4^4w_2^3 + 171w_3w_1v_1^4 - 153w_3w_1^2v_1^2w_2 + 225w_3c_2^3w_1v_2^2w_2^2 + 207w_3c_2^3w_1^2w_3^2 + 45w_3c_2^3w_1w_2^3 - 432w_3c_2^3w_1^3v_1^2w_2^2 - 423w_3c_2^3w_1^4w_2 - 6c_2^4w_1^3w_2 + \\ & 333w_3w_1v_1^4w_2^2 + 18c_2^4w_2^3v_1^2w_2 + 252w_3c_2^3w_1^2w_3^2 - 22w_3c_2^3w_1^2w_2^3 + 28w_3w_1v_1^2w_2^3 + 6w_3w_1^2w_2^4 + 45w_3c_2^3w_1^3w_2^2 - 2w_3w_1^3w_2^3 + 6w_3c_4^4w_2^3w_2 + 171w_3v_1^4w_2^3 - \\ & 2w_3c_4^4w_1^3w_2^3 - 6c_2^4w_1w_2^3 - 24w_3c_2^3w_1^3 + 310w_3w_1^2v_1^4w_2^2 + 8w_3w_1^3v_2^2w_2^2 + 18c_2^4w_1v_1^2w_2^2 - 154w_3c_2^3w_1^2w_2^2 + 18w_3c_2^3w_1^2w_2^3 - 432w_3c_2^3w_1v_1^2w_2^2 - 666w_3w_1^2v_1^4w_2^2 - \\ & 99w_3c_2^3w_1^3v_2^2 + 6w_3w_1^3 + 14w_3c_4^4w_1^3w_2^2 + 333w_3c_2^3w_1^4v_2^2 - 33w_3c_2^3w_1^3w_2 + 14w_3c_2^3w_1^2w_3^2 + 6w_3w_1^3 - 12w_3w_1^2w_2 + 4w_3c_2^3w_1^3w_2^3 + 8w_3w_1^2w_2^3 - 12w_3c_2^3w_1^2w_2^4 + \\ & 225w_3w_1v_1^2w_2^2 + 225w_3c_2^3w_1^2v_2^2w_2 - 153w_3w_1v_1^2w_2^2 + 6c_4^4w_1w_2^3 - 6w_3c_4^4w_2^2w_2^2 + 207w_3c_2^3w_1^2v_2^2 + 225w_3w_1v_1^2w_2^3 - 12w_3w_1^2w_2^2 - 22w_3c_2^3w_1^3w_2^2 - 24w_3c_2^3w_1^2w_2^4 \end{aligned}$$

$$\begin{aligned}
C_{11} = & 9w_3^2 u_1^2 v_1 w_2 v_2^2 - 45w_3^2 v_1^2 w_3^2 v_2^2 + 12c_s^2 w_1^2 w_3^2 v_2^2 - 54w_3^2 c_s^4 w_1^2 w_2 + 12c_s^2 w_3^3 w_2^2 + 45w_3^2 w_1^3 v_1^2 v_2^2 + 18w_3 c_s^2 w_1^3 w_2^2 v_2^2 - 6w_2^2 w_1 w_3^2 v_2^2 + 5w_3^2 c_s^2 w_1^2 w_3^2 + \\
& 75w_3^2 c_s^2 w_1^2 w_2^2 - w_3^2 w_1^2 w_3^2 - 6w_3^2 w_1^3 w_2 + 21w_3^2 c_s^2 w_1 w_2^2 v_2^2 + 6w_3^2 c_s^4 w_1^3 w_2^2 + 72w_3^2 w_1 v_1^2 w_3^2 v_2^2 + 108w_3^2 w_1^2 v_1^4 w_2^2 + 18w_3 c_s^4 w_1 w_2^3 + 6w_3 c_s^2 w_1 w_2^2 v_2^2 - \\
& w_3^2 w_1^2 w_2^2 v_2^2 + 72w_3^2 c_s^2 w_1^3 v_1^2 w_2^2 - 6w_3^2 w_2^3 + 36c_s^4 w_1^2 w_2^3 - 72w_3^2 c_s^2 w_1^2 w_2^2 - 24w_3^2 c_s^2 w_1^2 w_2^2 + 2w_3^2 c_s^2 w_1^2 w_2^2 v_2^2 + 6w_3^2 c_s^2 w_1 w_2^2 v_2^2 - 6w_3^2 w_1^2 w_2 + 63w_3^2 w_1 v_1^2 w_2^2 + \\
& 126w_3^2 w_1^3 v_1^2 w_2 + 60w_3^2 c_s^2 w_1^2 w_2 - 36c_s^4 w_1^3 w_2^2 - 72w_3^2 c_s^2 w_1^3 - 99w_3 c_s^4 w_1^3 w_2 + 18w_3^2 c_s^2 w_1^3 v_1^2 + 6w_3^2 w_1^3 - 6w_3^2 c_s^4 w_1^3 v_2^2 - 297w_3^2 c_s^2 w_1 v_1^2 w_2^2 - 6w_3^2 w_1 w_2^2 v_2^2 - \\
& 6w_3^2 w_1^2 v_2^2 + 54c_s^2 v_4^2 w_3^3 - 54w_3^2 w_2^2 v_4^2 w_2 - 486w_3^2 c_s^2 w_1^3 v_1^2 w_2 - 5w_3^2 c_s^2 w_3^3 w_2^2 - 18w_3^2 w_1 v_1^2 w_2^3 - 72w_3^2 w_1^3 v_1^2 w_2^2 - 6w_3^2 c_s^2 w_1^2 w_2^2 v_2^2 - 6w_3 c_s^2 w_1^2 w_3^2 + \\
& w_3^2 w_1^2 w_2 + 405w_3^2 c_s^2 w_1^3 v_1^2 + 72w_3^2 c_s^4 w_1^2 w_2^2 - 54w_3^2 c_s^2 w_1 v_1^2 w_2^3 - 12c_s^2 w_1^2 w_3^2 v_2^2 + 6w_3^2 w_1^2 w_2 v_2^2 - 99w_3^2 w_1^3 v_2^2 - 9w_3^2 w_1 v_1^2 w_2^2 v_2^2 + 18w_3 c_s^2 w_1^2 w_3^2 + \\
& 24w_3^2 c_s^2 v_2^2 w_2^2 - 108w_3^2 w_1^2 v_2^2 w_2^2 + 6w_3 c_s^2 w_1^3 w_2 - 18w_3^2 c_s^4 w_3^3 - 18w_3^2 c_s^2 w_2^3 v_2^2 + 540w_3^2 c_s^2 w_1^2 w_2^2 v_2^2 + 6w_3^2 w_2^3 v_2^2 - 18w_3^2 c_s^4 w_1^2 w_2^3 + 24w_3^2 c_s^2 w_1^2 w_3^2 + \\
& 54w_3^2 w_1^3 v_4^2 + 27w_3^2 c_s^4 w_1^2 w_3^2 - 72w_3^2 c_s^2 w_1^2 v_2^2 w_2^2 + 54w_3^2 c_s^2 w_1^3 w_2^2 - 24w_3^2 c_s^2 w_1^2 v_3^2 w_2^2 + w_3^2 w_2^3 w_3^2 v_2^2 - 54w_3^2 w_1^4 v_1^2 w_3^2 - 18w_3 c_s^4 w_3^3 w_2 + 6w_3^2 c_s^2 w_1^2 w_2^2 - \\
& 54w_3 c_s^2 w_1^2 w_3^2 - 2w_3^2 c_s^2 w_1^2 w_2^3 v_2^2 + 6w_3^2 w_1 w_2^3 - 21w_3^2 c_s^2 w_1^2 w_2^3 v_2^2 + 12w_3^2 c_s^2 w_1^2 w_2^2 - 18w_3 c_s^2 w_1^2 w_2^3 v_2^2 - 54w_3^2 w_1^4 v_1^2 w_2^2 - 12c_s^2 w_1^3 w_2^2 v_2^2 - 6w_3 c_s^2 w_1^2 w_2 v_2^2 - \\
& 243w_3^2 c_s^2 w_1^2 w_2 - 54w_3^2 w_1^3 v_1^2 w_2 + 90w_3^2 c_s^4 w_3^3 + 135w_3^2 c_s^2 v_2^2 w_3^2 - 3w_3^2 c_s^2 w_1 w_2^3 + 45w_3^2 v_1^2 w_2^2 - 9w_3^2 v_2^2 w_3^2 - 18w_3 c_s^2 w_1^2 w_2^2 + 6w_3^2 w_1 w_2^3
\end{aligned}$$

$$\begin{aligned}
C_{12} = & 54c_s^2w_1^3w_2^2 + 396w_3w_1^2v_1^2w_2^2 + 90w_3w_1w_2^3 + 45w_3w_1w_2^3 + 198w_3v_1^2w_3^2 + 18w_3^2w_2v_2^2 - 162w_3c_s^2w_1w_2^2 - 18w_1w_2^3 - 36w_3w_2^3v_2^2 - \\
& 45w_3w_1^3w_2v_2^2 - 10w_3w_1^2w_3^2v_2^2 + 18w_1w_3^2v_2^2 - 198w_3w_1^2v_1^2w_2 - 27w_3c_s^2w_1w_2^3 - 54c_s^2w_3^3w_2 - 54c_s^2w_1^2w_3^2 - 30w_3c_s^2w_1^2w_3^2 + 90w_3w_1^2w_2 - \\
& 297w_3c_s^2w_1^3w_2 + 36w_3w_1^3v_2^2 + 45w_3w_1w_3^2v_2^2 + 54c_s^2w_1w_2^3 + 270w_3c_s^2w_3^3 - 10w_3w_1^3w_2^2 + 324w_3c_s^2w_2^2w_2^2 - 18w_3^2w_2v_2^2 + 18w_1^3w_2 - 18w_2^2w_3^2v_2^2 + \\
& 18w_1^2w_3^2 + 198w_3w_1^3v_1^2 - 126w_3w_1^3 + 10w_3w_1^3w_2^2v_2^2 - 54w_3w_1^3 + 135w_3w_1^3w_2 + 10w_3w_1^2w_3^2 - 162w_3c_s^2w_1w_2^2 - 198w_3w_1^3v_1^2w_2 - 18w_3^2w_2^2 - \\
& 198w_3w_1v_1^2w_2^2 - 198w_3w_1v_1^2w_3^2 - 180w_3w_1^2w_2^2 + 30w_3c_s^2w_1^3w_2^2 + 54w_3c_s^2w_3^2
\end{aligned}$$

$$\begin{aligned}
C_{13} = & 27w_3^2w_1^2v_2^2w_2v_2^2 + 24w_3^2c_s^2w_1^3v_1^2w_3^2 - 135w_3^2v_2^2w_3^2v_2^2 + 7w_3^2w_1^4v_1w_3^2 + 72w_3w_1^3v_1^4w_2^2 - 30w_3^2c_4^4w_1^2w_2 + 135w_3^2w_1^3v_1^2v_2^2 - 36w_3c_s^2w_1^3w_2^2v_2^2 - 18w_3^2w_1w_3^2v_2^2 - 2w_3^2c_4^4w_1^3w_2^2 - 8w_3^2w_1^3v_1^2w_3^2 - 12w_3^2c_2^2w_1^2w_3^2 + 36w_3w_1^2v_1^2w_2^2 + 21w_3^2c_2^2w_1^3w_2^2 - w_3^2w_1^2w_3^2 + 198w_3c_s^2w_1^3v_1^2w_2^2 - 6w_3^2w_1^3w_2^2
\end{aligned}$$

$$\begin{aligned}
& 9w_3^2c_s^2w_1w_3^2v_2^2 - w_3^2c_s^4w_1^3w_2^2 + 216w_3^2w_1v_1^2w_3^2v_2^2 - 54w_3^2w_1v_1^4w_2^2 + 18w_3c_s^4w_1w_3^2 - 78w_3w_3^3v_1^4w_2^3 - 54w_3c_s^2w_1w_3^2v_2^2 - 3w_3^2w_1^3w_2^2v_2^2 + \\
& 84w_3^2c_s^2w_1^3v_1^2w_2^2 - 198w_3c_s^2w_1^3v_1^2w_2^3 - 6w_3^2w_1^3 - 12w_3^2c_s^2w_1^2w_2^2 - 43w_3^2w_1^3v_1^2w_2^2 - 6w_3^2c_s^2w_1^3w_2^2v_2^2 - 144w_3w_1^2v_1^2w_3^2 + 18w_3c_s^2w_1w_2^2v_2^2 - 6w_3^2w_1^2w_2 - \\
& 27w_3^2w_1v_1^2w_2^2 + 54w_3c_s^2w_1v_1^2w_2^3 + 135w_3^2c_s^2w_1^2v_1^2w_2 + 36w_3^2c_s^2w_1^2w_2^2 - 24w_3^2c_s^2w_1^2v_1^2w_2^3 - 18w_3w_1v_1^4w_2^3 - 15w_3c_s^4w_1^2w_2 + 18w_3c_s^2w_1^2v_1^2w_2^3 + \\
& 6w_3^2w_1^3 + 29w_3c_s^4w_1^2w_2^3 + 99w_3c_s^2w_1^2v_1^2w_2^2 - 18w_3w_1v_1^2w_2^2 - 18w_3w_1^3v_1^2w_2^2 + 36w_3^2c_s^2w_1^2v_1^2w_2^3 + 36w_3^2c_s^2w_1^2v_1^2w_2^4 - 297w_3^2c_s^2w_1^2v_1^2w_2^2 - 108w_1^2v_1^4w_2^3 - \\
& 9w_3^2w_1v_1^2w_2^3 - 216w_3^2w_1^3v_1^2w_2^2 - 324w_3^2c_s^2w_1^2v_1^2w_2^3 - 54w_3^2c_s^2w_1^2v_1^2w_2^2 - 18w_3c_s^2w_1^2v_1^2w_2^3 + w_3^2w_1^3w_2^2 - 54w_3c_s^2w_1^2v_1^2w_2^3 + 189w_3^2c_s^2w_1^2v_1^2w_2^3 - 18w_3w_1^3v_1^2w_2^2 + \\
& 12w_3^2c_s^4w_1^2w_2^2 - 171w_3^2c_s^2w_1v_1^2w_2^3 - 72w_3^2c_s^2w_1^2v_1^2w_2^3 + 18w_3c_s^2w_1^2v_1^2w_2^2 - 81w_3^2c_s^2w_1^2v_1^2w_2^3 - 27w_3^2c_s^2w_1^2v_1^2w_2^2 + 306w_3c_s^2w_1^2v_1^2w_2^3 + 18w_3^2c_s^2w_1^2v_1^2w_2^2 + \\
& 30w_3c_s^2c_1^2w_2^3 + 78w_3w_1^3v_1^2w_2^3 + 72w_3^2c_s^2w_1^2v_1^2w_2^2 + 54w_3^2c_s^2w_1^2v_1^2w_2^2 - 6w_3c_s^2w_1^2v_1^2w_2^2 + 30w_3^2c_s^2w_1^2v_1^2w_2^3 + 18w_3^2c_s^2w_1^2v_1^2w_2^2 + 12w_3c_s^4w_1^3w_2^3 - 144w_3^2c_s^2w_1^2v_1^2w_2^2 + \\
& 18w_3^2w_1^2v_2^2 + 144w_3w_1^2v_1^4w_2^3 + 6w_3^2c_s^2w_1^2v_1^2w_2^2 + 19w_3^2c_s^2w_1^2v_1^4w_2^2 - 72w_3w_1^3v_1^2w_2^2 - 24w_3c_s^2w_1^2v_1^2w_2^2 + 17w_3^2c_s^2w_1^2v_1^2w_2^3 - 72w_3c_s^2w_1^2v_1^2w_2^2 + 36w_3^2w_1^3v_1^2 - \\
& 36w_3w_1^2v_1^2w_2^2 + 8w_3^2c_s^3v_1^2w_2^3 - 57w_3^2c_s^2w_1^2v_1^2w_2^3 + 30w_3^2c_s^2w_1^2v_1^2w_2^2 + 36w_3c_s^2w_1^2v_1^2w_2^2 - 18w_3c_s^2w_1^2v_1^2w_2^3 - 72w_3^2c_s^2w_1^2v_1^2w_2^2 + 3w_3^2w_1^2v_1^2w_2^2 - 63w_2^2w_1v_1^4w_2^3 + \\
& 6w_3c_s^4w_1^2w_2^3 + 18w_3^2c_s^2w_1^2v_1^2w_2^2 + 108w_1^2v_1^2w_2^2 - 30w_3c_s^2w_1^2v_1^2w_2^3 + 6w_3^2c_s^2w_1^2v_1^2w_2^2 - 12w_3c_s^2w_1^2v_1^2w_2^3 + 6w_3^2w_1^2v_1^2w_2^2 + 18w_3w_1^3v_1^2w_2^2 - 9w_3^2c_s^2w_1^2v_1^2w_2^2 + \\
& 216c_s^2w_1^2v_1^2w_2^3 - 12w_3^2c_s^2w_1^2v_1^2w_2^2 + 72w_3^2c_s^2w_1^2v_1^2w_2^3 + 24w_3c_s^2w_1^2v_1^2w_2^2 + 36w_3c_s^2w_1^2v_1^2w_2^3 + 36w_3w_1^2v_1^4w_2^2 + 18w_3c_s^2w_1^2v_1^2w_2^2 + 81w_3^2c_s^2w_1^2v_1^2w_2^2 - 63w_3^2c_s^2w_1^2v_1^2w_2^3 + \\
& 18w_3^2c_s^4w_1^3 - 36w_3^2c_s^2v_1^4w_2^2 + 63w_3^2c_s^2v_1^2w_2^3 + 18w_3w_1v_1^2w_2^3 - 108w_3^2c_s^2v_1^2w_2^2 + 15w_3^2c_s^2w_1v_1^2w_2^3 - 45w_3^2c_s^2v_1^2w_2^2 + 9w_3^2v_1^2w_2^3 + 18w_3c_s^2w_1^2v_1^2w_2^3 + 6w_3^2w_1w_2^3
\end{aligned}$$

$$\begin{aligned}
C_{14} = & -45w_3^2w_1^2v_1^2w_2v_2^2 + 2w_3^2c_s^2w_1^2v_1^2w_2^3 + 45w_3^2v_1^2w_2v_2^2 - 18w_3^2c_s^4w_1^2w_2 + 12w_3^2c_s^2w_1^2w_2 + 45w_3^2w_1^3v_1^2v_2^2 - 9w_3^2w_1w_2^3v_2^2 + 10w_3^2c_s^4w_1^3w_2^3 - \\
& 27w_3^2w_1^2w_2^4 + 41w_3^2c_s^2w_1^2w_2^3 + 93w_3^2c_s^2w_1^3w_2 - 54w_3^2c_s^2w_1^2w_2v_2^2 - 6w_3^2w_1^3w_2 + 189w_3^2c_s^2w_1w_2^2v_2^2 + 35w_3^2c_s^4w_1^2w_2^2v_2^2 - 45w_3^2w_1v_1^2w_2^3v_2^2 - 24c_s^2w_1^3w_2^3 - \\
& 90w_3c_s^2w_1w_2^3 - 27w_3^2w_1^2w_2^2v_2^2 + 8w_3^2c_s^2w_1^2v_1^2w_2^2 - 12w_3c_s^2w_1^2v_1^2w_2^3 + 12w_3^2w_1^2w_2^2 + 6w_3^2w_1^3 - 108w_3^2c_s^2w_1^2w_2^3 - 54w_3^2c_s^2w_1^2w_2^2v_2^2 + 138w_3^2c_s^2w_1^2w_2^2v_2^2 + \\
& 27w_3^2c_s^2w_1^2w_2^2v_2^2 - 6w_3^2w_1^2w_2^2 + 6w_3^2w_1v_1^2w_2^2 - 2w_3^2c_s^2w_1^2w_2^3 - 30w_3^2c_s^2w_1v_1^2w_2^3 - 54w_3^2c_s^2w_1^2v_1^2w_2^2 + 6w_3^2w_1^2v_1^2w_2^2 + 24w_3^2c_s^2w_1^2v_1^2w_2^2 - 36c_s^4w_1^2w_2^2 - 72w_3^2c_s^2w_1^2v_1^2w_2^3 - \\
& 117w_3^2c_s^4w_1^2w_2^2 + 405w_3^2c_s^2w_1^2v_1^2w_2^3 + 6w_3^2w_1^3 - 91w_3^2c_s^2w_1^2v_1^2w_2^2 - 30w_3^2c_s^2w_1^2v_1^2w_2^3 - 9w_3^2w_1^2v_2^2 - 21w_3^2c_s^2w_1^2v_1^2w_2^2 + 72c_s^4w_1^2w_2^3 - \\
& 25w_3^2c_s^2w_1^2v_1^2w_2^2 + 6w_3^2w_1^2v_1^2w_2^3 - 45w_3^2c_s^2w_1^2v_1^2w_2^2 - 36c_s^2w_1^2v_1^2w_2^2 - 297w_3^2c_s^2w_1^2v_1^2w_2^2 + 30w_3c_s^2w_1^2v_1^2w_2^3 - 6w_3^2c_s^2w_1^2v_1^2w_2^2 + 18w_3^2c_s^2w_1^2v_1^2w_2^3 + 54w_3^2c_s^2w_1^2v_1^2w_2^2 + \\
& 3w_3^2c_s^2w_1^2v_1^2w_2^3 + 36c_s^2w_1^2v_1^2w_2^3 + 99w_3^2c_s^2w_1^2v_1^2w_2^2 - 6w_3^2w_1^2v_1^2v_1^2 - 45w_3^2c_s^2w_1^2v_1^2w_2^2 + 48w_3^2c_s^2w_1^2v_1^2w_2^3 + 270w_3^2c_s^2w_1^2v_1^2w_2^2 - 48w_3^2c_s^2w_1^2v_1^2w_2^3 + \\
& 54w_3^2c_s^2w_1^2v_1^2w_2^2 - 12w_3^2c_s^2w_1^2v_1^2w_2^3 + 6w_3c_s^2w_1^2v_1^2w_2^2 + 18w_3^2c_s^2w_1^2v_1^2w_2^3 - 135w_3^2c_s^2w_1^2v_1^2w_2^2 - 36w_3c_s^2w_1^2v_1^2w_2^3 + 18w_3^2c_s^2w_1^2v_1^2w_2^2 - 90w_3^2c_s^2w_1^2v_1^2w_2^3 + 9w_3^2w_1^2v_1^2w_2^2 - \\
& 90w_3^2c_s^4w_1^2w_2^2 - 36w_3c_s^2w_1^2w_2^2 + 36w_3c_s^2w_1^2v_1^2w_2^2 - 54w_3^2c_s^2w_1^2w_2v_2^2 + 63w_3^2c_s^2w_1^2v_1^2w_2^3 - 24w_3^2c_s^2w_1^2v_1^2w_2^2 + 27w_3^2c_s^2w_1^2v_1^2w_2^2 - 18w_3c_s^2w_1^2v_1^2w_2^2 + 99w_3^2c_s^2w_1^2w_2^2 + \\
& 144w_3c_s^4w_1^2w_2^2 - 138w_3^2c_s^2w_1^2v_1^2w_2^2 + 12w_3c_s^2w_1^2v_1^2w_2^3 - 6w_3^2w_1^2w_2^2 - 459w_3^2c_s^2w_1^2v_1^2w_2^2 + 24c_s^2w_1^2v_1^2w_2^3 + 48w_3^2c_s^2w_1^2v_1^2w_2^2 + 54w_3^2w_1^2v_1^2w_2^3 + 108w_3c_s^4w_1^2w_2^2 + \\
& 90w_3^2w_1^2v_1^2w_2^2 - 6w_3^2c_s^2w_1^2v_1^2w_2^2 + 90w_3^2c_s^2w_1^2v_1^2w_2^3 - 12c_s^2w_1^2v_1^2w_2^2 - 39w_3^2c_s^2w_1^2v_1^2w_2^3 + 6w_3^2c_s^2w_1^2v_1^2w_2^2 - 6w_3^2v_1^2w_2^3 + 27w_3^2c_s^2w_1^2v_1^2w_2^2 - 6w_3^2w_1w_2^3
\end{aligned}$$

$$\begin{aligned}
C_{15} = & -135w_3^2w_1^2v_1^2w_2v_2^2 + 135w_3^2v_1^2w_2^3v_2^2 - w_3^2w_1^2v_1^4w_2^3v_2^2 + 108w_3^2c_s^2w_1^2w_2v_2^2 - 30w_3^2c_s^4w_1^2w_2v_2^2 + 144w_3^2c_s^2w_1^3v_1^2v_2^2 - 2w_3^2c_s^4w_1^2w_2^3v_2^2 - \\
& 9w_3^2w_1^2w_2^3v_2^2 + 2w_3^2c_s^2w_1^2w_2^3v_2^2 + 21w_3^2c_s^2w_1^2w_2^4v_2^2 - 45w_3^2c_s^2w_1^2w_2v_2^4 - 90w_3c_s^2w_1^2v_1^2w_2^2v_2^2 + 2w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 - 135w_3^2w_1^2v_1^2w_2^2v_2^2 + \\
& 30w_3c_s^4w_1^2w_2^3v_2^2 + 18w_3c_s^2w_1^2w_2^2v_2^2 - 9w_3^2c_s^2w_1^2w_2^2v_2^2 + 6w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 272w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 12w_3^2c_s^2w_1^2w_2^2v_2^2 + 6w_3^2w_1^2v_1^2w_2^2v_2^2 - 42w_3^2c_s^2w_1^2w_2^2v_2^2 - w_3^2w_1^2v_1^2w_2^2v_2^2 + 30w_3^2c_s^2w_1^2w_2^2v_2^2 + \\
& 36w_3^2w_1^2w_2^2v_2^2 - 45w_3^2c_s^2w_1^2w_2^2v_2^2 + 18w_3w_1^2w_2v_2^2 - 6w_3^2w_1^2v_1^2w_2^2v_2^2 + 126w_3c_s^2w_1^2v_1^2w_2^2v_2^2 - 36w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 18w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 36w_3^2c_s^2w_1^2w_2^2v_2^2 - \\
& 24w_3^2c_s^2w_1^2v_1^2v_1^2 - 15w_3^2c_s^4w_1^2w_2^2 + 189w_3^2c_s^2w_1^2v_1^2v_2^2 + 18w_3w_1^2w_2v_2^2 + 6w_3^2w_1^3 - 10w_3^2c_s^4w_1^2w_2^2 - 90w_3^2c_s^2w_1^2v_1^2w_2^2 + 45w_3^2c_s^2w_1^2w_2^2v_2^2 - 81w_3^2c_s^2w_1^2v_1^2w_2^2 + 54w_3c_s^2w_1^2w_2^2v_2^2 - \\
& 9w_3^2c_s^2w_1^2v_1^2w_2^2 + 54w_3^2c_s^2w_1^2v_1^2w_2^3v_2^2 - 2w_3^2c_s^2w_1^2v_1^2w_2^2 + 18w_3^2c_s^2w_1^2v_1^2w_2^3v_2^2 - 30w_3c_s^2w_1^2w_2^2v_2^2 + 18w_3c_s^2w_1^2v_1^2w_2^2 + 18w_3^2c_s^2w_1^2v_1^2w_2^3v_2^2 + \\
& 30w_3^2c_s^4w_1^2w_2^2v_2^2 + 9w_3^2c_s^2w_1^2v_1^2w_2^2 + 36w_3^2c_s^2w_1^2v_1^2w_2^3v_2^2 - 18w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 - 135w_3^2c_s^2w_1^2v_1^2w_2^3v_2^2 - 18w_3w_1^2v_1^2w_2^2v_2^2 - 198w_3c_s^2w_1^2v_1^2w_2^2v_2^2 + 108w_3^2c_s^2w_1^2w_2^2v_2^2 - \\
& 54w_3^2c_s^2w_1^2v_1^2w_2^3v_2^2 + 60w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 - 36w_3^2c_s^2w_1^2v_1^2w_2^3v_2^2 - 6w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 - 30w_3^2c_s^2w_1^2v_1^2w_2^4v_2^2 - 36w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 - 18w_3w_1^2v_1^2w_2^3v_2^2 - 63w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 24w_3^2c_s^2w_1^2v_1^2w_2^3v_2^2 + \\
& 72w_3^2c_s^2w_1^2v_1^2w_2^2 - 90w_3^2c_s^2w_1^2w_2^2v_2^2 - 9w_3^2w_1^2w_2^2v_2^2 - 30w_3^2c_s^2w_1^2w_2^2v_2^2 - w_3^2w_1^2v_1^2w_2^2v_2^2 + 144w_3c_s^2w_1^2v_1^2w_2^2v_2^2 - 36w_1^2w_2^2v_2^2 + 45w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 - \\
& 54w_3^2c_s^2w_1^2w_2^2v_2^2 - 6w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 36w_3c_s^2w_1^2w_2^2v_2^2 - 36w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 9w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 6w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 90w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 - 60w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 36w_1^2w_2^2v_2^2 + 45w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 - \\
& 54w_3^2c_s^2w_1^2v_1^2w_2^2 - 6w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 216w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 36w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 45w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 60w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 - 144w_3c_s^2w_1^2v_1^2w_2^2v_2^2 - 108w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 270w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 - 54w_3c_s^2w_1^2v_1^2w_2^2v_2^2 + 9w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 36w_3c_s^2w_1^2v_1^2w_2^2v_2^2 - 6w_3^2w_1w_2^2v_2^2
\end{aligned}$$

$$\begin{aligned}
C_{16} = & -18w_1v_1^2w_2^3 + 54w_3w_1^2v_1^2w_2^2 - 18w_3w_1w_2^2 - 18w_3^2v_1^2w_2^2 - 81w_3w_1w_2^3 + 36w_3v_1^2w_2^3 - 54w_3c_s^2w_1w_2^2 + 18w_1w_2^3 - 198w_3w_1^2v_1^2w_2^2 + 108c_s^2w_1^2w_2^2 - 216w_3w_1^2w_2v_2^2 - \\
& 100w_3w_1^2w_2^2v_2^2 - 36w_3w_1^2v_1^2w_2^2 + 135w_3c_s^2w_1^2w_2^3 - 54c_s^2w_1^2w_2^2 - 36w_1^2w_2^2 - 84w_3c_s^2w_1^2w_2^3 + 90w_3w_1^2w_2^2v_2^2 - 297w_3c_s^2w_1^2w_2^3v_2^2 + 198w_3w_1^2v_1^2w_2^2 + 216w_3w_1^2w_2^2v_2^2 - \\
& 54c_s^2w_1^2w_2^3 + 270w_3c_s^2w_1^2w_2^3 - 46w_3w_1^2w_2^2 + 162w_3c_s^2w_1^2w_2^2 + 18w_3^2w_1^2w_2^3 + 36w_3c_s^2w_1^2w_2^2v_2^2 - 126w_3w_1^2 + 100w_3w_1^2w_2^2v_2^2 + 162w_3w_1^2w_2^2v_2^2 + 54w_3c_s^2w_1^2w_2^3 + 135w_3c_s^2w_1^2w_2^2 + \\
& 46w_3w_1^2w_2^3 - 162w_3c_s^2w_1^2w_2^2v_2^2 - 27w_3w_1^2v_1^2w_2^2 - 36w_3w_1v_1^2w_2^2v_2^2 - 162w_3c_s^2w_1^2w_2^2v_2^2 + 36w_1^2v_1^2w_2^2v_2^2 - 27w_3w_1v_1^2w_2^2v_2^2 - 54w_3w_1^2w_2^2v_2^2 + 84w_3c_s^2w_1^2w_2^2v_2^2 - 54w_3c_s^2w_1^2w_2^2v_2^2
\end{aligned}$$

$$\begin{aligned}
C_{17} = & -3w_3^2w_1^2v_1^4w_2^3v_2^2 - 24w_3^2c_s^2w_1^2w_2^2v_2^2 + 18w_3^2c_s^4w_1^2w_2^2v_2^2 - 18w_3c_s^2w_1^2v_1^2w_2^2v_2^2 - 102w_3^2c_s^2w_1^2w_2^2v_2^2 + 6w_3^2c_s^2w_1^2w_2^2v_2^2 - 39w_3^2w_1^2w_2^2v_2^2 + 81w_3^2c_s^2w_1^2w_2^2v_2^2 + \\
& 123w_3^2c_s^2w_1^2w_2^2v_2^2 - 7w_3^2w_1^2w_2^3v_2^2 - 90w_3^2c_s^2w_1^2w_2^2v_2^2 - 12w_3^2w_1^2w_2^2v_2^2 + 489w_3^2c_s^2w_1^2w_2^2v_2^2 + 72w_3^2c_s^2w_1^2w_2^2v_2^2 - 12c_s^2w_1^2w_2^2v_2^2 - 90w_3c_s^2w_1^2w_2^2v_2^2 - 30w_3c_s^2w_1^2w_2^2v_2^2 - \\
& 46w_3^2c_s^2w_1^2w_2^2v_2^2 + 9w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 - 72w_3^2c_s^2w_1^2w_2^2v_2^2 - 12w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 - 3w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 - 197w_3^2c_s^2w_1^2w_2^2v_2^2 - 165w_3^2c_s^2w_1^2w_2^2v_2^2 + 6w_3^2w_1^2w_2^2v_2^2 - \\
& 2w_3^2c_s^2w_1^2w_2^2v_2^2 - 45w_3^2c_s^2w_1^2w_2^2v_2^2 - 24w_3^2c_s^2w_1^2w_2^2v_2^2 - 72w_3^2c_s^2w_1^2w_2^2v_2^2 - 153w_3^2c_s^2w_1^2w_2^2v_2^2 + 261w_3^2c_s^2w_1^2w_2^2v_2^2 + 6w_3^2w_1^2w_2^2v_2^2 - 138w_3^2c_s^2w_1^2w_2^2v_2^2 - 6w_3^2c_s^2w_1^2w_2^2v_2^2 + 51w_3^2w_1^2w_2^2v_2^2 - \\
& 51w_3^2w_1^2w_2^2v_2^2 + 36c_s^4w_1^2w_2^2v_2^2 + 2w_3^2c_s^2w_1^2w_2^2v_2^2 - 59w_3^2c_s^2w_1^2w_2^2v_2^2 + 141w_3^2c_s^2w_1^2w_2^2v_2^2 + 30w_3c_s^2w_1^2w_2^2v_2^2 + 7w_3^2c_s^2w_1^2w_2^2v_2^2 + 36w_3^2c_s^2w_1^2w_2^2v_2^2 + 24c_s^2w_1^2w_2^2v_2^2 - 51w_3^2w_1^2w_2^2v_2^2 - \\
& 45w_3^2w_1^2w_2^2v_2^2 + 12w_3^2c_s^2w_1^2w_2^2v_2^2 - 42w_3^2c_s^2w_1^2w_2^2v_2^2 + 45w_3^2c_s^2w_1^2w_2^2v_2^2 - 6w_3^2c_s^2w_1^2w_2^2v_2^2 - 90w_3^2c_s^2w_1^2w_2^2v_2^2 - 261w_3^2c_s^2w_1^2w_2^2v_2^2 - 18w_3^2c_s^2w_1^2w_2^2v_2^2 + 51w_3^2c_s^2w_1^2w_2^2v_2^2 - 90w_3^2c_s^2w_1^2w_2^2v_2^2 + \\
& 3w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 - 24w_3c_s^2w_1^2v_1^2w_2^2v_2^2 + 45w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 225w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 - 9w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 24w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 - 54w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 12c_s^2w_1^2w_2^2v_2^2 + \\
& 46w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 18w_3c_s^2w_1^2v_1^2w_2^2v_2^2 + 102w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 - 219w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 6w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 - 6w_3^2w_1^2w_2^2v_2^2 - 465w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + 48w_3^2c_s^2w_1^2v_1^2w_2^2v_2^2 + \\
& 90w_3^2w_1^2v_1^2w_2^2v_2^2 + 72w_3c_s^2w_1^2v_1^2w_2^2v_2^2 + 42w_3c_s^2w_1^2v_1^2w_2^2v_2^2 + 6w_3^2c_s^$$

$$\begin{aligned}
C_{20} = & -3\omega_1\omega_2v_2^4 + c_s^4\omega_1 + 12c_s^2\omega_2v_2^2 + 3\omega_2v_2^4 - 3\omega_1v_2^2 + c_s^4\omega_2 - 12c_s^2\omega_1\omega_2v_2^2 + 12c_s^2\omega_1v_2^2 - c_s^4\omega_1\omega_2 + 3\omega_1v_2^4 - 3\omega_2v_2^2 - c_s^2\omega_1 + c_s^2\omega_1\omega_2 - c_s^2\omega_2 + 3\omega_1\omega_2v_2^2 \\
C_{21} = & 12\omega_3^2\omega_1^2v_2^2 - 12\omega_3^2c_s^2\omega_1^2v_2^2 + 12\omega_3^2c_s^2\omega_1 - 24\omega_3c_s^4\omega_1^2 + 72\omega_3\omega_1^2v_2^4 + 108c_s^2\omega_1^2v_2^2 + 24\omega_3^2c_s^4 + 6\omega_3c_s^4\omega_1^3 + 36\omega_3^3v_2^4 - 30\omega_3\omega_1^3v_2^4 + \omega_3^2c_s^2\omega_1^3 + \\
& 6\omega_3^2c_s^2\omega_1^3v_2^2 - 3\omega_3^2\omega_1^3v_2^2 + 24\omega_3c_s^4\omega_1 - 72\omega_3^2v_2^4 - 216c_s^2\omega_1^2v_2^2 + 72\omega_3c_s^2\omega_1v_2^2 - 8\omega_3^2c_s^2\omega_1^2 + 3\omega_3^2\omega_1^3v_2^4 + 144\omega_3c_s^2\omega_1^2v_2^2 + 30\omega_3\omega_1^3v_2^2 - 6\omega_3^2c_s^2\omega_1^3 - \\
& 48\omega_3^2c_s^4\omega_1 + 24\omega_3c_s^2\omega_1^2 + 72\omega_1^2v_2^2 - 24\omega_3c_s^2\omega_1 - 72\omega_3c_s^2\omega_1^3v_2^2 - 72\omega_3\omega_1^2v_2^2 - 12\omega_3^2\omega_1^2v_2^4 + 24\omega_3^2c_s^4\omega_1^2 - 36\omega_3^2c_s^2\omega_1v_2^2 - 36\omega_3^3v_2^2 - 3\omega_3^2c_s^4\omega_1^3
\end{aligned}$$

#### 2.7.4 Conservation of momentum: $\rho v_2$

 attached text file: output\_d2q9\_nse\_culbm2\_symbolic\_pde\_02.txt

$$\begin{aligned}
& v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{v_1 v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho v_2 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\rho v_1 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_1} + (c_s^2 + v_2^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{2\rho v_2 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_1) \frac{c_s^2 \delta_l^2}{2\delta_t \omega_1} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\
& (-3v_1^2 \omega_2 + 3\omega_1 v_1^2 - \omega_1 + 3c_s^2 \omega_1 + c_s^2 \omega_1 \omega_2 + \omega_2 - 5c_s^2 \omega_2) \frac{\delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (\omega_1 - \omega_2) \frac{3\rho v_1 \delta_l^2}{\delta_t \omega_1 \omega_2} \frac{\partial v_1}{\partial x_1} \frac{\partial v_1}{\partial x_2} + \\
& (-3v_1^2 \omega_2 + 3\omega_1 v_1^2 - \omega_1 + c_s^2 \omega_1 + \omega_2 - c_s^2 \omega_2) \frac{\delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_1} + \\
& (\omega_1 \omega_2 + 3\omega_1 v_1^2 - \omega_1 + 3\omega_2 v_2^2 + 2c_s^2 \omega_1 - 2c_s^2 \omega_1 \omega_2 - \omega_2 + 2c_s^2 \omega_2 - 3\omega_1 \omega_2 v_2^2) \frac{\delta_l^2}{\delta_t \omega_1 \omega_2} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + \\
& (-\omega_1 \omega_2 + \omega_1 + \omega_2) \frac{3\rho v_2 \delta_l^2}{\delta_t \omega_1 \omega_2} \left( \frac{\partial v_2}{\partial x_2} \right)^2 + (-2 + \omega_1) \frac{c_s^2 \rho \delta_l^2}{2\delta_t \omega_1} \frac{\partial^2 v_2}{\partial x_1^2} + \\
& (-v_1^2 \omega_2 + \omega_1 v_1^2 - \omega_1 + 3c_s^2 \omega_1 + \omega_2 - 3c_s^2 \omega_2) \frac{v_1 \delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial^2 \rho}{\partial x_1 \partial x_2} + \\
& (-3v_1^2 \omega_2 + 3\omega_1 v_1^2 - \omega_1 + c_s^2 \omega_1 + c_s^2 \omega_1 \omega_2 + \omega_2 - 3c_s^2 \omega_2) \frac{\rho \delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + \\
& (\omega_1 \omega_2 + \omega_1 v_1^2 - \omega_1 + \omega_2 v_2^2 + 3c_s^2 \omega_1 - 3c_s^2 \omega_1 \omega_2 - \omega_2 + 3c_s^2 \omega_2 - \omega_1 \omega_2 v_2^2) \frac{v_2 \delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial^2 \rho}{\partial x_2^2} + \\
& (\omega_1 \omega_2 + 3\omega_1 v_1^2 - \omega_1 + 3\omega_2 v_2^2 + c_s^2 \omega_1 - c_s^2 \omega_1 \omega_2 - \omega_2 + c_s^2 \omega_2 - 3\omega_1 \omega_2 v_2^2) \frac{\rho \delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial^2 v_2}{\partial x_2^2} + (-1 + 3c_s^2 + v_1^2) \frac{v_1 v_2 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_3^3} + \\
& (-1 + c_s^2 + 3v_1^2) \frac{\rho v_2 \delta_l^3}{12\delta_t} \frac{\partial^3 v_1}{\partial x_3^3} + C_1 \frac{\rho v_1 \delta_l^3}{6\omega_3 \delta_t \omega_1} \frac{\partial^3 v_2}{\partial x_3^3} + C_2 \frac{\delta_l^3}{12\delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + C_3 \frac{\rho v_1 \delta_l^3}{4\delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} - \frac{c_s^2 \rho v_2 \delta_l^3}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + \\
& C_4 \frac{3v_1 v_2 \delta_l^3}{4\delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + C_5 \frac{\rho v_2 \delta_l^3}{12\omega_3 \delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_6 \frac{\rho v_1 \delta_l^3}{4\omega_3 \delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_7 \frac{\delta_l^3}{12\delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 \rho}{\partial x_3^3} + C_8 \frac{\rho v_2 \delta_l^3}{12\delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 v_2}{\partial x_3^3} + \\
& C_9 \frac{v_2 \delta_l^4}{24\delta_t \omega_1 \omega_2} \frac{\partial^4 \rho}{\partial x_4^4} + (5v_1^2 \omega_2 + 2\omega_1 \omega_2 - 5\omega_1 v_1^2 \omega_2 + 5\omega_1 v_1^2 - 2\omega_1 + 3c_s^2 \omega_1 - 3c_s^2 \omega_1 \omega_2 - 2\omega_2 + 3c_s^2 \omega_2) \frac{\rho v_1 v_2 \delta_l^4}{12\delta_t \omega_1 \omega_2} \frac{\partial^4 v_1}{\partial x_4^4} + \\
& C_{10} \frac{\rho \delta_l^4}{24\omega_3^2 \delta_t \omega_1^3} \frac{\partial^4 v_2}{\partial x_1^4} + C_{11} \frac{v_1 \delta_l^4}{24\omega_3^2 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 \rho}{\partial x_3^3 \partial x_2} + C_{12} \frac{\rho \delta_l^4}{24\omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_1}{\partial x_3^3 \partial x_2} + C_{13} \frac{\rho v_1 v_2 \delta_l^4}{24\omega_3 \delta_t \omega_1 \omega_2} \frac{\partial^4 v_2}{\partial x_3^3 \partial x_2} + \\
& C_{14} \frac{v_2 \delta_l^4}{24\omega_3^2 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2^2} + C_{15} \frac{\rho v_1 v_2 \delta_l^4}{24\omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2^2} + C_{16} \frac{\rho \delta_l^4}{24\omega_3^2 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2^2} + C_{17} \frac{v_1 \delta_l^4}{24\omega_3^2 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + \\
& C_{18} \frac{\rho \delta_l^4}{24\omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{19} \frac{\rho v_1 v_2 \delta_l^4}{24\omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + C_{20} \frac{v_2 \delta_l^4}{24\omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 \rho}{\partial x_2^4} + C_{21} \frac{\rho \delta_l^4}{24\omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 = & 6 + 9\omega_3 c_s^2 - \omega_3 \omega_1 v_1^2 - 3\omega_3 - 18c_s^2 + 3\omega_1 v_1^2 - 3\omega_1 + 9c_s^2 \omega_1 - 3\omega_3 c_s^2 \omega_1 + \omega_3 \omega_1 + 3\omega_3 v_1^2 - 6v_1^2 \\
C_2 = & -30c_s^4 \omega_2^2 - 2c_s^4 \omega_1^2 \omega_2^2 + 6c_s^2 \omega_1^2 \omega_2 - 9\omega_1 v_1^2 \omega_2^2 - 45c_s^2 \omega_1^2 v_1^2 \omega_2 - 9\omega_1^2 v_1^4 \omega_2 + 9\omega_1^2 v_1^4 + 45c_s^2 \omega_1^2 \omega_1^2 v_1^2 + 9v_1^2 \omega_2^2 - 6c_s^4 \omega_1^2 \omega_2 + 6c_s^4 \omega_1^4 + 9\omega_1 v_1^4 \omega_2^2 + \\
C_3 = & -9c_s^2 \omega_1^2 \omega_2 + 11\omega_1 v_1^2 \omega_2^2 - 11v_1^2 \omega_2^2 - 5\omega_1 \omega_2^2 - 5\omega_1^2 + 11\omega_1 v_1^2 - 11\omega_1^2 v_1^2 \omega_2 - 9c_s^2 \omega_2^2 + 9c_s^2 \omega_1 \omega_2^2 + 9c_s^2 \omega_1^2 + 5\omega_2^2 + 5\omega_1^2 \omega_2 \\
C_4 = & 2\omega_1 \omega_2 - 3c_s^2 \omega_1^2 \omega_2 + \omega_1 v_1^2 \omega_2^2 - v_1^2 \omega_2^2 - \omega_1 \omega_2^2 + \omega_2^2 \omega_2^2 - 2\omega_1^2 + \omega_1^2 v_1^2 - \omega_1^2 v_1^2 \omega_2 + \omega_1^2 v_2^2 + 3c_s^2 \omega_1 \omega_2^2 - 6c_s^2 \omega_1 \omega_2 + 6c_s^2 \omega_1^2 - 2\omega_1 \omega_2 v_2^2 + \omega_1^2 \omega_2 \\
C_5 = & -3\omega_3 \omega_1^2 \omega_2^2 v_2^2 + 6\omega_3 \omega_1^2 v_2^2 - 6\omega_1^2 \omega_2 v_2^2 - 27\omega_3 v_1^2 \omega_2^2 - 12\omega_3 \omega_1 \omega_2^2 - 18c_s^2 \omega_1^2 \omega_2^2 - 18\omega_1 \omega_2^2 v_2^2 + 18\omega_3 c_s^2 \omega_1 \omega_2^2 + 36c_s^2 \omega_1^2 \omega_2^2 - 27\omega_3 \omega_1^2 \omega_2^2 + \\
& 18\omega_1 \omega_2^2 - 12\omega_1^2 \omega_2^2 + 12\omega_3 \omega_1^2 \omega_2 - 15\omega_3 \omega_1^2 - 11\omega_3 c_s^2 \omega_1^2 \omega_2^2 + 27\omega_3 c_s^2 \omega_1^2 - 54c_s^2 \omega_1 \omega_2^2 + 3\omega_3 \omega_1 \omega_2^2 v_2^2 + 6\omega_3 \omega_1^2 v_2^2 + 9\omega_3 c_s^2 \omega_2^2 - 18\omega_3 c_s^2 \omega_1^2 \omega_2 + \\
& 27\omega_3 \omega_1^2 \omega_2^2 - 3\omega_3 \omega_1^2 \omega_2 v_2^2 + 27\omega_3 \omega_1^2 v_1^2 + 6\omega_1^2 \omega_2^2 + 12\omega_1^2 \omega_2^2 v_2^2 + 3\omega_3 \omega_1^2 \omega_2^2 + 3\omega_3 \omega_1^2 v_1^2 \omega_2^2 + 2\omega_3 \omega_1^2 v_1^2 + 2\omega_1^2 \omega_2 - \omega_3 \omega_2^2 \\
C_6 = & 9\omega_3 \omega_2^2 v_2^2 - 2\omega_3 v_1^2 \omega_2^2 - \omega_3 \omega_1 \omega_2^2 - 6c_s^2 \omega_1^2 \omega_2 - 18\omega_3 \omega_1 \omega_2 v_2^2 + 2\omega_1 v_1^2 \omega_2^2 - 6\omega_3 c_s^2 \omega_1 \omega_2^2 + 3\omega_3 c_s^2 \omega_1 \omega_2^2 - \omega_3 \omega_1^2 v_1^2 \omega_2 - 2\omega_1 \omega_2^2 + 6\omega_3 \omega_1 \omega_2 + \\
& \omega_3 \omega_1^2 \omega_2 - 2\omega_1^2 v_1^2 \omega_2 - 5\omega_3 \omega_1^2 + 9\omega_3 c_s^2 \omega_1^2 + 6c_s^2 \omega_1 \omega_2^2 + 9\omega_3 \omega_1^2 v_2^2 - 3\omega_3 c_s^2 \omega_1^2 \omega_2^2 + \omega_3 \omega_1 \omega_2^2 + \omega_3 \omega_1 v_1^2 \omega_2^2 + 2\omega_3 \omega_1^2 v_1^2 + 2\omega_1^2 \omega_2 - \omega_3 \omega_2^2 \\
C_7 = & 6c_s^4 \omega_2^2 + 18\omega_1 \omega_2 v_2^4 + 18\omega_1^2 \omega_2 v_2^2 + c_s^4 \omega_1^2 \omega_2^2 + 6c_s^2 \omega_1^2 \omega_2 + 7\omega_1^2 \omega_2^2 v_2^4 + 18\omega_1 \omega_2^2 v_2^2 - c_s^2 \omega_1^2 \omega_2^2 + 24c_s^2 \omega_1^2 \omega_2^2 v_2^2 + 9\omega_1^2 v_2^4 + 45c_s^2 \omega_1^2 \omega_2^2 - \\
& 6c_s^4 \omega_1^2 \omega_2 + 6c_s^4 \omega_1^2 - 9\omega_2^2 \omega_2^2 + 54c_s^2 \omega_1 \omega_2 v_2^2 - 72c_s^2 \omega_1 \omega_2^2 v_2^2 + 9\omega_2^2 v_2^4 + 45c_s^2 \omega_2^2 v_2^2 - 6c_s^2 \omega_2^2 - 72c_s^2 \omega_1^2 \omega_2 v_2^2 - 9\omega_1^2 v_2^2 + 6c_s^2 \omega_1 \omega_2^2 - 18\omega_1^2 \omega_2 v_2^4 - \\
& 6c_s^2 \omega_1^2 - 18\omega_1 \omega_2 v_2^2 - 18\omega_1 \omega_2^2 v_2^4 - 6c_s^4 \omega_1 \omega_2^2 - 7\omega_1^2 \omega_2^2 v_2^2
\end{aligned}$$

$$\text{C}_8 = -60\omega_1^2\omega_2v_2^2 - 18\omega_1\omega_2 - 36c_s^2\omega_1^2\omega_2 - 60\omega_1\omega_2^2v_2 + 10c_s^2\omega_1^2\omega_2^2 + 24\omega_1\omega_2^2 + 33\omega_2^2v_2^2 - 8\omega_1^2\omega_2^2 - 15\omega_1^2 + 27c_s^2\omega_2^2 + 33\omega_1^2v_2^2 - 36c_s^2\omega_1\omega_2^2 + 18c_s^2\omega_1\omega_2 + 27c_s^2\omega_1^2 + 54\omega_1\omega_2v_2^2 - 15\omega_2^2 + 24\omega_1^2\omega_2 + 22\omega_1^2\omega_2^2v_2^2$$

$$C_9 = -3v_1^2\omega_2 + c_s^4\omega_1 + 3\omega_1 v_1^2\omega_2 - 3\omega_1 v_1^2 + c_s^4\omega_2 - c_s^4\omega_1\omega_2 + 12c_s^2\omega_1 v_1^2 + 3\omega_1 v_1^4 + 3v_1^4\omega_2 - c_s^2\omega_1 + 12c_s^2v_1^2\omega_2 + c_s^2\omega_1\omega_2 - c_s^2\omega_2 - 3\omega_1 v_1^4\omega_2 - 12c_s^2\omega_1 v_1^2\omega_2$$

$$\begin{aligned} C_{10} = & 12w_3^2c_s^2\omega_1 - 24w_3c_s^4\omega_1^2 + 108w_s^2\omega_1^3v^2 + 36w_3^3\omega_1^4 + 12w_3^2\omega_1^2v_1^2 - 12w_3^2\omega_1^2v_2^2 + 72w_3\omega_1^2v_4^4 + 24w_3^2c_s^4 + 6w_3c_s^4\omega_1^3 - 72w_1^2v_4^4 - 216c_s^2\omega_1^2v_1^2 + 72w_3c_s^2\omega_1^2v_1^2 + w_3^2c_s^2\omega_1^3 - 30w_3\omega_1^3v_1^4 + 24w_3c_s^4\omega_1 + 6w_3^2c_s^2\omega_1^3v_1^2 - 8w_3^2c_s^2\omega_1^2 - 3w_3^2\omega_1^3v_1^2 + 72w_1^2v_1^2 - 6w_3c_s^2\omega_1^3 - 48w_3^2c_s^4\omega_1 + 3w_3^2\omega_1^3v_1^4 + 30w_3\omega_1^3v_1^2 + 24w_3c_s^2\omega_1^2 + 144w_3c_s^2\omega_1^2v_1^2 - 24w_3c_s^2\omega_1^2 - 36w_3^2c_s^2\omega_1v_1^2 - 36\omega_1^3v_1^2 + 24w_3^2c_s^4\omega_1^2 - 72w_3\omega_1^2v_1^2 - 72w_3c_s^2\omega_1^2v_1^2 - 3w_3^2c_s^4\omega_1^3 - 12w_3^2\omega_1^2v_1^4 \end{aligned}$$

$$\begin{aligned}
C_{11} = & 2w_3^2 c_s^2 w_1^3 w_2^3 - 39w_3^2 w_1^4 w_2^3 + 18w_3^2 c_s^4 w_1^2 w_2 + 6w_3^2 c_s^6 w_1^3 w_2^3 - 3w_2^3 w_1^2 w_3^3 w_2^4 + 81w_3^2 c_s^2 w_1^2 w_2^3 + 123w_3^2 c_s^3 w_1^3 w_2 - 7w_2^3 w_1^2 w_2^3 - 18w_3 c_s^2 w_1^3 v_1^2 w_2^2 - \\
& 12w_2^3 s_1^2 w_2^3 + 72w_3^2 c_4^2 w_1^3 w_2^2 - 12c_s^2 w_1^3 w_2^3 - 90w_3 c_s^4 w_1 w_2^3 - 3w_2^3 w_1^2 w_2^2 v_2^2 + 197w_3^2 c_2^2 v_1^2 w_2^2 - 6w_3 c_s^2 w_1^3 v_1^2 w_2^3 + 72w_3^2 c_2^2 w_2^3 - 6w_3^2 w_2^3 - 72c_4^2 w_1^2 w_2^3 - \\
& 12w_3^2 c_s^2 w_1^2 w_2^2 - 46w_3^2 c_s^2 w_1^3 v_1^2 w_2 + 9w_3^2 c_s^2 w_3^2 v_2^2 + 6w_3^2 w_1^2 w_2^2 + 51w_3^2 w_1 v_1^2 w_2^2 - 2u_3^2 c_s^2 w_2^3 w_3^2 - 30w_3 c_s^2 w_1 v_1^2 w_2^3 + 102w_3^2 w_1^3 v_1^2 w_2^2 + 24w_3^2 c_2^2 w_1^2 w_2^3 - \\
& 72w_3^2 c_s^2 w_1^3 - 153w_3^2 c_4^2 w_1^3 w_2 + 6w_3^2 w_1^3 - 138w_3^2 c_4^2 w_1^2 w_2^3 - 165w_3^2 c_2^2 w_1 v_1^2 w_2^2 - 45w_3^2 c_1^4 w_1^2 w_2^3 + 45w_3^2 c_2^2 w_1^4 v_1^2 w_2 - 465w_3^2 c_3^2 w_1^3 v_1^2 w_2 + 36c_4^2 w_3^2 w_1^3 - 59w_3^2 c_3^2 w_1^2 w_2^3 - \\
& 102w_3^2 c_1^2 w_1^2 w_2^3 - 24c_4^2 w_2^2 v_1^2 w_2^3 + 30w_3^2 c_s^2 w_1^2 w_2^3 + 7w_2^2 w_1^2 w_2^2 + 6w_3^2 c_2^2 w_3^2 v_2^2 w_2 + 261w_3^2 c_s^2 w_1^2 v_1^2 + 36w_3^2 c_2^2 w_2^2 w_2^2 + 489w_3^2 c_3^2 w_1 v_1^2 w_2^3 + 24w_3^2 c_2^2 w_1^2 w_2^3 - \\
& 51w_3^2 w_2^3 v_1^2 + 42w_3^2 c_2^2 w_1^2 v_1^2 w_2^3 - 42w_3 c_2^2 w_1^2 w_2^3 - 6w_3 c_2^2 w_1^3 w_2 - 90w_3^2 c_4^2 w_2^3 - 18w_3 c_4^2 w_1^3 w_2^3 + 12w_3^2 c_2^2 w_1^2 v_1^2 w_2^2 - 90w_3^2 c_4^2 w_1 w_2^3 + 39w_3^2 w_1^3 v_1^2 w_2^2 - \\
& 24w_3 c_3^2 w_1^2 w_2^2 + 46w_3^2 c_2^2 w_1^2 w_2^3 + 24w_3 c_2^2 w_1^2 v_2^2 w_2^2 + 45w_3^2 c_3^2 w_1^3 v_1^4 + 225w_3^2 c_4^2 w_1 w_2^3 - 219w_3^2 c_2^2 w_1^2 v_1^2 w_2^3 - 54w_3 c_4^2 w_1^3 w_2^2 + 3w_3^2 c_2^2 w_1^2 w_2^2 + 90w_3^2 w_1 v_1^4 w_2^3 + \\
& 18w_3 c_4^2 w_1^3 w_2 + 126w_3 c_4^2 w_1^2 w_2^3 - 9w_3^2 c_2^2 w_1^2 w_2^3 v_2^2 + 6w_3 c_2^2 w_1^3 w_2^2 - 6w_3^2 w_1 w_2^2 + 12c_2^2 w_1^3 v_1^2 w_2^3 + 48w_3^2 c_2^2 w_1 w_2^2 + 72w_3 c_4^2 w_1^2 w_2^2 - 45w_3^2 w_1 v_1^4 w_2^2 + \\
& 141w_3^2 c_2^2 w_1^2 v_1^2 w_2 - 90w_3^2 c_3^2 w_1^3 v_1^4 w_2 + 90w_3^2 c_4^2 w_1^3 - 261w_3^2 c_2^2 v_1^2 w_2^3 - 147w_3^2 c_2^2 w_1 w_2^3 - 51w_3^2 c_2^2 w_1^2 v_1^2 w_2 + 51w_3^2 v_1^2 w_2^3 + 3w_3^2 c_2^2 w_1^3 v_2^2 w_4^2 + 18w_3 c_2^2 w_1^3 w_2^2 + 12w_3^2 c_2^2 w_1 w_2^3
\end{aligned}$$

$$\begin{aligned}
C_{12} = & 138w_3w_1^3w_1^4w_2^2 - 30w_3c_4^4w_1w_2^2 + 18c_8^2w_1^3w_2^2 + 3w_3c_8^2w_1^2w_2^2 + 24c_8^4w_1^2w_2^2 + 153w_3c_8^3w_1^3w_1^2w_2^2 - 6w_3w_1w_2^2 - 12c_8^2w_3^3w_2^3 + 69w_3c_4^4w_1w_2^3 + \\
& 12w_3w_1w_2^3 + 99w_3w_2^3w_3^2 - 30c_8^3w_1^2w_2^3 + 6c_8^4w_1w_2^3 + 18w_3c_8^4w_1^3 + 81w_3w_2^2w_1^2w_2^3 + 36w_3c_8^2w_1w_2^2 + 72c_8^2w_1^2w_2^2w_3^2 + 351w_3c_8^2w_1v_1^2w_2^2 - 18c_8^4w_3^3w_2^2 + \\
& 324w_3w_1v_1^4w_3^2 - 24c_8^2w_1^2w_2^3 + w_3w_2^3w_1^2v_2^2 - 42w_3c_8^4w_3^2 + 171w_3c_8^3w_1^3v_1^2 - 63w_3c_8^2w_1^2v_1^2w_2^2 + 12c_8^4w_1^3w_3^2 + 3w_3w_1^2w_2^2v_2^4 - 135w_3c_8^2w_1^2v_1^2w_2^2 - 207w_3c_8^2w_1^2v_1^2w_3^2 - \\
& 90c_8^2w_1^2v_1^2w_3^2 - 45w_3c_8^2w_1w_3^2 - 351w_3c_8^2w_1^3v_1^2w_2 - 324w_3w_1^3v_1^4w_2^2 - 6c_8^2w_1^2w_3^2 + 30c_8^2w_1^2w_2^3 - 135w_3w_1v_1^4w_2^2 + 18c_8^2w_1^3v_1^2w_2 - 153w_3c_8^2w_1^2v_1^2w_3^2 - \\
& w_3w_1^2w_2^2v_4^2 + 24w_3c_8^2w_1^2v_1^3 + 6w_3c_8^2w_1^2v_2^2 + 45w_3c_8^2w_1^2w_2 + 6w_3c_8^4w_2^2w_2^2 - 171w_3v_1^4w_3^2 - 2w_3c_8^2w_1^3w_2^3 - 18c_8^2w_1w_3^2 - 24w_3c_8^2w_1^3 - 138w_3c_8w_1^2v_1^2w_2^3 + \\
& w_3w_1^2v_1^2w_3^2 + 54c_8^2w_1v_1^2w_3^2 - 81w_3w_1^3v_1^2w_2^2 - 12w_3c_8^2w_1^2w_2^2 + 36w_3c_8^2w_1^2v_1^2w_2^2 - 99w_3w_1^3v_1^2 + 6w_3w_1^3 - w_3w_1^2v_2^2w_2^2 + 17w_3c_8^4w_1^3w_2^2 + 135w_3w_1^2v_1^4w_2 - \\
& 7w_3w_1^3w_2^2 + 25w_3c_8^4w_1^2w_3^2 - 6w_3w_2^3 - 12w_3w_1^2w_2 - 7w_3w_2^2v_3^2 - 12w_3c_8^2w_1^2w_2 + 180w_3w_1^3v_1^2w_2^2 + 36c_8^2w_1^3v_1^2w_2^2 + 63w_3c_8^2w_1^2v_1^2w_2 + \\
& 33w_3c_8^4w_1^2w_2 - 25w_3c_8^4w_1^2w_3^2 - 6w_3w_2^3 - 12w_3w_1^2w_2 - 7w_3w_2^2v_3^2 - 12w_3c_8^2w_1^2w_2 + 180w_3w_1^3v_1^2w_2^2 - 180w_3w_1v_1^2w_2^3 - 54c_8^2w_1^3v_1^2w_2^2 - 24w_3c_8^2w_1^2w_2^2 + 24w_3c_8^2w_1^2w_3^2
\end{aligned}$$

$$C_{13} = \omega_3 \omega_1 v_1^2 + 6\omega_1 \omega_2 + 9\omega_3 \omega_1 v_2^2 + 6\omega_3 c_s^2 \omega_1 \omega_2 - 6\omega_1 v_1^2 \omega_2 + \omega_3 v_1^2 \omega_2 - 2\omega_3 \omega_1 \omega_2 + 2\omega_3 \omega_2 + 2\omega_3 \omega_1 v_1^2 \omega_2 + 6\omega_3 c_s^2 \omega_1 - 18c_s^2 \omega_1 \omega_2 - 4\omega_3 \omega_1 - 9\omega_3 \omega_2 v_2^2$$

$$\begin{aligned}
C_{14} = & -45c_3^2w_1^2v_1^2w_2v_2^2 + 45w_3^2v_1^2w_3^2v_2^2 - 27w_3^2c_1^2v_1^2w_3^2 - 36c_2^2w_1^2w_3^2v_2^2 - 18w_3^2c_4^2w_1^2w_2 + 12c_2^2w_1^3w_2^2 + 45w_3^2w_1^1v_2^2 + 6w_3^2w_1w_3^2v_2^2 + 10w_3^2c_4^2w_1^3w_2^3 + \\
& 41w_3^2c_2^2w_1^2w_3^3 + 93w_3^2c_3^2w_1^3w_2^2 - 6w_3^2w_1^3w_2 + 3w_3^2c_5^2w_1w_3^2v_2^2 + 35w_2^2c_4^2w_1^3w_2^2 - 45w_3^2w_1v_1^2w_3^2v_2^2 - 24c_2^2w_1^3w_2^3 - 90w_3c_4^2w_1w_3^2 - 30w_3c_2^2w_1w_3^2v_2^2 + \\
& 138w_3^2c_2^2w_1^3v_1^2w_2^2 + 12w_3^2w_2^2w_3^2 + 6w_3^2w_3^2 - 108c_3^2w_1^2w_3^2 - 54w_3^2c_2^2w_1^2w_2^2 - 27w_3^2w_1^3v_1^2w_2^2 + 8w_3^2c_3^2w_1^3w_2^2v_2^2 - 30w_3^2c_2^2w_1w_2^2v_2^2 - 6w_3^2w_1^2w_2^2 - \\
& 9w_3^2w_1^2v_1^2w_2^2 - 2w_3^2c_5^2w_1^3w_3^3 + 99w_3^2w_1^3v_1^2w_2 + 24w_3^2c_3^2w_1^2w_2^2 - 36c_4^2w_1^3w_2^2 - 72w_3^2c_2^2w_1^3 - 117w_3^2c_4^2w_1^3w_2^2 + 18w_3^2c_5^2w_1^3v_2^2 + 6w_3^2w_1^3 - 91w_3^2c_4^2w_1^2w_3^2 + \\
& 27w_3^2c_2^2w_1^2v_1^2w_2^2 - 12w_3c_3^2w_1^3w_2^2v_2^2 + 6w_3^2w_1w_2^2v_2^2 - 6w_3^2w_1^3v_2^2 - 54w_3^2v_1^2w_3^2 - 54w_3^2w_1^2v_1^2w_2 + 459w_3^2c_3^2w_1^3v_1^2w_2 + 72c_4^2w_1^3w_3^2 + 2w_3^2c_2^2w_1^3w_2^2v_2^2 - \\
& 25w_3^2c_2^2w_1^2w_3^2 - 9w_3^2w_1v_1^2w_2^2 - 45w_3^2w_1^2v_1^2w_2^2v_2^2 - 6w_3^2c_3^2w_1^2w_2^2v_2^2 + 30w_3^2c_3^2w_1^3w_2^3 + 405w_3^2c_2^2w_1^3v_1^2 + 54w_3^2c_5^2w_1^2w_2^2 + 189w_3^2c_2^2w_1v_1^2w_3^2 + 36w_3^2w_1^2w_3^2 + \\
& 6w_3^2w_1^2w_2v_2^2 - 99w_3^2w_1^3v_1^2 - 45w_3^2w_1v_1^2w_2^2v_2^2 + 18w_3^2c_3^2w_1^2w_2^2v_2^2 - 48w_3^2c_5^2w_1^2w_3^2 - 90w_3^2w_1^2v_1^2w_2^2 + 6w_3^2c_2^2w_1^3w_2^3 + 18w_3^2c_4^2w_1^3w_2^2 + 18w_3^2c_5^2w_1^3v_2^2 - \\
& 36w_3c_3^2w_1^3w_3^3 + 270w_3^2c_2^2w_1^2v_1^2w_2^2 - 12w_3^2w_1^2w_2^2v_2^2 - 6w_3^2w_1^3w_2^2 - 90w_3^2c_4^2w_1^2w_2^2 + 27w_3^2w_1^3v_4^2w_2^2 - 36w_3c_3^2w_1^2w_2^2 + 27w_3^2c_2^2w_1^2v_1^2w_3^2 + 54w_3^2w_1^3v_4^2 + \\
& 63w_3^2c_2^2w_1^2v_1^2 - 138w_3^2c_3^2w_1^2v_1^2w_2^3 + 36w_3^2c_3^2w_1^2w_2^2v_2^2 + 24c_2^2w_1^3w_2^3v_2^2 + 54w_3^2w_1v_1^4w_2^3 - 18w_3c_4^2w_1^3w_2 + 6w_3^2w_1^2w_2v_2^2 + 144w_3c_3^2w_1^2w_2^3 - 24w_3^2c_2^2w_1^2w_2^3v_2^2 + \\
& 12w_3c_3^2w_1^2w_3^2 - 6w_3^2w_1w_2^2 - 21w_3^2c_3^2w_1^3w_2v_2^2 + 48w_3^2c_2^2w_1w_2^2 + 108w_3c_4^2w_1^2w_2^2 + 48w_3c_3^2w_1^2w_3^2v_2^2 + 54w_3^2w_1v_1^4w_2^2 - 12c_2^2w_1^3w_2^2v_2^2 + 90w_3^2w_1^2v_1^2w_2^2v_2^2 - \\
& 6w_3^2c_2^2w_1^2w_2v_2^2 - 297w_3^2c_3^2w_1^2v_1^2w_2 - 54w_3^2w_1^3v_1^2w_2 + 90w_3^2c_3^2w_1^3 - 135w_3^2c_3^2w_1^2w_3^2 - 39w_3^2c_2^2w_1w_3^2 + 99w_3^2w_1^2v_1^2w_2 + 9w_3^2v_1^1w_3^2 - 6w_3^2w_1^2w_3^2
\end{aligned}$$

$$\begin{aligned}
C_{15} = & 54w_3\omega_1^2\omega_2^2v_2^2 - 18w_3\omega_1\omega_2^2 - 81w_3\omega_1\omega_2^3 - 198w_3v_1^2\omega_2^3 - 100w_3c_s^2v_1^2\omega_2^3 - 54w_3c_s^2\omega_1\omega_2^2 + 18\omega_1\omega_2^3 + 36w_3\omega_2^3v_2^2 + 108c_s^2\omega_1^2\omega_2^2 - \\
& 27w_3\omega_1^3\omega_2v_2^2 - 18\omega_1\omega_2^3v_2^2 - 162w_3\omega_2^2v_1^2\omega_2 + 135w_3c_s^2\omega_1\omega_2^3 - 54c_s^2\omega_1^3\omega_2 - 36\omega_1^2\omega_2^2 - 84w_3c_s^2\omega_1^2\omega_2^3 + 90w_3\omega_1^2\omega_2 - 297w_3c_s^2\omega_1^3\omega_2 + 36w_3\omega_1^3v_2^2 - \\
& 27w_3\omega_1\omega_2^3v_2^2 - 54c_s^2\omega_1\omega_2^3 + 270w_3c_s^2\omega_1^3 - 46w_3\omega_1^3\omega_2^2 + 100w_3\omega_1^3v_1^2\omega_2^2 + 162w_3c_s^2\omega_1^2\omega_2^2 - 18w_1^3\omega_2v_2^2 + 18\omega_1^3\omega_2 + 198w_3\omega_1^3v_1^2 - 126w_3\omega_1^3 - \\
& 36w_3\omega_1\omega_2^2v_2^2 + 54w_3\omega_2^3 + 135w_3\omega_1^3\omega_2 + 46w_3\omega_2^3\omega_1^3 - 162w_3c_s^2\omega_1^2\omega_2 - 216w_3\omega_1^3v_1^2\omega_2 + 162w_3\omega_1v_1^2\omega_2^2 - 36w_3\omega_1^2\omega_2v_2^2 + 216w_3\omega_1v_1^2\omega_2^3 + \\
& 36\omega_1^2\omega_2^2v_2^2 - 54w_3\omega_1^2\omega_2^2 + 84w_3c_s^2\omega_1^3\omega_2^2 - 54w_3c_s^2\omega_1^3
\end{aligned}$$

$$\begin{aligned}
C_{16} = & -135w_3^2w_1^2v_2^2w_2v_2^2 + 135w_3^2v_1^2w_2^3v_2^2 - 9w_3^2w_1^4w_3^2 + 54w_3w_1^3v_4^2w_2^2 - 30w_3^2c_s^4w_1^2w_2 + 135w_3^2w_1^3v_1^2v_2^2 - 90w_3c_s^3w_3^2w_2^2v_2 + 18w_3^2w_1w_3^3v_2^2 - \\
& 2w_3^2c_s^6w_1^3w_2^2 - w_3^2w_1^2w_3^2v_2^4 + 2w_3^2c_s^8w_1^2w_3^2 + 21w_3^2c_s^6w_1^3v_2^2 + 144w_3c_s^8w_1^3v_1^2w_2^2 - 6w_3^2w_1^3w_2 + 9w_3^2c_s^4w_1w_3^2v_2^2 + 2w_3^2c_s^4w_1^2w_2^2 - 135w_2^2w_1v_1^2w_3^2v_2^2 + \\
& 30w_3c_s^4w_1w_3^2 + 126w_3c_s^6w_1w_3^2v_2^2 - w_3^2w_1^3v_2^2 + 30w_3^2c_s^8w_1^3v_1^2w_2^2 + 12w_3^2w_2^2w_2^2 + 6w_3^2w_3^2v_2^2 - 42w_3^2c_s^8w_1^2w_3^2v_2^2 - 9w_3^2w_1^3v_1^2w_2^2 + 6w_3^2c_s^6w_1^2w_3^2v_2^2 + 54w_3w_1^2v_1^2w_3^2 - \\
& 90w_3^2w_1^2w_3^2v_2^2 - 6w_3^2w_1^2w_2 + 45w_3^2w_1v_1^2w_2^2 + 18w_3c_s^4w_1v_1^2w_3^2 + 90w_2^2w_3^2v_1^2w_2 + 36w_3^2c_s^2w_2^2 + 36w_3^2v_1^2w_3^2 - 24w_2^2c_s^4w_1^2w_3^2 + 18w_3c_s^4w_1v_1^2w_2^2 - 15w_3^2c_s^4w_1^3w_2 + \\
& 18w_3^2c_s^6w_1^3v_2^2 + 6w_3^2w_1^2 - 10w_3^2c_s^4w_1^2w_3^2 - 45w_3^2c_s^6w_1v_1^2w_2^2 + 72w_3c_s^8w_1^3w_2^2v_2 + 18w_3^2w_1w_2^2v_2^2 - 18w_3^2w_1^3v_2^2 - 36w_3^2v_1^2w_2^2 - 216w_3^2c_s^6w_1^2v_2^2 - 2w_3^2c_s^4w_1^3w_2^2 + \\
& 36w_1^2c_s^4w_3^2 - 135w_3^2c_s^4v_1^2w_2v_2^2 + 108c_s^2w_1^2v_1^2w_3^2 - 54w_3^2c_s^2w_2^2w_2v_2^2 - 30w_3c_s^6w_1^3w_2^2 - 54w_3c_s^8w_1^3v_1^2w_2^2 + 189w_3c_s^2w_3^2v_1^2w_2^2 + 18w_3^2c_s^4w_1^2w_2^2 + 30w_3^2c_s^6w_1^2w_2^2 + \\
& 108w_3^2c_s^6w_1^2w_3^2 + 18w_3^2c_s^4w_1^2w_2v_2^2 - 81w_3^2w_1^3v_2^2 - 135w_3^2w_1^2v_1^2w_2^2v_2^2 - 144w_3c_s^8w_1^2v_1^2w_3^2 + 72w_3^2c_s^6w_1^2v_2^2w_2^2 + 60w_3c_s^2w_2^2v_2^3 - 90w_3^2w_1^2v_1^2w_2^2 - 6w_3c_s^6w_1^3w_2^2 + \\
& 30w_3^2c_s^4w_3^2 - 18w_3^2c_s^6w_1^3v_2^2 + 24w_3c_s^4w_1^3w_3^2 + 108w_3^2c_s^6w_1^2v_1^2w_2^2 - 36w_3^2w_1^2w_2^2v_2^2 - 18w_3^2w_1^3v_2^2 - 54w_3^2w_1^2v_1^4w_3^2 - 30w_3^2c_s^4w_1w_2^2 + 9w_3^2w_1^3v_4^2w_2^2 - \\
& 54w_3w_1^2v_1^2w_2^2 - 60w_3c_s^6w_1^2w_2^2 + 9w_3^2w_1^2v_2^2w_3^2 + 36w_3c_s^2w_1^2v_1^2w_2^2 + 36w_3^2w_1^3v_1^4 + 45w_3^2c_s^4w_1w_3^2 - 30w_3^2c_s^2w_1^2v_1^2w_3^2 + 144w_3c_s^2w_1^2w_2^2v_2^2 - 36w_3c_s^4w_1^3w_2^2 + \\
& w_3^2w_1^2w_3^2v_2^2 + 45w_3^2w_1v_1^4w_3^2 + 6w_3c_s^8w_1^3w_2^2 + 18w_3^2w_1^3w_2v_2^2 - 36w_1^2v_1^2w_3^2 - 60w_3c_s^4w_1^2w_3^2 - 6w_3^2c_s^2w_1^2w_3^2v_2^2 - 24w_3c_s^8w_1^3w_3^2 - 6w_3^2w_1w_2^2 + 18w_3w_1^3v_1^2w_2^2 - \\
& 9w_3^2c_s^6w_1^2w_2v_2^2 + 36w_3^2c_s^2w_1w_2^2 + 60w_3c_s^4w_1^2w_2^2 - 198w_3c_s^2w_1^2w_3^2v_2^2 + 270w_3^2w_1^2v_1^2w_2^2v_2^2 + 18w_3c_s^2w_1^3w_2v_2^2 - 81w_3^2c_s^2w_1^2v_1^2w_2^2 - 45w_3^2w_1^3v_4^2w_2 + 18w_3c_s^4w_1^3v_1^2w_2^2 - \\
& 36w_1^2c_s^4w_2^2 - 63w_3^2c_s^6w_1^2w_3^2 - 18w_3w_1v_1^2w_3^2 - 108c_s^2w_1^3v_1^2w_2^2 - 3w_3^2c_s^2w_1w_2^3 + 45w_3^2w_1^2v_1^2w_2^2 - 9w_3^2v_1^2w_3^2 + w_3^2w_1^3w_2^2v_4^2 + 36w_3c_s^2w_1^3w_2^2 - 6w_3^2w_1w_2^3
\end{aligned}$$

$$\begin{aligned}
C_{17} = & 9w_3^2 w_1^2 v_1^2 w_2 v_2^2 - 45w_3^2 v_1^2 w_3^2 v_2^2 - 54w_3^2 c_4^4 w_1^2 w_2 + 12c_2^2 w_3^3 v_1^2 w_2^2 + 45w_3^2 s_3^3 v_1^2 v_2^2 - 18w_3^2 w_1^3 w_3^2 v_2^2 + 5w_3^2 c_8^2 w_1^2 w_2^3 + 75w_3^2 c_2^2 w_3^1 w_2 - w_3^2 w_1^2 w_2^3 - 54w_3^2 w_1^3 w_2 v_2^4 + 18w_3 c_3^2 w_1^3 v_2^2 w_2^2 - 6w_3^2 w_1^3 w_2 - 54w_3^2 c_2^2 w_1 w_3^2 v_2^2 + 6w_3^2 c_4^4 w_3^1 w_2^2 + 72w_3^2 w_1 v_1^2 w_3^2 v_2^2 + 18w_3 c_4^4 w_1 w_2^3 - 24w_3^2 w_3^2 w_2^2 v_2^2 + 2w_3^2 c_8^2 w_3^1 v_1^2 w_2^2 - 6w_3^2 w_3^2 + 36c_4^4 w_1^2 w_2^3 - 72w_3^2 c_5^2 w_1^2 w_2^2 - w_3^2 w_3^2 v_1^2 w_2^2 + 72w_3^2 c_8^2 w_3^1 w_2^2 v_2^2 - 297w_3^2 c_5^2 w_1 w_2^2 v_2^2 - 6w_3^2 w_1^2 w_2 - 6w_3^2 w_1 v_1^2 w_2^2 + 108w_3^2 w_2^2 w_3^2 v_2^4 + 6w_3 c_8^2 w_1 v_1^2 w_2^3 + 54w_3^2 w_3^2 v_2^4 + 6w_3^2 w_3^1 v_1^2 w_2^2 + 60w_3^2 c_8^2 w_2^2 w_3^2 - 36c_4^4 w_3^1 w_2^2 - 72w_3^2 c_8^2 w_3^1 - 99w_3^2 c_4^4 w_3^1 w_2 + 405w_3^2 c_8^2 w_1^2 v_2^2 + 6w_3^2 w_1^3 - 6w_3^2 c_4^4 w_1^2 w_3^2 + 
\end{aligned}$$

$$\begin{aligned}
& 6\omega_3^2 c_s^2 \omega_1 v_1^2 \omega_2^2 + 63\omega_3^2 \omega_1 \omega_2^2 v_2^2 - 99\omega_3^2 \omega_1^3 v_1^2 \omega_2 - 21\omega_3^2 c_s^2 \omega_1^3 v_1^2 \omega_2^2 - 5\omega_3^2 c_s^2 \omega_1^3 \omega_2^2 - 6\omega_3^2 \omega_1 v_1^2 \omega_2^3 - 72\omega_3^2 \omega_1^3 v_1^2 \omega_2 v_2^2 + 12c_s^2 \omega_1^2 v_1^2 \omega_2^3 - 243\omega_3^2 c_s^2 \omega_1^2 \omega_2 v_2^2 - \\
& 6\omega_3 c_s^2 \omega_1 \omega_2^3 + \omega_3^2 \omega_1^3 \omega_2^2 - 6\omega_3 c_s^2 \omega_1^3 v_1^2 \omega_2 + 18\omega_3^2 c_s^2 \omega_1^3 v_1^2 + 72\omega_3^2 c_s^4 \omega_1^2 \omega_2^2 + 21\omega_3^2 c_s^2 \omega_1^2 v_1^2 \omega_2^3 - 12c_s^2 \omega_1^2 \omega_2^3 + 45\omega_3^2 \omega_1^2 \omega_2 v_2^2 - 6\omega_3^2 \omega_1^3 v_1^2 - \\
& 9\omega_3^2 \omega_1 v_1^2 \omega_2^2 - 54\omega_3^2 \omega_1 \omega_2^3 v_2^4 - 18\omega_3 c_s^2 \omega_1^2 v_1^2 \omega_2^3 + 540\omega_3^2 \omega_1^2 \omega_2^3 v_2^2 + 18\omega_3 c_s^2 \omega_1^2 \omega_2^3 + 54\omega_3^2 \omega_1^2 v_1^2 \omega_2^3 + 24\omega_3^2 \omega_1^2 v_1^2 \omega_2^3 v_2^2 + 6\omega_3^2 \omega_1^2 \omega_2^3 - 18\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 + \\
& 135\omega_3^2 c_s^2 \omega_1^2 \omega_2^2 - 108\omega_3^2 \omega_1^2 \omega_2^2 v_2^2 - 9\omega_3^2 \omega_1^2 \omega_2^2 - 18\omega_3^2 c_s^4 \omega_1^2 \omega_2^2 + \omega_3^2 \omega_1^2 v_1^2 \omega_2^3 - 54\omega_3^2 \omega_1^2 \omega_2^2 v_2^4 + 27\omega_3^2 c_s^4 \omega_1^2 \omega_2^3 - 2\omega_3^2 \omega_1^2 v_1^2 \omega_2^3 + 54\omega_3 c_s^4 \omega_1^2 \omega_2^2 - \\
& 24\omega_3^2 \omega_1^2 v_1^2 \omega_2^3 v_2^2 + 24\omega_3^2 \omega_1^2 \omega_2^3 v_2^2 - 18\omega_3 c_s^4 \omega_1^2 \omega_2^2 + 126\omega_3^2 \omega_1^2 \omega_2^2 - 54\omega_3 c_s^4 \omega_1^2 \omega_2^3 - 72\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^2 + 6\omega_3^2 \omega_1^2 \omega_2^2 - 486\omega_3^2 c_s^2 \omega_1^2 \omega_2^2 v_2^2 + 12\omega_3^2 c_s^2 \omega_1^2 \omega_2^2 - \\
& 54\omega_3^2 \omega_1 \omega_2^3 v_2^4 - 6\omega_3^2 c_s^2 \omega_1^2 v_1^2 \omega_2 + 90\omega_3^2 c_s^2 \omega_1^2 - 18\omega_3^2 c_s^2 v_1^2 \omega_2^3 - 12c_s^2 \omega_1^2 v_1^2 \omega_2^3 - 3\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 + 6\omega_3^2 \omega_1^2 v_1^2 \omega_2^3 - 18\omega_3 c_s^2 \omega_1^2 \omega_2^2 + 6\omega_3^2 \omega_1^2 \omega_2^2
\end{aligned}$$

$$\begin{aligned}
C_{18} = & 27\omega_3^2 \omega_1^2 v_1^2 \omega_2^2 v_2^2 + 36\omega_3 \omega_1^2 \omega_2^2 v_2^2 - 135\omega_3^2 v_1^2 \omega_2^3 v_2^2 - 324c_s^2 \omega_1^2 \omega_2^3 v_2^2 + 135\omega_3^2 \omega_1^2 v_1^2 v_2^2 + 198\omega_3 c_s^2 \omega_1^2 \omega_2^3 v_2^2 - 9\omega_3^2 \omega_1 \omega_2^3 v_2^2 - \\
& 2\omega_3^2 c_s^4 \omega_1^2 \omega_2^3 + 7\omega_3^2 \omega_1^2 \omega_2^3 v_4^2 - 12\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 + 21\omega_3^2 c_s^2 \omega_1^2 \omega_2 - \omega_3^2 \omega_1^2 \omega_2^3 - 63\omega_3^2 \omega_1^2 \omega_2 v_2^4 - 72\omega_3^2 \omega_1^2 \omega_2^3 v_2^2 - 36\omega_3 c_s^2 \omega_1^2 v_1^2 \omega_2^3 - 6\omega_3^2 \omega_1^2 \omega_2 - \\
& 171\omega_3^2 c_s^2 \omega_1 \omega_2^3 v_2^2 - \omega_3^2 c_s^4 \omega_1^2 \omega_2^2 + 216\omega_3 \omega_1^2 v_1^2 \omega_2^3 v_2^2 + 18\omega_3 c_s^4 \omega_1 \omega_2^3 + 54\omega_3 c_s^2 \omega_1^2 \omega_2^3 v_2^2 - 43\omega_3^2 \omega_1^2 \omega_2^3 v_2^2 - 6\omega_3^2 \omega_1^2 \omega_2^3 v_2^2 - 6\omega_3^2 \omega_2^3 - 12\omega_3^2 c_s^2 \omega_1^2 \omega_2^2 - \\
& 3\omega_3^2 \omega_1^2 \omega_2^2 - 78\omega_3 c_s^2 \omega_1^2 \omega_2^3 v_4^2 + 84\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^2 + 36\omega_3^2 \omega_1^2 \omega_2^3 + 99\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^2 - 18\omega_3 \omega_1^2 \omega_2^3 v_4^2 - 6\omega_3^2 \omega_1^2 \omega_2^2 - 18\omega_3 \omega_1^2 \omega_2^3 v_2^2 - 54\omega_3^2 \omega_1^2 \omega_2^3 v_2^4 - \\
& 54\omega_3 c_s^2 \omega_1^2 v_1^2 \omega_2^3 + 36\omega_3^2 \omega_1^2 v_2^4 + 18\omega_3^2 \omega_1^2 v_1^2 \omega_2 + 36\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 - 24\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 - 15\omega_3^2 c_s^4 \omega_1^2 \omega_2 + 189\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 + 18\omega_3 \omega_1^2 \omega_2^3 v_2^2 + 6\omega_3^2 \omega_1^2 \omega_2^3 + \\
& 29\omega_3^2 c_s^4 \omega_1^2 \omega_2^3 + 18\omega_3^2 c_s^2 \omega_1^2 \omega_2^2 - 198\omega_3 c_s^2 \omega_1^2 \omega_2^3 v_2^2 - 27\omega_3^2 \omega_1^2 \omega_2^3 v_2^2 - 81\omega_3^2 \omega_1^2 \omega_2^3 v_2^2 - 144\omega_3 c_s^2 \omega_1^2 \omega_2^3 v_2^2 - 9\omega_3^2 c_s^2 \omega_1^2 v_1^2 \omega_2 + 24\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^2 + \\
& 72\omega_3 \omega_1^2 \omega_2^3 v_4^2 - 18\omega_3^2 \omega_1^2 \omega_2^3 v_2^2 - 216\omega_3^2 \omega_1^2 v_1^2 \omega_2^3 v_2^2 + 81\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^2 - 18\omega_3 c_s^2 \omega_1^2 \omega_2^3 v_2^2 + 18\omega_3 c_s^2 \omega_1^2 v_1^2 \omega_2^3 v_2^2 + 18\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^2 - 8\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^2 + \\
& 12\omega_3^2 c_s^4 \omega_1^2 \omega_2^2 - 9\omega_3^2 c_s^2 \omega_1^2 v_1^2 \omega_2^3 - 108\omega_1^2 \omega_2^3 v_2^4 - 45\omega_3^2 \omega_1^2 \omega_2^3 v_2^2 - 18\omega_3^2 \omega_1^2 v_1^2 v_2^4 - 27\omega_3^2 \omega_1^2 v_1^2 \omega_2^3 v_2^2 - 18\omega_3 \omega_1^2 \omega_2^3 v_2^4 + 36\omega_3^2 \omega_1^2 \omega_2^3 v_2^2 + 36\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^2 - \\
& 144\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^2 + 144\omega_3^2 \omega_1^2 \omega_2^3 v_2^4 + 30\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 + 36\omega_3^2 \omega_1^2 v_1^2 v_2^4 + 72\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^2 - 6\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 + 30\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 - 36\omega_3^2 \omega_1^2 \omega_2^3 v_2^4 + 18\omega_3 \omega_1^2 \omega_2^3 v_2^2 + \\
& 63\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^2 + 12\omega_3 c_s^2 \omega_1^2 \omega_2^3 + 18\omega_3^2 c_s^2 \omega_1^2 v_1^2 \omega_2^3 + 54\omega_3^2 \omega_1^2 \omega_2^3 v_2^2 + 9\omega_3^2 \omega_1^2 \omega_2^3 + 6\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 - 24\omega_3 c_s^2 \omega_1^2 \omega_2^3 + 3\omega_3^2 \omega_1^2 v_1^2 \omega_2^3 + 8\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^4 + \\
& 36\omega_3 c_s^2 \omega_1^2 v_1^2 \omega_2^2 + 36\omega_3^2 \omega_1^2 \omega_2^3 v_2^4 + 108\omega_2^2 \omega_1^2 \omega_2^3 v_2^2 - 57\omega_3^2 c_s^4 \omega_1^2 \omega_2^3 - 72\omega_3 \omega_1^2 \omega_2^3 v_2^2 + 6\omega_3^2 \omega_1^2 v_1^2 \omega_2^3 - 72\omega_3 c_s^2 \omega_1^2 \omega_2^3 v_2^2 - 18\omega_3 c_s^4 \omega_1^2 \omega_2^3 - \\
& 72\omega_3^2 \omega_1^2 \omega_2^3 v_2^2 + 216\omega_3^2 \omega_1^2 \omega_2^3 v_2^2 + 17\omega_3^2 \omega_1^2 \omega_2^3 v_2^2 + 6\omega_3 c_s^4 \omega_1^2 \omega_2^3 + 72\omega_3^2 \omega_1^2 \omega_2^3 v_2^4 + 135\omega_3^2 \omega_1^2 \omega_2^3 v_2^2 - 36\omega_3 \omega_1^2 \omega_2^3 v_2^4 + 30\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^2 - \\
& 12\omega_3 c_s^2 \omega_1^2 \omega_2^3 v_2^2 + 6\omega_3^2 \omega_1^2 \omega_2^2 - 297\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^2 - 12\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^2 - 63\omega_3^2 \omega_1^2 \omega_2^3 v_2^4 + 24\omega_3 c_s^2 \omega_1^2 \omega_2^3 v_2^2 + 306\omega_3 c_s^2 \omega_1^2 \omega_2^3 v_2^2 - 108\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^2 - 54\omega_3 c_s^2 \omega_1^2 \omega_2^3 v_2^2 - \\
& 54\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^2 + 78\omega_3 c_s^2 \omega_1^2 \omega_2^3 v_2^2 + 18\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^2 + 15\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^2 + 18\omega_3^2 \omega_1^2 \omega_2^3 v_2^2 + 19\omega_3^2 c_s^2 \omega_1^2 \omega_2^3 v_2^2 + 6\omega_3^2 \omega_1^2 \omega_2^3
\end{aligned}$$

$$\begin{aligned}
C_{19} = & 396\omega_3 \omega_1^2 \omega_2^2 v_2^2 + 18\omega_1 v_1^2 \omega_2^3 + 54c_s^2 \omega_1^2 \omega_2^2 + 90\omega_3 \omega_1 \omega_2^2 - 18\omega_1^2 v_1^2 \omega_2 + 45\omega_3 \omega_1 \omega_2^3 - 36\omega_3 v_1^2 \omega_2^3 - 10\omega_3 \omega_1^2 v_1^2 \omega_2^3 - 162\omega_3 c_s^2 \omega_1 \omega_2^2 - \\
& 18\omega_1 \omega_2^3 + 18\omega_1^2 v_1^2 \omega_2^2 + 198\omega_3 \omega_1^2 \omega_2^3 v_2^2 - 198\omega_3 \omega_1^2 \omega_2^3 v_2^2 - 27\omega_3 c_s^2 \omega_1 \omega_2^3 - 54c_s^2 \omega_1^2 \omega_2^3 - 30\omega_3 c_s^2 \omega_1^2 \omega_2^3 + 90\omega_3 \omega_1^2 \omega_2^2 - 297\omega_3 c_s^2 \omega_1^2 \omega_2^3 + \\
& 198\omega_3 \omega_1^2 \omega_2^3 v_2^2 - 198\omega_3 \omega_1^2 \omega_2^3 v_2^2 + 54c_s^2 \omega_1^2 \omega_2^3 + 270\omega_3 c_s^2 \omega_1^2 \omega_2^3 - 10\omega_3 \omega_1^2 \omega_2^2 + 10\omega_3 \omega_1^2 \omega_2^3 v_2^2 + 324\omega_3 c_s^2 \omega_1^2 \omega_2^2 + 18\omega_1^2 \omega_2^3 + 36\omega_3 \omega_1^2 \omega_2^3 v_1^2 - \\
& 126\omega_3 \omega_1^2 \omega_2^3 - 18\omega_1^2 \omega_1^2 \omega_2^3 - 198\omega_3 \omega_1^2 \omega_2^2 v_2^2 - 54\omega_3 \omega_1^2 \omega_2^3 + 135\omega_3 \omega_1^2 \omega_2^3 v_2^2 + 10\omega_3 \omega_1^2 \omega_2^3 - 162\omega_3 c_s^2 \omega_1^2 \omega_2^2 - 45\omega_3 \omega_1^2 \omega_2^3 v_2^2 - 18\omega_1^2 \omega_2^3 - 198\omega_3 \omega_1^2 \omega_2^2 v_2^2 + \\
& 45\omega_3 \omega_1^2 \omega_2^3 - 180\omega_3 \omega_1^2 \omega_2^2 + 30\omega_3 c_s^2 \omega_1^2 \omega_2^3 + 54\omega_3 c_s^2 \omega_1^2 \omega_2^3
\end{aligned}$$

$$\begin{aligned}
C_{20} = & 210\omega_3 \omega_1^2 \omega_2^2 v_2^2 + 6c_s^2 \omega_1^2 \omega_2^2 v_2^2 + 54\omega_3 c_s^2 \omega_1^2 \omega_2^2 + 404\omega_3 c_s^2 \omega_1^2 \omega_2^2 v_2^2 + 261\omega_3 c_s^2 \omega_1^2 \omega_2^2 v_2^2 - 36c_s^4 \omega_1^2 \omega_2^2 + 6\omega_3 \omega_1 \omega_2^2 + 99\omega_3 \omega_1 \omega_2^2 v_2^4 - \\
& 171\omega_3 c_s^4 \omega_1^2 \omega_2^2 - 600\omega_3 c_s^2 \omega_1^2 \omega_2^2 v_2^2 - 12\omega_3 \omega_1 \omega_2^2 + 99\omega_3 \omega_1 \omega_2^2 v_2^4 + 18c_s^2 \omega_1^2 \omega_2^2 + 90\omega_3 c_s^4 \omega_1^2 \omega_2^2 - 18\omega_3 c_s^2 \omega_1^2 \omega_2^2 v_2^4 - 60\omega_3 c_s^2 \omega_1^2 \omega_2^2 v_2^2 - 117\omega_3 \omega_1 \omega_2^2 v_2^4 + \\
& 45\omega_3 \omega_1^2 \omega_2^2 v_2^2 - 51\omega_3 \omega_1 \omega_2^2 v_2^2 + 12c_s^2 \omega_1^2 \omega_2^2 v_2^2 + 129\omega_3 \omega_1^2 \omega_2^2 v_2^2 - 12c_s^2 \omega_1^2 \omega_2^2 v_2^2 - 68\omega_3 c_s^2 \omega_1^2 \omega_2^2 v_2^2 - 98\omega_3 \omega_1^2 \omega_2^2 v_2^2 + 90\omega_3 c_s^4 \omega_1^2 \omega_2^2 v_2^2 + 411\omega_3 c_s^2 \omega_1^2 \omega_2^2 v_2^2 + \\
& 90\omega_3 \omega_1^2 \omega_2^2 v_2^4 + 51\omega_3 \omega_1^2 \omega_2^2 v_2^2 + 12c_s^2 \omega_1^2 \omega_2^2 v_2^2 - 6c_s^2 \omega_1^2 \omega_2^2 + 45\omega_3 \omega_1^2 \omega_2^2 v_2^4 - 117\omega_3 c_s^2 \omega_1^2 \omega_2^2 v_2^2 + 90\omega_3 \omega_1^2 \omega_2^2 v_2^4 - 78\omega_3 c_s^2 \omega_1^2 \omega_2^2 v_2^2 + 6\omega_3 \omega_1^2 \omega_2^2 + \\
& 141\omega_3 c_s^2 \omega_1^2 \omega_2^2 - 2\omega_3 \omega_1^2 \omega_2^2 + 54c_s^2 \omega_1^2 \omega_2^2 v_2^2 - 51\omega_3 c_s^4 \omega_1^2 \omega_2^2 v_2^2 + 129\omega_3 \omega_1^2 \omega_2^2 v_2^2 - 10\omega_3 c_s^4 \omega_1^2 \omega_2^2 - 6c_s^2 \omega_1^2 \omega_2^2 - 72\omega_3 c_s^2 \omega_1^2 \omega_2^2 v_2^3 + 8\omega_3 \omega_1^2 \omega_2^2 v_2^2 + 114\omega_3 c_s^2 \omega_1^2 \omega_2^2 v_2^2 + \\
& 6\omega_3 \omega_1^2 \omega_2^2 - 12\omega_3 \omega_1^2 \omega_2^2 - 198\omega_3 \omega_1^2 \omega_2^2 v_2^4 + 12\omega_3 c_s^2 \omega_1^2 \omega_2^2 v_2^2 + 82\omega_3 c_s^2 \omega_1^2 \omega_2^2 v_2^2 - 171\omega_3 c_s^2 \omega_1^2 \omega_2^2 v_2^2 + 82\omega_3 c_s^2 \omega_1^2 \omega_2^2 v_2^2 - 105\omega_3 \omega_1^2 \omega_2^2 v_2^2 + \\
& 6\omega_3 \omega_1^2 \omega_2^2 - 12\omega_3 \omega_1^2 \omega_2^2 - 666\omega_3 \omega_1^2 \omega_2^2 v_2^4 + 4\omega_3 c_s^2 \omega_1^2 \omega_2^2 v_2^2 + 8\omega_3 \omega_1^2 \omega_2^2 - 12\omega_3 c_s^2 \omega_1^2 \omega_2^2 v_2^2 + 6c_s^4 \omega_1^2 \omega_2^2 v_2^2 + 252\omega_3 c_s^2 \omega_1^2 \omega_2^2 v_2^2 - 153\omega_3 \omega_1^2 \omega_2^2 v_2^2 - \\
& 432\omega_3 c_s^2 \omega_1^2 \omega_2^2 v_2^2 + 28\omega_3 \omega_1^2 \omega_2^2 v_2^2 - 12\omega_3 \omega_1^2 \omega_2^2 - 22\omega_3 c_s^2 \omega_1^2 \omega_2^2 - 24\omega_3 c_s^2 \omega_1^2 \omega_2^2
\end{aligned}$$

### 3 Comparison of SRT, MRT, CLBM, and CuLBM

#### 3.1 Conservation of mass: $\rho$

$$\begin{aligned}
& \frac{\partial \rho}{\partial t} + v_1 \frac{\delta_L}{\delta_t} \frac{\partial \rho}{\partial x_1} + \rho \frac{\delta_L}{\delta_t} \frac{\partial v_1}{\partial x_1} + v_2 \frac{\delta_L}{\delta_t} \frac{\partial \rho}{\partial x_2} + \rho \frac{\delta_L}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + v_1^2 + 3c_s^2) \frac{v_1}{12} \frac{\delta_L}{\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + 3v_1^2 + c_s^2) \frac{\rho}{12} \frac{\delta_L}{\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\
& \frac{\rho c_s^2}{6} \frac{\delta_L}{\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{\rho c_s^2}{6} \frac{\delta_L}{\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{v_2}{12} \frac{\delta_L}{\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + c_s^2 + 3v_2^2) \frac{\rho}{12} \frac{\delta_L}{\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_{D_x^4 \rho}^{(0)} \frac{\delta_L}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& C_{D_x^4 v_1}^{(0)} \frac{\delta_L}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_{D_x^3 D_y \rho}^{(0)} \frac{\delta_L}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{D_x^3 D_y v_1}^{(0)} \frac{\delta_L}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + C_{D_x^3 D_y v_2}^{(0)} \frac{\delta_L}{\delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_{D_x^2 D_y^2 \rho}^{(0)} \frac{\delta_L}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\
& C_{D_x^2 D_y^2 v_1}^{(0)} \frac{\delta_L}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{D_x^2 D_y^2 v_2}^{(0)} \frac{\delta_L}{\delta_t} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_{D_x D_y^3 \rho}^{(0)} \frac{\delta_L}{\delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{D_x D_y^3 v_1}^{(0)} \frac{\delta_L}{\delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{D_x D_y^3 v_2}^{(0)} \frac{\delta_L}{\delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\
& C_{D_y^4 \rho}^{(0)} \frac{\delta_L}{\delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{D_y^4 v_2}^{(0)} \frac{\delta_L}{\delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

**coefficient**  $C_{D_x^4 \rho}^{(0)}$  **at**  $\frac{\partial^4 \rho}{\partial x_1^4}$ :

$$C_{D_x^4 \rho}^{(0), SRT} = (-3v_1^4 \omega - 6v_1^2 - 2c_s^2 + c_s^2 \omega + 24v_1^2 c_s^2 - 12v_1^2 c_s^2 \omega + 3v_1^2 \omega + 2c_s^4 - c_s^4 \omega + 6v_1^4) \frac{1}{24\omega}$$

$$C_{D_x^4 \rho}^{(0), MRT1} = (3\omega_5 v_1^2 - 6v_1^2 - 2c_s^2 - 12\omega_5 v_1^2 c_s^2 + \omega_5 c_s^2 + 24v_1^2 c_s^2 + 2c_s^4 - \omega_5 c_s^4 - 3\omega_5 v_1^4 + 6v_1^4) \frac{1}{24\omega_5}$$

$$C_{D_x^4 \rho}^{(0), MRT2} = C_{D_x^4 \rho}^{(0), MRT1}$$

$$C_{D_x^4 \rho}^{(0), CLBM1} = C_{D_x^4 \rho}^{(0), MRT1}$$

$$C_{D_x^4 \rho}^{(0), CLBM2} = C_{D_x^4 \rho}^{(0), MRT1}$$

$$C_{D_x^4 \rho}^{(0), CuLBM1} = (-6v_1^2 - 3v_1^4 \omega_1 - 2c_s^2 - 12v_1^2 c_s^2 \omega_1 + c_s^2 \omega_1 + 24v_1^2 c_s^2 + 2c_s^4 + 3v_1^2 \omega_1 - c_s^4 \omega_1 + 6v_1^4) \frac{1}{24\omega_1}$$

$$C_{D_x^4 \rho}^{(0), CuLBM2} = (3\omega_2 v_1^4 - \omega_2 c_s^4 \omega_1 + 3v_1^4 \omega_1 + \omega_2 c_s^4 + 3\omega_2 v_1^2 \omega_1 + 12v_1^2 c_s^2 \omega_1 - c_s^2 \omega_1 + 12\omega_2 v_1^2 c_s^2 - \omega_2 c_s^2 + \omega_2 c_s^2 \omega_1 - 3v_1^2 \omega_1 - 12\omega_2 v_1^2 c_s^2 \omega_1 - 3\omega_2 v_1^4 \omega_1 + c_s^4 \omega_1 - 3\omega_2 v_1^2) \frac{1}{24\omega_2 \omega_1}$$

**coefficient**  $C_{D_x^4 v_1}^{(0)}$  **at**  $\frac{\partial^4 v_1}{\partial x_1^4}$ :

$$C_{D_x^4 v_1}^{(0), SRT} = (-4 + 10v_1^2 + 6c_s^2 - 3c_s^2 \omega - 5v_1^2 \omega + 2\omega) \frac{v_1 \rho}{12\omega}$$

$$C_{D_x^4 v_1}^{(0), MRT1} = (-4 - 5\omega_5 v_1^2 + 2\omega_5 + 10v_1^2 + 6c_s^2 - 3\omega_5 c_s^2) \frac{v_1 \rho}{12\omega_5}$$

$$C_{D_x^4 v_1}^{(0), MRT2} = C_{D_x^4 v_1}^{(0), MRT1}$$

$$C_{D_x^4 v_1}^{(0), CLBM1} = C_{D_x^4 v_1}^{(0), MRT1}$$

$$C_{D_x^4 v_1}^{(0), CLBM2} = C_{D_x^4 v_1}^{(0), MRT1}$$

$$C_{D_x^4 v_1}^{(0), CuLBM1} = (-4 + 10v_1^2 + 6c_s^2 - 3c_s^2 \omega_1 - 5v_1^2 \omega_1 + 2\omega_1) \frac{v_1 \rho}{12\omega_1}$$

$$C_{D_x^4 v_1}^{(0), CuLBM2} = (-2\omega_2 + 2\omega_2 \omega_1 - 5\omega_2 v_1^2 \omega_1 + 3c_s^2 \omega_1 + 3\omega_2 c_s^2 - 3\omega_2 c_s^2 \omega_1 + 5v_1^2 \omega_1 - 2\omega_1 + 5\omega_2 v_1^2) \frac{v_1 \rho}{12\omega_2 \omega_1}$$

**coefficient**  $C_{D_x^3 D_y \rho}^{(0)}$  **at**  $\frac{\partial^4 \rho}{\partial x_1^3 \partial x_2}$ :

$$C_{D_x^3 D_y \rho}^{(0), SRT} = 0$$

$$C_{D_x^3 D_y \rho}^{(0), MRT1} = (-\omega_5 v_1^2 + \omega_5 + v_1^2 \omega_7 - 3\omega_5 c_s^2 + 3c_s^2 \omega_7 - \omega_7) \frac{v_1 v_2}{4\omega_5 \omega_7}$$

$$C_{D_x^3 D_y \rho}^{(0), MRT2} = C_{D_x^3 D_y \rho}^{(0), MRT1}$$

$$C_{D_x^3 D_y \rho}^{(0), CLBM1} = 0$$

$$C_{D_x^3 D_y \rho}^{(0), CLBM2} = 0$$

$$C_{D_x^3 D_y \rho}^{(0), CuLBM1} = 0$$

$$C_{D_x^3 D_y \rho}^{(0), CuLBM2} = (-\omega_2 v_2^2 + \omega_2 + v_2^2 \omega_1 + 3c_s^2 \omega_1 - 3\omega_2 c_s^2 - \omega_1) \frac{v_1 v_2}{8\omega_2 \omega_1}$$

**coefficient**  $C_{D_x^3 D_y v_1}^{(0)}$  **at**  $\frac{\partial^4 v_1}{\partial x_1^3 \partial x_2}$ :

$$C_{D_x^3 D_y v_1}^{(0), SRT} = 0$$

$$C_{D_x^3 D_y v_1}^{(0), \text{MRT1}} = (-3\omega_5 v_1^2 + \omega_5 + 3v_1^2 \omega_7 - \omega_5 c_s^2 + c_s^2 \omega_7 - \omega_7) \frac{\rho v_2}{4\omega_5 \omega_7}$$

$$C_{D_x^3 D_y v_1}^{(0), \text{MRT2}} = C_{D_x^3 D_y v_1}^{(0), \text{MRT1}}$$

$$C_{D_x^3 D_y v_1}^{(0), \text{CLBM1}} = 0$$

$$C_{D_x^3 D_y v_1}^{(0), \text{CLBM2}} = 0$$

$$C_{D_x^3 D_y v_1}^{(0), \text{CuLBM1}} = 0$$

$$C_{D_x^3 D_y v_1}^{(0), \text{CuLBM2}} = (-\omega_2 v_2^2 + \omega_2 + v_2^2 \omega_1 + 3c_s^2 \omega_1 - 3\omega_2 c_s^2 - \omega_1) \frac{\rho v_2}{24\omega_2 \omega_1}$$

**coefficient**  $C_{D_x^3 D_y v_2}^{(0)}$  **at**  $\frac{\partial^4 v_2}{\partial x_1^3 \partial x_2}$ :

$$C_{D_x^3 D_y v_2}^{(0), \text{SRT}} = (2 - 2v_1^2 - 6c_s^2 + 3c_s^2 \omega + v_1^2 \omega - \omega) \frac{v_1 \rho}{12\omega}$$

$$C_{D_x^3 D_y v_2}^{(0), \text{MRT1}} = (v_1^2 \omega_7 \omega_4 - \omega_7 \omega_4 - \omega_5 \omega_7 \omega_4 + \omega_5 v_1^2 \omega_7 \omega_4 - 3\omega_5 c_s^2 \omega_4 - 6\omega_5 c_s^2 \omega_7 + 3\omega_5 \omega_4 - 3\omega_5 v_1^2 \omega_4 + 3c_s^2 \omega_7 \omega_4 + 3\omega_5 c_s^2 \omega_7 \omega_4) \frac{v_1 \rho}{12\omega_5 \omega_7 \omega_4}$$

$$C_{D_x^3 D_y v_2}^{(0), \text{MRT2}} = C_{D_x^3 D_y v_2}^{(0), \text{MRT1}}$$

$$C_{D_x^3 D_y v_2}^{(0), \text{CLBM1}} = (-3\omega_5 v_1^2 + 3\omega_5 + \omega_5 v_1^2 \omega_7 - \omega_5 \omega_7 + v_1^2 \omega_7 - 9\omega_5 c_s^2 + 3\omega_5 c_s^2 \omega_7 + 3c_s^2 \omega_7 - \omega_7) \frac{v_1 \rho}{12\omega_5 \omega_7}$$

$$C_{D_x^3 D_y v_2}^{(0), \text{CLBM2}} = C_{D_x^3 D_y v_2}^{(0), \text{CLBM1}}$$

$$C_{D_x^3 D_y v_2}^{(0), \text{CuLBM1}} = (3c_s^2 \omega_4 \omega_1 + 3c_s^2 \omega_4 - 9c_s^2 \omega_1 - 3v_1^2 \omega_1 + v_1^2 \omega_4 - \omega_4 + 3\omega_1 - \omega_4 \omega_1 + v_1^2 \omega_4 \omega_1) \frac{v_1 \rho}{12\omega_4 \omega_1}$$

$$C_{D_x^3 D_y v_2}^{(0), \text{CuLBM2}} = (6\omega_3 c_s^2 \omega_1 + 2\omega_2 \omega_3 - 4\omega_3 \omega_1 + \omega_2 v_1^2 \omega_3 + v_1^2 \omega_3 \omega_1 + 6\omega_2 \omega_1 - 6\omega_2 v_1^2 \omega_1 - 18\omega_2 c_s^2 \omega_1 + 9v_2^2 \omega_3 \omega_1 + 2\omega_2 v_1^2 \omega_3 \omega_1 - 2\omega_2 \omega_3 \omega_1 - 9\omega_2 v_2^2 \omega_3 + 6\omega_2 \omega_3 c_s^2 \omega_1) \frac{v_1 \rho}{24\omega_2 \omega_3 \omega_1}$$

**coefficient**  $C_{D_x^2 D_y^2 \rho}^{(0)}$  **at**  $\frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2}$ :

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{SRT}} = (-2 + \omega) \frac{c_s^4}{6\omega}$$

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{MRT1}} = (-2 + \omega_4) \frac{c_s^4}{6\omega_4}$$

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{MRT2}} = C_{D_x^2 D_y^2 \rho}^{(0), \text{MRT1}}$$

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{CLBM1}} = C_{D_x^2 D_y^2 \rho}^{(0), \text{MRT1}}$$

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{CLBM2}} = C_{D_x^2 D_y^2 \rho}^{(0), \text{MRT1}}$$

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{CuLBM1}} = (-2 + \omega_3) \frac{c_s^4}{6\omega_3}$$

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{CuLBM2}} = (-3\omega_2 v_2^2 + 2\omega_2 + 3v_2^2 \omega_1 + 2c_s^2 \omega_1 - 10\omega_2 c_s^2 + 4\omega_2 c_s^2 \omega_1 + 3v_1^2 \omega_1 - 2\omega_1 - 3\omega_2 v_1^2) \frac{c_s^2}{24\omega_2 \omega_1}$$

**coefficient**  $C_{D_x^2 D_y^2 v_1}^{(0)}$  **at**  $\frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2}$ :

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{SRT}} = 0$$

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{MRT1}} = (-\omega_7 + \omega_4) \frac{v_1 \rho c_s^2}{2\omega_7 \omega_4}$$

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{MRT2}} = C_{D_x^2 D_y^2 v_1}^{(0), \text{MRT1}}$$

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{CLBM1}} = 0$$

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{CLBM2}} = 0$$

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{CuLBM1}} = 0$$

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{CuLBM2}} = (\omega_2 + 3c_s^2\omega_1 - 3\omega_2 c_s^2 + v_1^2\omega_1 - \omega_1 - \omega_2 v_1^2) \frac{v_1 \rho}{24\omega_2 \omega_1}$$

**coefficient**  $C_{D_x^2 D_y^2 v_2}^{(0)}$  **at**  $\frac{\partial^4 v_2}{\partial x_1^4 \partial x_2^2}$ :

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{SRRT}} = 0$$

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{MRT1}} = (-\omega_8 + \omega_4) \frac{\rho v_2 c_s^2}{2\omega_8 \omega_4}$$

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{MRT2}} = C_{D_x^2 D_y^2 v_2}^{(0), \text{MRT1}}$$

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{CLBM1}} = 0$$

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{CLBM2}} = 0$$

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{CuLBM1}} = 0$$

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{CuLBM2}} = (-\omega_2 v_2^2 + \omega_2 + v_2^2 \omega_1 + 3c_s^2 \omega_1 - 3\omega_2 c_s^2 - \omega_1) \frac{\rho v_2}{24\omega_2 \omega_1}$$

**coefficient**  $C_{D_x D_y^3 \rho}^{(0)}$  **at**  $\frac{\partial^4 \rho}{\partial x_1 \partial x_2^3}$ :

$$C_{D_x D_y^3 \rho}^{(0), \text{SRRT}} = 0$$

$$C_{D_x D_y^3 \rho}^{(0), \text{MRT1}} = (-\omega_8 - v_2^2 \omega_6 + 3\omega_8 c_s^2 + \omega_6 + \omega_8 v_2^2 - 3\omega_6 c_s^2) \frac{v_1 v_2}{4\omega_8 \omega_6}$$

$$C_{D_x D_y^3 \rho}^{(0), \text{MRT2}} = C_{D_x D_y^3 \rho}^{(0), \text{MRT1}}$$

$$C_{D_x D_y^3 \rho}^{(0), \text{CLBM1}} = 0$$

$$C_{D_x D_y^3 \rho}^{(0), \text{CLBM2}} = 0$$

$$C_{D_x D_y^3 \rho}^{(0), \text{CuLBM1}} = 0$$

$$C_{D_x D_y^3 \rho}^{(0), \text{CuLBM2}} = (\omega_2 + 3c_s^2 \omega_1 - 3\omega_2 c_s^2 + v_1^2 \omega_1 - \omega_1 - \omega_2 v_1^2) \frac{v_1 v_2}{8\omega_2 \omega_1}$$

**coefficient**  $C_{D_x D_y^3 v_1}^{(0)}$  **at**  $\frac{\partial^4 v_1}{\partial x_1 \partial x_2^3}$ :

$$C_{D_x D_y^3 v_1}^{(0), \text{SRRT}} = (2 + v_2^2 \omega - 6c_s^2 + 3c_s^2 \omega - 2v_2^2 - \omega) \frac{\rho v_2}{12\omega}$$

$$C_{D_x D_y^3 v_1}^{(0), \text{MRT1}} = (-6\omega_8 \omega_6 c_s^2 + 3\omega_6 \omega_4 + 3\omega_8 c_s^2 \omega_4 - 3\omega_6 c_s^2 \omega_4 + \omega_8 v_2^2 \omega_6 \omega_4 - \omega_8 \omega_6 \omega_4 + \omega_8 v_2^2 \omega_4 - 3v_2^2 \omega_6 \omega_4 - \omega_8 \omega_4 + 3\omega_8 \omega_6 c_s^2 \omega_4) \frac{\rho v_2}{12\omega_8 \omega_6 \omega_4}$$

$$C_{D_x D_y^3 v_1}^{(0), \text{MRT2}} = C_{D_x D_y^3 v_1}^{(0), \text{MRT1}}$$

$$C_{\text{D}_x \text{D}_y^3 v_1}^{(0), \text{CLBM1}} = (-\omega_8 + 3\omega_8\omega_6c_s^2 - 3v_2^2\omega_6 + 3\omega_8c_s^2 + 3\omega_6 + \omega_8v_2^2 - \omega_8\omega_6 + \omega_8v_2^2\omega_6 - 9\omega_6c_s^2) \frac{\rho v_2}{12\omega_8\omega_6}$$

$$C_{\text{D}_x \text{D}_y^3 v_1}^{(0), \text{CLBM2}} = C_{\text{D}_x \text{D}_y^3 v_1}^{(0), \text{CLBM1}}$$

$$C_{\text{D}_x \text{D}_y^3 v_1}^{(0), \text{CuLBM1}} = (-3\omega_2v_2^2 - \omega_2\omega_6 + v_2^2\omega_6 + 3\omega_2 + 3\omega_2\omega_6c_s^2 - \omega_6 - 9\omega_2c_s^2 + 3\omega_6c_s^2 + \omega_2v_2^2\omega_6) \frac{\rho v_2}{12\omega_2\omega_6}$$

$$C_{\text{D}_x \text{D}_y^3 v_1}^{(0), \text{CuLBM2}} = (6\omega_3c_s^2\omega_1 + 2\omega_2\omega_3 - 4\omega_3\omega_1 - 9\omega_2v_1^2\omega_3 + 9v_1^2\omega_3\omega_1 + 2\omega_2v_2^2\omega_3\omega_1 + 6\omega_2\omega_1 - 18\omega_2c_s^2\omega_1 - 6\omega_2v_2^2\omega_1 + v_2^2\omega_3\omega_1 - 2\omega_2\omega_3\omega_1 + \omega_2v_2^2\omega_3 + 6\omega_2\omega_3c_s^2\omega_1) \frac{\rho v_2}{24\omega_2\omega_3\omega_1}$$

**coefficient**  $C_{\text{D}_x \text{D}_y^3 v_2}^{(0)}$  **at**  $\frac{\partial^4 v_2}{\partial x_1 \partial x_2^3}$ :

$$C_{\text{D}_x \text{D}_y^3 v_2}^{(0), \text{SRT}} = 0$$

$$C_{\text{D}_x \text{D}_y^3 v_2}^{(0), \text{MRT1}} = (-\omega_8 - 3v_2^2\omega_6 + \omega_8c_s^2 + \omega_6 + 3\omega_8v_2^2 - \omega_6c_s^2) \frac{v_1\rho}{4\omega_8\omega_6}$$

$$C_{\text{D}_x \text{D}_y^3 v_2}^{(0), \text{MRT2}} = C_{\text{D}_x \text{D}_y^3 v_2}^{(0), \text{MRT1}}$$

$$C_{\text{D}_x \text{D}_y^3 v_2}^{(0), \text{CLBM1}} = 0$$

$$C_{\text{D}_x \text{D}_y^3 v_2}^{(0), \text{CLBM2}} = 0$$

$$C_{\text{D}_x \text{D}_y^3 v_2}^{(0), \text{CuLBM1}} = 0$$

$$C_{\text{D}_x \text{D}_y^3 v_2}^{(0), \text{CuLBM2}} = (\omega_2 + 3c_s^2\omega_1 - 3\omega_2c_s^2 + v_1^2\omega_1 - \omega_1 - \omega_2v_1^2) \frac{v_1\rho}{24\omega_2\omega_1}$$

**coefficient**  $C_{\text{D}_y^4 \rho}^{(0)}$  **at**  $\frac{\partial^4 \rho}{\partial x_2^4}$ :

$$C_{\text{D}_y^4 \rho}^{(0), \text{SRT}} = (6v_2^4 + 3v_2^2\omega - 2c_s^2 + c_s^2\omega + 2c_s^4 + 24v_2^2c_s^2 - c_s^4\omega - 6v_2^2 - 12v_2^2c_s^2\omega - 3v_2^4\omega) \frac{1}{24\omega}$$

$$C_{\text{D}_y^4 \rho}^{(0), \text{MRT1}} = (6v_2^4 + 3v_2^2\omega_6 - 12v_2^2\omega_6c_s^2 - 2c_s^2 - \omega_6c_s^4 - 3v_2^4\omega_6 + 2c_s^4 + 24v_2^2c_s^2 + \omega_6c_s^2 - 6v_2^2) \frac{1}{24\omega_6}$$

$$C_{\text{D}_y^4 \rho}^{(0), \text{MRT2}} = C_{\text{D}_y^4 \rho}^{(0), \text{MRT1}}$$

$$C_{\text{D}_y^4 \rho}^{(0), \text{CLBM1}} = C_{\text{D}_y^4 \rho}^{(0), \text{MRT1}}$$

$$C_{\text{D}_y^4 \rho}^{(0), \text{CLBM2}} = C_{\text{D}_y^4 \rho}^{(0), \text{MRT1}}$$

$$C_{\text{D}_y^4 \rho}^{(0), \text{CuLBM1}} = (6v_2^4 + 3\omega_2v_2^2 - 12\omega_2v_2^2c_s^2 - 2c_s^2 - \omega_2c_s^4 + 2c_s^4 + \omega_2c_s^2 + 24v_2^2c_s^2 - 6v_2^2 - 3\omega_2v_2^4) \frac{1}{24\omega_2}$$

$$C_{\text{D}_y^4 \rho}^{(0), \text{CuLBM2}} = (-3\omega_2v_2^4\omega_1 - 12\omega_2v_2^2c_s^2\omega_1 - 3\omega_2v_2^2 - \omega_2c_s^4\omega_1 + 12\omega_2v_2^2c_s^2 + \omega_2c_s^4 - 3v_2^2\omega_1 - c_s^2\omega_1 - \omega_2c_s^2 + \omega_2c_s^2\omega_1 + 3\omega_2v_2^2\omega_1 + 3\omega_2v_2^4 + c_s^4\omega_1 + 12v_2^2c_s^2\omega_1 + 3v_2^4\omega_1) \frac{1}{24\omega_2\omega_1}$$

**coefficient**  $C_{\text{D}_y^4 v_2}^{(0)}$  **at**  $\frac{\partial^4 v_2}{\partial x_2^4}$ :

$$C_{\text{D}_y^4 v_2}^{(0), \text{SRT}} = (-4 - 5v_2^2\omega + 6c_s^2 - 3c_s^2\omega + 10v_2^2 + 2\omega) \frac{\rho v_2}{12\omega}$$

$$C_{\text{D}_y^4 v_2}^{(0), \text{MRT1}} = (-4 - 5v_2^2\omega_6 + 6c_s^2 + 2\omega_6 - 3\omega_6c_s^2 + 10v_2^2) \frac{\rho v_2}{12\omega_6}$$

$$C_{\text{D}_y^4 v_2}^{(0), \text{MRT2}} = C_{\text{D}_y^4 v_2}^{(0), \text{MRT1}}$$

$$C_{\text{D}_y^4 v_2}^{(0), \text{CLBM1}} = C_{\text{D}_y^4 v_2}^{(0), \text{MRT1}}$$

$$C_{D_y^4 v_2}^{(0), \text{CLBM2}} = C_{D_y^4 v_2}^{(0), \text{MRT1}}$$

$$C_{D_y^4 v_2}^{(0), \text{CuLBM1}} = (-4 - 5\omega_2 v_2^2 + 2\omega_2 + 6c_s^2 - 3\omega_2 c_s^2 + 10v_2^2) \frac{\rho v_2}{12\omega_2}$$

$$C_{D_y^4 v_2}^{(0), \text{CuLBM2}} = (5\omega_2 v_2^2 - 2\omega_2 + 5v_2^2 \omega_1 + 2\omega_2 \omega_1 + 3c_s^2 \omega_1 + 3\omega_2 c_s^2 - 3\omega_2 c_s^2 \omega_1 - 5\omega_2 v_2^2 \omega_1 - 2\omega_1) \frac{\rho v_2}{12\omega_2 \omega_1}$$

### 3.2 Conservation of momentum: $\rho v_1$

$$\begin{aligned} & v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (v_1^2 + c_s^2) \delta_t \frac{\partial \rho}{\partial x_1} + 2v_1 \rho \delta_t \frac{\partial v_1}{\partial x_1} + v_1 v_2 \delta_t \frac{\partial \rho}{\partial x_2} + \rho v_2 \delta_t \frac{\partial v_1}{\partial x_2} + v_1 \rho \delta_t \frac{\partial v_2}{\partial x_2} + C_{D_x \rho, D_x v_1}^{(1)} \frac{\delta_t^2}{\delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + \\ & C_{D_x v_1, D_x v_1}^{(1)} \frac{\delta_t^2}{\delta_t} \left( \frac{\partial v_1}{\partial x_1} \right)^2 + C_{D_x \rho, D_y v_2}^{(1)} \frac{\delta_t^2}{\delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_2} + C_{D_x v_2, D_y v_2}^{(1)} \frac{\delta_t^2}{\delta_t} \frac{\partial v_2}{\partial x_1} \frac{\partial v_2}{\partial x_2} + C_{D_y \rho, D_x v_2}^{(1)} \frac{\delta_t^2}{\delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\ & C_{D_y \rho, D_y v_1}^{(1)} \frac{\delta_t^2}{\delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + C_{D_x^2 \rho}^{(1)} \frac{\delta_t^2}{\delta_t} \frac{\partial^2 \rho}{\partial x_1^2} + C_{D_x^2 v_1}^{(1)} \frac{\delta_t^2}{\delta_t} \frac{\partial^2 v_1}{\partial x_1^2} + C_{D_x D_y \rho}^{(1)} \frac{\delta_t^2}{\delta_t} \frac{\partial^2 \rho}{\partial x_1 \partial x_2} + C_{D_x D_y v_2}^{(1)} \frac{\delta_t^2}{\delta_t} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + C_{D_y^2 v_1}^{(1)} \frac{\delta_t^2}{\delta_t} \frac{\partial^2 v_1}{\partial x_2^2} + \\ & + C_{D_x^3 \rho}^{(1)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + C_{D_x^3 v_1}^{(1)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + C_{D_x^2 D_y \rho}^{(1)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + C_{D_x^2 D_y v_1}^{(1)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} + C_{D_x^2 D_y v_2}^{(1)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + \\ & C_{D_x D_y^2 \rho}^{(1)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + C_{D_x D_y^2 v_1}^{(1)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_{D_x D_y^2 v_2}^{(1)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_{D_y^3 \rho}^{(1)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + C_{D_y^3 v_1}^{(1)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_2^3} + C_{D_y^3 v_2}^{(1)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & C_{D_x^4 \rho}^{(1)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + C_{D_x^4 v_1}^{(1)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_{D_x^3 D_y \rho}^{(1)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{D_x^3 D_y v_1}^{(1)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + C_{D_x^3 D_y v_2}^{(1)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_{D_x^2 D_y^2 \rho}^{(1)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\ & + C_{D_x^2 D_y^2 v_1}^{(1)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{D_x^2 D_y^2 v_2}^{(1)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_{D_x D_y^3 \rho}^{(1)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{D_x D_y^3 v_1}^{(1)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{D_x D_y^3 v_2}^{(1)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\ & C_{D_y^4 \rho}^{(1)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{D_y^4 v_1}^{(1)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_2^4} + C_{D_y^4 v_2}^{(1)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:

$$\text{coefficient } C_{D_x \rho, D_x v_1}^{(1)} \text{ at } \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1}:$$

$$C_{D_x \rho, D_x v_1}^{(1), \text{SRT}} = (-2 + 6v_1^2 + 4c_s^2 - 2c_s^2 \omega - 3v_1^2 \omega + \omega) \frac{1}{\omega}$$

$$C_{D_x \rho, D_x v_1}^{(1), \text{MRT1}} = (-2 - 3\omega_5 v_1^2 + \omega_5 + 6v_1^2 + 4c_s^2 - 2\omega_5 c_s^2) \frac{1}{\omega_5}$$

$$C_{D_x \rho, D_x v_1}^{(1), \text{MRT2}} = C_{D_x \rho, D_x v_1}^{(1), \text{MRT1}}$$

$$C_{D_x \rho, D_x v_1}^{(1), \text{CLBM1}} = C_{D_x \rho, D_x v_1}^{(1), \text{MRT1}}$$

$$C_{D_x \rho, D_x v_1}^{(1), \text{CLBM2}} = C_{D_x \rho, D_x v_1}^{(1), \text{MRT1}}$$

$$C_{D_x \rho, D_x v_1}^{(1), \text{CuLBM1}} = (-2 + 6v_1^2 + 4c_s^2 - 2c_s^2 \omega_1 - 3v_1^2 \omega_1 + \omega_1) \frac{1}{\omega_1}$$

$$C_{D_x \rho, D_x v_1}^{(1), \text{CuLBM2}} = (-\omega_2 + \omega_2 \omega_1 - 3\omega_2 v_1^2 \omega_1 + 2c_s^2 \omega_1 + 2\omega_2 c_s^2 - 2\omega_2 c_s^2 \omega_1 + 3v_1^2 \omega_1 - \omega_1 + 3\omega_2 v_1^2) \frac{1}{\omega_2 \omega_1}$$

$$\text{coefficient } C_{D_x v_1, D_x v_1}^{(1)} \text{ at } \left( \frac{\partial v_1}{\partial x_1} \right)^2:$$

$$C_{D_x v_1, D_x v_1}^{(1), \text{SRT}} = (2 - \omega) \frac{3v_1 \rho}{\omega}$$

$$C_{D_x v_1, D_x v_1}^{(1), \text{MRT1}} = (2 - \omega_5) \frac{3v_1 \rho}{\omega_5}$$

$$C_{D_x v_1, D_x v_1}^{(1), \text{MRT2}} = C_{D_x v_1, D_x v_1}^{(1), \text{MRT1}}$$

$$C_{D_x v_1, D_x v_1}^{(1), \text{CLBM1}} = C_{D_x v_1, D_x v_1}^{(1), \text{MRT1}}$$

$$C_{D_x v_1, D_x v_1}^{(1), \text{CLBM2}} = C_{D_x v_1, D_x v_1}^{(1), \text{MRT1}}$$

$$C_{D_x v_1, D_x v_1}^{(1), \text{CuLBM1}} = (2 - \omega_1) \frac{3v_1 \rho}{\omega_1}$$

$$C_{D_x v_1, D_x v_1}^{(1), \text{CuLBM2}} = (\omega_2 - \omega_2 \omega_1 + \omega_1) \frac{3v_1 \rho}{\omega_2 \omega_1}$$

**coefficient**  $C_{D_x \rho, D_y v_2}^{(1)}$  **at**  $\frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_2}$ :

$$C_{D_x \rho, D_y v_2}^{(1), SRT} = 0$$

$$C_{D_x \rho, D_y v_2}^{(1), MRT1} = 0$$

$$C_{D_x \rho, D_y v_2}^{(1), MRT2} = 0$$

$$C_{D_x \rho, D_y v_2}^{(1), CLBM1} = 0$$

$$C_{D_x \rho, D_y v_2}^{(1), CLBM2} = 0$$

$$C_{D_x \rho, D_y v_2}^{(1), CuLBM1} = 0$$

$$C_{D_x \rho, D_y v_2}^{(1), CuLBM2} = (-3\omega_2 v_2^2 + \omega_2 + 3v_2^2 \omega_1 + c_s^2 \omega_1 - \omega_2 c_s^2 - \omega_1) \frac{1}{2\omega_2 \omega_1}$$

**coefficient**  $C_{D_x v_2, D_y v_2}^{(1)}$  **at**  $\frac{\partial v_2}{\partial x_1} \frac{\partial v_2}{\partial x_2}$ :

$$C_{D_x v_2, D_y v_2}^{(1), SRT} = 0$$

$$C_{D_x v_2, D_y v_2}^{(1), MRT1} = 0$$

$$C_{D_x v_2, D_y v_2}^{(1), MRT2} = 0$$

$$C_{D_x v_2, D_y v_2}^{(1), CLBM1} = 0$$

$$C_{D_x v_2, D_y v_2}^{(1), CLBM2} = 0$$

$$C_{D_x v_2, D_y v_2}^{(1), CuLBM1} = 0$$

$$C_{D_x v_2, D_y v_2}^{(1), CuLBM2} = (-\omega_2 + \omega_1) \frac{3\rho v_2}{\omega_2 \omega_1}$$

**coefficient**  $C_{D_y \rho, D_x v_2}^{(1)}$  **at**  $\frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1}$ :

$$C_{D_y \rho, D_x v_2}^{(1), SRT} = (-2 + \omega) \frac{c_s^2}{2\omega}$$

$$C_{D_y \rho, D_x v_2}^{(1), MRT1} = (-2 + \omega_4) \frac{c_s^2}{2\omega_4}$$

$$C_{D_y \rho, D_x v_2}^{(1), MRT2} = C_{D_y \rho, D_x v_2}^{(1), MRT1}$$

$$C_{D_y \rho, D_x v_2}^{(1), CLBM1} = C_{D_y \rho, D_x v_2}^{(1), MRT1}$$

$$C_{D_y \rho, D_x v_2}^{(1), CLBM2} = C_{D_y \rho, D_x v_2}^{(1), MRT1}$$

$$C_{D_y \rho, D_x v_2}^{(1), CuLBM1} = (-2 + \omega_3) \frac{c_s^2}{2\omega_3}$$

$$C_{D_y \rho, D_x v_2}^{(1), CuLBM2} = (-3\omega_2 v_2^2 + \omega_2 + 3v_2^2 \omega_1 + 3c_s^2 \omega_1 - 5\omega_2 c_s^2 + \omega_2 c_s^2 \omega_1 - \omega_1) \frac{1}{2\omega_2 \omega_1}$$

**coefficient**  $C_{D_y \rho, D_y v_1}^{(1)}$  **at**  $\frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2}$ :

$$C_{D_y \rho, D_y v_1}^{(1), SRT} = (-2 + \omega) \frac{c_s^2}{2\omega}$$

$$C_{D_y \rho, D_y v_1}^{(1), MRT1} = (-2 + \omega_4) \frac{c_s^2}{2\omega_4}$$

$$C_{D_y \rho, D_y v_1}^{(1), MRT2} = C_{D_y \rho, D_y v_1}^{(1), MRT1}$$

$$C_{D_y \rho, D_y v_1}^{(1), CLBM1} = C_{D_y \rho, D_y v_1}^{(1), MRT1}$$

$$C_{D_y \rho, D_y v_1}^{(1), CLBM2} = C_{D_y \rho, D_y v_1}^{(1), MRT1}$$

$$C_{D_y \rho, D_y v_1}^{(1), CuLBM1} = (-2 + \omega_3) \frac{c_s^2}{2\omega_3}$$

$$C_{D_y \rho, D_y v_1}^{(1), CuLBM2} = (-2 + \omega_1) \frac{c_s^2}{2\omega_1}$$

**coefficient**  $C_{D_x^2 \rho}^{(1)}$  **at**  $\frac{\partial^2 \rho}{\partial x_1^2}$ :

$$C_{D_x^2 \rho}^{(1), SRT} = (-2 + 2v_1^2 + 6c_s^2 - 3c_s^2\omega - v_1^2\omega + \omega) \frac{v_1}{2\omega}$$

$$C_{D_x^2 \rho}^{(1), MRT1} = (-2 - \omega_5 v_1^2 + \omega_5 + 2v_1^2 + 6c_s^2 - 3\omega_5 c_s^2) \frac{v_1}{2\omega_5}$$

$$C_{D_x^2 \rho}^{(1), MRT2} = C_{D_x^2 \rho}^{(1), MRT1}$$

$$C_{D_x^2 \rho}^{(1), CLBM1} = C_{D_x^2 \rho}^{(1), MRT1}$$

$$C_{D_x^2 \rho}^{(1), CLBM2} = C_{D_x^2 \rho}^{(1), MRT1}$$

$$C_{D_x^2 \rho}^{(1), CuLBM1} = (-2 + 2v_1^2 + 6c_s^2 - 3c_s^2\omega_1 - v_1^2\omega_1 + \omega_1) \frac{v_1}{2\omega_1}$$

$$C_{D_x^2 \rho}^{(1), CuLBM2} = (-\omega_2 + \omega_2\omega_1 - \omega_2 v_1^2\omega_1 + 3c_s^2\omega_1 + 3\omega_2 c_s^2 - 3\omega_2 c_s^2\omega_1 + v_1^2\omega_1 - \omega_1 + \omega_2 v_1^2) \frac{v_1}{2\omega_2\omega_1}$$

**coefficient**  $C_{D_x^2 v_1}^{(1)}$  **at**  $\frac{\partial^2 v_1}{\partial x_1^2}$ :

$$C_{D_x^2 v_1}^{(1), SRT} = (-2 + 6v_1^2 + 2c_s^2 - c_s^2\omega - 3v_1^2\omega + \omega) \frac{\rho}{2\omega}$$

$$C_{D_x^2 v_1}^{(1), MRT1} = (-2 - 3\omega_5 v_1^2 + \omega_5 + 6v_1^2 + 2c_s^2 - \omega_5 c_s^2) \frac{\rho}{2\omega_5}$$

$$C_{D_x^2 v_1}^{(1), MRT2} = C_{D_x^2 v_1}^{(1), MRT1}$$

$$C_{D_x^2 v_1}^{(1), CLBM1} = C_{D_x^2 v_1}^{(1), MRT1}$$

$$C_{D_x^2 v_1}^{(1), CLBM2} = C_{D_x^2 v_1}^{(1), MRT1}$$

$$C_{D_x^2 v_1}^{(1), CuLBM1} = (-2 + 6v_1^2 + 2c_s^2 - c_s^2\omega_1 - 3v_1^2\omega_1 + \omega_1) \frac{\rho}{2\omega_1}$$

$$C_{D_x^2 v_1}^{(1), CuLBM2} = (-\omega_2 + \omega_2\omega_1 - 3\omega_2 v_1^2\omega_1 + c_s^2\omega_1 + \omega_2 c_s^2 - \omega_2 c_s^2\omega_1 + 3v_1^2\omega_1 - \omega_1 + 3\omega_2 v_1^2) \frac{\rho}{2\omega_2\omega_1}$$

**coefficient**  $C_{D_x D_y \rho}^{(1)}$  **at**  $\frac{\partial^2 \rho}{\partial x_1 \partial x_2}$ :

$$C_{D_x D_y \rho}^{(1), SRT} = 0$$

$$C_{D_x D_y \rho}^{(1), MRT1} = 0$$

$$C_{D_x D_y \rho}^{(1), MRT2} = 0$$

$$C_{D_x D_y \rho}^{(1), CLBM1} = 0$$

$$C_{D_x D_y \rho}^{(1), CLBM2} = 0$$

$$C_{D_x D_y \rho}^{(1), CuLBM1} = 0$$

$$C_{D_x D_y \rho}^{(1), \text{CuLBM2}} = (-\omega_2 v_2^2 + \omega_2 + v_2^2 \omega_1 + 3c_s^2 \omega_1 - 3\omega_2 c_s^2 - \omega_1) \frac{v_2}{2\omega_2 \omega_1}$$

**coefficient**  $C_{D_x D_y v_2}^{(1)}$  at  $\frac{\partial^2 v_2}{\partial x_1 \partial x_2}$ :

$$C_{D_x D_y v_2}^{(1), \text{SRT}} = (-2 + \omega) \frac{\rho c_s^2}{2\omega}$$

$$C_{D_x D_y v_2}^{(1), \text{MRT1}} = (-2 + \omega_4) \frac{\rho c_s^2}{2\omega_4}$$

$$C_{D_x D_y v_2}^{(1), \text{MRT2}} = C_{D_x D_y v_2}^{(1), \text{MRT1}}$$

$$C_{D_x D_y v_2}^{(1), \text{CLBMM1}} = C_{D_x D_y v_2}^{(1), \text{MRT1}}$$

$$C_{D_x D_y v_2}^{(1), \text{CLBMM2}} = C_{D_x D_y v_2}^{(1), \text{MRT1}}$$

$$C_{D_x D_y v_2}^{(1), \text{CuLBM1}} = (-2 + \omega_3) \frac{\rho c_s^2}{2\omega_3}$$

$$C_{D_x D_y v_2}^{(1), \text{CuLBM2}} = (-3\omega_2 v_2^2 + \omega_2 + 3v_2^2 \omega_1 + c_s^2 \omega_1 - 3\omega_2 c_s^2 + \omega_2 c_s^2 \omega_1 - \omega_1) \frac{\rho}{2\omega_2 \omega_1}$$

**coefficient**  $C_{D_y^2 v_1}^{(1)}$  at  $\frac{\partial^2 v_1}{\partial x_2^2}$ :

$$C_{D_y^2 v_1}^{(1), \text{SRT}} = (-2 + \omega) \frac{\rho c_s^2}{2\omega}$$

$$C_{D_y^2 v_1}^{(1), \text{MRT1}} = (-2 + \omega_4) \frac{\rho c_s^2}{2\omega_4}$$

$$C_{D_y^2 v_1}^{(1), \text{MRT2}} = C_{D_y^2 v_1}^{(1), \text{MRT1}}$$

$$C_{D_y^2 v_1}^{(1), \text{CLBMM1}} = C_{D_y^2 v_1}^{(1), \text{MRT1}}$$

$$C_{D_y^2 v_1}^{(1), \text{CLBMM2}} = C_{D_y^2 v_1}^{(1), \text{MRT1}}$$

$$C_{D_y^2 v_1}^{(1), \text{CuLBM1}} = (-2 + \omega_3) \frac{\rho c_s^2}{2\omega_3}$$

$$C_{D_y^2 v_1}^{(1), \text{CuLBM2}} = (-2 + \omega_1) \frac{\rho c_s^2}{2\omega_1}$$

**coefficient**  $C_{D_x^3 \rho}^{(1)}$  at  $\frac{\partial^3 \rho}{\partial x_1^3}$ :

$$C_{D_x^3 \rho}^{(1), \text{SRT}} = (24v_1^2 c_s^2 \omega^2 - c_s^2 \omega^2 - 36v_1^4 \omega - 36v_1^2 + 7v_1^4 \omega^2 - 12c_s^2 + 12c_s^2 \omega + 144v_1^2 c_s^2 - 144v_1^2 c_s^2 \omega + 36v_1^2 \omega + 12c_s^4 + c_s^4 \omega^2 - 12c_s^4 \omega - 7v_1^2 \omega^2 + 36v_1^4) \frac{1}{12\omega^2}$$

$$C_{D_x^3 \rho}^{(1), \text{MRT1}} =$$

$$(36\omega_5 v_1^2 - 36v_1^2 + 7\omega_5^2 v_1^4 + \omega_5^2 c_s^4 - 12c_s^2 - 144\omega_5 v_1^2 c_s^2 + 12\omega_5 c_s^2 + 144v_1^2 c_s^2 + 12c_s^4 - \omega_5^2 c_s^2 - 12\omega_5 c_s^4 + 24\omega_5^2 v_1^2 c_s^2 - 36\omega_5 v_1^4 - 7\omega_5^2 v_1^2 + 36v_1^4) \frac{1}{12\omega_5^2}$$

$$C_{D_x^3 \rho}^{(1), \text{MRT2}} = C_{D_x^3 \rho}^{(1), \text{MRT1}}$$

$$C_{D_x^3 \rho}^{(1), \text{CLBMM1}} = C_{D_x^3 \rho}^{(1), \text{MRT1}}$$

$$C_{D_x^3 \rho}^{(1), \text{CLBMM2}} = C_{D_x^3 \rho}^{(1), \text{MRT1}}$$

$$C_{D_x^3 \rho}^{(1), \text{CuLBM1}} =$$

$$(-c_s^2 \omega_1^2 + 24v_1^2 c_s^2 \omega_1^2 - 36v_1^2 - 36v_1^4 \omega_1 + 7v_1^4 \omega_1^2 - 12c_s^2 - 144v_1^2 c_s^2 \omega_1 + 12c_s^2 \omega_1 + 144v_1^2 c_s^2 + 12c_s^4 + 36v_1^2 \omega_1 + c_s^4 \omega_1^2 - 12c_s^4 \omega_1 - 7v_1^2 \omega_1^2 + 36v_1^4) \frac{1}{12\omega_1^2}$$

$$C_{D_x^3 \rho}^{(1), \text{CuLBM2}} = (-6c_s^2 \omega_1^2 - 18\omega_2^2 v_1^4 \omega_1 - 72\omega_2^2 v_1^2 c_s^2 \omega_1 + 18\omega_2 v_1^2 \omega_1^2 + 45v_1^2 c_s^2 \omega_1^2 - 9\omega_2^2 v_1^2 - \omega_2^2 c_s^2 \omega_1^2 + 6\omega_2^2 c_s^2 \omega_1 + 9v_1^4 \omega_1^2 + 45\omega_2^2 v_1^2 c_s^2 -$$

$$6\omega_2 c_s^4 \omega_1^2 - 18\omega_2 v_1^2 \omega_1 - 6\omega_2^2 c_s^2 + 7\omega_2^2 v_1^4 \omega_1^2 + 24\omega_2^2 v_1^2 c_s^2 \omega_1^2 + \omega_2^2 c_s^4 \omega_1^2 + 54\omega_2 v_1^2 c_s^2 \omega_1 + 18\omega_2^2 v_1^2 \omega_1 + 6c_s^4 \omega_1^2 - 18\omega_2 v_1^4 \omega_1^2 + 6\omega_2^2 c_s^4 + 18\omega_2 v_1^4 \omega_1 + 9\omega_2^2 v_1^4 - 7\omega_2^2 v_1^2 \omega_1^2 - 72\omega_2 v_1^2 c_s^2 \omega_1^2 - 9v_1^2 \omega_1^2 - 6\omega_2^2 c_s^4 \omega_1 + 6\omega_2 c_s^2 \omega_1^2) \frac{1}{12\omega_2^2 \omega_1^2}$$

**coefficient  $C_{D_x^3 v_1}^{(1)}$  at  $\frac{\partial^3 v_1}{\partial x_1^3}$ :**

$$C_{D_x^3 v_1}^{(1), SRT} = (-24 + 5c_s^2 \omega^2 + 60v_1^2 + 36c_s^2 - 36c_s^2 \omega - 60v_1^2 \omega + 24\omega - 4\omega^2 + 11v_1^2 \omega^2) \frac{v_1 \rho}{6\omega^2}$$

$$C_{D_x^3 v_1}^{(1), MRT1} = (-24 - 60\omega_5 v_1^2 + 24\omega_5 + 60v_1^2 - 4\omega_5^2 + 36c_s^2 - 36\omega_5 c_s^2 + 5\omega_5^2 c_s^2 + 11\omega_5^2 v_1^2) \frac{v_1 \rho}{6\omega_5^2}$$

$$C_{D_x^3 v_1}^{(1), MRT2} = C_{D_x^3 v_1}^{(1), MRT1}$$

$$C_{D_x^3 v_1}^{(1), CLBM1} = C_{D_x^3 v_1}^{(1), MRT1}$$

$$C_{D_x^3 v_1}^{(1), CLBM2} = C_{D_x^3 v_1}^{(1), MRT1}$$

$$C_{D_x^3 v_1}^{(1), CuLBM1} = (-24 + 5c_s^2 \omega_1^2 + 60v_1^2 + 36c_s^2 - 36c_s^2 \omega_1 - 60v_1^2 \omega_1 - 4\omega_1^2 + 24\omega_1 + 11v_1^2 \omega_1^2) \frac{v_1 \rho}{6\omega_1^2}$$

$$C_{D_x^3 v_1}^{(1), CuLBM2} = (27c_s^2 \omega_1^2 + 24\omega_2 \omega_1^2 - 60\omega_2 v_1^2 \omega_1^2 + 33\omega_2^2 v_1^2 - 15\omega_2^2 + 10\omega_2^2 c_s^2 \omega_1^2 - 36\omega_2^2 c_s^2 \omega_1 - 18\omega_2 \omega_1 + 54\omega_2 v_1^2 \omega_1 + 27\omega_2^2 c_s^2 + 24\omega_2^2 \omega_1 + 18\omega_2 c_s^2 \omega_1 - 60\omega_2^2 v_1^2 \omega_1 - 15\omega_2^2 + 22\omega_2^2 v_1^2 \omega_1^2 + 33v_1^2 \omega_1^2 - 8\omega_2^2 \omega_1^2 - 36\omega_2 c_s^2 \omega_1^2) \frac{v_1 \rho}{12\omega_2^2 \omega_1^2}$$

**coefficient  $C_{D_x^2 D_y \rho}^{(1)}$  at  $\frac{\partial^3 \rho}{\partial x_1^2 \partial x_2}$ :**

$$C_{D_x^2 D_y \rho}^{(1), SRT} = 0$$

$$C_{D_x^2 D_y \rho}^{(1), MRT1} = (v_1^2 \omega_7 \omega_4 - \omega_7 \omega_4 + \omega_5 \omega_7 \omega_4 - \omega_5 v_1^2 \omega_7 \omega_4 + \omega_5 v_1^2 \omega_7 - \omega_5 \omega_7 + \omega_5^2 - 3\omega_5 c_s^2 \omega_4 - \omega_5^2 \omega_4 + \omega_5^2 v_1^2 \omega_4 - 3\omega_5^2 c_s^2 \omega_4 + 3\omega_5^2 c_s^2 \omega_7 + \omega_5 \omega_4 - \omega_5 v_1^2 \omega_4 - \omega_5^2 v_1^2 + 3c_s^2 \omega_7 \omega_4 - 3\omega_5 c_s^2 \omega_7 \omega_4) \frac{v_1 v_2}{\omega_5^2 \omega_7 \omega_4}$$

$$C_{D_x^2 D_y \rho}^{(1), MRT2} = C_{D_x^2 D_y \rho}^{(1), MRT1}$$

$$C_{D_x^2 D_y \rho}^{(1), CLBM1} = 0$$

$$C_{D_x^2 D_y \rho}^{(1), CLBM2} = 0$$

$$C_{D_x^2 D_y \rho}^{(1), CuLBM1} = 0$$

$$C_{D_x^2 D_y \rho}^{(1), CuLBM2} = (6c_s^2 \omega_1^2 + \omega_2 \omega_1^2 + \omega_2^2 v_1^2 + v_2^2 \omega_1^2 + 3\omega_2^2 c_s^2 \omega_1 + 2\omega_2 \omega_1 - 2\omega_2 v_1^2 \omega_1 + \omega_2^2 v_2^2 \omega_1 - \omega_2^2 \omega_1 - 6\omega_2 c_s^2 \omega_1 - 2\omega_1^2 - \omega_2 v_2^2 \omega_1^2 + v_1^2 \omega_1^2 - 3\omega_2 c_s^2 \omega_1^2 - \omega_2^2 v_2^2) \frac{3v_1 v_2}{4\omega_2^2 \omega_1^2}$$

**coefficient  $C_{D_x^2 D_y v_1}^{(1)}$  at  $\frac{\partial^3 v_1}{\partial x_1^2 \partial x_2}$ :**

$$C_{D_x^2 D_y v_1}^{(1), SRT} = 0$$

$$C_{D_x^2 D_y v_1}^{(1), MRT1} = (3v_1^2 \omega_7 \omega_4 - \omega_7 \omega_4 + \omega_5 \omega_7 \omega_4 - 3\omega_5 v_1^2 \omega_7 \omega_4 + 3\omega_5 v_1^2 \omega_7 - \omega_5 \omega_7 + \omega_5^2 - \omega_5 c_s^2 \omega_4 - \omega_5^2 \omega_4 + 3\omega_5^2 v_1^2 \omega_4 - \omega_5^2 c_s^2 + \omega_5^2 c_s^2 \omega_4 + \omega_5 c_s^2 \omega_7 + \omega_5 \omega_4 - 3\omega_5 v_1^2 \omega_4 - 3\omega_5^2 v_1^2 + c_s^2 \omega_7 \omega_4 - \omega_5 c_s^2 \omega_7 \omega_4) \frac{\rho v_2}{\omega_5^2 \omega_7 \omega_4}$$

$$C_{D_x^2 D_y v_1}^{(1), MRT2} = C_{D_x^2 D_y v_1}^{(1), MRT1}$$

$$C_{D_x^2 D_y v_1}^{(1), CLBM1} = 0$$

$$C_{D_x^2 D_y v_1}^{(1), CLBM2} = 0$$

$$C_{D_x^2 D_y v_1}^{(1), CuLBM1} = 0$$

$$C_{\text{D}_x^2 \text{D}_y v_1}^{(1), \text{CuLBM2}} = (-\omega_2^2 \omega_3 \omega_1 + 2 \omega_2 \omega_1^2 - \omega_2 v_2^2 \omega_3 \omega_1^2 - 2 \omega_2^2 v_2^2 \omega_3 + 6 \omega_2^2 c_s^2 \omega_1 + 9 v_1^2 \omega_3 \omega_1^2 - 5 \omega_3 \omega_1^2 + 3 \omega_2^2 \omega_3 c_s^2 \omega_1 + 2 \omega_2^2 v_2^2 \omega_1 + 9 \omega_3 c_s^2 \omega_1^2 - 2 \omega_2^2 \omega_1 - 3 \omega_2 \omega_3 c_s^2 \omega_1^2 + \omega_2 \omega_3 \omega_1^2 - 18 \omega_2 v_2^2 \omega_3 \omega_1 - \omega_2^2 \omega_3 + \omega_2^2 v_2^2 \omega_3 \omega_1 - 2 \omega_2 v_2^2 \omega_1^2 + 2 v_2^2 \omega_3 \omega_1^2 + 6 \omega_2 \omega_3 \omega_1 + 9 \omega_2^2 v_1^2 \omega_3 - 6 \omega_2 c_s^2 \omega_1^2 - 6 \omega_2 \omega_3 c_s^2 \omega_1 - 3 \omega_2^2 \omega_3 c_s^2) \frac{\rho v_2}{4 \omega_2^2 \omega_3 \omega_1^2}$$

**coefficient**  $C_{\text{D}_x^2 \text{D}_y v_2}^{(1)}$  **at**  $\frac{\partial^3 v_2}{\partial x_1^2 \partial x_2}$ :

$$C_{\text{D}_x^2 \text{D}_y v_2}^{(1), \text{SRT}} = (12 - 11 c_s^2 \omega^2 - 12 v_1^2 - 36 c_s^2 + 36 c_s^2 \omega + 12 v_1^2 \omega - 12 \omega + 3 \omega^2 - 3 v_1^2 \omega^2) \frac{v_1 \rho}{12 \omega^2}$$

$$C_{\text{D}_x^2 \text{D}_y v_2}^{(1), \text{MRT1}} = (12 \omega_5^2 v_1^2 \omega_4^2 - 11 \omega_5^2 c_s^2 \omega_7 \omega_4^2 - 12 \omega_5 c_s^2 \omega_4^2 - 12 \omega_5^2 \omega_4^2 - 12 \omega_7 \omega_4^2 + 6 \omega_5 \omega_7 \omega_4^2 - 6 \omega_5 v_1^2 \omega_7 \omega_4^2 + 12 v_1^2 \omega_7 \omega_4^2 + 12 \omega_5^2 \omega_4 + 42 \omega_5^2 c_s^2 \omega_7 \omega_4 - 24 \omega_5^2 c_s^2 \omega_7 - 12 \omega_5^2 v_1^2 \omega_4 + 3 \omega_5^2 \omega_7 \omega_4^2 - 18 \omega_5 c_s^2 \omega_7 \omega_4^2 - 12 \omega_5^2 c_s^2 \omega_4 + 36 c_s^2 \omega_7 \omega_4^2 + 6 \omega_5^2 v_1^2 \omega_7 \omega_4 - 12 \omega_5 v_1^2 \omega_4^2 + 12 \omega_5 \omega_4^2 - 3 \omega_5^2 v_1^2 \omega_7 \omega_4^2 + 12 \omega_5^2 c_s^2 \omega_4^2 - 6 \omega_5^2 \omega_7 \omega_4 - 24 \omega_5 c_s^2 \omega_7 \omega_4) \frac{v_1 \rho}{12 \omega_5^2 \omega_7 \omega_4^2}$$

$$C_{\text{D}_x^2 \text{D}_y v_2}^{(1), \text{MRT2}} = C_{\text{D}_x^2 \text{D}_y v_2}^{(1), \text{MRT1}}$$

$$C_{\text{D}_x^2 \text{D}_y v_2}^{(1), \text{CLBM1}} = (12 v_1^2 \omega_7 \omega_4 - 12 \omega_7 \omega_4 + 6 \omega_5 \omega_7 \omega_4 - 6 \omega_5 v_1^2 \omega_7 \omega_4 + 12 \omega_5^2 - 36 \omega_5 c_s^2 \omega_4 - 12 \omega_5^2 \omega_4 - 11 \omega_5^2 c_s^2 \omega_7 \omega_4 + 18 \omega_5^2 c_s^2 \omega_7 + 12 \omega_5^2 v_1^2 \omega_4 + 6 \omega_5^2 v_1^2 \omega_7 - 36 \omega_5^2 c_s^2 + 36 \omega_5^2 c_s^2 \omega_4 - 6 \omega_5^2 \omega_7 - 3 \omega_5^2 v_1^2 \omega_7 \omega_4 + 12 \omega_5 \omega_4 - 12 \omega_5 v_1^2 \omega_4 - 12 \omega_5^2 v_1^2 + 36 c_s^2 \omega_7 \omega_4 + 3 \omega_5^2 \omega_7 \omega_4 - 18 \omega_5 c_s^2 \omega_7 \omega_4) \frac{v_1 \rho}{12 \omega_5^2 \omega_7 \omega_4}$$

$$C_{\text{D}_x^2 \text{D}_y v_2}^{(1), \text{CLBM2}} = C_{\text{D}_x^2 \text{D}_y v_2}^{(1), \text{CLBM1}}$$

$$C_{\text{D}_x^2 \text{D}_y v_2}^{(1), \text{CuLBM1}} = (-36 \omega_3 c_s^2 \omega_1 - 36 c_s^2 \omega_1^2 - 12 \omega_3 \omega_4 + 12 v_1^2 \omega_3 \omega_4 + 12 \omega_3 \omega_1 + 36 \omega_3 c_s^2 \omega_4 - 12 v_1^2 \omega_3 \omega_1 + 18 c_s^2 \omega_4 \omega_1^2 + 12 v_1^2 \omega_3 \omega_1^2 - 12 \omega_3 \omega_1^2 + 36 \omega_3 c_s^2 \omega_1^2 + 6 v_1^2 \omega_4 \omega_1^2 - 6 \omega_4 \omega_1^2 + 6 \omega_3 \omega_4 \omega_1 - 11 \omega_3 c_s^2 \omega_4 \omega_1^2 - 6 v_1^2 \omega_3 \omega_4 \omega_1 + 12 \omega_1^2 - 18 \omega_3 c_s^2 \omega_4 \omega_1 - 3 v_1^2 \omega_3 \omega_4 \omega_1^2 + 3 \omega_3 \omega_4 \omega_1^2 - 12 v_1^2 \omega_1^2) \frac{v_1 \rho}{12 \omega_3 \omega_4 \omega_1^2}$$

$$C_{\text{D}_x^2 \text{D}_y v_2}^{(1), \text{CuLBM2}} = (-12 \omega_2^2 \omega_3 \omega_1 - 11 \omega_2^2 \omega_3 c_s^2 \omega_1^2 + 6 \omega_2 \omega_1^2 - 6 \omega_2 v_1^2 \omega_1^2 - 27 \omega_2 v_2^2 \omega_3 \omega_1^2 - 3 \omega_2^2 v_1^2 \omega_3 \omega_1^2 - 27 \omega_2^2 v_2^2 \omega_3 + 36 \omega_2^2 c_s^2 \omega_1^2 - 54 \omega_2^2 c_s^2 \omega_1 + 6 v_1^2 \omega_3 \omega_1^2 - 15 \omega_3 \omega_1^2 + 3 \omega_2^2 v_1^2 \omega_3 \omega_1 + 3 \omega_2^2 \omega_3 \omega_1^2 + 18 \omega_2^2 c_s^2 \omega_1 + 27 \omega_3 c_s^2 \omega_1^2 + 18 \omega_2^2 \omega_1 - 18 \omega_2 \omega_3 c_s^2 \omega_1^2 - 18 \omega_2^2 v_1^2 \omega_1 + 12 \omega_2 \omega_3 \omega_1^2 - 3 \omega_2 v_1^2 \omega_3 \omega_1^2 + 3 \omega_2^2 \omega_3 + 27 \omega_2^2 v_2^2 \omega_3 \omega_1 + 12 \omega_2^2 v_1^2 \omega_1^2 + 27 v_2^2 \omega_3 \omega_1^2 + 6 \omega_2^2 v_1^2 \omega_3 - 12 \omega_2^2 \omega_1^2 - 18 \omega_2 c_s^2 \omega_1^2 + 9 \omega_2^2 \omega_3 c_s^2) \frac{v_1 \rho}{12 \omega_2^2 \omega_3 \omega_1^2}$$

**coefficient**  $C_{\text{D}_x \text{D}_y^2 \rho}^{(1)}$  **at**  $\frac{\partial^3 \rho}{\partial x_1 \partial x_2}$ :

$$C_{\text{D}_x \text{D}_y^2 \rho}^{(1), \text{SRT}} = (-12 + 12 \omega - \omega^2) \frac{c_s^4}{6 \omega^2}$$

$$C_{\text{D}_x \text{D}_y^2 \rho}^{(1), \text{MRT1}} = (-12 + 12 \omega_4 - \omega_4^2) \frac{c_s^4}{6 \omega_4^2}$$

$$C_{\text{D}_x \text{D}_y^2 \rho}^{(1), \text{MRT2}} = C_{\text{D}_x \text{D}_y^2 \rho}^{(1), \text{MRT1}}$$

$$C_{\text{D}_x \text{D}_y^2 \rho}^{(1), \text{CLBM1}} = C_{\text{D}_x \text{D}_y^2 \rho}^{(1), \text{MRT1}}$$

$$C_{\text{D}_x \text{D}_y^2 \rho}^{(1), \text{CLBM2}} = C_{\text{D}_x \text{D}_y^2 \rho}^{(1), \text{MRT1}}$$

$$C_{\text{D}_x \text{D}_y^2 \rho}^{(1), \text{CuLBM1}} = (-12 + 12 \omega_3 - \omega_3^2) \frac{c_s^4}{6 \omega_3^2}$$

$$C_{\text{D}_x \text{D}_y^2 \rho}^{(1), \text{CuLBM2}} = (-6 c_s^2 \omega_1^2 - 9 \omega_2^2 v_2^4 - 9 v_2^2 \omega_1^2 - 6 \omega_2^2 c_s^2 \omega_1 - 6 \omega_2 c_s^4 \omega_1^2 + 6 \omega_2^2 c_s^2 - 9 \omega_2^2 v_2^2 \omega_1 - 45 \omega_2 v_2^2 c_s^2 \omega_1^2 - 9 \omega_2 v_2^4 \omega_1^2 - 45 \omega_2^2 v_2^2 c_s^2 + 45 v_2^2 c_s^2 \omega_1^2 + 9 v_2^2 \omega_1^2 - 2 \omega_2^2 c_s^4 \omega_1^2 + 45 \omega_2^2 v_2^2 c_s^2 \omega_1 + 6 c_s^4 \omega_1^2 - 30 \omega_2^2 c_s^4 + 9 \omega_2^2 v_2^4 \omega_1 + 9 \omega_2 v_2^2 \omega_1^2 + 30 \omega_2^2 c_s^4 \omega_1 + 6 \omega_2 c_s^2 \omega_1^2 + 9 \omega_2^2 v_2^2) \frac{1}{12 \omega_2^2 \omega_1^2}$$

**coefficient**  $C_{\text{D}_x \text{D}_y^2 v_1}^{(1)}$  **at**  $\frac{\partial^3 v_1}{\partial x_1 \partial x_2}$ :

$$C_{\text{D}_x \text{D}_y^2 v_1}^{(1), \text{SRT}} = \frac{-v_1 \rho c_s^2}{6}$$

$$C_{\text{D}_x \text{D}_y^2 v_1}^{(1), \text{MRT1}} = (-12 \omega_7 \omega_4 + 12 \omega_5 \omega_7 \omega_4 - 12 \omega_5 \omega_7 - \omega_5 \omega_7 \omega_4^2 + 12 \omega_5 \omega_4 - 12 \omega_5 \omega_4^2 + 12 \omega_4^2) \frac{v_1 \rho c_s^2}{6 \omega_5 \omega_7 \omega_4^2}$$

$$C_{\text{D}_x \text{D}_y^2 v_1}^{(1), \text{MRT2}} = C_{\text{D}_x \text{D}_y^2 v_1}^{(1), \text{MRT1}}$$

$$C_{\text{D}_x \text{D}_y^2 v_1}^{(1), \text{CLBM1}} = C_{\text{D}_x \text{D}_y^2 v_1}^{(1), \text{SRT}}$$

$$C_{\text{D}_x \text{D}_y^2 v_1}^{(1), \text{CLBM2}} = C_{\text{D}_x \text{D}_y^2 v_1}^{(1), \text{SRT}}$$

$$C_{\text{D}_x \text{D}_y^2 v_1}^{(1), \text{CuLBM1}} = C_{\text{D}_x \text{D}_y^2 v_1}^{(1), \text{SRT}}$$

$$C_{\text{D}_x \text{D}_y^2 v_1}^{(1), \text{CuLBM2}} = C_{\text{D}_x \text{D}_y^2 v_1}^{(1), \text{SRT}}$$

**coefficient**  $C_{\text{D}_x \text{D}_y^2 v_2}^{(1)}$  **at**  $\frac{\partial^3 v_2}{\partial x_1 \partial x_2}$ :

$$C_{\text{D}_x \text{D}_y^2 v_2}^{(1), \text{SRT}} = 0$$

$$C_{\text{D}_x \text{D}_y^2 v_2}^{(1), \text{MRT1}} = (-2\omega_8 + 2\omega_4 - \omega_4^2 + \omega_8\omega_4) \frac{\rho v_2 c_s^2}{\omega_8 \omega_4^2}$$

$$C_{\text{D}_x \text{D}_y^2 v_2}^{(1), \text{MRT2}} = C_{\text{D}_x \text{D}_y^2 v_2}^{(1), \text{MRT1}}$$

$$C_{\text{D}_x \text{D}_y^2 v_2}^{(1), \text{CLBM1}} = 0$$

$$C_{\text{D}_x \text{D}_y^2 v_2}^{(1), \text{CLBM2}} = 0$$

$$C_{\text{D}_x \text{D}_y^2 v_2}^{(1), \text{CuLBM1}} = 0$$

$$C_{\text{D}_x \text{D}_y^2 v_2}^{(1), \text{CuLBM2}} = (9c_s^2 \omega_1^2 + 5\omega_2 \omega_1^2 + 5\omega_2^2 + 11v_2^2 \omega_1^2 + 9\omega_2^2 c_s^2 \omega_1 - 9\omega_2^2 c_s^2 + 11\omega_2^2 v_2^2 \omega_1 - 5\omega_2^2 \omega_1 - 5\omega_1^2 - 11\omega_2 v_2^2 \omega_1^2 - 9\omega_2 c_s^2 \omega_1^2 - 11\omega_2^2 v_2^2) \frac{\rho v_2}{4\omega_2^2 \omega_4^2}$$

**coefficient**  $C_{\text{D}_y^3 \rho}^{(1)}$  **at**  $\frac{\partial^3 \rho}{\partial x_2^3}$ :

$$C_{\text{D}_y^3 \rho}^{(1), \text{SRT}} = (-1 + 3c_s^2 + v_2^2) \frac{v_1 v_2}{12}$$

$$C_{\text{D}_y^3 \rho}^{(1), \text{MRT1}} = (-12\omega_8 - 12v_2^2 \omega_6 + 36\omega_8 c_s^2 + 12\omega_6 - 6\omega_6 \omega_4 + 12\omega_8 v_2^2 - 36\omega_6 c_s^2 - 18\omega_8 c_s^2 \omega_4 + 18\omega_6 c_s^2 \omega_4 + \omega_8 v_2^2 \omega_6 \omega_4 - \omega_8 \omega_6 \omega_4 - 6\omega_8 v_2^2 \omega_4 + 6v_2^2 \omega_6 \omega_4 + 6\omega_8 \omega_4 + 3\omega_8 \omega_6 c_s^2 \omega_4) \frac{v_1 v_2}{12\omega_8 \omega_6 \omega_4}$$

$$C_{\text{D}_y^3 \rho}^{(1), \text{MRT2}} = C_{\text{D}_y^3 \rho}^{(1), \text{MRT1}}$$

$$C_{\text{D}_y^3 \rho}^{(1), \text{CLBM1}} = C_{\text{D}_y^3 \rho}^{(1), \text{SRT}}$$

$$C_{\text{D}_y^3 \rho}^{(1), \text{CLBM2}} = C_{\text{D}_y^3 \rho}^{(1), \text{SRT}}$$

$$C_{\text{D}_y^3 \rho}^{(1), \text{CuLBM1}} = C_{\text{D}_y^3 \rho}^{(1), \text{SRT}}$$

**coefficient**  $C_{\text{D}_y^3 v_1}^{(1)}$  **at**  $\frac{\partial^3 v_1}{\partial x_2^3}$ :

$$C_{\text{D}_y^3 v_1}^{(1), \text{SRT}} = (6 - 3c_s^2 \omega^2 - v_2^2 \omega^2 + 6v_2^2 \omega - 18c_s^2 + 18c_s^2 \omega - 6v_2^2 - 6\omega + \omega^2) \frac{\rho v_2}{6\omega^2}$$

$$C_{\text{D}_y^3 v_1}^{(1), \text{MRT1}} = (3v_2^2 \omega_4^2 + 3c_s^2 \omega_4^2 - 12\omega_8 c_s^2 - 6c_s^2 \omega_4 - 6v_2^2 \omega_4 + \omega_8 \omega_4^2 - \omega_8 v_2^2 \omega_4^2 - 3\omega_8 c_s^2 \omega_4^2 + 15\omega_8 c_s^2 \omega_4 + 6\omega_4 + 3\omega_8 v_2^2 \omega_4 - 3\omega_4^2 - 3\omega_8 \omega_4) \frac{\rho v_2}{6\omega_8 \omega_4^2}$$

$$C_{\text{D}_y^3 v_1}^{(1), \text{MRT2}} = C_{\text{D}_y^3 v_1}^{(1), \text{MRT1}}$$

$$C_{\text{D}_y^3 v_1}^{(1), \text{CLBM1}} = (6 - 3\omega_8 + 9\omega_8 c_s^2 + 9c_s^2 \omega_4 - 18c_s^2 + 3v_2^2 \omega_4 + 3\omega_8 v_2^2 - 6v_2^2 - 3\omega_8 c_s^2 \omega_4 - 3\omega_4 - \omega_8 v_2^2 \omega_4 + \omega_8 \omega_4) \frac{\rho v_2}{6\omega_8 \omega_4}$$

$$C_{D_y^3 v_1}^{(1), \text{CLBM2}} = C_{D_y^3 v_1}^{(1), \text{CLBM1}}$$

$$C_{D_y^3 v_1}^{(1), \text{CuLBM1}} = (6 + 3v_2^2\omega_3 + \omega_6\omega_3 + 3v_2^2\omega_6 - 3\omega_6\omega_3c_s^2 - 18c_s^2 - 3\omega_6 - 3\omega_3 - v_2^2\omega_6\omega_3 + 9\omega_6c_s^2 - 6v_2^2 + 9\omega_3c_s^2) \frac{\rho v_2}{6\omega_6\omega_3}$$

$$C_{D_y^3 v_1}^{(1), \text{CuLBM2}} = (6 - 3\omega_3c_s^2\omega_1 + 3v_2^2\omega_3 + \omega_3\omega_1 - 18c_s^2 + 3v_2^2\omega_1 + 9c_s^2\omega_1 - 3\omega_3 - v_2^2\omega_3\omega_1 - 6v_2^2 + 9\omega_3c_s^2 - 3\omega_1) \frac{\rho v_2}{6\omega_3\omega_1}$$

**coefficient**  $C_{D_y^3 v_2}^{(1)}$  **at**  $\frac{\partial^3 v_2}{\partial x_2^3}$ :

$$C_{D_y^3 v_2}^{(1), \text{SRT}} = (-1 + c_s^2 + 3v_2^2) \frac{v_1\rho}{12}$$

$$C_{D_y^3 v_2}^{(1), \text{MRT1}} = (-12\omega_8 - 36v_2^2\omega_6 + 12\omega_8c_s^2 + 12\omega_6 - 6\omega_6\omega_4 + 36\omega_8v_2^2 - 12\omega_6c_s^2 - 6\omega_8c_s^2\omega_4 + 6\omega_6c_s^2\omega_4 + 3\omega_8v_2^2\omega_6\omega_4 - \omega_8\omega_6\omega_4 - 18\omega_8v_2^2\omega_4 + 18v_2^2\omega_6\omega_4 + 6\omega_8\omega_4 + \omega_8\omega_6c_s^2\omega_4) \frac{v_1\rho}{12\omega_8\omega_6\omega_4}$$

$$C_{D_y^3 v_2}^{(1), \text{MRT2}} = C_{D_y^3 v_2}^{(1), \text{MRT1}}$$

$$C_{D_y^3 v_2}^{(1), \text{CLBM1}} = C_{D_y^3 v_2}^{(1), \text{SRT}}$$

$$C_{D_y^3 v_2}^{(1), \text{CLBM2}} = C_{D_y^3 v_2}^{(1), \text{SRT}}$$

$$C_{D_y^3 v_2}^{(1), \text{CuLBM1}} = C_{D_y^3 v_2}^{(1), \text{SRT}}$$

$$C_{D_y^3 v_2}^{(1), \text{CuLBM2}} = C_{D_y^3 v_2}^{(1), \text{SRT}}$$

**coefficient**  $C_{D_x^4 \rho}^{(1)}$  **at**  $\frac{\partial^4 \rho}{\partial x_1^4}$ :

$$C_{D_x^4 \rho}^{(1), \text{SRT}} = (12 + 404v_1^2c_s^2\omega^2 - 78c_s^2\omega^2 - 216v_1^4\omega - 34v_1^2c_s^2\omega^3 - 156v_1^2 + 6c_s^2\omega^3 + 90v_1^4\omega^2 - 132c_s^2 + 198c_s^2\omega + 672v_1^2c_s^2\omega - 9v_1^4\omega^3 + 234v_1^2\omega + 144c_s^4 - 5c_s^4\omega^3 + 82c_s^4\omega^2 - 216c_s^4\omega - 18\omega + 10v_1^2\omega^3 + 8\omega^2 - 98v_1^2\omega^2 - \omega^3 + 144v_1^4) \frac{v_1}{12\omega^3}$$

$$C_{D_x^4 \rho}^{(1), \text{MRT1}} = (12 + 234\omega_5v_1^2 - 34\omega_5^3v_1^2c_s^2 - 18\omega_5 - 5\omega_5^3c_s^4 - 156v_1^2 + 90\omega_5^2v_1^4 + 82\omega_5^2c_s^4 + 8\omega_5^2 - 132c_s^2 - 9\omega_5^3v_1^4 - 1008\omega_5v_1^2c_s^2 + 198\omega_5c_s^2 + 672v_1^2c_s^2 - \omega_5^3 + 144c_s^4 - 78\omega_5^2c_s^2 - 216\omega_5c_s^4 + 404\omega_5^2v_1^2c_s^2 + 10\omega_5^3v_1^2 + 6\omega_5^3c_s^2 - 216\omega_5v_1^4 - 98\omega_5^2v_1^2 + 144v_1^4) \frac{v_1}{12\omega_5^3}$$

$$C_{D_x^4 \rho}^{(1), \text{MRT2}} = C_{D_x^4 \rho}^{(1), \text{MRT1}}$$

$$C_{D_x^4 \rho}^{(1), \text{CLBM1}} = C_{D_x^4 \rho}^{(1), \text{MRT1}}$$

$$C_{D_x^4 \rho}^{(1), \text{CLBM2}} = C_{D_x^4 \rho}^{(1), \text{MRT1}}$$

$$C_{D_x^4 \rho}^{(1), \text{CuLBM1}} = (12 - 78c_s^2\omega_1^2 + 404v_1^2c_s^2\omega_1^2 - 156v_1^2 + 6c_s^2\omega_1^3 - 216v_1^4\omega_1 - 34v_1^2c_s^2\omega_1^3 + 90v_1^4\omega_1^2 - 132c_s^2 - 1008v_1^2c_s^2\omega_1 - 9v_1^4\omega_1^3 + 198c_s^2\omega_1 + 672v_1^2c_s^2 + 144c_s^4 - 5c_s^4\omega_1^3 + 234v_1^2\omega_1 + 82c_s^4\omega_1^2 + 10v_1^2\omega_1^3 + 8\omega_1^2 - 216c_s^4\omega_1 - 18\omega_1 - 98v_1^2\omega_1^2 - \omega_1^3 + 144v_1^4) \frac{v_1}{12\omega_1^3}$$

$$C_{D_x^4 \rho}^{(1), \text{CuLBM2}} = (90v_2^3v_1^4\omega_3\omega_1^2 - 171\omega_2\omega_3c_s^4\omega_1^3 - 98\omega_2^2v_1^2\omega_3\omega_1^3 + 141\omega_3^2\omega_3c_s^2\omega_1 + 6\omega_2^2\omega_3\omega_1 + 114\omega_2^2\omega_3c_s^2\omega_1^2 + 99\omega_2^2v_1^4\omega_3\omega_1 + 45v_1^4\omega_3\omega_1^3 + 210\omega_2^2v_1^2\omega_3c_s^2\omega_1 - 600\omega_2^3v_1^2\omega_3c_s^2\omega_1 - 18\omega_2^3v_1^4\omega_3\omega_1^3 + 54\omega_2\omega_3c_s^4\omega_1^2 + 12\omega_2^2c_s^2\omega_1^2 + 129\omega_2^3v_1^2\omega_3\omega_1 - 78\omega_2^2\omega_3c_s^2\omega_1^3 + 90\omega_2^2v_1^4\omega_3\omega_1^3 - 98\omega_2^3v_1^2\omega_3\omega_1^2 - 72\omega_3^2\omega_3\omega_1^2 + 8\omega_2^2\omega_3\omega_1^3 + 18\omega_2^3c_s^2\omega_1 + 12\omega_2^2\omega_3c_s^2\omega_1^3 - 51\omega_2^3v_1^2\omega_3\omega_1 + 404\omega_2^3v_1^2\omega_3c_s^2\omega_1^2 - 105\omega_2^2v_1^2\omega_3\omega_1 - 72\omega_3c_s^2\omega_1^3 + 261v_1^2\omega_3c_s^2\omega_1^3 - 12\omega_2^2\omega_3\omega_1^2 - 51v_1^2\omega_3\omega_1^3 + 6\omega_2^3v_1^2c_s^2\omega_1 - 60\omega_2^3\omega_3c_s^2\omega_1 + 20\omega_2^2v_1^2\omega_3\omega_1^3 - 198\omega_2^2v_1^4\omega_3\omega_1^2 - 117\omega_2^3v_1^4\omega_3\omega_1 - 68\omega_2^3v_1^2\omega_3c_s^2\omega_1^3 + 18\omega_2^2c_s^4\omega_1^3 - 78\omega_2^3\omega_3c_s^2\omega_1^2 + 6\omega_3\omega_1^3 - 12\omega_2^2v_1^2c_s^2\omega_1^2 - 816\omega_2^2v_1^2\omega_3c_s^2\omega_1^2 - 12\omega_2\omega_3\omega_1^3 - 60\omega_2\omega_3c_s^2\omega_1^2 - 36\omega_2^2c_s^4\omega_1^2 + 90\omega_3^3\omega_3c_s^4 + 82\omega_2^2\omega_3c_s^4\omega_1^3 - 600\omega_2v_1^2\omega_3c_s^2\omega_1^2 + 129\omega_2v_1^2\omega_3\omega_1^3 + 6\omega_3^3\omega_3 + 141\omega_2\omega_3c_s^2\omega_1^3 - 12\omega_2^2\omega_3\omega_1 - 171\omega_2^3\omega_3c_s^4\omega_1 + 6\omega_2\omega_3\omega_1^2 + 404\omega_2^2v_1^2\omega_3c_s^2\omega_1^3 - 105\omega_2v_1^2\omega_3\omega_1^2 + 411\omega_2v_1^2\omega_3c_s^2\omega_1^2 - 90\omega_2^2\omega_3c_s^4\omega_1^2 + 54\omega_2^2\omega_3c_s^4\omega_1 + 6\omega_2v_1^2c_s^2\omega_1^3 + 8\omega_2^2\omega_3\omega_1^2 - 6\omega_2c_s^2\omega_1^3 - 117\omega_2v_1^4\omega_3\omega_1^3 + 261\omega_2^2v_1^2\omega_3c_s^2 + 82\omega_2^3\omega_3c_s^4\omega_1^2 + 411\omega_2^2v_1^2\omega_3c_s^2\omega_1 - 6\omega_2^3c_s^2\omega_1 + 99\omega_2v_1^4\omega_3\omega_1^2 + 45\omega_2^2v_1^4\omega_3 - 10\omega_2^3\omega_3c_s^4\omega_1^3 - 2\omega_2^3\omega_3\omega_1^3 + 90\omega_3c_s^4\omega_1^3) \frac{v_1}{24\omega_2^2\omega_3\omega_1^3}$$

**coefficient**  $C_{D_x^4 v_1}^{(1)}$  **at**  $\frac{\partial^4 v_1}{\partial x_1^4}$ :

$$C_{D_x^4 v_1}^{(1), \text{SRT}} = (12 + 252v_1^2c_s^2\omega^2 - 22c_s^2\omega^2 - 756v_1^4\omega - 18v_1^2c_s^2\omega^3 - 252v_1^2 + 2c_s^2\omega^3 + 310v_1^4\omega^2 - 36c_s^2 + 54c_s^2\omega + 432v_1^2c_s^2\omega - 648v_1^2c_s^2\omega - 29v_1^4\omega^3 +$$

$$378v_1^2\omega + 24c_s^4 - c_s^4\omega^3 + 14c_s^4\omega^2 - 36c_s^4\omega - 18\omega + 14v_1^2\omega^3 + 8\omega^2 - 154v_1^2\omega^2 - \omega^3 + 504v_1^4) \frac{\rho}{12\omega^3}$$

$$\begin{aligned} C_{D_x^4 v_1}^{(1), \text{MRT1}} = & (12 + 378\omega_5 v_1^2 - 18\omega_5^3 v_1^2 c_s^2 - 18\omega_5 - \omega_5^3 c_s^4 - 252v_1^2 + 310\omega_5^2 v_1^4 + 14\omega_5^2 c_s^4 + 8\omega_5^2 - 36c_s^2 - 29\omega_5^2 v_1^4 - 648\omega_5 v_1^2 c_s^2 + 54\omega_5 c_s^2 + \\ & 432v_1^2 c_s^2 - \omega_5^3 + 24c_s^4 - 22\omega_5^2 c_s^2 - 36\omega_5 c_s^4 + 252\omega_5^2 v_1^2 c_s^2 + 14\omega_5^3 v_1^2 + 2\omega_5^3 c_s^2 - 756\omega_5 v_1^4 - 154\omega_5^2 v_1^2 + 504v_1^4) \frac{\rho}{12\omega_5^3} \end{aligned}$$

$$C_{D_x^4 v_1}^{(1), \text{MRT2}} = C_{D_x^4 v_1}^{(1), \text{MRT1}}$$

$$C_{D_x^4 v_1}^{(1), \text{CLBMT1}} = C_{D_x^4 v_1}^{(1), \text{MRT1}}$$

$$C_{D_x^4 v_1}^{(1), \text{CLBMT2}} = C_{D_x^4 v_1}^{(1), \text{MRT1}}$$

$$\begin{aligned} C_{D_x^4 v_1}^{(1), \text{CuLBM1}} = & (12 - 22c_s^2\omega_1^2 + 252v_1^2 c_s^2\omega_1^2 - 252v_1^2 + 2c_s^2\omega_1^3 - 756v_1^4\omega_1 - 18v_1^2 c_s^2\omega_1^3 + 310v_1^4\omega_1^2 - 36c_s^2 - 648v_1^2 c_s^2\omega_1 - 29v_1^4\omega_1^3 + 54c_s^2\omega_1 + \\ & 432v_1^2 c_s^2 + 24c_s^4 - c_s^4\omega_1^3 + 378v_1^2\omega_1 + 14c_s^4\omega_1^2 + 14v_1^2\omega_1^3 + 8\omega_1^2 - 36c_s^4\omega_1 - 18\omega_1 - 154v_1^2\omega_1^2 - \omega_1^3 + 504v_1^4) \frac{\rho}{12\omega_1^3} \end{aligned}$$

$$\begin{aligned} C_{D_x^4 v_1}^{(1), \text{CuLBM2}} = & (310\omega_2^3 v_1^4\omega_3\omega_1^2 - 33\omega_2\omega_3 c_s^4\omega_1^3 - 154\omega_2^2 v_1^2\omega_3\omega_1^3 + 45\omega_2^3\omega_3 c_s^2\omega_1 + 6\omega_2^2\omega_3\omega_1 + 18\omega_2^2\omega_3 c_s^2\omega_1^2 + 333\omega_2^2 v_1^4\omega_3\omega_1^3 + \\ & 306\omega_2^2 v_1^2\omega_3\omega_1^2 - 432\omega_2^3 v_1^2\omega_3 c_s^2\omega_1 - 58\omega_2^3 v_1^4\omega_3\omega_1^3 + 6\omega_2\omega_3 c_s^4\omega_1^2 + 12\omega_2^2 c_s^2\omega_1^2 + 225\omega_2^3 v_1^2\omega_3\omega_1 + 22\omega_2^2\omega_3 c_s^2\omega_1^3 + 310\omega_2^2 v_1^4\omega_3\omega_1^3 - \\ & 154\omega_2^3 v_1^2\omega_3\omega_1^2 - 24\omega_2^3\omega_3 c_s^2 + 8\omega_2^3\omega_3\omega_1^3 + 6\omega_2^3 c_s^4\omega_1 + 4\omega_2^3\omega_3 c_s^2\omega_1^3 - 99\omega_2^3 v_1^2\omega_3\omega_1 + 252\omega_2^3 v_1^2\omega_3 c_s^2\omega_1^2 - 153\omega_2^3 v_1^2\omega_3\omega_1 - 24\omega_3 c_s^2\omega_1^3 + \\ & 207v_1^2\omega_2^3 c_s^2\omega_1^3 - 12\omega_2^3\omega_3 c_s^2\omega_1^2 - 12\omega_2^3\omega_3 c_s^2\omega_1^3 + 28\omega_2^3 v_1^2\omega_3\omega_1^2 - 66\omega_2^3 v_1^2\omega_3\omega_1^3 - 423\omega_2^3 v_1^2\omega_3\omega_1^4 - 36\omega_2^3 v_1^2\omega_3 c_s^2\omega_1^3 + \\ & 6\omega_2 c_s^4\omega_1^3 - 22\omega_2^3\omega_3 c_s^2\omega_1^2 + 6\omega_3\omega_1^3 - 36\omega_2^2 v_1^2 c_s^2\omega_1^2 - 432\omega_2^2 v_1^2\omega_3 c_s^2\omega_1^2 - 12\omega_2\omega_3\omega_1^3 - 12\omega_2\omega_3 c_s^2\omega_1^2 - 12\omega_2^2 c_s^4\omega_1^2 + 18\omega_2^3\omega_3 c_s^4\omega_1^3 + 14\omega_2^2\omega_3 c_s^4\omega_1^3 - \\ & 432v_1^2\omega_2^3 c_s^2\omega_1^3 + 225\omega_2 v_1^2\omega_3\omega_1^3 + 6\omega_2^3\omega_3\omega_1^3 - 45\omega_2\omega_3 c_s^2\omega_1^3 - 33\omega_2^3\omega_3 c_s^4\omega_1^2 + 6\omega_2\omega_3\omega_1^2 + 252\omega_2^2 v_1^2\omega_3 c_s^2\omega_1^3 - 153\omega_2 v_1^2\omega_3\omega_1^2 + \\ & 225\omega_2 v_1^2\omega_3 c_s^2\omega_1^2 - 6\omega_2^2\omega_3 c_s^4\omega_1^2 + 6\omega_2^2\omega_3 c_s^4\omega_1^3 + 18\omega_2 v_1^2 c_s^2\omega_1^3 + 8\omega_2^3\omega_3\omega_1^2 - 6\omega_2 c_s^2\omega_1^3 - 423\omega_2 v_1^2\omega_3\omega_1^3 + 207\omega_2^2 v_1^2\omega_3 c_s^2 + 14\omega_2^3\omega_3 c_s^4\omega_1^2 + \\ & 225\omega_2^2 v_1^2\omega_3 c_s^2\omega_1^1 - 6\omega_2^3\omega_3\omega_1^1 + 333\omega_2 v_1^2\omega_3\omega_1^2 + 171\omega_2^3 v_1^2\omega_3\omega_1^3 - 2\omega_2^3\omega_3 c_s^4\omega_1^3 - 2\omega_2^3\omega_3\omega_1^3 + 18\omega_3 c_s^4\omega_1^3) \frac{\rho}{24\omega_2^3\omega_3\omega_1^3} \end{aligned}$$

$$\text{coefficient } C_{D_x^3 D_y \rho}^{(1)} \text{ at } \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} :$$

$$C_{D_x^3 D_y \rho}^{(1), \text{SRT}} = 0$$

$$\begin{aligned} C_{D_x^3 D_y \rho}^{(1), \text{MRT1}} = & (4\omega_5^2 v_1^2\omega_4^2 - 20\omega_5^2 v_1^4\omega_7\omega_4 - 20\omega_5 v_1^4\omega_7\omega_4^2 - 8\omega_5^3 c_s^2\omega_7\omega_4 + 4\omega_5^3 v_1^4\omega_4^2 - 8\omega_5^2 c_s^2\omega_7\omega_4^2 + 72\omega_5 v_1^2 c_s^2\omega_7^2\omega_4 + 84\omega_5^3 v_1^2 c_s^2\omega_7\omega_4 - \\ & 4\omega_5 c_s^2\omega_7\omega_4 - 4\omega_5^2 c_s^2\omega_7^2 + 4\omega_5^3 c_s^2\omega_4 - 84\omega_5^2 v_1^2 c_s^2\omega_7\omega_4 - 12\omega_5 c_s^4\omega_7\omega_4^2 - 13\omega_5^2 v_1^2\omega_7\omega_4^2 + 8\omega_5^3 v_1^2\omega_7 - 4\omega_5^3 c_s^4\omega_7\omega_4^2 + 8c_s^4\omega_7\omega_4^2 + 20\omega_5^2 v_1^2\omega_7\omega_4 + \\ & 96v_1^2 c_s^2\omega_7\omega_4^2 - 4\omega_5^2 c_s^2\omega_4^2 + 36\omega_5^2 v_1^2 c_s^2\omega_7^2 + 20\omega_5 v_1^2\omega_7\omega_4^2 + 8\omega_5^2 c_s^2\omega_7\omega_4 + 8\omega_5^2 c_s^2\omega_7\omega_4^2 - 36\omega_5^3 v_1^2\omega_7\omega_4^2 - 4\omega_5^3 c_s^2\omega_7^2 + 51\omega_5^2 v_1^2 c_s^2\omega_7\omega_4^2 + \\ & 4\omega_5 c_s^4\omega_7\omega_4^2 + 8\omega_5^2 v_1^2\omega_7^2 - 4\omega_5^3 v_1^4\omega_7 - 144\omega_5 v_1^2 c_s^2\omega_7\omega_4 + 12\omega_5 c_s^2\omega_7\omega_4^2 - 51\omega_5^3 v_1^2 c_s^2\omega_7\omega_4^2 + 13\omega_5^2 v_1^2\omega_7\omega_4^2 + 4\omega_5^3 c_s^2\omega_7\omega_4^2 - 4\omega_5^3 c_s^4\omega_7 - 8c_s^2\omega_7\omega_4^2 + \\ & 4\omega_5^2 c_s^2\omega_7\omega_4^2 + 72\omega_5 v_1^2 c_s^2\omega_7\omega_4^2 - 4\omega_5^3 c_s^4\omega_4 + 4\omega_5 c_s^2\omega_7\omega_4^2 + 20\omega_5^3 v_1^4\omega_7\omega_4 + 32\omega_5^2 v_1^4\omega_7\omega_4^2 - 8\omega_5^3 v_1^4\omega_7 + 20\omega_5 v_1^4\omega_7\omega_4^2 - 4\omega_5^2 v_1^4\omega_4^2 + 16\omega_5^2 v_1^2\omega_7\omega_4 - \\ & 8\omega_5^2 c_s^2\omega_7\omega_4^2 - 72\omega_5 v_1^2 c_s^2\omega_7\omega_4^2 - 4\omega_5^3 c_s^4\omega_4 + 4\omega_5 c_s^2\omega_7\omega_4^2 + 20\omega_5^3 v_1^4\omega_7\omega_4^2 - 8\omega_5^3 v_1^4\omega_7 + 20\omega_5 v_1^4\omega_7\omega_4^2 - 4\omega_5^2 v_1^4\omega_4^2 + 16\omega_5^2 v_1^2\omega_7\omega_4 - \\ & 24\omega_5^3 v_1^2\omega_7\omega_4^2 + 36\omega_5 v_1^2\omega_7\omega_4^2 + 120\omega_5^2 v_1^2 c_s^2\omega_7\omega_4^2 - 4\omega_5^3 v_1^2\omega_4^2 - 24\omega_5^2 v_1^2 c_s^2\omega_7\omega_4^2 + 4\omega_5^2 c_s^2\omega_7\omega_4^2 + 13\omega_5^3 v_1^2\omega_7\omega_4^2 - 24\omega_5^2 v_1^2\omega_7\omega_4^2 + 4\omega_5^2 c_s^2\omega_7\omega_4^2 + 4\omega_5^2 v_1^2\omega_4^2 - \\ & 48\omega_5 v_1^2 c_s^2\omega_7\omega_4^2 - 8\omega_5^2 c_s^4\omega_7\omega_4^2 - 4\omega_5 c_s^4\omega_7\omega_4^2 - 20\omega_5^2 v_1^2\omega_7\omega_4^2 - 32\omega_5^2 v_1^2\omega_7\omega_4^2 - 20\omega_5 v_1^2\omega_7\omega_4^2 + 4\omega_5^3 c_s^2\omega_7 + 24\omega_5^2 v_1^2 c_s^2\omega_4^2 - 16\omega_5^2 v_1^4\omega_7\omega_4 + \\ & 4\omega_5^2 c_s^2\omega_4^2 - 36\omega_5 v_1^2\omega_7\omega_4^2 + 4\omega_5^3 c_s^4\omega_4 - 4\omega_5^2 c_s^2\omega_7\omega_4^2 - 13\omega_5^3 v_1^4\omega_7\omega_4^2 - 8\omega_5^2 v_1^2\omega_7^2 + 24v_1^4\omega_7\omega_4^2) \frac{\rho}{4\omega_5^3\omega_7\omega_4^2} \end{aligned}$$

$$C_{D_x^3 D_y \rho}^{(1), \text{MRT2}} = C_{D_x^3 D_y \rho}^{(1), \text{MRT1}}$$

$$C_{D_x^3 D_y \rho}^{(1), \text{CLBMT1}} = 0$$

$$C_{D_x^3 D_y \rho}^{(1), \text{CLBMT2}} = 0$$

$$C_{D_x^3 D_y \rho}^{(1), \text{CuLBM1}} = 0$$

$$\begin{aligned} C_{D_x^3 D_y \rho}^{(1), \text{CuLBM2}} = & (-18\omega_2\omega_3 c_s^4\omega_1^3 + 6\omega_2^3 v_1^2\omega_3 c_s^2\omega_1 - 18\omega_2^2\omega_3 c_s^4\omega_1 - 6\omega_2^2\omega_2^2\omega_3 c_s^2\omega_1 - 54\omega_2 v_1^4\omega_3\omega_1^2 - 6\omega_2^2\omega_3^2\omega_1^3 - 24\omega_2^3 v_1^2\omega_3\omega_1^2 - \\ & 6\omega_2^3\omega_2^2 c_s^4\omega_1^2 + 12\omega_2^2 c_s^2\omega_3^3 + 36\omega_2^3 c_s^4\omega_1^2 - 9\omega_2^3 v_1^2\omega_3\omega_1^2 - 6\omega_2^2 v_1^2\omega_3^2\omega_1^3 + 45v_1^2 v_2^2\omega_5^2\omega_1^3 + 90\omega_2^3 c_s^4\omega_1^3 - \omega_2^3\omega_2^2\omega_3^2\omega_1^2 - 18\omega_2^2\omega_3 c_s^2\omega_1^3 - \\ & 297\omega_2^2 v_1^2\omega_3^2 c_s^2\omega_1 - 54\omega_2 v_1^4\omega_3\omega_1^2 + 135\omega_2^3 v_1^2\omega_3^2\omega_1^2 + 6\omega_2^3\omega_3^2\omega_1 + 60\omega_2\omega_3 c_s^2\omega_1^2 - 6\omega_2^3 v_1^2\omega_3^2\omega_1^2 - 6\omega_2\omega_3^2\omega_1^2 + 540\omega_2^2 v_1^2\omega_3 c_s^2\omega_1^2 + 2\omega_2^2 v_1^2\omega_3^2 c_s^2\omega_1^3 - \\ & 6\omega_2 v_1^2\omega_3^2 c_s^2\omega_1^2 - 486\omega_2 v_1^2\omega_3^2 c_s^2\omega_1^3 + 45\omega_2 v_1^2\omega_3^2 c_s^2\omega_1^2 + 6\omega_2^2\omega_3^2 c_s^4\omega_1^3 + 72\omega_2^2 v_1^2\omega_3^2 c_s^2\omega_1^3 + 27\omega_2^3 v_1^2\omega_3^2 c_s^4\omega_1 - 6\omega_2\omega_3^2\omega_1^3 + 72\omega_2^3 v_1^2\omega_3^2 c_s^2\omega_1^2 + 18\omega_2\omega_3^2\omega_1^2 + 75\omega_2\omega_3^2 c_s^2\omega_1^3 - \\ & \omega_2^2\omega_3^2\omega_1^3 - 18\omega_2^3 v_1^2\omega_3 c_s^2\omega_1^2 + 18\omega_2^3 v_1^2\omega_3 c_s^2\omega_1^3 + 72\omega_2^2 v_1^2\omega_3^2 c_s^2\omega_1^2 + 126\omega_2 v_1^2\omega_3^2\omega_1^3 - 21\omega_2 v_1^2\omega_3^2 c_s^2\omega_1^3 - 243\omega_2 v_1^2\omega_3^2 c_s^2\omega_1^2 - 18\omega_2^3 v_1^2\omega_3^2 c_s^4 - 45\omega_2^3 v_1^2\omega_3^2\omega_1^2 - \\ & 12\omega_2^3 c_s^2\omega_1^2 + 18\omega_2^2 v_1^2\omega_3^2 c_s^2\omega_1^3 + 18\omega_2^2 v_1^2\omega_3^2 c_s^2\omega_1^2 - 99v_1^2\omega_3^2\omega_1^3 + 12\omega_2^3 v_1^2\omega_3^2 c_s^2\omega_1^2 + 54\omega_2^3 v_1^4\omega_3\omega_1^2 + 108\omega_2^2 v_1^2\omega_3^2 c_s^2\omega_1^2 - 72\omega_2^3 c_s^2\omega_1^3 - 54\omega_2^3 v_1^2\omega_3^2\omega_1^2 - \\ & 72\omega_2^3 v_1^2\omega_3^2 c_s^2\omega_1^2 + 6\omega_2^3\omega_3^2\omega_1^3 + 54\omega_2^3 v_1^2\omega_3^2 c_s^4\omega_1^2 + 6\omega_2 v_1^2\omega_3^2 c_s^2\omega_1^3 + 24\omega_2^3 v_1^2\omega_3^2 c_s^2\omega_1^2 + 18\omega_2^3 v_1^2\omega_3^2 c_s^4\omega_1^2 - 18\omega_2^3 v_1^2\omega_3^2 c_s^2\omega_1^3 + 12\omega_2^3 v_1^2\omega_3^2 c_s^2\omega_1^2 + 405v_1^2\omega_3^2 c_s^2\omega_1^3 - \\ & 6\omega_2 v_1^2\omega_3^2 c_s^2\omega_1^2 + 5\omega_2^3 v_1^2\omega_3^2 c_s^2\omega_1^2 - 2\omega_2^3 v_1^2\omega_3^2 c_s^2\omega_1^2 - 36\omega_2^2 c_s^4\omega_1^3 + 63\omega_2^2 v_1^2\omega_3^2\omega_1^2 - 9\omega_2^2 v_1^2\omega_3^2 c_s^2\omega_1^2 + 54v_1^4\omega_3^2 c_s^2\omega_1^2 + 6\omega_2 v_1^2\omega_3^2 c_s^2\omega_1^2 - 12\omega_2^2 v_1^2\omega_3^2 c_s^2\omega_1^3 - 3\omega_2^3 v_1^2\omega_3^2 c_s^2\omega_1^2 + \\ & 21\omega_2^3 v_1^2\omega_3^2 c_s^2\omega_1^2 - 99\omega_2 v_1^2\omega_3^2 c_s^2\omega_1^3 - 108\omega_2 v_1^2\omega_3^2 c_s^2\omega_1^2 - 72\omega_2 v_1^2\omega_3^2 c_s^2\omega_1^3 - 18\omega_2 v_1^2\omega_3^2 c_s^2\omega_1^2 - 72\omega_2 v_1^2\omega_3^2 c_s^2\omega_1^2 - 54\omega_2^3 v_1^2\omega_3^2 c_s^4\omega_1^2 + 24\omega_2^2 v_1^2\omega_3^2 c_s^2\omega_1^3 - 24\omega_2^2 v_1^2\omega_3^2 c_s^2\omega_1^2 - \\ & 54\omega_2 v_1^2\omega_3^2 c_s^2\omega_1^2 - 54\omega_2^3 v_1^2\omega_3^2 c_s^2\omega_1^2 - 6\omega_2^3\omega_3^2\omega_1^3 + 6\omega_2 v_1^2\omega_3^2 c_s^2\omega_1^2 - 5\omega_2^2\omega_3^2 c_s^2\omega_1^3 + 6\omega_2^2 v_1^2\omega_3^2 c_s^2\omega_1^2 - 54\omega_2 v_1^2\omega_3^2 c_s^2\omega_1^2 + 9\omega_2 v_1^2\omega_3^2 c_s^2\omega_1^2) \frac{\rho}{24\omega_2^3\omega_3^2\omega_1^3} \end{aligned}$$

**coefficient**  $C_{D_x^3 D_y v_1}^{(1)}$  at  $\frac{\partial^4 v_1}{\partial x_1^3 \partial x_2}$ :

$$C_{D_x^3 D_y v_1}^{(1), \text{SRT}} = 0$$

$$\begin{aligned} C_{D_x^3 D_y v_1}^{(1), \text{MRT1}} &= (-16\omega_5^2 v_1^2 \omega_4^2 + 8\omega_5^3 \omega_4 + 44\omega_5^3 c_s^2 \omega_7 \omega_4 + 28\omega_5^2 \omega_7^2 \omega_4 + 56\omega_5^2 c_s^2 \omega_7 \omega_4^2 + 32\omega_5 c_s^2 \omega_7^2 \omega_4 + 20\omega_5^2 c_s^2 \omega_7^2 - 16\omega_5^3 c_s^2 \omega_4 + 8\omega_5^2 \omega_4^2 + \\ &17\omega_5^3 \omega_7 \omega_4^2 + 43\omega_5^2 v_1^2 \omega_7^2 \omega_4^2 - 28\omega_5^3 v_1^2 \omega_7 \omega_4 - 68\omega_5^2 v_1^2 \omega_7^2 \omega_4 - 28\omega_5^3 \omega_7 \omega_4 + 24\omega_5 \omega_7 \omega_4^2 - 64\omega_5 v_1^2 \omega_7 \omega_4^2 + 16\omega_5^3 c_s^2 \omega_4^2 - 17\omega_5^2 \omega_7^2 \omega_4^2 - 16\omega_5^2 c_s^2 \omega_7 \omega_4 - \\ &72\omega_5 c_s^2 \omega_7^2 \omega_4^2 - 8\omega_5^3 \omega_4^2 - 25\omega_5^3 c_s^2 \omega_7 \omega_4^2 + 48c_s^2 \omega_7^2 \omega_4^2 - 44\omega_5^2 c_s^2 \omega_7^2 \omega_4 - 40\omega_5^2 \omega_7 \omega_4^2 - 32\omega_5 c_s^2 \omega_7 \omega_4^2 - 32\omega_5^2 \omega_4^2 - 48\omega_5^2 v_1^2 \omega_7 \omega_4 - 120\omega_5 v_1^2 \omega_7^2 \omega_4^2 + \\ &48\omega_5 \omega_7 \omega_4^2 + 16\omega_5^3 v_1^2 \omega_4^2 - 43\omega_5^3 v_1^2 \omega_7 \omega_4^2 + 80v_1^2 \omega_7^2 \omega_4^2 - 16\omega_5^3 v_1^2 \omega_4 + 68\omega_5^3 v_1^2 \omega_7 \omega_4 + 104\omega_5^2 v_1^2 \omega_7 \omega_4^2 - 12\omega_5^2 \omega_7^2 + 64\omega_5 v_1^2 \omega_7^2 \omega_4 - 20\omega_5^3 c_s^2 \omega_7 - \\ &24\omega_5 \omega_7 \omega_4 - 16\omega_5^2 c_s^2 \omega_4^2 + 25\omega_5^2 c_s^2 \omega_7^2 \omega_4^2 + 16\omega_5^2 \omega_7 \omega_4 + 28\omega_5^2 v_1^2 \omega_7^2 + 12\omega_5^3 \omega_7) \frac{v_1 \rho v_2}{4\omega_5^3 \omega_7^2 \omega_4^2} \end{aligned}$$

$$C_{D_x^3 D_y v_1}^{(1), \text{MRT2}} = C_{D_x^3 D_y v_1}^{(1), \text{MRT1}}$$

$$C_{D_x^3 D_y v_1}^{(1), \text{CLBM1}} = 0$$

$$C_{D_x^3 D_y v_1}^{(1), \text{CLBM2}} = 0$$

$$C_{D_x^3 D_y v_1}^{(1), \text{CuLBM1}} = 0$$

$$\begin{aligned} C_{D_x^3 D_y v_1}^{(1), \text{CuLBM2}} &= (-27\omega_2^3 \omega_3 c_s^2 \omega_1 - 45\omega_2 v_2^2 \omega_3 \omega_1^3 + 90\omega_2^2 \omega_3 \omega_1 + 324\omega_2^2 \omega_3 c_s^2 \omega_1^2 + 54\omega_2^2 c_s^2 \omega_3^3 + 396\omega_2^2 v_1^2 \omega_3 \omega_1^2 + 18\omega_2^2 v_2^2 \omega_1^3 - 198\omega_2^3 v_1^2 \omega_3 \omega_1 + \\ &18\omega_2^3 \omega_1^3 + 30\omega_2^2 \omega_3 c_s^2 \omega_1^3 + 54\omega_2^3 \omega_3 c_s^2 \omega_1^3 - 10\omega_2^2 \omega_3 \omega_1^3 + 198\omega_2^3 v_1^2 \omega_3 \omega_1 - 198\omega_2^2 v_1^2 \omega_3 \omega_1 + 270\omega_3 c_s^2 \omega_1^3 - 180\omega_2^2 \omega_3 \omega_1^2 + 198v_1^2 \omega_3 \omega_1^3 - 162\omega_2^2 \omega_3 c_s^2 \omega_1 - \\ &30\omega_3^2 \omega_1^2 + 126\omega_3 \omega_1^3 - 54\omega_3^2 c_s^2 \omega_1^2 + 135\omega_2 \omega_3 \omega_1^3 - 162\omega_2 \omega_3 c_s^2 \omega_1^2 + 10\omega_2^2 v_2^2 \omega_3 \omega_1^3 - 36\omega_3^2 v_2^2 \omega_3 - 198\omega_2 v_1^2 \omega_3 \omega_1^3 - 54\omega_2^3 \omega_3 - 297\omega_2 \omega_3 c_s^2 \omega_1^3 + \\ &45\omega_2^3 v_2^2 \omega_3 \omega_1 + 45\omega_2^2 \omega_3 \omega_1 + 18\omega_2^3 \omega_1^2 - 18\omega_2^2 v_2^2 \omega_1^2 + 90\omega_2 \omega_3 \omega_1^2 - 198\omega_2 v_1^2 \omega_3 \omega_1^2 - 10\omega_2^3 v_2^2 \omega_3 \omega_1^2 + 10\omega_2^3 \omega_3 \omega_1^2 - 54\omega_2 c_s^2 \omega_1^3 + 18\omega_2^2 v_2^2 \omega_1^2 - \\ &18\omega_2^3 \omega_1 - 18\omega_2^2 \omega_1^3 + 54\omega_2^3 c_s^2 \omega_1 + 36v_2^2 \omega_3 \omega_1^3 - 18\omega_2 v_2^2 \omega_1^3) \frac{v_1 \rho v_2}{24\omega_2^3 \omega_3 \omega_1^3} \end{aligned}$$

**coefficient**  $C_{D_x^3 D_y v_2}^{(1)}$  at  $\frac{\partial^4 v_2}{\partial x_1^3 \partial x_2}$ :

$$\begin{aligned} C_{D_x^3 D_y v_2}^{(1), \text{SRT}} &= (-42v_1^2 c_s^2 \omega^2 - 12c_s^2 \omega^2 + 54v_1^4 \omega + 12v_1^2 c_s^2 \omega^3 + 36v_1^2 - 26v_1^4 \omega^2 - 24c_s^2 + 36c_s^2 \omega - 36v_1^2 c_s^2 + 54v_1^2 c_s^2 \omega + 4v_1^4 \omega^3 - 54v_1^2 \omega + 36c_s^4 - \\ &c_s^4 \omega^3 + 20c_s^4 \omega^2 - 54c_s^4 \omega - 4v_1^2 \omega^3 + 26v_1^2 \omega^2 - 36v_1^4) \frac{\rho}{12\omega^3} \end{aligned}$$

$$\begin{aligned} C_{D_x^3 D_y v_2}^{(1), \text{MRT1}} &= (-12\omega_5 c_s^4 \omega_7^2 \omega_4^3 + 12\omega_5^3 c_s^4 \omega_7 \omega_4^2 - 12\omega_5^3 c_s^2 \omega_7 \omega_4 + 6\omega_5^3 c_s^4 \omega_7 \omega_4^3 - 12\omega_5^3 v_1^4 \omega_4^2 + 12c_s^4 \omega_7^2 \omega_4^3 - 12\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4 - \\ &18\omega_5^3 v_1^4 \omega_7^2 \omega_4^2 - 19\omega_5^2 v_1^2 \omega_7^2 \omega_4^3 - 48\omega_5^3 v_1^2 c_s^2 \omega_7^2 \omega_4 - 36\omega_5 v_1^4 \omega_7 \omega_4^3 - 48\omega_5^2 v_1^2 c_s^2 \omega_7^2 \omega_4 + 12\omega_5^3 v_1^2 \omega_7^2 \omega_4^2 - 12\omega_5^2 v_1^2 \omega_7^2 \omega_4^2 + \\ &4\omega_5^3 v_1^4 \omega_7^2 \omega_4^3 + 6\omega_5^2 c_s^2 \omega_7 \omega_4^3 - 18\omega_5^3 c_s^2 \omega_7 \omega_4^2 + 12\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^3 + 21\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^2 - 12\omega_5 c_s^2 \omega_7 \omega_4^3 - 306\omega_5 v_1^2 c_s^2 \omega_7 \omega_4^2 + 12\omega_5^3 c_s^4 \omega_7 \omega_4 + 12\omega_5^2 c_s^4 \omega_7 \omega_4^2 - \\ &6\omega_5^3 c_s^2 \omega_7 \omega_4^3 + 162\omega_5^2 v_1^2 c_s^2 \omega_7 \omega_4^2 - 12c_s^2 \omega_7 \omega_4^3 + 19\omega_5^2 v_1^4 \omega_7 \omega_4^2 + 18\omega_5^3 v_1^2 \omega_7 \omega_4^3 + 36\omega_5 v_1^2 c_s^2 \omega_7 \omega_4^2 - 108\omega_5 v_1^2 c_s^2 \omega_7 \omega_4^2 + 12\omega_5^3 v_1^4 \omega_7 \omega_4^2 + \\ &252v_1^2 c_s^2 \omega_7 \omega_4^3 + 12\omega_5 c_s^2 \omega_7 \omega_4^2 + 30\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^2 - 4\omega_5^2 v_1^2 \omega_7 \omega_4^2 + 18\omega_5^3 c_s^2 \omega_7 \omega_4^2 - 6\omega_5^2 c_s^4 \omega_7 \omega_4^2 + 90\omega_5 v_1^2 \omega_7 \omega_4^2 - 12\omega_5^2 v_1^2 c_s^2 \omega_7 \omega_4^2 - \\ &12\omega_5 v_1^4 \omega_4^3 - 81\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^2 - 24\omega_5^3 v_1^4 \omega_7 \omega_4^2 - 24\omega_5^2 v_1^4 \omega_7 \omega_4^2 + 27\omega_5^3 v_1^2 \omega_7 \omega_4^2 - 72v_1^2 \omega_7 \omega_4^3 + \omega_5^2 c_s^4 \omega_7 \omega_4^2 - 12\omega_5^2 v_1^2 c_s^2 \omega_7 \omega_4^2 + 54\omega_5^2 v_1^2 c_s^2 \omega_7 \omega_4^2 - \\ &5\omega_5^3 c_s^2 \omega_7 \omega_4^2 - 12\omega_5^3 v_1^2 \omega_7 \omega_4^3 + 12\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^3 - 24\omega_5^3 c_s^4 \omega_7 \omega_4^2 - 36\omega_5 v_1^2 c_s^2 \omega_7 \omega_4^2 - 12\omega_5^2 v_1^2 c_s^2 \omega_7 \omega_4^2 + 12\omega_5^3 v_1^2 \omega_7 \omega_4^2 + 6\omega_5^2 c_s^4 \omega_7 \omega_4^2 - 48\omega_5^3 v_1^2 \omega_7 \omega_4^2 + \\ &60\omega_5 v_1^4 \omega_7 \omega_4^3 - 90\omega_5 v_1^4 \omega_7 \omega_4^2 + 24\omega_5^3 v_1^2 \omega_7 \omega_4^2 - 27\omega_5^3 v_1^4 \omega_7 \omega_4^3 + 72v_1^4 \omega_7 \omega_4^3 + 24\omega_5^2 v_1^2 \omega_7 \omega_4^2 - 12\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^2 + 13\omega_5^3 c_s^4 \omega_7 \omega_4^2 - \omega_5^2 c_s^2 \omega_7 \omega_4^2 + \\ &6\omega_5^3 c_s^2 \omega_7 \omega_4^2 - 6\omega_5^2 c_s^2 \omega_7 \omega_4^2 - \omega_5^3 c_s^4 \omega_7 \omega_4^2 + 12\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^2 - 60\omega_5^2 v_1^2 \omega_7 \omega_4^3 + 48\omega_5^3 v_1^4 \omega_7 \omega_4^2 + 102\omega_5^2 v_1^2 c_s^2 \omega_7 \omega_4^2) \frac{\rho}{12\omega_5^3 \omega_7^2 \omega_4^3} \end{aligned}$$

$$C_{D_x^3 D_y v_2}^{(1), \text{MRT2}} = C_{D_x^3 D_y v_2}^{(1), \text{MRT1}}$$

$$\begin{aligned} C_{D_x^3 D_y v_2}^{(1), \text{CLBM1}} &= (-12\omega_5 c_s^4 \omega_7^2 \omega_4^3 + 12\omega_5^3 c_s^4 \omega_7 \omega_4^2 - 12\omega_5^3 c_s^2 \omega_7 \omega_4 + 6\omega_5^3 c_s^4 \omega_7 \omega_4^3 - 36\omega_5^3 v_1^4 \omega_4^2 + 12c_s^4 \omega_7^2 \omega_4^3 - 12\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4 - \\ &6\omega_5^3 v_1^4 \omega_7^2 \omega_4^2 - 19\omega_5^2 v_1^2 \omega_7 \omega_4^3 - 36\omega_5 v_1^4 \omega_7 \omega_4^2 + 36\omega_5^2 v_1^2 \omega_7 \omega_4^3 - 12\omega_5 c_s^4 \omega_7 \omega_4^2 + 4\omega_5^3 c_s^4 \omega_7 \omega_4^2 + 6\omega_5^2 c_s^2 \omega_7 \omega_4^3 - 18\omega_5^3 c_s^4 \omega_7 \omega_4^2 + 36\omega_5^3 v_1^4 \omega_4^3 - \\ &99\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^3 + 12\omega_5^2 c_s^2 \omega_7 \omega_4^3 - 306\omega_5 v_1^2 c_s^2 \omega_7 \omega_4^3 + 12\omega_5^3 c_s^4 \omega_7 \omega_4^2 + 12\omega_5^2 c_s^4 \omega_7 \omega_4^2 - 6\omega_5^3 c_s^2 \omega_7 \omega_4^3 + 18\omega_5^2 v_1^2 c_s^2 \omega_7 \omega_4^2 - 12\omega_5^2 c_s^2 \omega_7 \omega_4^3 + 19\omega_5^2 v_1^4 \omega_7 \omega_4^3 + \\ &6\omega_5^3 v_1^2 \omega_7 \omega_4^3 + 36\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^3 - 36\omega_5 v_1^2 c_s^2 \omega_7 \omega_4^2 + 252v_1^2 c_s^2 \omega_7 \omega_4^3 + 12\omega_5 c_s^2 \omega_7 \omega_4^3 + 54\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^2 - 4\omega_5^3 v_1^2 \omega_7 \omega_4^3 + 18\omega_5^3 c_s^2 \omega_7 \omega_4^2 + \\ &60\omega_5^2 v_1^2 c_s^2 \omega_7 \omega_4^3 - 6\omega_5^2 c_s^4 \omega_7 \omega_4^2 + 90\omega_5 v_1^2 \omega_7 \omega_4^2 - 36\omega_5^2 v_1^4 \omega_4^3 - 3\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^2 + 39\omega_5^3 v_1^2 \omega_7 \omega_4^3 - 72v_1^2 \omega_7 \omega_4^3 + \omega_5^2 c_s^4 \omega_7 \omega_4^2 - 108\omega_5^2 v_1^2 c_s^2 \omega_7 \omega_4^2 + \\ &198\omega_5^2 v_1^2 c_s^2 \omega_7 \omega_4^3 - 5\omega_5^3 c_s^2 \omega_7 \omega_4^2 - 36\omega_5^3 v_1^2 \omega_7 \omega_4^2 + 12\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^3 - 24\omega_5^3 c_s^4 \omega_7 \omega_4^2 - 108\omega_5 v_1^2 c_s^2 \omega_7 \omega_4^3 + 36\omega_5^2 v_1^2 c_s^2 \omega_7 \omega_4^2 + 36\omega_5^3 v_1^2 \omega_7 \omega_4^2 + 6\omega_5^2 c_s^4 \omega_7 \omega_4^2 - \\ &36\omega_5^3 v_1^2 \omega_7 \omega_4^3 + 72\omega_5^3 v_1^2 \omega_7 \omega_4^2 - 90\omega_5 v_1^2 \omega_7 \omega_4^3 - 39\omega_5^3 v_1^2 \omega_7 \omega_4^2 + 72v_1^4 \omega_7 \omega_4^3 - 108\omega_5^3 v_1^2 \omega_7 \omega_4^2 + 13\omega_5^3 c_s^4 \omega_7 \omega_4^2 - \omega_5^2 c_s^2 \omega_7 \omega_4^3 + 6\omega_5^3 c_s^2 \omega_7 \omega_4^2 - \\ &6\omega_5^2 c_s^2 \omega_7 \omega_4^3 - \omega_5^3 c_s^4 \omega_7 \omega_4^2 + 108\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^2 - 72\omega_5^2 v_1^2 \omega_7 \omega_4^3 + 36\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^2 - 18\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^2) \frac{\rho}{12\omega_5^3 \omega_7^2 \omega_4^3} \end{aligned}$$

$$C_{D_x^3 D_y v_2}^{(1), \text{CLBM2}} = C_{D_x^3 D_y v_2}^{(1), \text{CLBM1}}$$

$$\begin{aligned} C_{D_x^3 D_y v_2}^{(1), \text{CuLBM1}} &= (-99v_1^2 \omega_3^3 c_s^2 \omega_4 \omega_1^3 - 39v_1^4 \omega_3^3 \omega_4 \omega_1^3 - 6\omega_3^3 c_s^4 \omega_4 \omega_1^2 + 252v_1^2 \omega_3^3 c_s^2 \omega_4 \omega_1^2 + 18\omega_3^2 c_s^2 \omega_4 \omega_1^3 - 18v_1^2 \omega_3 \omega_4^2 \omega_1^2 + 108v_1^2 \omega_3^2 c_s^2 \omega_4 \omega_1^2 + \\ &198v_1^2 \omega_3^3 c_s^2 \omega_4 \omega_1^2 + 72v_1^4 \omega_3^3 \omega_4 \omega_1^2 - 12\omega_3^3 c_s^2 \omega_4 \omega_1^2 - 24\omega_3^4 c_s^2 \omega_4 \omega_1^3 + 90v_1^2 \omega_3^3 c_s^2 \omega_4 \omega_1^2 - 108v_1^2 \omega_3^2 c_s^2 \omega_4 \omega_1^2 - 12\omega_3^2 c_s^2 \omega_4 \omega_1^2 - 36v_1^2 \omega_3^2 c_s^2 \omega_4 \omega_1^3 + 6\omega_3^2 c_s^4 \omega_4 \omega_1^2 + \\ &36v_1^2 \omega_3^2 c_s^2 \omega_4 \omega_1^2 - 19v_1^2 \omega_3^3 c_s^2 \omega_4 \omega_1^2 - 18\omega_3^2 c_s^4 \omega_4 \omega_1^2 + 6\omega_3^3 c_s^2 \omega_4 \omega_1^2 - 6v_1^4 \omega_3^2 c_s^2 \omega_4 \omega_1^2 - 108v_1^2 \omega_3^3 c_s^2 \omega_4 \omega_1^2 + 54v_1^2 \omega_3^2 c_s^2 \omega_4 \omega_1^3 \end{aligned}$$

$$\begin{aligned}
& 6w_{3-s}^2w_4^2w_1^3 - 4v_1^2w_3^2w_4^2w_1^3 - 6w_3^3s_w w_4 w_3^3 + 12w_3^2s_w^4w_4 w_1^2 - 12w_3^3s_w^4w_4 w_1 + 36v_1^2w_3^2w_4 w_1^3 + 39v_1^2w_3^2w_4 w_1^3 - 12w_3c_s^2w_4 w_1^3 - 3v_1^2w_3^2s_w^4w_4 w_1^3 + \\
& 12w_3^2s_w^4w_4^2 + 72v_1^4w_3^2w_4^2 + 6w_3^2s_w^4w_4 w_1^2 + 36v_1^4w_3^2w_4^3 - 72v_1^2w_3^2w_4 w_1^2 + 18v_1^2w_3^2s_w^4w_4 w_1^2 + 12w_3^2s_w^2s_w^4w_1 - 108v_1^2w_3^2c_s^2w_4^3 + 36v_1^4w_3^2w_4 w_1^3 - \\
& w_3^2c_s^2w_4^2w_1^2 - 36v_1^4w_3^2w_4^3 + 13w_3^2s_w^4w_4^2w_1^3 - 306v_1^2w_3^2s_w^4w_4^2w_1 - 90v_4^4w_3^2w_4^2w_1 - 36v_4^4w_3^2w_4^3 + 12w_3^2c_s^2w_4^2w_1 + 12c_s^4w_4^2w_1^3 + 12w_3c_s^4w_4w_1^3 + \\
& 19v_4^2w_3^2w_4^2w_1^2 + 60v_1^2w_3^2s_w^2w_4^2w_1^3 - 36v_1^2w_3^2c_s^2w_4^2w_1 + 36v_1^2w_3^2w_4 w_1^2 - 72v_1^2w_3^2w_4^2 - w_3^2c_s^4w_4^2w_1^3 + 6v_1^2w_3^2w_4^2w_1^3 - 36v_1^2w_3^2w_4^3 - 6w_3^2c_s^2w_4^2w_1^2 + \\
& 4v_1^4w_3^2w_4^2w_1^2 + 12v_1^2w_3^2s_w^2w_4^2w_1^3 - 12w_3^2s_w^4w_4^2w_1 + 36v_1^2w_3^2c_s^2w_4^2w_1^3 - 5w_3^2c_s^2w_4^2w_1^2 + 36v_1^2w_3^2s_w^4w_4^2w_1^2) \frac{\rho}{12w_3^2w_4^2w_1^3}
\end{aligned}$$

$$\begin{aligned}
C^{(1), \text{CuLBM2}} &= (144w_3^2v_1^4w_3w_1^2 + 6w_2w_3c_s^4w_3^3 - 72w_2^2v_1^2w_3w_1^3 - 54w_3^2v_2^2w_3c_s^2w_1 + 6w_2^2w_3^2c_s^4w_1 - 18w_3^2w_3c_s^2w_1 + 18w_2^2v_2^2w_3^2c_s^2w_1 - 324w_3^2v_1^2c_s^2w_1 + \\
&\quad D_x^3v_2w_2) \\
&+ 36w_2w_3^2w_3^2w_1^2 - 18v_2^2w_3^2w_1^3 - 72w_3^2v_1^2v_2^2w_3^2w_1^2 - 24w_2^2w_3c_s^2w_1^2 + 29w_3^2w_2^2c_s^4w_1^2 + 9w_3^2v_1^2w_3^2 + 36w_2^2v_1^2w_3w_1^2 + 54w_3^2v_1^2w_3c_s^2w_1 - 18w_2^2v_2^2w_3^2w_1 - \\
&+ 78w_3^2v_1^4w_3w_1^3 + 135v_1^2v_2^2w_3^2w_1^3 + 18w_3^2c_s^4w_1^3 - w_3^2w_3^2w_1^2 + 3w_3^2v_2^2w_3^2w_1^2 + 18w_3^2v_1^2w_3w_1^3 + 18w_2^2w_3c_s^2w_1^3 - 2w_3^2w_3^2c_s^4w_1^3 + 216w_3^2v_1^2c_s^2w_1^3 + \\
&+ 99w_3^2v_1^2w_3^2c_s^2w_1^3 - 63w_2v_4^2w_3^2w_1^3 + 63w_3^2v_2^2w_3^2c_s^2 + 72w_2^2v_4^2w_3w_1^3 + 6w_3^2w_3^2w_1^3 + 36w_2w_3^2c_s^2w_1^2 - 144w_3^2v_1^2w_3w_1^2 - 18w_3^2v_2^2w_3^2w_1^2 - 6w_2w_3^2w_1^2 - \\
&- 72w_3^2v_1^2w_1^3 - 144w_2^2v_1^2w_3^2c_s^2w_1^2 - 6w_2^2v_2^2w_3^2c_s^2w_1^2 - 108w_2^2v_2^2c_s^2w_1^2 - 54w_2w_3^2v_2^2c_s^2w_1^2 - 297w_2v_2^2w_3^2c_s^2w_1^3 - 12w_3^2w_3c_s^2w_1^3 - 45w_2v_2^2w_3^2w_1^2 - w_3^2w_3^2c_s^4w_1^3 + \\
&+ 306w_3^2v_1^2w_3^2w_1^2 - 36w_2v_4^2w_1^3 + 108w_3^2v_1^2w_1^4 + 84w_2^2v_1^2w_3^2c_s^2w_1^2 + 18w_2^2v_2^2w_3^2c_s^2w_1^2 - 57w_3^2w_3^2c_s^2w_1^2 - 6w_2w_3^2w_1^2 + 216w_3^2v_1^2v_2^2w_3^2w_1^2 + 78w_2v_1^2w_3w_1^3 + \\
&+ 21w_2w_3^2c_s^2w_1^3 - 36w_2v_4^2w_3w_1^2 - 18w_3^2v_1^4w_3w_1^3 - 198w_3^2v_2^2w_3w_1^2c_s^2w_1^3 - 3w_2^2w_3^2c_s^2w_1^2 + 36w_3^2w_3^2c_s^2w_1^2 + 30w_2w_3^2c_s^2w_1^2 + 12w_2^2w_3^2c_s^4w_1^2 + 135w_2v_1^2w_3w_1^3 - \\
&+ 9w_2v_2^2w_3^2c_s^2w_1^3 + 81w_2w_2^2v_3^2w_1^2 + 30w_3^2w_3^2c_s^4 - 135w_3^2v_1^2w_2^2w_3^2 + 18w_2^2w_2^2c_s^2w_1^3 - 72w_2^2v_1^2w_3c_s^2w_1^2 - 36w_2^2v_2^2w_3c_s^2w_1^3 - 81w_1^2w_3w_1^3 + 36w_3^2v_1^2w_3^2 - \\
&- 54w_2v_1^2w_3^2w_1^2 - 8w_3^2v_1^2w_3^2w_1^3 - 24w_2^2c_s^2w_1^3 - 63w_3^2v_4^2w_3w_1^2 + 30w_3^2v_2^2w_3^2c_s^2w_1^2 + 6w_2^2w_3^2w_1^3 - 18w_2w_3^2c_s^4w_1^3 - 54w_2v_1^2w_3c_s^2w_1^3 + 18w_2v_1^2w_3w_1^3 - \\
&- 6w_2w_3c_s^2w_1^3 + 17w_3^2v_1^2w_3^2w_1^2 + 19w_2^2v_1^2c_s^2w_1^3 + 18w_3^2w_3c_s^4w_1 + 18w_3^2v_2^2w_3^2c_s^2w_1^2 + w_2^2w_3^2w_1^3 - 12w_2^2w_3^2c_s^2w_1 + 189w_1^2w_3^2c_s^2w_1^3 + 198w_2^2v_1^2w_3c_s^2w_1^3 + \\
&+ 36w_2^2w_3^2c_s^2w_1^2 + 18w_2v_2^2w_3c_s^2w_1^3 - 12w_3^2w_3^2c_s^2w_1^2 + 24w_2^2w_3c_s^4w_1^2 + 24w_3^2v_1^2w_3^2c_s^2w_1^3 + 6w_3^2v_2^2w_3^2c_s^2w_1^2 - 27w_2^2v_1^2w_3^2w_1^2 - 27w_2^2v_1^2v_2^2w_3^2w_1 + \\
&+ 36v_1^2w_3^2w_1^3 + 18w_2w_2^2w_3^2w_1^2 - 108w_3^2v_1^4w_2^2 + 15w_3^2w_3^2c_s^2w_1^2 + 8w_2^2v_1^4w_3w_1^3 - 9w_3^2v_2^2w_3^2c_s^2w_1^2 - 15w_2w_3^2c_s^4w_1^3 + 54w_2^2v_2^2w_3^2c_s^2w_1^2 - 216w_2v_1^2w_3^2w_1^3 - \\
&- 9w_3^2v_1^2w_3^2w_1^2 - 18w_2v_4^2w_3w_1^2 - 12w_2^2w_3^2c_s^2w_1^2 - 30w_3^2w_3c_s^4w_1^2 + 72w_2^2v_1^2w_3^2w_1^3 - 43w_2^2v_1^2w_3^2w_1^2 - 30w_2w_3^2c_s^4w_1^2 - 171w_3^2v_1^2w_3^2c_s^2w_1^2 + 7w_3^2v_1^4w_3^2w_1^2 - \\
&- 6w_3^2w_3^3 + 18w_2v_2^2w_3^2w_1^3 + 72w_2^3v_1^4w_1^3 + 18w_3^2v_2^2w_3^2 + 12w_3^2w_3c_s^4w_1^2 + 6w_2^2w_3^2w_1^3 + 36w_2^2v_1^2w_3^2w_1^2 + 36w_2^2v_1^2w_3^2w_1^3 + 27w_3v_1^2v_2^2w_3^2w_1^2) \frac{\rho}{24w_3^2v_1^2w_3^2w_1^3}
\end{aligned}$$

**coefficient**  $C_{D_x^2 D_y^2 \rho}^{(1)}$  **at**  $\frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2}$ :

$$C_{\frac{D_1}{D_2} \frac{D_3}{D_4} \rho}^{(1), \text{SRT}} = (24 - 46c_s^2\omega^2 - 24v_1^2 + 5c_s^2\omega^3 - 72c_s^2 + 108c_s^2\omega + 36v_1^2\omega - 36\omega + v_1^2\omega^3 + 14\omega^2 - 14v_1^2\omega^2 - \omega^3) \frac{v_1 c_s^2}{12\omega^3}$$

$$\begin{aligned}
& C_{\text{D}_x^2 \text{D}_y^2 \rho}^{(1), \text{MR11}} = (-24w_8 w_5^2 w_9 v_2^2 w_7 w_3^4 + 6w_5^2 v_1^2 w_9 v_2^2 w_7^2 w_4^3 - 12w_8 w_5^3 v_1^2 w_9 c_s^2 w_7 w_4 - 88w_8 w_5^3 w_9 c_s^4 w_2^2 w_4^2 + 12w_8 w_5^3 c_s^2 w_7 w_4^3 + 24w_8 w_5^2 v_1^2 w_9 v_2^2 w_7^2 w_4 - \\
& 12w_8 w_5^3 v_1^2 w_9 v_2^2 w_7 w_4^3 - 36w_8 w_5 w_9 v_2^2 c_s^2 w_7 w_3^4 + 6w_8 w_5^3 v_1^2 c_s^2 w_7^2 w_4^3 - 36w_8 w_5^3 v_1^2 w_9 c_s^2 w_7^2 w_4^3 - 12w_8 w_9 v_2^2 w_7^2 w_4^3 + 2w_8 w_5^2 v_1^2 w_9 c_s^2 w_7^2 w_4^3 - 12w_8 w_9 v_2^2 w_7^2 w_4^3 + \\
& 12w_8 w_5 v_1^2 w_9 v_2^2 w_7^2 w_4^2 - 42w_8 w_5^2 w_9 c_s^4 w_7 w_4^3 - 6w_5^2 w_9 c_s^2 w_7^2 w_4^3 + 12w_8 w_5^3 w_9 c_s^4 w_7^2 w_4^3 + 36w_8 w_5^2 v_2^2 c_s^2 w_7 w_4^3 + 36w_8 w_9 c_s^4 w_7^2 w_4^3 + 12w_8 w_5 w_9 c_s^4 w_7 w_4^3 - \\
& 12w_8 w_5^3 v_1^2 s_2^2 w_7 w_4^2 + 18w_8 w_5^3 v_2^2 c_s^2 w_7^2 w_4^3 + 18w_8 w_5^2 v_1^2 w_9 c_s^4 w_7^2 w_4^3 + 36w_8 w_5^3 v_1^2 w_9 v_2^2 w_7 w_4^3 + 36w_8 w_5^2 c_s^2 w_7 w_4^3 - 12w_5^2 v_1^2 w_9 v_2^2 w_7^2 w_4^2 + \\
& 12w_8 w_5^2 w_9 v_2^2 w_7 w_4^2 - 12w_8 w_5^3 v_1^2 s_2^2 w_7 w_4^3 + 5w_8 w_5^3 w_9 c_s^4 w_7 w_4^3 - 12w_8 w_5^3 w_9 c_s^4 w_7^2 w_4^3 + 12w_8 w_5^2 v_1^2 c_s^2 w_7 w_3^4 + 12w_8 w_5^2 w_9 c_s^2 w_7^2 w_4^3 + 12w_8 w_5 w_9 v_2^2 w_7 w_4^3 - \\
& 12w_8 w_5^2 w_9 c_s^2 w_7^2 w_4^2 - 12w_8 w_5^3 v_2^2 w_7 w_4^3 - 18w_8 w_5 v_1^2 w_9 v_2^2 w_7 w_4^3 - 36w_8 w_5^2 w_9 v_2^2 c_s^2 w_7 w_4^3 - 36w_8 w_5^3 c_s^4 w_7 w_4^3 + 12w_8 w_5 w_9 c_s^2 w_7^2 w_4^2 - 6w_5^2 w_9 v_2^2 w_7^2 w_4^3 - \\
& 12w_8 w_5^3 v_1^2 w_9 c_s^2 w_7^2 w_4^2 + 12w_8 w_5^3 v_2^2 w_9 c_s^2 w_7^2 w_4^3 + 12w_8 w_5^3 v_2^2 w_9 v_2^2 w_7 w_4^3 - 18w_8 w_5^2 w_9 c_s^2 w_7 w_4^3 + 6w_5^2 v_1^2 w_9 c_s^2 w_7^2 w_4^3 - 24w_8 w_5^3 v_1^2 w_9 v_2^2 w_7 w_4^3 - \\
& 12w_8 w_5^3 v_1^2 w_9 c_s^2 w_7^2 w_4^2 - 12w_8 w_5 c_s^2 w_7^2 w_4^3 + 6w_8 w_5^2 v_1^2 w_9 v_2^2 w_7^2 w_4^3 + 18w_2^2 w_9 c_s^4 w_7^2 w_4^3 + 36w_8 w_5^2 v_2^2 c_s^2 w_7 w_4^3 + 6w_8 w_5^3 v_1^2 w_9 v_2^2 w_7^2 w_4^3 + 12w_8 w_5^2 w_9 c_s^4 w_7^2 w_4^3 - \\
& 6w_8 w_5^3 v_1^2 w_9 c_s^2 w_7^2 w_4^2 - 36w_8 w_5 c_s^2 w_9 v_2^2 c_s^2 w_7^3 + 12w_8 w_5 v_1^2 w_9 v_2^2 w_7 w_4^3 + 12w_8 w_5^2 w_9 v_2^2 w_7^2 w_4^2 - 12w_8 w_5 v_1^2 w_9 c_s^2 w_7^2 w_4^3 - w_8 w_5^3 v_1^2 w_9 c_s^2 w_7^2 w_4^3 + \\
& 36w_8 w_5^3 c_s^4 w_7 w_4^2 + 72w_8 w_5^2 w_9 v_2^2 c_s^2 w_7^3 - 12w_8 w_5^2 c_s^2 w_7 w_4^3 + 180w_8 w_5^3 w_9 c_s^4 w_7^2 w_4^3 + 18w_8 w_5^3 v_1^2 w_9 c_s^2 w_7 w_4^2 - 36w_8 w_5^2 w_9 c_s^4 w_7^2 w_4^2 - 24w_8 w_5^2 v_1^2 w_9 v_2^2 w_7^2 w_4^2 - \\
& 12w_8 w_5^3 v_1^2 v_2^2 w_7 w_4^2 - 12w_8 w_5^3 v_2^2 w_7 w_4^2 + 12w_8 w_5^3 v_1^2 w_9 c_s^2 w_7^3 - 12w_5^2 v_1^2 w_9 c_s^2 w_7^2 w_4^2 - 12w_8 w_5^3 v_1^2 w_9 v_2^2 w_7^2 w_4^2 - 6w_8 w_5^2 v_1^2 v_2^2 w_7^2 w_4^3 + 12w_8 w_5 w_9 c_s^2 w_7^2 w_4^3 + \\
& 24w_8 w_5^3 w_9 v_2^2 w_7 w_4^2 - 72w_8 w_5^3 w_9 v_2^2 s_2^2 w_7 w_4 + 6w_8 w_5^2 c_s^2 w_7^2 w_4^3 + 12w_8 v_1^2 w_9 v_2^2 w_7^2 w_4^3 - 12w_8 w_5^3 v_1^2 w_9 v_2^2 w_7^2 w_4^3 + 18w_8 w_5^2 w_9 v_2^2 s_2^2 w_7^2 w_4^3 - 18w_5^3 v_2^2 c_s^2 w_7^2 w_4^3 + \\
& 12w_3^3 v_1^2 c_s^2 w_7^2 w_4^2 - 36w_8 w_5^3 s_4^2 w_7^2 w_4^2 + 6w_8 w_5^3 w_9 c_s^2 w_7 w_4^3 - 12w_8 w_5 v_1^2 w_9 c_s^2 w_7 w_4^3 + 12w_8 w_5^3 v_1^2 v_2^2 w_7 w_4^2 - 12w_8 w_5^2 v_1^2 w_9 v_2^2 w_7 w_4^2 - 12w_8 w_5^3 w_9 c_s^2 w_7^2 w_4^3 - \\
& 12w_8 w_5^3 v_1^2 w_9 c_s^2 w_7^2 w_4^2 - 36w_8 w_5^3 w_9 v_2^2 c_s^2 w_7^2 w_4^2 + 12w_8 w_5^2 w_9 v_2^2 w_7^2 w_4^2 + 12w_8 w_5^3 w_9 c_s^4 w_7 w_4 - 18w_8 w_5^2 w_9 c_s^2 w_7^2 w_4^2 - 12w_3^3 c_s^2 w_7^2 w_4^2 + 12w_8 w_5^3 v_2^2 w_7^2 w_4^2 - \\
& 18w_8 w_5^3 w_9 c_s^2 w_7 w_4^2 + 18w_8 w_5^3 c_s^2 w_7^2 w_4^3 + 36w_8^3 v_2^2 c_s^2 w_7^2 w_4^2 - 6w_5^2 v_1^2 c_s^2 w_7^2 w_4^3 - 12w_8 w_5^2 v_1^2 w_9 c_s^2 w_7^2 w_4^3 - 72w_8 w_5^2 w_9 v_2^2 c_s^2 w_7 w_4^2 - 84w_8 w_5^2 w_9 c_s^4 w_7 w_4 + \\
& 2w_8 w_5^2 w_9 c_s^2 w_7^2 w_4^3 - 6w_8 w_5^3 v_2^2 w_7^2 w_4^3 + 6w_3^2 c_s^2 w_7^2 w_4^3 - 12w_8 w_5^3 w_9 c_s^2 w_7^2 w_4^3 + 36w_8 w_5^3 w_9 v_2^2 c_s^2 w_7^2 w_4^3 + w_8 w_5^3 v_1^2 w_9 c_s^2 w_7^2 w_4^3 - 24w_8 w_5^2 w_9 v_2^2 w_7^2 w_4^2 - \\
& 12w_8 w_5^3 v_1^2 v_2^2 w_7 w_4^2 + 12w_8 w_5^3 w_9 c_s^2 w_7^2 w_4^2 + 24w_8 w_5^2 v_1^2 w_9 v_2^2 w_7 w_4^3 + 36w_8 w_5^3 w_9 v_2^2 c_s^2 w_7^2 w_4^3 - 36w_8 w_5^3 v_1^2 c_s^2 w_7 w_4^3 + 12w_8 w_5^3 v_2^2 c_s^2 w_7 w_4^2 - \\
& 36w_8 w_5 w_9 c_s^2 w_7^2 w_4^3 + 30w_8 w_5^3 w_9 c_s^4 w_7 w_4^3 + 12w_8 w_5^3 c_s^2 w_7^2 w_4^2 - 12w_5^2 v_1^2 w_9 c_s^2 w_7^2 w_4^2 + 24w_8 w_5^2 w_9 v_2^2 w_7^2 w_4^2 - 18w_8 w_5^2 c_s^4 w_7^2 w_4^3 + 18w_2^2 w_9 v_2^2 c_s^2 w_7^2 w_4^3 + \\
& 12w_8 w_5^3 v_1^2 w_9 c_s^2 w_7 w_4^2 + 72w_8 w_5^3 w_9 v_2^2 c_s^2 w_7^2 w_4^2 + 12w_8 w_5^2 w_9 v_2^2 w_7^2 w_4^3 + 18w_8 w_5 w_9 v_2^2 w_7^2 w_4^3 - 6w_8 w_5^2 v_1^2 c_s^2 w_7^2 w_4^3 + 150w_8 w_5^2 w_9 c_s^4 w_7^2 w_4^2 + 12w_5^2 v_1^2 v_2^2 w_7^2 w_4^2 + \\
& 12w_8 w_5^3 w_9 v_2^2 w_7 w_4^2 - 12w_8 w_5 v_1^2 w_9 v_2^2 w_7 w_4^3 + 36w_5^3 c_s^4 w_7 w_4^2 - 36w_8 w_5^3 w_9 v_2^2 c_s^2 w_7 w_4^3 + 12w_8 v_1^2 w_9 c_s^2 w_7 w_4^3 + 6w_8 w_5^2 v_2^2 w_7^2 w_4^3 + 6w_5^2 v_2^2 w_7^2 w_4^3 - \\
& 6w_8 w_5^3 c_s^2 w_7^2 w_4^3 - 42w_8 w_5^3 w_9 c_s^4 w_7 w_4^2 - 96w_8 w_5^3 w_9 c_s^4 w_7^2 - 6w_8 w_5^2 w_9 v_2^2 w_7^2 w_4^3 + 12w_8 w_5^3 v_1^2 w_9 c_s^2 w_7^2 w_4^2 + 18w_8 w_5^2 v_1^2 w_9 v_2^2 w_7 w_4^3 + 36w_8 w_5^3 v_2^2 c_s^2 w_7 w_4^2 - \\
& 48w_8 w_5 w_9 c_s^2 w_7^2 w_4^2 - 12w_8 w_5^3 v_2^2 c_s^2 w_7 w_4^3 + 12w_8 w_5^2 w_9 c_s^2 w_7^2 w_4^2 - 54w_8 w_5 w_9 v_2^2 c_s^2 w_7^2 w_4^3 - 6w_8 w_5^2 w_9 c_s^4 w_7^2 w_4^2 + 108w_8 w_5^3 w_9 v_2^2 c_s^2 w_7 w_4^2 - 18w_5^3 c_s^4 w_7^2 w_4^2 - \\
& 36w_8 w_5^3 w_9 v_2^2 w_7 w_4^2 - 6w_5^2 v_1^2 w_9 c_s^2 w_7^2 w_4^3 - 12w_8 w_5 w_9 v_2^2 w_7^2 w_4^2 - 18w_8 w_5^2 v_2^2 c_s^2 w_7^2 w_4^3 + 12w_8 w_5^2 w_9 c_s^2 w_7^2 w_4^3 - 36w_8 w_5^2 w_9 v_2^2 c_s^2 w_7^2 w_4^2) \frac{v_1^2}{12w_8 w_5^3 v_1^2 w_9 w_2^2 w_3^2}
\end{aligned}$$

$$C_{\mathrm{D}_x^2 \mathrm{D}_y^2 \rho}^{(1), \text{MRT2}} = C_{\mathrm{D}_x^2 \mathrm{D}_y^2 \rho}^{(1), \text{MRT1}}$$

$$\begin{aligned} C_{\substack{(1,1,CLBM1 \\ D_2^2 D_2^2 \nu}} &= (-6w_5^2 w_9 w_7^2 w_4^2 - 12w_8 w_3^2 v_1^2 w_9 w_4 - 2w_8 w_5^2 v_1^2 w_9 w_7^2 w_4^2 + 6w_8 w_3^2 v_1^2 w_7^2 w_4^2 + 12w_8 w_3^2 v_3 w_9 w_7 - 12w_8 w_5^2 v_1^2 w_9 w_4^2 + 12w_8 w_3^2 v_3 w_7 w_4^2 - \\ &12w_8 w_5^2 w_9 w_7^2 w_4 + 12w_8 w_5 w_9 w_7 w_4^2 - 36w_8 w_2^2 w_9 c_2^2 w_7^2 + 12w_5^3 v_2^2 w_7^2 w_4 - 36w_8 w_3^2 c_2^2 w_7 w_4^2 + 18w_5^2 w_9 c_2^2 w_7^2 w_4^2 - 40w_8 w_3^2 w_9 c_2^2 w_7^2 w_4 - 6w_8 w_3^2 v_1^2 w_9 w_7 w_4^2 + \\ &12w_8 w_5^2 w_9 w_4^2 - 36w_8 w_3^2 w_9 c_2^2 w_7 - 12w_8 w_5 v_1^2 w_9 w_7^2 w_4^2 - w_8 w_3^2 w_9 w_7^2 w_4^2 - 36w_5^2 w_9 c_2^2 w_7^2 w_4 + 36w_8 w_5^2 c_2^2 w_7 w_4^2 + 36w_8 w_3^2 w_9 c_2^2 w_7^2 + 18w_8 w_3^2 v_1^2 w_9 w_7 w_4 + \\ &5w_8 w_3^2 w_9 c_2^2 w_7^2 w_4^2 + 12w_8 w_3^2 w_9 w_7^2 w_4 - 36w_8 w_5 w_9 c_2^2 w_7 w_4^2 + 6w_8 w_5^2 w_7^2 w_4^2 + 2w_8 w_2^2 w_9 w_7^2 w_4^2 + 36w_8 w_3^2 c_2^2 w_7 w_4 - 6w_5^3 v_2^2 w_7^2 w_4^2 - 6w_8 w_5^2 v_1^2 w_7^2 w_4^2 - \\ &12w_8 w_5^3 w_7 w_4^2 + 12w_5^2 w_9 w_7^2 w_4 + 12w_8 w_3^2 v_1^2 w_9 w_4^2 + 36w_8 w_9 c_2^2 w_7^2 w_4^2 - 12w_8 w_5^3 w_9 w_7 - 12w_8 w_3^2 v_1^2 w_7^2 w_4 + 18w_8 w_5^2 v_1^2 w_9 w_7^2 w_4 + 54w_8 w_5^2 w_9 c_2^2 w_7 w_4^2 - \\ &12w_8 w_5^3 w_9 w_4^2 + 12w_8 w_5^2 v_1^2 w_7 w_4^2 + 12w_8 w_3^2 w_7^2 w_4 + 12w_8 w_3^2 v_1^2 w_9 w_7 + 12w_8 w_3^2 v_1^2 w_7 w_4 - 6w_8 w_5^2 w_9 c_2^2 w_7^2 w_4^2 - 18w_5^3 s_2^2 w_7 w_4^2 - 12w_8 w_3^2 v_1^2 w_9 w_7^2 w_4 - \\ &18w_8 w_3^2 w_9 c_2^2 w_7 w_4^2 - 12w_8 w_9 w_7^2 w_4^2 + 6w_5^2 v_1^2 w_9 w_7^2 w_4^2 - 12w_8 w_3^2 w_7 w_4 - 18w_8 w_5^2 c_2^2 w_7^2 w_4^2 - 12w_8 w_5^2 w_7 w_4^2 - 18w_8 w_3^2 w_9 w_7 w_4 - 36w_8 w_5 w_9 c_2^2 w_7^2 w_4^2 - \\ &18w_8 w_5^2 w_9 w_7 w_4^2 - 36w_8 w_3^2 c_2^2 w_7 w_4 + 36w_8 w_3^2 w_9 c_2^2 w_7^2 w_4^2 + 18w_8 w_3^2 c_2^2 w_7^2 w_4^2 + 12w_8 w_5 w_9 w_7^2 w_4^2 + 54w_8 w_5^2 w_9 c_2^2 w_7 w_4 + \end{aligned}$$

$$w_8 w_5^3 v_1^2 w_9 w_7^2 w_4^2 - 36 w_8 w_5^2 w_9 c_s^2 w_4^2 + 6 w_5^3 w_7^2 w_4^2 - 12 w_5^2 v_1^2 w_9 w_7^2 w_4 - 12 w_8 w_5 v_1^2 w_9 w_7 w_4^2 + 6 w_8 w_5^3 w_9 w_7 w_4^2 + 12 w_8 v_1^2 w_9 w_7^2 w_4^2 - 12 w_8 w_5^2 v_1^2 w_9 w_7^2 + 36 w_5^3 c_s^2 w_7^2 w_4 + 54 w_8 w_5^2 w_9 c_s^2 w_7^2 w_4 + 18 w_8 w_5^2 v_1^2 w_9 w_7 w_4^2 - 12 w_8 w_5^2 v_1^2 w_7 w_4^2 + 12 w_8 w_5^3 w_9 w_4 - 12 w_8 w_5^3 v_1^2 w_9 w_7 - 6 w_8 w_5^3 w_7 w_4^2) \frac{v_1 c_s^2}{12 w_8 w_5^3 w_9 w_7^2 w_4^2}$$

$$C_{\mathrm{D}_x^2 \mathrm{D}_y^2 \rho}^{(1), \text{CLBM2}} = C_{\mathrm{D}_x^2 \mathrm{D}_y^2 \rho}^{(1), \text{CLBM1}}$$

$$\begin{aligned} C_{\substack{(1,1), \text{CulBIM} \\ D_x^2 D_y^2}} &= (-6w_3 w_4^2 w_1^2 + 6w_3^2 w_4 w_1^3 - 12v_1^2 w_3 w_4^2 w_1^3 - 18w_3^2 c_s^2 w_4 w_1^3 + 6v_1^2 w_3^2 w_4 w_1^2 - 6w_3^2 w_4 w_1^2 + 12w_3 w_4^2 w_1^3 + 18w_3^2 c_s^2 w_4 w_1^2 - 6v_1^2 w_3^2 w_4 w_1^3 + 6v_1^2 w_3^2 w_4^2 w_1^2 + 18w_3 w_4^2 w_1^2 + 18w_3^2 c_s^2 w_4 w_1^2 + 12w_3 w_4^2 w_1^3 + 18w_3^2 c_s^2 w_4 w_1^2 + 12w_3 w_4^2 w_1^2 + 18w_3 w_4^2 w_1^2 + 12w_3 w_4^2 w_1^3 - 36w_3 c_s^2 w_4 w_1^3 - 36c_s^2 w_4 w_1^3 - 12v_1^2 w_3 w_4^2 w_1^3 - 12w_1^2 w_4^2 w_1^2 - 40w_3 c_s^2 w_4 w_1^3 + 12w_4^2 w_1^2 + 12w_3 w_4^3 - 18v_1^2 w_3^2 w_4^2 w_1^2 + 12v_1^2 w_3^2 w_4^2 w_1^3 + 36c_s^2 w_4^2 w_1^3 + 12v_1^2 w_3^2 w_4^2 w_1^2 + 54w_3 c_s^2 w_4 w_1^3 - 12w_3^2 w_4^2 + 36w_2^2 c_s^2 w_4^3 - 12w_3^2 w_4^3 + 36w_3^2 c_s^2 w_4^2 + 36w_3 c_s^2 w_4 w_1^2 - 12v_1^2 w_4 w_1^3 - 12v_1^2 w_3^2 w_4^2 w_1^3 - 36c_s^2 w_4^2 w_1^2 - 54w_3^2 c_s^2 w_4^2 w_1^2 + 12w_3^2 w_4^2 w_1^3 + 18w_3^2 w_4^2 w_1^2 - 36w_3^2 c_s^2 w_4^2 w_1^2 + 12w_4 w_1^3 - 18w_3 c_s^2 w_4^3 - 4w_3^2 w_4^2 w_1^2 + 12v_1^2 w_3 w_4 w_1^2 + v_1^2 w_3^2 w_4^2 w_1^3 + 12w_3^2 c_s^2 w_4^2 w_1^2 - w_3^2 w_4^2 w_1^3 - 12w_3 w_4 w_1^2 + 4v_1^2 w_3^2 w_4^2 w_1^2 + 5w_3^2 c_s^2 w_4^2 w_1^3 + 18v_1^2 w_3 w_4 w_1^3) \frac{v_1^2 w_3^2}{12w_3^2 c_s^2 w_4^2 w_1^3} \end{aligned}$$

$$\begin{aligned}
C_{D_x^2 D_y^2}^{(1), \text{CuLBMD2}} = & (-18w_2 w_3 c_s^4 w_1^3 + 54w_2^3 v_2^4 w_2^3 w_1 - 90w_2^5 w_3^2 c_s^4 w_1 + 30w_2^3 w_3 c_s^2 w_1 + 72w_2^3 c_s^4 w_1^3 + 27w_2^2 v_2^2 w_3^2 c_s^2 w_1 - 36w_2^3 v_1^2 c_s^2 w_1^2 - 99w_2^2 w_3^2 w_1^3 - \\
& 36w_2^6 w_3 c_s^2 w_1^2 - 91w_2^3 w_3^2 c_s^4 w_1^2 + 12w_2^2 c_s^2 w_1^3 - 108w_2^3 c_s^4 w_2^2 - 6w_2^3 v_1^2 w_2^2 w_1^2 - 30w_2^3 v_1^2 w_3 c_s^2 w_1 - 90w_2^2 v_2^2 w_3^2 w_1 + 45v_2^2 v_2^2 w_2^2 w_3^2 w_1^3 + 90w_2^2 c_s^4 w_3^4 w_1^2 + 27w_2^2 v_2^4 w_2^3 w_1^3 + 27w_2^3 v_2^2 w_2^2 w_1^2 + 10w_2^3 w_3^2 c_s^4 w_1^3 + 24w_3^2 v_2^2 c_s^2 w_1^3 - 30w_2^2 v_2^2 w_2^2 c_s^2 w_1 + 18w_2^3 v_1^2 w_3 c_s^2 w_1^2 - 6w_2^3 w_3^2 w_1^2 + 24w_2 w_3^2 c_s^2 w_1^2 - 9w_3^2 v_2^2 w_3^2 w_1 - 6w_2 w_3^2 w_1^2 - 54w_3^2 v_4^2 w_3^2 + 18w_2 v_2^2 w_2^2 c_s^2 w_1^2 + 138w_2^2 v_2^2 w_3^2 c_s^2 w_1^3 - 12w_2^3 v_2^2 c_s^2 w_1^3 - 297w_2 v_2^2 w_3^2 c_s^2 w_1^2 - 21w_2 v_1^2 w_3^2 c_s^2 w_1^3 + 12w_2 w_3^2 c_s^2 w_1^3 + 6w_2 v_2^2 w_3^2 w_1^2 + 54v_2^4 w_2^3 w_1^3 + 35w_2^5 w_3^2 c_s^4 w_1^3 - 90w_2^4 v_2^2 w_3^2 w_1^2 + 48w_3^2 v_1^2 w_3 c_s^2 w_1^2 + 8w_2^3 v_1^2 w_3^2 c_s^2 w_1^3 + 270w_2^3 v_2^2 w_3^2 c_s^2 w_1^2 + 63w_3^2 w_3^2 c_s^4 w_1 - 6w_2 w_3^2 w_1^3 - 45w_3^2 v_1^2 v_2^2 w_3^2 w_1 + 93w_2 w_3^2 c_s^2 w_1^3 + 54w_2 v_2^4 w_2^3 w_1^2 - 27w_3^2 v_4^2 w_3^2 w_1^2 - 12w_3^2 v_1^2 w_3 c_s^2 w_1^3 - 27w_2^2 v_2^2 w_2^2 w_1^3 - 48w_3^2 w_3 c_s^2 w_1^2 + 54w_2 w_3^2 c_s^4 w_1^2 + 6w_2 v_2^2 w_3^2 w_1^3 - 459w_2 v_2^2 w_3^2 c_s^2 w_1^3 - 6w_2 v_2^2 w_3^2 c_s^2 w_1^2 + 18w_3^2 w_3^2 c_s^4 + 45w_3^2 v_1^2 v_2^2 w_2^2 + 36w_3^2 c_s^2 w_1^2 + 405w_2^2 w_3^2 c_s^2 w_1^3 + 36w_2 v_1^2 w_3 c_s^2 w_1^2 + 12w_2^2 w_3^2 w_1^2 - 6v_1^2 w_3^2 w_1^3 - 54w_2 v_2^4 w_3^2 w_1^2 - 72w_2^3 c_s^2 w_1^3 - 24w_3^2 v_1^2 w_3^2 c_s^2 w_1^2 + 2w_3^2 w_2^2 c_s^2 w_1^3 + 6w_2^3 w_1^3 - 6w_2 v_2^2 w_3 c_s^2 w_1^3 + 6w_2 w_3 c_s^2 w_1^3 - 90w_2^3 w_3 c_s^4 w_1 - 54w_2 v_2^4 w_2^2 w_1^3 - 135w_3^2 v_2^2 w_3^2 c_s^2 + 48w_2 v_3^2 c_s^2 w_1 + 18w_2^3 v_2^2 c_s^2 w_1^2 - 24w_3^2 v_2^2 c_s^2 w_1^3 + 41w_3^2 w_3^2 c_s^2 w_1^2 + 108w_2^2 w_3 c_s^4 w_1^2 + 2w_3^2 v_1^2 w_3^2 c_s^2 w_1^3 - 138w_3^2 v_2^2 w_3^2 c_s^2 w_1^2 - 36w_2^2 c_s^4 w_3^4 w_1^3 + 6w_2^2 v_2^2 w_3^2 w_1^2 - 45w_2^2 v_1^2 v_2^2 w_3^2 w_1 + 99w_2 v_2^2 w_3^2 c_s^2 w_1^2 - 39w_3^2 w_3^2 c_s^2 w_1^2 + 189w_2^3 v_2^2 w_3^2 c_s^2 w_1^2 - 117w_2 w_3^2 c_s^4 w_1^3 - 12w_3^2 v_1^2 w_3^2 w_1^2 + 90w_3^2 v_1^2 v_2^2 w_3^2 w_1^2 - 45w_2 v_1^2 v_2^2 w_3^2 w_1^3 + 6w_2^3 v_1^2 w_3^2 w_1^2 - 54w_2^2 w_3^2 c_s^2 w_1^2 + 144w_3^2 w_3^2 c_s^4 w_1^2 - 18w_2 w_3^2 c_s^4 w_1^2 + 3w_3^2 v_1^2 w_3^2 c_s^2 w_1^2 + 6w_3^2 w_1^3 + 99w_2 v_2^2 w_3^2 w_1^3 + 9w_3^2 v_2^2 w_2^2 - 25w_2^2 w_3^2 c_s^2 w_1^3 - 36w_3^2 w_3 c_s^4 w_1^3 - 6w_2^2 w_3^2 w_1^2 - 45w_2 v_1^2 v_2^2 w_3^2 w_1^2) \frac{v_1}{24w_3^2 w_3^2 c_s^2 w_1^3}
\end{aligned}$$

**coefficient**  $C_{D_x^2 D_y^2 v_1}^{(1)}$  **at**  $\frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2}$ :

$$C_{\frac{D^2_x}{D^2_y} v_1}^{(1), \text{SRT}} = (-24 + 8c_s^2\omega^2 + 72v_1^2 - c_s^2\omega^3 + 12c_s^2 - 18c_s^2\omega - 108v_1^2\omega + 36\omega - 12\omega^2 + 36v_1^2\omega^2) \frac{\rho c_s^2}{12\omega^3}$$

$$\begin{aligned}
C_{D_2^2 D_2^2 v_1}^{(1),MRT1} = & -24w_8 w_5^2 w_9 v_2^2 w_7 w_3^4 + 18w_8^2 v_1^2 w_9 v_2^2 w_7 w_3^4 + 60w_8 w_3^3 v_1^2 w_9 c_s^4 w_7 w_4 - 4w_8 w_3^2 w_9 c_4^4 w_7^2 w_4 + 12w_8 w_3^2 c_s^2 w_7 w_3^4 + 72w_8 w_3^2 v_1^2 w_9 v_2^2 w_7^2 w_4 - \\
& 36w_8 w_5^2 v_1^2 w_9 v_2^2 w_7 w_4^3 - 12w_8 w_5 w_9 v_2^2 c_s^2 w_7 w_3^4 + 18w_8 w_3^2 v_1^2 c_2^2 w_7 w_4^3 - 12w_8 w_5^3 v_2^2 c_s^2 w_7^2 w_4^2 - 18w_8 w_5^2 v_1^2 w_9 c_s^2 w_7^2 w_4^3 - 12w_8 w_9 v_2^2 w_7^2 w_4^3 + \\
& 36w_8 w_5 v_1^2 w_9 v_2^2 w_7^2 w_4^2 - 12w_8 w_5^2 w_9 c_2^2 w_7 w_4^3 - 6w_5^2 w_9 v_2^2 w_7 w_4^2 + 12w_8 w_5^2 v_2^2 c_s^2 w_7 w_3^4 - 36w_8 w_5^3 v_1^2 c_2^2 w_7^2 w_4^2 + 6w_8 w_5^3 v_2^2 c_s^2 w_7^2 w_4^3 + \\
& 180w_8 w_5^2 v_1^2 w_9 c_2^2 w_7^2 w_4^2 + 108w_8 w_3^2 v_1^2 w_9 v_2^2 w_7 w_4^2 + 12w_8 w_5^2 c_4^2 w_7 w_4^3 - 36w_8 w_1^2 v_1^2 w_9 v_2^2 w_7^2 w_4^2 + 12w_8 w_5^2 w_9 v_2^2 w_7 w_4^2 - 12w_8 w_5^3 c_s^2 w_7 w_4^2 - w_8 w_5^3 w_9 c_4^2 w_7^2 w_4^2 + \\
& 36w_8 w_5^2 v_1^2 s_2 w_7 w_4^3 + 12w_8 w_5^2 w_9 c_2^2 w_7^2 w_4^2 + 12w_8 w_5 w_9 v_2^2 w_7 w_4^3 - 12w_8 w_5^2 v_2^2 w_7 w_4^3 - 54w_8 w_5 v_1^2 w_9 v_2^2 w_7^2 w_4^3 + 12w_8 w_5^2 w_9 c_4^2 w_7 w_4^2 - 12w_8 w_5^2 w_9 v_2^2 c_2^2 w_7 w_4^2 - \\
& 12w_8 w_5^3 c_s^2 w_7 w_4^3 - 108w_8 w_5 v_1^2 w_9 v_2^2 w_7^2 w_4^2 - 6w_5^2 w_9 v_2^2 w_7^2 w_4^3 - 84w_8 w_5^2 v_1^2 w_9 c_2^2 w_7^2 w_4^2 + 36w_8 w_5^3 v_1^2 w_9 v_2^2 w_7^2 w_4^3 + 12w_8 w_5^3 v_2^2 w_7 w_4^3 + 12w_8 w_5^2 w_9 v_2^2 c_2^2 w_7 w_4^2 + \\
& 18w_8^2 v_1^2 w_9 c_2^2 w_7^2 w_4^3 - 72w_8 w_3^2 v_1^2 w_9 v_2^2 w_7 w_4^2 + 24w_8 w_5^2 v_1^2 w_9 c_2^2 w_7^2 w_4^2 + 18w_8 w_5^2 v_1^2 w_9 v_2^2 w_7^2 w_4^3 + 6w_5^2 w_9 c_4^2 w_7^2 w_4^3 + 12w_8 w_9 v_2^2 c_2^2 w_7 w_4^3 + \\
& 18w_8 w_3^2 v_1^2 w_9 c_2^2 w_7^2 w_4^3 + 78w_8 w_3^2 v_1^2 w_9 c_2^2 w_7 w_4^2 - 12w_8 w_5^2 w_9 c_2^2 c_2^2 w_4^3 + 36w_8 w_5^2 v_1^2 w_9 v_2^2 w_7 u_4^3 + 12w_5^2 w_9 v_2^2 w_7^2 w_4^2 + 18w_8 w_5 v_1^2 w_9 c_2^2 w_7^2 w_4^3 + 12w_8 w_5^2 c_4^2 w_7 w_4^2 + \\
& 24w_8 w_5^2 w_9 c_2^2 s_2 w_7 w_4^3 - 12w_8 w_5^2 c_4^2 w_7 w_4^3 + 18w_8 w_5^2 v_1^2 w_9 c_2^2 w_7^2 w_4 - 132w_8 w_5^2 v_1^2 w_9 c_2^2 w_7^2 w_4^2 - 12w_5^2 w_9 c_4^2 w_7^2 w_4^2 - 72w_8 w_5 v_1^2 w_9 v_2^2 w_7^2 w_4^2 - \\
& 36w_8 w_5^2 v_1^2 w_9 c_2^2 w_7^2 w_4^2 - 12w_8 w_5^2 v_1^2 w_9 v_2^2 w_7^2 w_4^2 - 24w_8 w_5^2 v_1^2 w_9 c_2^2 w_7^2 w_4^3 - 36w_8 w_5^2 v_1^2 w_9 c_2^2 w_7^2 w_4^2 - 12w_8 w_5^2 w_9 c_2^2 w_7^2 w_4^3 - 18w_8 w_5^2 v_1^2 w_9 v_2^2 w_7^2 w_4^3 - \\
& 6w_8 w_5 w_9 c_2^2 w_7^2 w_4^3 + 24w_8 w_5^2 w_9 v_2^2 w_7 w_4^4 - 24w_8 w_5^2 w_9 v_2^2 c_2^2 w_7 w_4^2 + 6w_8 w_5^2 c_2^2 w_7^2 w_4^3 + 36w_8 w_5^2 v_1^2 w_9 v_2^2 w_7^2 w_4^2 - 36w_8 w_5^2 v_1^2 w_9 v_2^2 w_7^2 w_4^3 + 6w_8 w_5^2 w_9 v_2^2 c_2^2 w_7^2 w_4^3 - \\
& 6w_5^2 v_2^2 c_2^2 w_7^2 w_4^3 + 36w_5^2 v_1^2 c_2^2 w_7^2 w_4^2 - 12w_8 w_5^2 c_4^2 w_7^2 w_4^2 - 12w_8 w_5^2 w_9 c_2^2 w_7 w_4^3 + 72w_8 w_5 v_1^2 w_9 c_2^2 w_7 w_4^3 + 36w_8 w_5^2 v_1^2 w_9 v_2^2 w_7 w_4^2 - 36w_8 w_5^2 v_1^2 w_9 v_2^2 w_7 w_4^3 - \\
& 42w_8 w_5^2 v_1^2 w_9 c_2^2 w_7^2 w_4^2 - 12w_8 w_5^2 w_9 c_2^2 c_2^2 w_4^2 + 12w_8 w_5^2 c_3^2 w_9 v_2^2 w_4^2 + 12w_8 w_5^2 w_9 c_4^2 w_7 w_4^2 - 24w_8 w_5^2 w_9 c_2^2 w_7^2 w_4^2 - 12w_5^2 s_2^2 w_7^2 w_4^2 + 12w_8 w_5^2 v_1^2 w_9 c_2^2 w_7^2 w_4^2 + \\
& 24w_8 w_5^2 w_9 c_2^2 w_7 w_4^2 + 6w_8 w_5^2 s_2^2 w_7^2 w_4^2 + 12w_5^2 s_2^2 c_2^2 w_7^2 w_4^2 - 18w_5^2 v_1^2 s_2 w_7^2 w_4^2 + 24w_8 w_5^2 v_2^2 w_9 c_2^2 w_7^2 w_4^2 - 24w_8 w_5^2 w_9 v_2^2 s_2 w_7^2 w_4^2 - 12w_8 w_5^2 w_9 c_2^2 w_7^2 w_4^2 - \\
& 48w_8 w_5^2 v_1^2 w_9 c_2^2 w_7^2 w_4^2 + 12w_8 w_5 w_9 c_2^2 s_2 w_7^2 w_4^2 + 6w_8 w_5^2 w_9 c_2^2 w_7^2 w_4^3 - 6w_8 w_5^2 v_2^2 w_7^2 w_4^3 + 6w_5^2 c_2^2 w_7^2 w_4^3 - 12w_8 w_5^2 w_9 v_2^2 w_7^2 w_4^3 + 12w_8 w_5^2 w_9 v_2^2 c_2^2 w_7^2 w_4^3 - \\
& 24w_8 w_5^2 w_9 v_2^2 w_7^2 w_4^2 - 36w_8 w_5^2 v_1^2 w_9 c_2^2 w_7^2 w_4^2 + 72w_8 w_5^2 v_1^2 w_9 v_2^2 w_7^2 w_4^2 + 12w_8 w_5 w_9 v_2^2 c_2^2 w_7^2 w_4^2 + 60w_8 w_5^2 v_1^2 w_9 c_2^2 w_7^2 w_4^2 - 12w_8 w_5^2 v_2^2 s_2^2 w_7^2 w_4^2 + \\
& 36w_8 w_5^2 v_1^2 w_9 c_2^2 w_7^2 w_4^2 + 6w_8 w_5^2 v_1^2 c_2^2 w_7^2 w_4^2 + 12w_8 w_5^2 w_9 c_4^2 w_7 w_4^3 + 12w_8 w_5^2 c_3^2 w_7^2 w_4^2 - 12w_5^2 v_2^2 w_7^2 w_4^2 + 24w_8 w_5^2 w_9 v_2^2 w_7^2 w_4^2 - 6w_8 w_5^2 c_4^2 w_7^2 w_4^3 + \\
& 6w_5^2 w_9 v_2^2 c_2^2 w_7^2 w_4^3 - 12w_8 w_5^2 w_9 c_2^2 w_7 w_4^4 + 24w_8 w_5^2 w_9 v_2^2 c_2^2 w_7^2 w_4^2 + 12w_8 w_5^2 w_9 v_2^2 w_7^2 w_4^3 - 18w_8 w_5^2 v_1^2 s_2 w_7^2 w_4^2 + 24w_8 w_5^2 v_1^2 w_9 c_2^2 w_7^2 w_4^2 + 36w_8 w_5^2 v_1^2 w_9 v_2^2 w_7^2 w_4^2 - \\
& 36w_8 w_5^2 v_1^2 w_9 c_2^2 w_7^2 w_4^2 + 12w_8 w_5^2 w_9 v_2^2 c_2^2 w_7^2 w_4^2 - 12w_8 w_5^2 w_9 v_2^2 w_7^2 w_4^3 + 84w_8 w_5^2 v_1^2 w_9 c_2^2 w_7^2 w_4^2 - 144w_8 w_5^2 v_1^2 w_9 c_2^2 w_7 w_4^3 + 12w_8 w_5^2 v_2^2 c_2^2 w_7 w_4^2 - \\
& 12w_8 w_5 w_9 c_4^2 w_7^2 w_4^2 - 36w_8 w_5^2 v_1^2 c_2^2 w_7 w_4^3 + 12w_8 w_5^2 w_9 c_2^2 w_7^2 w_4^2 - 18w_8 w_5 w_9 v_2^2 c_2^2 w_7^2 w_4^3 - 6w_8 w_5^2 w_9 c_4^2 w_7^2 w_4^3 + 36w_8 w_5^2 w_9 v_2^2 c_2^2 w_7 w_4^2 - 6w_5^2 c_4^2 w_7^2 w_4^3 - \\
& 36w_8 w_5^2 w_9 v_2^2 w_7 w_4^2 - 18w_5^2 v_1^2 v_2^2 w_7 w_4^3 - 12w_8 w_5 w_9 v_2^2 w_7^2 w_4^2 - 6w_8 w_5^2 v_2^2 c_2^2 w_7^2 w_4^3 - 12w_5^2 w_9 v_2^2 c_2^2 w_7^2 w_4^2) \frac{\rho}{12w_8 w_5^2 w_9 w_2^2 w_3^2}
\end{aligned}$$

$$C_{\mathrm{D}_x^2 \mathrm{D}_y^2 v_1}^{(1), \text{MRT2}} = C_{\mathrm{D}_x^2 \mathrm{D}_y^2 v_1}^{(1), \text{MRT1}}$$

$$C_{\frac{D_2^2 D_3}{D_2 D_3^2} v_1}^{(1), \text{CLBM1}} = (-36 w_8 w_5^2 v_1^2 w_3^3 + 6 w_8 w_5 w_9 w_7 w_4^3 - 36 w_8 v_1^2 w_9 w_7 w_4^2 + 18 w_8 w_5^2 w_9 c_s^2 w_7 w_4 + 12 w_5^2 c_s^2 w_7 w_4^2 - 72 w_8 w_5^2 v_1^2 w_9 w_4^2 - 12 w_8 w_5^2 w_9 w_3^4 - 24 w_8 w_5 w_9 w_7 w_4^2 + 18 w_8 v_1^2 w_9 w_7 w_4^3 + 12 w_8 w_5 c_s^2 w_3^4 + 36 w_8 w_5^2 v_1^2 w_4^2 + 36 w_8 w_5^2 v_1^2 w_9 w_3^4 + 24 w_8 w_5^2 w_9 w_4^2 - 18 w_8 w_5 v_1^2 w_7 w_4^3 - 6 w_5^2 c_s^2 w_7 w_3^4 - 12 w_8 w_5 w_9 c_s^2 w_7 w_4 - 12 w_8 w_5^2 c_s^2 w_7 w_4^2 + 12 w_8 w_5 w_9 w_7 w_4^2 - 12 w_8 w_5^2 w_9 w_4 + 24 w_8 w_5 w_9 c_s^2 w_7 w_4^2 + 12 w_8 w_5 w_9 c_s^2 w_7 w_4^3 + 6 w_8 w_5 w_7 w_4^3 - 6 w_5 w_9 w_7 w_4^3 - w_8 w_5 w_9 c_s^2 w_7 w_4^3 - 12 w_8 w_5^2 w_9 c_s^2 w_7 + 12 w_8 w_5 w_9 w_7 w_4 + 18 w_5 v_1^2 w_9 w_7 w_4^3 - 6 w_5 w_9 w_7 c_s^2 w_7 w_4^3 - 12 w_8 w_5 w_9 c_s^2 w_7 w_4^3 - 6 w_5 w_9 w_7 w_4^3 + 36 w_8 v_1^2 v_2 w_9 w_4 - 6 w_8 w_5^2 c_s^2 w_7 w_4^3 - 36 w_5 v_1^2 w_9 w_7 w_4^2 - 12 w_8 w_5 w_4^3 - 4 w_8 w_5^2 w_9 c_s^2 w_7 w_4^2 + 12 w_5 w_9 w_7 w_4^2 - 6 w_8 w_5^2 w_7 w_4^3 + 36 w_8 w_5 v_1^2 w_9 w_4^2 - 36 w_8 w_5^2 v_1^2 w_7 w_4^2 - 12 w_5^2 w_7 w_4^3)$$

$$12\omega_8\omega_9c_s^2\omega_7\omega_4^2 + 12\omega_8\omega_5^2\omega_9c_s^2\omega_4^3 + 12\omega_8\omega_5^2\omega_9c_s^2\omega_4 + 6\omega_5^2\omega_7\omega_4^3 + 12\omega_8\omega_5^2\omega_7\omega_4^2 - 36\omega_8\omega_5v_1^2\omega_9\omega_7\omega_4^3 - 36\omega_8\omega_5v_1^2\omega_9\omega_7\omega_4 + 18\omega_8\omega_5^2v_1^2\omega_7\omega_4^3 - 12\omega_8\omega_5^2\omega_4^2 + 6\omega_8\omega_9c_s^2\omega_7\omega_4^3 + 6\omega_5\omega_9c_s^2\omega_7\omega_4^3 + 36\omega_8\omega_5v_1^2\omega_4^3 + 12\omega_8\omega_5c_s^2\omega_4^2 - 24\omega_8\omega_5^2\omega_9c_s^2\omega_4^2 + 36\omega_5^2v_1^2\omega_7\omega_4^2 + 12\omega_8\omega_5\omega_9\omega_4^2 + 72\omega_8\omega_5v_1^2\omega_9\omega_7\omega_4^2 - 12\omega_8\omega_5^2c_s^2\omega_4^3 - 12\omega_5\omega_9c_s^2\omega_7\omega_4^3 - 18\omega_8\omega_5v_1^2\omega_9\omega_7\omega_4^3 - 6\omega_8\omega_5c_s^2\omega_7\omega_4^3 - 18\omega_5^2v_1^2\omega_7\omega_4^3 - 12\omega_8\omega_5\omega_9\omega_4^2 + 12\omega_8\omega_5^2\omega_9c_s^2\omega_4^3) \frac{\rho c_s^2}{12\omega_8\omega_5^2\omega_9\omega_7\omega_4^3}$$

$$C_{D_x^2 D_y^2 v_1}^{(1), \text{CLBM2}} = C_{D_x^2 D_y^2 v_1}^{(1), \text{CLBM1}}$$

$$\begin{aligned} C_{D_x^2 D_y^2 v_1}^{(1), \text{CuLBM1}} &= (12\omega_3^3c_s^2\omega_1^2 - 4\omega_3^2c_s^2\omega_4\omega_1 + 12\omega_3^2c_s^2\omega_4\omega_1^2 - 12c_s^2\omega_4\omega_1^2 + 36v_1^2\omega_3\omega_1^2 - 12\omega_3\omega_1^2 - \omega_3^3c_s^2\omega_4\omega_1^2 - 12\omega_3^2\omega_4\omega_1 + 36v_1^2\omega_3^2\omega_4\omega_1 + \\ 12\omega_3^2c_s^2\omega_1^2 &+ 12\omega_3c_s^2\omega_1^2 - 24\omega_3^3c_s^2\omega_1 + 72v_1^2\omega_3^2\omega_1 + 36v_1^2\omega_3^3 + 12\omega_3\omega_4\omega_1 + 24\omega_3^3\omega_1 + 18\omega_3c_s^2\omega_4\omega_1^2 - 12\omega_3^2 - 72v_1^2\omega_3^2\omega_1^2 - 36v_1^2\omega_3\omega_4\omega_1 + \\ 24\omega_3^2\omega_1^2 &- 24\omega_3^2c_s^2\omega_1^2 - 24\omega_3^2\omega_1 + 24\omega_3^2c_s^2\omega_1 - 12\omega_3c_s^2\omega_4\omega_1 + 72v_1^2\omega_3^2\omega_1 + 12\omega_3^2\omega_4\omega_1^2 - 12\omega_3^2c_s^2\omega_1^2 + 36v_1^2\omega_3^2\omega_1^2 - 36v_1^2\omega_3^2\omega_4\omega_1^2) \frac{\rho c_s^2}{12\omega_3^2\omega_4\omega_1^2} \end{aligned}$$

$$\begin{aligned} C_{D_x^2 D_y^2 v_1}^{(1), \text{CuLBM2}} &= (6\omega_2\omega_3c_s^4\omega_1^3 + 45\omega_2^3v_1^2\omega_3^2\omega_1 + 18\omega_2^3v_2^2\omega_3c_s^2\omega_1 + 30\omega_2^2\omega_3^2c_s^4\omega_1 + 36\omega_2^3v_2^4\omega_1^2 - 30\omega_2^3\omega_3c_s^2\omega_1 + 18\omega_2v_2^2\omega_3\omega_1^3 - 45\omega_2^2v_2^2\omega_3^2c_s^2\omega_1 - \\ 81v_2^2\omega_3^2\omega_1^3 &- 60\omega_2^2\omega_3c_s^2\omega_1^2 - 10\omega_2^3\omega_2^2c_s^4\omega_1^2 - 18\omega_2^3v_1^2\omega_3^2\omega_1^2 + 126\omega_2^3v_1^2\omega_3c_s^2\omega_1 + 45\omega_2^2v_2^2\omega_3^2\omega_1^3 + 135v_1^2v_2^2\omega_3^2\omega_1^3 + 18\omega_2^3c_s^4\omega_1^3 + 36\omega_2^2v_2^2\omega_3^2\omega_1^3 + \\ 9\omega_2^2v_2^2\omega_3^2\omega_1^3 &+ 9\omega_2^3v_2^2\omega_3^2\omega_1^2 + 36\omega_2^2\omega_3c_s^2\omega_1^2 - 2\omega_2^3\omega_3^2c_s^4\omega_1^2 - 90\omega_2^2v_1^2\omega_3^2\omega_1^2 - 18\omega_2^3v_1^2\omega_3^2\omega_1^2 - 6\omega_2^3\omega_3^2\omega_1^2 + 36\omega_2v_2^2\omega_3^2c_s^2\omega_1^2 - 6\omega_2^2v_2^2\omega_3^2\omega_1^2 - 36\omega_2^2v_2^2\omega_3^2\omega_1^2 + \\ 72\omega_2^2v_2^2\omega_3^2c_s^2\omega_1^2 &- 18\omega_2v_2^2\omega_3^2\omega_1^2 + 30\omega_2^2v_1^2\omega_3^2c_s^2\omega_1^2 - 81\omega_2v_2^2\omega_3^2c_s^2\omega_1^2 - 9\omega_2v_2^2\omega_3^2c_s^2\omega_1^2 + 24\omega_2^3\omega_3c_s^2\omega_1^2 + 18\omega_2v_1^2\omega_3^2c_s^2\omega_1^2 + 36v_1^2\omega_3^2\omega_1^2 + 2\omega_2^2v_3^2\omega_1^2 - \\ 90\omega_2^2v_2^2\omega_3^2\omega_1^2 &- 198\omega_2^3v_1^2\omega_3^2c_s^2\omega_1^2 + 6\omega_2^3v_1^2\omega_3^2c_s^2\omega_1^2 + 108\omega_2^2v_2^2\omega_3^2c_s^2\omega_1^2 + 45\omega_2^3\omega_3c_s^2\omega_1^2 - 6\omega_2\omega_2^2\omega_3^2\omega_1^2 - 135\omega_2^3v_1^2\omega_3^2\omega_1^2 + 21\omega_2\omega_3^2c_s^2\omega_1^2 - \\ 9\omega_2^3v_2^4\omega_1^2 &- 72\omega_2^3v_1^2\omega_3^2c_s^2\omega_1^2 - 9\omega_2^3v_2^2\omega_3^2\omega_1^2 - 144\omega_2^3v_2^2\omega_3c_s^2\omega_1^2 + 30\omega_2^2\omega_3c_s^2\omega_1^2 + 18\omega_2v_2^2\omega_3^2\omega_1^2 - 216\omega_2v_2^2\omega_3c_s^2\omega_1^2 - \\ 54\omega_2v_2^2\omega_3^2c_s^2\omega_1^2 &- 30\omega_2^3v_1^2v_2^2\omega_3^2\omega_1^2 + 135\omega_2^3v_1^2v_2^2\omega_3^2\omega_1^2 + 189\omega_2^2v_3^2\omega_3^2\omega_1^2 + 144\omega_2v_1^2\omega_3c_s^2\omega_1^2 + 144\omega_2v_2^2\omega_3^2c_s^2\omega_1^2 + 12\omega_2\omega_3^2\omega_1^2 - 18v_1^2\omega_3^2\omega_1^2 + \\ 108\omega_2^3v_2^2\omega_3^2\omega_1^2 &- 54\omega_2^2v_2^2\omega_3\omega_1^2 - 36\omega_2^2v_2^2\omega_3\omega_1^3 - 24\omega_3^2c_s^2\omega_1^3 - 6\omega_2^3v_2^2\omega_3^2c_s^2\omega_1^2 + 60\omega_2^3\omega_3^2c_s^2\omega_1^2 + 144\omega_2v_2^2\omega_3^2c_s^2\omega_1^2 + 36\omega_2^2\omega_3^2\omega_1^2 - 18v_2^2\omega_3^2c_s^2\omega_1^2 - \\ 6\omega_2\omega_3c_s^2\omega_1^3 &- 18\omega_2^3v_2^2\omega_3\omega_1^2 + \omega_2^3v_1^2\omega_3^2\omega_1^2 + \omega_2^3v_2^4\omega_3^2\omega_1^2 + 30\omega_2^3\omega_3c_s^4\omega_1^2 - 45\omega_2v_2^2\omega_3^2\omega_1^2 - 63\omega_2^3v_2^2\omega_3^2c_s^2\omega_1^2 - 36\omega_2^3v_2^2\omega_3^2\omega_1^2 + 36\omega_2^2v_2^2\omega_3^2c_s^2\omega_1^2 + \\ 6\omega_2\omega_3c_s^2\omega_1^3 &- 18\omega_2^3v_2^2\omega_3\omega_1^2 + 90\omega_2^2v_1^2\omega_3c_s^2\omega_1^2 + 36\omega_2^2v_2^2\omega_3c_s^2\omega_1^2 - 54\omega_2v_2^2\omega_3c_s^2\omega_1^2 + 2\omega_2^3\omega_3^2c_s^2\omega_1^2 + 60\omega_2^3\omega_3c_s^2\omega_1^2 - 30\omega_2^3v_2^2\omega_3^2c_s^2\omega_1^2 + 18\omega_2^2v_1^2\omega_3^2\omega_1^2 - \\ 18v_1^2\omega_3^2\omega_1^2 &- 90\omega_2^2v_1^2\omega_3c_s^2\omega_1^2 + 135\omega_2^2v_1^2\omega_3c_s^2\omega_1^2 + 45\omega_2v_2^2\omega_3^2\omega_1^2 - 108\omega_2^2v_2^2\omega_3^2\omega_1^2 - 3\omega_2^3\omega_3^2c_s^2\omega_1^2 + 108\omega_2^3v_2^2\omega_3^2c_s^2\omega_1^2 - 15\omega_2\omega_3^2c_s^4\omega_1^2 - 36\omega_2^2v_1^2\omega_3^2\omega_1^2 + 270\omega_2^2v_1^2\omega_3^2\omega_1^2 - \\ 135\omega_2v_1^2\omega_3^2\omega_1^2 &+ 18\omega_2^3v_1^2\omega_3^2\omega_1^2 + 54\omega_2^3\omega_3^2\omega_3\omega_1^2 + 54\omega_2^2v_2^2\omega_3\omega_1^2 - 42\omega_2^2\omega_3^2c_s^2\omega_1^2 - 60\omega_2^3\omega_3c_s^2\omega_1^2 - \omega_2^2v_1^2\omega_3^2\omega_1^2 - 30\omega_2\omega_3^2c_s^4\omega_1^2 + 18\omega_2^3v_4^2\omega_3\omega_1^2 + \\ 9\omega_2^3v_1^2\omega_3^2c_s^2\omega_1^2 &- \omega_2^2v_1^2\omega_3^2\omega_1^2 + 6\omega_2^3\omega_3^2\omega_1^2 + 90\omega_2v_2^2\omega_3^2\omega_1^2 - 9\omega_2^3v_2^2\omega_3^2\omega_1^2 - 2\omega_2\omega_3^2c_s^2\omega_1^2 + 24\omega_2^3\omega_3c_s^4\omega_1^2 - 6\omega_2^2\omega_3^2\omega_1^2 - 135\omega_2v_1^2v_2^2\omega_3^2\omega_1^2) \frac{\rho c_s^2}{24\omega_2^3\omega_3^2\omega_1^2} \end{aligned}$$

coefficient  $C_{D_x^2 D_y^2 v_2}^{(1)}$  at  $\frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2}$ :

$$C_{D_x^2 D_y^2 v_2}^{(1), \text{SRT}} = 0$$

$$\begin{aligned} C_{D_x^2 D_y^2 v_2}^{(1), \text{MRT1}} &= (-16\omega_8^2\omega_3^3\omega_9\omega_6c_s^2\omega_7^2 - 4\omega_8^2\omega_5^2\omega_6\omega_7\omega_4^3 - \omega_8^2\omega_5^3v_1^2\omega_9\omega_6\omega_6^2\omega_4^2 - 24\omega_8\omega_5^3\omega_9\omega_6c_s^2\omega_7\omega_4^2 - 5\omega_8^2\omega_5^3\omega_9\omega_6c_s^2\omega_7\omega_4^3 + 4\omega_8^2\omega_5^2v_1^2\omega_6\omega_7\omega_4^3 - \\ 6\omega_8^2\omega_5^2\omega_6c_s^2\omega_7\omega_4^2 &- 8\omega_8^2\omega_5^2\omega_6\omega_6c_s^2\omega_7\omega_4^2 - 2\omega_8^2\omega_5^2\omega_9\omega_6\omega_6c_s^2\omega_7\omega_4^2 + 4\omega_8\omega_5^2\omega_6c_s^2\omega_7\omega_4^2 - 4\omega_8^2\omega_5^3v_1^2\omega_6\omega_7\omega_4^2 + 2\omega_8\omega_5^3\omega_9\omega_6c_s^2\omega_7\omega_4^2 - \\ 4\omega_8\omega_5v_1^2\omega_9\omega_6\omega_7\omega_4^3 &- 4\omega_8^2\omega_5^3v_1^2\omega_9\omega_6\omega_4^2 + 2\omega_8\omega_5^2\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 11\omega_8\omega_5^2\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 8\omega_8\omega_5^3\omega_9\omega_6c_s^2\omega_7\omega_4^2 + \\ 12\omega_8^2\omega_9\omega_6c_s^2\omega_7\omega_4^3 &- 15\omega_8^2\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 4\omega_8^2\omega_5^3v_1^2\omega_9\omega_6\omega_4^2 + 2\omega_8^2\omega_5^3\omega_9\omega_6\omega_7\omega_4^2 - 2\omega_8\omega_5^3\omega_9\omega_6c_s^2\omega_7\omega_4^2 - \omega_8\omega_5^3\omega_9\omega_6\omega_7\omega_4^3 - 4\omega_8\omega_9\omega_6\omega_7\omega_4^3 - \\ 4\omega_8^2\omega_5^3\omega_6\omega_7\omega_4^3 &+ 2\omega_8^2\omega_5^2\omega_9\omega_6\omega_7\omega_4^2 + 4\omega_8\omega_5^3v_1^2\omega_9\omega_6\omega_7\omega_4^2 - 4\omega_8\omega_5^3\omega_9\omega_6\omega_7\omega_4^2 + 2\omega_8\omega_5^3\omega_6c_s^2\omega_7\omega_4^2 - 2\omega_8\omega_5^3v_1^2\omega_6\omega_7\omega_4^2 - 2\omega_8\omega_5^2\omega_9\omega_6\omega_7\omega_4^2 + \\ 3\omega_8^2\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4^3 &- 6\omega_8^2\omega_5^2\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 4\omega_8^2\omega_5\omega_9\omega_6\omega_7\omega_4^2 + 4\omega_8^2\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4^2 - 2\omega_8\omega_5^2\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 4\omega_8\omega_5^3\omega_6c_s^2\omega_7\omega_4^2 - \\ 7\omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^3 &- 2\omega_8^3\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 2\omega_8^2\omega_5^3v_1^2\omega_9\omega_6\omega_7\omega_4^2 + 12\omega_8\omega_5^3v_1^2\omega_9\omega_6c_s^2\omega_7\omega_4^2 - \omega_8\omega_5^2\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 4\omega_8\omega_5^2v_1^2\omega_6\omega_7\omega_4^2 - \\ 4\omega_8^2\omega_5^2\omega_6c_s^2\omega_7\omega_4^2 &+ 7\omega_8^2\omega_5^2v_1^2\omega_9\omega_6\omega_7\omega_4^2 + \omega_8^2\omega_5^3v_1\omega_9\omega_6\omega_7\omega_4^2 - \omega_8^2\omega_5^2\omega_9\omega_6\omega_7\omega_4^2 - 2\omega_8^2\omega_5^3\omega_6c_s^2\omega_7\omega_4^2 - 4\omega_8^2\omega_5^3\omega_9\omega_6\omega_4^3 - \\ 4\omega_8\omega_5^2\omega_6c_s^2\omega_7\omega_4^2 &- 6\omega_8\omega_5^2v_1^2\omega_9\omega_6\omega_7\omega_4^2 - 2\omega_8\omega_5^2\omega_5\omega_9\omega_6\omega_7\omega_4^2 + 2\omega_8\omega_5^2v_1^2\omega_9\omega_6\omega_7\omega_4^2 - 9\omega_8\omega_5^2\omega_9\omega_6\omega_7\omega_4^2 + 4\omega_8\omega_5^2\omega_6c_s^2\omega_7\omega_4^2 + 2\omega_8\omega_5^2\omega_9\omega_6c_s^2\omega_7\omega_4^2 - \\ 4\omega_8\omega_5^2\omega_9\omega_6c_s^2\omega_7\omega_4^2 &+ 4\omega_8\omega_5^2\omega_9\omega_6\omega_7\omega_4^2 + 3\omega_8\omega_5^2\omega_9\omega_6\omega_7\omega_4^2 + 4\omega_8\omega_5^2\omega_9\omega_6\omega_7\omega_4^2 + 8\omega_8\omega_5^2\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 13\omega_8\omega_5^2\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 3\omega_8\omega_5^2\omega_9\omega_6\omega_7\omega_4^2 - \\ 3\omega_8^2\omega_5^2v_1^2\omega_9\omega_6\omega_7\omega_4^2 &- 4\omega_8^2\omega_5^2\omega_6c_s^2\omega_7\omega_4^2 + 2\omega_8\omega_5^2\omega_6\omega_7\omega_4^2 + 5\omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^2 + 26\omega_8\omega_5^2\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 2\omega_8^2\omega_5^2v_1^2\omega_9\omega_6\omega_7\omega_4^2 - \\ 3\omega_8^2\omega_5^2v_1^2\omega_9\omega_6\omega_7\omega_4^2 &- 2\omega_8\omega_5^2v_1^2\omega_9\omega_6\omega_7\omega_4^2 - 8\omega_8^2\omega_5^2\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 2\omega_8^2\omega_5^2v_1^2\omega_9\omega_6\omega_7\omega_4^2 - 4\omega_8^2\omega_5^2v_1^2\omega_6\omega_7\omega_4^2 - 2\omega_8^2\omega_5^2\omega_9\omega_6\omega_7\omega_4^2 - \\ 8\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4^2 &+ 8\omega_8^2\omega_5^2\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 4\omega_8^2\omega_5^2\omega_9\omega_6\omega_7\omega_4^2 + \omega_8\omega_5^2v_1^2\omega_9\omega_6\omega_7\omega_4^2 + 9\omega_8\omega_5^2v_1^2\omega_9\omega_6\omega_7\omega_4^2 + 4\omega_8^2v_1^2\omega_9\omega_6\omega_7\omega_4^2 + 2\omega_8^2\omega_5^2\omega_6\omega_7\omega_4^2 - \\ 4\omega_8^2\omega_5^2v_1^2\omega_9\omega_6\omega_4^2 &+ 6\omega_8^2\omega_5^2\omega_9\omega_6\omega_7\omega_4^2 - 2\omega_8\omega_5^2v_1^2\omega_6\omega_7\omega_4^2 - 8\omega_8\omega_5^2v_1^2\omega_9\omega_6c_s^2\omega_4^2 - 5\omega_8\omega_5^2v_1^2\omega_9\omega_6\omega_7\omega_4^2 + 4\omega_8\omega_5^2v_1^2\omega_6\omega_7\omega_4^2) \frac{\rho v_2}{2\omega_8^2\omega_5^2\omega_9\omega_6\omega_7\omega_4^2} \end{aligned}$$

$$C_{D_x^2 D_y^2 v_2}^{(1), \text{MRT2}} = C_{D_x^2 D_y^2 v_2}^{(1), \text{MRT1}}$$

$$C_{D_x^2 D_y^2 v_2}^{(1), \text{CLBM1}} = 0$$

$$C_{D_x^2 D_y^2 v_2}^{(1), \text{CLBM2}} = 0$$

$$C_{D_x^2 D_y^2 v_2}^{(1), \text{CuLBM1}} = 0$$

$$\begin{aligned} C_{D_x^2 D_y^2 v_2}^{(1), \text{CuLBM2}} &= (135\omega_3^2\omega_3c_s^2\omega_1^2 - 216\omega_2v_2^2\omega_3\omega_1^2 - 18\omega_2^2\omega_3\omega_1^2 - 18\omega_2^3v_1^2\omega_1^2 + 162\omega_2^2\omega_3c_s^2\omega_1^2 - 162\omega_2v_2^2\omega_3\omega_1^2 + 54\omega_2^2v_1^2\omega_3\omega_1^2 + 108\omega_2^2c_s^2\omega_1^2 - \\ 18\omega_2v_1^2\omega_3^3 &- 27\omega_3^2v_1^2\omega_3\omega_1^2 + 18\omega_2\omega_3^3 + 84\omega_2^2\omega_3c_s^2\omega_1^2 - 54\omega_2^2\omega_3c_s^2 - 46\omega_2^2\omega_3\omega_1^2 + 36\omega_2^3v_1^2\omega_3\omega_1^2 + 270\omega_3c_s^2\omega_1^2 - 54\omega_2^2\omega_3\omega_1^2 + \\ 36v_1^2\omega_3\omega_1^2 &- 54\omega_2^2\omega_3c_s^2\omega_1^2 - 84\omega_2^3\omega_3c_s^2\omega_1^2 - 126\omega_3\omega_1^2 + 135\omega_2\omega_3c_s^2\omega_1^2 - 162\omega_2\omega_3c_s^2\omega_1^2 + 100\omega_2^2v_2^2\omega_3\omega_1^2 - 198\omega_2^3v_2^2\omega_3\omega_1^2 - 27\omega_2v_1^2\omega_3\omega_1^2 + 54\omega_2^3\omega_3 - \\ 297\omega_2\omega_3c_s^2\omega_1^2 &+ 216\omega_2^3v_2^2\omega_3\omega_1^2 - 81\omega_2^3\omega_3\omega_1^2 + 90\omega_2\omega_3\omega_1^2 - 36\omega_2v_1^2\omega_3\omega_1^2 + 162\omega_2^2v_2^2\omega_3\omega_1^2 - 100\omega_2^3v_2^2\omega_3\omega_1^2 + 46\omega_2^3\omega_3\omega_1^2 + 36\omega_2^2v_1^2\omega_1^2 \end{aligned}$$

$$54\omega_2c_s^2\omega_1^3 + 18\omega_2^3\omega_1 - 36\omega_2^2\omega_1^2 - 54\omega_2^3c_s^2\omega_1 + 198v_2^2\omega_3\omega_1^3) \frac{v_1\rho v_2}{24\omega_2^3\omega_3\omega_1^3}$$

coefficient  $C_{D_x D_y^3 \rho}^{(1)}$  at  $\frac{\partial^4 \rho}{\partial x_1 \partial x_2^3}$ :

$$C_{\text{D}_x \text{D}_y^3 \rho}^{(1), \text{SRT}} = (24 - 42c_s^2\omega^2 + v_2^2\omega^3 - 14v_2^2\omega^2 + 3c_s^2\omega^3 + 36v_2^2\omega - 72c_s^2 + 108c_s^2\omega - 24v_2^2 - 36\omega + 14\omega^2 - \omega^3) \frac{v_2 c_s^2}{12w^3}$$

$$\begin{aligned}
C_{D_x D_y}^{(1),MRT1} = & (6w_8^2 v_1^2 w_9 w_6 w_7 w_4^3 - 12w_8 w_5 w_9 v_2^2 w_6^2 c_s^2 w_7 w_4 + 72w_8^2 w_5 v_1^2 w_9 w_6 c_s^2 w_7 w_4 - 72w_8 w_5 v_1^2 w_9 w_6^2 c_s^2 w_7 w_4 + 3w_8^2 w_5 w_9 w_6^2 c_s^4 w_7 w_4^3 + \\
& 24w_8^2 v_1^2 w_9^2 w_6^2 w_7 w_4^2 - 24w_8 v_1^2 w_9 v_2^2 w_6^2 w_7 w_4^2 - 12w_8^2 w_5 w_9 w_6^2 c_s^2 w_7 w_4 - 18w_8^2 w_6^4 c_s^4 w_7 w_4^3 + 9w_8^2 w_5 v_1^2 w_9 w_6 w_7 w_4^3 - 36w_8^2 w_9 w_6 c_s^4 w_4^3 - \\
& 12w_8^2 w_5 w_9 w_6^2 c_s^2 w_7 w_4 - 6w_8^2 w_5 v_1^2 w_9 v_2^2 w_7 w_4^3 - 6w_8 w_5 v_2^2 w_6^2 c_s^2 w_7 w_4^3 - 72w_8 v_1^2 w_9 w_6^2 c_s^2 w_7 w_4^3 + 12w_8^2 w_5 w_9 w_6 c_s^2 w_4^2 - 24w_8 w_5 v_2^2 w_9 w_6^2 w_7 w_4^2 + \\
& 12w_8^2 v_1^2 v_2^2 w_6^2 w_7 w_4^3 + 36w_8 v_1^2 w_2^2 c_s^2 w_4^3 - 9w_8^2 w_5 v_1^2 w_9 w_6 w_7 w_4^3 - 12w_8^2 w_5 w_6^2 c_s^2 w_4^2 + 15w_8 w_5 v_1^2 w_9 w_6^2 w_7 w_4^3 + 12w_8 v_1^2 w_9 v_2^2 w_6^2 w_7 w_4^3 - \\
& 6w_8^2 v_1^2 w_9 v_2^2 w_6 w_7 w_4^3 - 60w_8^2 w_5 w_9 w_6^2 c_s^4 w_7 w_4^2 - 24w_8^2 v_1^2 w_9 w_6 w_7 w_4^2 - 48w_8 w_5 v_1^2 w_9 w_6^2 w_7 w_4^2 + 12w_8^2 v_2^2 w_6^2 c_s^2 w_4^3 + 36w_8 w_5 v_1^2 w_9 w_6 w_7 w_4^2 + \\
& 12w_8^2 w_5 w_6^2 c_s^2 w_4^3 + 36w_8 v_1^2 w_9 w_6^2 c_s^2 w_7 w_4^3 + 12w_8^2 w_5 v_1^2 w_9 v_2^2 w_7 w_4^2 + 12w_8 w_5 v_2^2 w_6^2 s w_7 w_4^2 - 36w_8^2 w_5 v_1^2 w_9 v_2^2 w_6 w_7 w_4^2 - 12w_8^2 w_5 w_9 w_6 c_s^2 w_4^3 + \\
& 24w_8 w_5 v_2^2 w_9 w_6^2 w_7 w_4 + 72w_8^2 v_1^2 w_9 w_6 c_s^2 w_7 w_4^2 - 6w_5 v_2^2 w_9 w_6^2 w_7 w_4^3 - 24w_8^2 w_5 v_1^2 w_9 w_6 w_7 w_4 - 12w_8^2 v_1^2 w_9 v_2^2 w_6 w_7 w_4^3 - 18w_8^2 w_5 v_1^2 w_9 c_s^2 w_7 w_4^3 - \\
& 18w_8 w_5 v_1^2 w_2^2 c_s^2 w_7 w_4^3 - 15w_8 w_5 v_1^2 w_9 v_2^2 w_6^2 w_7 w_4^3 + 6w_8^2 w_6^2 c_s^2 w_7 w_4^3 + 24w_8^2 w_5 v_1^2 w_9 v_2^2 w_6 w_7 w_4 - w_8^2 w_5 w_9 w_6^2 c_s^2 w_7 w_4^3 - 5w_8^2 w_5 w_9 v_2^2 w_6 s w_7 w_4^3 + \\
& 36w_8^2 w_6 c_s^4 w_4^3 + 15w_8^2 w_5 w_9 w_6^2 c_s^4 w_7 w_4 - 36w_8 w_5 v_1^2 w_9 w_6 c_s^2 w_4^3 + 12w_8^2 v_1^2 w_9 w_6 w_7 w_4^3 - 45w_8 w_5 v_1^2 w_9 w_6 c_s^2 w_7 w_4^3 + \\
& 27w_8^2 w_5 v_1^2 w_9 w_6 c_s^2 w_7 w_4^3 + 12w_8^2 w_5 v_1^2 w_9 v_2^2 w_6 w_4^3 - 3w_8 w_5 v_1^2 w_9 w_6^2 c_s^2 w_7 w_4^3 - 36w_8^2 v_1^2 w_9 w_6 c_s^2 w_4^3 + 12w_8^2 w_9 w_6^2 w_7 w_4^3 + 6w_8^2 w_9 v_2^2 w_6 c_s^2 w_7 w_4^3 + \\
& 48w_8 w_5 v_2^2 w_9 w_6^2 w_7 w_4^2 + 36w_8 w_5 v_1^2 w_9 w_6^2 c_s^2 w_7 w_4^2 - 12w_8^2 v_1^2 w_6 w_4^3 + 12w_8 v_1^2 w_9 w_6^2 w_7 w_4^3 + 36w_8^2 w_5 v_2^2 w_9 c_s^2 w_7 w_4^2 - 18w_8^2 v_1^2 w_9 w_6 c_s^2 w_7 w_4^3 - \\
& 12w_8^2 w_5 v_1^2 w_9 v_2^2 w_6 w_4^2 + 18w_8 w_5 v_9 w_6^2 c_s^2 w_7 w_4^2 - 108w_8^2 w_5 v_1^2 w_9 w_6 c_s^2 w_7 w_4^2 + 36w_8^2 w_5 v_1^2 w_9 w_6 c_s^2 w_4^3 + 6w_8^2 v_1^2 w_6 w_7 w_4^3 + 144w_8 w_5 v_1^2 w_9 w_6^2 c_s^2 w_7 w_4^2 + \\
& 12w_8 w_5 v_1^2 v_2^2 w_6^2 w_7 w_4^2 + 18w_8^2 w_5 v_9 w_6^2 c_s^2 w_7 w_4^2 + 18w_8^2 w_5 v_1^2 w_6^2 c_s^2 w_7 w_4^3 + 18w_8^2 w_5 w_6^2 c_s^4 w_7 w_4^3 - 12w_8^2 w_5 v_1^2 w_6^2 w_4^2 + 3w_8 w_5 w_9 w_6^2 c_s^2 w_7 w_4^3 + \\
& 24w_8 v_1^2 w_9 w_6^2 w_7 w_4^2 - 6w_8^2 v_1^2 w_6^2 w_7 w_4^3 + 6w_8 w_5 v_1^2 w_9 v_2^2 w_6 w_7 w_4^3 + 18w_8^2 w_9 w_6 c_s^4 w_7 w_4^3 + 12w_8^2 w_5 w_9 w_6^2 c_s^2 w_7 w_4^3 - 12w_8 w_5 w_6^2 c_s^2 w_7 w_4^3 + \\
& 6w_8 w_5 v_1^2 w_6^2 w_7 w_4^3 - 15w_8^2 w_5 w_9 w_6 c_s^4 w_7 w_4^3 + 6w_8^2 w_5 v_1^2 w_9 w_7 w_4^3 - 36w_8^2 w_5 w_6^2 c_s^4 w_4^3 - 36w_8^2 w_5 v_1^2 w_6^2 c_s^2 w_4^3 + 12w_8^2 w_5 v_2^2 w_6^2 c_s^2 w_4^3 + \\
& 12w_8^2 w_5 w_9 v_2^2 w_6^2 c_s^2 w_7 w_4 - 6w_5 w_9 w_6^2 c_s^4 w_7 w_4^3 - 18w_8^2 v_1^2 w_9 w_6^2 c_s^2 w_7 w_4^3 - 12w_8^2 w_5 v_2^2 w_6^2 w_4^3 + 36w_8^2 w_5 w_9 w_6 c_s^4 w_4^3 + 6w_8^2 w_5 v_1^2 w_6^2 w_7 w_4^3 + \\
& 12w_8 w_5 w_9 w_6^2 c_s^4 w_7 w_4^2 + 6w_8 w_5 w_6^2 c_s^2 w_7 w_4^3 - 12w_8 v_1^2 w_9 w_6^2 w_7 w_4^3 - 18w_8 w_5 w_9 w_6^2 c_s^2 w_7 w_4^3 - 12w_8 w_5 v_1^2 w_9 v_2^2 w_6 w_7 w_4^3 - 36w_8^2 w_5 v_1^2 w_6^2 c_s^2 w_7 w_4^3 + \\
& 36w_8^2 w_5 w_6^2 c_s^4 w_7 w_4^2 + 12w_8^2 w_5 v_1^2 w_6^2 w_4^3 - 12w_8^2 w_5 v_2^2 w_6^2 c_s^2 w_4^3 + 36w_8^2 w_5 w_6^2 c_s^4 w_4^2 - 12w_8^2 w_5 w_9 w_6^2 c_s^4 w_4^3 + 12w_8^2 w_5 w_6^2 c_s^2 w_4^3 + 12w_8^2 w_5 w_9 w_6^2 c_s^4 w_4^3 + 12w_8^2 w_5 v_1^2 w_6^2 w_7 w_4^3 + \\
& 36w_8^2 w_5 v_1^2 w_9 w_6^2 c_s^4 w_7 w_4^3 - 12w_8 w_5 v_2^2 w_6^2 c_s^2 w_7 w_4^3 - 6w_8^2 w_5 w_6^2 c_s^2 w_7 w_4^3 + w_8^2 w_5 w_9 v_2^2 w_6^2 c_s^2 w_7 w_4^3 - 36w_5 v_1^2 w_9 w_6^2 c_s^2 w_7 w_4^3 + 36w_8 w_5 v_2^2 w_6^2 c_s^2 w_7 w_4^3 + \\
& 36w_8^2 w_5 w_9 w_6 c_s^4 w_7 w_4^2 - 6w_8^2 w_5 w_6 c_s^2 w_7 w_4^3 + 18w_8 w_5 v_1^2 w_9 w_6^2 c_s^2 w_7 w_4^3 - 6w_8 w_5 v_1^2 w_9 w_6 w_7 w_4^3 - 12w_8^2 w_5 w_9 v_2^2 w_6^2 c_s^2 w_4^2 + 5w_8^2 w_5 w_9 w_6 c_s^2 w_7 w_4^3 - \\
& 12w_8 v_1^2 w_9 v_2^2 w_6^2 w_7 w_4^2 - 12w_8^2 w_5 v_1^2 w_9 w_6 w_4^3 + 12w_8^2 w_5 v_1^2 w_6^2 w_7 w_4^2 + 12w_8 w_5 w_9 w_6^2 c_s^2 w_7 w_4^3 + 6w_8^2 w_5 v_2^2 w_6^2 c_s^2 w_7 w_4^3 - 6w_5 w_9 w_6^2 c_s^2 w_7 w_4^3 - \\
& 36w_8 w_5 v_1^2 w_9 w_6 c_s^2 w_7 w_4^2 - 12w_8^2 w_5 w_9 v_2^2 w_6^2 c_s^2 w_7 w_4^3 - 12w_8^2 w_9 v_2^2 w_6^2 c_s^2 w_4^3 - 18w_8 w_5 w_6^2 c_s^4 w_7 w_4^3 + 18w_5 v_1^2 w_9 w_6^2 c_s^2 w_7 w_4^3 + 12w_8^2 w_5 w_6^2 c_s^2 w_7 w_4^2 - \\
& 42w_8 w_5 w_9 w_6^2 c_s^4 w_7 w_4^2 + 6w_5 w_9 v_2^2 w_6^2 c_s^2 w_7 w_4^3 - 12w_8^2 w_5 w_9 w_6^2 c_s^2 w_7 w_4^2 - 96w_8^2 w_5 w_9 w_6^2 c_s^4 w_7 + 12w_8^2 w_5 v_1^2 w_9 w_6 w_4^2 - 12w_8^2 w_5 v_2^2 w_6^2 c_s^2 w_7 w_4^2 - \\
& 6w_8^2 w_5 v_1^2 w_6^2 w_7 w_4^3 + 6w_5 v_1^2 w_9 v_2^2 w_6^2 w_7 w_4^3 - 18w_8^2 w_5 w_9 w_6 c_s^2 w_7 w_4^3 + 12w_8^2 w_5 w_9 v_2^2 w_6^2 c_s^2 w_4^3 + 12w_8 w_5 v_1^2 w_9 w_6 w_7 w_4^3) \frac{v_2}{12w_8^2 w_5 w_9 w_6^2 c_s^2 w_7 w_4^3}
\end{aligned}$$

$$C_{\mathrm{D}_x \mathrm{D}_y^3 \rho}^{(1), \text{MRT2}} = C_{\mathrm{D}_x \mathrm{D}_y^3 \rho}^{(1), \text{MRT1}}$$

$$\begin{aligned}
C_{\text{D}_x \text{D}_y}^{(1), \text{CLBM1}} = & (-36w_8^2 w_5 w_6 c_s^2 w_7 w_4 + 12w_8^2 w_5 w_9 w_6 w_7 w_4 + 18w_8^2 w_5 w_9 v_2^2 w_7 w_4 + 18w_8 w_5 w_9 v_2^2 w_6 w_7 w_4 - 36w_8^2 w_5 w_9 c_s^2 w_4 + 12w_8^2 w_5 w_9 w_4 + \\
& 36w_8^2 w_6 c_s^2 w_4^2 + 3w_8 w_5 w_9 w_6 w_7 w_4^2 - 18w_8 w_5 w_6 c_s^2 w_7 w_4^2 + 5w_8^2 w_5 w_9 w_7 w_4^2 - 36w_5 w_9 w_6 c_s^2 w_7 w_4 - 12w_8^2 w_5 w_9 v_2^2 w_6 w_7 w_4^2 - \\
& 15w_8^2 w_5 w_9 c_s^2 w_7 w_4^2 + 12w_8 w_5 v_2^2 w_6 w_7 w_4 - 9w_8 w_5 w_9 w_6 c_s^2 w_7 w_4^2 + 12w_8^2 v_2^2 w_6 w_4^2 + 6w_8^2 w_5 v_2^2 w_6 w_7 w_4^2 + 54w_8 w_5 w_9 w_6 c_s^2 w_7 w_4 - 12w_8^2 w_5 v_2^2 w_6 w_7 w_4 \\
& 12w_5 w_9 v_2^2 w_6 w_7 w_4 + 12w_8^2 w_5 w_9 v_2^2 w_4^2 - 12w_8^2 w_6 w_4^2 - 6w_8 w_5 v_2^2 w_6 w_7 w_4^2 + 54w_8^2 w_5 w_9 c_s^2 w_7 w_4 - 12w_8^2 w_5 w_9 w_4^2 + 36w_8 w_5 w_6 c_s^2 w_7 w_4 - \\
& 18w_8 w_5 w_9 w_6 w_7 w_4 + 18w_8 w_5 w_9 w_6 c_s^2 w_7 w_4^2 - 18w_8^2 w_5 w_9 w_7 w_4 - w_8^2 w_5 w_9 w_6 w_7 w_4^2 + 18w_8^2 w_5 w_6 c_s^2 w_7 w_4^2 + 6w_8^2 w_6 w_7 w_4^2 + 36w_8^2 w_5 w_9 c_s^2 w_4^2 - \\
& 3w_8 w_5 w_9 v_2^2 w_6 w_7 w_4^2 - 5w_8^2 w_5 w_9 v_2^2 w_7 w_4^2 + 12w_8^2 w_9 w_4^2 + 12w_8^2 w_5 v_2^2 w_6 w_4 + 6w_8 w_5 w_6 w_7 w_4^2 - 36w_8^2 w_5 w_9 w_6 c_s^2 w_7 w_4 - 12w_8^2 w_9 v_2^2 w_4^2 + \\
& 36w_8^2 w_5 w_9 w_6 c_s^2 w_7 + 18w_8^2 w_9 c_s^2 w_7 w_4^2 + 36w_8^2 w_5 w_6 c_s^2 w_4 + w_8^2 w_5 w_9 v_2^2 w_6 w_7 w_4^2 + 12w_8^2 w_5 w_6 w_4^2 - 6w_8^2 w_5 w_6 w_7 w_4^2 - 6w_8^2 v_2^2 w_6 w_7 w_4^2 - \\
& 6w_5 w_9 w_6 w_7 w_4^2 + 12w_8^2 w_5 w_9 v_2^2 w_6 w_7 - 36w_8^2 w_9 c_s^2 w_4^2 - 12w_8^2 w_5 w_9 v_2^2 w_6 w_7 w_4 + 12w_8^2 w_5 w_6 w_7 w_4 - 12w_8^2 w_5 w_6 w_4 + 12w_5 w_9 w_6 w_7 w_4 - \\
& 36w_8^2 w_5 w_6 c_s^2 w_4^2 - 12w_8^2 w_5 w_9 v_2^2 w_7 - 12w_8 w_5 w_9 v_2^2 w_6 w_7 - 6w_8^2 w_9 w_7 w_4^2 - 12w_8^2 w_5 w_9 w_6 w_7 + 12w_8^2 w_5 w_9 w_6 w_7 - 36w_8^2 w_5 w_9 c_s^2 w_4^2 \\
& 12w_8^2 w_5 v_2^2 w_6 w_4^2 - 12w_8 w_5 w_6 w_7 w_4 + 6w_8^2 w_9 v_2^2 w_7 w_4^2 + 3w_8^2 w_5 w_9 w_6 c_s^2 w_7 w_4^2 - 18w_8^2 w_6 c_s^2 w_7 w_4^2 - 36w_8 w_5 w_9 w_6 c_s^2 w_4^2) \frac{v_2 c_s}{12w_8^2 w_5 w_9 w_6 w_7 w_4^2}
\end{aligned}$$

$$C_{\mathrm{D}_x \mathrm{D}_y^3 \rho}^{(1), \text{CLBM2}} = C_{\mathrm{D}_x \mathrm{D}_y^3 \rho}^{(1), \text{CLBM1}}$$

$$\begin{aligned} C_{\substack{\text{D}_x \text{D}_y \\ \text{D}_z}}^{(1), \text{CuLBM1}} = & (v_2^2 w_6^2 w_3^2 + 12 w_2 w_6^2 w_3 - 3 w_2 v_2^2 w_6 w_3^2 - 12 w_6 w_3 + 12 w_2 w_6 + 12 w_2 w_3 - 36 w_2 w_3 c_s^2 - 36 w_2 w_6 c_s^2 + 36 w_6 w_3 c_s^2 - 36 w_6^2 c_s^2 - 6 w_2 w_3^2 - 12 w_2^2 w_6^2 - 9 w_2 w_6 w_3^2 c_s^2 + 3 w_6^2 w_3^2 c_s^2 + 18 w_2 v_2^2 w_6 w_3 - 12 w_2 w_6^2 + 6 w_6 w_3^2 + 6 v_2^2 w_6^2 w_3 - w_2 w_6^2 w_3^2 - 36 w_2 w_6^2 w_3 c_s^2 + 18 w_2^2 w_3 c_s^2 - w_6^2 w_3^2 - 12 w_2 v_2^2 w_6^2 w_3 + 3 w_2 w_6 w_3^2 + 6 w_2 v_2^2 w_3^2 + 12 v_2^2 w_6 w_3 + 3 w_2 w_6^2 w_3^2 + 12 w_2 v_2^2 w_6^2 - 18 w_6 w_3^2 c_s^2 + 36 w_2 w_6 c_s^2 + 54 w_2 w_6 w_3 c_s^2 + 12 w_6^2 + 18 w_2 w_3^2 c_s^2 - 12 w_2 v_2^2 w_6 - 18 w_2 w_6 w_3 - 12 w_2 v_2^2 w_3 - 6 v_2^2 w_6 w_3^2 - 6 w_6^2 w_3 + w_2 v_2^2 w_6 w_3^2) \frac{v_2^2 c_s^2}{12 w_2 w_6^2 w_3^2} \end{aligned}$$

$$C_{\text{D}_{\text{xx}}^{\text{L}} \text{G}_\mu}^{(1), \text{CuLBMe}^2} = (18w_2 w_3 c_s^4 w_1^3 + 90w_3^2 v_2^2 w_3^2 w_1 - 30w_3^2 v_2^2 w_3 c_s^2 w_1 - 90w_2^2 w_3^2 c_s^4 w_1 + 30w_3^2 w_3 c_s^2 w_1 + 36w_3^2 c_s^4 w_1^3 - 165w_3^2 v_2^2 w_3^2 c_s^2 w_1 - 51v_2^2 w_3^2 w_1^3 - 24w_2^2 w_3 c_s^2 w_1^2 - 138w_3^2 w_3^2 c_s^4 w_1^2 - 72w_3^2 c_s^4 w_1^2 + 51w_2^2 w_3^2 w_3^2 w_1 + 90w_3^2 c_s^4 w_1^3 + 39w_2^2 v_2^2 w_3^2 w_1^3 - 7w_3^2 w_2^2 w_2^2 + 46w_3^2 v_2^2 w_3^2 w_2^2 + 18w_2^2 w_3 c_s^2 w_3^2 w_1^3 + 6w_3^2 c_s^2 w_3^2 c_s^4 w_1^3 + 12w_3^2 w_3^2 c_s^2 w_1^2 - 24w_2^2 w_3^2 c_s^2 w_1^2 - 102w_3^2 v_2^2 w_3^2 w_1 + 6w_2^2 w_3^2 w_1^2 - 45w_3^2 v_4^2 w_3^2 + 197w_2^2 v_2^2 w_3^2 c_s^2 w_3^2 + 141w_2 v_3^2 w_3^2 c_s^2 w_3^2 + 6w_3^2 w_3 c_s^2 w_3^2 + 45w_4^2 w_3^2 w_3^1 + 72w_2^2 w_3^2 c_s^2 w_3^1 - 6w_3^2 v_2^2 w_3 c_s^2 w_3^1 + 9w_2^2 v_1^2 w_3^2 c_s^2 w_3^1 + 12w_2^2 v_2^2 w_3^2 c_s^2 w_3^1 + 225w_3^2 w_3^2 c_s^4 w_1 - 12w_2 w_3^2 w_3^1 + 123w_2 w_3^2 c_s^2 w_3^1 - 45w_2^2 v_4^2 c_s^2 w_3^1 - 39w_3^2 v_4^2 w_3^2 w_1^2 - 46w_2^2 v_2^2 w_3^2 w_3^1 + 42w_3^2 v_2^2 w_3 c_s^2 w_3^1 - 42w_3^2 w_3 c_s^2 w_3^2 + 36w_2^2 w_3^2 c_s^4 w_1 - 465w_2 v_2^2 w_3^2 c_s^2 w_3^1 - 90w_3^2 w_3^2 c_s^4 + 24w_3^2 c_s^2 w_1^2 + 261v_2^2 w_3^2 c_s^2 w_3^1 - 18w_2^2 v_2^2 w_3 c_s^2 w_3^2 + 45w_2 v_4^2 w_3^2 w_1^2 -$$

$$24\omega_3^3v_2^2c_s^2\omega_1^2 - 72\omega_3^2c_s^2\omega_3^3 - 9\omega_3^2v_1^2\omega_3^2c_s^2\omega_1^2 + 2\omega_3^2v_2^2\omega_3^2c_s^2\omega_3^3 - 2\omega_3^2\omega_3^2c_s^2\omega_3^2 - 6\omega_3^2\omega_1^3 - 54\omega_2^2\omega_3c_s^4\omega_1^3 - 6\omega_2\omega_3c_s^2\omega_3^3 + 3\omega_3^2v_1^2\omega_3^2\omega_1^2 + 3\omega_3^2v_1^4\omega_3^2\omega_1^3 - 90\omega_3^2\omega_3c_s^4\omega_1 - 90\omega_2v_2^2\omega_3^2\omega_1^3 - 261\omega_3^2v_2^2\omega_3^2c_s^2 + 7\omega_2^2\omega_3^2\omega_1^3 + 48\omega_2^2\omega_3^2c_s^2\omega_1 + 12\omega_2^3v_2^2c_s^2\omega_1^3 - 12\omega_2^3\omega_3^2\omega_1^3 + 72\omega_2^3\omega_3^2c_s^2 + 24\omega_2^2v_2^2\omega_3c_s^2\omega_1^2 + 6\omega_2v_2^2\omega_3c_s^2\omega_1^3 + 81\omega_2^3\omega_3^2c_s^2\omega_1^2 + 72\omega_2^3\omega_3c_s^4\omega_1^2 - 219\omega_2^3v_2^2\omega_3^2c_s^2\omega_1^2 - 51\omega_2v_2^2\omega_3^2\omega_1^2 - 147\omega_2^3\omega_3^2c_s^2\omega_1 + 489\omega_2^3v_2^2\omega_3^2c_s^2\omega_1^2 - 153\omega_2\omega_3^2c_s^4\omega_1^2 - 12\omega_2^2\omega_3^2c_s^2\omega_1^2 + 126\omega_2^3\omega_3c_s^4\omega_1^2 - 3\omega_2^2v_1^2\omega_3^2\omega_1^3 + 18\omega_2\omega_3^2c_s^4\omega_1^2 - 3\omega_2^3v_1^4\omega_3^2\omega_1^2 - 6\omega_2^3\omega_3^2 + 102\omega_2v_2^2\omega_3^2\omega_1^3 + 51\omega_2^3v_2^2\omega_3^2 + 59\omega_2^2\omega_3^2c_s^2\omega_1^3 - 18\omega_2^3\omega_3c_s^4\omega_1^3 - 6\omega_2^2\omega_3^2\omega_1) \frac{v_2}{24\omega_2^3\omega_3^2\omega_1^3}$$

**coefficient  $C_{D_x D_y^3 v_1}^{(1)}$  at  $\frac{\partial^4 v_1}{\partial x_1 \partial x_2}$ :**

$$C_{D_x D_y^3 v_1}^{(1),\text{SRT}} = (2 + v_2^2\omega - 6c_s^2 + 3c_s^2\omega - 2v_2^2 - \omega) \frac{v_1 \rho v_2}{12\omega}$$

$$\begin{aligned} C_{D_x D_y^3 v_1}^{(1),\text{MRT1}} &= (24\omega_8^2\omega_5^2\omega_6^2\omega_7^2\omega_4^2 - 132\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 - 24\omega_8\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^3 + 12\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 - 66\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + \\ &12\omega_8^2\omega_5^2\omega_6^2\omega_7^2\omega_4^3 + 12\omega_8^2\omega_5^2\omega_6^2c_s^2\omega_7^2\omega_4^3 - 12\omega_8\omega_5\omega_9\omega_6^2\omega_7^2\omega_4^3 - 96\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 + 12\omega_8\omega_5^2\omega_6^2\omega_7^2\omega_4^3 - 12\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - \\ &18\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 - 12\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 4\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 - 12\omega_8^2\omega_5^2\omega_6^2c_s^2\omega_7^2\omega_4^3 + 18\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 - 24\omega_8^2\omega_5^2\omega_6^2\omega_7^2\omega_4^2 + \\ &90\omega_8\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^3 + 60\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^3 - 12\omega_8^2\omega_5^2\omega_6^2\omega_7^2\omega_4^3 - 18\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - 24\omega_8\omega_5^2\omega_6^2\omega_7^2\omega_4^2 + 12\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 + \\ &66\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - 12\omega_8^2\omega_5^2\omega_6^2\omega_7^2\omega_4^3 + 24\omega_8\omega_5\omega_9\omega_6^2\omega_7^2\omega_4^2 - 24\omega_8^2\omega_5^2\omega_6^2c_s^2\omega_7^2\omega_4^2 + 12\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 12\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 - \\ &\omega_8^2\omega_5\omega_9\omega_6^2\omega_7^2\omega_4^2 - 36\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 24\omega_8\omega_5\omega_9\omega_6^2\omega_7^2\omega_4^2 - 24\omega_8^2\omega_5^2\omega_6^2\omega_7^2\omega_4^2 + 6\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 - 84\omega_8^2\omega_5\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 - 24\omega_8\omega_5\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 - \\ &24\omega_8\omega_5\omega_9\omega_6^2\omega_7^2\omega_4^2 - 12\omega_8^2\omega_5\omega_9\omega_6^2\omega_7^2\omega_4^3 - 24\omega_8^2\omega_5\omega_6^2\omega_7^2\omega_4^2 + 24\omega_8^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 + 12\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^3 - 36\omega_8\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 + \\ &24\omega_8^2\omega_5^2\omega_6^2\omega_7^2\omega_4^2 - 12\omega_8^2\omega_5\omega_9\omega_6^2\omega_7^2\omega_4^3 + 24\omega_8^2\omega_5\omega_6^2\omega_7^2\omega_4^3 + 12\omega_8\omega_5\omega_9\omega_6^2\omega_7^2\omega_4^2 + 48\omega_8^2\omega_5\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 - 12\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 + \\ &12\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - 12\omega_8^2\omega_5^2\omega_6^2\omega_7^2\omega_4^3 - 24\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - 48\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - 72\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - 12\omega_8\omega_5^2\omega_6^2\omega_7^2\omega_4^3 - \\ &42\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 + 12\omega_8\omega_5\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^3 + 36\omega_8\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 - 6\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 - 84\omega_8^2\omega_5\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 - 4\omega_8^2\omega_5\omega_9\omega_6^2\omega_7^2\omega_4^3 - \\ &12\omega_8^2\omega_5\omega_9\omega_6^2\omega_7^2\omega_4^3 + 12\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 12\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 + 156\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 + 12\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 + \\ &6\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^3 - 24\omega_8^2\omega_5\omega_9\omega_6^2\omega_7^2\omega_4^2 + 24\omega_8^2\omega_5\omega_6^2\omega_7^2\omega_4^3 - 12\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 + \\ &12\omega_8\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 - 24\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 + 12\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 24\omega_8^2\omega_5\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 + 3\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^3 + \\ &24\omega_8^2\omega_5^2\omega_6^2\omega_7^2\omega_4^3 + \omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^3 - 18\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 - 12\omega_8^2\omega_5\omega_9\omega_6^2\omega_7^2\omega_4^2 + 6\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 + 24\omega_8^2\omega_5\omega_6^2\omega_7^2\omega_4^2 + \\ &24\omega_8^2\omega_5^2\omega_6^2\omega_7^2\omega_4^3 - 24\omega_8^2\omega_5^2\omega_6^2\omega_7^2\omega_4^2 - 66\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^3 - 24\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 - 6\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 + 24\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^3 + \\ &24\omega_8^2\omega_5^2\omega_6^2\omega_7^2\omega_4^3 - 72\omega_8^2\omega_5\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 + 36\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^3 - 12\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2) \frac{v_1 \rho v_2}{12\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3}$$

$$C_{D_x D_y^3 v_1}^{(1),\text{MRT1}} = C_{D_x D_y^3 v_1}^{(1),\text{MRT2}}$$

$$C_{D_x D_y^3 v_1}^{(1),\text{CLBM1}} = (-\omega_8 + 3\omega_8\omega_6c_s^2 - 3v_2\omega_6 + 3\omega_8c_s^2 + 3\omega_6 + \omega_8v_2^2 - \omega_8\omega_6 + \omega_8v_2^2\omega_6 - 9\omega_6c_s^2) \frac{v_1 \rho v_2}{12\omega_8\omega_6}$$

$$C_{D_x D_y^3 v_1}^{(1),\text{CLBM2}} = C_{D_x D_y^3 v_1}^{(1),\text{CLBM1}}$$

$$C_{D_x D_y^3 v_1}^{(1),\text{CuLBM1}} = (-3\omega_2v_2^2 - \omega_2\omega_6 + v_2^2\omega_6 + 3\omega_2 + 3\omega_2\omega_6c_s^2 - \omega_6 - 9\omega_2c_s^2 + 3\omega_6c_s^2 + \omega_2v_2^2\omega_6) \frac{v_1 \rho v_2}{12\omega_2\omega_6}$$

$$C_{D_x D_y^3 v_1}^{(1),\text{CuLBM2}} = (6\omega_3c_s^2\omega_1 + 2\omega_2\omega_3 - 4\omega_3\omega_1 - 9\omega_2v_1^2\omega_3 + 9v_1^2\omega_3\omega_1 + 2\omega_2v_2^2\omega_3\omega_1 + 6\omega_2\omega_1 - 18\omega_3c_s^2\omega_1 - 6\omega_2v_2^2\omega_1 + v_2^2\omega_3\omega_1 - 2\omega_3\omega_3\omega_1 + \\ \omega_2v_2^2\omega_3 + 6\omega_2\omega_3c_s^2\omega_1) \frac{v_1 \rho v_2}{24\omega_2\omega_3\omega_1}$$

**coefficient  $C_{D_x D_y^3 v_2}^{(1)}$  at  $\frac{\partial^4 v_2}{\partial x_1 \partial x_3}$ :**

$$C_{D_x D_y^3 v_2}^{(1),\text{SRT}} = (-12 + 2c_s^2\omega^2 + 18v_2^2\omega^2 - c_s^2\omega^3 - 54v_2^2\omega + 36v_2^2 + 18\omega - 6\omega^2) \frac{\rho c_s^2}{12\omega^3}$$

$$\begin{aligned} C_{D_x D_y^3 v_2}^{(1),\text{MRT1}} &= (6\omega_8^2v_1^2\omega_9\omega_6\omega_7\omega_4^3 + 60\omega_8\omega_5\omega_9v_2^2\omega_6^2c_s^2\omega_7\omega_4^2 + 24\omega_8^2\omega_5v_1^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^2 - 24\omega_8\omega_5v_1^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^3 - \omega_8^2\omega_5\omega_9\omega_6^2c_s^4\omega_7\omega_4^3 + \\ &72\omega_8^2v_1^2\omega_9v_2^2\omega_6\omega_7\omega_4^2 - 72\omega_8^2v_1^2\omega_9v_2^2\omega_6^2\omega_7\omega_4^2 - 36\omega_8^2\omega_5\omega_9v_2^2\omega_6^2c_s^2\omega_7\omega_4^2 - 6\omega_8^2\omega_6^2c_s^4\omega_7\omega_4^3 + 27\omega_8^2\omega_5v_1^2\omega_9v_2^2\omega_6\omega_7\omega_4^3 - 12\omega_8^2\omega_9\omega_6^2c_s^4\omega_7\omega_4^3 - \\ &18\omega_8^2\omega_5v_1^2\omega_9v_2^2\omega_7\omega_4^3 - 18\omega_8\omega_5v_1^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^3 - 24\omega_8v_1^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^2 + 12\omega_8^2\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4^3 + 36\omega_8^2v_1^2v_2^2\omega_6^2\omega_7\omega_4^3 + \\ &12\omega_8^2v_1^2\omega_6^2c_s^2\omega_7\omega_4^3 - 9\omega_8^2\omega_5v_1^2\omega_9\omega_6\omega_7\omega_4^3 - 12\omega_8^2\omega_5\omega_6^2\omega_7\omega_4^2 + 15\omega_8\omega_5v_1^2\omega_9\omega_6^2\omega_7\omega_4^3 + 36\omega_8v_1^2\omega_9v_2^2\omega_6^2\omega_7\omega_4^3 - 18\omega_8^2v_1^2\omega_9\omega_6^2\omega_7\omega_4^3 - \\ &5\omega_8^2\omega_5\omega_9\omega_6^2c_s^4\omega_7\omega_4^2 - 24\omega_8^2v_1^2\omega_9\omega_6\omega_7\omega_4^2 - 48\omega_8\omega_5v_1^2\omega_9\omega_6^2\omega_7\omega_4^2 + 36\omega_8^2v_1^2\omega_9\omega_6^2\omega_7\omega_4^3 + 36\omega_8^2v_1^2\omega_9\omega_6^2\omega_7\omega_4^2 + 12\omega_8^2\omega_5\omega_6^2\omega_7\omega_4^3 + \\ &12\omega_8v_1^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^3 + 36\omega_8^2\omega_5v_1^2\omega_9v_2^2\omega_7\omega_4^2 + 36\omega_8\omega_5v_1^2\omega_9\omega_6^2\omega_7\omega_4^3 - 108\omega_8^2\omega_5v_1^2\omega_9v_2^2\omega_6\omega_7\omega_4^2 - 12\omega_8^2\omega_5\omega_9\omega_6^2c_s^2\omega_7\omega_4^3 + 24\omega_8\omega_5v_1^2\omega_9\omega_6^2\omega_7\omega_4^2 + \\ &24\omega_8^2v_1^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^2 - 6\omega_5v_1^2\omega_9\omega_6^2\omega_7\omega_4^3 - 24\omega_8^2\omega_5v_1^2\omega_9\omega_6\omega_7\omega_4^2 - 36\omega_8^2v_1^2\omega_9v_2^2\omega_6\omega_7\omega_4^3 - 6\omega_8^2\omega_5v_1^2\omega_9c_s^2\omega_7\omega_4^3 - 6\omega_8\omega_5v_1^2\omega_9\omega_6^2\omega_7\omega_4^2 - \\ &45\omega_8\omega_5v_1^2\omega_9v_2^2\omega_6^2\omega_7\omega_4^3 + 6\omega_8^2\omega_6^2\omega_7\omega_4^3 + 72\omega_8^2\omega_5v_1^2\omega_9v_2^2\omega_6\omega_7\omega_4^2 - 15\omega_8\omega_5\omega_9v_2^2\omega_6c_s^2\omega_7\omega_4^3 + 12\omega_8^2\omega_6^2c_s^4\omega_7\omega_4^3 + 18\omega_8^2\omega_5\omega_9\omega_6^2c_s^4\omega_7\omega_4^2 - \\ &12\omega_8^2\omega_5v_1^2\omega_9\omega_6c_s^2\omega_7\omega_4^3 + 12\omega_8^2v_2^2\omega_9\omega_6\omega_7\omega_4^3 - 18\omega_8\omega_5v_1^2v_2^2\omega_6^2\omega_7\omega_4^3 - 15\omega_8\omega_5v_1^2\omega_9\omega_6^2\omega_7\omega_4^3 + 9\omega_8^2\omega_5v_1^2\omega_9\omega_6^2\omega_7\omega_4^2 + 36\omega_8^2\omega_5v_1^2\omega_9\omega_6^2\omega_7\omega_4^3 + \\ &30\omega_8\omega_5\omega_9v_2^2\omega_6^2c_s^2\omega_7\omega_4^3 - 12\omega_8^2v_1^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^3 + \omega_8^2\omega_5\omega_9\omega_6^2c_s^2\omega_7\omega_4^2 + 18\omega_8^2\omega_5v_2^2\omega_6c_s^2\omega_7\omega_4^3 + 144\omega_8\omega_5v_1^2\omega_9v_2^2\omega_6^2\omega_7\omega_4^2 + 12\omega_8\omega_5v_1^2\omega_9\omega_6^2\omega_7\omega_4^3 - \\ &12\omega_8^2v_1^2\omega_6^2\omega_7\omega_4^3 + 12\omega_5v_1^2\omega_9\omega_6^2\omega_7\omega_4^2 + 12\omega_8^2\omega_5v_1^2\omega_9c_s^2\omega_7\omega_4^2 - 6\omega_8^2v_1^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^3 - 36\omega_8^2\omega_5v_1^2\omega_9v_2^2\omega_6^2\omega_7\omega_4^2 - 102\omega_8\omega_5\omega_9v_2^2\omega_6^2\omega_7\omega_4^2 - \\ &36\omega_8^2\omega_5v_1^2\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 12\omega_8^2\omega_5v_1^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^3 + 6\omega_8^2\omega_5v_1^2\omega_9\omega_6^2\omega_7\omega_4^2 + 36\omega_8\omega_5v_1^2\omega_9\omega_6^2\omega_7\omega_4^3 + 54\omega_8\omega_5\omega_9v_2^2\omega_6c_s^2\omega_7\omega_4^2 + \\ &6\omega_8^2\omega_5v_1^2\omega_9\omega_6^2\omega_7\omega_4^3 + 6\omega_8^2\omega_9\omega_6^2\omega_7\omega_4^2 + 12\omega_8^2\omega_5\omega_9\omega_6^2c_s^2\omega_7\omega_4^3 - 12\omega_8\omega_5\omega_9\omega_6^2c_s^2\omega_7\omega_4^2 + 6\omega_8\omega_5v_1^2\omega_9\omega_6^2\omega_7\omega_4^3 - 5\omega_8\omega_5\omega_9\omega_6^2c_s^4\omega_7\omega_4^3 + \\ &6\omega_8\omega_5v_1^2\omega_9\omega_7\omega_4^3 - 12\omega_8^2\omega_5v_1^2\omega_6^2c_s^2\omega_7\omega_4^3 - 48\omega_8^2\omega_5\omega_9v_2^2\omega_6^2c_s^2\omega_7\omega_4^2 + 36\omega_8^2\omega_5v_2^2\omega_6^2c_s^2\omega_7\omega_4^2 + 60\omega_8^2\omega_5\omega_9v_2^2\omega_6^2c_s^2\omega_7\omega_4^3 - 6\omega_8^2v_1^2\omega_6^2c_s^2\omega_7\omega_4^3 -$$

$$\begin{aligned}
& 36w_8^2 w_5 v_1^2 v_2^2 w_6^2 w_4^3 + 12w_8^2 w_5 w_9 w_6 c_s^4 w_4^3 + 18w_8^2 w_5 v_1^2 v_2^2 w_6^2 \omega_7 w_4^3 + 12w_8 w_5 w_9 w_6^2 c_s^4 \omega_7 w_4 + 6w_8 w_5 w_6^2 c_s^2 \omega_7 w_4^3 - 12w_8 v_1^2 w_9 w_6^2 \omega_7 w_4^3 + \\
& 18w_8 w_5 w_9 w_6^2 c_s^2 \omega_7 w_4^2 - 36w_8 w_5 v_1^2 w_9 v_2^2 w_6 \omega_7 w_4^2 - 12w_8^2 w_5 v_2^2 c_s^2 \omega_7 w_4^2 - 12w_8^2 w_5 w_6^2 c_s^4 \omega_7 w_4^2 + 12w_8^2 w_5 v_1^2 w_6^2 \omega_4^3 - 36w_8^2 w_5 v_2^2 c_s^2 \omega_7 w_4^2 + \\
& 12w_8^2 w_5 w_9 w_6 c_s^4 w_4^2 + 12w_8^2 w_9 w_6 c_s^2 w_4^3 + 36w_8^2 w_5 v_1^2 v_2^2 w_6^2 w_4^2 + 12w_8^2 w_5 v_1^2 w_6^2 c_s^2 w_4^3 + 12w_8^2 w_5 v_2^2 c_s^2 \omega_7 w_4^2 - 6w_8^2 w_5 w_6^2 c_s^2 \omega_7 w_4^2 - \\
& 18w_8^2 w_5 w_9 w_6 c_s^4 \omega_7 w_4^2 - 12w_8 w_5 v_1^2 w_6^2 \omega_7 w_4^2 - 18w_8^2 v_2^2 w_6^2 c_s^2 \omega_7 w_4^3 + 6w_8 w_5 w_9 w_6^2 c_s^4 \omega_7 w_4^3 + 24w_5 w_9 v_2^2 w_6^2 c_s^2 \omega_7 w_4^2 - 6w_8 w_5 w_6^2 c_s^4 \omega_7 w_4^2 + \\
& 12w_5 v_1^2 w_9 w_6^2 c_s^2 \omega_7 w_4^2 + 12w_8 w_5 w_6^2 c_s^4 \omega_7 w_4^2 - 12w_8^2 w_5 w_9 w_6^2 c_s^4 \omega_7 w_4^2 - 6w_8^2 w_9 w_6 c_s^2 \omega_7 w_4^3 + 6w_8 w_5 v_1^2 w_9 w_6 c_s^2 \omega_7 w_4^3 - 6w_8 w_5 v_1^2 w_9 w_6 \omega_7 w_4^2 - \\
& 36w_8^2 w_5 w_9 w_6^2 c_s^2 \omega_7 w_4^3 + 5w_8^2 w_5 w_9 w_6^2 c_s^2 \omega_7 w_4^2 - 36w_8^2 v_1^2 w_9 v_2^2 w_6^2 \omega_7 w_4^2 - 12w_8^2 w_5 v_1^2 w_9 w_6 w_4^3 + 12w_8^2 w_5 v_2^2 w_6^2 \omega_7 w_4^2 - 12w_8 w_5 w_6^2 c_s^2 \omega_7 w_4^2 + \\
& 18w_8^2 w_5 v_2^2 w_6^2 c_s^2 \omega_7 w_4^3 - 12w_8 w_5 v_1^2 w_9 w_6^2 c_s^2 \omega_7 w_4^2 - 15w_8^2 w_5 w_9 v_2^2 w_6^2 c_s^2 \omega_7 w_4^2 - 36w_8^2 w_9 v_2^2 w_6^2 c_s^2 \omega_7 w_4^3 - 6w_8 w_5 w_6^2 c_s^4 \omega_7 w_4^2 + 6w_5 v_1^2 w_9 w_6 c_s^2 \omega_7 w_4^2 + \\
& 12w_8^2 w_5 w_6^2 c_s^2 \omega_7 w_4^2 - 18w_8 w_5 w_9 w_6^2 c_s^4 \omega_7 w_4^2 - 12w_5 w_9 v_2^2 w_6^2 c_s^2 \omega_7 w_4^3 - 12w_8^2 w_6^2 c_s^2 \omega_7 w_4^3 - 12w_8^2 w_5 v_1^2 w_9 w_6 c_s^4 \omega_7 + 12w_8^2 w_5 v_1^2 w_9 w_6 \omega_4^2 - 36w_8^2 w_5 v_2^2 w_6^2 c_s^2 \omega_7 w_4^2 - \\
& 6w_8^2 w_5 v_1^2 w_6^2 \omega_7 w_4^3 + 18w_5 v_1^2 w_9 v_2^2 w_6^2 \omega_7 w_4^3 - 18w_8^2 w_5 w_9 w_6 c_s^2 \omega_7 w_4^2 + 36w_8^2 w_5 w_9 v_2^2 w_6 c_s^2 \omega_7 w_4^3 + 12w_8 w_5 v_1^2 w_9 w_6 \omega_7 w_4^2 - \frac{\rho}{12w_8^2 w_5 w_9 w_6^2 \omega_7 w_4^3}
\end{aligned}$$

$$C_{D_x D_y^3 v_2}^{(1), MRT2} = C_{D_x D_y^3 v_2}^{(1), MRT1}$$

$$\begin{aligned}
C_{D_x D_y^3 v_2}^{(1), CLBM1} &= (12w_8 w_5 w_6 w_4^3 + 5w_8 w_5 w_9 w_7 w_4^3 - 12w_8 w_9 c_s^2 w_4^3 + 18w_5 w_9 v_2^2 w_6 \omega_7 w_4^2 - 12w_8 w_5 w_6 c_s^2 \omega_7 w_4^2 + 12w_5 w_9 w_6 c_s^2 \omega_7 w_4 - \\
& w_8 w_5 w_9 w_6^2 c_s^2 \omega_7 w_4^3 + 54w_8 w_5 w_9 v_2^2 w_7 w_4^2 - 54w_5 w_9 v_2^2 w_6 \omega_7 w_4^2 - 36w_8 w_9 v_2^2 w_4^3 - 18w_8 w_5 w_9 w_7 w_4^2 - 12w_8 w_5 w_6 \omega_4^2 - 5w_8 w_5 w_9 w_6 c_s^2 \omega_7 w_4^2 - \\
& 15w_8 w_5 w_9 v_2^2 w_7 w_4^3 - 12w_8 w_5 w_9 c_s^2 \omega_7 w_4^2 + 6w_8 w_5 w_6 c_s^2 \omega_7 w_4^3 + 18w_8 w_5 w_9 w_6 c_s^2 \omega_7 w_4^2 + 6w_5 w_9 w_6 c_s^2 \omega_7 w_4^3 + 18w_8 w_5 w_9 c_s^2 \omega_7 w_4^2 - 12w_8 w_5 w_9 c_s^2 \omega_4^2 + \\
& 36w_8 w_5 w_9 v_2^2 w_4^3 + 36w_5 w_9 v_2^2 w_6 \omega_7 w_4^2 + 12w_8 w_5 w_9 w_7 w_4^2 - 36w_8 w_5 v_2^2 w_6 \omega_7 w_4^2 + 12w_8 w_9 w_4^3 - 36w_8 w_5 w_9 v_2^2 w_4^2 - 5w_8 w_5 w_9 c_s^2 \omega_7 w_4^2 + 12w_8 w_5 w_9 c_s^2 \omega_4^2 - \\
& 6w_8 w_9 w_7 w_4^3 - 18w_5 w_9 w_6 c_s^2 \omega_7 w_4^2 - 36w_8 w_5 w_9 v_2^2 w_7 w_4^2 + 18w_8 w_5 v_2^2 w_6 \omega_7 w_4^3 - 3w_8 w_5 w_9 v_2^2 w_6 \omega_7 w_4^2 + 36w_8 v_2^2 w_6 \omega_4^3 + 12w_8 w_5 w_6 c_s^2 \omega_7 w_4^2 - \\
& 6w_5 w_9 w_6 \omega_7 w_4^3 - 6w_5 w_6 c_s^2 w_7 w_4^3 + 12w_8 w_6 c_s^2 w_4^3 - 18w_8 v_2^2 w_6 \omega_7 w_4^3 - 6w_8 w_5 w_6 \omega_7 w_4^3 - 12w_8 w_5 w_9 w_6 w_7 w_4^2 + 12w_5 w_6 c_s^2 \omega_7 w_4^2 + 6w_8 w_9 c_s^2 \omega_7 w_4^3 + \\
& 36w_8 w_5 v_2^2 w_6 \omega_4^2 + 6w_8 w_6 \omega_7 w_4^3 - 12w_8 w_5 w_6 c_s^2 w_4^3 - 12w_5 w_9 w_6 \omega_7 w_4^2 - 12w_5 w_6 w_7 w_4^2 - 12w_8 w_5 w_9 w_3^3 - 18w_5 v_2^2 w_6 \omega_7 w_4^3 - 6w_8 w_6 c_s^2 \omega_7 w_4^3 + \\
& 12w_8 w_5 w_6 c_s^2 w_4^2 + 18w_8 w_9 v_2^2 w_7 w_4^3 - 36w_8 w_5 v_2^2 w_6 \omega_4^3 + 36w_5 v_2^2 w_6 \omega_7 w_4^2 + 12w_8 w_5 w_9 w_2^2 + 6w_5 w_6 \omega_7 w_4^3 - 12w_8 w_5 w_9 w_6 c_s^2 \omega_7) \frac{\rho c_s^8}{12w_8 w_5 w_9 w_6 \omega_7 w_4^3}
\end{aligned}$$

$$C_{D_x D_y^3 v_2}^{(1), CLBM2} = C_{D_x D_y^3 v_2}^{(1), CLBM1}$$

$$\begin{aligned}
C_{D_x D_y^3 v_2}^{(1), CuLBM1} &= (-w_2 w_6 w_3^2 c_s^2 - 3w_2 v_2^2 w_6 \omega_3^2 + 12w_6 w_3 - 12w_2 w_3 + 12w_2^2 c_s^2 - w_6 \omega_3^3 - 18v_2^2 \omega_3^3 - 12w_2 w_6 c_s^2 - 12w_6 w_3 c_s^2 + \\
& 18w_2 w_3^2 - 5w_2 w_6 w_3 c_s^2 + 36v_2^2 \omega_3^2 - 6w_6 \omega_3^2 - 6w_3^2 c_s^2 - 6w_2 \omega_3^2 + 6w_2 w_3 c_s^2 - 12w_3^2 + w_2 w_6 \omega_3^2 - 54w_2 v_2^2 \omega_3^2 - 36v_2^2 w_6 \omega_3 + 6w_3^3 + 6w_6 w_3^2 c_s^2 + \\
& 18w_2 w_6 w_3 c_s^2 + 18w_2 v_2^2 \omega_3^2 + 3v_2^2 w_6 \omega_3^3 - 18w_2 w_3^2 c_s^2 + 36w_2 v_2^2 w_3 + 18v_2^2 w_6 \omega_3^2 + w_6 \omega_3^3 c_s^2) \frac{\rho c_s^2}{12w_2 w_6 \omega_3^3}
\end{aligned}$$

$$\begin{aligned}
C_{D_x D_y^3 v_2}^{(1), CuLBM2} &= (-w_3^3 v_4^4 w_3 \omega_1^2 - 33w_2 w_3 c_s^4 \omega_1^3 - w_2^2 v_1^2 w_3 \omega_1^3 + 351w_3^2 v_2^2 w_3 c_s^2 \omega_1 - 45w_3^2 w_3 c_s^2 \omega_1 + 180w_2 v_2^2 w_3 \omega_1^3 + 12w_3^3 c_s^4 \omega_1^3 - 6w_2^2 w_3 \omega_1 - \\
& 12w_2^2 w_3^2 c_s^2 \omega_1^2 + 18w_2^2 c_s^2 \omega_1^3 - 30w_3^2 c_s^4 \omega_1^2 - 63w_2 v_2^2 w_3 \omega_1^2 + 6w_2 w_3 c_s^4 \omega_1^2 - 24w_2^2 c_s^2 \omega_1^2 - 24w_2^2 w_3 c_s^2 \omega_1^3 - 171w_2^3 v_2^4 w_3 + w_2^2 v_1^4 w_3 \omega_1^3 + w_2^3 v_1^2 w_3 \omega_1^2 + \\
& 24w_2^3 w_3 c_s^2 \omega_1^2 + 207v_2^2 w_3 c_s^2 \omega_1^3 - 324w_2 v_2^4 w_3 \omega_1^3 - 207w_3^2 v_2^2 w_3 c_s^2 \omega_1^2 + 7w_2^2 w_3 \omega_1^3 + 18w_3^2 c_s^4 \omega_1^2 - 3w_3^2 v_1^2 w_3 c_s^2 \omega_1^2 - 24w_3 c_s^2 \omega_1^3 + 18w_2 v_2^2 c_s^2 \omega_1^3 + 135w_2 v_2^4 w_3 \omega_1^2 + \\
& 36w_2^2 w_3 c_s^2 \omega_1^2 - 153w_3^2 v_2^2 w_3 c_s^2 \omega_1^2 + 6w_2 c_s^4 \omega_1^3 + 24w_3^2 w_3 c_s^2 \omega_1^2 + 6w_3 \omega_1^3 + 30w_3^2 c_s^2 \omega_1^2 + 153w_2^2 v_2^2 w_3 c_s^2 \omega_1^3 - 12w_2 w_3 \omega_1^3 - 90w_3^2 v_2^2 c_s^2 \omega_1^2 - 135w_2 v_2^4 w_3 \omega_1^2 - \\
& 12w_2 w_3 c_s^2 \omega_1^2 - 81w_2^2 v_2^2 w_3 \omega_1^3 + 24w_2^2 c_s^4 \omega_1^2 - 138w_3^2 v_2^4 w_3 \omega_1^2 - 42w_3^2 w_3 c_s^4 \omega_1^2 + 17w_2^2 w_3 c_s^2 \omega_1^3 + 99w_3^2 v_2^2 w_3 + 63w_2 v_2^2 w_3 c_s^2 \omega_1^2 - 6w_3^2 w_3 + 45w_2 w_3 c_s^2 \omega_1^2 - \\
& 180w_3^2 v_2^2 w_3 \omega_1^2 + 12w_3^2 w_3 \omega_1^3 + 69w_2^2 w_3 c_s^4 \omega_1^2 + 36w_3^2 v_2^2 c_s^2 \omega_1^3 - 12w_3^2 c_s^2 \omega_1^3 + 6w_2 w_3 \omega_1^2 + 3w_2^2 v_2^2 w_3 c_s^2 \omega_1^3 + 36w_2^2 v_2^2 w_3 c_s^2 \omega_1^2 + 171v_2^4 w_3 \omega_1^3 - \\
& 351w_2^2 v_2^2 w_3 c_s^2 \omega_1^3 + 12w_2^2 w_3 c_s^4 \omega_1^2 - 18w_2^2 c_s^4 \omega_1^3 - 30w_2^2 w_3 c_s^4 \omega_1^2 - 54w_2^2 v_2^2 c_s^2 \omega_1^3 + 63w_2^2 v_2^2 w_3 \omega_1^2 + 81w_2^2 v_2^2 w_3 c_s^2 \omega_1^2 - 7w_2^2 w_3 \omega_1^2 - 6w_2 c_s^2 \omega_1^3 + 138w_2^2 v_2^4 w_3 \omega_1^2 - \\
& 25w_2^2 w_3 c_s^4 \omega_1^2 - 135w_2^2 v_2^2 w_3 c_s^2 \omega_1^3 + 324w_2^2 v_2^2 w_3 \omega_1^2 + 72w_2^2 v_2^2 c_s^2 \omega_1^2 - 18w_2^2 c_s^2 \omega_1^3 + 54w_2^2 v_2^2 c_s^2 \omega_1^2 - 99w_2^2 w_3 \omega_1^2 - 2w_2^2 w_3 c_s^4 \omega_1^2 + 18w_3^2 v_2^4 \omega_1^2) \frac{\rho}{24w_2^2 w_3 \omega_1^3}
\end{aligned}$$

coefficient  $C_{D_y^4 \rho}^{(1)}$  at  $\frac{\partial^4 \rho}{\partial x_2^4}$ :

$$C_{D_y^4 \rho}^{(1), SRT} = (6v_2^4 + 3v_2^2 \omega - 2c_s^2 + c_s^2 \omega + 2c_s^4 + 24v_2^2 c_s^2 - c_s^4 \omega - 6v_2^2 - 12v_2^2 c_s^2 \omega - 3v_2^4 \omega) \frac{v_1}{24\omega}$$

$$\begin{aligned}
C_{D_y^4 \rho}^{(1), MRT1} &= (-96w_8^2 v_4^2 w_6 \omega_4 - 24w_8^2 c_s^2 \omega_4 - 30w_8 v_2^4 w_6^2 \omega_4^2 - 144v_2^2 w_6^2 c_s^2 \omega_4 - 24v_2^2 w_6^2 \omega_4 - 24w_8 w_6^2 c_s^4 \omega_4 - 12w_8^2 v_2^2 w_6^2 c_s^2 \omega_4^2 - \\
& 144w_8^2 v_2^2 c_s^2 \omega_4^2 + 24w_2^2 c_s^2 \omega_4 - 72w_8^2 v_2^2 \omega_4 + 12w_4^2 w_6^2 \omega_4^2 - 126w_8 v_2^2 w_6^2 c_s^2 \omega_4^2 - 36w_8^2 v_2^2 w_6 \omega_4^2 - 48w_8 w_6^2 c_s^2 \omega_4 - 96w_8 v_2^2 w_6^2 \omega_4 + 24w_8^2 w_6 c_s^4 \omega_4 + \\
& 12w_8 w_6^2 c_s^2 \omega_4^2 + 96w_8^2 v_2^2 w_6 \omega_4 + 30w_8 v_2^2 w_6^2 \omega_4^2 + 36w_8^2 v_2^2 w_6^2 \omega_4^2 - 24v_4^2 w_6^2 \omega_4 + 432w_8 v_2^2 w_6^2 c_s^2 \omega_4 - 48w_8 v_2^4 w_6^2 \omega_4^2 - 12w_2^2 w_6^2 \omega_4^2 + 48w_8^2 w_6 c_s^2 \omega_4 + \\
& 288w_8^2 v_2^2 c_s^2 \omega_4 - 12w_6^2 c_s^2 \omega_4^2 + 36w_8^2 v_2^4 w_6 \omega_4^2 + 12w_8^2 c_s^2 \omega_4^2 + 96w_8 v_2^2 w_6^2 \omega_4^2 + 72w_2^2 w_6^2 c_s^2 \omega_4^2 - 48w_8^2 v_2^2 w_6 \omega_4^2 - w_8 w_6^2 c_s^4 \omega_4^2 + 72w_8^2 v_2^4 w_6 \omega_4^2 + 216w_8^2 v_2^2 w_6 c_s^2 \omega_4^2 - \\
& 144w_8^2 v_2^2 w_6 c_s^2 \omega_4 + 48w_8 w_6^2 c_s^4 \omega_4 - 3w_8^2 v_2^4 w_6^2 \omega_4^2 - 48w_8 v_2^4 w_6 \omega_4^2 - 432w_8 v_2^2 w_6^2 c_s^2 \omega_4 + 24w_8^2 c_s^4 \omega_4 - 24w_8 v_2^2 w_6 \omega_4^2 + 14w_8^2 w_6 c_s^4 \omega_4^2 - 24w_6^2 c_s^4 \omega_4 - \\
& 48w_8^2 w_6 c_s^4 \omega_4 + 48w_8 v_2^2 w_6^2 + 24w_8 w_6^2 c_s^2 + 12w_6^2 c_s^4 \omega_4^2 + 3w_8^2 v_2^2 w_6^2 \omega_4^2 + 150w_8^2 v_2^2 w_6 c_s^2 \omega_4^2 - 12w_8^2 c_s^2 \omega_4^2 + w_8^2 w_6^2 c_s^2 \omega_4^2 + 48w_8 v_2^2 w_6 \omega_4 + 48w_8^2 v_2^4 \omega_6 + 72w_8 v_2^2 w_6 c_s^2 \omega_4^2 - \\
& 72w_8^2 v_2^2 w_6 c_s^2 \omega_4^2 - 12w_8 w_6^2 c_s^4 \omega_4^2 + 24w_8 v_2^4 w_6 \omega_4^2 - 24w_8^2 w_6 c_s^2 \omega_4^2 - 36w_8^2 v_2^4 w_6^2 \omega_4^2 - 216w_8 v_2^2 w_6^2 c_s^2 \omega_4^2) \frac{v_1}{24w_8^2 w_6^2 \omega_4^2}
\end{aligned}$$

$$C_{D_y^4 \rho}^{(1), MRT2} = C_{D_y^4 \rho}^{(1), MRT1}$$

$$C_{D_y^4 \rho}^{(1), CLBM1} = (6v_2^4 + 3v_2^2 \omega - 12v_2^2 w_6 c_s^2 - 2c_s^2 - \omega_6 c_s^4 - 3v_2^4 \omega_6 + 2c_s^4 + 24v_2^2 c_s^2 + \omega_6 c_s^2 - 6v_2^2) \frac{v_1}{24\omega_6}$$

$$C_{D_y^4 \rho}^{(1), CLBM2} = C_{D_y^4 \rho}^{(1), CLBM1}$$

$$C_{D_y^4 \rho}^{(1), CuLBM1} = (6v_2^4 + 3w_2 v_2^2 - 12w_2 v_2^2 c_s^2 - 2c_s^2 - \omega_2 c_s^4 + 2c_s^4 + w_2 c_s^2 + 24v_2^2 c_s^2 - 6v_2^2 - 3w_2 v_2^4) \frac{v_1}{24\omega_2}$$

$$C_{D_y^4 \rho}^{(1), \text{CuLBM2}} = (-3\omega_2 v_2^4 \omega_1 - 12\omega_2 v_2^2 c_s^2 \omega_1 - 3\omega_2 v_2^2 - \omega_2 c_s^4 \omega_1 + 12\omega_2 v_2^2 c_s^2 + \omega_2 c_s^4 - 3v_2^2 \omega_1 - c_s^2 \omega_1 - \omega_2 c_s^2 + \omega_2 c_s^2 \omega_1 + 3\omega_2 v_2^2 \omega_1 + 3\omega_2 v_2^4 + c_s^4 \omega_1 + 12v_2^2 c_s^2 \omega_1 + 3v_2^4 \omega_1) \frac{v_1}{24\omega_2 \omega_1}$$

**coefficient**  $C_{D_y^4 v_1}^{(1)}$  at  $\frac{\partial^4 v_1}{\partial x_2^4}$ :

$$C_{D_y^4 v_1}^{(1), \text{SRT}} = (-14c_s^2 \omega^2 - 72v_2^4 - 3v_2^2 \omega^3 + 42v_2^2 \omega^2 + c_s^2 \omega^3 - 108v_2^2 \omega - 24c_s^2 + 36c_s^2 \omega + 48c_s^4 - 84v_2^2 c_s^2 \omega^2 - 42v_2^4 \omega^2 - 3c_s^4 \omega^3 + 30c_s^4 \omega^2 - 144v_2^2 c_s^2 + 3v_2^4 \omega^3 + 6v_2^2 c_s^2 \omega^3 - 72c_s^4 \omega + 72v_2^2 + 216v_2^2 c_s^2 \omega + 108v_2^4 \omega) \frac{\rho}{24\omega^3}$$

$$C_{D_y^4 v_1}^{(1), \text{MRT1}} = (12\omega_8^2 c_s^2 \omega_4 + 24\omega_8^2 c_s^4 + 24v_2^2 \omega_4^2 - 18\omega_8 v_2^4 \omega_4^3 - 24\omega_8 c_s^4 \omega_4^2 - 72\omega_8 v_2^2 c_s^2 \omega_4^2 - 24\omega_8^2 v_2^2 \omega_4^2 - 12v_2^2 \omega_4^3 + 6\omega_8 v_2^2 c_s^2 \omega_4^3 + 6\omega_8 c_s^4 \omega_4^3 + 72\omega_8 v_2^4 \omega_4^2 - 48\omega_8 v_2^4 \omega_4 + 24\omega_8^2 v_2^2 \omega_4^2 + \omega_8^2 c_s^2 \omega_4^3 + 24\omega_8 c_s^4 \omega_4 + 156\omega_8^2 v_2^2 c_s^2 \omega_4 - 8\omega_8^2 c_s^2 \omega_4^3 - 3\omega_8^2 v_2^2 \omega_4^3 + 48\omega_8 v_2^2 c_s^2 \omega_4^2 + 24\omega_8^2 v_2^4 \omega_4 + 12v_2^2 c_s^2 \omega_4^3 + 12v_2^4 \omega_4^3 - 6\omega_8 c_s^2 \omega_4^3 - 72\omega_8 v_2^2 \omega_4^2 - 48\omega_8^2 c_s^2 \omega_4^2 - 24v_2^4 \omega_4^2 - 24v_2^2 c_s^2 \omega_4^2 - 12\omega_8 v_2^2 c_s^2 \omega_4^3 + 18\omega_8 v_2^2 \omega_4^3 + 24\omega_8 c_s^2 \omega_4^2 - 24\omega_8 c_s^2 \omega_4 + 24\omega_8^2 c_s^4 \omega_4^2 + 3\omega_8^2 v_2^4 \omega_4^2 - 96\omega_8^2 v_2^2 c_s^2 + 48\omega_8 v_2^2 \omega_4^2 - 24\omega_8^2 v_2^2 c_s^2 \omega_4 - 3\omega_8^2 c_s^4 \omega_4^2) \frac{\rho}{24\omega_8^2 \omega_4^3}$$

$$C_{D_y^4 v_1}^{(1), \text{MRT2}} = C_{D_y^4 v_1}^{(1), \text{MRT1}}$$

$$C_{D_y^4 v_1}^{(1), \text{CLBM1}} = (12\omega_8^2 c_s^2 \omega_4 + 24\omega_8^2 c_s^4 + 72v_2^2 \omega_4^2 - 30\omega_8 v_2^4 \omega_4^3 - 24\omega_8 c_s^4 \omega_4^2 - 12\omega_8^2 v_2^2 c_s^2 \omega_4^2 - 36v_2^2 \omega_4^3 + 6\omega_8 v_2^2 c_s^2 \omega_4^3 + 6\omega_8 c_s^4 \omega_4^3 + 72\omega_8 v_2^4 \omega_4^2 - 8\omega_8^2 v_2^2 c_s^2 \omega_4^2 + 24\omega_8 c_s^4 \omega_4^2 - 36\omega_8^2 v_2^2 c_s^2 \omega_4 - 8\omega_8^2 c_s^2 \omega_4^2 - 3\omega_8^2 v_2^2 \omega_4^3 + 144\omega_8 v_2^2 c_s^2 \omega_4^2 + 108v_2^2 c_s^2 \omega_4^3 + 36v_2^4 \omega_4^3 - 6\omega_8 c_s^2 \omega_4^3 - 72\omega_8 v_2^2 \omega_4^2 - 48\omega_8^2 c_s^4 \omega_4^2 - 72v_2^2 c_s^2 \omega_4^2 - 216v_2^2 c_s^2 \omega_4^2 - 72\omega_8 v_2^2 c_s^2 \omega_4^3 + 30\omega_8 v_2^2 \omega_4^2 + 24\omega_8 c_s^2 \omega_4^2 - 24\omega_8 v_2^2 c_s^2 \omega_4 + 24\omega_8^2 c_s^4 \omega_4^2 + 3\omega_8^2 v_2^4 \omega_4^3 - 12\omega_8^2 v_2^2 \omega_4^2 + 72\omega_8 v_2^2 c_s^2 \omega_4 - 3\omega_8^2 c_s^4 \omega_4^3) \frac{\rho}{24\omega_8^2 \omega_4^3}$$

$$C_{D_y^4 v_1}^{(1), \text{CLBM2}} = C_{D_y^4 v_1}^{(1), \text{CLBM1}}$$

$$C_{D_y^4 v_1}^{(1), \text{CuLBM1}} = (12v_2^2 \omega_6^2 \omega_3^2 + 6\omega_6 \omega_3^3 c_s^4 - 3v_2^2 \omega_6^2 \omega_3^3 + 6v_2^2 \omega_6^2 \omega_3^2 c_s^2 + \omega_6^2 \omega_3^2 c_s^2 - 12v_2^4 \omega_6^2 \omega_3^2 - 36v_2^2 \omega_6^3 - 24\omega_6 \omega_3 c_s^2 - 48\omega_6^2 \omega_3 c_s^4 + 72v_2^2 \omega_6 \omega_3 c_s^2 - 8\omega_6^2 \omega_3^2 c_s^2 + 3v_2^4 \omega_6^2 \omega_3^3 + 72v_2^2 \omega_6^2 \omega_3^2 - 12v_2^2 \omega_6^2 \omega_3^2 c_s^2 - 24\omega_6 \omega_3^2 c_s^4 + 24\omega_6^2 \omega_3^2 c_s^2 + 12\omega_6^2 \omega_3 c_s^2 - 30v_2^4 \omega_6 \omega_3^3 - 36v_2^2 \omega_6^2 \omega_3^2 + 24\omega_6 \omega_3 c_s^4 - 216v_2^2 \omega_6^2 \omega_3^2 + 72v_2^4 \omega_6 \omega_3^2 + 24\omega_6 \omega_3^2 c_s^2 + 24\omega_6^2 \omega_3^2 c_s^4 + 144v_2^2 \omega_6 \omega_3^2 c_s^2 - 72v_2^4 \omega_3^2 + 30v_2^2 \omega_6 \omega_3^3 - 72v_2^2 \omega_6 \omega_3^2 c_s^2 + 36v_2^4 \omega_3^3 - 72v_2^2 \omega_6 \omega_3^2 - 3\omega_6^2 \omega_3^2 c_s^4 - 6\omega_6 \omega_3^2 \omega_3^2 + 108v_2^2 \omega_6 \omega_3^2 c_s^2) \frac{\rho}{24\omega_6^2 \omega_3^3}$$

$$C_{D_y^4 v_1}^{(1), \text{CuLBM2}} = (-24\omega_3 c_s^2 \omega_1 - 36v_2^2 \omega_3^3 + 72v_2^2 \omega_3 c_s^2 \omega_1 - 3v_2^2 \omega_3^2 \omega_1^3 + 24\omega_3^2 c_s^4 \omega_1^2 + 72v_2^2 \omega_1^2 - 3\omega_3^2 c_s^4 \omega_1^3 + 12v_2^2 \omega_3^2 \omega_1^2 - 72v_2^2 \omega_3 c_s^2 \omega_1^3 + 3v_2^4 \omega_3^2 \omega_1^3 - 6\omega_3 c_s^2 \omega_1^3 + 144v_2^2 \omega_3 c_s^2 \omega_1^2 - 48\omega_3^2 c_s^4 \omega_1 + 24\omega_3 c_s^2 \omega_1^2 - 12v_2^4 \omega_3^2 \omega_1^2 + 6v_2^2 \omega_3^2 c_s^2 \omega_1^3 - 216v_2^2 c_s^2 \omega_1^2 - 72v_2^4 \omega_1^2 + \omega_3^2 c_s^2 \omega_1^3 + 72v_2^4 \omega_3 \omega_1^2 + 36v_2^4 \omega_1^3 + 108v_2^2 c_s^2 \omega_1^2 + 24\omega_3 c_s^4 \omega_1^2 - 12v_2^2 \omega_3^2 c_s^2 \omega_1^2 - 30v_2^4 \omega_3 \omega_1^2 - 8\omega_3^2 c_s^2 \omega_1^2 + 12\omega_3^2 c_s^2 \omega_1 - 24\omega_3 c_s^4 \omega_1^2 - 72v_2^2 \omega_3 \omega_1^2 - 36v_2^2 \omega_3^2 c_s^2 \omega_1 + 30v_2^2 \omega_3 \omega_1^3 + 24\omega_3^2 c_s^4 + 6\omega_3 c_s^4 \omega_1^2) \frac{\rho}{24\omega_3^2 \omega_1^3}$$

**coefficient**  $C_{D_y^4 v_2}^{(1)}$  at  $\frac{\partial^4 v_2}{\partial x_2^4}$ :

$$C_{D_y^4 v_2}^{(1), \text{SRT}} = (-4 - 5v_2^2 \omega + 6c_s^2 - 3c_s^2 \omega + 10v_2^2 + 2\omega) \frac{v_1 \rho v_2}{12\omega}$$

$$C_{D_y^4 v_2}^{(1), \text{MRT1}} = (72\omega_8^2 c_s^2 \omega_4 - 72\omega_8 \omega_6^2 \omega_4 - 48\omega_8^2 \omega_4 + 39\omega_8^2 \omega_6 c_s^2 \omega_4^2 - 48\omega_8^2 c_s^2 \omega_4 + 120\omega_8^2 v_2^2 \omega_4 + 72\omega_8^2 \omega_6 \omega_4 + 61\omega_8^2 v_2^2 \omega_6 \omega_4^2 +$$

$$120\omega_8 \omega_6^2 \omega_4 + 168\omega_8 v_2^2 \omega_6 \omega_4 - 33\omega_8 \omega_6^2 c_s^2 \omega_4^2 - 168\omega_8^2 v_2^2 \omega_6 \omega_4 - 51\omega_8 v_2^2 \omega_6 \omega_4^2 - 36\omega_8^2 \omega_6 \omega_4^2 - 60\omega_8^2 v_2^2 \omega_6 \omega_4^2 - 25\omega_8^2 \omega_6 \omega_4^2 + 24v_2^2 \omega_6 \omega_4^2 - 120\omega_8^2 \omega_6 c_s^2 \omega_4 + 24\omega_8^2 \omega_6^2 \omega_4^2 + 24\omega_8^2 c_s^2 \omega_4^2 - 36\omega_8^2 c_s^2 \omega_4^2 + 84\omega_8^2 v_2^2 \omega_6 + 21\omega_8 \omega_6^2 \omega_4^2 + 12\omega_8 \omega_6 c_s^2 \omega_4^2 - 12\omega_8 \omega_6 \omega_4^2 + 24\omega_8^2 \omega_4 + 36\omega_8 v_2^2 \omega_6 \omega_4^2 + 2\omega_8^2 \omega_6 \omega_4^2 - 84\omega_8 v_2^2 \omega_6^2 - 60\omega_8 \omega_6^2 c_s^2 - 5\omega_8^2 v_2^2 \omega_6^2 \omega_4^2 + 36\omega_8 \omega_6^2 - 3\omega_8^2 \omega_6^2 c_s^2 \omega_4^2 - 72\omega_8 v_2^2 \omega_6 \omega_4 + 24\omega_8 \omega_6 \omega_4 - 12\omega_8^2 \omega_4^2 + 60\omega_8^2 \omega_6 c_s^2 - 24\omega_8 \omega_6 c_s^2 \omega_4) \frac{v_1 \rho v_2}{12\omega_8^2 \omega_6^2}$$

$$C_{D_y^4 v_2}^{(1), \text{MRT2}} = C_{D_y^4 v_2}^{(1), \text{MRT1}}$$

$$C_{D_y^4 v_2}^{(1), \text{CLBM1}} = (-4 - 5v_2^2 \omega_6 + 6c_s^2 + 2\omega_6 - 3\omega_6 c_s^2 + 10v_2^2) \frac{v_1 \rho v_2}{12\omega_6}$$

$$C_{D_y^4 v_2}^{(1), \text{CLBM2}} = C_{D_y^4 v_2}^{(1), \text{CLBM1}}$$

$$C_{D_y^4 v_2}^{(1), \text{CuLBM1}} = (-4 - 5\omega_2 v_2^2 + 2\omega_2 + 6c_s^2 - 3\omega_2 c_s^2 + 10v_2^2) \frac{v_1 \rho v_2}{12\omega_2}$$

$$C_{D_y^4 v_2}^{(1), \text{CuLBM2}} = (5\omega_2 v_2^2 - 2\omega_2 + 5v_2^2 \omega_1 + 2\omega_2 \omega_1 + 3c_s^2 \omega_1 + 3\omega_2 c_s^2 - 3\omega_2 c_s^2 \omega_1 - 5\omega_2 v_2^2 \omega_1 - 2\omega_1) \frac{v_1 \rho v_2}{12\omega_2 \omega_1}$$

### 3.3 Conservation of momentum: $\rho v_2$

$$\begin{aligned}
& v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + v_1 v_2 \frac{\delta_t}{\delta_t} \frac{\partial \rho}{\partial x_1} + \rho v_2 \frac{\delta_t}{\delta_t} \frac{\partial v_1}{\partial x_1} + v_1 \rho \frac{\delta_t}{\delta_t} \frac{\partial v_2}{\partial x_1} + (c_s^2 + v_2^2) \frac{\delta_t}{\delta_t} \frac{\partial \rho}{\partial x_2} + 2\rho v_2 \frac{\delta_t}{\delta_t} \frac{\partial v_2}{\partial x_2} + C_{D_x \rho, D_x v_2}^{(2)} \frac{\delta_t^2}{\delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\
& C_{D_x \rho, D_y v_1}^{(2)} \frac{\delta_t^2}{\delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + C_{D_x v_1, D_y v_1}^{(2)} \frac{\delta_t^2}{\delta_t} \frac{\partial v_1}{\partial x_1} \frac{\partial v_1}{\partial x_2} + C_{D_y \rho, D_x v_1}^{(2)} \frac{\delta_t^2}{\delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_1} + C_{D_y \rho, D_y v_2}^{(2)} \frac{\delta_t^2}{\delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + \\
& C_{D_y v_2, D_y v_2}^{(2)} \frac{\delta_t^2}{\delta_t} \left( \frac{\partial v_2}{\partial x_2} \right)^2 + C_{D_x^2 v_2}^{(2)} \frac{\delta_t^2}{\delta_t} \frac{\partial^2 v_2}{\partial x_1^2} + C_{D_x D_y \rho}^{(2)} \frac{\delta_t^2}{\delta_t} \frac{\partial^2 \rho}{\partial x_1 \partial x_2} + C_{D_x D_y v_1}^{(2)} \frac{\delta_t^2}{\delta_t} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + C_{D_y^2 \rho}^{(2)} \frac{\delta_t^2}{\delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + C_{D_y^2 v_2}^{(2)} \frac{\delta_t^2}{\delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + \\
& + C_{D_x^3 \rho}^{(2)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + C_{D_x^3 v_1}^{(2)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + C_{D_x^3 v_2}^{(2)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_1^3} + C_{D_x^2 D_y \rho}^{(2)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + C_{D_x^2 D_y v_1}^{(2)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} + C_{D_x^2 D_y v_2}^{(2)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + \\
& + C_{D_x D_y^2 \rho}^{(2)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + C_{D_x D_y^2 v_1}^{(2)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_{D_x D_y^2 v_2}^{(2)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_{D_y^3 \rho}^{(2)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + C_{D_y^3 v_2}^{(2)} \frac{\delta_t^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_{D_x^4 \rho}^{(2)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^4} \\
& + C_{D_x^4 v_1}^{(2)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_{D_x^4 v_2}^{(2)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1^4} + C_{D_x^3 D_y \rho}^{(2)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{D_x^3 D_y v_1}^{(2)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + C_{D_x^3 D_y v_2}^{(2)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& C_{D_x^2 D_y^2 \rho}^{(2)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_{D_x^2 D_y^2 v_1}^{(2)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{D_x^2 D_y^2 v_2}^{(2)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_{D_x D_y^3 \rho}^{(2)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{D_x D_y^3 v_1}^{(2)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& C_{D_x D_y^3 v_2}^{(2)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + C_{D_y^4 \rho}^{(2)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{D_y^4 v_2}^{(2)} \frac{\delta_t^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

**coefficient**  $C_{D_x \rho, D_x v_2}^{(2)}$  at  $\frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1}$ :

$$C_{D_x \rho, D_x v_2}^{(2), \text{SRT}} = (-2 + \omega) \frac{c_s^2}{2\omega}$$

$$C_{D_x \rho, D_x v_2}^{(2), \text{MRT1}} = (-2 + \omega_4) \frac{c_s^2}{2\omega_4}$$

$$C_{D_x \rho, D_x v_2}^{(2), \text{MRT2}} = C_{D_x \rho, D_x v_2}^{(2), \text{MRT1}}$$

$$C_{D_x \rho, D_x v_2}^{(2), \text{CLB1}} = C_{D_x \rho, D_x v_2}^{(2), \text{MRT1}}$$

$$C_{D_x \rho, D_x v_2}^{(2), \text{CLB2}} = C_{D_x \rho, D_x v_2}^{(2), \text{MRT1}}$$

$$C_{D_x \rho, D_x v_2}^{(2), \text{CuLBM1}} = (-2 + \omega_3) \frac{c_s^2}{2\omega_3}$$

$$C_{D_x \rho, D_x v_2}^{(2), \text{CuLBM2}} = (-2 + \omega_1) \frac{c_s^2}{2\omega_1}$$

**coefficient**  $C_{D_x \rho, D_y v_1}^{(2)}$  at  $\frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2}$ :

$$C_{D_x \rho, D_y v_1}^{(2), \text{SRT}} = (-2 + \omega) \frac{c_s^2}{2\omega}$$

$$C_{D_x \rho, D_y v_1}^{(2), \text{MRT1}} = (-2 + \omega_4) \frac{c_s^2}{2\omega_4}$$

$$C_{D_x \rho, D_y v_1}^{(2), \text{MRT2}} = C_{D_x \rho, D_y v_1}^{(2), \text{MRT1}}$$

$$C_{D_x \rho, D_y v_1}^{(2), \text{CLB1}} = C_{D_x \rho, D_y v_1}^{(2), \text{MRT1}}$$

$$C_{D_x \rho, D_y v_1}^{(2), \text{CLB2}} = C_{D_x \rho, D_y v_1}^{(2), \text{MRT1}}$$

$$C_{D_x \rho, D_y v_1}^{(2), \text{CuLBM1}} = (-2 + \omega_3) \frac{c_s^2}{2\omega_3}$$

$$C_{D_x \rho, D_y v_1}^{(2), \text{CuLBM2}} = (\omega_2 + 3c_s^2\omega_1 - 5\omega_2 c_s^2 + \omega_2 c_s^2 \omega_1 + 3v_1^2 \omega_1 - \omega_1 - 3\omega_2 v_1^2) \frac{1}{2\omega_2 \omega_1}$$

**coefficient**  $C_{D_x v_1, D_y v_1}^{(2)}$  at  $\frac{\partial v_1}{\partial x_1} \frac{\partial v_1}{\partial x_2}$ :

$$C_{D_x v_1, D_y v_1}^{(2), \text{SRT}} = 0$$

$$C_{D_x v_1, D_y v_1}^{(2), \text{MRT1}} = 0$$

$$C_{D_x v_1, D_y v_1}^{(2), \text{MRT2}} = 0$$

$$C_{D_x v_1, D_y v_1}^{(2), \text{CLBM1}} = 0$$

$$C_{D_x v_1, D_y v_1}^{(2), \text{CLBM2}} = 0$$

$$C_{D_x v_1, D_y v_1}^{(2), \text{CuLBM1}} = 0$$

$$C_{D_x v_1, D_y v_1}^{(2), \text{CuLBM2}} = (-\omega_2 + \omega_1) \frac{3v_1 \rho}{\omega_2 \omega_1}$$

**coefficient**  $C_{D_y \rho, D_x v_1}^{(2)}$  **at**  $\frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_1}$ :

$$C_{D_y \rho, D_x v_1}^{(2), \text{SRT}} = 0$$

$$C_{D_y \rho, D_x v_1}^{(2), \text{MRT1}} = 0$$

$$C_{D_y \rho, D_x v_1}^{(2), \text{MRT2}} = 0$$

$$C_{D_y \rho, D_x v_1}^{(2), \text{CLBM1}} = 0$$

$$C_{D_y \rho, D_x v_1}^{(2), \text{CLBM2}} = 0$$

$$C_{D_y \rho, D_x v_1}^{(2), \text{CuLBM1}} = 0$$

$$C_{D_y \rho, D_x v_1}^{(2), \text{CuLBM2}} = (\omega_2 + c_s^2 \omega_1 - \omega_2 c_s^2 + 3v_1^2 \omega_1 - \omega_1 - 3\omega_2 v_1^2) \frac{1}{2\omega_2 \omega_1}$$

**coefficient**  $C_{D_y \rho, D_y v_2}^{(2)}$  **at**  $\frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2}$ :

$$C_{D_y \rho, D_y v_2}^{(2), \text{SRT}} = (-2 - 3v_2^2 \omega + 4c_s^2 - 2c_s^2 \omega + 6v_2^2 + \omega) \frac{1}{\omega}$$

$$C_{D_y \rho, D_y v_2}^{(2), \text{MRT1}} = (-2 - 3v_2^2 \omega_6 + 4c_s^2 + \omega_6 - 2\omega_6 c_s^2 + 6v_2^2) \frac{1}{\omega_6}$$

$$C_{D_y \rho, D_y v_2}^{(2), \text{MRT2}} = C_{D_y \rho, D_y v_2}^{(2), \text{MRT1}}$$

$$C_{D_y \rho, D_y v_2}^{(2), \text{CLBM1}} = C_{D_y \rho, D_y v_2}^{(2), \text{MRT1}}$$

$$C_{D_y \rho, D_y v_2}^{(2), \text{CLBM2}} = C_{D_y \rho, D_y v_2}^{(2), \text{MRT1}}$$

$$C_{D_y \rho, D_y v_2}^{(2), \text{CuLBM1}} = (-2 - 3\omega_2 v_2^2 + \omega_2 + 4c_s^2 - 2\omega_2 c_s^2 + 6v_2^2) \frac{1}{\omega_2}$$

$$C_{D_y \rho, D_y v_2}^{(2), \text{CuLBM2}} = (3\omega_2 v_2^2 - \omega_2 + 3v_2^2 \omega_1 + \omega_2 \omega_1 + 2c_s^2 \omega_1 + 2\omega_2 c_s^2 - 2\omega_2 c_s^2 \omega_1 - 3\omega_2 v_2^2 \omega_1 - \omega_1) \frac{1}{\omega_2 \omega_1}$$

**coefficient**  $C_{D_y v_2, D_y v_2}^{(2)}$  **at**  $\left(\frac{\partial v_2}{\partial x_2}\right)^2$ :

$$C_{D_y v_2, D_y v_2}^{(2), \text{SRT}} = (2 - \omega) \frac{3\rho v_2}{\omega}$$

$$C_{D_y v_2, D_y v_2}^{(2), \text{MRT1}} = (2 - \omega_6) \frac{3\rho v_2}{\omega_6}$$

$$C_{D_y v_2, D_y v_2}^{(2), \text{MRT2}} = C_{D_y v_2, D_y v_2}^{(2), \text{MRT1}}$$

$$C_{D_y v_2, D_y v_2}^{(2), \text{CLBM1}} = C_{D_y v_2, D_y v_2}^{(2), \text{MRT1}}$$

$$C_{D_y v_2, D_y v_2}^{(2), \text{CLBM2}} = C_{D_y v_2, D_y v_2}^{(2), \text{MRT1}}$$

$$C_{D_y v_2, D_y v_2}^{(2), \text{CuLBM1}} = (2 - \omega_2) \frac{3\rho v_2}{\omega_2}$$

$$C_{D_y v_2, D_y v_2}^{(2), \text{CuLBM2}} = (\omega_2 - \omega_2 \omega_1 + \omega_1) \frac{3\rho v_2}{\omega_2 \omega_1}$$

**coefficient**  $C_{D_x^2 v_2}^{(2)}$  **at**  $\frac{\partial^2 v_2}{\partial x_1^2}$ :

$$C_{D_x^2 v_2}^{(2), SRT} = (-2 + \omega) \frac{\rho c_s^2}{2\omega}$$

$$C_{D_x^2 v_2}^{(2), MRT1} = (-2 + \omega_4) \frac{\rho c_s^2}{2\omega_4}$$

$$C_{D_x^2 v_2}^{(2), MRT2} = C_{D_x^2 v_2}^{(2), MRT1}$$

$$C_{D_x^2 v_2}^{(2), CLBM1} = C_{D_x^2 v_2}^{(2), MRT1}$$

$$C_{D_x^2 v_2}^{(2), CLBM2} = C_{D_x^2 v_2}^{(2), MRT1}$$

$$C_{D_x^2 v_2}^{(2), CuLBM1} = (-2 + \omega_3) \frac{\rho c_s^2}{2\omega_3}$$

$$C_{D_x^2 v_2}^{(2), CuLBM2} = (-2 + \omega_1) \frac{\rho c_s^2}{2\omega_1}$$

**coefficient**  $C_{D_x D_y \rho}^{(2)}$  **at**  $\frac{\partial^2 \rho}{\partial x_1 \partial x_2}$ :

$$C_{D_x D_y \rho}^{(2), SRT} = 0$$

$$C_{D_x D_y \rho}^{(2), MRT1} = 0$$

$$C_{D_x D_y \rho}^{(2), MRT2} = 0$$

$$C_{D_x D_y \rho}^{(2), CLBM1} = 0$$

$$C_{D_x D_y \rho}^{(2), CLBM2} = 0$$

$$C_{D_x D_y \rho}^{(2), CuLBM1} = 0$$

$$C_{D_x D_y \rho}^{(2), CuLBM2} = (\omega_2 + 3c_s^2\omega_1 - 3\omega_2 c_s^2 + v_1^2\omega_1 - \omega_1 - \omega_2 v_1^2) \frac{v_1}{2\omega_2 \omega_1}$$

**coefficient**  $C_{D_x D_y v_1}^{(2)}$  **at**  $\frac{\partial^2 v_1}{\partial x_1 \partial x_2}$ :

$$C_{D_x D_y v_1}^{(2), SRT} = (-2 + \omega) \frac{\rho c_s^2}{2\omega}$$

$$C_{D_x D_y v_1}^{(2), MRT1} = (-2 + \omega_4) \frac{\rho c_s^2}{2\omega_4}$$

$$C_{D_x D_y v_1}^{(2), MRT2} = C_{D_x D_y v_1}^{(2), MRT1}$$

$$C_{D_x D_y v_1}^{(2), CLBM1} = C_{D_x D_y v_1}^{(2), MRT1}$$

$$C_{D_x D_y v_1}^{(2), CLBM2} = C_{D_x D_y v_1}^{(2), MRT1}$$

$$C_{D_x D_y v_1}^{(2), CuLBM1} = (-2 + \omega_3) \frac{\rho c_s^2}{2\omega_3}$$

$$C_{D_x D_y v_1}^{(2), CuLBM2} = (\omega_2 + c_s^2\omega_1 - 3\omega_2 c_s^2 + \omega_2 c_s^2\omega_1 + 3v_1^2\omega_1 - \omega_1 - 3\omega_2 v_1^2) \frac{\rho}{2\omega_2 \omega_1}$$

**coefficient**  $C_{D_y^2 \rho}^{(2)}$  **at**  $\frac{\partial^2 \rho}{\partial x_2^2}$ :

$$C_{D_y^2 \rho}^{(2), SRT} = (-2 - v_2^2\omega + 6c_s^2 - 3c_s^2\omega + 2v_2^2 + \omega) \frac{v_2}{2\omega}$$

$$C_{D_y^2 \rho}^{(2), MRT1} = (-2 - v_2^2\omega_6 + 6c_s^2 + \omega_6 - 3\omega_6 c_s^2 + 2v_2^2) \frac{v_2}{2\omega_6}$$

$$C_{D_y^2 \rho}^{(2), \text{MRT2}} = C_{D_y^2 \rho}^{(2), \text{MRT1}}$$

$$C_{D_y^2 \rho}^{(2), \text{CLBM1}} = C_{D_y^2 \rho}^{(2), \text{MRT1}}$$

$$C_{D_y^2 \rho}^{(2), \text{CLBM2}} = C_{D_y^2 \rho}^{(2), \text{MRT1}}$$

$$C_{D_y^2 \rho}^{(2), \text{CuLBM1}} = (-2 - \omega_2 v_2^2 + \omega_2 + 6c_s^2 - 3\omega_2 c_s^2 + 2v_2^2) \frac{v_2}{2\omega_2}$$

$$C_{D_y^2 \rho}^{(2), \text{CuLBM2}} = (\omega_2 v_2^2 - \omega_2 + v_2^2 \omega_1 + \omega_2 \omega_1 + 3c_s^2 \omega_1 + 3\omega_2 c_s^2 - 3\omega_2 c_s^2 \omega_1 - \omega_2 v_2^2 \omega_1 - \omega_1) \frac{v_2}{2\omega_2 \omega_1}$$

**coefficient**  $C_{D_y^2 v_2}^{(2)}$  **at**  $\frac{\partial^2 v_2}{\partial x_2^2}$ :

$$C_{D_y^2 v_2}^{(2), \text{SRT}} = (-2 - 3v_2^2 \omega + 2c_s^2 - c_s^2 \omega + 6v_2^2 + \omega) \frac{\rho}{2\omega}$$

$$C_{D_y^2 v_2}^{(2), \text{MRT1}} = (-2 - 3v_2^2 \omega_6 + 2c_s^2 + \omega_6 - \omega_6 c_s^2 + 6v_2^2) \frac{\rho}{2\omega_6}$$

$$C_{D_y^2 v_2}^{(2), \text{MRT2}} = C_{D_y^2 v_2}^{(2), \text{MRT1}}$$

$$C_{D_y^2 v_2}^{(2), \text{CLBM1}} = C_{D_y^2 v_2}^{(2), \text{MRT1}}$$

$$C_{D_y^2 v_2}^{(2), \text{CLBM2}} = C_{D_y^2 v_2}^{(2), \text{MRT1}}$$

$$C_{D_y^2 v_2}^{(2), \text{CuLBM1}} = (-2 - 3\omega_2 v_2^2 + \omega_2 + 2c_s^2 - \omega_2 c_s^2 + 6v_2^2) \frac{\rho}{2\omega_2}$$

$$C_{D_y^2 v_2}^{(2), \text{CuLBM2}} = (3\omega_2 v_2^2 - \omega_2 + 3v_2^2 \omega_1 + \omega_2 \omega_1 + c_s^2 \omega_1 + \omega_2 c_s^2 - \omega_2 c_s^2 \omega_1 - 3\omega_2 v_2^2 \omega_1 - \omega_1) \frac{\rho}{2\omega_2 \omega_1}$$

**coefficient**  $C_{D_x^3 \rho}^{(2)}$  **at**  $\frac{\partial^3 \rho}{\partial x_1^3}$ :

$$C_{D_x^3 \rho}^{(2), \text{SRT}} = (-1 + v_1^2 + 3c_s^2) \frac{v_1 v_2}{12}$$

$$C_{D_x^3 \rho}^{(2), \text{MRT1}} = (-12\omega_5 v_1^2 + 12\omega_5 - 6v_1^2 \omega_7 \omega_4 + 6\omega_7 \omega_4 - \omega_5 \omega_7 \omega_4 + \omega_5 v_1^2 \omega_7 \omega_4 + 12v_1^2 \omega_7 + 18\omega_5 c_s^2 \omega_4 - 36\omega_5 c_s^2 + 36c_s^2 \omega_7 - 12\omega_7 - 6\omega_5 \omega_4 + 6\omega_5 v_1^2 \omega_4 - 18c_s^2 \omega_7 \omega_4 + 3\omega_5 c_s^2 \omega_7 \omega_4) \frac{v_1 v_2}{12\omega_5 \omega_7 \omega_4}$$

$$C_{D_x^3 \rho}^{(2), \text{MRT2}} = C_{D_x^3 \rho}^{(2), \text{MRT1}}$$

$$C_{D_x^3 \rho}^{(2), \text{CLBM1}} = C_{D_x^3 \rho}^{(2), \text{SRT}}$$

$$C_{D_x^3 \rho}^{(2), \text{CLBM2}} = C_{D_x^3 \rho}^{(2), \text{SRT}}$$

$$C_{D_x^3 \rho}^{(2), \text{CuLBM1}} = C_{D_x^3 \rho}^{(2), \text{SRT}}$$

$$C_{D_x^3 \rho}^{(2), \text{CuLBM2}} = C_{D_x^3 \rho}^{(2), \text{SRT}}$$

**coefficient**  $C_{D_x^3 v_1}^{(2)}$  **at**  $\frac{\partial^3 v_1}{\partial x_1^3}$ :

$$C_{D_x^3 v_1}^{(2), \text{SRT}} = (-1 + 3v_1^2 + c_s^2) \frac{\rho v_2}{12}$$

$$C_{D_x^3 v_1}^{(2), \text{MRT1}} = (-36\omega_5 v_1^2 + 12\omega_5 - 18v_1^2 \omega_7 \omega_4 + 6\omega_7 \omega_4 - \omega_5 \omega_7 \omega_4 + 3\omega_5 v_1^2 \omega_7 \omega_4 + 36v_1^2 \omega_7 + 6\omega_5 c_s^2 \omega_4 - 12\omega_5 c_s^2 + 12c_s^2 \omega_7 - 12\omega_7 - 6\omega_5 \omega_4 + 18\omega_5 v_1^2 \omega_4 - 6c_s^2 \omega_7 \omega_4 + \omega_5 c_s^2 \omega_7 \omega_4) \frac{\rho v_2}{12\omega_5 \omega_7 \omega_4}$$

$$C_{D_x^3 v_1}^{(2), \text{MRT2}} = C_{D_x^3 v_1}^{(2), \text{MRT1}}$$

$$C_{D_x^3 v_1}^{(2), CLBM1} = C_{D_x^3 v_1}^{(2), SRT}$$

$$C_{D_x^3 v_1}^{(2), CLBM2} = C_{D_x^3 v_1}^{(2), SRT}$$

$$C_{D_x^3 v_1}^{(2), CuLBM1} = C_{D_x^3 v_1}^{(2), SRT}$$

$$C_{D_x^3 v_1}^{(2), CuLBM2} = C_{D_x^3 v_1}^{(2), SRT}$$

**coefficient**  $C_{D_x^3 v_2}^{(2)}$  **at**  $\frac{\partial^3 v_2}{\partial x^3}$ :

$$C_{D_x^3 v_2}^{(2), SRT} = (6 - 3c_s^2\omega^2 - 6v_1^2 - 18c_s^2 + 18c_s^2\omega + 6v_1^2\omega - 6\omega + \omega^2 - v_1^2\omega^2) \frac{v_1\rho}{6\omega^2}$$

$$C_{D_x^3 v_2}^{(2), MRT1} = (3c_s^2\omega_4^2 + 3v_1^2\omega_7\omega_4 - 3\omega_7\omega_4 + \omega_7\omega_4^2 - 6c_s^2\omega_4 - v_1^2\omega_7\omega_4^2 - 3c_s^2\omega_7\omega_4^2 - 12c_s^2\omega_7 - 6v_1^2\omega_4 + 3v_1^2\omega_4^2 + 6\omega_4 - 3\omega_4^2 + 15c_s^2\omega_7\omega_4) \frac{v_1\rho}{6\omega_7\omega_4^2}$$

$$C_{D_x^3 v_2}^{(2), MRT2} = C_{D_x^3 v_2}^{(2), MRT1}$$

$$C_{D_x^3 v_2}^{(2), CLBM1} = (6 - 6v_1^2 - v_1^2\omega_7\omega_4 + \omega_7\omega_4 + 3v_1^2\omega_7 + 9c_s^2\omega_4 - 18c_s^2 + 9c_s^2\omega_7 + 3v_1^2\omega_4 - 3\omega_7 - 3\omega_4 - 3c_s^2\omega_7\omega_4) \frac{v_1\rho}{6\omega_7\omega_4}$$

$$C_{D_x^3 v_2}^{(2), CLBM2} = C_{D_x^3 v_2}^{(2), CLBM1}$$

$$C_{D_x^3 v_2}^{(2), CuLBM1} = (6 + \omega_3\omega_4 - v_1^2\omega_3\omega_4 - 3\omega_3c_s^2\omega_4 - 6v_1^2 + 9c_s^2\omega_4 - 18c_s^2 - 3\omega_3 + 3v_1^2\omega_4 - 3\omega_4 + 9\omega_3c_s^2 + 3v_1^2\omega_3) \frac{v_1\rho}{6\omega_3\omega_4}$$

$$C_{D_x^3 v_2}^{(2), CuLBM2} = (6 - 3\omega_3c_s^2\omega_1 + \omega_3\omega_1 - 6v_1^2 - v_1^2\omega_3\omega_1 - 18c_s^2 + 9c_s^2\omega_1 - 3\omega_3 + 3v_1^2\omega_1 + 9\omega_3c_s^2 + 3v_1^2\omega_3 - 3\omega_1) \frac{v_1\rho}{6\omega_3\omega_1}$$

**coefficient**  $C_{D_x^2 D_y \rho}^{(2)}$  **at**  $\frac{\partial^3 \rho}{\partial x_1^2 \partial x_2}$ :

$$C_{D_x^2 D_y \rho}^{(2), SRT} = (-12 + 12\omega - \omega^2) \frac{c_s^4}{6\omega^2}$$

$$C_{D_x^2 D_y \rho}^{(2), MRT1} = (-12 + 12\omega_4 - \omega_4^2) \frac{c_s^4}{6\omega_4^2}$$

$$C_{D_x^2 D_y \rho}^{(2), MRT2} = C_{D_x^2 D_y \rho}^{(2), MRT1}$$

$$C_{D_x^2 D_y \rho}^{(2), CLBM1} = C_{D_x^2 D_y \rho}^{(2), MRT1}$$

$$C_{D_x^2 D_y \rho}^{(2), CLBM2} = C_{D_x^2 D_y \rho}^{(2), MRT1}$$

$$C_{D_x^2 D_y \rho}^{(2), CuLBM1} = (-12 + 12\omega_3 - \omega_3^2) \frac{c_s^4}{6\omega_3^2}$$

$$C_{D_x^2 D_y \rho}^{(2), CuLBM2} = (-6c_s^2\omega_1^2 + 9\omega_2^2v_1^4\omega_1 + 45\omega_2^2v_1^2c_s^2\omega_1 + 9\omega_2v_1^2\omega_1^2 + 45v_1^2c_s^2\omega_1^2 + 9\omega_2^2v_1^2 - 6\omega_2^2c_s^2\omega_1 + 9v_1^4\omega_1^2 - 45\omega_2^2v_1^2c_s^2 - 6\omega_2c_s^4\omega_1^2 + 6\omega_2^2c_s^2 - 2\omega_2^2c_s^4\omega_1^2 - 9\omega_2^2v_1^2\omega_1 + 6c_s^4\omega_1^2 - 9\omega_2v_1^4\omega_1^2 - 30\omega_2^2c_s^4 - 9\omega_2^2v_1^4 - 45\omega_2v_1^2c_s^2\omega_1^2 - 9v_1^2\omega_1^2 + 30\omega_2^2c_s^4\omega_1 + 6\omega_2c_s^2\omega_1^2) \frac{1}{12\omega_2^2\omega_1^2}$$

**coefficient**  $C_{D_x^2 D_y v_1}^{(2)}$  **at**  $\frac{\partial^3 v_1}{\partial x_1^2 \partial x_2}$ :

$$C_{D_x^2 D_y v_1}^{(2), SRT} = 0$$

$$C_{D_x^2 D_y v_1}^{(2), MRT1} = (\omega_7\omega_4 - 2\omega_7 + 2\omega_4 - \omega_4^2) \frac{v_1\rho c_s^2}{\omega_7\omega_4^2}$$

$$C_{D_x^2 D_y v_1}^{(2), MRT2} = C_{D_x^2 D_y v_1}^{(2), MRT1}$$

$$C_{D_x^2 D_y v_1}^{(2), CLBM1} = 0$$

$$C_{D_x^2 D_y v_1}^{(2), CLBM2} = 0$$

$$C_{D_x^2 D_y v_1}^{(2), \text{CuLBM1}} = 0$$

$$C_{D_x^2 D_y v_1}^{(2), \text{CuLBM2}} = (9c_s^2\omega_1^2 + 5\omega_2\omega_1^2 - 11\omega_2v_1^2\omega_1^2 - 11\omega_2^2v_1^2 + 5\omega_2^2 + 9\omega_2^2c_s^2\omega_1 - 9\omega_2^2c_s^2 - 5\omega_2^2\omega_1 + 11\omega_2^2v_1^2\omega_1 - 5\omega_1^2 + 11v_1^2\omega_1^2 - 9\omega_2c_s^2\omega_1^2) \frac{v_1\rho}{4\omega_2^2\omega_1^2}$$

**coefficient**  $C_{D_x^2 D_y v_2}^{(2)}$  at  $\frac{\partial^3 v_2}{\partial x_1^2 \partial x_2}$ :

$$C_{D_x^2 D_y v_2}^{(2), \text{SRT}} = \frac{-\rho v_2 c_s^2}{6}$$

$$C_{D_x^2 D_y v_2}^{(2), \text{MRT1}} = (-12\omega_6\omega_4^2 + 12\omega_6\omega_4 - \omega_8\omega_6\omega_4^2 - 12\omega_8\omega_6 + 12\omega_8\omega_6\omega_4 + 12\omega_4^2 - 12\omega_8\omega_4) \frac{\rho v_2 c_s^2}{6\omega_8\omega_6\omega_4^2}$$

$$C_{D_x^2 D_y v_2}^{(2), \text{MRT2}} = C_{D_x^2 D_y v_2}^{(2), \text{MRT1}}$$

$$C_{D_x^2 D_y v_2}^{(2), \text{CLBM1}} = C_{D_x^2 D_y v_2}^{(2), \text{SRT}}$$

$$C_{D_x^2 D_y v_2}^{(2), \text{CLBM2}} = C_{D_x^2 D_y v_2}^{(2), \text{SRT}}$$

$$C_{D_x^2 D_y v_2}^{(2), \text{CuLBM1}} = C_{D_x^2 D_y v_2}^{(2), \text{SRT}}$$

$$C_{D_x^2 D_y v_2}^{(2), \text{CuLBM2}} = C_{D_x^2 D_y v_2}^{(2), \text{SRT}}$$

**coefficient**  $C_{D_x D_y^2 \rho}^{(2)}$  at  $\frac{\partial^3 \rho}{\partial x_1 \partial x_2}$ :

$$C_{D_x D_y^2 \rho}^{(2), \text{SRT}} = 0$$

$$C_{D_x D_y^2 \rho}^{(2), \text{MRT1}} = (v_2^2\omega_6^2\omega_4 + 3\omega_6^2c_s^2\omega_4 + 3\omega_8\omega_6c_s^2 - 3\omega_6^2c_s^2 - v_2^2\omega_6^2 + \omega_6\omega_4 - \omega_6^2\omega_4 - \omega_8\omega_6 + \omega_8v_2^2\omega_6 + \omega_6^2 + 3\omega_8c_s^2\omega_4 - 3\omega_6c_s^2\omega_4 - \omega_8v_2^2\omega_6\omega_4 + \omega_8\omega_6\omega_4 + \omega_8v_2^2\omega_4 - v_2^2\omega_6\omega_4 - \omega_8\omega_4 - 3\omega_8\omega_6c_s^2\omega_4) \frac{v_1 v_2}{\omega_8\omega_6^2\omega_4}$$

$$C_{D_x D_y^2 \rho}^{(2), \text{MRT2}} = C_{D_x D_y^2 \rho}^{(2), \text{MRT1}}$$

$$C_{D_x D_y^2 \rho}^{(2), \text{CLBM1}} = 0$$

$$C_{D_x D_y^2 \rho}^{(2), \text{CLBM2}} = 0$$

$$C_{D_x D_y^2 \rho}^{(2), \text{CuLBM1}} = 0$$

$$C_{D_x D_y^2 \rho}^{(2), \text{CuLBM2}} = (6c_s^2\omega_1^2 + \omega_2\omega_1^2 - \omega_2v_1^2\omega_1^2 - \omega_2^2v_1^2 + v_2^2\omega_1^2 + 3\omega_2^2c_s^2\omega_1 + 2\omega_2\omega_1 - \omega_2^2\omega_1 - 6\omega_2c_s^2\omega_1 - 2\omega_2v_2^2\omega_1 + \omega_2^2v_1^2\omega_1 - 2\omega_1^2 + v_1^2\omega_1^2 - 3\omega_2c_s^2\omega_1^2 + \omega_2^2v_2^2) \frac{3v_1 v_2}{4\omega_2^2\omega_1^2}$$

**coefficient**  $C_{D_x D_y^2 v_1}^{(2)}$  at  $\frac{\partial^3 v_1}{\partial x_1 \partial x_2}$ :

$$C_{D_x D_y^2 v_1}^{(2), \text{SRT}} = (12 - 11c_s^2\omega^2 - 3v_2^2\omega^2 + 12v_2^2\omega - 36c_s^2 + 36c_s^2\omega - 12v_2^2 - 12\omega + 3\omega^2) \frac{\rho v_2}{12\omega^2}$$

$$C_{D_x D_y^2 v_1}^{(2), \text{MRT1}} = (-6\omega_8\omega_6^2\omega_4 + 12\omega_6\omega_4^2 - 12v_2^2\omega_6^2\omega_4 - 12\omega_6^2c_s^2\omega_4 + 42\omega_8\omega_6^2c_s^2\omega_4 + 6\omega_8v_2^2\omega_6^2\omega_4 - 11\omega_8\omega_6^2c_s^2\omega_4^2 - 3\omega_8v_2^2\omega_6^2\omega_4^2 + 12v_2^2\omega_6^2\omega_4^2 + 12\omega_6^2c_s^2\omega_4^2 + 3\omega_8\omega_6^2\omega_4^2 - 12\omega_8\omega_6^2 - 12v_2^2\omega_6\omega_4^2 - 18\omega_8\omega_6c_s^2\omega_4^2 + 6\omega_8\omega_6\omega_4^2 + 12\omega_6^2\omega_4 + 12\omega_8v_2^2\omega_4^2 - 12\omega_6c_s^2\omega_4^2 - 6\omega_8v_2^2\omega_6\omega_4^2 + 36\omega_8c_s^2\omega_4^2 - 24\omega_8\omega_6^2c_s^2\omega_4) \frac{\rho v_2}{12\omega_8\omega_6^2\omega_4^2}$$

$$C_{D_x D_y^2 v_1}^{(2), \text{MRT2}} = C_{D_x D_y^2 v_1}^{(2), \text{MRT1}}$$

$$C_{D_x D_y^2 v_1}^{(2), \text{CLBM1}} = (3\omega_8\omega_6^2\omega_4 + 12v_2^2\omega_6^2\omega_4 + 36\omega_6^2c_s^2\omega_4 - 11\omega_8\omega_6^2c_s^2\omega_4 - 3\omega_8v_2^2\omega_6^2\omega_4 - 36\omega_6^2c_s^2 - 12v_2^2\omega_6^2 + 12\omega_6\omega_4 - 12\omega_6^2\omega_4 + 12\omega_6^2 + 6\omega_8v_2^2\omega_6^2 + 18\omega_8\omega_6^2c_s^2 + 36\omega_8c_s^2\omega_4 - 36\omega_6c_s^2\omega_4 - 6\omega_8\omega_6^2 - 6\omega_8v_2^2\omega_6\omega_4 + 6\omega_8\omega_6\omega_4 + 12\omega_8v_2^2\omega_4 - 12v_2^2\omega_6\omega_4 - 12\omega_8\omega_4 - 18\omega_8\omega_6c_s^2\omega_4) \frac{\rho v_2}{12\omega_8\omega_6^2\omega_4}$$

$$C_{\text{D}_x \text{D}_y^2 v_1}^{(2), \text{CLBM2}} = C_{\text{D}_x \text{D}_y^2 v_1}^{(2), \text{CLBM1}}$$

$$C_{\text{D}_x \text{D}_y^2 v_1}^{(2), \text{CuLBM1}} = (6\omega_2^2 v_2^2 \omega_6 - 12\omega_6 \omega_3 + 12\omega_2 \omega_3 - 11\omega_2^2 \omega_6 \omega_3 c_s^2 + 12\omega_2^2 v_2^2 \omega_3 + 12\omega_2^2 + 3\omega_2^2 \omega_6 \omega_3 - 36\omega_2 \omega_3 c_s^2 + 36\omega_6 \omega_3 c_s^2 - 6\omega_2 v_2^2 \omega_6 \omega_3 - 36\omega_2^2 c_s^2 + 18\omega_2^2 \omega_6 c_s^2 + 12v_2^2 \omega_6 \omega_3 - 18\omega_2 \omega_6 \omega_3 c_s^2 - 12\omega_2^2 \omega_3 + 6\omega_2 \omega_6 \omega_3 - 12\omega_2 v_2^2 \omega_3 - 3\omega_2^2 v_2^2 \omega_6 \omega_3 - 6\omega_2^2 \omega_6 - 12\omega_2^2 v_2^2 + 36\omega_2^2 \omega_3 c_s^2) \frac{\rho v_2}{12\omega_2^2 \omega_6 \omega_3}$$

$$C_{\text{D}_x \text{D}_y^2 v_1}^{(2), \text{CuLBM2}} = (-12\omega_2^2 \omega_3 \omega_1 - 11\omega_2^2 \omega_3 c_s^2 \omega_1 + 6\omega_2 \omega_1^2 + 12\omega_2^2 v_2^2 \omega_1^2 - 3\omega_2 v_2^2 \omega_3 \omega_1^2 + 6\omega_2^2 v_2^2 \omega_3 + 36\omega_2^2 c_s^2 \omega_1^2 - 54\omega_2^2 c_s^2 \omega_1 + 27v_1^2 \omega_3 \omega_1^2 - 15\omega_3 \omega_1^2 + 27\omega_2^2 v_1^2 \omega_3 \omega_1 + 3\omega_2^2 \omega_3 \omega_1^2 + 18\omega_2^2 \omega_3 c_s^2 \omega_1 - 18\omega_2^2 v_2^2 \omega_1 + 27\omega_3 c_s^2 \omega_1^2 + 18\omega_2^2 \omega_1 - 18\omega_2 \omega_3 c_s^2 \omega_1^2 + 12\omega_2 \omega_3 \omega_1^2 - 27\omega_2 v_1^2 \omega_3 \omega_1^2 - 3\omega_2^2 v_2^2 \omega_3 \omega_1^2 + 3\omega_2^2 \omega_3 + 3\omega_2^2 v_2^2 \omega_3 \omega_1 - 6\omega_2 v_2^2 \omega_1^2 + 6v_2^2 \omega_3 \omega_1^2 - 27\omega_2^2 v_1^2 \omega_3 - 12\omega_2^2 \omega_1^2 - 18\omega_2 c_s^2 \omega_1^2 + 9\omega_2^2 \omega_3 c_s^2) \frac{\rho v_2}{12\omega_2^2 \omega_3 \omega_1^2}$$

coefficient  $C_{\text{D}_x \text{D}_y^2 v_2}^{(2)}$  at  $\frac{\partial^3 v_2}{\partial x_1 \partial x_2^2}$ :

$$C_{\text{D}_x \text{D}_y^2 v_2}^{(2), \text{SRT}} = 0$$

$$C_{\text{D}_x \text{D}_y^2 v_2}^{(2), \text{MRT1}} = (3v_2^2 \omega_6^2 \omega_4 + \omega_6^2 c_s^2 \omega_4 + \omega_8 \omega_6 c_s^2 - \omega_6^2 c_s^2 - 3v_2^2 \omega_6^2 + \omega_6 \omega_4 - \omega_6^2 \omega_4 - \omega_8 \omega_6 + 3\omega_8 v_2^2 \omega_6 + \omega_6^2 + \omega_8 c_s^2 \omega_4 - \omega_6 c_s^2 \omega_4 - 3\omega_8 v_2^2 \omega_6 \omega_4 + \omega_8 \omega_6 \omega_4 + 3\omega_8 v_2^2 \omega_4 - 3v_2^2 \omega_6 \omega_4 - \omega_8 \omega_4 - \omega_8 \omega_6 c_s^2 \omega_4) \frac{v_1 \rho}{\omega_8 \omega_6 \omega_4}$$

$$C_{\text{D}_x \text{D}_y^2 v_2}^{(2), \text{MRT2}} = C_{\text{D}_x \text{D}_y^2 v_2}^{(2), \text{MRT1}}$$

$$C_{\text{D}_x \text{D}_y^2 v_2}^{(2), \text{CLBM1}} = 0$$

$$C_{\text{D}_x \text{D}_y^2 v_2}^{(2), \text{CLBM2}} = 0$$

$$C_{\text{D}_x \text{D}_y^2 v_2}^{(2), \text{CuLBM1}} = 0$$

$$C_{\text{D}_x \text{D}_y^2 v_2}^{(2), \text{CuLBM2}} = (-\omega_2^2 \omega_3 \omega_1 + 2\omega_2 \omega_1^2 - 2\omega_2 v_1^2 \omega_1^2 + 9\omega_2^2 v_2^2 \omega_3 + 6\omega_2^2 c_s^2 \omega_1 + 2v_1^2 \omega_3 \omega_1^2 - 5\omega_3 \omega_1^2 - 18\omega_2 v_2^2 \omega_3 \omega_1 + \omega_2^2 v_2^2 \omega_3 \omega_1 + 3\omega_2^2 \omega_3 c_s^2 \omega_1 + 9\omega_3 c_s^2 \omega_1^2 - 2\omega_2^2 \omega_1 - 3\omega_2 \omega_3 c_s^2 \omega_1^2 + 2\omega_2^2 v_1^2 \omega_1 + \omega_2 \omega_3 \omega_1^2 - \omega_2 v_1^2 \omega_3 \omega_1^2 - \omega_2^2 \omega_3 + 9v_2^2 \omega_3 \omega_1^2 + 6\omega_2 \omega_3 \omega_1 - 2\omega_2^2 v_1^2 \omega_3 - 6\omega_2 c_s^2 \omega_1^2 - 6\omega_2 \omega_3 c_s^2 \omega_1 - 3\omega_2^2 \omega_3 c_s^2) \frac{v_1 \rho}{4\omega_2^2 \omega_3 \omega_1^2}$$

coefficient  $C_{\text{D}_y^3 \rho}^{(2)}$  at  $\frac{\partial^3 \rho}{\partial x_2^3}$ :

$$C_{\text{D}_y^3 \rho}^{(2), \text{SRT}} = (-c_s^2 \omega^2 + 36v_2^4 - 7v_2^2 \omega^2 + 36v_2^2 \omega - 12c_s^2 + 12c_s^2 \omega + 12c_s^4 + 24v_2^2 c_s^2 \omega^2 + 7v_2^4 \omega^2 + c_s^4 \omega^2 + 144v_2^2 c_s^2 - 12c_s^4 \omega - 36v_2^2 - 144v_2^2 c_s^2 \omega - 36v_2^4 \omega) \frac{1}{12\omega^2}$$

$$C_{\text{D}_y^3 \rho}^{(2), \text{MRT1}} =$$

$$(36v_2^4 + 36v_2^2 \omega_6 - 144v_2^2 \omega_6 c_s^2 - \omega_6^2 c_s^2 - 12c_s^2 - 7v_2^2 \omega_6^2 - 12\omega_6 c_s^4 + \omega_6^2 c_s^4 - 36v_2^4 \omega_6 + 12c_s^4 + 144v_2^2 c_s^2 + 12\omega_6 c_s^2 - 36v_2^2 + 24v_2^2 \omega_6^2 c_s^2 + 7v_2^4 \omega_6^2) \frac{1}{12\omega_6^2}$$

$$C_{\text{D}_y^3 \rho}^{(2), \text{MRT2}} = C_{\text{D}_y^3 \rho}^{(2), \text{MRT1}}$$

$$C_{\text{D}_y^3 \rho}^{(2), \text{CLBM1}} = C_{\text{D}_y^3 \rho}^{(2), \text{MRT1}}$$

$$C_{\text{D}_y^3 \rho}^{(2), \text{CLBM2}} = C_{\text{D}_y^3 \rho}^{(2), \text{MRT1}}$$

$$C_{\text{D}_y^3 \rho}^{(2), \text{CuLBM1}} =$$

$$(36v_2^4 + 36\omega_2 v_2^2 + 7\omega_2^2 v_2^4 - 144\omega_2 v_2^2 c_s^2 - 12c_s^2 - 12\omega_2 c_s^4 - \omega_2^2 c_s^2 + 24\omega_2^2 v_2^2 c_s^2 + 12c_s^4 + 12\omega_2 c_s^2 + 144v_2^2 c_s^2 + \omega_2^2 c_s^4 - 36v_2^2 - 36\omega_2 v_2^4 - 7\omega_2^2 v_2^2) \frac{1}{12\omega_2^2}$$

$$C_{\text{D}_y^3 \rho}^{(2), \text{CuLBM2}} = (18\omega_2 v_2^4 \omega_1 + 54\omega_2 v_2^2 c_s^2 \omega_1 - 6c_s^2 \omega_1^2 - 7\omega_2^2 v_2^2 \omega_1^2 + 9\omega_2^2 v_2^4 - 9v_2^2 \omega_1^2 - \omega_2^2 c_s^2 \omega_1^2 + 6\omega_2^2 c_s^2 \omega_1 - 6\omega_2 c_s^4 \omega_1^2 - 6\omega_2^2 c_s^2 + 18\omega_2^2 v_2^2 \omega_1 - 72\omega_2^2 c_s^2 \omega_1^2 - 18\omega_2 v_2^2 \omega_1 + 6c_s^4 \omega_1^2 + 6\omega_2^2 c_s^4 + 7\omega_2^2 v_2^4 \omega_1^2 - 18\omega_2^2 v_2^4 \omega_1 + 18\omega_2 v_2^2 \omega_1^2 - 6\omega_2^2 c_s^4 \omega_1 + 24\omega_2^2 v_2^2 c_s^2 \omega_1^2 + 6\omega_2 c_s^2 \omega_1^2 - 9\omega_2^2 v_2^2) \frac{1}{12\omega_2^2 \omega_1^2}$$

coefficient  $C_{\text{D}_y^3 v_2}^{(2)}$  at  $\frac{\partial^3 v_2}{\partial x_2^3}$ :

$$C_{\text{D}_y^3 v_2}^{(2), \text{SRT}} = (-24 + 5c_s^2 \omega^2 + 11v_2^2 \omega^2 - 60v_2^2 \omega + 36c_s^2 - 36c_s^2 \omega + 60v_2^2 + 24\omega - 4\omega^2) \frac{\rho v_2}{6\omega^2}$$

$$C_{D_y^3 v_2}^{(2), \text{MRT1}} = (-24 - 60v_2^2\omega_6 + 5\omega_6^2c_s^2 + 36c_s^2 + 11v_2^2\omega_6^2 + 24\omega_6 - 4\omega_6^2 - 36\omega_6c_s^2 + 60v_2^2) \frac{\rho v_2}{6\omega_6^2}$$

$$C_{D_y^3 v_2}^{(2), \text{MRT2}} = C_{D_y^3 v_2}^{(2), \text{MRT1}}$$

$$C_{D_y^3 v_2}^{(2), \text{CLBM1}} = C_{D_y^3 v_2}^{(2), \text{MRT1}}$$

$$C_{D_y^3 v_2}^{(2), \text{CLBM2}} = C_{D_y^3 v_2}^{(2), \text{MRT1}}$$

$$C_{D_y^3 v_2}^{(2), \text{CuLBM1}} = (-24 - 60\omega_2v_2^2 + 24\omega_2 - 4\omega_2^2 + 36c_s^2 + 5\omega_2^2c_s^2 - 36\omega_2c_s^2 + 60v_2^2 + 11\omega_2^2v_2^2) \frac{\rho v_2}{6\omega_2^2}$$

$$C_{D_y^3 v_2}^{(2), \text{CuLBM2}} = (27c_s^2\omega_1^2 + 24\omega_2\omega_1^2 + 22\omega_2^2v_2^2\omega_1^2 - 15\omega_2^2 + 33v_2^2\omega_1^2 + 10\omega_2^2c_s^2\omega_1^2 - 36\omega_2^2c_s^2\omega_1 - 18\omega_2\omega_1 + 27\omega_2^2c_s^2 - 60\omega_2^2v_2^2\omega_1 + 24\omega_2^2\omega_1 + 18\omega_2c_s^2\omega_1 + 54\omega_2v_2^2\omega_1 - 15\omega_1^2 - 60\omega_2v_2^2\omega_1^2 - 8\omega_2^2\omega_1^2 - 36\omega_2c_s^2\omega_1^2 + 33\omega_2^2v_2^2) \frac{\rho v_2}{12\omega_2^2\omega_1^2}$$

coefficient  $C_{D_x^4 \rho}^{(2)}$  at  $\frac{\partial^4 \rho}{\partial x_1^4}$ :

$$C_{D_x^4 \rho}^{(2), \text{SRT}} = (-3v_1^4\omega - 6v_1^2 - 2c_s^2 + c_s^2\omega + 24v_1^2c_s^2 - 12v_1^2c_s^2\omega + 3v_1^2\omega + 2c_s^4 - c_s^4\omega + 6v_1^4) \frac{v_2}{24\omega}$$

$$\begin{aligned} C_{D_x^4 \rho}^{(2), \text{MRT1}} &= (-12\omega_2^2v_1^2\omega_4^2 + 24\omega_5v_1^4\omega_7\omega_4^2 - 24c_s^2\omega_7^2\omega_4 + 12\omega_2^2c_s^2\omega_7\omega_4^2 - 432\omega_5v_1^2c_s^2\omega_7^2\omega_4 + 48\omega_5c_s^2\omega_7^2\omega_4 + 48\omega_5^2c_s^4\omega_7\omega_4 - 48\omega_5^2v_1^4\omega_7 + \\ &14\omega_5c_s^4\omega_7^2\omega_4^2 - 48\omega_5v_1^2\omega_7^2 + 3\omega_5^2v_1^2\omega_7^2\omega_4^2 - 24\omega_5^2c_s^4\omega_4 + 288v_1^2c_s^2\omega_7^2\omega_4 + 48\omega_5v_1^2\omega_7\omega_4 - 12c_s^4\omega_7^2\omega_4^2 - 216\omega_5^2v_1^2c_s^2\omega_7^2\omega_4^2 - 144v_1^2c_s^2\omega_7^2\omega_4^2 + \\ &12\omega_5^2c_s^4\omega_4^2 + 24c_s^4\omega_7^2\omega_4 - 24\omega_5v_1^2\omega_7\omega_4^2 - 12\omega_5^2c_s^4\omega_7\omega_4^2 - 12\omega_5^2v_1^2c_s^2\omega_7^2\omega_4^2 - 48\omega_5c_s^4\omega_7\omega_4 + 150\omega_5v_1^2c_s^2\omega_7^2\omega_4^2 - 48\omega_5^2c_s^2\omega_7\omega_4 - 14\omega_5c_s^2\omega_7^2\omega_4^2 + \\ &24\omega_5^2c_s^2\omega_7^2\omega_4^2 + 24\omega_5^2c_s^4\omega_7^2\omega_4^2 + 24\omega_5^2v_1^2\omega_4^2 - 3\omega_5^2v_1^4\omega_7^2\omega_4^2 + 12c_s^2\omega_7^2\omega_4^2 - 48\omega_5v_1^4\omega_7\omega_4 + 72\omega_5v_1^2c_s^2\omega_7\omega_4^2 + 48\omega_5^2v_1^2\omega_7 + 72v_1^4\omega_7^2\omega_4 + 48\omega_5v_1^4\omega_7^2\omega_4^2 - \\ &30\omega_5^2v_1^4\omega_7\omega_4^2 + 24\omega_5^2c_s^2\omega_4 - 96\omega_5v_1^4\omega_7^2\omega_4 + 12\omega_5^2v_1^2\omega_7^2\omega_4^2 - 96\omega_5^2v_1^2\omega_7\omega_4 - 36\omega_5v_1^2\omega_7^2\omega_4^2 - 126\omega_5^2v_1^2c_s^2\omega_7\omega_4^2 + 72\omega_5^2v_1^2c_s^2\omega_4^2 - \omega_5^2c_s^4\omega_7^2\omega_4^2 + \\ &36v_1^2\omega_7^2\omega_4^2 + 432\omega_5^2v_1^2c_s^2\omega_7\omega_4 - 144\omega_5^2v_1^2c_s^2\omega_4 - 72v_1^2\omega_7^2\omega_4 - 24\omega_5^2c_s^4\omega_7 - 24\omega_5c_s^2\omega_7^2 + 30\omega_5^2v_1^2\omega_7\omega_4^2 + 216\omega_5v_1^2c_s^2\omega_7^2 - 24\omega_5^2v_1^4\omega_4 + \\ &96\omega_5v_1^2\omega_7^2\omega_4 + 96\omega_5^2v_1^4\omega_7\omega_4 - 12\omega_5^2c_s^2\omega_4^2 + 36\omega_5v_1^4\omega_7^2\omega_4^2 - 144\omega_5v_1^2c_s^2\omega_7\omega_4 + \omega_5^2c_s^2\omega_7^2\omega_4^2 - 36v_1^4\omega_7^2\omega_4^2) \frac{v_2}{24\omega_5^2\omega_7^2\omega_4^2} \end{aligned}$$

$$C_{D_x^4 \rho}^{(2), \text{MRT2}} = C_{D_x^4 \rho}^{(2), \text{MRT1}}$$

$$C_{D_x^4 \rho}^{(2), \text{CLBM1}} = (3\omega_5v_1^2 - 6v_1^2 - 2c_s^2 - 12\omega_5v_1^2c_s^2 + \omega_5c_s^2 + 24v_1^2c_s^2 + 2c_s^4 - \omega_5c_s^4 - 3\omega_5v_1^4 + 6v_1^4) \frac{v_2}{24\omega_5}$$

$$C_{D_x^4 \rho}^{(2), \text{CLBM2}} = C_{D_x^4 \rho}^{(2), \text{CLBM1}}$$

$$C_{D_x^4 \rho}^{(2), \text{CuLBM1}} = (-6v_1^2 - 3v_1^4\omega_1 - 2c_s^2 - 12v_1^2c_s^2\omega_1 + c_s^2\omega_1 + 24v_1^2c_s^2 + 2c_s^4 + 3v_1^2\omega_1 - c_s^4\omega_1 + 6v_1^4) \frac{v_2}{24\omega_1}$$

$$\begin{aligned} C_{D_x^4 \rho}^{(2), \text{CuLBM2}} &= (3\omega_2v_1^4 - \omega_2c_s^4\omega_1 + 3v_1^4\omega_1 + \omega_2c_s^4 + 3\omega_2v_1^2\omega_1 + 12v_1^2c_s^2\omega_1 - c_s^2\omega_1 + 12\omega_2v_1^2c_s^2 - \omega_2c_s^2 + \omega_2c_s^2\omega_1 - 3v_1^2\omega_1 - 12\omega_2v_1^2c_s^2\omega_1 - \\ &3\omega_2v_1^4\omega_1 + c_s^4\omega_1 - 3\omega_2v_1^2) \frac{v_2}{24\omega_2\omega_1} \end{aligned}$$

coefficient  $C_{D_x^4 v_1}^{(2)}$  at  $\frac{\partial^4 v_1}{\partial x_1^4}$ :

$$C_{D_x^4 v_1}^{(2), \text{SRT}} = (-4 + 10v_1^2 + 6c_s^2 - 3c_s^2\omega - 5v_1^2\omega + 2\omega) \frac{v_1\rho v_2}{12\omega}$$

$$\begin{aligned} C_{D_x^4 v_1}^{(2), \text{MRT1}} &= (24\omega_5^2v_1^2\omega_4^2 + 72c_s^2\omega_7\omega_4 - 33\omega_5^2c_s^2\omega_7\omega_4^2 - 120\omega_5c_s^2\omega_7^2\omega_4 - 12\omega_5^2\omega_4^2 - 36\omega_5\omega_7^2 + 84\omega_5v_1^2\omega_7^2 - 5\omega_5^2v_1^2\omega_7^2\omega_4^2 + 24\omega_5\omega_7\omega_4 - \\ &72\omega_5v_1^2\omega_7\omega_4 - 12\omega_5\omega_7\omega_4^2 + 36\omega_5v_1^2\omega_7\omega_4^2 + 24\omega_5^2\omega_4 + 2\omega_5^2\omega_7\omega_4^2 + 120\omega_5^2c_s^2\omega_7\omega_4 - 39\omega_5c_s^2\omega_7^2\omega_4^2 - 60\omega_5^2c_s^2\omega_7 - 48\omega_5^2v_1^2\omega_4 - 36c_s^2\omega_7^2\omega_4^2 + \\ &21\omega_5^2\omega_7\omega_4^2 + 12\omega_5c_s^2\omega_7\omega_4^2 - 84\omega_5^2v_1^2\omega_7 - 48\omega_5^2c_s^2\omega_4 + 24\omega_7^2\omega_4^2 + 36\omega_5^2\omega_7 + 168\omega_5^2v_1^2\omega_7\omega_4 + 61\omega_5v_1^2\omega_7\omega_4^2 - 25\omega_5\omega_7\omega_4^2 - 60v_1^2\omega_7\omega_4^2 + 120v_1^2\omega_7\omega_4 + \\ &60\omega_5c_s^2\omega_7^2 - 51\omega_5^2v_1^2\omega_7\omega_4^2 - 48\omega_7^2\omega_4 - 168\omega_5v_1^2\omega_7\omega_4 + 72\omega_5\omega_7\omega_4 + 24\omega_5^2c_s^2\omega_4^2 - 3\omega_5^2c_s^2\omega_7\omega_4^2 - 72\omega_5^2\omega_7\omega_4 - 24\omega_5c_s^2\omega_7\omega_4) \frac{v_1\rho v_2}{12\omega_5^2\omega_7^2\omega_4^2} \end{aligned}$$

$$C_{D_x^4 v_1}^{(2), \text{MRT2}} = C_{D_x^4 v_1}^{(2), \text{MRT1}}$$

$$C_{D_x^4 v_1}^{(2), \text{CLBM1}} = (-4 - 5\omega_5v_1^2 + 2\omega_5 + 10v_1^2 + 6c_s^2 - 3\omega_5c_s^2) \frac{v_1\rho v_2}{12\omega_5}$$

$$C_{D_x^4 v_1}^{(2), \text{CLBM2}} = C_{D_x^4 v_1}^{(2), \text{CLBM1}}$$

$$C_{D_x^4 v_1}^{(2), \text{CuLBM1}} = (-4 + 10v_1^2 + 6c_s^2 - 3c_s^2\omega_1 - 5v_1^2\omega_1 + 2\omega_1) \frac{v_1\rho v_2}{12\omega_1}$$

$$C_{\substack{(2), \text{CuLBMB2} \\ D_x^v v_1}} = (-2\omega_2 + 2\omega_2\omega_1 - 5\omega_2 v_1^2 \omega_1 + 3c_s^2 \omega_1 + 3\omega_2 c_s^2 - 3\omega_2 c_s^2 \omega_1 + 5v_1^2 \omega_1 - 2\omega_1 + 5\omega_2 v_1^2) \frac{v_1 \rho v_2}{12\omega_2 \omega_1}$$

**coefficient**  $C_{D_x^4 v_2}^{(2)}$  **at**  $\frac{\partial^4 v_2}{\partial x_1^4}$ :

$$C_{\frac{D_4}{v_2}, \text{SRT}}^{v_1} = (-84v_1^2c_s^2\omega^2 - 14c_s^2\omega^2 + 108v_1^4\omega + 6v_1^2c_s^2\omega^3 + 72v_1^2 + c_s^2\omega^3 - 42v_1^4\omega^2 - 24c_s^2 + 36c_s^2\omega - 144v_1^2c_s^2 + 216v_1^2c_s^2\omega + 3v_1^4\omega^3 - 108v_1^2\omega + 48c_s^4 - 3c_s^4\omega^3 + 30c_s^4\omega^2 - 72c_s^4\omega - 3v_1^2\omega^3 + 42v_1^2\omega^2 - 72v_1^4) \frac{\rho}{24\omega^3}$$

$$\begin{aligned} C_{\substack{(2,4), \text{MRT1} \\ \text{D}_x^4 v_2}} &= (12c_s^2\omega_7^2\omega_4 - 3c_s^4\omega_7^2\omega_4^3 + 12v_1^2c_s^2\omega_7^3 + 72v_4^4\omega_7\omega_4^2 + 48v_2^1\omega_7\omega_4 - 18v_4^4\omega_7\omega_4^3 + 156v_1^2c_s^2\omega_7^2\omega_4 - 24v_1^2c_s^2\omega_7^4 + 24c_s^4\omega_7^2\omega_4^2 - \\ &72v_1^2c_s^2\omega_7^2\omega_4^2 - 48c_s^4\omega_7^2\omega_4 + 12v_1^4\omega_7^3 + c_s^2\omega_7^2\omega_4^3 - 72v_2^1\omega_7\omega_4^2 - 24v_4^1\omega_4^2 + 6v_1^2c_s^2\omega_7^2\omega_4^3 - 48v_4^1\omega_7\omega_4 + 18v_1^2\omega_7\omega_4^3 + 24c_s^4\omega_7^2 - 8c_s^2\omega_7^2\omega_4^2 + \\ &24v_1^2\omega_7^2\omega_4 - 12v_2^2c_s^2\omega_7\omega_4^3 - 3v_2^2\omega_7^2\omega_4^3 + 24c_s^2\omega_7\omega_4^2 + 48v_1^2c_s^2\omega_7\omega_4^2 + 24c_s^4\omega_7\omega_4 - 6c_s^2\omega_7\omega_4^3 + 24v_1^2\omega_7^2\omega_4^2 + 24v_1^2\omega_7^4 - 96v_1^2c_s^2\omega_7^2 - 24v_1^2\omega_7^2\omega_4 - \\ &24v_1^2c_s^2\omega_7\omega_4 + 3v_1^4\omega_7^2\omega_4^3 - 24c_s^4\omega_7\omega_4^2 - 12v_1^2\omega_7^4 - 24c_s^2\omega_7\omega_4 + 6c_s^4\omega_7\omega_4^3 - 24v_1^4\omega_7^2\omega_4^2) \frac{\rho}{24\omega_7^2\omega_4^3} \end{aligned}$$

$$C_{\mathrm{D}_x^4 v_2}^{(2), \text{MRT2}} = C_{\mathrm{D}_x^4 v_2}^{(2), \text{MRT1}}$$

$$\begin{aligned} C_{\substack{D_4 \\ x \\ v_2}}^{(2), \text{CLBM1}} = & (12c_s^2w_7^2w_4 - 3c_s^4w_7^2w_4^3 + 108v_1^2c_s^2w_4^3 + 72v_1^4w_7w_4^2 - 30v_1^4w_7w_4^3 - 36v_1^2c_s^2w_7^2w_4 - 216v_1^2c_s^2w_4^2 + 24c_s^4w_7^2w_4^2 - 12v_1^2c_s^2w_7^2w_4^2 - 48c_s^4w_7^2w_4 + 36v_1^4w_4^3 + c_s^2w_7^2w_4^3 - 72v_1^2w_7w_4^2 - 72v_1^4w_4^2 + 6v_1^2c_s^2w_7^2w_4^3 + 30v_1^2w_7w_4^3 + 24c_s^4w_7^2 - 8c_s^2w_7^2w_4^2 - 72v_1^2c_s^2w_7w_4^3 - 3v_1^2w_7^2w_4^3 + 24c_s^2w_7w_4^2 + 144v_1^2c_s^2w_7w_4^2 + 24c_s^4w_7w_4 - 6c_s^2w_7w_4^3 + 12v_1^2w_7^2w_4^2 + 72v_1^2w_1^2 + 72v_1^2c_s^2w_7w_4 + 3v_1^4w_7^2w_4^3 - 24c_s^4w_7w_4^2 - 36v_1^2w_4^3 - 24c_s^2w_7w_4 + 6c_s^4w_7w_4^3 - 12v_1^2w_7^2w_4^2) \frac{\rho}{24w_7^2w_4^3} \end{aligned}$$

$$C_{\mathrm{D}_x^4 v_2}^{(2), \text{CLBM2}} = C_{\mathrm{D}_x^4 v_2}^{(2), \text{CLBM1}}$$

$$\begin{aligned} C_{\frac{D_4^2}{D_x^2}v_2}^{(4), \text{CuLBM1}} = & (-72v_1^4w_3^2 + 6v_1^2w_3^3c_s^2w_4^2 + 72v_1^2w_3c_s^2w_4 - 24w_3c_s^2w_4 + w_3^3c_s^2w_4^2 + 36v_1^4w_3^3 - 216v_1^2w_3^2c_s^2 + 24w_3^2c_s^4w_4^2 - 24w_3^2c_s^4w_4 + 12w_3c_s^2w_4^2 - \\ & 6w_3^3c_s^2w_4 - 72v_1^2w_3^3c_s^2w_4 - 36v_1^2w_3c_s^2w_4^2 + 108v_1^2w_3^3c_s^2 + 24c_4^4w_4^2 + 24w_3c_s^4w_4 - 36v_1^2w_3^3 + 12v_1^2w_3^2w_4^2 - 3w_3^3c_s^4w_4^2 + 3v_1^4w_3^3w_4^2 - 8w_3^2c_s^2w_4^2 + 72v_1^2w_3^2 - \\ & 12v_1^2w_3^2c_s^2w_4^2 + 30v_1^2w_3^3w_4 + 72v_1^4w_3^2w_4 - 12v_1^4w_3^2w_4^2 - 3v_1^2w_3^3w_4^2 + 144v_1^2w_3^2c_s^2w_4 - 30v_1^4w_3^2w_4 + 24w_3^2c_s^4w_4 - 48w_3c_s^4w_4^2 + 6w_3^3c_s^4w_4 - 72v_1^2w_3^2w_4) \frac{\rho}{24w_3^2c_s^2w_4^2} \end{aligned}$$

$$C_{\substack{D_x \\ v_2}}^{(2), \text{CuLBMB2}} = (-24w_3c_s^2w_1 + 72v_1^4w_3w_1^2 - 216v_1^2c_s^2w_1^2 + 24w_3^2c_s^4w_1^2 - 30v_1^4w_3w_1^3 - 3w_2^3c_s^4w_1^3 + 108v_1^2c_s^2w_1^3 + 72v_1^2w_3c_s^2w_1 - 72v_1^4w_1^2 + 144v_1^2w_3c_s^2w_1^2 - 72v_1^2w_3w_1^2 - 6w_3c_s^2w_1^3 - 72v_1^2w_3c_s^2w_1^3 + 30v_1^2w_3w_1^3 + 36v_1^4w_1^3 - 48w_3^2c_s^4w_1 + 24w_3c_s^2w_1^2 - 12v_1^2w_3c_s^2w_1^2 - 3v_1^2w_3^2w_1^3 + w_3^2c_s^2w_1^3 + 24w_3c_s^4w_1 + 12v_1^2w_3^2w_1^2 + 6v_1^2w_3c_s^2w_1^3 - 8w_3^2c_s^2w_1^2 + 3v_1^4w_3^2w_1^3 - 36v_1^2w_1^3 + 12w_3^2c_s^4w_1 - 24w_3c_s^4w_1^2 + 72v_1^2w_1^2 - 12v_1^2w_3^2w_1^2 - 36v_1^2w_3c_s^2w_1 + 24w_3c_s^4w_1^3 + 6w_3c_s^4w_1) \frac{\rho}{24w_3^2w_1^3}$$

coefficient  $C_{D_x^3 D_y \rho}^{(2)}$  at  $\frac{\partial^4 \rho}{\partial x_1^3 \partial x_2}$ :

$$C_{\substack{D_x^2 \\ x^\rho}}^{(2), \text{SRT}} = (24 - 42c_s^2\omega^2 - 24v_1^2 + 3c_s^2\omega^3 - 72c_s^2 + 108c_s^2\omega + 36v_1^2\omega - 36\omega + v_1^2\omega^3 + 14\omega^2 - 14v_1^2\omega^2 - \omega^3) \frac{v_1 c_s^2}{12\omega^3}$$

$$\begin{aligned}
& C_{(2),MRT1} = (-12w_8w_5^2w_9v_2^2w_7w_4^3 + 18w_8w_5v_1^2w_9w_6c_s^2w_7w_4^2 - 3w_8w_5^2v_1^2w_9w_6c_s^2w_7w_4^3 - 36w_5^2w_6c_s^4w_7w_4^3 + 6w_8w_9v_2^2w_6w_7w_4^3 - \\
& D_{x,y,p}^{3,2,1} \\
& 12w_8^2w_5^2w_6c_s^2w_7w_3^3 + 6w_8w_5^2w_6c_s^2w_7w_4^3 - 6w_8v_2^2w_9v_2^2w_6w_7w_3^3 + 18w_8w_5^2v_2^2w_6c_s^2w_7w_3^3 + 12w_5^2v_2^2w_6w_7w_4^3 - 108w_8w_5w_9v_2^2w_6c_s^2w_7w_4^2 - \\
& 45w_8w_5^2w_9v_2^2w_6c_s^2w_7w_4^3 - 42w_8w_5^2w_9w_6c_s^4w_7w_4^2 + 24w_8w_5v_1^2w_9v_2^2w_7w_4^2 - 24w_8w_5^2v_1^2w_9v_2^2w_6w_7w_4 + 6w_8w_5^2v_2^2w_6w_7w_4^3 - 48w_8w_5^2w_9v_2^2w_6w_7w_4^2 - \\
& 96w_8w_5^2w_9w_6c_s^4w_7w_4^2 + 144w_8w_5^2w_9v_2^2w_6c_s^2w_7w_4^2 + 27w_8w_5w_9v_2^2w_6c_s^2w_7w_4^3 - 12w_8w_5^2w_6c_s^2w_7w_4^2 + 12w_5^2v_2^2w_6c_s^2w_7w_4^2 - 12w_5^2v_2^2w_6w_7w_4^2 + \\
& 12w_8w_5^2w_9w_6c_s^2w_7w_4 - 36w_8w_5^2v_2^2w_6c_s^2w_7w_4^2 + 12w_8v_1^2w_9v_2^2w_6w_7w_4^2 + 18w_8w_5^2v_1^2w_9w_6c_s^2w_7w_4^2 - 5w_8w_5v_1^2w_9w_6c_s^2w_7w_4^3 + 36w_5^2w_6c_s^4w_7w_4^2 - \\
& 12w_8w_9v_2^2w_6w_7w_4^2 + 12w_5^2v_2^2w_7w_4^3 + 24w_8w_5^2w_9v_2^2w_7w_4^2 + 12w_5w_9c_s^2w_7w_4^3 + 15w_8w_5^2w_9v_2^2w_6w_7w_4^3 - 12w_5v_2^2w_9w_6c_s^2w_7w_4^2 - 12w_8w_5^2v_1^2v_2^2w_6w_7w_4^2 - \\
& 6w_8w_5v_1^2w_9v_2^2w_7w_4^3 + 15w_8w_5^2w_9w_6c_s^4w_7w_4^2 + 24w_8w_5v_1^2w_9v_2^2w_6w_7w_4^2 - 12w_5^2v_1^2v_2^2w_6w_7w_4^3 - 72w_8w_5^2w_9v_2^2c_s^2w_7w_4^2 + 6w_8w_5^2v_2^2w_6w_7w_4^2 + \\
& 12w_5^2w_6c_s^2w_7w_4^3 + 12w_8w_5^2w_9w_6c_s^2w_7w_4^2 - 18w_8w_9v_2^2w_6c_s^2w_7w_4^3 - 36w_8w_5v_1^2w_9v_2^2w_6w_7w_4^2 - 18w_8w_5^2w_6c_s^4w_7w_4^2 - 6w_8w_5^2w_9v_2^2w_6w_7w_4^3 - \\
& 15w_8w_5^2v_1^2w_9v_2^2w_6w_7w_4^2 + 6w_8w_5v_1^2w_9w_6c_s^2w_7w_4^3 - 72w_8w_5^2w_9v_2^2w_6c_s^2w_7w_4^2 - 36w_5^2v_2^2w_6c_s^2w_7w_4^3 + 18w_8w_5^2w_9v_2^2w_6w_7w_4^2 - 6w_8w_5w_9v_2^2w_6w_7w_4^3 - \\
& 18w_8w_5^2w_9w_6c_s^2w_7w_4^2 + 36w_5^2v_2^2c_s^2w_7w_4^3 - 12w_8w_5^2v_1^2w_9w_6c_s^2w_7w_4^2 + 48w_8w_5^2v_2^2w_9v_2^2w_6w_7w_4^2 + 12w_8w_5^2w_9w_6c_s^2w_7w_4^3 + 36w_8w_5^2w_6c_s^4w_7w_4^2 + \\
& 9w_8w_5v_1^2w_9v_2^2w_6w_7w_4^3 + 12w_8w_5^2w_9w_6c_s^2w_7w_4^2 - 12w_5v_2^2w_9v_2^2w_6w_7w_4^3 + 36w_8w_5v_2^2w_6c_s^2w_7w_4^2 - 6w_8w_5^2w_9w_6c_s^4w_7w_4^3 - 36w_5w_9c_s^4w_7w_4^2 + \\
& 6w_8w_5v_1^2w_9c_s^2w_7w_4^3 - 12w_8w_5^2v_2^2w_6w_7w_4^2 + 36w_8w_5v_2^2w_9v_2^2w_7w_4^3 + 12w_5^2v_1^2v_2^2w_6w_7w_4^2 - 12w_5^2w_6c_s^2w_7w_4^2 + 24w_8w_5^2w_9v_2^2w_6w_7w_4^2 - \\
& 12w_8w_5v_1^2w_9w_6c_s^2w_7w_4^2 + 3w_8w_5^2w_9w_6c_s^2w_7w_4^3 + 12w_5w_9v_2^2w_7w_4^3 + 12w_8w_5w_9v_2^2w_6w_7w_4^2 + 12w_5^2v_1^2v_2^2w_7w_4^3 - 12w_8w_5^2v_1^2w_6c_s^2w_7w_4^2 - \\
& 36w_8w_5^2w_9v_2^2w_6c_s^2w_7w_4^3 + 72w_8w_5w_9v_2^2w_6c_s^2w_7w_4^2 + 36w_5^2v_2^2w_6c_s^2w_7w_4^2 - 6w_8w_5^2v_1^2v_2^2w_7w_4^3 - 6w_8w_5w_9c_s^2w_7w_4^3 + 156w_8w_5^2w_9w_6c_s^4w_7w_4^2 - \\
& 36w_8w_5^2w_6c_s^4w_7w_4^2 + 6w_8w_5v_1^2w_9v_2^2w_6w_7w_4^3 + 6w_8w_5^2c_s^2w_7w_4^3 - 12w_5^2v_2^2w_7w_4^3 + 12w_8w_5^2v_2^2w_6w_7w_4^2 + 12w_5w_9w_6c_s^2w_7w_4^2 + 36w_5w_9v_2^2w_6c_s^2w_7w_4^3 - \\
& 24w_8w_5^2v_1^2w_9v_2^2w_7w_4^2 + 54w_8w_5w_9w_6c_s^2w_7w_4^2 + 12w_5v_1^2w_9w_6c_s^2w_7w_4^3 + 12w_8w_5^2v_1^2w_6c_s^2w_7w_4^2 + 36w_5^2c_s^2w_7w_4^3 - 8w_8w_5^2w_9w_6c_s^2w_7w_4^3 + \\
& 36w_8w_5w_9v_2^2w_6w_7w_4^2 - 12w_5w_9w_6c_s^2w_7w_4^3 + 12w_8w_5w_9w_6c_s^2w_7w_4^2 - 6w_8w_5^2v_2^2w_6w_7w_4^2 - 12w_8w_5v_1^2w_9v_2^2w_6w_7w_4^2 + 18w_8w_5^2w_6c_s^2w_7w_4^3 - \\
& 9w_8w_5w_9v_2^2w_6w_7w_4^3 + 12w_8w_5^2w_9w_6c_s^2w_7w_4^2 - 12w_5v_1^2w_9w_6c_s^2w_7w_4^3 - 6w_8w_5^2v_1^2w_6c_s^2w_7w_4^2 - 6w_8w_5^2v_2^2w_6c_s^2w_7w_4^3 - 36w_5w_9v_2^2c_s^2w_7w_4^2 + \\
& 36w_5w_9v_2^2w_6c_s^2w_7w_4^2 - 15w_8w_5w_9w_6c_s^4w_7w_4^2 + 12w_8w_5^2v_1^2w_9v_2^2w_7w_4^2 - 12w_8w_5^2w_9w_6c_s^2w_7w_4^2 + 36w_8w_5^2v_2^2w_6c_s^2w_7w_4^2 + 72w_8w_5w_9v_2^2c_s^2w_7w_4^2 + \\
& 12w_8w_5^2w_6c_s^2w_7w_4^2 + 18w_8w_5w_9v_2^2w_6c_s^2w_7w_4^3 + 12w_5w_9v_2^2w_6w_7w_4^2 + 18w_8w_5^2w_9c_s^4w_7w_4^2 - 6w_8w_5^2w_9w_6c_s^4w_7w_4^2 - 36w_5w_9w_6c_s^2w_7w_4^2 - \\
& 18w_8w_5^2c_s^4w_7w_4^3 - 12w_8w_5^2v_1^2w_9w_6c_s^2w_7w_4^2 + 12w_5v_1^2w_9v_2^2w_6w_7w_4^3 + 12w_8w_5^2v_1^2v_2^2w_6w_7w_4^2 + 6w_8w_5w_9v_2^2w_7w_4^3 - 18w_8w_5w_9w_6c_s^2w_7w_4^2 - \\
& 6w_8w_5^2v_1^2c_s^2w_7w_4^3 + 6w_8w_5v_1^2w_9w_6c_s^2w_7w_4^3 + 3w_8w_5^2w_9w_6c_s^4w_7w_4^2 - 12w_5^2c_s^2w_7w_4^3 - 12w_8w_5^2v_1^2w_9v_2^2w_6w_7w_4^2 + 6w_8w_5^2v_2^2w_7w_4^3 - 36w_8w_5w_9w_6c_s^4w_7w_4^2 + \\
& w_8w_5^2v_1^2w_9w_6c_s^2w_7w_4^3 + 36w_5w_9w_6c_s^4w_7w_4^2 + 12w_8w_5^2w_9w_6c_s^4w_7w_4^2 - 12w_5w_9v_2^2w_6w_7w_4^3 - 36w_8w_5w_9v_2^2w_6c_s^2w_7w_4^2 - 24w_8w_5w_9v_2^2w_6w_7w_4^2 - \\
& 18w_8w_5^2v_2^2w_6c_s^2w_7w_4^3 - 6w_8w_5^2w_6c_s^2w_7w_4^3 - 18w_8w_5w_9v_2^2c_s^2w_7w_4^3 + 6w_8w_5^2v_1^2w_9v_2^2w_6w_7w_4^2 - 60w_8w_5^2w_9w_6c_s^4w_7w_4^2 - 24w_8w_5w_9v_2^2w_7w_4^2 -
\end{aligned}$$

$$18w_8w_5^2v_2^2c_s^2\omega_7^2\omega_4^3 - 12w_8w_5^2v_1^2\omega_9w_6c_s^2\omega_4^2 + 5w_8w_5w_9w_6c_s^2\omega_7^2\omega_4^3 - 12w_5v_1^2\omega_9v_2^2\omega_6\omega_7^2\omega_4^2 - 6w_8w_5^2v_1^2v_2^2\omega_6\omega_7\omega_4^3) \frac{v_1}{12w_8w_5^2w_9w_6\omega_7^2\omega_4^3}$$

$$C_{D_x^3 D_y \rho}^{(2), \text{MRT2}} = C_{D_x^3 D_y \rho}^{(2), \text{MRT1}}$$

$$\begin{aligned} C_{D_x^3 D_y \rho}^{(2), \text{CLBIM1}} &= (12w_8w_5v_1^2\omega_6\omega_7w_4 + 12v_1^2\omega_9w_6\omega_7^2\omega_4^2 - 36w_8w_5w_9w_6c_s^2\omega_7^2w_4 + 12w_9w_6\omega_7^2w_4 + 54w_8w_9w_6c_s^2\omega_7^2w_4 + 12w_8w_5w_6\omega_7^2w_4 + \\ 3w_8w_5w_9w_6\omega_7w_4^2 + 12w_8w_5w_9w_6\omega_4 - 18w_8w_5w_6c_s^2\omega_7w_4^2 - 12w_8w_5v_1^2\omega_9w_6\omega_7 + w_8w_5v_1^2\omega_9w_6\omega_7^2w_4^2 - 36w_9c_s^2\omega_7^2w_4^2 + 5w_8w_9w_6\omega_7^2w_4^2 + \\ 36w_5w_6c_s^2\omega_7^2w_4^2 - 9w_8w_5w_9w_6c_s^2\omega_7w_4^2 + 12w_5w_6\omega_7^2w_4^2 - 12w_5v_1^2\omega_6\omega_7^2w_4^2 - 18w_8w_5c_s^2\omega_7^2w_4^2 + 54w_8w_5w_9w_6c_s^2\omega_7w_4 - 12w_5w_6\omega_7^2w_4 + \\ 12w_5v_1^2\omega_6\omega_7^2w_4^2 - 36w_8w_9w_6c_s^2\omega_7^2 - 18w_8w_9w_6\omega_7^2w_4 - 36w_5w_6c_s^2\omega_7^2w_4^2 - 6w_8w_5w_6\omega_7^2w_4^2 - 15w_8w_9w_6c_s^2\omega_7^2w_4^2 - 6w_8w_5w_9w_6\omega_4^2 + \\ 36w_8w_5w_6c_s^2\omega_7w_4^2 - 18w_8w_5w_9w_6\omega_7w_4 - 12w_8w_5v_1^2\omega_9w_6\omega_7^2w_4 + 12w_8w_5v_1^2\omega_9w_6\omega_7^2 + 36w_5c_s^2\omega_7^2w_4^2 - 12v_1^2\omega_9w_6\omega_7^2w_4 - 6w_8w_5v_1^2\omega_6\omega_7w_4^2 + \\ 18w_8w_9c_s^2\omega_7^2w_4^2 + 18w_8w_5w_9w_6c_s^2\omega_4^2 - 12w_9w_6\omega_7^2w_4^2 + 12w_8w_5w_9w_6\omega_7^2w_4 - 36w_8w_5w_6c_s^2\omega_7^2w_4^2 + 6w_8w_5w_6\omega_7^2w_4^2 + 12w_9w_6\omega_7^2w_4^2 + \\ 36w_8w_5w_6c_s^2\omega_7^2w_4^2 + 18w_8w_5v_1^2\omega_9w_6\omega_7w_4 + 6w_8w_5v_1^2\omega_6\omega_7^2w_4^2 - 12w_8w_5w_9w_6\omega_7^2 - 6w_8w_5w_9w_6c_s^2\omega_7^2w_4^2 - 36w_8w_5w_9w_6c_s^2\omega_7^2w_4^2 + \\ 12w_5v_1^2\omega_7^2w_4^2 - 12w_5w_6\omega_7^2w_4^2 - 5w_8v_1^2\omega_9w_6\omega_7^2w_4^2 - 12w_8v_1^2\omega_9w_6\omega_7^2 - 12w_1^2\omega_9w_6\omega_7^2w_4^2 - 36w_9w_6c_s^2\omega_7^2w_4^2 + 3w_8w_5w_9w_6c_s^2\omega_7^2w_4^2 + \\ 18w_8v_1^2\omega_9w_6\omega_7^2w_4 + 6w_8v_1^2\omega_9w_7^2w_4^2 - 12w_8w_5v_1^2\omega_6\omega_7^2w_4 + 12w_8w_5w_9w_6\omega_7^2 + 18w_8w_5w_6c_s^2\omega_7^2w_4^2 - w_8w_5w_9w_6\omega_7^2w_4^2 + \\ 12w_8w_9w_6\omega_7^2 - 12w_8w_5w_6\omega_7w_4 - 6w_8w_5v_1^2\omega_7^2w_4^2 - 3w_8w_5v_1^2\omega_9w_6\omega_7w_4^2 - 36w_8w_5w_6c_s^2\omega_7w_4 \frac{v_1 c_s^2}{12w_8w_5w_9w_6\omega_7^2w_4^2} \end{aligned}$$

$$C_{D_x^3 D_y \rho}^{(2), \text{CLBIM2}} = C_{D_x^3 D_y \rho}^{(2), \text{CLBIM1}}$$

$$\begin{aligned} C_{D_x^3 D_y \rho}^{(2), \text{CuLBM1}} &= (-36w_3c_s^2\omega_1 - 12w_4^2\omega_1 - 12w_3\omega_4 + 12v_1^2\omega_4^2\omega_1 + 12v_1^2\omega_3\omega_4 - 36c_s^2\omega_4^2 + 12\omega_3\omega_1 + 36w_3c_s^2\omega_4^2\omega_1 - 36w_3c_s^2\omega_4^2\omega_1 - 12v_1^2\omega_3\omega_1 - \\ 36c_s^2\omega_4\omega_1 - 9w_3c_s^2\omega_4\omega_1 - 12v_1^2\omega_3\omega_4^2\omega_1 + 18w_3c_s^2\omega_4^2 + 3w_3^2\omega_4\omega_1 - 3v_1^2\omega_3^2\omega_4\omega_1 + 6v_1^2\omega_3\omega_4^2 + 12w_3\omega_1^2\omega_1 - 6w_3\omega_4^2 + v_1^2\omega_3^2\omega_4^2\omega_1 + v_1^2\omega_3^2\omega_4^2 - \\ w_3^2\omega_4^2 - 18w_3\omega_4\omega_1 + 3w_3^2c_s^2\omega_4^2 + 18v_1^2\omega_3\omega_4\omega_1 + 3w_3^2c_s^2\omega_4^2\omega_1 - w_3^2\omega_4^2\omega_1 - 12v_1^2\omega_2^2 - 6w_3^2\omega_1 + 18w_3c_s^2\omega_4\omega_1 + 36c_s^2\omega_4^2\omega_1 + \\ 6v_1^2\omega_3^2\omega_1 + 6w_3^2\omega_4 + 12w_4\omega_1 - 18w_3^2c_s^2\omega_4 + 12\omega_4^2 - 6v_1^2\omega_3^2\omega_4 - 12v_1^2\omega_4\omega_1) \frac{v_1 c_s^2}{12w_2^2v_4^2\omega_1} \end{aligned}$$

$$C_{D_x^3 D_y \rho}^{(2), \text{CuLBM2}} =$$

$$\begin{aligned} &(18w_2w_3c_s^4\omega_1^3 - 90w_2^2w_3^2c_s^4\omega_1 + 30w_2^3w_3c_s^2\omega_1^3 + 36w_2^3c_s^4\omega_1^3 - 24w_2^3v_1^2c_s^2\omega_1^2 - 24w_2^2w_3c_s^2\omega_1^2 - 138w_2^3w_3c_s^2\omega_1^2 + 51w_2^3v_1^2\omega_1^2 - \\ 30w_2^3v_1^2\omega_2^2\omega_1^2 + 90w_2^3c_s^2\omega_1^3 + 3w_2^2v_1^2\omega_2^2\omega_1^3 - 7w_3^3\omega_3^2\omega_1^2 + 3w_2^2v_1^2\omega_2^2\omega_1^2 + 18w_2^2w_3c_s^2\omega_1^3 + 6w_2^3w_3c_s^4\omega_1^3 + 12w_2^3v_1^2\omega_2^2\omega_1^3 - 165w_2^2v_1^2\omega_3^2c_s^2\omega_1^2 - \\ 90w_2v_1^4\omega_2^2\omega_1^3 - 261w_2^3v_1^2\omega_2^2\omega_1^3 + 12w_2^3\omega_3^2\omega_1^2 - 24w_2w_3^2c_s^2\omega_1^2 + 6w_2^2v_1^2\omega_2^2\omega_1^2 + 9w_2^2v_2^2\omega_2^2\omega_1^2 - 465w_2v_1^2\omega_2^2\omega_1^2 + 6w_2^3w_3c_s^2\omega_1^3 - \\ 51w_2v_1^2\omega_3^2\omega_1^2 + 72w_2^2\omega_2^2\omega_1^2 + 42w_2^3v_1^2\omega_3^2\omega_1^2 + 42w_2^3v_1^2\omega_3^2\omega_1^2 + 197w_2^2v_1^2\omega_3^2\omega_1^2 + 225w_2^3w_3c_s^2\omega_1^2 - 12w_2w_3^2\omega_1^3 + 123w_2w_3^2\omega_1^3 - 3w_2^3v_2^2\omega_3^2\omega_1^2 - 6w_2^3v_1^2\omega_3^2\omega_1^2 - \\ 3w_2^2v_2^2\omega_3^2\omega_1^3 - 42w_2^3w_3c_s^2\omega_1^2 + 36w_2^3v_1^2\omega_3^2\omega_1^2 + 102w_2v_1^2\omega_3^2\omega_1^2 + 141w_2v_1^2\omega_3^2\omega_1^2 - 90w_3^2w_3^2\omega_1^3 + 24w_2^3c_s^2\omega_1^2 + 24w_2^2v_1^2\omega_3^2\omega_1^2 - 51v_1^2\omega_3^2\omega_1^2 - \\ 45w_3^3v_1^2\omega_3^2\omega_1^3 - 72w_2^3\omega_2^2\omega_1^3 + 90w_3^2v_1^2\omega_3^2\omega_1^3 - 219w_3^3v_1^2\omega_3^2\omega_1^3 - 2w_2^3\omega_2^2\omega_1^3 + 6w_2^3\omega_2^2\omega_1^3 + 54w_2^2\omega_3^2c_s^4\omega_1^3 + 6w_2v_1^2\omega_3^2\omega_1^3 - 6w_2w_3c_s^2\omega_1^3 + 46w_3^3v_1^2\omega_3^2\omega_1^2 + \\ 39w_2v_1^2\omega_3^2\omega_1^3 - 90w_2^3w_3c_s^2\omega_1^3 + 7w_2^3\omega_3^2\omega_1^3 + 48w_2^2\omega_2^2\omega_1^3 + 261v_1^2\omega_3^2c_s^2\omega_1^3 - 12w_2^3\omega_3^2\omega_1^3 - 18w_2^2v_1^2\omega_3^2\omega_1^3 + 72w_3^3\omega_3^2\omega_1^3 + 81w_3^2\omega_3^2c_s^2\omega_1^2 + 72w_2^2\omega_3c_s^4\omega_1^2 + \\ 2w_2^3v_1^2\omega_3^2\omega_1^3 - 9w_2^2v_2^2\omega_3^2\omega_1^2 + 51w_2^2v_1^2\omega_3^2\omega_1^2 + 45v_1^2\omega_3^2\omega_1^3 - 147w_2^3\omega_3^2\omega_1^2 - 153w_2\omega_3^2c_s^4\omega_1^3 - 102w_2^2v_1^2\omega_3^2\omega_1^2 - 12w_2^2\omega_3^2c_s^2\omega_1^2 + 126w_2^2w_3c_s^4\omega_1^2 - \\ 46w_2^2v_1^2\omega_3^2\omega_1^3 + 18w_2w_3c_s^2\omega_1^2 + 489w_2^3v_1^2\omega_3^2c_s^2\omega_1^2 - 39w_3^2v_1^2\omega_3^2\omega_1^2 - 6w_2^3\omega_3^2\omega_1^2 - 59w_2^2w_3c_s^2\omega_1^2 - 18w_2^3w_3c_s^4\omega_1^2 - 6w_2^2\omega_3^2\omega_1^2 - 45w_2^2v_1^2\omega_3^2\omega_1^2) \frac{v_1}{24w_2^3\omega_3^2\omega_1^2} \end{aligned}$$

coefficient  $C_{D_x^3 D_y v_1}^{(2)}$  at  $\frac{\partial^4 v_1}{\partial x_1^3 \partial x_2}$ :

$$C_{D_x^3 D_y v_1}^{(2), \text{SRIT}} = (-12 + 2c_s^2\omega^2 + 36v_1^2 - c_s^2\omega^3 - 54v_1^2\omega + 18\omega - 6\omega^2 + 18v_1^2\omega^2) \frac{\rho c_s^2}{12w_3^3}$$

$$\begin{aligned} C_{D_x^3 D_y v_1}^{(2), \text{MRT1}} &= (-48w_8w_5^2v_1^2\omega_9w_6c_s^2\omega_7^2 - 12w_8w_5^2w_9v_2^2\omega_7w_4^3 + 54w_8w_5v_1^2\omega_9w_6c_s^2\omega_7^2w_4^2 + 30w_8w_5^2v_1^2\omega_9w_6c_s^2\omega_7w_4^3 - 12w_5^2w_6c_s^4\omega_7^2w_4^2 + \\ 6w_8w_9v_2^2w_6\omega_7w_3^2 - 36w_5^2v_1^2\omega_6c_s^2\omega_7^2w_3^3 + 6w_8w_5^2w_6c_s^2\omega_7w_3^2 - 18w_8v_1^2\omega_9v_2^2\omega_6\omega_7^2w_4^3 + 6w_8w_5^2v_2^2\omega_6c_s^2\omega_7^2w_4^3 + 12w_2^2v_2^2\omega_6\omega_7^2w_4^3 - \\ 36w_8w_5w_9v_2^2w_6c_s^2\omega_7^2w_4^2 - 15w_8w_5^2w_9v_2^2w_6c_s^2\omega_7w_4^3 - 18w_8w_5^2w_9w_6c_s^2\omega_7^2w_4^2 + 72w_8w_5v_1^2\omega_9v_2^2\omega_6\omega_7^2w_4^2 - 72w_8w_5v_1^2\omega_9v_2^2\omega_6\omega_7w_4^3 + 18w_8w_5^2v_1^2v_2^2\omega_6\omega_7^2w_4^3 - \\ 48w_8w_5^2w_9v_2^2w_6\omega_7w_4^2 - 12w_8w_5^2w_9w_6c_s^2\omega_7^2w_4^2 + 48w_8w_5^2w_9v_2^2w_6c_s^2\omega_7^2w_4^2 + 9w_8w_5w_9v_2^2w_6c_s^2\omega_7^2w_4^3 - 12w_8w_5^2w_6c_s^2\omega_7^2w_4^2 + 36w_5^2v_1^2\omega_6c_s^2\omega_7^2w_4^2 - \\ 12w_2^2v_2^2\omega_6\omega_7^2w_4^2 - 12w_8w_5^2w_9w_6c_s^2\omega_7^2w_4^2 - 12w_8w_5^2v_2^2\omega_6c_s^2\omega_7^2w_4^2 + 36w_8v_1^2\omega_9v_2^2\omega_6\omega_7^2w_4^2 - 102w_8w_5^2v_1^2\omega_9w_6c_s^2\omega_7^2w_4^2 - 15w_8w_5v_1^2\omega_9w_6c_s^2\omega_7^2w_4^2 + \\ 12w_5^2w_6c_s^4\omega_7^2w_4^2 - 12w_8w_9v_2^2\omega_6\omega_7^2w_4^2 + 36w_5^2v_1^2v_2^2\omega_7^2w_4^3 + 24w_8w_5^2w_9v_2^2\omega_7^2w_4^2 + 12w_5w_9c_s^2\omega_7^2w_4^2 + 15w_8w_5^2w_9v_2^2\omega_6\omega_7^2w_4^2 - 36w_5v_1^2\omega_9c_s^2\omega_7^2w_4^2 - \\ 36w_8w_5^2v_2^2\omega_6\omega_7^2w_4^2 - 18w_8w_5v_1^2\omega_9v_2^2\omega_7^2w_4^2 + 6w_8w_5^2w_9w_6c_s^4\omega_7^2w_4^3 + 72w_8w_5v_1^2\omega_9v_2^2\omega_6\omega_7^2w_4^2 - 36w_5^2v_1^2\omega_9v_2^2\omega_6\omega_7^2w_4^2 + 24w_8w_5^2w_9v_2^2c_s^2\omega_7^2w_4^2 + \\ 6w_8w_5^2v_2^2\omega_6\omega_7w_4^3 - 6w_8w_5^2w_6c_s^2\omega_7^2w_4^3 - 6w_8w_5^2v_2^2\omega_6\omega_7^2w_4^2 - 6w_8w_5^2w_6c_s^2\omega_7^2w_4^3 - 6w_8w_5w_9v_2^2\omega_6\omega_7^2w_4^3 - \\ 45w_8w_5^2v_1^2\omega_9v_2^2\omega_6\omega_7w_4^3 + 24w_8w_5^2v_1^2\omega_9v_2^2\omega_6\omega_7^2w_4^2 + 6w_8w_5^2v_1^2\omega_9v_2^2\omega_6\omega_7^2w_4^3 - 6w_8w_5w_9v_2^2\omega_6\omega_7^2w_4^3 + 45w_8w_5^2v_1^2\omega_9v_2^2\omega_6\omega_7^2w_4^2 + \\ 18w_8w_5w_9v_2^2\omega_6\omega_7w_4^2 + 12w_5^2v_2^2c_s^2\omega_7^2w_4^3 + 60w_8w_5^2v_1^2\omega_9w_6c_s^2\omega_7^2w_4^2 + 144w_8w_5^2v_1^2\omega_9v_2^2\omega_6\omega_7^2w_4^2 + 12w_8w_5^2w_5v_2^2\omega_6\omega_7^2w_4^2 + 12w_8w_5^2w_6c_s^4\omega_7^2w_4^2 + \\ 27w_8w_5v_1^2\omega_9v_2^2\omega_6\omega_7^2w_4^3 + 12w_8w_5^2w_9w_6c_s^4\omega_7^2w_4^2 - 36w_5v_1^2\omega_9v_2^2\omega_6\omega_7^2w_4^3 + 12w_8w_5v_1^2\omega_9c_s^2\omega_7^2w_4^3 + 18w_8w_5v_1^2\omega_9v_2^2\omega_6\omega_7^2w_4^2 - \\ 12w_8w_5^2v_2^2\omega_6\omega_7w_4^2 + 12w_8w_5^2w_9v_2^2\omega_6\omega_7^2w_4^3 + 36w_5^2v_1^2v_2^2\omega_7^2w_4^2 - 12w_5^2w_6c_s^2\omega_7^2w_4^2 + 24w_8w_5^2v_1^2\omega_9v_2^2\omega_6\omega_7^2w_4^2 - 36w_8w_5v_1^2\omega_9w_6c_s^2\omega_7^2w_4^2 - \\ 6w_8w_5^2w_9w_6c_s^2\omega_7^2w_4^2 + 12w_5w_9v_2^2\omega_6\omega_7^2w_4^3 + 12w_8w_5w_9v_2^2\omega_6\omega_7w_4^2 + 36w_2^2v_1^2c_s^2\omega_7^2w_4^3 - 36w_8w_5^2v_1^2\omega_9w_6c_s^2\omega_7^2w_4^2 - 12w_8w_5^2w_9w_6c_s^2\omega_7^2w_4^2 + \\ 24w_8w_5w_9v_2^2\omega_6\omega_7^2w_4^2 + 12w_5^2v_2^2\omega_6c_s^2\omega_7^2w_4^2 - 18w_8w_5^2v_1^2\omega_9v_2^2\omega_6\omega_7^2w_4^3 - 6w_8w_5w_9w_6c_s^2\omega_7^2w_4^2 + 18w_8w_5^2w_9w_6c_s^2\omega_7^2w_4^2 - 12w_8w_5^2w_6c_s^2\omega_7^2w_4^2 + \\ 18w_8w_5v_1^2\omega_9v_2^2\omega_6\omega_7^2w_4^2 + 6w_8w_5^2c_s^2\omega_7^2w_4^3 - 12w_5^2v_2^2\omega_6\omega_7^2w_4^3 + 12w_8w_5^2v_2^2\omega_6\omega_7^2w_4^2 + 12w_5w_9v_2^2\omega_6\omega_7^2w_4^2 + 12w_8w_5^2v_1^2\omega_9v_2^2\omega_6\omega_7^2w_4^2 + \\ 18w_8w_5w_9w_6c_s^2\omega_7^2w_4^2 + 36w_5^2v_1^2\omega_9w_6c_s^2\omega_7^2w_4^2 + 36w_8w_5^2v_1^2\omega_9w_6c_s^2\omega_7^2w_4^2 + 12w_5^2c_s^2\omega_7^2w_4^2 + 36w_8w_5w_9w_6c_s^2\omega_7^2w_4^2 - 12w_5w_9w_6c_s^2\omega_7^2w_4^2 + \\ 12w_8w_5w_9w_6c_s^2\omega_7^2w_4^2 - 6w_8w_5^2v_2^2\omega_6\omega_7^2w_4^3 - 36w_8w_5v_1^2\omega_9v_2^2\omega_6\omega_7^2w_4^2 + 6w_8w_5^2w_6c_s^4\omega_7^2w_4^2 - 9w_8w_5w_9v_2^2\omega_6\omega_7^2w_4^3 + w_8w_5^2w_9w_6c_s^2\omega_7^2w_4^2 - \\ 36w_5v_1^2\omega_9w_6c_s^2\omega_7^2w_4^2 - 18w_8w_5^2v_1^2\omega_9w_6c_s^2\omega_7^2w_4^3 - 12w_5w_9v_2^2\omega_6\omega_7^2w_4^3 + 60w_8w_5^2v_1^2\omega_9w_6c_s^2\omega_7^2w_4^2 - 12w_5w_9v_2^2\omega_6\omega_7^2w_4^3 - 5w_8w_5w_9w_6c_s^2\omega_7^2w_4^2 + \\ 36w_8w_5^2v_2^2\omega_6\omega_7^2w_4^2 + 12w_8w_5^2v_1^2\omega_9w_6c_s^2\omega_7^2w_4^2 + 24w_8w_5w_9v_2^2\omega_6\omega_7^2w_4^2 + 12w_8w_5^2w_6c_s^2\omega_7^2w_4^2 + 6w_8w_5w_9v_2^2\omega_6\omega_7^2w_4^2 + 12w_5w_9w_6c_s^2\omega_7^2w_4^2 + \\ 6w_8w_5w_9c_s^2\omega_7^2w_4^2 - 12w_5w_9w_6c_s^2\omega_7^2w_4^2 - 6w_8w_5^2c_s^2\omega_7^2w_4^2 - 15w_8w_5v_1^2\omega_9w_6c_s^2\omega_7^2w_4^2 + 36w_5v_1^2\omega_9v_2^2\omega_6\omega_7^2w_4^2 + 36w_8w_5^2v_1^2\omega_9v_2^2\omega_6\omega_7^2w_4^2 + \end{aligned}$$

$$6\omega_8\omega_5\omega_9v_2^2\omega_7^2\omega_4^3 - 18\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7^2\omega_4^2 - 18\omega_8\omega_5^2v_1^2c_s^2\omega_7^2\omega_4^3 - 12\omega_8\omega_5^2v_1^2\omega_9\omega_6c_s^2\omega_7^2\omega_4^3 - \omega_8\omega_5^2\omega_9\omega_6c_s^4\omega_7^2\omega_4^3 - 12\omega_8\omega_5^2v_1^2\omega_9\omega_6c_s^4\omega_7^2\omega_4^3 - 12\omega_5\omega_9\omega_6c_s^4\omega_7^2\omega_4^3 - 12\omega_5\omega_9\omega_6c_s^4\omega_7^2\omega_4^3 - 12\omega_5\omega_9\omega_6c_s^4\omega_7^2\omega_4^3 - 12\omega_5\omega_9\omega_6c_s^4\omega_7^2\omega_4^3 - 12\omega_5\omega_9\omega_6c_s^4\omega_7^2\omega_4^3 - 24\omega_8\omega_5\omega_9v_2^2\omega_6\omega_7^2\omega_4 - 6\omega_8\omega_5^2v_2^2\omega_6\omega_7^2\omega_4^3 - 6\omega_8\omega_5^2\omega_6c_s^2\omega_7^2\omega_4^3 - 6\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7^2\omega_4^3 + 18\omega_8\omega_5^2v_1^2\omega_9v_2^2\omega_6\omega_7^2\omega_4^3 - 5\omega_8\omega_5^2\omega_9\omega_6c_s^4\omega_7^2\omega_4^3 - 24\omega_8\omega_5\omega_9v_2^2\omega_7^2\omega_4^3 - 6\omega_8\omega_5^2v_2^2\omega_6\omega_7^2\omega_4^3 + 24\omega_8\omega_5^2v_1^2\omega_9\omega_6c_s^2\omega_7^2\omega_4^3 + 5\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7^2\omega_4^3 - 36\omega_5v_1^2\omega_9v_2^2\omega_6\omega_7^2\omega_4^3 - 18\omega_8\omega_5^2v_1^2v_2^2\omega_6\omega_7^2\omega_4^3 - 18\omega_8\omega_5^2v_1^2v_2^2\omega_6\omega_7^2\omega_4^3) \frac{\rho}{12\omega_8\omega_5^2\omega_9\omega_6\omega_7^2\omega_4^3}$$

$$C_{D_x^3 D_y v_1}^{(2), \text{MRT2}} = C_{D_x^3 D_y v_1}^{(2), \text{MRT1}}$$

$$\begin{aligned} C_{D_x^3 D_y v_1}^{(2), \text{CLBM1}} &= (6\omega_8\omega_5\omega_6\omega_4^3 + 12\omega_9\omega_6c_s^2\omega_7\omega_4^3 + 12\omega_8\omega_5\omega_9\omega_6c_s^2\omega_4 + \omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^2 - 12\omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^2 - 12\omega_8\omega_5\omega_6c_s^2\omega_7\omega_4^2 - \\ &\quad \omega_8\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4^3 - 12\omega_9\omega_6c_s^2\omega_7\omega_4^3 + 18\omega_8v_1^2\omega_9\omega_7\omega_4^3 - 12\omega_8\omega_5\omega_6\omega_7\omega_4^2 - 5\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4^2 - 18\omega_8\omega_5v_1^2\omega_7\omega_3^3 + 6\omega_8\omega_5\omega_6c_s^2\omega_7\omega_4^3 - \\ &\quad 36\omega_8v_1^2\omega_9\omega_6\omega_7\omega_4^2 + 12\omega_9\omega_7\omega_4^3 + 18\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 54\omega_8v_1^2\omega_9\omega_6\omega_7\omega_4^2 - 6\omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^2 + 6\omega_8\omega_5\omega_7\omega_4^3 + 36\omega_8\omega_5v_1^2\omega_6\omega_4^2 + 6\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4^2 - \\ &\quad 18\omega_8\omega_5v_1^2\omega_6\omega_7\omega_4^2 + 36\omega_5v_1^2\omega_7\omega_4^3 + 18\omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^2 - 15\omega_8v_1^2\omega_9\omega_6\omega_7\omega_4^2 - 12\omega_5\omega_7\omega_4^3 - 6\omega_8\omega_9\omega_7\omega_4^2 - 36\omega_8\omega_5v_1^2\omega_6\omega_7\omega_4^2 + 12\omega_9\omega_6\omega_7\omega_4^2 + 12\omega_5c_s^2\omega_7\omega_4^3 - \\ &\quad 18\omega_8\omega_5v_1^2\omega_6\omega_7\omega_4^3 - 36v_1^2\omega_9\omega_7\omega_4^3 + 18\omega_8\omega_5\omega_6\omega_7\omega_4^2 - 54\omega_8v_1^2\omega_9\omega_6\omega_7\omega_4^2 + 18\omega_8\omega_5v_1^2\omega_9\omega_6\omega_7\omega_4^3 + 12\omega_8\omega_5\omega_6\omega_7\omega_4^2 + 6\omega_8\omega_9c_s^2\omega_7\omega_4^3 - 6\omega_8\omega_5\omega_6c_s^2\omega_7\omega_4^3 + \\ &\quad 36v_1^2\omega_9\omega_6\omega_7\omega_4^2 - 18\omega_8\omega_9\omega_6\omega_7\omega_4^2 - 12\omega_5\omega_6\omega_7\omega_4^2 + 36\omega_5v_1^2\omega_6\omega_7\omega_4^2 + 5\omega_8\omega_9\omega_6\omega_7\omega_4^3 + 36\omega_8\omega_5v_1^2\omega_9\omega_6\omega_7\omega_4^2 - 36v_1^2\omega_9\omega_6\omega_7\omega_4^2 + 12\omega_8\omega_5\omega_6c_s^2\omega_7\omega_4^2 - \\ &\quad 12\omega_9c_s^2\omega_7\omega_4^3 - 36\omega_5v_1^2\omega_6\omega_7\omega_4^3 - 12\omega_8\omega_9\omega_6c_s^2\omega_7\omega_4^2 - 6\omega_8\omega_5c_s^2\omega_7\omega_4^3 + 12\omega_5\omega_6\omega_7\omega_4^3 - 3\omega_8\omega_5v_1^2\omega_9\omega_6\omega_7\omega_4^2 - 12\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4^2) \frac{\rho c_s^2}{12\omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^3} \end{aligned}$$

$$C_{D_x^3 D_y v_1}^{(2), \text{CLBM2}} = C_{D_x^3 D_y v_1}^{(2), \text{CLBM1}}$$

$$\begin{aligned} C_{D_x^3 D_y v_1}^{(2), \text{CuLBM1}} &= (12\omega_3c_s^2\omega_1 + 12\omega_3\omega_4 - 36v_1^2\omega_3\omega_4 - 12\omega_3\omega_1 - 12\omega_3c_s^2\omega_4 - \omega_3^2c_s^2\omega_4\omega_1 + 12\omega_3^2c_s^2 + 36v_1^2\omega_3\omega_1 - 12c_s^2\omega_4\omega_1 - 5\omega_3^2c_s^2\omega_4\omega_1 + \\ &\quad \omega_3^2\omega_4\omega_1 + \omega_3^3c_s^2\omega_4 - 3v_1^2\omega_3^2\omega_4\omega_1 - 6\omega_3^3c_s^2 + 6\omega_3^3c_s^2\omega_1 + 18v_1^2\omega_3^2\omega_1 - 18v_1^2\omega_3^2\omega_3 - 12\omega_3^2 - 6\omega_3^3\omega_1 + 36v_1^2\omega_3^2 + 6\omega_3^3 + 3v_1^2\omega_3^2\omega_4 - \omega_3^3\omega_4 + 18\omega_3^2\omega_1 - \\ &\quad 18\omega_3^2c_s^2\omega_1 + 18\omega_3c_s^2\omega_4\omega_1 - 54v_1^2\omega_3^2\omega_1 - 6\omega_3^2\omega_4 + 6\omega_3^2c_s^2\omega_4 + 18v_1^2\omega_3^2\omega_4) \frac{\rho c_s^2}{12\omega_3^2\omega_4\omega_1} \end{aligned}$$

$$\begin{aligned} C_{D_x^3 D_y v_1}^{(2), \text{CuLBM2}} &= (-138\omega_3^2v_1^4\omega_3\omega_1^2 - 33\omega_2\omega_3c_s^4\omega_1^3 - 81\omega_2^2v_1^2\omega_3\omega_1^3 - 45\omega_2^3\omega_3c_s^2\omega_1 + 12\omega_2^3c_s^4\omega_1^3 - 90\omega_2^3v_1^2c_s^2\omega_1^2 - 6\omega_2^2\omega_3\omega_1 - 12\omega_2^2\omega_3c_s^2\omega_1^2 + \\ &\quad 18\omega_2^2c_s^2\omega_1^3 - 135\omega_2^2v_1^4\omega_3\omega_1 - 30\omega_2^3c_s^4\omega_1^2 + 171v_1^4\omega_3\omega_1^3 + 351\omega_2^3v_1^2\omega_3c_s^2\omega_1 + 6\omega_2\omega_3c_s^4\omega_1^2 - 24\omega_2^2c_s^2\omega_1^2 - 180\omega_2^3v_1^2\omega_3\omega_1 - 24\omega_2^2\omega_3c_s^2\omega_1^3 + \\ &\quad 36\omega_2^3v_1^2\omega_3^2\omega_1^3 + 138\omega_2^2v_1^4\omega_3\omega_1^3 + 81\omega_2^3v_1^2\omega_3\omega_1^2 + 24\omega_2^3\omega_3c_s^2 + 7\omega_2^2\omega_3\omega_1^3 - 54\omega_2^2v_1^2c_s^2\omega_1^3 + 18\omega_2^3c_s^4\omega_1^2 + 99\omega_2^3v_1^2\omega_3c_s^2\omega_1^2 + 63\omega_2^2v_1^2\omega_3\omega_1 - \\ &\quad 24\omega_3c_s^2\omega_1^3 + 207v_1^2\omega_3c_s^2\omega_1^3 - 99v_1^2\omega_3\omega_1^3 + 54\omega_3^2v_1^2c_s^2\omega_1 + 36\omega_2^2\omega_3c_s^2\omega_1 + 324\omega_2^3v_1^4\omega_3\omega_1 - 3\omega_3^2v_1^2\omega_3c_s^2\omega_1^2 + 6\omega_2c_s^4\omega_1^3 + 24\omega_2^3\omega_3c_s^2\omega_1^2 + 6\omega_3\omega_1^3 + \\ &\quad 72\omega_2^2v_1^2\omega_3\omega_1^2 + 30\omega_2^3c_s^2\omega_1^2 + 36\omega_2^2v_1^2\omega_3c_s^2\omega_1^2 + 3\omega_2^2v_1^2\omega_3c_s^2\omega_1^3 - 12\omega_2\omega_3\omega_1^3 - 12\omega_2\omega_3c_s^2\omega_1^2 - \omega_2^2v_1^2\omega_3\omega_1^3 + 24\omega_2^2c_s^4\omega_1^2 - \omega_2^3v_1^2\omega_3\omega_1^2 - 42\omega_3^2\omega_3c_s^4 + \\ &\quad 17\omega_2^2\omega_3c_s^4\omega_1^3 - 351\omega_2v_1^2\omega_3c_s^2\omega_1^3 + 180\omega_2v_1^2\omega_3\omega_1^3 - 6\omega_2^3\omega_3 + 45\omega_2\omega_3c_s^2\omega_1^3 + 12\omega_2^3\omega_3\omega_1 + 69\omega_2^3\omega_3c_s^4\omega_1^2 - 12\omega_2^3c_s^2\omega_1^3 + 6\omega_2\omega_3\omega_1^2 + 153\omega_2^2v_1^2\omega_3c_s^2\omega_1^2 - \\ &\quad 63\omega_2v_1^2\omega_3\omega_1^2 + 63\omega_2v_1^2\omega_3c_s^2\omega_1^2 + 12\omega_2^2\omega_3c_s^4\omega_1^2 - 18\omega_2^2c_s^4\omega_1^3 - 30\omega_2^2\omega_3c_s^4\omega_1^2 + 18\omega_2v_1^2c_s^2\omega_1^3 + \omega_2^2v_1^2\omega_3\omega_1^2 - 7\omega_2^2\omega_3\omega_1^2 - 6\omega_2^2c_s^2\omega_1^3 + \omega_2^2v_1^2\omega_3\omega_1^2 - \\ &\quad 324\omega_2v_1^2\omega_3\omega_1^3 - 207\omega_2^3v_1^2\omega_3c_s^2\omega_1^2 - 25\omega_2^3\omega_3c_s^4\omega_1^2 - 135\omega_2v_1^2\omega_3c_s^2\omega_1^2 - 18\omega_2^3c_s^2\omega_1^3 + 135\omega_2v_1^2\omega_3\omega_1^2 - 2\omega_2^3\omega_3c_s^4\omega_1^2 + 18\omega_3c_s^4\omega_1^2) \frac{\rho}{24\omega_2^3\omega_3\omega_1^3} \end{aligned}$$

coefficient  $C_{D_x^3 D_y v_2}^{(2)}$  at  $\frac{\partial^4 v_2}{\partial x_1^3 \partial x_2}$ :

$$C_{D_x^3 D_y v_2}^{(2), \text{SRT}} = (2 - 2v_1^2 - 6c_s^2 + 3c_s^2\omega + v_1^2\omega - \omega) \frac{v_1\rho v_2}{12\omega}$$

$$\begin{aligned} C_{D_x^3 D_y v_2}^{(2), \text{MRT1}} &= (-12\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_4^3 + 24\omega_8^2\omega_5^2\omega_6^2\omega_7^2\omega_4^2 + 90\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^2 + 12\omega_8^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 60\omega_8\omega_5^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^3 + \\ &\quad 12\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - 72\omega_8^2\omega_5^2\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 12\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 12\omega_8^2\omega_5^2v_1^2\omega_6\omega_7^2\omega_4^2 + 12\omega_8^2\omega_5^2\omega_6^2c_s^2\omega_7\omega_4^3 - \\ &\quad 96\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^2 + 24\omega_8\omega_5^2\omega_6^2\omega_7^2\omega_4^2 - 12\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - 12\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 - 24\omega_8^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + \\ &\quad 12\omega_8^2v_1^2\omega_6\omega_7^2\omega_4^3 - 12\omega_8^2\omega_5\omega_9\omega_6^2\omega_7^2\omega_4^2 + 24\omega_8^2\omega_5^2\omega_6^2\omega_7^2\omega_4^2 - 132\omega_8\omega_5^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^2 - 24\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - 6\omega_8^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - \\ &\quad 12\omega_8^2\omega_5^2\omega_6^2\omega_7^2\omega_4^3 + 4\omega_8^2\omega_5^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - 24\omega_8\omega_5^2\omega_6^2\omega_7^2\omega_4^2 - 24\omega_8^2\omega_5^2\omega_6^2\omega_7^2\omega_4^2 + 24\omega_8^2\omega_5^2\omega_6^2\omega_7^2\omega_4^2 - \omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - 24\omega_8\omega_5^2v_1^2\omega_6\omega_7^2\omega_4^3 + \\ &\quad 6\omega_8^2\omega_5\omega_9\omega_6^2\omega_7^2\omega_4^2 - 12\omega_8\omega_5^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^3 - 24\omega_8^2\omega_5^2v_1^2\omega_9\omega_6\omega_7\omega_4^2 + \omega_8^2\omega_5^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^3 + 72\omega_8\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 + 24\omega_8\omega_5^2\omega_6^2\omega_7^2\omega_4^2 + \\ &\quad 18\omega_8^2\omega_5\omega_9\omega_6^2\omega_7^2\omega_4^2 - 24\omega_8^2\omega_5\omega_6^2\omega_7^2\omega_4^3 - 12\omega_8^2\omega_5\omega_9\omega_6^2\omega_7^2\omega_4^2 + 12\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 24\omega_8\omega_5^2v_1^2\omega_6\omega_7^2\omega_4^2 - 12\omega_8^2\omega_5^2\omega_9\omega_6\omega_7^2\omega_4^3 + \\ &\quad 12\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 24\omega_8^2\omega_5^2\omega_6^2\omega_7^2\omega_4^2 - 36\omega_8^2\omega_5^2\omega_6\omega_7^2\omega_4^2 - 4\omega_8^2\omega_5\omega_9\omega_6^2\omega_7^2\omega_4^2 - 84\omega_8^2\omega_5^2\omega_9\omega_6c_s^2\omega_7^2\omega_4^2 - \\ &\quad 12\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 12\omega_8^2\omega_5^2\omega_6^2\omega_7^2\omega_4^2 - 42\omega_8^2\omega_5^2\omega_6\omega_7^2\omega_4^2 + 24\omega_8\omega_5^2v_1^2\omega_6\omega_7^2\omega_4^2 - 12\omega_8^2\omega_5^2\omega_6^2c_s^2\omega_7^2\omega_4^2 - 66\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - \\ &\quad 6\omega_8^2\omega_5\omega_9\omega_6^2\omega_7^2\omega_4^3 + 156\omega_8^2\omega_5^2\omega_6\omega_7^2\omega_4^2 - 24\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 - 84\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 36\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 + 66\omega_8^2\omega_5^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + \\ &\quad 12\omega_8\omega_5^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^3 + 18\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - 12\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 12\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 - 12\omega_8\omega_5^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - \\ &\quad 18\omega_8\omega_5\omega_9\omega_6^2\omega_7^2\omega_4^3 - 18\omega_8\omega_5^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 48\omega_8\omega_5^2\omega_9\omega_6c_s^2\omega_7^2\omega_4^2 + 12\omega_8\omega_5^2\omega_9\omega_6\omega_7^2\omega_4^2 + 12\omega_8\omega_5^2\omega_6^2\omega_7^2\omega_4^2 - 24\omega_8\omega_5^2\omega_6\omega_7^2\omega_4^3 - 12\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + \\ &\quad 24\omega_8\omega_5\omega_9\omega_6^2c_s^2\omega_4^3 - 12\omega_8\omega_5^2v_1^2\omega_6\omega_7^2\omega_4^2 + 24\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 3\omega_8\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 + 12\omega_8\omega_5^2\omega_6^2\omega_7^2\omega_4^2 - 24\omega_8\omega_5^2\omega_6\omega_7^2\omega_4^3 - \\ &\quad 6\omega_8\omega_5^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 24\omega_8\omega_5^2\omega_6^2c_s^2\omega_7^2\omega_4^2 - 24\omega_8\omega_5^2\omega_6^2\omega_7^2\omega_4^2 - 66\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 12\omega_8\omega_5^2\omega_6^2\omega_7^2\omega_4^2 + 24\omega_8\omega_5^2v_1^2\omega_6\omega_7^2\omega_4^2 - \\ &\quad 36\omega_8\omega_5^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 12\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - 48\omega_8\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 + 24\omega_8\omega_5^2\omega_6^2c_s^2\omega_7^2\omega_4^2 - 12\omega_8\omega_5^2\omega_6^2\omega_7^2\omega_4^2 + 12\omega_8\omega_5^2\omega_6^2c_s^2\omega_7^2\omega_4^2 + \\ &\quad 12\omega_8\omega_5\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 - 24\omega_8\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 - 12\omega_8\omega_5^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 6\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 36\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2) \frac{\rho}{12\omega_8\omega_5^2\omega_9\omega_6\omega_7^2\omega_4^2} \end{aligned}$$

$$C_{D_x^3 D_y v_2}^{(2), \text{MRT2}} = C_{D_x^3 D_y v_2}^{(2), \text{MRT1}}$$

$$C_{D_x^3 D_y v_2}^{(2), \text{CLBM1}} = (-3\omega_5v_1^2 + 3\omega_5 + \omega_5v_1^2\omega_7 - \omega_5\omega_7 + v_1^2\omega_7 - 9\omega_5c_s^2 + 3\omega_5c_s^2\omega_7 + 3c_s^2\omega_7 - \omega_7) \frac{v_1\rho v_2}{12\omega_5\omega_7}$$

$$C_{\mathrm{D}_x^3 \mathrm{D}_y v_2}^{(2), \text{CLBM2}} = C_{\mathrm{D}_x^3 \mathrm{D}_y v_2}^{(2), \text{CLBM1}}$$

$$C_{\substack{D_x^3 D_y v_2}}^{(2), \text{CuLBMBI}} = (3c_s^2 \omega_4 \omega_1 + 3c_s^2 \omega_4 - 9c_s^2 \omega_1 - 3v_1^2 \omega_1 + v_1^2 \omega_4 - \omega_4 + 3\omega_1 - \omega_4 \omega_1 + v_1^2 \omega_4 \omega_1) \frac{v_1 \rho v_2}{12 \omega_4 \omega_1}$$

$$C_{D_x^3 D_y v_2}^{(2), \text{CuLBM2}} = (6w_3 c_s^2 \omega_1 + 2\omega_2 \omega_3 - 4\omega_3 \omega_1 + \omega_2 v_1^2 \omega_3 + v_1^2 \omega_3 \omega_1 + 6\omega_2 \omega_1 - 6\omega_2 v_1^2 \omega_1 - 18\omega_2 c_s^2 \omega_1 + 9v_2^2 \omega_3 \omega_1 + 2\omega_2 v_1^2 \omega_3 \omega_1 - 2\omega_2 \omega_3 \omega_1 - 9\omega_2 v_2^2 \omega_3 + 6\omega_2 \omega_3 c_s^2 \omega_1) \frac{v_1 \rho v_2}{24\omega_2 \omega_3 \omega_1}$$

coefficient  $C_{D_x^2 D_y^2 \rho}^{(2)}$  at  $\frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2}$ :

$$C_{\frac{D_2}{D_2 D_2} \rho}^{(2), \text{SRT}} = (24 - 46c_s^2\omega^2 + v_2^2\omega^3 - 14v_2^2\omega^2 + 5c_s^2\omega^3 + 36v_2^2\omega - 72c_s^2 + 108c_s^2\omega - 24v_2^2 - 36\omega + 14\omega^2 - \omega^3) \frac{v_2 c_s^2}{12\omega^3}$$

$$\begin{aligned}
C_{(2),MRT1}^{(2)} &= (18w_8^2 v_1^2 w_9 w_6 w_7 w_4^3 + 12 w_9 v_2^2 w_6^3 c_2^2 w_7 w_4^3 + 12 w_8 w_9 w_3^2 c_2^2 w_7 w_4 + 12 v_1^2 w_9 w_6^2 w_7 w_4^3 - 12 w_8 w_9 v_2^2 w_6^3 c_3^2 w_7 w_4 - 12 w_8^2 w_9 v_2^2 w_6^2 c_3^2 w_7 w_4^2 + \\
&36v_1^2 w_9 w_6^3 c_3^2 w_7 w_4^3 + 12 w_8^2 v_1^2 w_9 v_2^2 w_7 w_4^3 + 12 w_8^2 w_9 w_6^2 c_2^2 w_7 w_4 + 12 w_8^2 v_1^2 w_9 v_2^2 w_6 w_7 w_4^2 + 12 w_8^2 w_9 w_6^3 c_2^2 w_7 w_4^2 - 12 w_8 v_1^2 w_9 v_2^2 w_6^2 w_7 w_4^2 - \\
&18 w_8^2 w_9^2 c_3^2 w_7 w_4^3 - 72 w_8 v_1^2 w_9 w_6^3 c_3^2 w_7 w_4 - 6 w_8^2 w_9 w_6^2 c_3^4 w_7 w_4^3 - 36 w_8 v_2^2 w_9 w_6^2 c_2^2 w_7 w_4^2 + 12 w_8 w_9^3 c_2^2 w_7 w_4^3 + 12 w_8^2 w_9 w_6^3 c_2^2 w_7 w_4^2 + 30 w_8 w_9 w_6^3 c_4^2 w_7 w_4^3 - \\
&12 w_8^2 v_1^2 w_9 w_7 w_4^3 - 12 w_8^2 v_1^2 v_2^2 w_6^3 c_2^2 w_7 w_4 - 12 w_8^2 v_1^2 w_6^3 c_2^2 w_7 w_4^3 + 24 w_8 v_1^2 w_9 w_6^3 c_2^2 w_7 w_4^3 + 180 w_8^2 w_9 w_6^3 c_3^2 w_7 w_4 - 36 v_1^2 w_9 w_6^3 c_2^2 w_7 w_4^3 - \\
&6 w_8 w_6^3 c_2^2 w_7 w_4^3 - 18 w_8^2 v_1^2 w_9 v_2^2 w_6 w_7 w_4^3 + 6 w_8 w_9 v_2^2 w_6^2 c_2^2 w_7 w_4^3 - 12 w_8^2 v_1^2 w_9 w_6 w_7 w_4^2 - 12 w_9 v_2^2 w_6^2 c_2^2 w_7 w_4^2 + 6 w_8 v_1^2 v_2^2 w_6^3 c_2^2 w_7 w_4^3 - 18 w_8 w_9 w_6^2 c_2^2 w_7 w_4^3 - \\
&36 w_8 v_1^2 w_9 w_6^3 c_2^2 w_7 w_4^3 + 6 w_8^2 v_1^2 w_6^3 c_2^2 w_7 w_4^3 - 42 w_8 w_9 w_6^3 c_4^2 w_7 w_4^3 + 18 w_8 w_9 v_2^2 w_6^2 c_2^2 w_7 w_4^3 - w_8^2 w_9 w_6^3 c_2^2 w_7 w_4^3 - 12 w_8 w_9^3 c_2^2 w_7 w_4^2 + 36 w_8 w_9^2 c_2^2 w_7 w_4^3 + \\
&72 w_8 v_2^2 w_9 w_6^2 c_2^2 w_7 w_4^3 + 150 w_8^2 w_9 w_6^2 c_3^2 w_7 w_4^3 + 24 w_8 v_1^2 w_9 w_6^3 c_2^2 w_7 w_4^3 + 36 w_8^2 v_1^2 w_9 w_6 c_2^2 w_7 w_4^3 + 6 w_8^2 w_9 w_6^3 c_2^2 w_7 w_4^3 + 12 w_8 w_9 w_6^3 c_3^2 w_7 w_4 - 12 w_8 v_1^2 w_6^2 c_2^2 w_7 w_4^3 + \\
&6 w_8 w_6^2 c_2^2 w_7 w_4^3 - 6 w_8^2 v_1^2 v_2^2 w_6^3 c_2^2 w_7 w_4^3 - 36 w_8^2 w_6^3 c_4^2 w_7 w_4^3 - 84 w_8^2 w_9 w_6^3 c_4^2 w_7 w_4 - 88 w_8^2 w_9 w_6^3 c_4^2 w_7 w_4^3 - 36 w_8 v_1^2 w_9 w_6^3 c_2^2 w_7 w_4^3 - 36 w_8 w_6^3 c_4^2 w_7 w_4^3 + \\
&12 w_8^2 w_9 w_6^3 c_2^2 w_7 w_4^3 + 2 w_8 w_9 w_6^3 c_2^2 w_7 w_4^3 - 12 w_8 v_1^2 v_2^2 w_6^3 c_2^2 w_7 w_4^3 - 6 w_8 w_9 w_6^3 c_2^2 w_7 w_4^3 + 36 w_8 v_1^2 w_9 w_6^3 c_2^2 w_7 w_4^3 + 12 w_8^2 v_1^2 v_2^2 w_6^3 c_2^2 w_7 w_4^3 + \\
&18 w_8^2 w_9^3 c_2^2 w_7 w_4^3 - 36 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 + 12 w_8 v_1^2 w_9 w_6 w_7 w_4^3 - 12 w_8^2 w_9 w_6^2 c_2^2 w_7 w_4^3 - 12 w_8^2 w_9 w_6^3 c_2^2 w_7 w_4^3 - 12 w_9 v_2^2 w_6^2 c_2^2 w_7 w_4^3 - \\
&54 w_8 v_1^2 w_9 w_6 w_7 w_4^3 - 18 w_8 w_9 w_6^3 c_2^2 w_7 w_4^3 - 42 w_8 w_9 w_6^2 c_3^2 w_7 w_4^3 + 18 w_8 w_9 v_2^2 w_6^2 c_2^2 w_7 w_4^3 - 12 w_8^2 w_9 w_6^3 c_2^2 w_7 w_4^3 - 12 w_8^2 w_9 w_6^3 c_2^2 w_7 w_4^3 - \\
&18 w_8 w_9 w_6^2 c_2^2 w_7 w_4^3 - 12 w_8 w_9^2 c_2^2 w_7 w_4^3 + 6 w_8^2 v_1^2 w_6^2 w_7 w_4^3 + 108 w_8 v_1^2 w_9 w_6^3 c_2^2 w_7 w_4^3 + 36 w_8 w_9^3 c_2^2 w_7 w_4^3 + 5 w_8 w_9 w_6^3 c_4^2 w_7 w_4^3 - 6 w_8^2 w_9 w_6^3 c_2^2 w_7 w_4^3 - \\
&12 w_8^2 v_2^2 w_6^3 c_2^2 w_7 w_4^3 + 36 w_8^2 v_1^2 w_6^3 c_2^2 w_7 w_4^3 - 36 w_8^2 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 - 6 w_8^2 v_2^2 w_6^3 c_2^2 w_7 w_4^3 + 12 w_8 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 - 6 w_8 v_1^2 v_2^2 w_6^3 c_2^2 w_7 w_4^3 - 36 w_8^2 w_9 w_6 c_4^2 w_7 w_4^3 - \\
&12 w_8 w_9 c_2^2 w_7 w_4^3 - 12 w_8^2 v_1^2 w_9 v_2^2 w_6^2 c_2^2 w_7 w_4^3 - 24 w_8^2 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4 - 36 w_8 v_1^2 w_6^3 c_2^2 w_7 w_4^3 + u_8^2 w_9 v_2^2 w_6^2 c_2^2 w_7 w_4^3 - 12 w_8 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 + 12 w_9 w_6^3 c_4^2 w_7 w_4^3 - \\
&18 w_8 v_1^2 w_6^2 c_2^2 w_7 w_4^3 + 12 w_9 w_6^2 c_2^2 w_7 w_4^3 + 12 w_9^2 w_6^2 c_2^2 w_7 w_4^3 + 12 w_8^2 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 - 12 w_8^2 w_9 v_2^2 w_6^2 c_2^2 w_7 w_4^3 + 6 w_8^2 v_1^2 w_9 v_2^2 w_6^2 c_2^2 w_7 w_4^3 - 12 w_8 w_9^2 c_2^2 w_6^2 c_2^2 w_7 w_4^3 - \\
&36 w_8 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 - 48 w_8^2 w_9 w_6 c_2^2 w_7 w_4^3 + 12 w_8 w_9 w_6 c_2^2 w_7 w_4^3 - 24 w_8 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 + 72 w_8^2 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 - 12 w_8 w_9 v_2^2 w_6^2 c_2^2 w_7 w_4^3 + 6 w_8^2 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 - 12 w_8^2 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 + 24 w_8 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 - \\
&6 w_8^2 v_2^2 w_6^2 c_2^2 w_7 w_4^3 - 18 w_8^2 v_1^2 w_6^2 c_2^2 w_7 w_4^3 + 18 w_8^2 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 + 12 w_8^2 v_2^2 w_6^2 c_2^2 w_7 w_4^3 - 6 w_8^2 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 - 12 w_8^2 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 + 24 w_8^2 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 - \\
&12 w_8 w_9^3 c_2^2 w_7 w_4^3 - 36 w_8 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 - 12 w_8^2 w_9 w_6^2 c_2^2 w_7 w_4^3 + 36 w_8^2 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 - 6 w_8^2 v_2^2 w_6^2 c_2^2 w_7 w_4^3 - 24 w_8^2 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 - \\
&24 w_8 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 + 36 w_8^2 w_9^3 c_2^2 w_7 w_4^3 - 12 w_8 w_9 v_2^2 w_6^2 c_2^2 w_7 w_4^3 + 36 w_8^2 w_9 w_6^2 c_2^2 w_7 w_4^3 - 6 w_8^2 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 + 12 w_8^2 w_9 w_6^2 c_2^2 w_7 w_4^3 + \\
&12 w_8 w_9 w_6 c_2^2 w_7 w_4^3 + 12 w_9 v_2^2 w_6^2 c_2^2 w_7 w_4^3 - 96 w_8^2 w_9 w_6^3 c_2^2 w_7 w_4^3 + 18 w_8^2 w_9 v_2^2 w_6^2 c_2^2 w_7 w_4^3 - 12 w_8 v_1^2 w_6^2 c_2^2 w_7 w_4^3 + 12 w_9 w_6^2 c_4^2 w_7 w_4^3 + 12 w_8 w_9 v_2^2 c_2^2 w_7 w_4^3 - \\
&72 w_8^2 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 - 36 w_8^2 v_1^2 w_6^2 c_2^2 w_7 w_4^3 + 12 w_9 w_6^3 c_2^2 w_7 w_4^3 + 18 w_8^2 w_9 w_6^3 c_2^2 w_7 w_4^3 + 24 w_8^2 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 + 12 w_8^2 v_1^2 w_6^3 c_2^2 w_7 w_4^3 - 12 w_8 v_1^2 w_9 v_2^2 w_6^2 c_2^2 w_7 w_4^3 + \\
&12 w_8 w_9 v_2^2 w_6^2 c_2^2 w_7 w_4^3 - 18 w_8^2 w_9 w_6^3 c_2^2 w_7 w_4^3 + 12 w_8 w_9 v_2^2 w_6^3 c_2^2 w_7 w_4^3 + 6 w_8^2 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 - 12 w_8 v_1^2 w_9 v_2^2 w_6^2 c_2^2 w_7 w_4^3 + 12 w_8 w_9 w_6 c_4^2 w_7 w_4^3 - 36 w_8^2 w_9 w_6^3 c_2^2 w_7 w_4^3 - \\
&12 w_9 w_6^3 c_2^2 w_7 w_4^3 + 18 w_8^2 v_1^2 w_6^2 c_2^2 w_7 w_4^3 + 18 w_8^2 v_1^2 w_9 w_6^2 c_2^2 w_7 w_4^3 - 2 w_8^2 w_9 w_6^2 c_2^2 w_7 w_4^3 + 12 w_8 v_1^2 w_6^2 c_2^2 w_7 w_4^3 + 36 w_8 w_9^2 v_1^2 w_6^2 c_2^2 w_7 w_4^3) \frac{1}{12 w_8^2 w_9^2 w_7 w_4^3}
\end{aligned}$$

$$C_{D_x^2 D_y^2 \rho}^{(2), \text{MRT2}} = C_{D_x^2 D_y^2 \rho}^{(2), \text{MRT1}}$$

$$C_{\mathrm{D}_x^2 \mathrm{D}_y^2 \rho}^{(2), \text{CLBM2}} = C_{\mathrm{D}_x^2 \mathrm{D}_y^2 \rho}^{(2), \text{CLBM1}}$$

$$\begin{aligned}
C_{D_2^2 D_2^2 y}^{(2), \text{CuLBMI}} = & (12w_2^2 w_6^2 w_3^2 + 12w_2^2 w_6^2 w_3 c_s^2 + w_2^3 v_2^2 w_6^2 w_3^2 - w_2^3 w_6^2 w_3^2 + 6w_2^2 v_2^2 w_6^2 w_3 - 36w_2^2 w_3^2 c_s^2 + 36w_2^3 w_6 w_3 c_s^2 - 12w_2^2 w_6 w_3 - 36w_2^3 w_6 c_s^2 + \\
& 54w_2^3 w_6 w_3 c_s^2 - 36w_2^3 w_3 c_s^2 - 12w_2^2 v_2^2 w_3^2 - 6w_2^2 w_6 w_3^2 - 36w_2^2 w_6^2 c_s^2 - 12w_2^2 v_2^2 w_6^2 + 36w_2^2 w_3^2 c_s^2 + 4w_2^2 v_2^2 w_6^2 w_3^2 + 5w_2^3 w_6^2 w_3^2 c_s^2 + 18w_2 w_6^2 w_3^2 - \\
& 12w_2^3 v_2^2 w_6^2 w_3 + 12w_2^3 w_6^2 w_3 + 6w_2^2 v_2^2 w_6 w_3^2 - 12w_2^2 w_6^2 w_3^2 - 40w_2^2 w_6^2 w_3 c_s^2 + 12w_2^2 w_6^2 - 18w_2^3 w_6 w_3 - 54w_2 w_6^2 w_3 c_s^2 - 12w_2^3 v_2^2 w_3 + 12w_2^3 w_3 + \\
& 18w_2^3 v_2^2 w_6 w_3 - 12w_2^2 v_2^2 w_6 + 12w_2^3 w_6 - 18w_2^3 w_6 w_3 c_s^2 + 36w_2^3 w_3^2 c_s^2 + 12w_2^2 w_3^2 - 4w_2^2 w_6^2 w_3^2 - 6w_2^2 w_6^2 w_3 + 18w_2^3 w_6 w_3 c_s^2 - 12w_2^3 w_6^2 + 12w_2^3 v_2^2 w_6^2 + \\
& 36w_2^3 w_6^2 c_s^2 + 18w_2^2 w_6^2 w_3 c_s^2 + 6w_2^3 w_6 w_3^2 - 12w_2^3 w_3^2 - 6w_2^3 v_2^2 w_6 w_3^2 + 12w_2^3 v_2^2 w_3^2 + 12w_2^2 v_2^2 w_6 w_3 - 18w_2 v_2^2 w_6^2 w_3^2) \frac{v_2^2 c_s^2}{12w_2^2 w_6^2 w_3^2}
\end{aligned}$$

$$\begin{aligned} C_{\substack{(2), \text{CuLBMe2} \\ -D_2^2 D_y}} &= (-18\omega_2 w_3 c_s^4 w_3^3 - 30w_3^2 v_2^2 w_3 c_s^2 w_1 - 90w_2^2 w_3^2 c_s^4 w_1 + 30w_3^2 w_3 c_s^2 w_1 + 72w_3^2 c_s^4 w_3^3 - 30w_2^2 v_2^2 w_3^2 c_s^2 w_1 - 54w_2 v_1^4 w_3^2 w_1^2 - 6v_2^2 w_3^2 w_1^3 - 36w_2^2 w_3 c_s^2 w_1^2 - 91w_3^2 w_3^2 c_s^4 w_1^2 + 12w_2^2 c_s^2 w_1^3 - 108w_3^2 c_s^4 w_1^2 + 9w_3^2 v_1^2 w_3^2 + 6w_2^2 v_2^2 w_3^2 w_1 + 45v_1^2 v_2^2 c_s^2 w_1^3 + 90w_3^2 c_s^4 w_1^3 + 10w_3^2 w_3^2 c_s^4 w_1^3 + 27w_2^2 v_1^2 w_3^2 c_s^2 w_1 - 54w_4 v_1^2 w_3^2 w_1^3 - 135w_3^2 v_1^2 w_3^2 c_s^2 - 6w_3^2 w_3^2 w_1 + 24w_2 w_3^2 c_s^2 w_1^2 + 6w_3^2 v_2^2 w_3^2 w_1 - 6w_2 w_3^2 w_1^2 + 270w_2 v_1^2 w_3^2 c_s^2 w_1^2 + 8w_2 v_2^2 w_3^2 c_s^2 w_1^3 - 6w_2 v_2^2 w_3^2 c_s^2 w_1^2 - 459w_2 v_1^2 w_3^2 c_s^2 w_1^3 + 12w_2^2 w_3 c_s^2 w_1^3 + 99w_2 v_1^2 w_3^2 w_1^2 + 35w_2^2 w_3^2 c_s^4 w_1^3 - 12w_2^2 v_2^2 w_3^2 w_1^2 - 12w_3^2 v_2^2 w_3 c_s^2 w_1^3 + 138w_2 v_1^2 w_3^2 c_s^2 w_1^3 + 18w_2 v_2^2 w_3^2 c_s^2 w_1^2 + 63w_3^2 c_s^2 c_s^4 w_1 - 6w_2 w_3^2 w_1^3 - 45w_3^2 v_1^2 v_2^2 w_3^2 w_1 + 93w_2 w_3^2 c_s^2 c_s^2 w_1^3 + 48w_3^2 v_2^2 w_3 c_s^2 w_1^2 - 48w_3^2 w_3 c_s^2 w_1^2 + 54w_2^2 w_3^2 c_s^4 w_1^2 + 99w_2 v_1^2 w_3^2 w_1^3 - 21w_2 v_2^2 w_3^2 c_s^2 w_1^3 - 297w_2 v_1^2 w_3^2 c_s^2 w_1^2 + 18w_3^2 w_3^2 c_s^4 w_1^2 + 45w_3^2 v_1^2 v_2^2 w_3^2 + 36w_3^2 c_s^2 c_s^2 w_1^2 + 18w_2 v_2^2 w_3^2 c_s^2 w_1^3 + 12w_2 w_3^2 w_1^2 - 99w_1^2 w_3^2 w_1^3 - 36w_3^2 v_2^2 c_s^2 w_1^2 - 54w_3^2 v_4^4 w_3^2 - 72w_3^2 c_s^2 w_1^3 + 54w_3^2 c_s^4 w_1^2 w_3^2 w_1 - 138w_3^2 v_1^2 v_2^2 c_s^2 c_s^2 w_1^2 + 2w_3^2 v_2^2 w_3^2 c_s^2 w_1^3 - 2w_3^2 w_3^2 c_s^2 w_1^3 + 6w_3^2 w_3^2 w_1^3 + 6w_2 w_3 c_s^2 w_1^2 + 27w_3^2 v_1^2 w_3^2 w_1^2 + 27w_2^2 v_4^4 w_3^2 w_1^3 - 90w_3^2 w_3 c_s^4 w_1^2 + 18w_3^2 v_2^2 w_3^2 c_s^2 w_1^2 + 48w_2 v_2^2 c_s^2 w_1^2 + 24w_2^2 v_2^2 c_s^2 w_1^3 + 405w_1^2 w_3^2 c_s^2 w_1^3 - 24w_3^2 c_s^4 w_1^3 + 36w_2^2 v_2^2 w_3^2 c_s^2 w_1^2 - 6w_2 v_2^2 w_3^2 c_s^2 w_1^3 + 41w_3^2 c_s^2 c_s^2 w_1^2 + 108w_2^2 w_3 c_s^2 w_1^2 - 24w_3^2 v_2^2 w_3^2 c_s^2 w_1^2 - 36w_2^2 c_s^2 w_1^3 - 9w_2^2 v_1^2 w_3^2 w_1^2 - 45w_3^2 v_1^2 v_2^2 w_3^2 w_1 + 54v_1^2 w_3^2 c_s^2 w_1^3 + 6w_2 v_2^2 w_3^2 c_s^2 w_1^2 - 12w_2^2 v_2^2 c_s^2 w_1^3 - 38w_2^2 w_3^2 c_s^2 w_1^2 + 3w_3^2 v_2^2 w_3^2 c_s^2 w_1^3 - 117w_2 w_3^2 c_s^2 w_1^2 - 90w_2^2 v_1^2 w_3^2 w_1^2 + 90w_3^2 v_1^2 v_2^2 w_3^2 w_1^2 - 45w_2 v_1^2 v_2^2 w_3^2 w_1^3 - 9w_3^2 v_1^2 w_3^2 w_1^2 - 54w_2^2 w_3^2 c_s^2 w_1^2 + 144w_3^2 w_3 c_s^4 w_1^2 - 27w_2^2 v_1^2 w_3^2 c_s^2 w_1^3 - 18w_2 w_3^2 c_s^4 w_1^2 + 189w_3^2 v_1^2 w_3^2 c_s^2 w_1^2 - 27w_3^2 v_1^2 w_3^2 w_1^2 + 6w_3^2 w_3^2 + 6w_2 v_2^2 w_3^2 c_s^2 w_1^2 - 6w_3^2 v_2^2 w_3^2 w_1^2 - 25w_2^2 w_3^2 c_s^2 w_1^3 - 36w_3^2 w_3 c_s^4 w_1^2 - 6w_2^2 w_3^2 w_1^2 + 54w_2 v_1^4 w_3^2 w_1^2 - 45w_2 v_1^2 v_2^2 w_3^2 c_s^2 w_1^2) \frac{v_2}{24w_3^2 w_3^2 c_s^2 w_1^3} \end{aligned}$$

coefficient  $C_{D_x^2 D_y^2 v_1}^{(2)}$  at  $\frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2}$ :

$$C_{D_x^2 D_y^2 v_1}^{(2), \text{SRT}} = 0$$

$$\begin{aligned}
C_{D_x^2 D_y^2}^{(2),MRT1} = & (-2w_8^2 w_5 w_3^3 w_7^2 w_4^3 + 4w_8 w_5 w_3^3 c_s^2 w_7^2 w_4^2 - 4w_5 w_9 w_3^3 w_7^2 w_4^3 + 2w_8^2 w_5 w_9 w_3^3 w_7^2 w_4^2 - 24w_8 w_5 w_9 w_3^3 c_s^2 w_7^2 w_4^2 - 5w_8 w_5 w_9 w_3^3 c_s^2 w_7^2 w_4^3 - \\
& 7w_8 w_5 w_9 w_3^2 w_7^2 w_4^3 + 4w_8^2 w_5 w_9 w_3^2 c_s^2 w_7^2 w_4^2 - 4w_8 w_5 w_9 w_3^2 w_7^2 w_4^3 + 2w_8^2 w_5 w_9 w_6^2 s_8^2 w_7^2 w_4^2 - 2w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 + 11w_8 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^2 + 8w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 + \\
& 4w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^2 - 4w_5 w_9 v_2^2 w_6^2 w_7^2 w_4^3 - w_8^2 w_5 w_9 w_6^2 w_7^2 w_4^3 - 2w_8^2 w_5 w_9 w_6^2 w_7^2 w_4^2 - 4w_8 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 + 4w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^2 - w_8^2 w_5 w_9 v_2^2 w_6^2 w_7^2 w_4^2 - \\
& 15w_8^2 w_5 w_9 w_6^2 s_8^2 w_7^2 w_4^3 + 4w_8 w_5 w_9 w_6^2 s_8^2 w_7^2 w_4^3 - 2w_8^2 w_5 w_9 w_6^2 s_8^2 w_7^2 w_4^3 - 8w_5 w_9 w_6^2 s_8^2 w_7^2 w_4^3 - 2w_8^2 w_5 v_2^2 w_6^2 w_7^2 w_4^3 + 2w_8 w_5 w_9 w_6^2 w_7^2 w_4^2 + 4w_8^2 w_5 v_2^2 w_6^2 w_7^2 w_4^3 - \\
& 2w_8 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 + 4w_8 w_5 w_6^2 c_s^2 w_7^2 w_4^3 + 2w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 - 4w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 + 2w_8 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^2 - 6w_8 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 - \\
& 2w_8 w_5 w_9 v_2^2 w_6^2 w_7^2 w_4^3 + 2w_8^2 w_5 w_9 v_2^2 w_6^2 w_7^2 w_4^3 + 8w_5 w_9 w_6^2 s_8^2 w_7^2 w_4^3 + 4w_8 w_5 v_2^2 w_6^2 w_7^2 w_4^3 + 3w_8^2 w_5 w_9 w_6^2 w_7^2 w_4^2 + 2w_8 w_5 w_9 v_2^2 w_6^2 w_7^2 w_4^3 - \\
& 4w_8 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 + 13w_8 w_5 w_9 w_6^2 s_8^2 w_7^2 w_4^3 + 8w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 + 4w_8^2 w_5 w_9 w_6^2 s_8^2 w_7^2 w_4^3 - 2w_8^2 w_5 v_2^2 w_6^2 w_7^2 w_4^3 - w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 - \\
& 5w_8 w_5 w_9 w_6^2 v_2^2 w_6^2 w_7^2 w_4^3 - 8w_5 w_9 w_6^2 s_8^2 w_7^2 w_4^3 + 12w_8^2 w_5 w_9 w_6^2 s_8^2 w_7^2 w_4^3 + 7w_8 w_5 w_9 w_6^2 w_7^2 w_4^3 - 4w_8 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 - 4w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 - w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^2 + \\
& 4w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 - 9w_8 w_5 w_9 w_6^2 s_8^2 w_7^2 w_4^3 + 2w_8^2 w_5 w_9 v_2^2 w_6^2 w_7^2 w_4^3 + 4w_5 w_9 v_2^2 w_6^2 w_7^2 w_4^3 - 16w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 - 6w_8 w_5 w_9 w_6^2 v_2^2 w_6^2 w_7^2 w_4^2 - 2w_8^2 w_5 w_9 w_6^2 w_7^2 w_4^3 - \\
& 3w_8 w_5 w_9 v_2^2 w_6^2 w_7^2 w_4^2 + 26w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 - 2w_8^2 w_5 w_9 w_6^2 w_7^2 w_4^3 + 3w_8 w_5 w_9 w_6^2 s_8^2 w_7^2 w_4^3 - 4w_8^2 w_5 v_2^2 w_6^2 c_s^2 w_7^2 w_4^3 + 3w_8 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 - \\
& 2w_8 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 + 4w_8^2 w_5 w_9 v_2^2 w_6^2 w_7^2 w_4^3 + 2w_8^2 w_5 w_9 w_6^2 w_7^2 w_4^3 - 4w_8 w_5 w_9 v_2^2 w_6^2 w_7^2 w_4^3 + w_8^2 w_5 w_9 v_2^2 w_6^2 w_7^2 w_4^3 + 5w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 - 4w_5 w_9 v_2^2 w_6^2 w_7^2 w_4^2 + \\
& 4w_5 w_9 w_6^2 w_7^2 w_4^3 + 4w_8 w_5 w_9 v_2^2 w_6^2 w_7^2 w_4^3 - 3w_8 w_5 w_9 v_2^2 w_6^2 w_7^2 w_4^3 - 2w_8^2 w_5 w_9 v_2^2 w_6^2 w_7^2 w_4^3 - 4w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 - 4w_8^2 w_5 w_9 w_6^2 w_7^2 w_4^3 + 4w_8 w_5 w_9 w_6^2 w_7^2 w_4^3 + \\
& 2w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 + w_8^2 w_5 w_9 v_2^2 w_6^2 w_7^2 w_4^3 + 9w_8 w_5 w_9 v_2^2 w_6^2 w_7^2 w_4^3 + 12w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 - 8w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^2 + 2w_8^2 w_5 w_9 v_2^2 w_6^2 w_7^2 w_4^3 + \\
& w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 - 4w_8 w_5 v_2^2 w_6^2 w_7^2 w_4^3 + 2w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^2 - 6w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3 + 6w_8 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^2 - 8w_8 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3) \frac{v_1 v_2}{2w_8^2 w_5 w_9 w_6^2 c_s^2 w_7^2 w_4^3}
\end{aligned}$$

$$C_{\mathrm{D}_x^2 \mathrm{D}_y^2 v_1}^{(2), \text{MRT2}} = C_{\mathrm{D}_x^2 \mathrm{D}_y^2 v_1}^{(2), \text{MRT1}}$$

$$C_{D_x^2 D_y^2 v_1}^{(2), \text{CLBM1}} = 0$$

$$C_{D_x^2 D_y^2 v_1}^{(2), \text{CLBM2}} = 0$$

$$C_{D_x^2 D_y^2 v_1}^{(2), \text{CuLBM1}} = 0$$

$$\begin{aligned} C_{(2),\text{ChLBM2}}^{(2)} = & (100\omega_2^2 v_1^2 \omega_3 w_1^3 + 135\omega_3^2 w_3 c_s^2 w_1 - 27w_2 v_2^2 w_3 w_1^3 - 18w_2^2 w_3 s_1 + 162w_2^2 w_3 c_s^2 w_1^2 + 36w_2^2 v_2^2 w_1^2 - 36w_2 v_2^2 w_3 w_1^2 + 108w_2^2 c_s^2 w_1^2 + \\ & 216w_3^3 v_1^2 \omega_3 w_1 + 18w_2 w_3^3 + 84w_2^2 w_3 c_s^2 w_3^2 - 100w_3^2 v_1^2 \omega_3 w_1^2 - 54w_3^2 w_3 c_s^2 - 46w_2^2 w_3 w_3^3 - 198w_3^2 v_1^2 w_3 + 162w_2^2 v_1^2 \omega_3 w_1 + 270w_3 c_s^2 w_3^3 - 54w_2^2 \omega_3 w_1^2 + \\ & 198v_1^2 \omega_3 w_1^3 - 54w_2^2 w_3 c_s^2 w_1 - 84w_3^2 w_3 c_s^2 w_1^2 - 126w_3 w_1^3 + 135w_2 w_3 w_1^3 - 162w_2 w_3 c_s^2 w_1^2 + 36w_3^2 v_2^2 w_3 - 216w_2 v_2^2 w_3 w_1^3 + 54w_3^2 w_3 - \\ & 297w_2 w_3 c_s^2 w_1^3 - 27w_3^2 v_2^2 w_3 w_1 - 81w_3^2 w_3 w_1 + 90w_2 w_3 w_1^2 - 162w_2 v_2^2 w_3 w_1^2 + 54w_2^2 v_2^2 w_3 w_1^2 - 36w_2^2 v_2^2 w_3 w_1 + 46w_3^2 w_3 w_1^2 - 54w_2 c_s^2 w_1^3 - \\ & 18w_3^2 v_2^2 w_1 + 18w_3^2 w_1 - 36w_2^2 w_1^2 - 54w_3^2 c_s^2 w_1 + 36v_2^2 w_3 w_1^3 - 18w_2 v_2^2 w_1^3) \frac{v_1 v_2 v_3}{24w_3^2 w_2 w_1^3} \end{aligned}$$

coefficient  $C_{D_x^2 D_y^2 v_2}^{(2)}$  at  $\frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2}$ :

$$C_{\substack{D_x^2 D_y^2 v_2}}^{(2), \text{SRT}} = (-24 + 8c_s^2\omega^2 + 36v_2^2\omega^2 - c_s^2\omega^3 - 108v_2^2\omega + 12c_s^2 - 18c_s^2\omega + 72v_2^2 + 36\omega - 12\omega^2) \frac{\rho c_s^2}{12\omega^3}$$

$$\begin{aligned} C_{\frac{D_2^2 D_2^2 v_2}{v_2}}^{(2), \text{MRT1}} = & (18 w_8^2 v_2^2 w_9 w_6 w_7 w_3^3 - 24 w_9 v_2^2 w_3^2 c_s^2 w_7 w_3^3 - 12 w_8 w_9 w_3^2 c_s^2 w_7 w_4 + 12 v_1^2 w_9 w_6^2 w_7 w_4^3 + 60 w_8 w_9 v_2^2 w_3^2 c_s^2 w_7 w_4 - 36 w_8^2 w_9 v_2^2 w_6^2 c_s^2 w_4^2 + \\ & 12 v_2^2 w_9 w_3^2 c_s^2 w_7 w_4^3 + 36 w_8^2 v_2^2 w_9 v_2^2 w_7 w_4^3 + 12 w_8^2 w_9 w_6^2 c_s^2 w_7 w_4 + 36 w_8^2 v_1^2 w_9 v_2^2 w_6 w_7 w_4^2 + 12 w_8^2 w_9 w_6^2 c_s^2 w_7 w_4^2 - 36 w_8 v_1^2 w_9 v_2^2 w_6^2 w_7 w_4^2 - 6 w_8^2 w_9 w_6^2 c_s^4 w_7 w_4^3 - 12 w_8 v_1^2 w_9 w_6^2 c_s^2 w_7 w_4^2 + 12 w_8 w_9 w_6^2 c_s^2 w_7 w_4^3 + 60 w_8 w_9 v_2^2 w_6^2 c_s^2 w_7 w_4^2 + 12 w_8 w_9 w_6^2 c_s^4 w_7 w_4^3 - \\ & 12 w_8^2 v_1^2 w_9 w_7 w_4^3 - 12 w_8 w_9 w_6^2 c_s^2 w_7 w_4^2 - 36 w_8^2 v_1^2 v_2^2 w_3^2 w_7 w_4^2 - 12 w_8^2 v_1^2 w_9 w_6^2 c_s^2 w_7 w_4^2 + 12 w_8 v_1^2 w_9 w_6^2 c_s^2 w_7 w_4^3 + 72 w_8 v_1^2 w_9 v_2^2 w_6^2 w_7 w_4^3 + 18 w_8^2 w_9 w_6^2 c_s^4 w_7 w_4 - \\ & 12 v_1^2 w_9 w_3^2 c_s^2 w_7 w_4^2 - 6 w_8^2 w_9 w_6^2 c_s^2 w_7 w_4^3 - 54 w_8^2 v_2^2 w_9 v_2^2 w_6 w_7 w_4^3 + 18 w_8^2 w_9 v_2^2 w_6^2 c_s^2 w_4^3 - 12 w_8^2 v_1^2 w_9 w_6 w_7 w_4^2 + 24 w_9 v_2^2 w_6^2 c_s^2 w_7 w_4^2 + 18 w_8^2 v_1^2 v_2^2 w_3^2 w_7 w_4^3 + 12 w_8 w_9 w_6^2 c_s^2 w_7 w_4^3 - 36 w_8 v_1^2 w_9 w_6^2 w_7 w_4^2 + 6 w_8^2 v_2^2 w_3^2 w_4^3 - 24 w_8 w_9 w_6^2 c_s^4 w_7 w_4^2 - 144 w_8 w_9 v_2^2 w_6^2 c_s^2 w_7 w_4^3 - 48 w_8^2 w_9 v_2^2 w_6^2 c_s^3 w_7 - 12 w_8 w_6^2 c_s^2 w_7 w_4^2 + \\ & 12 w_8 w_6^2 c_s^4 w_7 w_4^3 + 24 w_8 v_1^2 w_9 w_6^2 c_s^2 w_7 w_4^3 + 24 w_8^2 w_9 w_6^2 c_s^4 w_7 w_4^2 + 24 w_8 v_1^2 w_9 w_6^3 c_s^2 w_7 w_4 + 12 w_8^2 v_1^2 w_9 w_6 c_s^2 w_7 w_4^2 + 6 w_8^2 w_9 w_6^2 c_s^3 w_4^3 + 12 w_8 w_9 w_6^3 c_s^4 w_7 w_4 - \end{aligned}$$

$$\begin{aligned}
& 12w_8v_1^2w_6^2w_7w_4^3 - 108w_8^2w_9v_2^2w_6c_s^2w_7w_4^2 + 6w_2^2w_6^2c_s^2w_7w_4^3 - 18w_8^2v_1^2v_2^2w_6^3w_4^3 - 12w_8^2w_6c_s^4w_7w_4^2 - 4w_8^2w_9w_6^3c_s^4w_7w_4^2 - \\
& 12w_8v_1^2w_9w_6^3c_s^2w_7w_4^3 - 12w_8w_6^3c_s^4w_7w_4^3 + 12w_8^2w_9w_6^2c_s^2w_7w_4^2 + 6w_8^2w_9w_6^2c_s^2w_7w_4^3 - 36w_8v_1^2v_2^2w_6^3w_7w_4^3 + 78w_8w_9v_2^2w_6^3c_s^2w_7w_4^3 + 12w_8w_9w_6^2c_s^4w_7w_4^2 - \\
& 12w_8w_9w_6^3c_s^2w_7w_4^3 + 12w_2^2v_2^2w_6c_s^2w_7w_4^3 + 36w_8^2v_1^2v_2^2w_6^3w_7w_4^3 + 6w_8^2w_6^3c_s^2w_7w_4^3 - 12w_1^2w_9w_6^2c_s^2w_7w_4^3 + 12w_8v_1^2w_9w_6w_7w_4^3 + 18w_8w_9v_2^2w_6c_s^2w_7w_4^3 - \\
& 12w_8w_9w_6^3c_s^2w_7w_4^3 + 24w_9w_7^2w_6^2c_s^2w_7w_4^3 - 18w_8^2v_1^2w_9w_6c_s^2w_7w_4^3 + 24w_8w_9w_6^3c_s^2w_7w_4^3 - 12w_8w_9w_6^2c_s^4w_7w_4^3 - 132w_8w_9w_6^2w_6^2c_s^2w_7w_4^2 - \\
& 36w_8^2v_1^2w_9w_6^2c_s^2w_7w_4^3 + 36w_8v_1^2w_2^2w_6^2w_7w_4^2 - 12w_8w_6^2c_s^2w_7w_4^3 + 6w_8^2v_1^2w_6^2w_7w_4^3 + 36w_8v_1^2w_9w_6^3c_s^2w_7w_4^3 + 12w_8w_6^3c_s^4w_7w_4^2 - \\
& 8w_8w_9w_6^3c_s^4w_7w_4^3 - 6w_8w_9w_6^2c_s^2w_7w_4^3 - 36w_8w_2^2w_6^2c_s^2w_7w_4^2 + 12w_8v_1^2w_9w_6^2c_s^2w_7w_4^2 - 18w_8^2v_2^2w_6^3c_s^2w_7w_4^3 + 12w_8v_1^2w_9w_6^2w_7w_4^2 - \\
& 18w_8v_1^2v_2^2w_6^2c_s^2w_7w_4^3 + 6w_8^2v_1w_9w_6c_s^4w_7w_4^3 - 36w_8v_1^2w_9w_6^2c_s^2w_7w_4^2 - 24w_8v_1^2w_9w_6^2w_7w_4^3 - 12w_8v_1^2w_6^3c_s^2w_7w_4^3 - 36w_8v_1^2w_9w_6^2w_7w_4^3 - \\
& 6w_8v_1^2w_6^2c_s^2w_7w_4^3 + 12w_1^2w_9w_6^3c_s^2w_7w_4^2 + 12w_8^2v_1^2w_9w_6^2w_7w_4^2 - 84w_8w_9v_2^2w_6^2c_s^2w_7w_4^3 + 18w_8^2v_1^2w_9v_2^2w_6^2c_s^2w_7w_4^3 + 36w_8w_2^2w_6^2c_s^2w_7w_4^3 - 12w_8v_1^2w_9w_6c_s^4w_7w_4^3 - \\
& 12w_8^2w_9w_6c_s^4w_7w_4^2 - 24w_8v_1^2w_9w_6^2w_7w_4^3 + 24w_8^2v_1^2w_9w_6^2w_6^2s^2w_7w_4 + 72w_8w_9v_2^2w_6c_s^2w_7w_4^3 + 18w_8^2v_2^2w_6^3c_s^2w_7w_4^3 - 6w_8^2v_1^2w_6^3c_s^2w_7w_4^3 + \\
& 6w_8^2v_1^2w_9w_6^2c_s^2w_7w_4^3 + 36w_8^2w_2^2w_6^2c_s^2w_7w_4^2 - 6w_8^2v_1^2w_9w_6^2w_7w_4^3 - 12w_1^2w_9w_6^3c_s^2w_7w_4^3 + 72w_8v_1^2w_9w_6^2c_s^2w_7w_4^2 + 108w_8v_1^2w_9w_6^2w_6^2s^2w_7w_4 + 42w_8^2w_9v_2^2w_6^3c_s^2w_7w_4^2 - \\
& 12w_8v_1^2w_6^3c_s^2w_7w_4^2 - 18w_8^2v_2^2w_6^2c_s^2w_7w_4^3 - 72w_8^2v_1^2w_9w_6^2w_6^2s^2w_7w_4^2 - 72w_8v_1^2w_9w_6^2v_2^2w_6^3c_s^2w_7w_4 + 12w_8^2w_6^3c_s^4w_7w_4^2 - 36w_8w_2^2w_6^3c_s^2w_7w_4^3 - 6w_8^2v_1^2w_9w_6^2w_7w_4^2 - \\
& 6w_8v_1^2w_6^3w_7w_4^3 + 84w_8^2w_9w_6^2w_6^2s^2w_7w_4^2 - 6w_8^2w_9w_6^2s^2w_7w_4^3 + 36w_7^2w_9w_6^2w_6^2s^2w_7w_4^3 - 12w_8w_9w_6^2c_s^4w_7w_4^2 + 180w_8^2w_9w_6^2w_6^2s^2w_7w_4^2 - 12w_8v_1^2w_6^3w_7w_4^2 - \\
& 24w_8^2v_1^2w_9w_6^2w_6^2s^2w_7w_4^2 - 12w_8^2v_1^2w_6^3c_s^2w_7w_4^2 + 6w_8^2w_9w_6^2w_6^2s^2w_7w_4^2 + 24w_8^2v_1^2w_9w_6^2w_7w_4^2 + 12w_8^2v_1^2w_6^3c_s^2w_7w_4^2 - 36w_8v_1^2w_9w_6^2w_6^2s^2w_7w_4^2 - 36w_8v_1^2w_6^3c_s^2w_7w_4^2 + \\
& 36w_8v_1^2v_2^2w_6^2c_s^2w_7w_4^3 - 6w_8^2w_6^3c_s^4w_7w_4^3 + 36w_8v_2^2w_6^3s^2w_7w_4^2 + 18w_8^2v_1^2w_9w_6^2w_6^2s^2w_7w_4^3 - 36w_8v_1^2w_9w_6^2w_6^2s^2w_7w_4^3 - 12w_8^2w_9w_6^2c_s^4w_7w_4^2 + 6w_8^2v_1^2w_6^3c_s^2w_7w_4^3 + \\
& 6w_8^2v_1^2w_9w_6^2c_s^2w_7w_4^3 - 18w_8^2w_9v_2^2w_6^2c_s^2w_7w_4^2 + 12w_8v_1^2w_6^3w_7w_4^3 + 12w_8v_1^2w_6^2c_s^2w_7w_4^3) \frac{\rho}{12w_2^2w_9w_6^3w_7w_4^3}
\end{aligned}$$

$$C_{\mathrm{D}_x^2 \mathrm{D}_y^2 v_2}^{(2), \text{MRT2}} = C_{\mathrm{D}_x^2 \mathrm{D}_y^2 v_2}^{(2), \text{MRT1}}$$

$$\begin{aligned}
& C_{(2),CLBM1}^{(2)} = (12w_6^2c_5^2w_7w_4^2 - 12w_9w_6c_s^2w_7w_4^3 + 12w_8w_9w_6w_4^2 - 6w_8w_6^2w_7w_3^4 - 12w_6^2w_7w_4^2 - 4w_8w_9w_6^2c_s^2w_7w_4^2 + 36v_2^2w_6^2w_7w_4^2 + 12w_6^2w_7w_4^3 - \\
& 6w_8w_9w_6w_4^3 + 12w_8w_6^2w_7w_4^2 - 12w_9w_6^2w_7w_4 + 12w_9w_6c_5^2w_7w_4^2 - 12w_6^2c_s^2w_7w_4^3 - w_8w_9w_6^2c_s^2w_7w_4^3 - 36v_2^2w_6^2w_7w_4^3 + 12w_8w_6^2c_s^2w_7w_4^2 + 12w_8w_9w_7w_4^2 - \\
& 36w_8v_2^2w_6^2w_7w_4^2 + 36w_8v_2^2w_6^2w_4^2 - 36w_9v_2^2w_6w_7w_4^3 + 6w_8w_6^2w_4^3 - 12w_8w_6^2c_s^2w_7w_4^2 + 24w_9w_6^2w_7w_4^2 + 36w_9v_2^2w_6w_7w_4^2 + 18w_8v_2^2w_6^2w_7w_4^3 - \\
& 18w_8v_2^2w_6^2w_3^3 - 6w_8w_9w_7w_4^3 - 6w_8w_9w_6^2c_s^2w_7w_4^3 + 18w_8w_9w_6^2c_s^2w_7w_4^2 - 12w_9w_6^2w_7w_4^3 + 6w_8w_6^2c_s^2w_7w_4^3 - 12w_8w_6^2w_4^3 + 6w_8w_9w_6c_s^2w_3^4 + 36w_9v_2^2w_6^2w_7w_4^3 + \\
& 24w_8w_9w_6c_5^2w_7w_4^2 - 36w_8w_9v_2^2w_6w_4^2 - 12w_8w_9c_s^2w_7w_4^2 - 12w_9w_6w_7w_4^2 + 12w_9w_6^2c_s^2w_7w_4^3 + 18w_8w_9v_2^2w_6w_4^3 - 6w_8w_9w_6c_s^2w_7w_4^3 - \\
& 18w_8v_2^2w_6w_7w_4^3 - 72w_9v_2^2w_6^2w_7w_4^2 - 12w_8w_9w_6c_s^2w_4^2 - 24w_9w_6^2c_s^2w_7w_4^2 + 12w_9w_6w_7w_4^3 + 12w_6c_5^2w_7w_4^3 + 12w_8w_9w_6w_7w_4^2 + 6w_8w_9v_2^2w_7w_4^3 - \\
& 36w_8w_9v_2^2w_6w_7w_4 + 12w_9w_6^2c_s^2w_7w_4 - 36w_8w_9v_2^2w_7w_4^2 + 6w_8w_6w_7w_4^3 + 72w_8w_9v_2^2w_6w_7w_4^2 - 24w_8w_9w_6w_7w_4^2 + 36w_9v_2^2w_6^2w_7w_4^2 - 6w_8w_6c_s^2w_7w_4^3 - \\
& 18w_8w_9v_2^2w_6w_7w_4^3 + 6w_8w_9w_6w_7w_4^3 + 18w_8w_9v_2^2w_7w_4^3 - 12w_8w_9w_6^2c_s^2w_7 - 12w_6w_7w_4^3 - 12w_8w_9w_6c_s^2w_7w_4 + 36v_2^2w_6w_7w_4^3) \frac{\rho c_s^2}{12w_8w_9w_6^2w_7w_4^3}
\end{aligned}$$

$$C_{\mathrm{D}_x^2 \mathrm{D}_y^2 v_2}^{(2), \text{CLBM2}} = C_{\mathrm{D}_x^2 \mathrm{D}_y^2 v_2}^{(2), \text{CLBM1}}$$

$$\begin{aligned} C_{(2),\text{CuLBMI}}^{(2)} = & (36\omega_2 v_2^2 \omega_6 w_3^2 - 24\omega_2^2 w_3^2 c_s^2 + 18\omega_2^2 \omega_6 w_3 c_s^2 + 36\omega_2^2 v_2^2 w_3 + 12\omega_2^2 w_3^3 c_s^2 + 36v_2^2 w_3^3 - 24\omega_2 w_3^2 + 12\omega_2 w_6 w_3^2 c_s^2 - 72w_2^2 v_2^2 w_3^2 - \\ & 36w_2 v_2^2 w_6 w_3 + 12\omega_6 w_3^2 + 12w_3^3 c_s^2 + 36\omega_2^2 v_2^2 w_3^3 + 24\omega_2 w_3^3 - 12w_2^2 w_6 c_s^2 - 24\omega_2 w_3^3 c_s^2 - 12\omega_2 w_6 w_3^2 + 72w_2 v_2^2 w_3^2 - w_2^2 w_6 w_3^2 c_s^2 - 12w_2^2 w_3^3 - 12\omega_3^3 - \\ & 12\omega_6 w_3^2 c_s^2 - 12\omega_2 w_6 w_3 c_s^2 + 24\omega_2^2 w_3^2 - 72w_2 v_2^2 w_3^3 - 12\omega_2^2 w_3 - 4\omega_2^2 w_6 w_3^2 c_s^2 + 24\omega_2 w_3^2 c_s^2 + 12\omega_2 w_6 w_3 - 36v_2^2 w_6 w_3^2 + 12\omega_2^2 w_3 c_s^2) \frac{\rho c_s^2}{12w_2^2 w_6 w_3^2} \end{aligned}$$

$$\begin{aligned}
C_{D_2^2 y}^{(2), \text{CuLBM2}} = & (-54w_3^2 v_4^4 w_3 w_1^2 + 6w_2 w_3 c_s^4 w_1^3 - 54w_2^2 v_1^2 w_3 w_1^3 + 126w_3^2 v_2^2 w_3 c_s^4 w_1 - 30w_2^2 w_3^2 c_s^4 w_1 - 30w_3^2 w_3 c_s^2 w_1 - 90w_2^2 v_2^2 w_3^2 c_s^2 w_1 + \\
& 108w_3^2 v_1^2 c_s^2 w_1^2 - 18w_2^2 w_3^2 w_1^3 - 60w_2^2 w_3 c_s^2 w_1^2 - 10w_3^2 w_3^4 c_s^4 w_1^2 - 9w_3^2 v_2^2 w_3^2 w_1^2 + 18w_3^2 v_1^2 w_3 c_s^2 w_1 + 18w_2^2 v_2^2 w_3^2 w_1 + 135w_1^2 v_2^2 w_3^2 c_s^3 w_1^3 + 18w_3^2 c_s^4 w_3^3 + \\
& w_2^2 v_4^2 w_3 w_1^3 + w_3^2 v_2^2 w_3^2 w_1^2 - 18w_3^2 v_1^2 w_3 w_1 + 36w_2^2 w_3 c_s^3 w_1^3 - 2w_3^2 w_3^2 c_s^4 w_1^3 - 45w_2^2 v_1^2 w_3^2 c_s^2 w_1^3 - 45w_2 v_1^4 w_3 w_1^3 - 63w_2^2 v_2^2 w_3^2 c_s^2 w_1^3 + 54w_2^2 v_1^4 w_3 w_1^3 - \\
& 6w_2^2 w_3^2 w_1 + 36w_2 w_3 c_s^2 w_1^2 + 54w_3^2 v_1^2 w_3 w_1^2 + 18w_3^2 v_2^2 w_3^2 w_1^2 - 6w_2 w_3^2 w_1^3 + 108w_2^2 v_1^2 w_3^2 c_s^2 w_1^2 + 6w_2^2 v_2^2 w_3^2 c_s^2 w_1^3 - 108w_2^2 v_2^2 c_s^2 w_1^3 - 54w_2^2 v_2^2 w_3^2 c_s^2 w_1^2 - \\
& 216w_2 v_1^2 w_3^2 c_s^2 w_1^3 - 24w_3^2 w_3 c_s^2 w_1^3 + 45w_2 v_1^2 w_3^2 w_1^2 + 2w_3^2 w_3^2 c_s^4 w_1^3 - 36w_2^2 v_2^2 w_3^2 w_1^2 - 144w_3^2 v_1^2 w_3 c_s^2 w_1^2 + 72w_2^2 v_2^2 w_3 c_s^3 w_1^3 - 36w_2^2 v_1^4 w_3^3 - 36w_3^2 v_1^2 w_1^2 - \\
& 30w_2^2 v_1^2 w_3^2 c_s^2 w_1^3 + 72w_2^2 v_2^2 w_3^2 c_s^2 w_1^2 + 45w_3^2 w_3 c_s^2 w_1^3 - 6w_2 w_3^2 w_1^3 - 135w_3^2 v_1^2 v_2^2 w_3 w_1 + 21w_2 w_3^2 c_s^2 w_1^3 - w_3^2 v_4^2 w_3^2 w_1^2 + 18w_3^2 v_1^4 w_3 w_1 - w_2^2 v_2^2 w_3^2 w_1^3 - \\
& 198w_3^2 v_2^2 w_3 c_s^2 w_1^2 + 60w_3^2 w_3 c_s^2 w_1^3 + 30w_2^2 w_3 c_s^4 w_1^2 + 90w_2 v_2^2 w_3^2 w_1^3 - 9w_2 v_2^2 w_3^2 c_s^2 w_1^3 - 81w_2 v_1^2 v_3^2 c_s^2 w_1^2 - 30w_3^2 w_3 c_s^4 w_1 + 135w_3^2 v_1^2 v_2^2 w_3^2 + \\
& 18w_2^2 w_3 c_s^2 w_1^3 + 36w_2^2 v_1^2 w_3 c_s^2 w_1^2 - 90w_2^2 v_2^2 w_3 c_s^2 w_1^3 + 12w_2^2 w_3^2 w_1^2 - 81v_1^2 w_3^2 w_1^3 - 36w_3^2 v_4^2 w_3^2 - 24w_3^2 c_s^2 w_1^3 + 45w_3^2 v_1^2 w_3^2 w_1 - 30w_3^2 v_1^2 w_3^2 c_s^2 w_1^2 + \\
& 6w_3^2 w_3^3 - 36w_2 w_3 c_s^4 w_1^3 - 54w_2 v_1^2 w_3 c_s^3 w_1^3 + 18w_2 v_1^2 w_3 w_1^3 - 6w_2 w_3 c_s^2 w_1^3 + 9w_3^2 v_1^2 w_3^2 w_1^2 + 9w_2^2 v_1^4 w_2^2 w_3^2 w_1^3 + 30w_3^2 w_3 c_s^4 w_1 - 18w_3^2 v_2^2 w_3^2 c_s^2 w_1^2 + \\
& 36w_2^2 w_3^2 c_s^2 w_1^2 + 189w_1^2 w_3^2 c_s^2 w_1^3 + 144w_2^2 v_1^2 w_3 c_s^2 w_1^3 + 144w_2^2 v_2^2 w_3 c_s^2 w_1^3 + 18w_2 v_2^2 w_3 c_s^2 w_1^3 + 2w_3^2 w_3^2 c_s^2 w_1^2 + 60w_2^2 w_3 c_s^4 w_1^2 - 6w_3^2 v_2^2 w_3^2 c_s^2 w_1^2 + \\
& 45w_2 v_1^2 w_3^2 w_1^3 - 135w_2^2 v_1^2 v_2^2 w_3^2 w_1^3 + 36w_1^2 w_3^2 w_1^3 + 18w_2 v_2^2 w_3^2 w_1^2 + 36w_3^2 v_4^2 w_1^2 - 3w_3^2 w_3^2 c_s^2 w_1^3 + 9w_3^2 v_2^2 w_3^2 c_s^2 w_1^2 - 15w_2 w_3^2 c_s^4 w_1^3 - 90w_2^2 v_1^2 w_3^2 w_1^2 + \\
& 270w_2^2 v_1^2 v_2^2 w_3^2 w_1^2 - 135w_2 v_1^2 v_2^2 w_3^2 w_1^3 - 18w_2 v_1^4 w_3 w_1^3 - 42w_2^2 w_3^2 c_s^2 w_1^2 - 60w_3^2 w_3 c_s^4 w_1^2 - 9w_2^2 v_1^2 w_3^2 c_s^2 w_1^3 - 30w_2 w_3^2 c_s^4 w_1^2 + 108w_2^2 v_1^2 w_3^2 c_s^2 w_1^2 - \\
& 9w_3^2 v_1^2 w_3^2 w_1^2 + 6w_2^3 w_3^2 + 18w_2 v_2^2 w_3^2 w_1^3 - 18w_3^2 v_2^2 w_3^2 - 2w_2^2 w_3^2 c_s^2 w_1^3 + 24w_3^2 w_3 c_s^4 w_1^2 - 6w_2^2 w_3^2 w_1^2 + 36w_2^2 v_1^2 w_3^2 w_1^3 - 135w_2 v_1^2 v_2^2 w_3^2 c_s^2 w_1^2) \frac{\rho}{24w_3^2 v_3^2 c_s^2 w_1^3}
\end{aligned}$$

**coefficient**  $C_{D_x D_y^3 \rho}^{(2)}$  **at**  $\frac{\partial^4 \rho}{\partial x_1 \partial x_2^3}$ :

$$C_{D_x D_y^3 \rho}^{(2), \text{SRT}} = 0$$

$$\begin{aligned} C_{\text{Dx3DyP}}^{(2), \text{MRT1}} = & (20w_8^2 v_4^2 w_6 w_4 - 4w_6^3 c_s^4 w_4 - 4w_8^2 w_6^2 c_s^2 - 8w_8^2 w_6^2 c_s^4 w_4 - 4v_4^2 w_6^3 w_4 - 8w_8^2 v_2^2 w_6^2 + 32w_8 v_4^2 w_6^2 w_4^2 + 13w_8 v_2^2 w_6^3 w_4^2 + 12w_8^2 w_6 c_s^2 w_4^2 + \\ & 11w_8^2 v_2^2 w_6^2 c_s^2 w_4^2 + 96w_5^2 v_2^2 c_s^2 w_4^2 + 8w_8 w_6^3 c_s^4 w_4 - 4v_4^2 w_6^2 w_4^2 + 20w_8 v_4^2 w_6^3 w_4 - 4w_8 w_6 c_s^4 w_4^2 + 120w_8 v_2^2 w_6^2 c_s^2 w_4^2 + 36w_8^2 v_2^2 w_6 w_4^2 + 16w_8 v_2^2 w_6^2 w_4^2 - \\ & 15w_4^2 v_2^2 w_6^2 c_s^2 w_4^2 + 96w_5^2 v_2^2 c_s^2 w_4^2 + 8w_8 w_6^3 c_s^4 w_4 - 4v_4^2 w_6^2 w_4^2 + 20w_8 v_4^2 w_6^3 w_4 - 4w_8 w_6 c_s^4 w_4^2 + 120w_8 v_2^2 w_6^2 c_s^2 w_4^2 + 36w_8^2 v_2^2 w_6 w_4^2 + 16w_8 v_2^2 w_6^2 w_4^2 - \\ & 4v_2^2 w_6^3 w_4^2 - 8w_8 w_6^2 c_s^2 w_4^2 - 20w_5^2 v_2^2 w_6 w_4 + 4v_2^2 w_6^3 w_4^2 - 32w_8 v_2^2 w_6^2 w_4^2 - 8w_8 v_2^4 w_6^3 - 4w_8 w_6^3 c_s^4 - 4w_8 w_6^3 c_s^4 w_4^2 - 24w_8 v_2^2 w_6^2 w_4^2 - 13w_8 v_4^2 w_6^3 w_4^2 - \\ & 48w_8 v_2^2 w_6^2 c_s^2 w_4 + 4v_2^2 w_6^2 w_4^2 - 20w_8 v_2^2 w_6^3 w_4 - 4w_8 w_6^2 c_s^4 w_4 - 84w_8 v_2^2 w_6^2 c_s^2 w_4 + 36w_8^2 v_2^2 w_6^2 c_s^2 + 4w_6^2 c_s^2 w_4^2 - 36w_8^2 v_4^2 w_6 w_4^2 - 8w_8^2 c_s^2 w_4^2 + 4w_6^3 c_s^4 w_4^2 - \\ & 16w_8 v_2^4 w_6^2 w_4 - 24v_2^2 w_6^2 c_s^2 w_4^2 + 4v_2^2 w_6^3 c_s^4 w_4 - 8w_8 w_6^3 c_s^2 w_4 + 4w_8 w_6 c_s^2 w_4^2 + 13w_8 v_4^2 w_6^2 w_4^2 - 51w_8 v_2^2 w_6^3 c_s^2 w_4^2 + 4w_8 w_6^3 c_s^2 + \\ & 20w_8 v_2^2 w_6^2 w_4 + 72w_8 v_2^2 w_6 c_s^3 w_4 - 36w_8 v_2^2 w_6^3 c_s^2 + 4w_6^3 c_s^2 w_4 + 20w_8 v_2^2 w_6 w_4^2 + 8w_8^2 v_2^4 w_6^2 + 8w_8^2 w_6^2 c_s^2 w_4 - 12w_8^2 w_6 c_s^4 w_4^2 - 24v_2^2 w_6 c_s^2 w_4 + \end{aligned}$$

$$+ 4w_8^2 w_6 c_s^4 w_4 + 24 v_2^2 w_6^3 c_s^2 w_4^2 - 4w_6^2 c_4^4 w_4^2 - 13w_8^2 v_2^2 w_6^2 w_2^2 - 144v_2^2 w_2^2 w_6 c_s^2 w_4^2 + 4w_8^2 w_6^2 c_s^4 - 4w_6^3 c_s^2 w_4^2 + 8w_2^2 c_s^4 w_4^2 - 4w_8^2 w_6^2 c_s^2 w_4^2 - 20w_8^2 v_4^2 w_6^2 w_4 - 72w_8 v_2^2 w_6 c_s^2 w_4^2 + 8w_8 w_6^2 c_s^4 w_4 - 20w_8 v_4^2 w_6 w_4^2 + 84w_8 v_2^2 w_6^3 c_s^2 w_4 + 8w_8 v_2^2 w_3^3 + 24w_8^2 v_2^2 w_4^2 + 4w_8 w_6^2 c_s^2 w_4^2) \frac{v_1}{4w_8^2 w_6^3 c_s^2 w_4^2}$$

$$C_{\mathrm{D}_x \mathrm{D}_y^3 \rho}^{(2), \text{MRT2}} = C_{\mathrm{D}_x \mathrm{D}_y^3 \rho}^{(2), \text{MRT1}}$$

$$C_{D_x D_y^3 \rho}^{(2), \text{CLBM1}} = 0$$

$$C_{DxD_y^3\rho}^{(2),\text{CLBM2}} = 0$$

$$C_{D_x D_y^3 \rho}^{(2), \text{CuLBM1}} = 0$$

$$\begin{aligned} C_{\text{D}_{\text{xx}} \text{LBM2}}^{(2), \text{CuLBMD2}} = & (-18w_2s_3c_s^4w_1^3 - 54w_3^2v_4^2w_3^2w_1 - 18w_2^2w_3^2c_s^4w_1 - 6w_3^2w_3c_s^2w_1 - 297w_2^2v_2^2w_3^2c_s^2w_1 + 12w_3^2v_1^2c_s^2w_1^2 - 99w_2^2w_3^2w_1^3 - 24w_3^2v_1^2v_2^2w_3^2w_1^2 - \\ & 6w_3^2w_3^2c_s^4w_1^2 + 12w_2^2c_s^2w_3^1 + 108w_2^2v_4^2w_3^2w_1^2 + 36w_3^2c_s^4w_1^2 + 6w_3^2v_1^2w_3^2w_1^2 + 6w_3^2v_1^2w_3c_s^2w_1 + 63w_2^2v_2^2w_3^2w_1 + 45w_1^2v_2^2w_3^2w_1^3 + 90w_3^2c_s^4w_1^3 - w_3^2w_3^2w_1^2 + \\ & 24w_3^2v_2^2w_3^2w_1^2 - 18w_2^2w_3c_s^3w_1^3 + 6w_2^2v_1^2w_3^2c_s^2w_1 - 18w_2^3v_1^2w_3^2c_s^2 + 6w_3^3w_3^2w_1 + 60w_2w_3^2c_s^2w_1^2 - 18w_3^2v_2^2w_3^2w_1 - 6w_2w_3^2w_1^3 + 54w_3^2v_2^2w_3^2w_1^2 + 72w_2^2v_3^2w_3^2c_s^2w_1^3 - \\ & 12w_2^2v_1^2c_s^3w_1^3 - 243w_2v_2^2w_3^2c_s^2w_1^2 - 21w_2v_1^2w_3^2c_s^2w_1^3 + 6w_2v_1^2w_3^2w_1^2 + 54w_4^2w_3^2w_1^3 + 6w_2^2w_3^2c_s^4w_1^3 - 108w_2^2v_2^2w_3^2w_1^2 - 18w_3^2v_1^2w_3^2c_s^2w_1^2 + 2w_2v_1^2w_3^2c_s^2w_1^3 + \\ & 54w_3^2v_2^2w_3^2c_s^2w_1^2 + 27w_3^2w_2^2c_s^4w_1 - 6w_2w_3^2w_1^3 + 72w_3^2v_1^2v_2^2w_3^2w_1 + 75w_2w_3^2c_s^2w_1^3 - 54w_2^2v_4^2w_3^2w_1 - 24w_2^2v_2^2w_3^2c_s^3w_1^3 + 18w_3^2w_3c_s^2w_1^2 + 72w_2^2w_3^2c_s^4w_1^2 + \\ & 6w_2v_1^2w_3^2c_s^3w_1^3 - 486w_2v_2^2w_3^2c_s^2w_1^2 - 6w_2v_1^2w_3^2c_s^2w_1^2 - 18w_3^2w_3^2c_s^4w_1^3 - 45w_3^2v_2^2v_2^2w_3^2 - 12w_3^2c_s^2w_1^2 + 405w_2^2v_3^2c_s^2w_1^3 - 6w_1^2v_2^2w_3^2w_1^3 - 54w_3^2v_4^2w_3^2w_1^2 - \\ & 72w_3^2c_s^2w_1^3 - 2w_3^2v_1^2w_3^2c_s^2w_1^2 + 6w_3^2w_3^2w_1^3 + 54w_2w_3c_s^4w_1^3 - 6w_2v_1^2w_3c_s^2w_1^3 + 6w_2w_3c_s^2w_1^3 + w_2^2v_1^2w_3^2w_1^2 + 18w_3^2w_3c_s^4w_1^3 - 54w_2v_4^2w_3^2w_1^3 + 135w_3^2v_2^2w_3^2c_s^2w_1^2 + \\ & w_2^2w_3^2w_1^3 + 12w_2^2w_3^2c_s^2w_1^2 + 18w_1^2v_2^2w_3^2c_s^2w_1^3 + 18w_2^2v_1^2w_3c_s^2w_1^3 + 5w_3^2w_3^2c_s^2w_1^2 - 72w_3^2v_2^2w_3^2c_s^2w_1^2 - 36w_2^2c_s^4w_1^3 - 6w_2^2v_1^2w_3^2w_1^3 - 9w_2^2v_1^2v_2^2w_3^2w_1^2 + \\ & 45w_2v_2^2w_3^2w_1^2 - 3w_2^2w_3^2c_s^4w_1^2 - 54w_3^2v_2^2w_3^2c_s^2w_1^2 - 99w_2w_3^2c_s^4w_1^3 - 72w_2v_1^2v_2^2w_3^2c_s^2w_1^2 - 6w_2^2v_1^2w_3^2w_1^2 - 72w_2^2w_3^2c_s^2w_1^2 - 54w_2^2w_3^2c_s^4w_1^2 + 24w_3^2v_1^2v_2^2w_3^2c_s^2w_1^2 - \\ & w_2^2v_1^2w_3^2c_s^3w_1^3 - 54w_2w_3^2c_s^4w_1^2 + 21w_3^2v_1^2w_3^2c_s^2w_1^2 - 6w_3^2w_3^2 + 126w_2v_2^2w_3^2w_1^3 - 9w_3^2v_2^2w_3^2 - 5w_2^2w_3^2c_s^3w_1^3 + 6w_3^2w_3^2w_1^2 + 9w_2v_1^2v_2^2w_3^2w_1^2) \frac{v_1}{24w_3^2w_3^2w_1^3} \end{aligned}$$

coefficient  $C_{D_x D_y^3 v_1}^{(2)}$  at  $\frac{\partial^4 v_1}{\partial x_1 \partial x_2^3}$ :

$$C_{\substack{(2), \text{SRT} \\ \text{D}_x \text{D}_y^3 v_1}} = (-12c_s^2\omega^2 - 36v_2^4 - 4v_2^2\omega^3 + 26v_2^2\omega^2 - 54v_2^2\omega - 24c_s^2 + 36c_s^2\omega + 36c_s^4 - 42v_2^2c_s^2\omega^2 - 26v_2^4\omega^2 - c_s^4\omega^3 + 20c_s^4\omega^2 - 36v_2^2c_s^2 + 4v_2^4\omega^3 + 12v_2^2c_s^2\omega^3 - 54c_s^4\omega + 36v_2^2 + 54v_2^2c_s^2\omega + 54v_2^4\omega) \frac{\rho}{12\omega^3}$$

$$\begin{aligned}
C_{\text{D}_x^2 \text{D}_y^3 v_1}^{(2), \text{MRT1}} = & (54 w_8 v_2^2 w_6^2 c_s^2 w_4 - 12 w_4^2 w_6^2 w_4^3 - 24 w_8 v_4^2 w_6^2 w_4^2 - 48 w_8 v_2^2 w_6^3 w_4^2 + 12 w_8^2 w_6 c_s^2 w_4^2 - 12 v_2^2 w_6^3 w_4^3 + 162 w_8^2 v_2^2 w_6^2 c_s^2 w_4 + 90 w_8^2 v_2^2 w_6 w_4^3 + \\
& 6 w_8 w_6^3 s_w^2 w_4 + 60 w_8 v_4^2 w_6^2 w_4^3 + 12 w_8 w_6^3 s_w^4 w_4 - 24 w_8 v_4^2 w_6^3 w_4 - 12 w_8 v_2^2 w_6^2 s_w^2 w_4^2 - 48 w_8^2 v_2^2 w_6^3 c_s^2 + 60 w_8^2 v_2^2 w_6^2 c_s^2 w_4^3 + 252 w_8^2 v_2^2 w_6^2 c_s^2 w_4^3 + 12 v_2^2 w_6^3 w_4^2 + \\
& 12 w_6^2 w_6 c_s^2 w_4^3 + 27 w_8 v_2^2 w_6^3 w_4^3 - 12 w_8 w_6^2 s_w^2 w_4^2 + 24 w_8 v_2^2 w_6^2 w_4^2 + 12 v_2^2 w_6^2 w_4^3 - 18 w_8 w_6^2 c_s^4 w_4^2 - 12 v_2^2 w_6^2 c_s^2 w_4^3 + w_8^2 w_6^2 c_s^2 w_4^3 + 12 w_4^2 w_6^3 w_4^3 - \\
& 12 w_2^2 c_s^2 w_4^3 + 48 w_8 v_4^2 w_6^3 w_4^3 - 90 w_8^2 v_2^2 w_6^4 w_4^3 + 24 w_8 v_2^2 w_6^3 w_4 - 60 w_8 v_2^2 w_6^4 w_4 + 6 w_8 w_6^2 c_s^2 w_4^3 - 48 w_8^2 v_2^2 w_6^3 c_s^2 w_4^2 - 5 w_8^2 w_6^3 c_s^2 w_4^2 - 27 w_8 v_2^2 w_6^3 w_4^3 + \\
& 6 w_8 w_6^3 s_w^4 w_4 - 72 w_8^2 v_2^2 w_4^3 - 12 v_2^2 w_6^3 w_4^2 + 6 w_8^2 w_6^2 c_s^4 w_4^2 - 12 w_8 w_6^3 c_s^2 w_4 + 36 w_8 v_2^2 w_6 w_4^3 + 12 w_8^2 w_6^3 c_s^4 + 12 w_8^2 v_2^2 w_6^3 c_s^2 w_4^3 + 18 w_8^2 v_2^2 w_6^3 w_4^2 + \\
& 12 w_8^2 c_s^4 w_6^2 w_4^2 + 30 w_8 v_2^2 w_6^3 c_s^2 w_4^2 - 12 w_2^2 w_6 c_s^4 w_4^3 - 4 w_8^2 v_2^2 w_6^3 w_4^3 - 81 w_8^2 v_2^2 w_6^3 c_s^2 w_4^2 - 21 w_8 v_2^2 w_6^3 c_s^2 w_4^3 - 12 w_8^2 w_6 c_s^4 w_4^2 - 24 w_8^2 w_6^3 c_s^4 w_4 + \\
& 19 w_8^2 c_s^4 w_6^2 w_4^3 + 12 w_8^2 v_2^2 w_6^3 w_4^3 - 36 w_8 v_2^2 w_6 w_4^3 + 13 w_8^2 w_6^3 c_s^2 w_4^2 - 12 v_2^2 w_6^3 c_s^2 w_4^2 - 6 w_8 w_6^2 c_s^4 w_4^3 - 36 w_8 v_2^2 w_6 c_s^2 w_4^3 - 18 w_8^2 v_2^2 w_6^3 w_4^2 - 12 w_8^2 v_2^2 w_6^2 c_s^2 w_4^2 - \\
& 108 w_8^2 v_2^2 w_6 c_s^2 w_4^2 - 6 w_8^2 w_6^2 c_s^2 w_4^3 + 102 w_8^2 v_2^2 w_6^3 c_s^2 w_4 + 72 w_8^2 v_2^2 w_6^4 - 6 w_8 w_6^2 c_s^4 w_4^3 + 4 w_8 v_4^2 w_6^3 w_4^3 - w_8^2 w_6^3 c_s^4 w_4^3 + 12 w_8 w_6^2 c_s^4 w_4 + 12 v_2^2 w_6^3 c_s^2 w_4^2 - \\
& 12 w_8 v_2^2 w_6^3 c_s^2 w_4 - w_8^2 w_6^2 c_s^2 w_4^3 + 18 w_8 w_6^3 c_s^2 w_4^2 + 12 w_8^2 c_s^4 w_4 - 306 w_8 v_2^2 w_6 c_s^2 w_4^3 - 12 w_8^2 v_2^2 w_6^3 c_s^2 w_4 - 19 w_8^2 v_2^2 w_6^2 c_s^2 w_4) \frac{\rho}{12 w_8^2 s_w^6 c_w^4}
\end{aligned}$$

$$C_{\mathrm{D}_x \mathrm{D}_y^3 v_1}^{(2), \text{MRT2}} = C_{\mathrm{D}_x \mathrm{D}_y^3 v_1}^{(2), \text{MRT1}}$$

$$\begin{aligned}
C_{\substack{(2), CLBM1 \\ \text{D}_x \text{D}_y v_1}}^{(2)} = & (198 w_8 v_2^2 w_6^2 c_s^2 w_4^3 - 36 v_4^2 w_6^2 w_4^3 - 36 w_8 v_2^3 w_6^3 w_4^2 + 12 w_8^2 w_6 c_s^2 w_4^2 - 36 v_2^2 w_6^3 w_4^3 + 18 w_8^2 v_2^2 w_6^2 c_s^2 w_4^2 + 90 w_8^2 v_2^2 w_6 w_4^3 + 6 w_8 w_6^3 c_s^2 w_4 + \\
& 72 w_8 v_2^4 w_6^2 w_4^3 + 12 w_8 w_6^3 c_s^4 w_4 + 36 w_8 v_2^2 w_6^2 c_s^2 w_4^2 + 60 w_8^2 v_2^2 w_6^2 c_s^2 w_4^3 + 252 w_8^2 v_2^2 c_s^2 w_4^3 + 36 v_2^2 w_6^3 w_4^2 + 12 w_8^2 w_6 c_s^2 w_4^3 + 39 w_8 v_2^2 w_6^3 w_4^3 - 12 w_8 w_6^2 c_s^2 w_4^2 - \\
& 36 v_2^2 w_6^3 w_4^3 - 18 w_8 w_6^3 c_s^2 w_4^2 - 108 v_2^2 w_6^2 c_s^2 w_4^3 + w_8^2 w_6^2 c_s^2 w_4^3 + 36 v_4^2 w_6^3 w_4^3 - 12 w_8^2 c_s^2 w_4^3 + 36 w_8 v_4^2 w_6^3 w_4^2 - 90 w_8^2 v_4^2 w_6 w_4^3 - 72 w_8 v_2^2 w_6^2 w_4^3 + \\
& 6 w_8 w_6^3 c_s^2 w_4^3 - 5 w_8 w_6^3 c_s^2 w_4^2 - 39 w_8 v_2^2 w_6^3 c_s^4 + 6 w_8 w_6^3 c_s^4 w_4^3 - 72 w_8^2 v_2^2 w_6^3 - 36 v_2^2 w_6^3 c_s^2 w_4^2 + 6 w_8 w_6^2 c_s^4 w_4^2 - 12 w_8 w_6^3 c_s^2 w_4^3 + 36 w_8 v_2^2 w_6^3 w_4^3 + 12 w_8^2 w_6^3 c_s^4 - \\
& 12 w_8^2 w_6^2 c_s^2 w_4^3 + 6 w_8^2 v_2^2 w_6^3 w_4^2 + 54 w_8 v_2^2 w_6^3 c_s^2 w_4^2 - 12 w_8 w_6 c_s^4 w_4^3 - 4 w_8^2 v_2^2 w_6^3 w_4^3 - 3 w_8^2 v_2^2 c_s^2 w_4^2 - 99 w_8 v_2^2 w_6^3 c_s^2 w_4^3 - 12 w_8 w_6 c_s^4 w_4^2 - \\
& 24 w_8 w_6^3 c_s^4 w_4 + 19 w_8^2 v_2^2 w_6^2 w_4^3 - 36 w_8 v_2^2 w_6 w_4^3 + 13 w_8^2 w_6^3 c_s^4 w_4^2 - 108 v_2^2 w_6^3 c_s^2 w_4^2 - 6 w_8 w_6^2 c_s^4 w_4^3 - 108 w_8 v_2^2 w_6 c_s^2 w_4^3 - 6 w_8 v_2^2 w_6^3 w_4^2 - \\
& 36 w_8^2 w_6^2 c_s^2 w_4^2 - 6 w_8^2 w_6^2 c_s^2 w_4^3 - 18 w_8 v_2^2 w_6^3 c_s^2 w_4^2 + 72 w_8^2 v_4^2 w_4^3 - 6 w_8 w_6^3 c_s^2 w_4^3 + 4 w_8^2 v_2^2 w_6^3 w_4^3 - w_8 w_6^3 c_s^4 w_4^3 + 12 w_8 w_6^2 c_s^4 w_4^2 + 108 v_2^2 w_6^3 c_s^2 w_4^3 + \\
& 36 w_8 v_2^2 w_6^3 c_s^2 w_4^2 - w_8^2 w_6^2 c_s^3 w_4^3 + 18 w_8 w_6^3 c_s^2 w_4^2 + 12 w_8^2 c_s^4 w_4^3 - 306 w_8^2 v_2^2 w_6 c_s^2 w_4^3 - 19 w_8^2 v_2^2 w_6^2 w_4^3) \frac{\rho}{12 w_8^2 w_6^3 w_4^3}
\end{aligned}$$

$$C_{\mathrm{D}_x \mathrm{D}_y^3 v_1}^{(2), \text{CLBM2}} = C_{\mathrm{D}_x \mathrm{D}_y^3 v_1}^{(2), \text{CLBM1}}$$

$$\begin{aligned}
C_{D_3^2 v_1}^{(2), \text{CuLBMI}} = & (-6w_2^2 w_6^2 w_3^2 c_s^2 + 12w_3^2 w_6^2 c_s^4 + 6w_3^2 v_2^2 w_6^2 w_3^2 + 19w_2^2 v_4^2 w_6^2 w_3^3 - 12w_2 w_6^2 w_3^4 c_s^4 - 108w_2 v_2^2 w_6 w_3^3 c_s^2 + 12w_2^2 w_6 w_3^2 c_s^4 - 4w_3^2 v_2^2 w_6^2 w_3^3 + \\
& 6w_3^2 w_6 w_3^2 c_s^4 - 72v_2^2 w_6^2 w_3^2 + 252v_2^2 w_6^2 w_3^2 s^2 + 198w_2^2 v_2^2 w_6 w_3^2 c_s^2 + 36w_2 v_2^2 w_6 w_3^2 + 12w_3^2 v_2^2 w_6^2 w_3^2 c_s^2 - 12w_2^2 w_3^2 c_s^2 - 6w_2^2 w_6 w_3^2 c_s^4 - 12w_3^2 w_6 w_3^2 c_s^2 + \\
& 36w_3^2 v_2^2 w_6 w_3^2 s^2 - 12w_2 w_6^2 w_3^2 c_s^4 - 19w_2^2 v_2^2 w_6^2 w_3^3 - 36w_2^2 v_4^2 w_6^2 w_3^2 - 6w_3^2 v_2^2 w_6^2 w_3^2 - 24w_3^2 w_6^2 w_3^2 c_s^4 - w_2^2 w_6^2 w_3^2 s^2 + 36w_3^2 v_2^4 w_6^2 w_3^3 - 3w_3^2 v_2^2 w_6^2 w_3^2 c_s^2 + \\
& 4w_3^2 v_4^2 w_6^2 w_3^3 + 72v_2^2 w_6^2 w_3^3 - 5w_3^2 w_6^2 w_3^2 c_s^2 - 36w_2 v_4^2 w_6 w_3^3 - 18w_3^2 w_6 w_3^2 s^2 + 36w_2^2 v_2^2 w_6^2 w_3^3 + 36w_2^2 v_2^2 w_6^2 w_3^2 c_s^2 - 18w_3^2 v_2^2 w_6 w_3^2 c_s^2 - 39w_3^2 v_4^2 w_6 w_3^3 + \\
& w_2^2 w_6^2 w_3^2 s^2 + 6w_3^2 w_6^2 w_3^2 s^2 - 36w_2 v_2^2 w_6^2 w_3^2 s^2 + 12w_3^2 w_6 w_3^2 c_s^4 + 6w_2^2 w_6 w_3^2 c_s^2 - 90w_2 v_4^2 w_6^2 w_3^3 + 12w_2 w_6^2 w_3^2 c_s^4 - 108w_2 v_2^2 w_6 w_3^2 c_s^2 + 18w_3^2 w_6 w_3^2 c_s^2 + 
\end{aligned}$$

$$36\omega_3^3v_2^4\omega_6\omega_3^2 - 108\omega_3^3v_2^3\omega_6^2c_s^2 - 72\omega_2^2v_2^2\omega_6\omega_3^2 + 54\omega_2^3v_2^2\omega_6\omega_3^2c_s^2 + 18\omega_2^3v_2^2\omega_6^2\omega_3^2c_s^2 + 13\omega_3^3\omega_6^2\omega_3^2c_s^4 + 12\omega_2\omega_6^2\omega_3^2c_s^2 + 39\omega_2^3v_2^2\omega_6\omega_3^3 - 36\omega_2^3v_2^2\omega_6\omega_3^2 - 12\omega_2^2\omega_6\omega_3^2c_s^2 - 306\omega_2v_2^2\omega_6^2\omega_3^2c_s^2 + 6\omega_2^2\omega_6^2\omega_3^2c_s^4 + 90\omega_2v_2^2\omega_6^2\omega_3^3 - 36\omega_2^2v_2^4\omega_3^3 + 60\omega_2^2v_2^2\omega_6^2\omega_3^2c_s^2 - \omega_2^3\omega_6^2\omega_3^2c_s^4 + 72\omega_2^2v_2^4\omega_6\omega_3^3 - 36\omega_2^3v_2^2\omega_6\omega_3^2 + 36\omega_2^3v_2^2\omega_3^2 + 12\omega_6^2\omega_3^2c_s^4 + 108\omega_2^3\omega_2^2\omega_3^2c_s^2 - 99\omega_2^3v_2^2\omega_6\omega_3^2c_s^2 - 6\omega_2^3\omega_6\omega_3^2c_s^2) \frac{\rho}{12\omega_2^3\omega_6^2\omega_3^3}$$

$$\begin{aligned} C_{D_x D_y^3 v_2}^{(2), \text{CuLBM2}} = & (6\omega_2\omega_3c_s^4\omega_1^3 - 63\omega_2^3v_2^4\omega_3^2\omega_1 + 54\omega_2^3v_2^2\omega_3c_s^2\omega_1 + 6\omega_2^2\omega_3^2c_s^4\omega_1 - 108\omega_2^3v_2^4\omega_2^2 - 18\omega_2^3\omega_3c_s^2\omega_1 + 18\omega_2v_2^2\omega_3\omega_1^3 + 99\omega_2^2v_2^2\omega_3^2c_s^2\omega_1 - \\ & 81\omega_2^2v_2^3\omega_3^3 - 72\omega_2^3v_1^2v_2^2\omega_3^2\omega_1^2 - 24\omega_2^2\omega_3c_s^2\omega_1^2 + 29\omega_2^3\omega_3^2c_s^4\omega_1^2 - 8\omega_2^3v_2^2\omega_3^2\omega_1^3 - 54\omega_2^2v_2^4\omega_3^2\omega_1^2 + 18\omega_2^3v_1^2\omega_3^2\omega_1^2 + 72\omega_2^3v_2^4\omega_3^1 - 54\omega_2^3v_2^2\omega_3c_s^2\omega_1 - \\ & 27\omega_2^2v_2^3\omega_3^2\omega_1 + 135\omega_1^2v_2^2\omega_3^2\omega_1^3 + 18\omega_3^2c_s^4\omega_3^3 + 36\omega_2^2v_2^2\omega_3^2\omega_1^3 - \omega_3^3\omega_6^2\omega_3^2\omega_1^2 + 17\omega_2^3v_2^2\omega_3^2\omega_1^2 + 18\omega_2^2\omega_3c_s^2\omega_1^3 - 2\omega_2^3\omega_3^2c_s^4\omega_1^3 + 18\omega_2^2v_2^2\omega_3^2\omega_1^2 + \\ & 18\omega_2^3v_1^2\omega_3^2\omega_1^2 + 6\omega_2^3\omega_3^2\omega_1^3 + 36\omega_2\omega_3^2c_s^2\omega_1^2 - 9\omega_2^3v_2^2\omega_3^2\omega_1^2 - 6\omega_2\omega_3^2\omega_1^2 + 36\omega_2^3v_2^2\omega_3^2\omega_1^2 + 18\omega_2^3v_1^2\omega_3^2c_s^2\omega_1^2 - 18\omega_2v_2^2\omega_3\omega_1^3 + 84\omega_2^2v_2^2\omega_3^2\omega_1^2 + \\ & 81\omega_2^2v_2^3\omega_3^2\omega_1^2 - 9\omega_2v_2^2\omega_3^2c_s^2\omega_1^2 - 12\omega_2^2\omega_3c_s^2\omega_1^2 + 18\omega_2v_2^2\omega_3^2\omega_1^2 + 36v_2^4\omega_2^2\omega_3^2\omega_1^2 - \omega_2^2\omega_3^2c_s^2\omega_1^3 + 54\omega_2^2v_2^2\omega_3^2\omega_1^2 + 36\omega_2^3v_1^2\omega_3c_s^2\omega_1^2 - 198\omega_2^3v_2^2\omega_3c_s^2\omega_1^2 + \\ & 8\omega_2^3v_2^4\omega_3^2\omega_1^2 - 6\omega_2^2v_2^2\omega_3^2c_s^2\omega_1^2 - 144\omega_2^2v_2^2\omega_3^2c_s^2\omega_1^2 - 57\omega_2^3\omega_3^2c_s^4\omega_1^2 - 6\omega_2\omega_3^2\omega_1^3 + 216\omega_2^3v_1^2v_2^2\omega_3^2\omega_1^2 + 36\omega_2^2v_2^4\omega_3^2\omega_1^2 + 7\omega_2^3v_2^4\omega_3^2\omega_1^2 - \\ & 43\omega_2^2v_2^2\omega_3^2\omega_1^3 + 306\omega_2^3v_2^2\omega_3^2c_s^2\omega_1^2 + 30\omega_2^3v_2^2\omega_3^2c_s^2\omega_1^2 + 12\omega_2^2v_2^2\omega_3^2c_s^2\omega_1^2 + 18\omega_2v_2^2\omega_3^2\omega_1^3 - 297\omega_2v_2^2\omega_3^2c_s^2\omega_1^2 - 54\omega_2v_1^2\omega_3^2c_s^2\omega_1^2 + 30\omega_2^3v_2^2\omega_3^2\omega_1^2 - 135\omega_2^3v_2^2\omega_3^2\omega_1^2 + \\ & 189\omega_2^2v_2^3\omega_3^2\omega_1^3 + 36\omega_2v_1^2\omega_3^2c_s^2\omega_1^2 + 198\omega_2^2v_2^2\omega_3^2c_s^2\omega_1^2 - 18\omega_1^2\omega_3^2\omega_1^3 + 36\omega_2v_2^2\omega_3^2\omega_1^2 - 324\omega_2^3v_2^2\omega_3^2\omega_1^2 - 72\omega_2^3v_2^2\omega_3\omega_1^3 - 36\omega_2^2v_2^4\omega_3^2\omega_1^2 - \\ & 24\omega_2^3c_s^2\omega_1^3 + 6\omega_2^3v_1^2\omega_3^2c_s^2\omega_1^2 + 24\omega_2^3v_2^2\omega_3^2c_s^2\omega_1^2 + 144\omega_2^3v_2^4\omega_3^2c_s^2\omega_1^2 + 6\omega_2^2\omega_1^3 - 18\omega_2^2\omega_3c_s^4\omega_1^2 + 18\omega_2v_1^2\omega_3c_s^2\omega_1^2 - 6\omega_2\omega_3c_s^2\omega_1^3 + 18\omega_2^3v_2^2\omega_3\omega_1^2 + \\ & 3\omega_2^3v_1^2\omega_3^2\omega_1^2 + 18\omega_2^3v_3c_s^4\omega_1^2 - 63\omega_2v_2^4\omega_3^2\omega_1^2 + 63\omega_2^3v_2^2\omega_3^2c_s^2\omega_1^2 + \omega_2^2\omega_3^2\omega_1^3 + 108\omega_2^3v_2^2\omega_1^2 - 12\omega_2^2\omega_3c_s^2\omega_1^2 + 216\omega_2^3v_2^2c_s^2\omega_1^2 + 18\omega_2^3v_2^2c_s^2\omega_1^3 - \\ & 36\omega_2^2v_1^2\omega_3c_s^2\omega_1^3 - 72\omega_2^2v_2^2\omega_3c_s^2\omega_1^2 - 54\omega_2v_2^2\omega_3c_s^2\omega_1^2 - 12\omega_2^3\omega_3^2c_s^2\omega_1^2 + 24\omega_2^2\omega_3c_s^4\omega_1^2 + 30\omega_2^3v_2^2\omega_3^2c_s^2\omega_1^2 - 78\omega_2^3v_2^2\omega_3c_s^2\omega_1^2 - 18\omega_2^2v_1^2\omega_3^2\omega_1^2 - \\ & 27\omega_2^2v_1^2\omega_3^2\omega_1^2 + 36\omega_2^2v_2^2\omega_3\omega_1^2 - 45\omega_2v_2^2\omega_3^2\omega_1^2 - 108\omega_2^2v_2^2\omega_3^2\omega_1^2 + 15\omega_2^3\omega_3^2c_s^2\omega_1^2 - 171\omega_2^3v_2^2\omega_3^2c_s^2\omega_1^2 - 15\omega_2^2v_2^2\omega_3^2c_s^4\omega_1^2 - 216\omega_2v_1^2v_2^2\omega_3^2\omega_1^2 - \\ & 18\omega_2^3v_1^2\omega_3^2\omega_1^2 - 144\omega_2^3v_2^2\omega_3\omega_1^2 + 72\omega_2^2v_2^2\omega_3\omega_1^2 - 12\omega_2^3\omega_3^2c_s^2\omega_1^2 - 30\omega_2^3v_3c_s^4\omega_1^2 + 72\omega_2^3v_1^2v_2^2\omega_3^2\omega_1^2 - 3\omega_2^3v_1^2\omega_3^2\omega_1^2 - 30\omega_2\omega_3^2c_s^4\omega_1^2 - 18\omega_2^3v_2^2\omega_3\omega_1^2 - \\ & 9\omega_2^3v_1^2\omega_3^2\omega_1^2 - 6\omega_2^3\omega_3^2 + 135\omega_2v_2^2\omega_3^2\omega_1^2 + 9\omega_2^3v_2^2\omega_3^2 + 12\omega_2^3v_3c_s^4\omega_1^2 + 6\omega_2^2\omega_3^2\omega_1^2 - 36\omega_2^2v_2^2\omega_3\omega_1^2 + 27\omega_2v_1^2v_2^2\omega_3^2\omega_1^2 + 78\omega_2^3v_2^2\omega_3\omega_1^2) \frac{\rho}{24\omega_2^3\omega_3^2\omega_1^2} \end{aligned}$$

coefficient  $C_{D_x D_y^3 v_2}^{(2)}$  at  $\frac{\partial^4 v_2}{\partial x_1 \partial x_2}$ :

$$C_{D_x D_y^3 v_2}^{(2), \text{SRT}} = 0$$

$$\begin{aligned} C_{D_x D_y^3 v_2}^{(2), \text{MRT1}} = & (20\omega_8^2\omega_6^2c_s^2 + 28\omega_8^2v_2^2\omega_6^2 + 16\omega_8\omega_6^2\omega_4 - 43\omega_8v_2^2\omega_6^3\omega_4 - 72\omega_8^2\omega_6c_s^2\omega_4^2 - 24\omega_8^2\omega_6\omega_4 + 17\omega_8\omega_6^3\omega_4^2 - 120\omega_8^2v_2^2\omega_6\omega_4^2 - 16\omega_8\omega_6^2c_s^2\omega_4 - \\ & 48\omega_8v_2^2\omega_6^2\omega_4 + 16\omega_8^2v_2^3\omega_6^2\omega_4 - 12\omega_8^2\omega_6^2 + 56\omega_8\omega_6^2c_s^2\omega_4^2 + 64\omega_8^2v_2^2\omega_6\omega_4 - 16\omega_2^2v_2^3\omega_6^3\omega_4 + 104\omega_8v_2^2\omega_6^2\omega_4^2 + 80\omega_8^2v_2^2\omega_4^2 - 28\omega_8\omega_6^3\omega_4 + 48\omega_8^2\omega_6\omega_4^2 - \\ & 16\omega_2^2v_2^2\omega_6^2\omega_4 + 68\omega_2v_2^2\omega_6^3\omega_4 + 32\omega_2^3v_6c_s^2\omega_4 - 32\omega_2^2\omega_4^2 - 16\omega_2^2\omega_6^2\omega_4^2 + 48\omega_2^2c_s^2\omega_4^2 - 40\omega_8\omega_6^2\omega_4^2 + 44\omega_8\omega_6^3\omega_4^2 - 32\omega_8\omega_6c_s^2\omega_4^2 + 24\omega_8\omega_6\omega_4^2 - \\ & 20\omega_8\omega_6^2c_s^2\omega_4^2 - 68\omega_2^3v_2^2\omega_6\omega_4 - 16\omega_2^3\omega_6^2\omega_4 - 64\omega_2^3v_2^2\omega_6\omega_4 - 44\omega_2^3\omega_6^2c_s^2\omega_4 - 17\omega_2\omega_6^2\omega_4^2 - 8\omega_6^3\omega_4^2 + 8\omega_6^2\omega_4 + 28\omega_8\omega_6^2\omega_4 + 43\omega_2^2v_2^2\omega_6\omega_4^2 + \\ & 16\omega_6^3c_s^2\omega_4^2 + 25\omega_8^2\omega_6^2c_s^2\omega_4^2 + 8\omega_6^2\omega_4^2 - 28\omega_8v_2^2\omega_6^3 - 25\omega_8\omega_6^3c_s^2\omega_4^2 + 12\omega_8\omega_6^3\omega_4^3) \frac{v_1 \rho v_2}{4\omega_8^2\omega_6^2\omega_4^2} \end{aligned}$$

$$C_{D_x D_y^3 v_2}^{(2), \text{MRT2}} = C_{D_x D_y^3 v_2}^{(2), \text{MRT1}}$$

$$C_{D_x D_y^3 v_2}^{(2), \text{CLBM1}} = 0$$

$$C_{D_x D_y^3 v_2}^{(2), \text{CLBM2}} = 0$$

$$C_{D_x D_y^3 v_2}^{(2), \text{CuLBM1}} = 0$$

$$\begin{aligned} C_{D_x D_y^3 v_2}^{(2), \text{CuLBM2}} = & (10\omega_2^2v_1^2\omega_3\omega_1^3 - 27\omega_2^3\omega_3c_s^2\omega_1 - 198\omega_2v_2^2\omega_3\omega_1^3 + 90\omega_2^3\omega_3\omega_1 + 18\omega_2^3v_1^2\omega_1 + 324\omega_2^2\omega_3c_s^2\omega_1^2 + 54\omega_2^2c_s^2\omega_1^3 - 198\omega_2v_2^2\omega_3\omega_1^2 - \\ & 18\omega_2v_1^2\omega_3^2\omega_1^3 + 45\omega_2^3v_1^2\omega_3\omega_1^2 + 18\omega_2\omega_3^2\omega_1^3 + 30\omega_2^3\omega_3c_s^2\omega_1^2 - 10\omega_2^3v_1^2\omega_3\omega_1^2 + 54\omega_2^3\omega_3c_s^2\omega_1^2 - 10\omega_2^2\omega_3\omega_1^3 - 36\omega_2^3v_1^2\omega_3^2\omega_1^2 + 270\omega_3c_s^2\omega_1^3 - 18\omega_2^3v_1^2\omega_3^2\omega_1^2 - \\ & 180\omega_2^2\omega_3\omega_1^3 + 36\omega_1^2\omega_3\omega_1^3 - 162\omega_2^2\omega_3c_s^2\omega_1^2 - 30\omega_2^3\omega_3c_s^2\omega_1^2 - 126\omega_3\omega_1^3 - 54\omega_2^3c_s^2\omega_1^2 + 135\omega_2\omega_3\omega_1^3 - 162\omega_2\omega_3c_s^2\omega_1^2 + 198\omega_2^3v_2^2\omega_3\omega_1^2 - \\ & 45\omega_2v_2^2\omega_3\omega_1^3 - 54\omega_2^3\omega_3 - 297\omega_2\omega_3c_s^2\omega_1^2 - 198\omega_2^3v_2^2\omega_3\omega_1^2 + 45\omega_2^3\omega_3\omega_1 + 18\omega_2^3\omega_1^2 + 90\omega_2\omega_3\omega_1^2 + 396\omega_2^2v_2^2\omega_3\omega_1^2 - 198\omega_2^2v_2^2\omega_3\omega_1^2 + 10\omega_2^3\omega_3\omega_1^2 - \\ & 54\omega_2c_s^2\omega_1^3 - 18\omega_2^3\omega_1 - 18\omega_2^2\omega_3^2 + 54\omega_2^3c_s^2\omega_1 + 198\omega_2^2v_2\omega_3\omega_1^2 + 18\omega_2^2\omega_3\omega_1^2) \frac{v_1 \rho v_2}{24\omega_2^2\omega_3\omega_1^2} \end{aligned}$$

coefficient  $C_{D_y^4 \rho}^{(2)}$  at  $\frac{\partial^4 \rho}{\partial x_2^4}$ :

$$C_{D_y^4 \rho}^{(2), \text{SRT}} = (12 - 78c_s^2\omega^2 + 144v_2^4 + 10v_2^2\omega^3 - 98v_2^2\omega^2 + 6c_s^2\omega^3 + 234v_2^2\omega - 132c_s^2 + 198c_s^4\omega + 404v_2^2c_s^2\omega^2 + 90v_2^4\omega^2 - 5c_s^4\omega^3 + 82c_s^4\omega^2 + 672v_2^2c_s^2 - 9v_2^4\omega^3 - 34v_2^2c_s^2\omega^3 - 216c_s^4\omega - 156v_2^2 - 18\omega + 8\omega^2 - 1008v_2^2c_s^2\omega - 216v_2^4\omega - \omega^3) \frac{v_2}{12\omega^3}$$

$$C_{D_y^4 \rho}^{(2), \text{MRT1}} = (12 + 6\omega_6^2c_s^2 + 144v_2^4 + 234v_2^2\omega_6 - 1008v_2^2\omega_6c_s^2 - 78\omega_6^2c_s^2 - 132c_s^2 - 98v_2^2\omega_6^2 - 18\omega_6 - 216\omega_6c_s^4 + 10v_2^2\omega_6^3 + 82\omega_6^2c_s^4 - 34v_2^2\omega_6^2c_s^2 - 216v_2^4\omega_6 + 144c_s^4 - \omega_6^3 + 8\omega_6^2 + 672v_2^2c_s^2 + 198\omega_6c_s^2 - 156v_2^2 - 9v_2^4\omega_6^3 + 404v_2^2\omega_6^2c_s^2 - 5\omega_6^3c_s^4 + 90v_2^4\omega_6^2) \frac{v_2}{12\omega_6^3}$$

$$C_{D_y^4 \rho}^{(2), \text{MRT2}} = C_{D_y^4 \rho}^{(2), \text{MRT1}}$$

$$C_{D_y^4 \rho}^{(2), \text{CLBM1}} = C_{D_y^4 \rho}^{(2), \text{MRT1}}$$

$$C_{\text{D}_y^4 \rho}^{(2), \text{CLBM2}} = C_{\text{D}_y^4 \rho}^{(2), \text{MRT1}}$$

$$C_{\text{D}_y^4 \rho}^{(2), \text{CuLBM1}} = (12 - \omega_2^3 + 144v_2^4 + 234\omega_2 v_2^2 - 18\omega_2 + 90\omega_2^2 v_2^4 + 8\omega_2^2 + 6\omega_2^3 c_s^2 - 34\omega_2^3 v_2^2 c_s^2 - 1008\omega_2 v_2^2 c_s^2 - 132c_s^2 - 216\omega_2 c_s^4 - 78\omega_2^2 c_s^2 - 9\omega_2^3 v_2^4 + 404\omega_2^2 v_2^2 c_s^2 + 144c_s^4 + 198\omega_2 c_s^2 + 10\omega_2^3 v_2^2 + 672v_2^2 c_s^2 + 82\omega_2^2 c_s^4 - 156v_2^2 - 216\omega_2 v_2^4 - 5\omega_2^3 c_s^4 - 98\omega_2^2 v_2^2) \frac{v_2}{12\omega_2^3}$$

$$C_{\text{D}_y^4 \rho}^{(2), \text{CuLBM2}} = (-171\omega_2 \omega_3 c_s^4 \omega_1^3 - 600\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1 + 141\omega_2^3 \omega_3 c_s^2 \omega_1 + 129\omega_2 v_2^2 \omega_3 \omega_1^3 + 6\omega_2^2 \omega_3 \omega_1 + 114\omega_2^2 \omega_3 c_s^2 \omega_1^2 - 105\omega_2 v_2^2 \omega_3 \omega_1^2 + 54\omega_2 \omega_3 c_s^4 \omega_1^2 + 12\omega_2^2 c_s^2 \omega_1^2 - 78\omega_2^2 \omega_3 c_s^2 \omega_1^3 + 45\omega_2^3 v_2^4 \omega_3 - 72\omega_2^3 \omega_3 c_s^2 + 261v_2^2 \omega_3 c_s^2 \omega_1^3 - 117\omega_2 v_2^4 \omega_3 \omega_1^3 + 261\omega_2^3 v_2^2 \omega_3 c_s^2 + 8\omega_2^2 \omega_3 \omega_1^3 + 18\omega_2^3 c_s^4 \omega_1 + 12\omega_2^3 \omega_3 c_s^2 \omega_1^3 - 68\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^3 - 72\omega_3 c_s^2 \omega_1^3 + 6\omega_2 v_2^2 c_s^2 \omega_1^3 - 12\omega_2^2 \omega_3 \omega_1^2 + 99\omega_2 v_2^4 \omega_3 \omega_1^2 - 60\omega_2^2 \omega_3 c_s^2 \omega_1 + 404\omega_3^3 v_2^2 \omega_3 c_s^2 \omega_1^2 + 18\omega_2 c_s^4 \omega_1^3 - 78\omega_2^3 v_2^2 \omega_3 \omega_1^2 + 6\omega_3 \omega_1^3 + 404\omega_2^2 v_2^2 \omega_3 c_s^2 \omega_1^3 - 12\omega_2 \omega_3 \omega_1^3 + 99\omega_2^2 v_2^4 \omega_3 \omega_1 - 60\omega_2 \omega_3 c_s^2 \omega_1^2 - 98\omega_2^2 v_2^2 \omega_3 \omega_1^3 - 36\omega_2^3 c_s^4 \omega_1^2 + 90\omega_2^3 v_2^4 \omega_3 \omega_1^2 + 90\omega_2^3 \omega_3 \omega_1^2 + 82\omega_2^2 \omega_3 c_s^2 \omega_1^3 - 51\omega_2^2 v_2^2 \omega_3 + 411\omega_2 v_2^2 \omega_3 c_s^2 \omega_1^2 + 6\omega_2^2 \omega_3 + 141\omega_2 \omega_3 c_s^2 \omega_1^3 + 129\omega_2^3 v_2^2 \omega_3 \omega_1 - 12\omega_2^3 \omega_3 \omega_1 - 171\omega_2^3 \omega_3 c_s^2 \omega_1 + 6\omega_2 \omega_3 \omega_1^2 - 816\omega_2^2 v_2^2 \omega_3 c_s^2 \omega_1^2 + 45\omega_2^4 \omega_3 \omega_1^3 - 600\omega_2 v_2^2 \omega_3 c_s^2 \omega_1^3 - 90\omega_2^2 \omega_3 c_s^4 \omega_1^2 - 18\omega_2^3 v_2^4 \omega_3 \omega_1^3 + 210\omega_2^2 v_2^2 \omega_3 \omega_1^2 + 54\omega_2^2 \omega_3 c_s^4 \omega_1 - 105\omega_2^2 v_2^2 \omega_3 \omega_1 - 98\omega_2^3 v_2^2 \omega_3 \omega_1^2 + 8\omega_2^3 \omega_3 \omega_1^2 - 6\omega_2 c_s^4 \omega_1^3 + 90\omega_2^2 v_2^4 \omega_3 \omega_1^3 + 82\omega_2^2 \omega_3 c_s^4 \omega_1^2 + 411\omega_2^2 v_2^2 \omega_3 c_s^2 \omega_1 - 117\omega_2^3 v_2^4 \omega_3 \omega_1 - 12\omega_2^2 v_2^2 c_s^2 \omega_1^2 - 6\omega_2^3 c_s^2 \omega_1 + 6\omega_2^3 v_2^2 c_s^2 \omega_1 - 51\omega_2^2 v_2 \omega_3 \omega_1^3 - 10\omega_2^3 \omega_3 c_s^4 \omega_1^3 - 2\omega_2^3 \omega_3 \omega_1^3 - 198\omega_2^2 v_2^4 \omega_3 \omega_1^2 + 90\omega_3 c_s^4 \omega_1^3 + 20\omega_2^2 v_2^2 \omega_3 \omega_1^3) \frac{v_2}{24\omega_2^3 \omega_3 \omega_1^3}$$

coefficient  $C_{\text{D}_y^4 v_2}^{(2)}$  at  $\frac{\partial^4 v_2}{\partial x_2^4}$ :

$$C_{\text{D}_y^4 v_2}^{(2), \text{SR1T}} = (12 - 22c_s^2 \omega^2 + 504v_2^4 + 14v_2^2 \omega^3 - 154\omega_2^2 \omega^2 + 2c_s^2 \omega^3 + 378v_2^2 \omega - 36c_s^2 + 54c_s^2 \omega + 24c_s^4 + 252v_2^2 c_s^2 \omega^2 + 310v_2^4 \omega^2 - c_s^4 \omega^3 + 14c_s^4 \omega^2 + 432v_2^2 c_s^2 - 29v_2^4 \omega^3 - 18v_2^2 c_s^2 \omega^3 - 36c_s^4 \omega - 252v_2^2 - 18\omega + 8\omega^2 - 648v_2^2 c_s^2 \omega - 756v_2^4 \omega - \omega^3) \frac{\rho}{12\omega^3}$$

$$C_{\text{D}_y^4 v_2}^{(2), \text{MRT1}} = (12 + 2\omega_6^3 c_s^2 + 504v_2^4 + 378v_2^2 \omega_6 - 648v_2^2 \omega_6 c_s^2 - 22\omega_6^2 c_s^2 - 36c_s^2 - 154v_2^2 \omega_6^2 - 18\omega_6 - 36\omega_6 c_s^4 + 14v_2^2 \omega_6^3 + 14\omega_6^2 c_s^4 - 18v_2^2 \omega_6^3 c_s^2 - 756v_2^4 \omega_6 + 24c_s^4 - \omega_6^3 + 8\omega_6^2 + 432v_2^2 c_s^2 + 54\omega_6 c_s^2 - 252v_2^2 - 29v_2^4 \omega_6^3 + 252v_2^2 \omega_6^2 c_s^2 - \omega_6^3 c_s^4 + 310v_2^4 \omega_6^2) \frac{\rho}{12\omega_6^3}$$

$$C_{\text{D}_y^4 v_2}^{(2), \text{MRT2}} = C_{\text{D}_y^4 v_2}^{(2), \text{MRT1}}$$

$$C_{\text{D}_y^4 v_2}^{(2), \text{CLBM1}} = C_{\text{D}_y^4 v_2}^{(2), \text{MRT1}}$$

$$C_{\text{D}_y^4 v_2}^{(2), \text{CLBM2}} = C_{\text{D}_y^4 v_2}^{(2), \text{MRT1}}$$

$$C_{\text{D}_y^4 v_2}^{(2), \text{CuLBM1}} = (12 - \omega_2^3 + 504v_2^4 + 378\omega_2 v_2^2 - 18\omega_2 + 310\omega_2^2 v_2^4 + 8\omega_2^2 + 2\omega_2^3 c_s^2 - 18\omega_2^3 v_2^2 c_s^2 - 648\omega_2 v_2^2 c_s^2 - 36c_s^2 - 36\omega_2 c_s^4 - 22\omega_2^2 c_s^2 - 29\omega_2^3 v_2^4 + 252\omega_2^2 v_2^2 c_s^2 + 24c_s^4 + 54\omega_2 c_s^2 + 14\omega_2^3 v_2^2 + 432v_2^2 c_s^2 + 14\omega_2^2 c_s^4 - 252v_2^2 - 756\omega_2 v_2^4 - \omega_2^3 c_s^4 - 154\omega_2^2 v_2^2) \frac{\rho}{12\omega_2^3}$$

$$C_{\text{D}_y^4 v_2}^{(2), \text{CuLBM2}} = (-33\omega_2 \omega_3 c_s^4 \omega_1^3 - 432\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1 + 45\omega_2^3 \omega_3 c_s^2 \omega_1 + 225\omega_2 v_2^2 \omega_3 \omega_1^3 + 6\omega_2^2 \omega_3 \omega_1 + 18\omega_2^2 \omega_3 c_s^2 \omega_1^2 - 153\omega_2 v_2^2 \omega_3 \omega_1^2 + 6\omega_2 \omega_3 c_s^4 \omega_1^2 + 12\omega_2^2 c_s^2 \omega_1^2 - 22\omega_2^2 \omega_3 c_s^2 \omega_1^3 + 171\omega_2^3 v_2^4 \omega_3 - 24\omega_2^3 \omega_3 c_s^2 + 207v_2^2 \omega_3 c_s^2 \omega_1^3 - 423\omega_2 v_2^4 \omega_3 \omega_1^3 + 207\omega_2^3 v_2^2 \omega_3 c_s^2 + 8\omega_2^2 \omega_3 \omega_1^3 + 6\omega_2^3 c_s^4 \omega_1 + 4\omega_2^3 \omega_3 c_s^2 \omega_1^3 - 36\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^3 - 24\omega_3 c_s^2 \omega_1^3 + 18\omega_2 v_2^2 c_s^2 \omega_1^3 - 12\omega_2^2 \omega_3 \omega_1^2 + 333\omega_2 v_2^4 \omega_3 \omega_1^2 - 12\omega_2^3 \omega_3 c_s^2 \omega_1^2 + 252\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^2 + 6\omega_2 c_s^4 \omega_1^3 - 22\omega_2^3 \omega_3 c_s^2 \omega_1^2 + 6\omega_3 \omega_1^3 + 252\omega_2^2 v_2^2 \omega_3 c_s^2 \omega_1^3 - 12\omega_2 \omega_3 \omega_1^3 + 333\omega_2^2 v_2^4 \omega_3 \omega_1 - 12\omega_2 \omega_3 c_s^2 \omega_1^2 - 154\omega_2^2 v_2^2 \omega_3 \omega_1^3 - 12\omega_2^2 c_s^4 \omega_1^2 + 310\omega_2^3 v_2^4 \omega_3 \omega_1^2 + 18\omega_2^2 \omega_3 c_s^4 + 14\omega_2^2 \omega_3 c_s^4 \omega_1^3 - 99\omega_2^2 v_2^2 \omega_3 + 225\omega_2 v_2^2 \omega_3 c_s^2 \omega_1^2 + 6\omega_2^2 \omega_3 + 45\omega_2 \omega_3 c_s^2 \omega_1^3 + 225\omega_2^3 v_2^2 \omega_3 \omega_1 - 12\omega_2^3 \omega_3 \omega_1^3 - 33\omega_2^3 \omega_3 c_s^4 \omega_1 + 6\omega_2 \omega_3 \omega_1^2 - 432\omega_2^2 v_2^2 \omega_3 c_s^2 \omega_1^2 + 171\omega_2^4 \omega_3 \omega_1^3 - 432\omega_2 v_2^2 \omega_3 c_s^2 \omega_1^3 - 6\omega_2^2 \omega_3 c_s^4 \omega_1^2 - 58\omega_2^3 v_2^4 \omega_3 \omega_1^3 + 306\omega_2^2 v_2^2 \omega_3 \omega_1^2 + 6\omega_2^2 \omega_3 c_s^4 \omega_1 - 153\omega_2^2 v_2^2 \omega_3 \omega_1^2 - 154\omega_2^3 v_2^2 \omega_3 \omega_1^2 + 8\omega_2^3 \omega_3 \omega_1^2 - 6\omega_2 c_s^4 \omega_1^3 + 310\omega_2^2 v_2^4 \omega_3 \omega_1^3 + 14\omega_2^3 \omega_3 c_s^4 \omega_1^2 + 225\omega_2^2 v_2^2 \omega_3 c_s^2 \omega_1^2 - 423\omega_2^3 v_2^4 \omega_3 \omega_1^2 - 36\omega_2^2 v_2^2 c_s^2 \omega_1^2 - 6\omega_2^3 c_s^2 \omega_1 + 18\omega_2^3 v_2^2 c_s^2 \omega_1^3 - 99\omega_2^2 \omega_3 \omega_1^3 - 2\omega_2^3 \omega_3 c_s^4 \omega_1^3 - 2\omega_2^3 \omega_3 \omega_1^3 - 666\omega_2^2 v_2^4 \omega_3 \omega_1^2 + 18\omega_3 c_s^4 \omega_1^3 + 28\omega_2^2 v_2^2 \omega_3 \omega_1^3) \frac{\rho}{24\omega_2^3 \omega_3 \omega_1^3}$$

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