

D2Q9 NSE, a supplementary material for Lattice Boltzmann Method Analysis Tool (LBMAT)

Radek Fučík[†], Pavel Eichler[†], Jakub Klinkovský[†], Robert Straka^{‡,†}, and Tomáš Oberhuber[†]

[†]Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague,
Trojanova 13, 120 00 Prague, Czech Republic

[‡]AGH University of Science and Technology, al. Mickiewicza 30, 30-059 Krakow, Poland

Contents

1	Global definitions	2
1.1	Discrete velocity vectors	2
1.2	Raw and central moments	3
1.3	Transformation matrix \mathbf{M}	3
1.4	Equilibrium	4
2	Spatial EPDEs	4
2.1	SRT	4
2.1.1	Definitions	4
2.1.2	Conservation of mass: ρ	4
2.1.3	Conservation of momentum: ρv_1	5
2.1.4	Conservation of momentum: ρv_2	6
2.2	MRT	7
2.2.1	Definitions	7
2.2.2	Conservation of mass: ρ	7
2.2.3	Conservation of momentum: ρv_1	7
2.2.4	Conservation of momentum: ρv_2	11
2.3	MRT2	15
2.3.1	Definitions	15
2.3.2	Conservation of mass: ρ	16
2.3.3	Conservation of momentum: ρv_1	16
2.3.4	Conservation of momentum: ρv_2	20
2.4	CLBM1	23
2.4.1	Definitions	23
2.4.2	Conservation of mass: ρ	24
2.4.3	Conservation of momentum: ρv_1	24
2.4.4	Conservation of momentum: ρv_2	26
2.5	CLBM2	28
2.5.1	Definitions	28

2.5.2	Conservation of mass: ρ	28
2.5.3	Conservation of momentum: ρv_1	29
2.5.4	Conservation of momentum: ρv_2	30
2.6	CuLBM1	32
2.6.1	Definitions	32
2.6.2	Conservation of mass: ρ	33
2.6.3	Conservation of momentum: ρv_1	33
2.6.4	Conservation of momentum: ρv_2	34
2.7	CuLBM2	35
2.7.1	Definitions	35
2.7.2	Conservation of mass: ρ	36
2.7.3	Conservation of momentum: ρv_1	37
2.7.4	Conservation of momentum: ρv_2	39
3	Comparison of SRT, MRT, CLBM, and CuLBM	42
3.1	Conservation of mass: ρ	42
3.2	Conservation of momentum: ρv_1	48
3.3	Conservation of momentum: ρv_2	69

1 Global definitions

In \mathbb{R}^2 , the position and velocity vectors are given by $\mathbf{x} = (x_1, x_2)^T$ and $\mathbf{v} = (v_1, v_2)^T$, respectively.

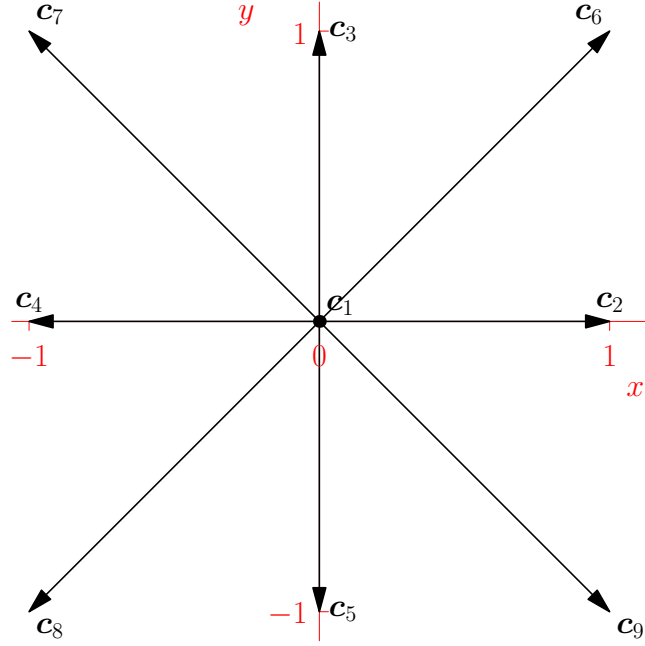
1.1 Discrete velocity vectors

Discrete velocity vectors and the lattice speed of sound are defined by

$$\{\mathbf{c}_i\}_{i=1}^9 = \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right),$$

$$c_s = \frac{1}{\sqrt{3}},$$

respectively [1].



1.2 Raw and central moments

The raw and central moments are defined by

$$m_{\alpha} := \sum_{i=1}^9 f_i c_i^{\alpha},$$

and

$$k_{\alpha} := \sum_{i=1}^9 f_i (c_i - v)^{\alpha},$$

respectively, where $\alpha = (\alpha_1, \alpha_2) \in \mathbb{Z}^2$ denotes a multi-index (as a row vector) and $c_i^{\alpha} := \prod_{j=1}^2 [c_i]_j^{\alpha_j}$.

1.3 Transformation matrix M

Matrix \mathbf{M} , that defines macroscopic quantities (moments) $\boldsymbol{\mu}$ by

$$\boldsymbol{\mu} = \mathbf{M} \mathbf{f},$$

with $\mathbf{f} = (f_1, f_2, \dots, f_9)^T$, is selected such that

$$\boldsymbol{\mu} = \left(m_{(0,0)}, m_{(1,0)}, m_{(0,1)}, m_{(1,1)}, m_{(2,0)}, m_{(0,2)}, m_{(2,1)}, m_{(1,2)}, m_{(2,2)} \right)^T,$$

i.e., \mathbf{M} is given by

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

1.4 Equilibrium

The corresponding equilibrium raw moments are defined using the continuous Maxwell–Boltzmann distribution function [1]

$$f^{(eq)}(\boldsymbol{\xi}) = \frac{\rho}{2\pi c_s^2} \exp\left(-\frac{\|\boldsymbol{\xi} - \mathbf{v}\|^2}{2c_s^2}\right)$$

as

$$m_{\boldsymbol{\alpha}}^{(eq)} = \int_{\mathbb{R}^2} \boldsymbol{\xi}^{\boldsymbol{\alpha}} f^{(eq)}(\boldsymbol{\xi}) d\boldsymbol{\xi},$$

where $\alpha_i \in \{0, 1, 2\}$, $i = 1, 2$. Hence, the equilibrium moments $\boldsymbol{\mu}^{(eq)}$ satisfy

$$\boldsymbol{\mu}^{(eq)} = \begin{pmatrix} \rho \\ \rho v_1 \\ \rho v_2 \\ \rho v_1 v_2 \\ \rho(v_1^2 + c_s^2) \\ \rho(v_2^2 + c_s^2) \\ \rho v_2(c_s^2 + v_1^2) \\ \rho v_1(c_s^2 + v_2^2) \\ \rho(c_s^4 + c_s^2(v_1^2 + v_2^2) + v_1^2 v_2^2) \end{pmatrix}.$$

2 Spatial EPDEs

2.1 SRT

2.1.1 Definitions

Collision operator \mathcal{C} :

$$\mathcal{C}(\mathbf{f}) = \omega \left(\mathbf{M}^{-1} \boldsymbol{\mu}^{(eq)} - \mathbf{f} \right),$$

$\omega \in (0, 2)$.

2.1.2 Conservation of mass: ρ



attached text file: output_d2q9_nse_srt_symbolic_pde_00.txt

$$\begin{aligned}
& \frac{\partial \rho}{\partial t} + \frac{v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + v_1^2 + 3cs^2) \frac{v_1 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + 3v_1^2 + cs^2) \frac{\rho \delta_l^3}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\
& \frac{\rho \delta_l^3 cs^2}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{\rho \delta_l^3 cs^2}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + v_2^2 + 3cs^2) \frac{v_2 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + 3v_2^2 + cs^2) \frac{\rho \delta_l^3}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& (-12v_1^2 cs^2 \omega - 6v_1^2 - 3v_1^4 \omega - cs^4 \omega + 2cs^4 - 2cs^2 + 3v_1^2 \omega + cs^2 \omega + 24v_1^2 cs^2 + 6v_1^4) \frac{\delta_l^4}{24\delta_t \omega} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& (-4 + 10v_1^2 + 6cs^2 - 5v_1^2 \omega - 3cs^2 \omega + 2\omega) \frac{v_1 \rho \delta_l^4}{12\delta_t \omega} \frac{\partial^4 v_1}{\partial x_1^4} + (2 - 2v_1^2 - 6cs^2 + v_1^2 \omega + 3cs^2 \omega - \omega) \frac{v_1 \rho \delta_l^4}{12\delta_t \omega} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& (-2 + \omega) \frac{\delta_l^4 cs^4}{6\delta_t \omega} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + (2 - 2v_2^2 + v_2^2 \omega - 6cs^2 + 3cs^2 \omega - \omega) \frac{v_2 \rho \delta_l^4}{12\delta_t \omega} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& (-6v_2^2 - 3v_2^4 \omega - cs^4 \omega + 2cs^4 - 12v_2^2 cs^2 \omega + 3v_2^2 \omega + 24v_2^2 cs^2 - 2cs^2 + 6v_2^4 + cs^2 \omega) \frac{\delta_l^4}{24\delta_t \omega} \frac{\partial^4 \rho}{\partial x_2^4} + \\
& (-4 + 10v_2^2 - 5v_2^2 \omega + 6cs^2 - 3cs^2 \omega + 2\omega) \frac{v_2 \rho \delta_l^4}{12\delta_t \omega} \frac{\partial^4 v_2}{\partial x_2^4} = 0.
\end{aligned}$$

2.1.3 Conservation of momentum: ρv_1



attached text file: output_d2q9_nse_srt_symbolic_pde_01.txt

$$\begin{aligned}
& v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (v_1^2 + cs^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2v_1 \rho \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{v_2 \rho \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{v_1 \rho \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\
& (-2 + 6v_1^2 + 4cs^2 - 3v_1^2 \omega - 2cs^2 \omega + \omega) \frac{\delta_l^2}{\delta_t \omega} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + (2 - \omega) \frac{3v_1 \rho \delta_l^2}{\delta_t \omega} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + (-2 + \omega) \frac{\delta_l^2 cs^2}{2\delta_t \omega} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\
& (-2 + \omega) \frac{\delta_l^2 cs^2}{2\delta_t \omega} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (-2 + 2v_1^2 + 6cs^2 - v_1^2 \omega - 3cs^2 \omega + \omega) \frac{v_1 \delta_l^2}{2\delta_t \omega} \frac{\partial^2 \rho}{\partial x_1^2} + \\
& (-2 + 6v_1^2 + 2cs^2 - 3v_1^2 \omega - cs^2 \omega + \omega) \frac{\rho \delta_l^2}{2\delta_t \omega} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega) \frac{\rho \delta_l^2 cs^2}{2\delta_t \omega} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega) \frac{\rho \delta_l^2 cs^2}{2\delta_t \omega} \frac{\partial^2 v_1}{\partial x_2^2} + C_1 \frac{\delta_l^3}{12\delta_t \omega^2} \frac{\partial^3 \rho}{\partial x_1^3} + \\
& (-24 + 60v_1^2 + 5cs^2 \omega^2 + 36cs^2 + 11v_1^2 \omega^2 - 60v_1^2 \omega - 4\omega^2 - 36cs^2 \omega + 24\omega) \frac{v_1 \rho \delta_l^3}{6\delta_t \omega^2} \frac{\partial^3 v_1}{\partial x_1^3} + \\
& (12 - 12v_1^2 - 11cs^2 \omega^2 - 36cs^2 - 3v_1^2 \omega^2 + 12v_1^2 \omega + 3\omega^2 + 36cs^2 \omega - 12\omega) \frac{v_1 \rho \delta_l^3}{12\delta_t \omega^2} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + \\
& (-12 - \omega^2 + 12\omega) \frac{\delta_l^3 cs^4}{6\delta_t \omega^2} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} - \frac{v_1 \rho \delta_l^3 cs^2}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + v_2^2 + 3cs^2) \frac{v_2 v_1 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\
& (6 - 6v_2^2 - 3cs^2 \omega^2 + 6v_2^2 \omega - 18cs^2 + \omega^2 + 18cs^2 \omega - 6\omega - v_2^2 \omega^2) \frac{v_2 \rho \delta_l^3}{6\delta_t \omega^2} \frac{\partial^3 v_1}{\partial x_2^3} + (-1 + 3v_2^2 + cs^2) \frac{v_1 \rho \delta_l^3}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& C_2 \frac{v_1 \delta_l^4}{12\delta_t \omega^3} \frac{\partial^4 \rho}{\partial x_1^4} + C_3 \frac{\rho \delta_l^4}{12\delta_t \omega^3} \frac{\partial^4 v_1}{\partial x_1^4} + C_4 \frac{\rho \delta_l^4}{12\delta_t \omega^3} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_5 \frac{v_1 \delta_l^4 cs^2}{12\delta_t \omega^3} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\
& (-24 + 72v_1^2 + 8cs^2 \omega^2 - cs^2 \omega^3 + 12cs^2 + 36v_1^2 \omega^2 - 108v_1^2 \omega - 12\omega^2 - 18cs^2 \omega + 36\omega) \frac{\rho \delta_l^4 cs^2}{12\delta_t \omega^3} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + \\
& C_6 \frac{v_2 \delta_l^4 cs^2}{12\delta_t \omega^3} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + (2 - 2v_2^2 + v_2^2 \omega - 6cs^2 + 3cs^2 \omega - \omega) \frac{v_2 v_1 \rho \delta_l^4}{12\delta_t \omega} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& (-12 + 36v_2^2 + 2cs^2 \omega^2 - 54v_2^2 \omega - cs^2 \omega^3 - 6\omega^2 + 18\omega + 18v_2^2 \omega^2) \frac{\rho \delta_l^4 cs^2}{12\delta_t \omega^3} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\
& (-6v_2^2 - 3v_2^4 \omega - cs^4 \omega + 2cs^4 + 3v_2^2 \omega - 2cs^2 - 12v_2^2 cs^2 \omega + 24v_2^2 cs^2 + 6v_2^4 + cs^2 \omega) \frac{v_1 \delta_l^4}{24\delta_t \omega} \frac{\partial^4 \rho}{\partial x_2^4} + C_7 \frac{\rho \delta_l^4}{24\delta_t \omega^3} \frac{\partial^4 v_1}{\partial x_2^4} + \\
& (-4 + 10v_2^2 - 5v_2^2 \omega + 6cs^2 - 3cs^2 \omega + 2\omega) \frac{v_2 v_1 \rho \delta_l^4}{12\delta_t \omega} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= 7v_1^4 \omega^2 + cs^4 \omega^2 - 144v_1^2 cs^2 \omega - 36v_1^2 + 24v_1^2 cs^2 \omega^2 - 12cs^4 \omega - 36v_1^4 \omega + 12cs^4 - cs^2 \omega^2 - 12cs^2 - 7v_1^2 \omega^2 + 36v_1^2 \omega + 36v_1^4 + 144v_1^2 cs^2 + 12cs^2 \omega \\
C_2 &= 12 - 5cs^4 \omega^3 + 90v_1^4 \omega^2 + 82cs^4 \omega^2 - 9v_1^4 \omega^3 - 1008v_1^2 cs^2 \omega - 156v_1^2 + 404v_1^2 cs^2 \omega^2 - 216cs^4 \omega - 34v_1^2 cs^2 \omega^3 - 216v_1^4 \omega + 144cs^4 - \\
& 78cs^2 \omega^2 + 10v_1^2 \omega^3 + 6cs^2 \omega^3 - 132cs^2 - 98v_1^2 \omega^2 - \omega^3 + 234v_1^2 \omega + 144v_1^4 + 8\omega^2 + 672v_1^2 cs^2 + 198cs^2 \omega - 18\omega \\
C_3 &= 12 - cs^4 \omega^3 + 310v_1^4 \omega^2 + 14cs^4 \omega^2 - 29v_1^4 \omega^3 - 648v_1^2 cs^2 \omega - 252v_1^2 + 252v_1^2 cs^2 \omega^2 - 36cs^4 \omega - 18v_1^2 cs^2 \omega^3 - 756v_1^4 \omega + 24cs^4 - 22cs^2 \omega^2 + \\
& 14v_1^2 \omega^3 + 2cs^2 \omega^3 - 36cs^2 - 154v_1^2 \omega^2 - \omega^3 + 378v_1^2 \omega + 504v_1^4 + 8\omega^2 + 432v_1^2 cs^2 + 54cs^2 \omega - 18\omega \\
C_4 &= -cs^4 \omega^3 - 26v_1^4 \omega^2 + 20cs^4 \omega^2 + 4v_1^4 \omega^3 + 54v_1^2 cs^2 \omega + 36v_1^2 - 42v_1^2 cs^2 \omega^2 - 54cs^4 \omega + 12v_1^2 cs^2 \omega^3 + 54v_1^4 \omega + 36cs^4 - 12cs^2 \omega^2 - 4v_1^2 \omega^3 - \\
& 24cs^2 + 26v_1^2 \omega^2 - 54v_1^2 \omega - 36v_1^4 - 36v_1^2 cs^2 + 36cs^2 \omega \\
C_5 &= 24 - 24v_1^2 - 46cs^2 \omega^2 + v_1^2 \omega^3 + 5cs^2 \omega^3 - 72cs^2 - 14v_1^2 \omega^2 - \omega^3 + 36v_1^2 \omega + 14\omega^2 + 108cs^2 \omega - 36\omega
\end{aligned}$$

$$C_6 = 24 - 24v_2^2 - 42cs^2\omega^2 + 36v_2^2\omega + 3cs^2\omega^3 - 72cs^2 - \omega^3 + v_2^2\omega^3 + 14\omega^2 + 108cs^2\omega - 36\omega - 14v_2^2\omega^2$$

$$C_7 = -3cs^4\omega^3 + 72v_2^2 + 30cs^4\omega^2 + 108v_2^4\omega - 72cs^4\omega - 42v_2^4\omega^2 + 3v_2^4\omega^3 + 48cs^4 - 14cs^2\omega^2 - 108v_2^2\omega + cs^2\omega^3 - 24cs^2 + 216v_2^2cs^2\omega - 144v_2^2cs^2 - 84v_2^2cs^2\omega^2 - 72v_2^4 - 3v_2^2\omega^3 + 6v_2^2cs^2\omega^3 + 36cs^2\omega + 42v_2^2\omega^2$$

2.1.4 Conservation of momentum: ρv_2



attached text file: output_d2q9_nse_srt_symbolic_pde_02.txt

$$\begin{aligned} & v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{v_2 v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{v_2 \rho \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_1 \rho \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_1} + (v_2^2 + cs^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{2v_2 \rho \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega) \frac{\delta_l^2 cs^2}{2\delta_t \omega} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\ & (-2 + \omega) \frac{\delta_l^2 cs^2}{2\delta_t \omega} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (-2 + 6v_2^2 + 4cs^2 - 3v_2^2\omega - 2cs^2\omega + \omega) \frac{\delta_l^2}{\delta_t \omega} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (2 - \omega) \frac{3v_2 \rho \delta_l^2}{\delta_t \omega} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + \\ & (-2 + \omega) \frac{\rho \delta_l^2 cs^2}{2\delta_t \omega} \frac{\partial^2 v_2}{\partial x_1^2} + (-2 + \omega) \frac{\rho \delta_l^2 cs^2}{2\delta_t \omega} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + (-2 + 2v_2^2 + 6cs^2 - v_2^2\omega - 3cs^2\omega + \omega) \frac{v_2 \delta_l^2}{2\delta_t \omega} \frac{\partial^2 \rho}{\partial x_2^2} + \\ & (-2 + 6v_2^2 + 2cs^2 - 3v_2^2\omega - cs^2\omega + \omega) \frac{\rho \delta_l^2}{2\delta_t \omega} \frac{\partial^2 v_2}{\partial x_2^2} + (-1 + v_1^2 + 3cs^2) \frac{v_2 v_1 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + 3v_1^2 + cs^2) \frac{v_2 \rho \delta_l^3}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + \\ & (6 - 6v_1^2 - 18cs^2 + 18cs^2\omega - v_1^2\omega^2 + 6v_1^2\omega - 3cs^2\omega^2 + \omega^2 - 6\omega) \frac{v_1 \rho \delta_l^3}{6\delta_t \omega^2} \frac{\partial^3 v_2}{\partial x_1^3} + (-12 - \omega^2 + 12\omega) \frac{\delta_l^3 cs^4}{6\delta_t \omega^2} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} - \\ & \frac{v_2 \rho \delta_l^3 cs^2}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + (12 - 12v_2^2 - 36cs^2 + 12v_2^2\omega + 36cs^2\omega - 11cs^2\omega^2 + 3\omega^2 - 12\omega - 3v_2^2\omega^2) \frac{v_2 \rho \delta_l^3}{12\delta_t \omega^2} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + \\ & C_1 \frac{\delta_l^3}{12\delta_t \omega^2} \frac{\partial^3 \rho}{\partial x_2^3} + (-24 + 60v_2^2 + 36cs^2 - 60v_2^2\omega - 36cs^2\omega + 5cs^2\omega^2 - 4\omega^2 + 24\omega + 11v_2^2\omega^2) \frac{v_2 \rho \delta_l^3}{6\delta_t \omega^2} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & (-cs^4\omega - 6v_1^2 + 2cs^4 - 12v_1^2cs^2\omega - 3v_1^4\omega - 2cs^2 + cs^2\omega + 3v_1^2\omega + 24v_1^2cs^2 + 6v_1^4) \frac{v_2 \delta_l^4}{24\delta_t \omega} \frac{\partial^4 \rho}{\partial x_1^4} + \\ & (-4 + 10v_1^2 + 6cs^2 - 3cs^2\omega - 5v_1^2\omega + 2\omega) \frac{v_2 v_1 \rho \delta_l^4}{12\delta_t \omega} \frac{\partial^4 v_1}{\partial x_1^4} + C_2 \frac{\rho \delta_l^4}{24\delta_t \omega^3} \frac{\partial^4 v_2}{\partial x_1^4} + C_3 \frac{v_1 \delta_l^4 cs^2}{12\delta_t \omega^3} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + \\ & (-12 + 36v_1^2 + 18v_1^2\omega^2 - 54v_1^2\omega + 2cs^2\omega^2 - 6\omega^2 - cs^2\omega^3 + 18\omega) \frac{\rho \delta_l^4 cs^2}{12\delta_t \omega^3} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + \\ & (2 - 2v_1^2 - 6cs^2 + 3cs^2\omega + v_1^2\omega - \omega) \frac{v_2 v_1 \rho \delta_l^4}{12\delta_t \omega} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_4 \frac{v_2 \delta_l^4 cs^2}{12\delta_t \omega^3} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\ & (-24 + 72v_2^2 + 12cs^2 - 108v_2^2\omega - 18cs^2\omega + 8cs^2\omega^2 - 12\omega^2 - cs^2\omega^3 + 36\omega + 36v_2^2\omega^2) \frac{\rho \delta_l^4 cs^2}{12\delta_t \omega^3} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + \\ & C_5 \frac{\rho \delta_l^4}{12\delta_t \omega^3} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_6 \frac{v_2 \delta_l^4}{12\delta_t \omega^3} \frac{\partial^4 \rho}{\partial x_2^4} + C_7 \frac{\rho \delta_l^4}{12\delta_t \omega^3} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:

$$C_1 = -12cs^4\omega - 36v_2^2 - 36v_2^2\omega + 12cs^4 + 7v_2^4\omega^2 + cs^4\omega^2 - 12cs^2 + 36v_2^2\omega + 24v_2^2cs^2\omega^2 + 144v_2^2cs^2 + 12cs^2\omega + 36v_2^4 - cs^2\omega^2 - 144v_2^2cs^2\omega - 7v_2^2\omega^2$$

$$C_2 = -72cs^4\omega - 42v_1^4\omega^2 - 84v_1^2cs^2\omega^2 + 6v_1^2cs^2\omega^3 + 3v_1^4\omega^3 + 72v_1^2 - 3cs^4\omega^3 + 48cs^4 + 216v_1^2cs^2\omega + 108v_1^4\omega + 30cs^4\omega^2 - 24cs^2 - 3v_1^2\omega^3 + 36cs^2\omega + 42v_1^2\omega^2 - 108v_1^2\omega - 144v_1^2cs^2 - 14cs^2\omega^2 - 72v_1^4 + cs^2\omega^3$$

$$C_3 = 24 - 24v_1^2 - 72cs^2 + v_1^2\omega^3 + 108cs^2\omega - 14v_1^2\omega^2 - \omega^3 + 36v_1^2\omega - 42cs^2\omega^2 + 14\omega^2 + 3cs^2\omega^3 - 36\omega$$

$$C_4 = 24 - 24v_2^2 - 72cs^2 + 36v_2^2\omega + 108cs^2\omega - \omega^3 + v_2^2\omega^3 - 46cs^2\omega^2 + 14\omega^2 + 5cs^2\omega^3 - 36\omega - 14v_2^2\omega^2$$

$$C_5 = -54cs^4\omega + 36v_2^2 + 54v_2^4\omega - cs^4\omega^3 + 36cs^4 - 26v_2^4\omega^2 + 4v_2^4\omega^3 + 20cs^4\omega^2 - 24cs^2 - 54v_2^2\omega - 42v_2^2cs^2\omega^2 - 36v_2^2cs^2 + 36cs^2\omega + 12v_2^2cs^2\omega^3 - 36v_2^4 - 4v_2^2\omega^3 - 12cs^2\omega^2 + 54v_2^2cs^2\omega + 26v_2^2\omega^2$$

$$C_6 = 12 - 216cs^4\omega - 156v_2^2 - 216v_2^4\omega - 5cs^4\omega^3 + 144cs^4 + 90v_2^4\omega^2 - 9v_2^4\omega^3 + 82cs^4\omega^2 - 132cs^2 + 234v_2^2\omega + 404v_2^2cs^2\omega^2 + 672v_2^2cs^2 + 198cs^2\omega - 34v_2^2cs^2\omega^3 - \omega^3 + 144v_2^4 + 10v_2^2\omega^3 - 78cs^2\omega^2 - 1008v_2^2cs^2\omega + 8\omega^2 + 6cs^2\omega^3 - 18\omega - 98v_2^2\omega^2$$

$$C_7 = 12 - 36cs^4\omega - 252v_2^2 - 756v_2^4\omega - cs^4\omega^3 + 24cs^4 + 310v_2^4\omega^2 - 29v_2^4\omega^3 + 14cs^4\omega^2 - 36cs^2 + 378v_2^2\omega + 252v_2^2cs^2\omega^2 + 432v_2^2cs^2 + 54cs^2\omega - 18v_2^2cs^2\omega^3 - \omega^3 + 504v_2^4 + 14v_2^2\omega^3 - 22cs^2\omega^2 - 648v_2^2cs^2\omega + 8\omega^2 + 2cs^2\omega^3 - 18\omega - 154v_2^2\omega^2$$

2.2 MRT

2.2.1 Definitions

Collision operator \mathbf{C} :

$$\mathbf{C}(f) = \mathbf{M}^{-1} \mathbf{S} \left(\mu^{(eq)} - \mathbf{M} f \right),$$

where

$$\mathbf{S} = \text{diag}(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9),$$

$$\omega_1, \omega_2, \dots, \omega_9 \in (0, 2).$$

2.2.2 Conservation of mass: ρ



attached text file: output_d2q9_nse_mrt1_symbolic_pde_00.txt

$$\begin{aligned} & \frac{\partial \rho}{\partial t} + \frac{v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + v_1^2 + 3cs^2) \frac{v_1 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + 3v_1^2 + cs^2) \frac{\rho \delta_l^3}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\ & \frac{\rho cs^2 \delta_l^3}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{\rho cs^2 \delta_l^3}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + v_2^2 + 3cs^2) \frac{v_2 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + 3v_2^2 + cs^2) \frac{\rho \delta_l^3}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & (-cs^4 \omega_5 - 6v_1^2 + 2cs^4 - 3v_1^4 \omega_5 - 12v_1^2 cs^2 \omega_5 - 2cs^2 + cs^2 \omega_5 + 24v_1^2 cs^2 + 3v_1^2 \omega_5 + 6v_1^4) \frac{\delta_l^4}{24\delta_t \omega_5} \frac{\partial^4 \rho}{\partial x_1^4} + \\ & (-4 + 10v_1^2 + 6cs^2 - 3cs^2 \omega_5 + 2\omega_5 - 5v_1^2 \omega_5) \frac{v_1 \rho \delta_l^4}{12\delta_t \omega_5} \frac{\partial^4 v_1}{\partial x_1^4} + (3\omega_7 cs^2 + v_1^2 \omega_7 - \omega_7 - 3cs^2 \omega_5 + \omega_5 - v_1^2 \omega_5) \frac{v_2 v_1 \delta_l^4}{4\delta_t \omega_7 \omega_5} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} \\ & + (\omega_7 cs^2 + 3v_1^2 \omega_7 - \omega_7 - cs^2 \omega_5 + \omega_5 - 3v_1^2 \omega_5) \frac{v_2 \rho \delta_l^4}{4\delta_t \omega_7 \omega_5} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + (v_1^2 \omega_7 \omega_4 \omega_5 - 3\omega_4 cs^2 \omega_5 - \omega_7 \omega_4 \omega_5 - 6\omega_7 cs^2 \omega_5 + \\ & 3\omega_7 \omega_4 cs^2 - 3v_1^2 \omega_4 \omega_5 - \omega_7 \omega_4 + v_1^2 \omega_7 \omega_4 + 3\omega_7 \omega_4 cs^2 \omega_5 + 3\omega_4 \omega_5) \frac{v_1 \rho \delta_l^4}{12\delta_t \omega_7 \omega_4 \omega_5} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + (-2 + \omega_4) \frac{cs^4 \delta_l^4}{6\delta_t \omega_4} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\ & (-\omega_7 + \omega_4) \frac{v_1 \rho cs^2 \delta_l^4}{2\delta_t \omega_7 \omega_4} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + (\omega_4 - \omega_8) \frac{v_2 \rho cs^2 \delta_l^4}{2\delta_t \omega_4 \omega_8} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + (\omega_6 - 3\omega_6 cs^2 + v_2^2 \omega_8 + 3cs^2 \omega_8 - \omega_8 - \omega_6 v_2^2) \frac{v_2 v_1 \delta_l^4}{4\omega_6 \delta_t \omega_8} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} \\ & + (v_2^2 \omega_4 \omega_8 + 3\omega_4 cs^2 \omega_8 - 3\omega_6 v_2^2 \omega_4 + \omega_6 v_2^2 \omega_4 \omega_8 + 3\omega_6 \omega_4 cs^2 \omega_8 - \omega_4 \omega_8 - 6\omega_6 cs^2 \omega_8 - \omega_6 \omega_4 \omega_8 + 3\omega_6 \omega_4 - \\ & 3\omega_6 \omega_4 cs^2) \frac{v_2 \rho \delta_l^4}{12\omega_6 \delta_t \omega_4 \omega_8} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + (\omega_6 - \omega_6 cs^2 + 3v_2^2 \omega_8 + cs^2 \omega_8 - \omega_8 - 3\omega_6 v_2^2) \frac{v_1 \rho \delta_l^4}{4\omega_6 \delta_t \omega_8} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\ & (-6v_2^2 - 3\omega_6 v_2^4 + 2cs^4 + \omega_6 cs^2 - 2cs^2 - \omega_6 cs^4 + 24v_2^2 cs^2 + 6v_2^4 + 3\omega_6 v_2^2 - 12\omega_6 v_2^2 cs^2) \frac{\delta_l^4}{24\omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\ & (-4 + 10v_2^2 + 2\omega_6 - 3\omega_6 cs^2 + 6cs^2 - 5\omega_6 v_2^2) \frac{v_2 \rho \delta_l^4}{12\omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0. \end{aligned}$$

2.2.3 Conservation of momentum: ρv_1



attached text file: output_d2q9_nse_mrt1_symbolic_pde_01.txt

$$\begin{aligned} & v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (v_1^2 + cs^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2v_1 \rho \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{v_2 \rho \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{v_1 \rho \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\ & (-2 - 2\omega_5 cs^2 + 6v_1^2 + 4cs^2 + \omega_5 - 3v_1^2 \omega_5) \frac{\delta_l^2}{\delta_t \omega_5} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + (2 - \omega_5) \frac{3v_1 \rho \delta_l^2}{\delta_t \omega_5} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + (-2 + \omega_4) \frac{cs^2 \delta_l^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\ & (-2 + \omega_4) \frac{cs^2 \delta_l^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (-2 - 3\omega_5 cs^2 + 2v_1^2 + 6cs^2 + \omega_5 - v_1^2 \omega_5) \frac{v_1 \delta_l^2}{2\delta_t \omega_5} \frac{\partial^2 \rho}{\partial x_1^2} + \\ & (-2 - \omega_5 cs^2 + 6v_1^2 + 2cs^2 + \omega_5 - 3v_1^2 \omega_5) \frac{\rho \delta_l^2}{2\delta_t \omega_5} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega_4) \frac{\rho cs^2 \delta_l^2}{2\delta_t \omega_4} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_4) \frac{\rho cs^2 \delta_l^2}{2\delta_t \omega_4} \frac{\partial^2 v_1}{\partial x_2^2} + \\ & C_1 \frac{\delta_l^3}{12\delta_t \omega_5^2} \frac{\partial^3 \rho}{\partial x_1^3} + (-24 - 36\omega_5 cs^2 + 60v_1^2 + 36cs^2 + 11v_1^2 \omega_5^2 + 24\omega_5 + 5\omega_5^2 cs^2 - 4\omega_5^2 - 60v_1^2 \omega_5) \frac{v_1 \rho \delta_l^3}{6\delta_t \omega_5^2} \frac{\partial^3 v_1}{\partial x_1^3} + \\ & C_2 \frac{v_2 v_1 \delta_l^3}{\delta_t \omega_7 \omega_4 \omega_5^2} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + C_3 \frac{v_2 \rho \delta_l^3}{\delta_t \omega_7 \omega_4 \omega_5^2} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} + C_4 \frac{v_1 \rho \delta_l^3}{12\delta_t \omega_7 \omega_4 \omega_5^2} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + (-12 + 12\omega_4 - \omega_4^2) \frac{cs^4 \delta_l^3}{6\delta_t \omega_4^2} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + \\ & (-12\omega_7 \omega_5 - 12\omega_4^2 \omega_5 + 12\omega_7 \omega_4 \omega_5 + 12\omega_4^2 - \omega_7 \omega_4^2 \omega_5 - 12\omega_7 \omega_4 + 12\omega_4 \omega_5) \frac{v_1 \rho cs^2 \delta_l^3}{6\delta_t \omega_7 \omega_4 \omega_5} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + \\ & (2\omega_4 - \omega_4^2 + \omega_4 \omega_8 - 2\omega_8) \frac{v_2 \rho cs^2 \delta_l^3}{\delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_5 \frac{v_2 v_1 \delta_l^3}{12\omega_6 \delta_t \omega_4 \omega_8} \frac{\partial^3 \rho}{\partial x_2^3} + C_6 \frac{v_2 \rho \delta_l^3}{6\delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_1}{\partial x_2^3} + C_7 \frac{v_1 \rho \delta_l^3}{12\omega_6 \delta_t \omega_4 \omega_8} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & C_8 \frac{v_1 \delta_l^4}{12\delta_t \omega_5^3} \frac{\partial^4 \rho}{\partial x_1^4} + C_9 \frac{\rho \delta_l^4}{12\delta_t \omega_5^3} \frac{\partial^4 v_1}{\partial x_1^4} + C_{10} \frac{v_2 \delta_l^4}{4\delta_t \omega_7^2 \omega_4^2 \omega_5^3} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{11} \frac{v_2 v_1 \rho \delta_l^4}{4\delta_t \omega_7^2 \omega_4^2 \omega_5^3} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + C_{12} \frac{\rho \delta_l^4}{12\delta_t \omega_7^2 \omega_4^2 \omega_5^3} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \end{aligned}$$

$$\begin{aligned}
& C_{13} \frac{v_1 \delta_l^4}{12 \delta_t \omega_7^2 \omega_4^3 \omega_8 \omega_5^3 \omega_9} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_{14} \frac{\rho \delta_l^4}{12 \delta_t \omega_7^2 \omega_4^3 \omega_8 \omega_5^3 \omega_9} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{15} \frac{v_2 v_1 \rho \delta_l^4}{2 \omega_6 \delta_t \omega_7^2 \omega_4^3 \omega_8 \omega_5^3 \omega_9} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + \\
& C_{16} \frac{v_2 \delta_l^4}{12 \omega_6^2 \delta_t \omega_7 \omega_4^3 \omega_8^2 \omega_5 \omega_9} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^2} + C_{17} \frac{v_2 v_1 \rho \delta_l^4}{12 \omega_6^2 \delta_t \omega_7^2 \omega_4^3 \omega_8^2 \omega_5^2 \omega_9} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^2} + C_{18} \frac{\rho \delta_l^4}{12 \omega_6^2 \delta_t \omega_7 \omega_4^3 \omega_8^2 \omega_5 \omega_9} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^2} + C_{19} \frac{v_1 \delta_l^4}{24 \omega_6^2 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 \rho}{\partial x_2^4} \\
& + C_{20} \frac{\rho \delta_l^4}{24 \delta_t \omega_4^3 \omega_8^2} \frac{\partial^4 v_1}{\partial x_2^4} + C_{21} \frac{v_2 v_1 \rho \delta_l^4}{12 \omega_6^2 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$C_1 = -144v_1^2\omega_5cs^2 + \omega_5^2cs^4 + 7v_1^4\omega_5^2 + 12\omega_5cs^2 - 36v_1^2 - 12cs^2 - 36v_1^4\omega_5 + 144v_1^2cs^2 - 7v_1^2\omega_5^2 - \omega_5^2cs^2 + 12cs^4 - 12\omega_5cs^4 + 36v_1^2\omega_5 + 36v_1^4 + 24v_1^2\omega_5^2cs^2$$

$$C_2 = -v_1^2\omega_7\omega_4\omega_5 - \omega_7\omega_5 + v_1^2\omega_7\omega_5 - 3\omega_7\omega_4\omega_5cs^2 + \omega_7\omega_4\omega_5 + 3\omega_4\omega_5^2cs^2 - v_1^2\omega_5^2 - v_1^2\omega_4\omega_5 - 3\omega_4\omega_5cs^2 - \omega_7\omega_4 + v_1^2\omega_7\omega_4 + \omega_4\omega_5 + 3\omega_7\omega_4cs^2 - 3\omega_5^2cs^2 - \omega_4\omega_5^2 + \omega_5^2 + 3\omega_7\omega_5cs^2 + v_1^2\omega_4\omega_5^2$$

$$C_3 = -3v_1^2\omega_7\omega_4\omega_5 - \omega_7\omega_5 + 3v_1^2\omega_7\omega_5 - \omega_7\omega_4\omega_5cs^2 + \omega_7\omega_4\omega_5 + \omega_4\omega_5^2cs^2 - 3v_1^2\omega_5^2 - 3v_1^2\omega_4\omega_5 - \omega_4\omega_5cs^2 - \omega_7\omega_4 + 3v_1^2\omega_7\omega_4 + \omega_4\omega_5 + \omega_7\omega_4cs^2 - \omega_5^2cs^2 - \omega_4\omega_5^2 + \omega_5^2 + \omega_7\omega_5cs^2 + 3v_1^2\omega_4\omega_5^2$$

$$C_4 = -6\omega_7\omega_4\omega_5^2 - 11\omega_7\omega_4\omega_5^2cs^2 + 12v_1^2\omega_5^2 - 24\omega_7\omega_5^2cs^2 + 36\omega_7\omega_4cs^2 - 12\omega_5^2\omega_5^2 - 24\omega_7\omega_4\omega_5cs^2 - 12\omega_4\omega_5cs^2 + 12\omega_4\omega_5 - 12\omega_4\omega_5^2cs^2 - 12v_1^2\omega_4\omega_5 + 6v_1^2\omega_7\omega_4\omega_5^2 + 6\omega_7\omega_4\omega_5 - 3v_1^2\omega_7\omega_4\omega_5^2 + 12\omega_4\omega_5^2cs^2 + 12\omega_4\omega_5^2 + 12v_1^2\omega_7\omega_4^2 + 42\omega_7\omega_4\omega_5^2cs^2 - 6v_1^2\omega_7\omega_4\omega_5 - 12\omega_7\omega_4^2 - 18\omega_7\omega_4\omega_5cs^2 + 3\omega_7\omega_4\omega_5^2 - 12v_1^2\omega_4\omega_5^2$$

$$C_5 = 12\omega_6 - 6v_2^2\omega_4\omega_8 + 18\omega_6\omega_4cs^2 + 3\omega_6\omega_4\omega_8cs^2 + 36\omega_8cs^2 + 6\omega_6v_2^2\omega_4 + \omega_6v_2^2\omega_4\omega_8 + 12v_2^2\omega_8 + 6\omega_4\omega_8 - 12\omega_8 - 18\omega_4\omega_8cs^2 - \omega_6\omega_4\omega_8 - 12\omega_6v_2^2 - 6\omega_6\omega_4 - 36\omega_6cs^2$$

$$C_6 = -3\omega_4^2\omega_8cs^2 - 6\omega_4cs^2 + 3v_2^2\omega_4\omega_8 - 6v_2^2\omega_4 + 3v_2^2\omega_4^2 - 12\omega_8cs^2 + 6\omega_4 - 3\omega_4^2 + \omega_4^2\omega_8 - 3\omega_4\omega_8 - v_2^2\omega_4^2\omega_8 + 15\omega_4\omega_8cs^2 + 3\omega_4^2cs^2$$

$$C_7 = 12\omega_6 - 18v_2^2\omega_4\omega_8 + 6\omega_6\omega_4cs^2 + \omega_6\omega_4\omega_8cs^2 + 12\omega_8cs^2 + 18\omega_6v_2^2\omega_4 + 3\omega_6v_2^2\omega_4\omega_8 + 36v_2^2\omega_8 + 6\omega_4\omega_8 - 12\omega_8 - 6\omega_4\omega_8cs^2 - \omega_6\omega_4\omega_8 - 36\omega_6v_2^2 - 6\omega_6\omega_4 - 12\omega_6cs^2$$

$$C_8 = 12 - 9v_1^4\omega_5^3 - 1008v_1^2\omega_5cs^2 + 82\omega_5^2cs^4 + 90v_1^4\omega_5^2 + 198\omega_5cs^2 - 156v_1^2 - 132cs^2 - 5\omega_5^3cs^4 - 216v_1^4\omega_5 + 672v_1^2cs^2 - 98v_1^2\omega_5^2 + 6\omega_5^3cs^2 + 10v_1^2\omega_5^3 - 34v_1^2\omega_5^3cs^2 - 18\omega_5 - \omega_5^3 - 78\omega_5^2cs^2 + 8\omega_5^2 + 144cs^4 - 216\omega_5cs^4 + 234v_1^2\omega_5 + 144v_1^4 + 404v_1^4\omega_5^2cs^2$$

$$C_9 = 12 - 29v_1^4\omega_5^3 - 648v_1^2\omega_5cs^2 + 14\omega_5^2cs^4 + 310v_1^4\omega_5^2 + 54\omega_5cs^2 - 252v_1^2 - 36cs^2 - \omega_5^3cs^4 - 756v_1^4\omega_5 + 432v_1^2cs^2 - 154v_1^2\omega_5^2 + 2\omega_5^3cs^2 + 14v_1^2\omega_5^3 - 18v_1^2\omega_5^3cs^2 - 18\omega_5 - \omega_5^3 - 22\omega_5^2cs^2 + 8\omega_5^2 + 24cs^4 - 36\omega_5cs^4 + 378v_1^2\omega_5 + 504v_1^4 + 252v_1^2\omega_5^2cs^2$$

$$\begin{aligned}
C_{10} = & 4\omega_7^2\omega_5^2cs^4 - 8\omega_7\omega_5^2cs^2 - 48v_1^2\omega_7\omega_4\omega_5^2cs^2 + 4v_1^2\omega_5^2\omega_5^2 + 32v_1^4\omega_7\omega_4\omega_5^2 - 4\omega_7\omega_4\omega_5cs^4 - 36v_1^2\omega_7\omega_5^3cs^2 + 36v_1^2\omega_7\omega_4\omega_5^2 - 8v_1^4\omega_7\omega_5^3 + \\
& 4\omega_4^2\omega_5^3cs^4 + 4\omega_4\omega_5^3cs^2 - 13v_1^4\omega_7\omega_4\omega_5^3 - 20v_1^4\omega_7\omega_4\omega_5^2 - 4v_1^2\omega_5^2\omega_5^2 - 24v_1^2\omega_7\omega_4^2 - 72v_1^2\omega_7\omega_4\omega_5cs^2 + 8\omega_7^2\omega_4^2cs^4 + 51v_1^2\omega_7^2\omega_4^2\omega_5^2cs^2 + \\
& 8\omega_7^2\omega_4\omega_5^2cs^2 + 4\omega_7\omega_5^3cs^2 + 20v_1^4\omega_7^2\omega_4\omega_5 + 84v_1^2\omega_7\omega_4\omega_5^2cs^2 + 4\omega_7\omega_4\omega_5^3cs^2 + 8\omega_7\omega_4\omega_5^3cs^4 - 24v_1^2\omega_4\omega_5^3cs^2 - 13v_1^2\omega_7^2\omega_4^2\omega_5^2 - 4v_1^4\omega_7^2\omega_5^2 - \\
& 20v_1^2\omega_7\omega_4\omega_5^3 + 4\omega_7^2\omega_4^2\omega_5^2cs^4 + 12\omega_7^2\omega_5^2cs^2 + 72v_1^2\omega_7\omega_4\omega_5cs^2 + 8v_1^2\omega_7\omega_5^3 - 20v_1^4\omega_7\omega_4\omega_5 + 4v_1^4\omega_4\omega_5^3 + 16v_1^2\omega_7\omega_4\omega_5^2 - 4\omega_4^2\omega_5^2cs^4 + \\
& 4\omega_7^2\omega_4\omega_5cs^4 + 24v_1^4\omega_7^2\omega_5^2 - 51v_1^2\omega_7\omega_4\omega_5^3cs^2 - 8\omega_7\omega_4\omega_5^3cs^2 + 96v_1^2\omega_7^2\omega_4^2cs^2 - 84v_1^2\omega_7^2\omega_4\omega_5^2cs^2 - 4\omega_7^2\omega_4^2\omega_5^2cs^2 - 8\omega_7^2\omega_4\omega_5^2cs^4 - 4\omega_7\omega_5^3cs^4 - \\
& 32v_1^2\omega_7\omega_4^2\omega_5^2 - 4v_1^4\omega_4\omega_5^3 + 24v_1^2\omega_4^2\omega_5^3cs^2 - 4\omega_7\omega_4^2\omega_5^3cs^4 - 36v_1^4\omega_7^2\omega_4\omega_5 - 8v_1^2\omega_7^2\omega_5^2 - 4\omega_7^2\omega_4\omega_5cs^2 - 144v_1^2\omega_7^2\omega_4\omega_5cs^2 + 4\omega_4^2\omega_5^2cs^2 + \\
& 36v_1^2\omega_7^2\omega_5^2cs^2 + 13v_1^2\omega_7\omega_4\omega_5^3 - 12\omega_7^2\omega_4\omega_5cs^4 + 20v_1^2\omega_7^2\omega_4\omega_5^2cs^2 + 8\omega_7\omega_4\omega_5^3cs^4 - 20v_1^2\omega_7^2\omega_4\omega_5 + 4v_1^2\omega_4\omega_5^3 - 4\omega_7^2\omega_5^2cs^2 + \\
& 13v_1^4\omega_7^2\omega_4\omega_5^2 + 20v_1^4\omega_7\omega_4\omega_5^3 + 120v_1^2\omega_7\omega_4^2\omega_5^2cs^2 - 4\omega_4\omega_5^3cs^4 + 20v_1^2\omega_7\omega_4^2\omega_5 + 8v_1^4\omega_7^2\omega_5^2 - 8\omega_7^2\omega_4^2cs^2 + 4\omega_7\omega_4^2\omega_5cs^2 - 16v_1^4\omega_7\omega_4\omega_5^2 - 4\omega_4^2\omega_5^3cs^2
\end{aligned}$$

$$\begin{aligned}
C_{11} = & -8\omega_4^2\omega_5^3 - 32\omega_7\omega_5^2 + 16\omega_7\omega_4\omega_5^2 + 56\omega_7\omega_4\omega_5^2cs^2 - 16v_1^2\omega_4^2\omega_5^2 + 8\omega_4^2\omega_5^2 - 120v_1^2\omega_7^2\omega_4\omega_5 - 16\omega_4\omega_5^3cs^2 + 16v_1^2\omega_5^2\omega_5^3 - 28\omega_7\omega_4\omega_5^3 + \\
& 80v_1^2\omega_7^2\omega_4^2 - 17\omega_7^2\omega_5^2 - 44\omega_7^2\omega_4\omega_5^2cs^2 - 20\omega_7\omega_5^3cs^2 - 25\omega_7\omega_4^2\omega_5^3cs^2 + 48\omega_7^2\omega_4^2\omega_5 + 43v_1^2\omega_7^2\omega_4^2\omega_5^2 + 12\omega_7\omega_5^3 + 68v_1^2\omega_7\omega_4\omega_5^3 - \\
& 72\omega_7^2\omega_4\omega_5cs^2 - 28v_1^2\omega_7\omega_5^3 - 48v_1^2\omega_7\omega_4\omega_5^2 + 24\omega_7\omega_4^2\omega_5 + 44\omega_7\omega_4\omega_5^3cs^2 + 25\omega_7^2\omega_4^2\omega_5^2cs^2 - 12\omega_7^2\omega_5^2 + 104v_1^2\omega_7\omega_4\omega_5^2 + 28v_1^2\omega_7^2\omega_5^2 + \\
& 32\omega_7^2\omega_4\omega_5cs^2 - 16\omega_4^2\omega_5^3cs^2 - 24\omega_7^2\omega_4\omega_5 - 43v_1^2\omega_7\omega_4\omega_5^3 - 68v_1^2\omega_7^2\omega_4\omega_5^2 + 64v_1^2\omega_7^2\omega_4\omega_5 - 16v_1^2\omega_4\omega_5^3 + 20\omega_7^2\omega_5^2cs^2 + 28\omega_7^2\omega_4\omega_5^2 - \\
& 16\omega_7\omega_4\omega_5^2cs^2 + 17\omega_7\omega_4^2\omega_5^3 + 8\omega_4\omega_5^3 - 64v_1^2\omega_7\omega_4^2\omega_5 + 48\omega_7^2\omega_4^2cs^2 - 32\omega_7\omega_4^2\omega_5cs^2 - 40\omega_7\omega_4^2\omega_5^2 + 16\omega_4^2\omega_5^3cs^2
\end{aligned}$$

$$\begin{aligned}
C_{12} = & -19v_1^2\omega_7^2\omega_4^3\omega_5^2 + 13\omega_7^2\omega_4^3\omega_5^3cs^4 - 12v_1^4\omega_4^3\omega_5^2 - 12\omega_7^2\omega_4^3\omega_5cs^4 - 72v_1^2\omega_7^2\omega_4^3 - 306v_1^2\omega_7^2\omega_4^3\omega_5cs^2 - 12\omega_7\omega_4^2\omega_5^2cs^2 + 12v_1^4\omega_7^2\omega_4\omega_5^3 + \\
& 6\omega_7^2\omega_4\omega_5^3cs^2 - 81v_1^2\omega_7^2\omega_4^2\omega_5^3cs^2 - 24v_1^4\omega_7\omega_4^2\omega_5^2 + 12v_1^4\omega_4^3\omega_5^3 + 6\omega_7\omega_4^3\omega_5^3cs^4 + 12v_1^2\omega_4^3\omega_5^3cs^2 - 4v_1^2\omega_7^2\omega_4^3\omega_5^3 + 48v_1^4\omega_7\omega_4^2\omega_5^3 + 12v_1^2\omega_7^2\omega_4^2\omega_5^3 - \\
& 36v_1^4\omega_7\omega_4^2\omega_5 - 21v_1^2\omega_7\omega_4^2\omega_5^3cs^2 - \omega_7^2\omega_4^2\omega_5^3cs^2 + 12v_1^2\omega_4^3\omega_5^2 + 162v_1^2\omega_7^2\omega_4^2\omega_5^2cs^2 + 60v_1^4\omega_7\omega_4^2\omega_5^2 - 12v_1^2\omega_7\omega_4\omega_5^3cs^2 + 18\omega_7\omega_4^2\omega_5^3cs^2 + \\
& 12\omega_7\omega_4\omega_5^3cs^4 + 72v_1^4\omega_7^2\omega_4 + 12\omega_7^2\omega_5^3cs^4 - 12v_1^2\omega_7^2\omega_4\omega_5^2 + 24v_1^2\omega_7\omega_4\omega_5^3 + 6\omega_7^2\omega_4^2\omega_5^2cs^4 + 12\omega_7^2\omega_4^3cs^4 + 12\omega_7^2\omega_4^2\omega_5cs^2 - 27v_1^4\omega_7\omega_4^2\omega_5^3 + \\
& 54v_1^2\omega_7\omega_4^2\omega_5^2cs^2 - 12v_1^2\omega_4^3\omega_5^2 - 6\omega_7\omega_4^2\omega_5^2cs^4 - 12v_1^4\omega_4^3\omega_5^3 + 90v_1^2\omega_7^2\omega_4^3\omega_5 + 18v_1^2\omega_7^2\omega_4^3\omega_5^2 - 12v_1^2\omega_4^3\omega_5^2cs^2 + 30v_1^2\omega_7\omega_4^2\omega_5^3cs^2 - 12\omega_7\omega_4\omega_5^3cs^2 + \\
& 19v_1^4\omega_7^2\omega_4^3\omega_5^2 - 36v_1^2\omega_7\omega_4^2\omega_5cs^2 - 48v_1^2\omega_7^2\omega_4\omega_5^2cs^2 - 6\omega_7^2\omega_4^2\omega_5^2cs^2 - 12v_1^2\omega_7^2\omega_4\omega_5^3 + 24v_1^2\omega_7\omega_4^2\omega_5^2 - 12v_1^2\omega_4^3\omega_5^3cs^2 - 18\omega_7\omega_4^2\omega_5^3cs^4 + \\
& 6\omega_7\omega_4^2\omega_5^2cs^2 + 4v_1^4\omega_7^2\omega_4^3\omega_5^3 - 108v_1^2\omega_7^2\omega_4^2\omega_5cs^2 + 12v_1^2\omega_7^2\omega_4^3\omega_5^2cs^2 - \omega_7^2\omega_4^3\omega_5^3cs^4 - 12\omega_7^2\omega_4^3cs^2 - 48v_1^2\omega_7\omega_4^2\omega_5^3 - 12\omega_7^2\omega_4^2\omega_5cs^4 + 36v_1^2\omega_7\omega_4^3\omega_5 + \\
& 12\omega_7\omega_4^2\omega_5^2cs^4 - 24\omega_7^2\omega_4\omega_5^3cs^4 - 60v_1^2\omega_7\omega_4^3\omega_5^2 + 12v_1^4\omega_7^2\omega_4^2\omega_5^2 + 252v_1^2\omega_7^2\omega_4^2\omega_5^3cs^2 - 5\omega_7^2\omega_4^2\omega_5^3cs^2 + 102v_1^2\omega_7^2\omega_4\omega_5^3cs^2 - 24v_1^4\omega_7\omega_4\omega_5^3 - \\
& 12v_1^2\omega_7\omega_4^2\omega_5^2cs^2 + 12\omega_7^2\omega_4^3\omega_5cs^2 + 27v_1^2\omega_7\omega_4^3\omega_5^3 + \omega_7^2\omega_4^3\omega_5^2cs^4 - 48v_1^2\omega_7^2\omega_5^3cs^2 + 60v_1^2\omega_7^2\omega_4^3\omega_5^2cs^2 - 6\omega_7\omega_4^3\omega_5^3cs^2 - 90v_1^4\omega_7^2\omega_4^3\omega_5 - 18v_1^4\omega_7^2\omega_4^2\omega_5^3
\end{aligned}$$

$$\begin{aligned}
C_{13} = & 36\omega_7^2\omega_4^3\omega_5^3cs^4 + 12v_2^2\omega_7\omega_4^3\omega_8\omega_5^3 + 18\omega_7^2\omega_4^3\omega_5^2cs^4\omega_9 - 36v_2^2\omega_4^3\omega_8\omega_5^2cs^2\omega_9 - 2v_1^2\omega_7^2\omega_4^3\omega_8\omega_5^2cs^2\omega_9 + 12\omega_7\omega_4^3\omega_8\omega_5^3cs^2 + \\
& 36v_2^2v_1^2\omega_7\omega_4^2\omega_8\omega_5^3\omega_9 + 36v_2^2\omega_7^2\omega_4^3\omega_8cs^2\omega_9 - 12v_1^2\omega_4^3\omega_8\omega_5^2cs^2\omega_9 + 6v_2^2v_1^2\omega_7^2\omega_4^3\omega_8\omega_5^2\omega_9 + 72v_2^2\omega_7^2\omega_4\omega_8\omega_5^2cs^2\omega_9 + 12v_1^2\omega_7^2\omega_4^2\omega_5^3cs^2 - \\
& 12v_2^2v_1^2\omega_4^2\omega_8\omega_5^3\omega_9 + 18v_1^2\omega_7\omega_4^2\omega_8\omega_5^3cs^2\omega_9 + 12\omega_7\omega_4\omega_8\omega_5^3cs^2\omega_9 - 36v_2^2\omega_7^2\omega_4^2\omega_5^3cs^2\omega_9 + 36v_2^2\omega_7^2\omega_4^2\omega_5^3cs^2 + 2\omega_7^2\omega_4^3\omega_8\omega_5^2cs^2\omega_9 - 6\omega_7^2\omega_4^3\omega_8\omega_5^3cs^2 - \\
& 36v_2^2\omega_7\omega_4^2\omega_8\omega_5^3cs^2\omega_9 + 24v_2^2v_1^2\omega_7\omega_4^2\omega_8\omega_5^2\omega_9 - 12v_1^2\omega_7\omega_4\omega_8\omega_5^3cs^2\omega_9 - 12v_2^2\omega_7\omega_4^2\omega_8\omega_5^2 - 18\omega_7\omega_4^2\omega_8\omega_5^3cs^2\omega_9 + 24v_2^2\omega_7\omega_4\omega_8\omega_5^3\omega_9 + \\
& 12\omega_7\omega_4^2\omega_8\omega_5cs^4\omega_9 + 12v_2^2\omega_7^2\omega_4^2\omega_8\omega_5^3 - 72v_2^2\omega_7^2\omega_4^2\omega_8\omega_5^2cs^2\omega_9 + 12\omega_4^3\omega_8\omega_5^2cs^4\omega_9 - 12v_2^2v_1^2\omega_4^3\omega_8\omega_5^2\omega_9 + 6v_2^2\omega_7^2\omega_4^3\omega_8\omega_5^3cs^4\omega_9 -
\end{aligned}$$

$$\begin{aligned}
& 12w_4^2w_8w_5^2cs^2w_9 + 6v_2^2v_1^2w_7^2w_4^2w_5^2w_9 - 12v_2^2w_7^2w_4^2w_8w_5w_9 + 6v_1^2w_7^2w_4^2w_5^2cs^2w_9 - 54v_2^2w_7^2w_4^2w_8w_5cs^2w_9 + 12w_4^2w_8w_5^2cs^4w_9 - 24v_2^2w_7^2w_4^2w_8w_5^2w_9 \\
& - 18w_7^2w_4^2w_8w_5^2cs^2w_9 - 12v_2^2w_7^2w_4^2w_8w_5w_9 - 12w_7^2w_4^2w_8w_5^2cs^2 - 12v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 + 12w_7^2w_4^2w_8w_5^2cs^4w_9 + 6v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 + 12w_7^2w_4^2w_8w_5^2cs^2 + \\
& + 18v_2^2w_7^2w_4^2w_8w_5^2cs^2w_9 + 12v_2^2w_7^2w_4^2w_8w_5^2cs^4w_9 - 36v_2^2w_7^2w_4^2w_8w_5^2cs^2w_9 - 36w_7^2w_4^2w_8w_5^2cs^4w_9 + 30w_7^2w_4^2w_8w_5^2cs^4w_9 - 12v_1^2w_7^2w_4^2w_8w_5^2cs^2w_9 + \\
& + 18v_2^2w_7^2w_4^2w_8w_5^2w_9 + 150w_7^2w_4^2w_8w_5^2cs^4w_9 - 36v_2^2w_7^2w_4^2w_8w_5^2cs^2w_9 + 36w_7^2w_4^2w_8w_5^2cs^4w_9 + 12v_2^2w_7^2w_4^2w_8w_5^2w_9 - 6v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 + 12v_2^2w_7^2w_4^2w_8w_5^2w_9 + \\
& + 12v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 + 6w_7^2w_4^2w_8w_5^2cs^2 + 12v_2^2w_7^2w_4^2w_8w_5^2w_9 - 36v_2^2w_7^2w_4^2w_8w_5^2cs^2w_9 - 12v_2^2w_7^2w_4^2w_8w_5^2w_9 - 36w_7^2w_4^2w_8w_5^2cs^4w_9 - 84w_7^2w_4^2w_8w_5^2cs^4w_9 + \\
& + 12v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 + 18v_1^2w_7^2w_4^2w_8w_5^2cs^2w_9 - 24v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 - 12v_2^2w_7^2w_4^2w_8w_5^2cs^2w_9 + 6w_7^2w_4^2w_8w_5^2cs^2 - 12v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 - \\
& + 12w_7^2w_4^2w_8w_5^2cs^2 - 12w_7^2w_4^2w_8w_5^2cs^2w_9 - 12v_2^2w_7^2w_4^2w_8w_5^2cs^2w_9 + 12v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 - 6w_7^2w_4^2w_8w_5^2cs^4w_9 - 24v_2^2w_7^2w_4^2w_8w_5^2w_9 + \\
& + 12w_7^2w_4^2w_8w_5^2cs^2w_9 - 12v_2^2v_1^2w_7^2w_4^2w_8w_5^2cs^2 - 24v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 - 42w_7^2w_4^2w_8w_5^2cs^4w_9 + 12w_7^2w_4^2w_8w_5^2cs^2w_9 - 12v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 - \\
& + 18v_2^2w_7^2w_4^2w_8w_5^2cs^2w_9 - 48w_7^2w_4^2w_8w_5^2cs^4w_9 + 72v_2^2w_7^2w_4^2w_8w_5^2cs^2w_9 + 12v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 + 36v_2^2w_7^2w_4^2w_8w_5^2cs^2 - w_7^2w_4^2w_8w_5^2cs^2w_9 - \\
& + 18v_2^2w_7^2w_4^2w_8w_5^2cs^2 + 36w_7^2w_4^2w_8w_5^2cs^4 - 12w_4^2w_8w_5^2cs^4w_9 - 6v_2^2w_7^2w_4^2w_8w_5^2w_9 + 12v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 - 96w_7^2w_4^2w_8w_5^2cs^4w_9 - 36w_7^2w_4^2w_8w_5^2cs^4 + \\
& + 36v_2^2w_7^2w_4^2w_8w_5^2cs^2 + 12v_2^2w_7^2w_4^2w_8w_5^2cs^2w_9 + v_1^2w_7^2w_4^2w_8w_5^2cs^2w_9 - 36v_2^2w_7^2w_4^2w_8w_5^2cs^2w_9 - 6w_7^2w_4^2w_8w_5^2cs^2w_9 - 12v_2^2w_7^2w_4^2w_8w_5^2w_9 - 18w_7^2w_4^2w_8w_5^2cs^4 - \\
& + 6v_2^2w_7^2w_4^2w_8w_5^2cs^2 + 36w_7^2w_4^2w_8w_5^2cs^4 + 6v_2^2w_7^2w_4^2w_8w_5^2w_9 - 18w_7^2w_4^2w_8w_5^2cs^4 - 12v_2^2w_7^2w_4^2w_8w_5^2w_9 - 36v_2^2w_7^2w_4^2w_8w_5^2w_9 - 6v_2^2w_7^2w_4^2w_8w_5^2w_9 + \\
& + 6v_7^2w_4^2w_8w_5^2cs^2 + 12w_7^2w_4^2w_8w_5^2cs^4w_9 - 12v_1^2w_7^2w_4^2w_8w_5^2w_9 - 6v_2^2w_7^2w_4^2w_8w_5^2cs^2w_9 - 36w_7^2w_4^2w_8w_5^2cs^4 - 12v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 + \\
& + 12w_7^2w_4^2w_8w_5^2cs^2w_9 + 12w_7^2w_4^2w_8w_5^2cs^2w_9 + 18v_2^2w_7^2w_4^2w_8w_5^2cs^2 + 18v_1^2w_7^2w_4^2w_8w_5^2cs^2w_9 - 72v_2^2w_7^2w_4^2w_8w_5^2cs^2w_9 + 12v_2^2w_4^2w_8w_5^2w_9 + \\
& + 36v_2^2w_7^2w_4^2w_8w_5^2cs^2w_9 + 12v_1^2w_7^2w_4^2w_8w_5^2cs^2 - 12w_7^2w_4^2w_8w_5^2cs^2 - 12v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 - 12v_1^2w_7^2w_4^2w_8w_5^2cs^2 + 24v_2^2w_7^2w_4^2w_8w_5^2w_9 - 6v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 + \\
& + 180w_7^2w_4^2w_8w_5^2cs^4w_9 + 12v_2^2w_7^2w_4^2w_8w_5^2w_9 - 12v_1^2w_7^2w_4^2w_8w_5^2cs^2w_9 + 12w_4^2w_8w_5^2cs^2w_9 + 24v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 - 6v_2^2w_7^2w_4^2w_8w_5^2cs^2 - \\
& + 12w_7^2w_4^2w_8w_5^2cs^4w_9 + 12v_2^2w_7^2w_4^2w_8w_5^2cs^2 + 36v_2^2w_7^2w_4^2w_8w_5^2cs^2w_9 + 12v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 - 6v_2^2w_7^2w_4^2w_8w_5^2w_9 + 12w_7^2w_4^2w_8w_5^2cs^2w_9 - \\
& + 88w_7^2w_4^2w_8w_5^2cs^4w_9 + 18v_2^2w_7^2w_4^2w_8w_5^2cs^2w_9 + 18w_7^2w_4^2w_8w_5^2cs^2w_9 - 18v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 + 6w_7^2w_4^2w_8w_5^2cs^4w_9 + 108v_2^2w_7^2w_4^2w_8w_5^2cs^2w_9 + \\
& + 12v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 - 12v_2^2v_1^2w_7^2w_4^2w_8w_5^2w_9 - 18w_7^2w_4^2w_8w_5^2cs^2w_9 - 12v_2^2w_7^2w_4^2w_8w_5^2cs^2w_9 - 36v_2^2w_7^2w_4^2w_8w_5^2cs^2 + 12v_2^2w_4^2w_8w_5^2w_9 - 12w_7^2w_4^2w_8w_5^2w_9
\end{aligned}$$

[illegible]

$$96\omega_2^2v_2^2\omega_4\omega_8 - 24\omega_4\omega_8^2cs^2 + 24\omega_6v_2^4\omega_4^2\omega_8 - 24\omega_6^2\omega_8cs^4 + 48\omega_6^2\omega_4\omega_8cs^4 + 12\omega_6^2\omega_4^2\omega_8cs^2 + 72\omega_6v_2^2\omega_4^2\omega_8cs^2 - 12\omega_4^2\omega_8^2cs^4 - 14\omega_6\omega_4^2\omega_8^2cs^2 - 24\omega_6^2\omega_4cs^4 - 48\omega_6\omega_4\omega_8^2cs^4 - 72v_2^2\omega_4\omega_8^2 + 36\omega_6v_2^2\omega_4^2\omega_8^2 + 288v_2^2\omega_4\omega_8^2cs^2 + 96\omega_6^2v_2^2\omega_4\omega_8 - 24\omega_6v_2^2\omega_4^2\omega_8 + 12\omega_6^2v_2^4\omega_4^2 - 12\omega_6^2\omega_4^2cs^2 + 72\omega_6^2v_2^2\omega_4^2cs^2 + 24\omega_6^2\omega_4cs^2 + 48\omega_6\omega_4\omega_8^2cs^2 + 48\omega_6v_2^2\omega_4\omega_8 + 3\omega_6^2v_2^2\omega_4^2\omega_8^2 + 216\omega_6v_2^2\omega_8^2cs^2 - 144v_2^2\omega_4^2\omega_8^2cs^2 - 144\omega_6^2v_2^2\omega_4cs^2 + 14\omega_6\omega_4^2\omega_8^2cs^4 - 48\omega_6^2v_2^4\omega_8 - 48\omega_6v_2^2\omega_8^2 - 96\omega_6v_2^4\omega_4\omega_8^2 + 36v_2^2\omega_4^2\omega_8^2 + 24\omega_6^2v_2^2\omega_4 + 12\omega_6^2\omega_4^2cs^4 - 30\omega_6^2v_2^4\omega_4\omega_8 - 48\omega_6v_2^2\omega_4\omega_8 - 12\omega_6^2v_2^2\omega_4^2 + 432\omega_6^2v_2^2\omega_4\omega_8cs^2 - 3\omega_6^2v_2^4\omega_4\omega_8^2 + 150\omega_6v_2^2\omega_4^2\omega_8^2cs^2 + \omega_6^2\omega_4^2\omega_8^2cs^2 - 24\omega_6\omega_8^2cs^2 - 12\omega_6^2\omega_4^2\omega_8cs^4 - 12\omega_6^2v_2^2\omega_4^2\omega_8^2cs^2 - 216\omega_6^2v_2^2\omega_8cs^2 + 12\omega_4^2\omega_8^2cs^2 + 24\omega_4\omega_8^2cs^4 + 96\omega_6v_2^2\omega_4\omega_8^2 + 24\omega_6^2\omega_8cs^2 - 36v_2^4\omega_4^2\omega_8^2 + 30\omega_6^2v_2^2\omega_4\omega_8 - 48\omega_6^2\omega_4\omega_8cs^2 - 144\omega_6v_2^2\omega_4\omega_8cs^2$$

$$C_{20} = 24\omega_4^2\omega_8cs^2 + 24v_2^4\omega_4\omega_8^2 + 24\omega_4\omega_8cs^4 + 12v_2^2\omega_4^2cs^2 + 6v_2^2\omega_4^3\omega_8^2cs^2 + 12\omega_4\omega_8^2cs^2 + 48v_2^2\omega_4\omega_8 - 96v_2^2\omega_8^2cs^2 + 24\omega_4^2\omega_8^2cs^4 - 24v_2^2\omega_4^2cs^2 - 24v_2^2\omega_4\omega_8^2 + \omega_4^3\omega_8^2cs^2 + 156v_2^2\omega_4\omega_8^2cs^2 + 24v_2^2\omega_4^2 + 24\omega_8^2cs^4 - 48v_2^2\omega_4\omega_8 + 6\omega_4^3\omega_8cs^4 + 48v_2^2\omega_4^2\omega_8cs^2 - 12v_2^2\omega_4^3 + 72v_2^4\omega_4^3\omega_8 - 3\omega_4^3\omega_8^2cs^4 - 72v_2^2\omega_4^2\omega_8^2cs^2 + 18v_2^2\omega_4^3\omega_8 + 3v_2^4\omega_4^3\omega_8^2 + 24v_2^2\omega_4^2\omega_8^2 - 24v_2^2\omega_4\omega_8cs^2 - 6\omega_4^3\omega_8cs^2 - 72v_2^2\omega_4^2\omega_8 - 24\omega_4\omega_8cs^2 - 12v_2^2\omega_4^3\omega_8cs^2 + 12v_2^4\omega_4^3 - 24\omega_4^3\omega_8cs^4 - 18v_2^4\omega_4^3\omega_8 - 3v_2^2\omega_4^3\omega_8^2 - 8\omega_4^2\omega_8^2cs^2 - 48\omega_4\omega_8^2cs^4 - 24v_2^4\omega_4^2\omega_8^2 - 24v_2^4\omega_4^2$$

$$C_{21} = -72\omega_2^2\omega_4\omega_8 - 84\omega_6^2v_2^2\omega_8 + 24\omega_4^2\omega_8^2 + 61\omega_6v_2^2\omega_4^2\omega_8^2 - 36\omega_6\omega_8^2 + 168\omega_6^2v_2^2\omega_4\omega_8 + 72\omega_4\omega_8^2cs^2 - 12\omega_6\omega_4^2\omega_8 - 33\omega_6^2\omega_4\omega_8cs^2 + 24\omega_6^2\omega_4 - 12\omega_6^2\omega_4^2 + 39\omega_6\omega_4^2\omega_8^2cs^2 + 120v_2^2\omega_4\omega_8^2 - 25\omega_6\omega_4^2\omega_8^2 + 36\omega_6v_2^2\omega_4^2\omega_8 - 24\omega_6\omega_4\omega_8cs^2 + 24\omega_6^2\omega_4^2cs^2 - 48\omega_6^2\omega_4cs^2 + 36\omega_6^2\omega_8 - 120\omega_6\omega_4\omega_8^2cs^2 - 72\omega_6v_2^2\omega_4\omega_8 - 5\omega_6^2v_2^2\omega_4^2\omega_8^2 + 2\omega_6^2\omega_4^2\omega_8^2 + 12\omega_6\omega_4^2\omega_8cs^2 + 84\omega_6v_2^2\omega_8^2 - 60v_2^2\omega_4^2\omega_8^2 - 48\omega_6^2v_2^2\omega_4 + 72\omega_6\omega_4\omega_8^2 + 24\omega_6^2v_2^2\omega_4^2 + 24\omega_6\omega_4\omega_8 - 3\omega_6^2\omega_4^2\omega_8^2cs^2 + 60\omega_6\omega_8^2cs^2 + 21\omega_6^2\omega_4\omega_8 - 48\omega_4\omega_8^2 - 36\omega_4^2\omega_8^2cs^2 - 168\omega_6v_2^2\omega_4\omega_8^2 - 60\omega_6^2\omega_8cs^2 - 51\omega_6^2v_2^2\omega_4\omega_8 + 120\omega_6^2\omega_4\omega_8cs^2$$

2.2.4 Conservation of momentum: ρv_2



attached text file: output_d2q9_nse_mrt1_symbolic_pde_02.txt

$$\begin{aligned} & v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{v_2 v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{v_2 \rho \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_1 \rho \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_1} + (v_2^2 + cs^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{2v_2 \rho \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_4) \frac{cs^2 \delta_l^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\ & (-2 + \omega_4) \frac{cs^2 \delta_l^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (-2 + 6v_2^2 + \omega_6 - 2\omega_6 cs^2 + 4cs^2 - 3\omega_6 v_2^2) \frac{\delta_l^2}{\omega_6 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (2 - \omega_6) \frac{3v_2 \rho \delta_l^2}{\omega_6 \delta_t} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + \\ & (-2 + \omega_4) \frac{\rho cs^2 \delta_l^2}{2\delta_t \omega_4} \frac{\partial^2 v_2}{\partial x_1^2} + (-2 + \omega_4) \frac{\rho cs^2 \delta_l^2}{2\delta_t \omega_4} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + (-2 + 2v_2^2 + \omega_6 - 3\omega_6 cs^2 + 6cs^2 - \omega_6 v_2^2) \frac{v_2 \delta_l^2}{2\omega_6 \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + \\ & (-2 + 6v_2^2 + \omega_6 - \omega_6 cs^2 + 2cs^2 - 3\omega_6 v_2^2) \frac{\rho \delta_l^2}{2\omega_6 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + C_1 \frac{v_2 v_1 \delta_l^3}{12\delta_t \omega_7 \omega_4 \omega_5} \frac{\partial^3 \rho}{\partial x_1^3} + C_2 \frac{v_2 \rho \delta_l^3}{12\delta_t \omega_7 \omega_4 \omega_5} \frac{\partial^3 v_1}{\partial x_1^3} + C_3 \frac{v_1 \rho \delta_l^3}{6\delta_t \omega_7 \omega_4} \frac{\partial^3 v_2}{\partial x_1^3} + \\ & (-12 + 12\omega_4 - \omega_4^2) \frac{cs^4 \delta_l^3}{6\delta_t \omega_4^2} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + (-2\omega_7 + 2\omega_4 - \omega_4^2 + \omega_7 \omega_4) \frac{v_1 \rho cs^2 \delta_l^3}{\delta_t \omega_7 \omega_4^2} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} + \\ & (-\omega_6 \omega_4^2 \omega_8 - 12\omega_6 \omega_8 + 12\omega_4^2 - 12\omega_4 \omega_8 - 12\omega_6 \omega_4^2 + 12\omega_6 \omega_4 \omega_8 + 12\omega_6 \omega_4) \frac{v_2 \rho cs^2 \delta_l^3}{6\omega_6 \delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + C_4 \frac{v_2 v_1 \delta_l^3}{\omega_6^2 \delta_t \omega_4 \omega_8} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + \\ & C_5 \frac{v_2 \rho \delta_l^3}{12\omega_6^2 \delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_6 \frac{v_1 \rho \delta_l^3}{\omega_6^2 \delta_t \omega_4 \omega_8} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_7 \frac{\delta_l^3}{12\omega_6^2 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\ & (-24 + 60v_2^2 + 24\omega_6 + 11\omega_6^2 v_2^2 - 36\omega_6 cs^2 - 4\omega_6^2 + 36cs^2 + 5\omega_6^2 cs^2 - 60\omega_6 v_2^2) \frac{v_2 \rho \delta_l^3}{6\omega_6^2 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_8 \frac{v_2 \delta_l^4}{24\delta_t \omega_7^2 \omega_4^2 \omega_5^2} \frac{\partial^4 \rho}{\partial x_1^4} + \\ & C_9 \frac{v_2 v_1 \rho \delta_l^4}{12\delta_t \omega_7^2 \omega_4^2 \omega_5^2} \frac{\partial^4 v_1}{\partial x_1^4} + C_{10} \frac{\rho \delta_l^4}{24\delta_t \omega_7^2 \omega_4^2} \frac{\partial^4 v_2}{\partial x_1^4} + C_{11} \frac{v_1 \delta_l^4}{12\omega_6 \delta_t \omega_7^2 \omega_4^2 \omega_8 \omega_5^2 \omega_9} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{12} \frac{\rho \delta_l^4}{12\omega_6 \delta_t \omega_7^2 \omega_4^2 \omega_8 \omega_5^2 \omega_9} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + \\ & C_{13} \frac{v_2 v_1 \rho \delta_l^4}{12\omega_6^2 \delta_t \omega_7^2 \omega_4^2 \omega_8^2 \omega_5^2 \omega_9} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_{14} \frac{v_2 \delta_l^4}{12\omega_6^2 \delta_t \omega_7^2 \omega_4^2 \omega_8^2 \omega_5^2 \omega_9} \frac{\partial^4 \rho}{\partial x_2^4} + C_{15} \frac{v_2 v_1 \rho \delta_l^4}{2\omega_6^2 \delta_t \omega_7^2 \omega_4^2 \omega_8^2 \omega_5^2 \omega_9} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + \\ & C_{16} \frac{\rho \delta_l^4}{12\omega_6^2 \delta_t \omega_7^2 \omega_4^2 \omega_8^2 \omega_5^2 \omega_9} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_{17} \frac{v_1 \delta_l^4}{4\omega_6^3 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{18} \frac{\rho \delta_l^4}{12\omega_6^3 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{19} \frac{v_2 v_1 \rho \delta_l^4}{4\omega_6^3 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\ & C_{20} \frac{v_2 \delta_l^4}{12\omega_6^3 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{21} \frac{\rho \delta_l^4}{12\omega_6^3 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:

$$C_1 = v_1^2 \omega_7 \omega_4 \omega_5 + 36\omega_7 cs^2 + 18\omega_4 cs^2 \omega_5 + 12v_1^2 \omega_7 - 12\omega_7 - 18\omega_7 \omega_4 cs^2 - \omega_7 \omega_4 \omega_5 - 36cs^2 \omega_5 + 6v_1^2 \omega_4 \omega_5 + 3\omega_7 \omega_4 cs^2 \omega_5 + 6\omega_7 \omega_4 - 6v_1^2 \omega_7 \omega_4 - 6\omega_4 \omega_5 + 12\omega_5 - 12v_1^2 \omega_5$$

$$C_2 = 3v_1^2 \omega_7 \omega_4 \omega_5 + 12\omega_7 cs^2 + 6\omega_4 cs^2 \omega_5 + 36v_1^2 \omega_7 - 12\omega_7 - 6\omega_7 \omega_4 cs^2 - \omega_7 \omega_4 \omega_5 - 12cs^2 \omega_5 + 18v_1^2 \omega_4 \omega_5 + \omega_7 \omega_4 cs^2 \omega_5 + 6\omega_7 \omega_4 - 18v_1^2 \omega_7 \omega_4 - 6\omega_4 \omega_5 + 12\omega_5 - 36v_1^2 \omega_5$$

$$C_3 = -12\omega_7 cs^2 + 3v_1^2 \omega_4^2 + 3\omega_4^2 cs^2 + 15\omega_7 \omega_4 cs^2 - 6v_1^2 \omega_4 + 6\omega_4 - 3\omega_4^2 - 3\omega_7 \omega_4^2 cs^2 - 3\omega_7 \omega_4 + 3v_1^2 \omega_7 \omega_4 - 6\omega_4 cs^2 - v_1^2 \omega_7 \omega_4^2 + \omega_7 \omega_4^2$$

$$C_4 = 3\omega_4 cs^2 \omega_8 + 3\omega_6^2 \omega_4 cs^2 + v_2^2 \omega_4 \omega_8 - \omega_6^2 \omega_4 - \omega_6^2 v_2^2 - \omega_6 \omega_8 - \omega_6 v_2^2 \omega_4 - \omega_6 v_2^2 \omega_4 \omega_8 + \omega_6^2 - \omega_4 \omega_8 - 3\omega_6^2 cs^2 - 3\omega_6 \omega_4 cs^2 \omega_8 + \omega_6^2 v_2^2 \omega_4 + \omega_6 v_2^2 \omega_8 + \omega_6 \omega_4 \omega_8 + \omega_6 \omega_4 - 3\omega_6 \omega_4 cs^2 + 3\omega_6 cs^2 \omega_8$$

$$C_5 = -6\omega_6^2 \omega_4 \omega_8 - 12\omega_6 \omega_4^2 cs^2 - 12\omega_6 v_2^2 \omega_4^2 + 6\omega_6^2 v_2^2 \omega_4 \omega_8 - 12\omega_6^2 \omega_4 cs^2 + 6\omega_6 \omega_4^2 \omega_8 + 42\omega_6^2 \omega_4 cs^2 \omega_8 + 12\omega_6^2 \omega_4 - 24\omega_6^2 cs^2 \omega_8 - 12\omega_6^2 \omega_4^2 - 18\omega_6 \omega_4^2 cs^2 \omega_8 - 6\omega_6 v_2^2 \omega_4^2 \omega_8 - 12\omega_4^2 \omega_8 - 24\omega_6 \omega_4 cs^2 \omega_8 + 12\omega_6 \omega_4^2 - 12\omega_6^2 v_2^2 \omega_4 + 12v_2^2 \omega_4^2 \omega_8 + 36\omega_4^2 cs^2 \omega_8 + 12\omega_6^2 v_2^2 \omega_4^2 + 12\omega_6^2 \omega_4^2 cs^2 + 3\omega_6^2 \omega_4^2 \omega_8 - 3\omega_6^2 v_2^2 \omega_4^2 \omega_8 - 11\omega_6^2 \omega_4^2 cs^2 \omega_8$$

$$3\omega_6^3\omega_7^2\omega_4^3\omega_8\omega_5\omega_9 - 4\omega_6^3v_2^2\omega_7^2\omega_5\omega_9 - 4\omega_6^2v_2^2\omega_7^2\omega_5\omega_9 - 2\omega_6^3\omega_7\omega_4^3cs^2\omega_8\omega_9 - 2\omega_6^2\omega_7^2\omega_4^3cs^2\omega_8\omega_5 + 4\omega_6^3\omega_4^3cs^2\omega_8\omega_5\omega_9 - 6\omega_6^2\omega_7^2\omega_4^3cs^2\omega_8\omega_5\omega_9 - 2\omega_6^3v_2^2\omega_7\omega_4^3\omega_8\omega_5 - 2\omega_6^2v_2^2\omega_7^2\omega_4^3\omega_8\omega_5\omega_9 - 3\omega_6^3v_2^2\omega_7^2\omega_4^3\omega_8\omega_5\omega_9 + 5\omega_6\omega_7^2\omega_4^3\omega_8\omega_5\omega_9 + 2\omega_6v_2^2\omega_7^2\omega_4^3\omega_8\omega_5\omega_9 + 3\omega_6^2\omega_7^2\omega_4^3\omega_8\omega_5\omega_9$$

$$\begin{aligned} C_{16} = & -12v_1^2\omega_7\omega_4^3\omega_8\omega_9 - 24\omega_6^2v_1^2\omega_7\omega_4^3\omega_8\omega_9 - 36\omega_6^3v_1^2\omega_7\omega_4^3\omega_8\omega_9 + 84\omega_6^3v_2^2\omega_7\omega_4^3cs^2\omega_8\omega_9 + 18\omega_6^2v_2^2\omega_7^2\omega_4^3\omega_8\omega_9 + 36\omega_6^3v_1^2\omega_7\omega_4^3cs^2\omega_8\omega_9 - \\ & 12\omega_6^3\omega_7\omega_4^3cs^4\omega_8^2 + 24\omega_6^2v_1^2\omega_7\omega_4^3cs^2\omega_8\omega_9 - 36\omega_6v_2^2\omega_1^2\omega_7\omega_4^3\omega_8\omega_9 - 12\omega_6^3\omega_7\omega_4^3cs^2\omega_8\omega_9 - 12\omega_6\omega_7\omega_4^3cs^4\omega_8^2\omega_9 + 18\omega_6^3v_2^2\omega_7\omega_4^3cs^2\omega_8^2 + \\ & 78\omega_6^3v_2^2\omega_7\omega_4^3cs^2\omega_8\omega_9 + 12\omega_6^2v_2^2\omega_7\omega_4^3cs^4\omega_8 - 36\omega_6^3v_2^2\omega_1^2\omega_7\omega_4^3\omega_8 + 60\omega_6^2v_2^2\omega_7\omega_4^3cs^2\omega_8\omega_9 + 108\omega_6^3v_2^2\omega_1^2\omega_7\omega_4^3\omega_8\omega_9 + 12\omega_6^3v_1^2\omega_7\omega_4^3\omega_8^2 + \\ & 24\omega_6^2v_1^2\omega_7\omega_4^3cs^2\omega_8\omega_9 - 6\omega_6^2\omega_7\omega_4^3cs^4\omega_8\omega_9 - 6\omega_6^3\omega_7\omega_4^3cs^2\omega_8^2 + 12v_1^2\omega_7\omega_4^3cs^2\omega_8^2\omega_9 - 18\omega_6v_1^2\omega_7\omega_4^3cs^2\omega_8\omega_9 + 18\omega_6^2v_2^2\omega_1^2\omega_7\omega_4^3\omega_8\omega_9 - \\ & 24\omega_6^3\omega_7\omega_4^3cs^4\omega_8\omega_9 - 36\omega_6^2v_2^2\omega_1^2\omega_7\omega_4^3\omega_9 - 12\omega_6^3\omega_7cs^4\omega_8^2\omega_9 + 6\omega_6^2v_1^2\omega_7\omega_4^3cs^2\omega_8^2 + 12\omega_6^3v_1^2\omega_7\omega_4^3\omega_8 + 6\omega_6^3v_1^2\omega_4^3\omega_8^2 - 24\omega_6^2\omega_7\omega_4^3cs^2\omega_8^2\omega_9 - \\ & 12\omega_6v_1^2\omega_7\omega_4^3\omega_8\omega_9 - 108\omega_6v_2^2\omega_7\omega_4^3cs^2\omega_8\omega_9 - 6\omega_6\omega_7\omega_4^3cs^2\omega_8^2\omega_9 - 36\omega_6^3v_2^2\omega_1^2\omega_7\omega_4^3\omega_8 - 6\omega_6^2v_1^2\omega_7\omega_4^3\omega_8^2 + 12\omega_6^3\omega_4^3cs^4\omega_8^2 + 12\omega_6^2v_1^2\omega_7\omega_4^3\omega_9 + \\ & 12\omega_6^3\omega_7\omega_4^3cs^4\omega_8\omega_9 + 12\omega_6v_1^2\omega_7\omega_4^3cs^2\omega_8^2\omega_9 - 12\omega_6^3v_1^2\omega_7\omega_4^3cs^2\omega_8\omega_9 - 12\omega_6^2v_1^2\omega_7\omega_4^3cs^2\omega_8 + 18\omega_6^3\omega_7\omega_4^3cs^4\omega_8^2\omega_9 + 36\omega_6v_2^2\omega_1^2\omega_7\omega_4^3\omega_8\omega_9 + 72\omega_6^2v_2^2\omega_1^2\omega_7\omega_4^3\omega_8\omega_9 + \\ & 6\omega_6^2\omega_7\omega_4^3cs^2\omega_8^2\omega_9 - 36\omega_6^3v_2^2\omega_7\omega_4^3cs^2\omega_8 + 36\omega_6^2v_2^2\omega_7\omega_4^3cs^2\omega_8^2 + 24\omega_6^3\omega_7\omega_4^3cs^2\omega_8\omega_9 - 48\omega_6^3v_2^2\omega_7cs^2\omega_8\omega_9 - 12\omega_6^3v_1^2\omega_7\omega_4^3cs^2\omega_8\omega_9 + \\ & 36\omega_6^3v_2^2\omega_1^2\omega_7\omega_4^3\omega_8 - 12\omega_6^2v_1^2\omega_7\omega_4^3cs^2\omega_8\omega_9 - 6\omega_6^2\omega_7\omega_4^3cs^4\omega_8^2 + 12\omega_6v_1^2\omega_7\omega_4^3\omega_8\omega_9 - 12\omega_6^3v_2^2\omega_7\omega_4^3cs^2\omega_8\omega_9 - 12\omega_6^2v_2^2\omega_7\omega_4^3cs^2\omega_8 + 84\omega_6^2v_2^2\omega_7\omega_4^3cs^2\omega_8^2\omega_9 - 6\omega_6^2v_1^2\omega_4^3\omega_8^2\omega_9 - \\ & 12\omega_6^3v_1^2\omega_7\omega_4^3\omega_8 + 12\omega_6^2\omega_7\omega_4^3cs^2\omega_8 + 24\omega_6^3v_2^2\omega_7\omega_4^3cs^2\omega_8\omega_9 - 36\omega_6^2v_2^2\omega_1^2\omega_7\omega_4^3\omega_8\omega_9 + 24\omega_6^3\omega_7\omega_4^3cs^4\omega_8\omega_9 - 24\omega_6^2v_1^2\omega_7\omega_4^3cs^2\omega_8\omega_9 + \\ & 132\omega_6^3v_2^2\omega_7\omega_4^3cs^2\omega_8\omega_9 + 12\omega_6^2\omega_7\omega_4^3cs^4\omega_8 - 144\omega_6^3v_2^2\omega_7\omega_4^3cs^2\omega_8\omega_9 + 12\omega_6^2v_1^2\omega_4^3\omega_8^2 + 6\omega_6\omega_7\omega_4^3cs^4\omega_8\omega_9 - 24\omega_6^2v_1^2\omega_7\omega_4^3cs^2\omega_8\omega_9 + \\ & 18\omega_6v_1^2\omega_7\omega_4^3\omega_8\omega_9 - 12\omega_6^2\omega_4^3cs^4\omega_8^2\omega_9 + 36\omega_6^2v_2^2\omega_1^2\omega_7\omega_4^3\omega_8 - 72\omega_6^2v_2^2\omega_1^2\omega_7\omega_4^3\omega_8^2\omega_9 + 12\omega_6^2\omega_7\omega_4^3cs^4\omega_8\omega_9 - 24\omega_6^3v_1^2\omega_7\omega_4^3cs^2\omega_8\omega_9 - \\ & 12\omega_6^2v_1^2\omega_7\omega_4^3\omega_9 + 6\omega_6^2v_1^2\omega_7\omega_4^3\omega_8^2 - 12\omega_6^2v_1^2\omega_4^3cs^2\omega_8\omega_9 - 18\omega_6^2v_2^2\omega_7\omega_4^3cs^2\omega_8\omega_9 - 6\omega_6^2v_1^2\omega_7\omega_4^3cs^2\omega_8^2 - 12\omega_6^3v_1^2\omega_7\omega_4^3cs^2\omega_8^2 - 24\omega_6^2v_1^2\omega_7\omega_4^3\omega_8^2\omega_9 + \\ & 36\omega_6^3v_2^2\omega_1^2\omega_7\omega_4^3\omega_8 - 42\omega_6^2v_2^2\omega_7\omega_4^3cs^2\omega_8\omega_9 - 36\omega_6^2v_2^2\omega_7\omega_4^3cs^2\omega_8^2\omega_9 - 18\omega_6^2v_2^2\omega_7\omega_4^3cs^2\omega_8 - 36\omega_6^3v_2^2\omega_7\omega_4^3cs^2\omega_8^2 - 12\omega_6^3\omega_7\omega_4^3cs^2\omega_8^2 + \\ & 12\omega_6^2\omega_7\omega_4^3cs^2\omega_8\omega_9 - \omega_6^2\omega_7\omega_4^3cs^2\omega_8\omega_9 - 6\omega_6^2\omega_4^3cs^2\omega_8^2\omega_9 - 36\omega_6^3v_2^2\omega_1^2\omega_7\omega_4^3\omega_8\omega_9 + 72\omega_6v_2^2\omega_7\omega_4^3cs^2\omega_8\omega_9 + 12\omega_6^2v_1^2\omega_7\omega_4^3\omega_8\omega_9 + \\ & 12\omega_6^3v_1^2\omega_7\omega_4^3\omega_8\omega_9 - 18\omega_6^3v_2^2\omega_1^2\omega_4^3\omega_8^2 - 12\omega_6^2\omega_7\omega_4^3cs^2\omega_8 + 12\omega_6^2\omega_7\omega_4^3cs^2\omega_8\omega_9 + 6\omega_6^3\omega_7\omega_4^3cs^4\omega_8^2 - 36\omega_6^2v_2^2\omega_1^2\omega_4^3\omega_8^2\omega_9 + 12\omega_6^3\omega_7\omega_4^3cs^4\omega_8\omega_9 - \\ & 12\omega_6v_1^2\omega_7\omega_4^3cs^2\omega_8\omega_9 + 72\omega_6^2v_2^2\omega_1^2\omega_7\omega_4^3\omega_8\omega_9 - 4\omega_6^2\omega_7\omega_4^3cs^4\omega_8^2\omega_9 - 24\omega_6^3v_2^2\omega_7\omega_4^3cs^2\omega_8\omega_9 + 12\omega_6^2\omega_4^3cs^2\omega_8^2\omega_9 - 36\omega_6^3v_2^2\omega_1^2\omega_7\omega_4^3\omega_8\omega_9 + \\ & 6\omega_6^2\omega_7\omega_4^3cs^2\omega_8^2 + 12\omega_6^3v_1^2\omega_7\omega_4^3\omega_8 - 6\omega_6^3v_1^2\omega_4^3cs^2\omega_8^2 + 24\omega_6^3v_2^2\omega_7\omega_4^3\omega_8\omega_9 - 12\omega_6^3\omega_7\omega_4^3cs^4\omega_8 + 18\omega_6^2v_2^2\omega_4^3cs^2\omega_8^2\omega_9 - 72\omega_6^3v_2^2\omega_1^2\omega_7\omega_4^3\omega_8\omega_9 - \\ & 12\omega_6^2\omega_7\omega_4^3cs^2\omega_8\omega_9 - \omega_6^2\omega_7\omega_4^3cs^2\omega_8\omega_9 + 36\omega_6^3v_2^2\omega_7\omega_4^3cs^2\omega_8 + 36\omega_6^2v_2^2\omega_7\omega_4^3cs^2\omega_8\omega_9 - 12\omega_6^2\omega_7\omega_4^3cs^2\omega_8 + 12\omega_6^2v_1^2\omega_4^3\omega_8^2\omega_9 + \\ & 60\omega_6^3v_2^2\omega_7\omega_4^3cs^2\omega_8\omega_9 - 6\omega_6^2v_1^2\omega_7\omega_4^3\omega_8\omega_9 - 12\omega_6^3\omega_4^3cs^2\omega_8^2 + 36v_2^2\omega_1^2\omega_7\omega_4^3\omega_8\omega_9 + 6\omega_6^2\omega_4^3cs^4\omega_8\omega_9 - 12\omega_6^2v_1^2\omega_7\omega_4^3\omega_8 + 6\omega_6^2v_1^2\omega_7\omega_4^3cs^2\omega_8^2\omega_9 - \\ & 54\omega_6v_2^2\omega_7\omega_4^3\omega_8\omega_9 + 12\omega_6^3v_2^2\omega_7\omega_4^3cs^2\omega_8 + 36\omega_6^2v_2^2\omega_7\omega_4^3\omega_8^2 - 12\omega_6^2\omega_7\omega_4^3cs^4\omega_8^2\omega_9 + 12\omega_6^2v_2^2\omega_7\omega_4^3cs^2\omega_8 + 180\omega_6^3v_2^2\omega_7\omega_4^3cs^2\omega_8\omega_9 - \\ & 6\omega_6^3\omega_4^3cs^4\omega_8^2 + 6\omega_6^2v_1^2\omega_4^3cs^2\omega_8\omega_9 - 18\omega_6^2v_2^2\omega_7\omega_4^3\omega_8^2 + 12\omega_6^3v_1^2\omega_7\omega_4^3cs^2\omega_8\omega_9 - 12\omega_6^3\omega_7\omega_4^3cs^2\omega_8\omega_9 - 12\omega_6^2\omega_7\omega_4^3cs^2\omega_8\omega_9 - 12\omega_6^2\omega_7\omega_4^3cs^2\omega_8\omega_9 \end{aligned}$$

$$\begin{aligned} C_{17} = & 4\omega_6^3v_2^2\omega_4 - 48\omega_6^2v_2^2\omega_4cs^2\omega_8 + 8\omega_4^3cs^4\omega_8^2 + 12\omega_6\omega_4^2cs^2\omega_8^2 + 36\omega_6v_2^2\omega_4^2\omega_8^2 - 51\omega_6^3v_2^2\omega_4^2cs^2\omega_8 - 4\omega_6^2\omega_4^2cs^4 - 8\omega_6^3v_2^4\omega_8 + 16\omega_6^2v_2^2\omega_4\omega_8 - \\ & 20\omega_6v_2^2\omega_4^2\omega_8 - 20\omega_6^2v_2^2\omega_4\omega_8^2 + 4\omega_6^2\omega_4^2cs^4\omega_8^2 - 4\omega_6^3cs^4\omega_8 + 20\omega_6^3v_2^2\omega_4\omega_8 - 4\omega_6^2\omega_4^2cs^4\omega_8 + 72\omega_6v_2^2\omega_4cs^2\omega_8^2 - 4\omega_6^3\omega_4cs^4 + 8\omega_6^2\omega_4cs^2\omega_8^2 - \\ & 36\omega_6^3v_2^2cs^2\omega_8 - 4\omega_6^2cs^2\omega_8^2 + 36\omega_6^2v_2^2cs^2\omega_8^2 - 24\omega_6^3v_2^2\omega_4cs^2 - 36\omega_6v_2^2\omega_4^2\omega_8^2 - 8\omega_6^3\omega_4cs^2\omega_8 + 8\omega_6^2\omega_4^2cs^4\omega_8 - 16\omega_6^2v_2^4\omega_4\omega_8 + 4\omega_6\omega_4^2cs^2\omega_8 + \\ & 20\omega_6v_2^2\omega_4^2\omega_8 - 4\omega_6^2v_2^4\omega_4^2 + 20\omega_6^2v_2^2\omega_4\omega_8^2 - 4\omega_6^3\omega_4^2cs^2 - 4\omega_6^2v_2^2cs^2 - 4\omega_6^3v_2^2\omega_4 - 20\omega_6^2v_2^2\omega_4 - 84\omega_6^2v_2^2\omega_4cs^2\omega_8^2 - 8\omega_6^2v_2^2\omega_4 + 4\omega_6\omega_4cs^4\omega_8 - \\ & 144\omega_6v_2^2\omega_4^2cs^2\omega_8^2 + 4\omega_6^3cs^2\omega_8 - 13\omega_6^2v_2^2\omega_4\omega_8^2 - 4\omega_6^2\omega_4^2cs^2\omega_8^2 + 4\omega_6^3\omega_4^2cs^2\omega_8 + 13\omega_6^3v_2^2\omega_4^2\omega_8 + 4\omega_6^3\omega_4cs^2 - 4\omega_6^3v_2^4\omega_4 + 4\omega_6^3\omega_4^2cs^4 - \\ & 12\omega_6\omega_4^2cs^4\omega_8^2 + 20\omega_6v_2^4\omega_4\omega_8^2 - 24v_2^2\omega_4^2\omega_8^2 - 8\omega_4^2cs^2\omega_8^2 + 24\omega_6^3v_2^2\omega_4^2cs^2 + 8\omega_6^3v_2^2\omega_8 + 51\omega_6^2v_2^2\omega_4^2cs^2\omega_8^2 + 32\omega_6^2v_2^2\omega_4^2\omega_8 - 4\omega_6^2\omega_4^2cs^4\omega_8 + 4\omega_6^2v_2^2\omega_4^2 + \\ & 4\omega_6^2\omega_4^2cs^2 + 120\omega_6^2v_2^2\omega_4^2cs^2\omega_8 + 96v_2^2\omega_4^2cs^2\omega_8^2 + 13\omega_6^3v_2^2\omega_4^2\omega_8^2 - 13\omega_6^3v_2^2\omega_4^2\omega_8 + 4\omega_6^3v_2^2\omega_4^2 - 4\omega_6\omega_4cs^2\omega_8^2 + 8\omega_6^2v_2^4\omega_8^2 + 84\omega_6^3v_2^2\omega_4cs^2\omega_8 - \\ & 24\omega_6^2v_2^2\omega_4^2cs^2 - 8\omega_6^2\omega_4cs^4\omega_8^2 + 4\omega_6^2cs^4\omega_8^2 - 20\omega_6v_2^2\omega_4\omega_8^2 + 24v_2^2\omega_4^2\omega_8^2 - 32\omega_6^2v_2^2\omega_4\omega_8 - 8\omega_6^2\omega_4^2cs^2\omega_8 - 72\omega_6v_2^2\omega_4^2cs^2\omega_8 + 8\omega_6^3\omega_4cs^4\omega_8 \end{aligned}$$

$$\begin{aligned} C_{18} = & -12\omega_6^3v_2^2\omega_4\omega_8^2 - 12\omega_4^3cs^2\omega_8^2 - 12\omega_6\omega_4^3cs^4\omega_8^2 - 90\omega_6v_2^4\omega_4^3\omega_8^2 + 60\omega_6^2v_2^2\omega_4^3cs^2\omega_8^2 + 12\omega_6\omega_4^3cs^2\omega_8^2 + 30\omega_6^3v_2^2\omega_4^3cs^2\omega_8 - 6\omega_6^3\omega_4^3cs^2\omega_8 + \\ & 6\omega_6^2\omega_4^3cs^4\omega_8^2 + 6\omega_6^3\omega_4cs^2\omega_8^2 - 24\omega_6^3v_2^2\omega_4\omega_8 - 18\omega_6^2\omega_4^3cs^4\omega_8 - 48\omega_6^3v_2^2cs^2\omega_8^2 + 36\omega_6v_2^2\omega_3\omega_8 - \omega_6^2\omega_3cs^2\omega_8^2 - 306\omega_6v_2^2\omega_4^3cs^2\omega_8^2 + 13\omega_6^3\omega_4^3cs^4\omega_8^2 + \\ & 12\omega_6^3v_2^4\omega_4\omega_8^2 - 12\omega_6^2v_2^4\omega_4^3 + 90\omega_6v_2^2\omega_4^3\omega_8^2 - 36\omega_6v_2^2\omega_4^3cs^2\omega_8 + 6\omega_6^2\omega_4^3cs^2\omega_8 + 12\omega_6^3cs^4\omega_8^2 - 12\omega_6^3\omega_4cs^2\omega_8 + 12\omega_6^2\omega_4^3cs^4\omega_8 - 12\omega_6^2v_2^2\omega_4^3 - \\ & 81\omega_6^3v_2^2\omega_3cs^2\omega_8^2 + 12\omega_6^3v_2^2\omega_4^3 + 24\omega_6^3v_2^2\omega_4\omega_8 - 48\omega_6^3v_2^2\omega_4cs^2\omega_8^2 + 252v_2^2\omega_3^2cs^2\omega_8^2 + 54\omega_6^2v_2^2\omega_4^3cs^2\omega_8 - 36\omega_6v_2^4\omega_4^3\omega_8 + 6\omega_6^3\omega_4^3cs^4\omega_8 - \\ & 12\omega_6^2v_2^4\omega_4^3cs^2 - 24\omega_6^3\omega_4cs^4\omega_8^2 - 108\omega_6v_2^2\omega_4^2cs^2\omega_8^2 - 27\omega_6^3v_2^4\omega_4^3\omega_8 - 12\omega_6^2v_2^2\omega_4^3\omega_8^2 - 6\omega_6^2\omega_3^2cs^2\omega_8^2 + 18\omega_6^3\omega_4^3cs^2\omega_8 - 48\omega_6^3v_2^2\omega_4^3\omega_8 + \\ & 19\omega_6^3v_2^4\omega_4^3\omega_8^2 + \omega_6^2\omega_3^2cs^4\omega_8^2 - 60\omega_6^2v_2^2\omega_4^3\omega_8 - 18\omega_6^3v_2^4\omega_4^3\omega_8 + 12\omega_6\omega_3^2cs^2\omega_8^2 - 21\omega_6^3v_2^2\omega_4^3cs^2\omega_8 + 12\omega_4^3cs^4\omega_8^2 + 102\omega_6^3v_2^2\omega_4cs^2\omega_8^2 + 72v_2^4\omega_4^3\omega_8^2 - \\ & 12\omega_6\omega_4^3cs^4\omega_8^2 - 12\omega_6^3v_2^2\omega_4^2cs^2 + 162\omega_6^2v_2^2\omega_4^2cs^2\omega_8^2 - 24\omega_6^2v_2^4\omega_4^3\omega_8 - 4\omega_6^3v_2^2\omega_4^3\omega_8 - 12\omega_6^2v_2^2\omega_4^2cs^2\omega_8 + 27\omega_6^3v_2^2\omega_4^3\omega_8 + 12\omega_6^2v_2^4\omega_4^3\omega_8^2 + \\ & 48\omega_6^3v_2^2\omega_4^3\omega_8 - 19\omega_6^2v_2^2\omega_4^3\omega_8^2 - 12\omega_6^3v_2^2\omega_4^3 + 12\omega_6^2v_2^2\omega_4^3cs^2\omega_8^2 - 12\omega_6^3v_2^2\omega_4cs^2\omega_8 - 5\omega_6^3\omega_4^3cs^2\omega_8^2 + 60\omega_6^2v_2^4\omega_4^3\omega_8 + 18\omega_6^3v_2^2\omega_4^3\omega_8^2 - 72v_2^2\omega_4^3\omega_8^2 + \\ & 12\omega_6^3v_2^2\omega_4^3cs^2 + 12\omega_6^2v_2^2\omega_4^3 - 6\omega_6^2\omega_4^3cs^4\omega_8 - \omega_6^2\omega_3^2cs^4\omega_8^2 + 24\omega_6^2v_2^2\omega_4^3\omega_8 + 4\omega_6^3v_2^4\omega_4^3\omega_8^2 - 12\omega_6^2\omega_4^3cs^2\omega_8 + 12\omega_6^3\omega_4cs^4\omega_8 + 12\omega_6^3v_2^4\omega_4^3 \end{aligned}$$

$$\begin{aligned} C_{19} = & -16\omega_6^3v_2^2\omega_4 + 16\omega_6^2\omega_4\omega_8 - 32\omega_4^2\omega_8^2 - 8\omega_6^3\omega_4^2 - 72\omega_6\omega_4^2cs^2\omega_8^2 - 120\omega_6v_2^2\omega_4^2\omega_8^2 - 48\omega_6^2v_2^2\omega_4\omega_8 + 24\omega_6\omega_4^2\omega_8 - 16\omega_6^2\omega_4cs^2\omega_8 - \\ & 44\omega_6^2\omega_4cs^2\omega_8^2 + 8\omega_6^2\omega_4^2 + 20\omega_6^2cs^2\omega_8^2 + 17\omega_6^3\omega_4^2\omega_8 + 44\omega_6^3\omega_4cs^2\omega_8 + 48\omega_6\omega_4^2\omega_8^2 - 32\omega_6\omega_4^2cs^2\omega_8 - 64\omega_6v_2^2\omega_4^2\omega_8 + 8\omega_6^3\omega_4 - 68\omega_6^2v_2^2\omega_4\omega_8^2 + \\ & 16\omega_6^3\omega_4^2cs^2 + 16\omega_6^3v_2^2\omega_4^2 + 68\omega_6^3v_2^2\omega_4\omega_8 + 28\omega_6^2\omega_4\omega_8^2 + 28\omega_6^2v_2^2\omega_8^2 - 20\omega_6^3cs^2\omega_8 + 43\omega_6^2v_2^2\omega_4^2\omega_8^2 + 25\omega_6^2\omega_4^2cs^2\omega_8^2 - 17\omega_6^2\omega_4^2\omega_8^2 - \\ & 25\omega_6^3\omega_4^2cs^2\omega_8 - 43\omega_6^3v_2^2\omega_4\omega_8 - 16\omega_6^3\omega_4cs^2 - 28\omega_6^3\omega_4\omega_8 + 80v_2^2\omega_4^2\omega_8^2 + 48\omega_4^2cs^2\omega_8^2 - 24\omega_6\omega_4\omega_8^2 - 28\omega_6^3v_2^2\omega_8 - 16\omega_6^2v_2^2\omega_4^2 - 16\omega_6^2\omega_4^2cs^2 + \\ & 12\omega_6^3\omega_8 + 32\omega_6\omega_4cs^2\omega_8^2 - 40\omega_6^2\omega_4^2\omega_8 - 12\omega_6^2\omega_8^2 + 64\omega_6v_2^2\omega_4\omega_8^2 + 104\omega_6^2v_2^2\omega_4\omega_8 + 56\omega_6^2\omega_4^2cs^2\omega_8 \end{aligned}$$

$$C_{20} = 12 - 156v_2^2 - 216\omega_6v_2^4 - 18\omega_6 + 404\omega_6^2v_2^2cs^2 - 5\omega_6^3cs^4 - 98\omega_6^2v_2^2 + 198\omega_6cs^2 + 144cs^4 + 10\omega_6^3v_2^2 + 82\omega_6^2cs^4 + 8\omega_6^2 - 216\omega_6cs^4 - 132cs^2 - 34\omega_6^3v_2^2cs^2 - \omega_6^3 - 78\omega_6^2cs^2 + 672v_2^2cs^2 - 9\omega_6^3v_2^2 + 144v_2^4 + 234\omega_6v_2^2 - 1008\omega_6v_2^2cs^2 + 90\omega_6^2v_2^4 + 6\omega_6^3cs^2$$

$$C_{21} = 12 - 252v_2^2 - 756\omega_6v_2^4 - 18\omega_6 + 252\omega_6^2v_2^2cs^2 - \omega_6^3cs^4 - 154\omega_6^2v_2^2 + 54\omega_6cs^2 + 24cs^4 + 14\omega_6^3v_2^2 + 14\omega_6^2cs^4 + 8\omega_6^2 - 36\omega_6cs^4 - 36cs^2 - 18\omega_6^3v_2^2cs^2 - \omega_6^3 - 22\omega_6^2cs^2 + 432v_2^2cs^2 - 29\omega_6^3v_2^4 + 504v_2^4 + 378\omega_6v_2^2 - 648\omega_6v_2^2cs^2 + 310\omega_6^2v_2^4 + 2\omega_6^3cs^2$$

2.3 MRT2

2.3.1 Definitions

Collision operator \mathbf{C} :

$$\mathbf{C}(\mathbf{f}) = \mathbf{M}_2^{-1} \mathbf{S} \left(\boldsymbol{\mu}_2^{(eq)} - \mathbf{M}_2 \mathbf{f} \right),$$

where

$$\mathbf{S} = \text{diag}(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9),$$

$\omega_1, \omega_2, \dots, \omega_9 \in (0, 2)$.

Matrix \mathbf{M}_2 corresponds to the transformation matrix to the raw moment basis defined by

$$\boldsymbol{\mu}_2 = \begin{pmatrix} m_{(0,0)} \\ m_{(1,0)} \\ m_{(0,1)} \\ m_{(1,1)} \\ m_{(2,0)} + m_{(0,2)} \\ m_{(2,0)} - m_{(0,2)} \\ m_{(2,1)} \\ m_{(1,2)} \\ m_{(2,2)} \end{pmatrix},$$

and is given by

$$\mathbf{M}_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

The equilibrium moments $\boldsymbol{\mu}_2^{(eq)}$ are defined by

$$\boldsymbol{\mu}_2^{(eq)} = \mathbf{M}_2 \mathbf{M}^{-1} \boldsymbol{\mu}^{(eq)},$$

i.e.,

$$\boldsymbol{\mu}_2^{(eq)} = \begin{pmatrix} \rho \\ \rho v_1 \\ \rho v_2 \\ \rho v_1 v_2 \\ \rho(v_2^2 + v_1^2 + 2c_s^2) \\ \rho(v_1^2 - v_2^2) \\ \rho v_2(v_1^2 + c_s^2) \\ \rho v_1(v_2^2 + c_s^2) \\ \rho(v_1^2 v_2^2 + c_s^2 v_2^2 + c_s^2 v_1^2 + c_s^4) \end{pmatrix}.$$

2.3.2 Conservation of mass: ρ



attached text file: output_d2q9_nse_mrt2_symbolic_pde_00.txt

$$\begin{aligned}
& \frac{\partial \rho}{\partial t} + \frac{\delta_I v_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\delta_I \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_I v_2}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\delta_I \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + v_1^2 + 3cs^2) \frac{\delta_I^3 v_1}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + 3v_1^2 + cs^2) \frac{\delta_I^3 \rho}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\
& \frac{\delta_I^3 \rho cs^2}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{\delta_I^3 \rho cs^2}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + v_2^2 + 3cs^2) \frac{\delta_I^3 v_2}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + 3v_2^2 + cs^2) \frac{\delta_I^3 \rho}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& (-3v_1^4 \omega_5 + 2cs^4 - 6v_1^2 - cs^4 \omega_5 - 12cs^2 v_1^2 \omega_5 + 3v_1^2 \omega_5 + 6v_1^4 + 24cs^2 v_1^2 + cs^2 \omega_5 - 2cs^2) \frac{\delta_I^4}{24\delta_t \omega_5} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& (-4 + 10v_1^2 - 5v_1^2 \omega_5 - 3cs^2 \omega_5 + 6cs^2 + 2\omega_5) \frac{\delta_I^4 \rho v_1}{12\delta_t \omega_5} \frac{\partial^4 v_1}{\partial x_1^4} + (3\omega_7 cs^2 + \omega_7 v_1^2 - \omega_7 - v_1^2 \omega_5 - 3cs^2 \omega_5 + \omega_5) \frac{\delta_I^4 v_1 v_2}{4\delta_t \omega_7 \omega_5} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} \\
& + (\omega_7 cs^2 + 3\omega_7 v_1^2 - \omega_7 - 3v_1^2 \omega_5 - cs^2 \omega_5 + \omega_5) \frac{\delta_I^4 \rho v_2}{4\delta_t \omega_7 \omega_5} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + (3\omega_7 \omega_4 cs^2 - \omega_7 \omega_4 - 3\omega_4 cs^2 \omega_5 - 3\omega_4 v_1^2 \omega_5 + \\
& \omega_7 \omega_4 v_1^2 - 6\omega_7 cs^2 \omega_5 + \omega_7 \omega_4 v_1^2 \omega_5 + 3\omega_4 \omega_5 - \omega_7 \omega_4 \omega_5 + 3\omega_7 \omega_4 cs^2 \omega_5) \frac{\delta_I^4 \rho v_1}{12\delta_t \omega_7 \omega_4 \omega_5} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + (-2 + \omega_4) \frac{\delta_I^4 cs^4}{6\delta_t \omega_4} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\
& (-\omega_7 + \omega_4) \frac{\delta_I^4 \rho cs^2 v_1}{2\delta_t \omega_7 \omega_4} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + (\omega_4 - \omega_8) \frac{\delta_I^4 \rho cs^2 v_2}{2\delta_t \omega_4 \omega_8} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + (\omega_6 - \omega_6 v_2^2 + 3cs^2 \omega_8 - \omega_8 - 3\omega_6 cs^2 + \omega_8 v_2^2) \frac{\delta_I^4 v_1 v_2}{4\omega_6 \delta_t \omega_8} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^2} \\
& + (\omega_4 \omega_8 v_2^2 - 3\omega_6 \omega_4 v_2^2 - \omega_6 \omega_4 \omega_8 - 6\omega_6 cs^2 \omega_8 + 3\omega_4 cs^2 \omega_8 - 3\omega_6 \omega_4 cs^2 + 3\omega_6 \omega_4 cs^2 \omega_8 + \omega_6 \omega_4 \omega_8 v_2^2 + 3\omega_6 \omega_4 - \\
& \omega_4 \omega_8) \frac{\delta_I^4 \rho v_2}{12\omega_6 \delta_t \omega_4 \omega_8} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^2} + (\omega_6 - 3\omega_6 v_2^2 + cs^2 \omega_8 - \omega_8 - \omega_6 cs^2 + 3\omega_8 v_2^2) \frac{\delta_I^4 \rho v_1}{4\omega_6 \delta_t \omega_8} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^2} + \\
& (2cs^4 - \omega_6 cs^4 + 3\omega_6 v_2^2 - 6v_2^2 - 12\omega_6 cs^2 v_2^2 + 24cs^2 v_2^2 - 3\omega_6 v_2^4 + 6v_2^4 - 2cs^2 + \omega_6 cs^2) \frac{\delta_I^4}{24\omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\
& (-4 + 2\omega_6 - 5\omega_6 v_2^2 + 10v_2^2 + 6cs^2 - 3\omega_6 cs^2) \frac{\delta_I^4 \rho v_2}{12\omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0.
\end{aligned}$$

2.3.3 Conservation of momentum: ρv_1



attached text file: output_d2q9_nse_mrt2_symbolic_pde_01.txt

$$\begin{aligned}
& v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (v_1^2 + cs^2) \frac{\delta_I}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2\delta_I \rho v_1}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_I v_1 v_2}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\delta_I \rho v_2}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{\delta_I \rho v_1}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\
& (-2 - 2cs^2 \omega_5 + 6v_1^2 - 3v_1^2 \omega_5 + 4cs^2 + \omega_5) \frac{\delta_I^2}{\delta_t \omega_5} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + (2 - \omega_5) \frac{3\delta_I^2 \rho v_1}{\delta_t \omega_5} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + (-2 + \omega_4) \frac{cs^2 \delta_I^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\
& (-2 + \omega_4) \frac{cs^2 \delta_I^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (-2 - 3cs^2 \omega_5 + 2v_1^2 - v_1^2 \omega_5 + 6cs^2 + \omega_5) \frac{\delta_I^2 v_1}{2\delta_t \omega_5} \frac{\partial^2 \rho}{\partial x_1^2} + \\
& (-2 - cs^2 \omega_5 + 6v_1^2 - 3v_1^2 \omega_5 + 2cs^2 + \omega_5) \frac{\delta_I^2 \rho}{2\delta_t \omega_5} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega_4) \frac{cs^2 \delta_I^2 \rho}{2\delta_t \omega_4} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_4) \frac{cs^2 \delta_I^2 \rho}{2\delta_t \omega_4} \frac{\partial^2 v_1}{\partial x_2^2} + \\
& C_1 \frac{\delta_I^3}{12\delta_t \omega_5^2} \frac{\partial^3 \rho}{\partial x_1^3} + (-24 - 36cs^2 \omega_5 + 5cs^2 \omega_5^2 + 60v_1^2 - 4\omega_5^2 - 60v_1^2 \omega_5 + 36cs^2 + 11v_1^2 \omega_5^2 + 24\omega_5) \frac{\delta_I^3 \rho v_1}{6\delta_t \omega_5^2} \frac{\partial^3 v_1}{\partial x_1^3} + \\
& C_2 \frac{\delta_I^3 v_1 v_2}{\delta_t \omega_7 \omega_4 \omega_5^2} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + C_3 \frac{\delta_I^3 \rho v_2}{\delta_t \omega_7 \omega_4 \omega_5^2} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} + C_4 \frac{\delta_I^3 \rho v_1}{12\delta_t \omega_7 \omega_4 \omega_5^2} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + (-12 + 12\omega_4 - \omega_4^2) \frac{cs^4 \delta_I^3}{6\delta_t \omega_4^2} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + \\
& (-12\omega_7 \omega_4 - \omega_7 \omega_4^2 \omega_5 - 12\omega_4^2 \omega_5 + 12\omega_4^4 + 12\omega_4 \omega_5 + 12\omega_7 \omega_4 \omega_5 - 12\omega_7 \omega_5) \frac{cs^2 \delta_I^3 \rho v_1}{6\delta_t \omega_7 \omega_4^2 \omega_5} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + \\
& (2\omega_4 - \omega_4^2 + \omega_4 \omega_8 - 2\omega_8) \frac{cs^2 \delta_I^3 \rho v_2}{\delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_5 \frac{\delta_I^3 v_1 v_2}{12\omega_6 \delta_t \omega_4 \omega_8} \frac{\partial^3 \rho}{\partial x_2^3} + C_6 \frac{\delta_I^3 \rho v_2}{6\delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_1}{\partial x_2^3} + C_7 \frac{\delta_I^3 \rho v_1}{12\omega_6 \delta_t \omega_4 \omega_8} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& C_8 \frac{\delta_I^4 v_1}{12\delta_t \omega_5^3} \frac{\partial^4 \rho}{\partial x_1^4} + C_9 \frac{\delta_I^4 \rho}{12\delta_t \omega_5^3} \frac{\partial^4 v_1}{\partial x_1^4} + C_{10} \frac{\delta_I^4 v_2}{4\delta_t \omega_7^2 \omega_4^2 \omega_5^3} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{11} \frac{\delta_I^4 \rho v_1 v_2}{4\delta_t \omega_7^2 \omega_4^2 \omega_5^3} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + C_{12} \frac{\delta_I^4 \rho}{12\delta_t \omega_7^2 \omega_4^2 \omega_5^3} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& C_{13} \frac{\delta_I^4 v_1}{12\omega_9 \delta_t \omega_7^2 \omega_4^3 \omega_8 \omega_5^3} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_{14} \frac{\delta_I^4 \rho}{12\omega_9 \delta_t \omega_7^2 \omega_4^3 \omega_8 \omega_5^3} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{15} \frac{\delta_I^4 \rho v_1 v_2}{2\omega_9 \omega_6 \delta_t \omega_7^2 \omega_4^3 \omega_8 \omega_5^3} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + \\
& C_{16} \frac{\delta_I^4 v_2}{12\omega_9 \omega_6^2 \delta_t \omega_7 \omega_4^3 \omega_8 \omega_5^3} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{17} \frac{\delta_I^4 \rho v_1 v_2}{12\omega_9 \omega_6^2 \delta_t \omega_7 \omega_4^3 \omega_8 \omega_5^3} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{18} \frac{\delta_I^4 \rho}{12\omega_9 \omega_6^2 \delta_t \omega_7 \omega_4^3 \omega_8 \omega_5^3} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + C_{19} \frac{\delta_I^4 v_1}{24\omega_6^2 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 \rho}{\partial x_1^2} \\
& + C_{20} \frac{\delta_I^4 \rho}{24\delta_t \omega_4^3 \omega_8^2} \frac{\partial^4 v_1}{\partial x_2^4} + C_{21} \frac{\delta_I^4 \rho v_1 v_2}{12\omega_6^2 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= 24cs^2 v_1^2 \omega_5^2 - 36v_1^4 \omega_5 + 12cs^2 \omega_5 - cs^2 \omega_5^2 + 7v_1^4 \omega_5^2 + 144cs^2 v_1^2 - 36v_1^2 + 12cs^4 - 144cs^2 v_1^2 \omega_5 + 36v_1^2 \omega_5 - 12cs^4 \omega_5 + 36v_1^4 - 12cs^2 + cs^4 \omega_5^2 - 7v_1^2 \omega_5^2 \\
C_2 &= -3cs^2 \omega_7 \omega_4 \omega_5 + \omega_4 v_1^2 \omega_5^2 - \omega_7 \omega_4 + 3cs^2 \omega_7 \omega_5 + 3cs^2 \omega_4 \omega_5^2 - 3cs^2 \omega_5^2 + \omega_5^2 - 3cs^2 \omega_4 \omega_5 - \omega_4 v_1^2 \omega_5 + \omega_7 \omega_4 v_1^2 - \omega_7 \omega_4 v_1^2 \omega_5 + \omega_4 \omega_5 - \\
& \omega_4 \omega_5^2 + \omega_7 \omega_4 \omega_5 - v_1^2 \omega_5^2 - \omega_7 \omega_5 + 3cs^2 \omega_7 \omega_4 + \omega_7 v_1^2 \omega_5
\end{aligned}$$

$$C_3 = -cs^2\omega_7\omega_4\omega_5 + 3\omega_4v_1^2\omega_5^2 - \omega_7\omega_4 + cs^2\omega_7\omega_5 + cs^2\omega_4\omega_5^2 - cs^2\omega_5^2 + \omega_5^2 - cs^2\omega_4\omega_5 - 3\omega_4v_1^2\omega_5 + 3\omega_7\omega_4v_1^2 - 3\omega_7\omega_4v_1^2\omega_5 + \omega_4\omega_5 - \omega_4\omega_5^2 + \omega_7\omega_4\omega_5 - 3v_1^2\omega_5^2 - \omega_7\omega_5 + cs^2\omega_7\omega_4 + 3\omega_7v_1^2\omega_5$$

$$C_4 = -3\omega_7\omega_4^2v_1^2\omega_5^2 - 24cs^2\omega_7\omega_4\omega_5 - 12\omega_4v_1^2\omega_5^2 - 12cs^2\omega_4\omega_5^2 - 12\omega_4^2\omega_5^2 + 6\omega_7\omega_4^2\omega_5 + 12\omega_7\omega_4^2v_1^2 + 3\omega_7\omega_4^2\omega_5^2 + 12\omega_4^2\omega_5 - 12\omega_7\omega_4^2 - 24cs^2\omega_7\omega_5^2 - 6\omega_7\omega_4^2v_1^2\omega_5 + 42cs^2\omega_7\omega_4\omega_5^2 + 36cs^2\omega_7\omega_4^2 - 11cs^2\omega_7\omega_4^2\omega_5^2 - 6\omega_7\omega_4\omega_5^2 - 12\omega_4^2v_1^2\omega_5 - 12cs^2\omega_4^2\omega_5 + 12cs^2\omega_4^2\omega_5^2 + 6\omega_7\omega_4v_1^2\omega_5^2 + 12\omega_4\omega_5^2 + 12\omega_4^2v_1^2\omega_5^2 - 18cs^2\omega_7\omega_4^2\omega_5$$

$$C_5 = -6\omega_4\omega_8v_2^2 + 6\omega_6\omega_4v_2^2 - \omega_6\omega_4\omega_8 - 18cs^2\omega_4\omega_8 + 12\omega_6 + 18cs^2\omega_6\omega_4 + 36cs^2\omega_8 - 12\omega_6v_2^2 - 36cs^2\omega_6 + \omega_6\omega_4\omega_8v_2^2 - 6\omega_6\omega_4 + 6\omega_4\omega_8 - 12\omega_8 + 3cs^2\omega_6\omega_4\omega_8 + 12\omega_8v_2^2$$

$$C_6 = 3\omega_4\omega_8v_2^2 + 3\omega_4^2v_2^2 + \omega_4^2\omega_8 + 15cs^2\omega_4\omega_8 - 12cs^2\omega_8 + 6\omega_4 + 3cs^2\omega_4^2 - 3\omega_4^2 - \omega_4^2\omega_8v_2^2 - 6cs^2\omega_4 - 3\omega_4\omega_8 - 3cs^2\omega_4^2\omega_8 - 6\omega_4v_2^2$$

$$C_7 = -18\omega_4\omega_8v_2^2 + 18\omega_6\omega_4v_2^2 - \omega_6\omega_4\omega_8 - 6cs^2\omega_4\omega_8 + 12\omega_6 + 6cs^2\omega_6\omega_4 + 12cs^2\omega_8 - 36\omega_6v_2^2 - 12cs^2\omega_6 + 3\omega_6\omega_4\omega_8v_2^2 - 6\omega_6\omega_4 + 6\omega_4\omega_8 - 12\omega_8 + cs^2\omega_6\omega_4\omega_8 + 36\omega_8v_2^2$$

$$C_8 = 12 + 404cs^2v_1^2\omega_5^2 - 34cs^2v_1^2\omega_5^3 - 216v_1^4\omega_5 + 198cs^2\omega_5 - 78cs^2\omega_5^2 + 90v_1^4\omega_5^2 + 672cs^2v_1^2 - 156v_1^2 + 144cs^4 + 8\omega_5^2 - \omega_5^3 - 9v_1^4\omega_5^3 + 6cs^2\omega_5^3 - 1008cs^2v_1^2\omega_5 + 234v_1^2\omega_5 - 216cs^4\omega_5 + 144v_1^4 - 132cs^2 + 10v_1^2\omega_5^3 - 5cs^4\omega_5^3 + 82cs^4\omega_5^2 - 98v_1^2\omega_5^2 - 18\omega_5$$

$$C_9 = 12 + 252cs^2v_1^2\omega_5^2 - 18cs^2v_1^2\omega_5^3 - 756v_1^4\omega_5 + 54cs^2\omega_5 - 22cs^2\omega_5^2 + 310v_1^4\omega_5^2 + 432cs^2v_1^2 - 252v_1^2 + 24cs^4 + 8\omega_5^2 - \omega_5^3 - 29v_1^4\omega_5^3 + 2cs^2\omega_5^3 - 648cs^2v_1^2\omega_5 + 378v_1^2\omega_5 - 36cs^4\omega_5 + 504v_1^4 - 36cs^2 + 14v_1^2\omega_5^3 - cs^4\omega_5^3 + 14cs^4\omega_5^2 - 154v_1^2\omega_5^2 - 18\omega_5$$

$$C_{10} = -8\omega_7v_1^4\omega_5^3 - 4cs^4\omega_7\omega_5^3 - 32\omega_7\omega_4^2v_1^2\omega_5^2 + 4cs^2\omega_4\omega_5^3 - 24cs^2\omega_4^2v_1^2\omega_5^2 + 8cs^4\omega_7\omega_4^2\omega_5^2 + 24cs^2\omega_4^2v_1^2\omega_5^3 - 20\omega_7\omega_4v_1^2\omega_5 + 8cs^4\omega_7^2\omega_4^2 + 12cs^2\omega_7^2\omega_4\omega_5 + 13\omega_7\omega_4^2v_1^2\omega_5^3 + 51cs^2\omega_7^2\omega_4^2v_1^2\omega_5^2 - 8cs^4\omega_7^2\omega_4\omega_5^2 - 4cs^4\omega_7\omega_4\omega_5^3 + 13\omega_7^2\omega_4^2v_1^4\omega_5^2 + 4\omega_4v_1^2\omega_5^3 + 20\omega_7\omega_4v_1^4\omega_5^3 + 4cs^4\omega_7^2\omega_4\omega_5 - 144cs^2\omega_7\omega_4^2v_1^2\omega_5 - 8\omega_7^2v_1^2\omega_5^2 + 4cs^2\omega_7\omega_5^3 + 84cs^2\omega_7\omega_4v_1^2\omega_5^3 - 36\omega_7^2\omega_4^2v_1^4\omega_5 - 4cs^4\omega_4\omega_5^3 + 4\omega_4^2v_1^4\omega_5^3 + 20\omega_7^2\omega_4v_1^2\omega_5^2 + 96cs^2\omega_7^2\omega_4^2v_1^2 - 8cs^2\omega_7\omega_4\omega_5^3 + 24\omega_7^2\omega_4^2v_1^4 - 4cs^2\omega_7^2\omega_4\omega_5^2 - 48cs^2\omega_7\omega_4v_1^2\omega_5^2 - 4cs^4\omega_7\omega_4^2\omega_5 - 16\omega_7\omega_4v_1^4\omega_5^2 - 8cs^2\omega_7^2\omega_4^2 + 20\omega_7\omega_4^2v_1^2\omega_5 - 36cs^2\omega_7v_1^2\omega_5^3 - 4\omega_4^2v_1^4\omega_5^2 - 51cs^2\omega_7\omega_4^2v_1^2\omega_5^3 - 4cs^4\omega_4^2\omega_5^2 + 20\omega_7^2\omega_4v_1^4\omega_5 - 24\omega_7^2\omega_4^2v_1^2 - 13\omega_7\omega_4^2v_1^4\omega_5^3 - 4cs^2\omega_7^2\omega_5^2 + 72cs^2\omega_7^2\omega_4v_1^2\omega_5 - 8cs^2\omega_7\omega_4^2\omega_5^2 + 36cs^2\omega_7^2v_1^2\omega_5^2 - 13\omega_7^2\omega_4^2v_1^2\omega_5^2 - 4\omega_4v_1^4\omega_5^3 + 32\omega_7\omega_4^2v_1^2\omega_5^2 + 8\omega_7v_1^2\omega_5^3 - 12cs^4\omega_7^2\omega_4^2\omega_5 + 4cs^4\omega_4^2\omega_5^3 + 120cs^2\omega_7\omega_4^2v_1^2\omega_5^2 + 8cs^2\omega_7^2\omega_4\omega_5^2 + 4cs^2\omega_7\omega_4^2\omega_5^3 + 4cs^2\omega_4^2\omega_5^2 + 4cs^4\omega_7^2\omega_5^2 - 4cs^2\omega_7^2\omega_4\omega_5 + 16\omega_7\omega_4v_1^2\omega_5^2 - 20\omega_7\omega_4^2v_1^4\omega_5 + 8cs^4\omega_7\omega_4\omega_5^3 + 4cs^4\omega_7^2\omega_4^2\omega_5^2 - 72cs^2\omega_7\omega_4^2v_1^2\omega_5 + 4\omega_4^2v_1^4\omega_5^2 + 8\omega_7^2v_1^4\omega_5^2 + 4cs^2\omega_7\omega_4^2\omega_5 - 20\omega_7\omega_4v_1^2\omega_5^3 - 24cs^2\omega_4v_1^2\omega_5^3 + 36\omega_7^2\omega_4^2v_1^2\omega_5 - 4cs^2\omega_4^2\omega_5^3 - 4\omega_4^2v_1^4\omega_5^2 - 20\omega_7^2\omega_4v_1^4\omega_5^2 - 84cs^2\omega_7^2\omega_4v_1^2\omega_5^2$$

$$C_{11} = 104\omega_7\omega_4^2v_1^2\omega_5^2 - 8\omega_4^2\omega_5^3 - 16cs^2\omega_4\omega_5^3 - 24\omega_7^2\omega_4\omega_5 + 64\omega_7^2\omega_4v_1^2\omega_5 + 8\omega_4^2\omega_5^2 - 72cs^2\omega_7^2\omega_4^2\omega_5 - 43\omega_7\omega_4^2v_1^2\omega_5^3 + 24\omega_7\omega_4^2\omega_5 - 16\omega_4v_1^2\omega_5^3 + 28\omega_7^2v_1^2\omega_5^2 - 40\omega_7\omega_4^2\omega_5^2 - 20cs^2\omega_7\omega_5^3 - 68\omega_7^2\omega_4v_1^2\omega_5^2 + 44cs^2\omega_7\omega_4\omega_5^3 + 25cs^2\omega_7^2\omega_4^2\omega_5^2 - 12\omega_7^2\omega_5^2 + 17\omega_7\omega_4^2\omega_5^3 + 28\omega_7^2\omega_4\omega_5^2 + 48cs^2\omega_7^2\omega_4^2 - 64\omega_7\omega_4^2v_1^2\omega_5 - 16cs^2\omega_7\omega_4\omega_5^2 - 32\omega_7^2\omega_4^2 - 17\omega_7^2\omega_4^2\omega_5^2 + 80\omega_7^2\omega_4^2v_1^2 + 20cs^2\omega_7^2\omega_5^2 - 28\omega_7\omega_4\omega_5^3 + 56cs^2\omega_7\omega_4^2\omega_5^2 + 43\omega_7^2\omega_4^2v_1^2\omega_5^2 + 16\omega_7\omega_4\omega_5^2 - 28\omega_7v_1^2\omega_5^3 + 12\omega_7\omega_5^3 - 44cs^2\omega_7^2\omega_4\omega_5^2 - 25cs^2\omega_7\omega_4^2\omega_5^3 - 16cs^2\omega_4^2\omega_5^2 + 32cs^2\omega_7^2\omega_4\omega_5 - 48\omega_7\omega_4v_1^2\omega_5^2 - 16\omega_4^2v_1^2\omega_5^2 - 32cs^2\omega_7\omega_4^2\omega_5 + 8\omega_4\omega_5^3 + 68\omega_7\omega_4v_1^2\omega_5^3 - 12\omega_7^2\omega_4^2v_1^2\omega_5 + 16cs^2\omega_4^2\omega_5^3 + 16\omega_4^2v_1^2\omega_5^3 + 48\omega_7^2\omega_4^2\omega_5$$

$$C_{12} = -4\omega_7^3\omega_3^2v_1^2\omega_5^3 + 24\omega_7\omega_4^2v_1^2\omega_5^2 + 252cs^2\omega_7^3\omega_3^2v_1 - cs^2\omega_7^3\omega_4^2\omega_5^2 + 12cs^4\omega_7^3\omega_3^2 + 72\omega_7^3\omega_4^2v_1^4 - 18\omega_7^3\omega_4^2v_1^4\omega_5^3 + 60\omega_7\omega_4^3v_1^4\omega_5^2 + 12cs^4\omega_7\omega_4^2\omega_5^2 + 54cs^2\omega_7\omega_4^3v_1^2\omega_5^2 - 24cs^4\omega_7\omega_4\omega_5^3 - 81cs^2\omega_7^2\omega_4^2v_1^2\omega_5^3 - 12cs^2\omega_4^2v_1^2\omega_5^3 - 19\omega_7^2\omega_4^2v_1^2\omega_5^2 + 12cs^2\omega_7^2\omega_4^2\omega_5 - 48\omega_7\omega_4^2v_1^2\omega_5^3 - 21cs^2\omega_7\omega_4^3v_1^2\omega_5^3 + 162cs^2\omega_7^2\omega_4^2v_1^2\omega_5^2 - 18cs^4\omega_7\omega_4^2\omega_5^3 + 12\omega_7^2\omega_4^2v_1^2\omega_5^2 - 27\omega_7\omega_4^3v_1^2\omega_5^3 - 24\omega_7\omega_4v_1^4\omega_5^3 - 108cs^2\omega_7^2\omega_4^2v_1^2\omega_5 - 6cs^4\omega_7\omega_4^3\omega_5^2 - 12cs^2\omega_7^2\omega_4^2 - 12\omega_3^2v_1^2\omega_5^3 - 12cs^2\omega_7\omega_4v_1^2\omega_5^3 - 12\omega_3^2v_1^4\omega_5^3 - 12cs^2\omega_7\omega_4\omega_5^3 - 6cs^2\omega_7^2\omega_4^2\omega_5^2 + 90\omega_7^3\omega_4^2v_1^2\omega_5 - 36\omega_7\omega_4^3v_1^4\omega_5 + 12\omega_3^2v_1^2\omega_5^2 - 36cs^2\omega_7\omega_4^3v_1^2\omega_5 + 6cs^4\omega_7\omega_4^3v_1^2\omega_5^2 - 5cs^2\omega_7^2\omega_4^2\omega_5^2 + 12cs^2\omega_7^2\omega_4^2\omega_5 - 12\omega_7^2\omega_4^2v_1^2\omega_5^3 + 60cs^2\omega_7^3\omega_4^2v_1^2\omega_5^2 + 30cs^2\omega_7\omega_4^3v_1^2\omega_5^3 + 48\omega_7\omega_4^2v_1^4\omega_5^3 + cs^4\omega_7^2\omega_4^2\omega_5^2 + 19\omega_7^2\omega_4^3v_1^4\omega_5^2 - 12cs^2\omega_7\omega_4^2\omega_5^2 + 12cs^2\omega_3^2v_1^2\omega_5^3 + 6cs^2\omega_7^2\omega_4\omega_5^3 + 27\omega_7\omega_3^2v_1^2\omega_5^3 - 12\omega_7^2\omega_4^2v_1^2\omega_5^2 - 24\omega_7\omega_4^3v_1^4\omega_5^2 + 4\omega_7^3\omega_4^3v_1^4\omega_5^3 - cs^4\omega_7^2\omega_4^3\omega_5^3 - 12cs^4\omega_7^2\omega_4^2\omega_5 + 12cs^2\omega_7^2\omega_4^3v_1^4\omega_5^3 - 12cs^2\omega_7\omega_4^3v_1^2\omega_5^2 - 60\omega_7\omega_4^3v_1^2\omega_5^2 + 18\omega_7^2\omega_4^2v_1^2\omega_5^3 - 48cs^2\omega_7^2v_1^2\omega_5^3 + 18cs^2\omega_7\omega_4^2\omega_5^3 - 12cs^2\omega_3^3v_1^2\omega_5^2 + 36\omega_7\omega_4^3v_1^2\omega_5 - 12\omega_3^3v_1^4\omega_5^2 + 6cs^2\omega_7\omega_3^3\omega_5^2 + 12cs^4\omega_7\omega_4\omega_5^3 + 102cs^2\omega_7^2\omega_4^2v_1^2\omega_5^3 + 6cs^4\omega_7^2\omega_4^2\omega_5^2 - 72\omega_7^3\omega_4^3v_1^2 + 12\omega_7^2\omega_4^4v_1^4\omega_5^3 + 24\omega_7\omega_4^3v_1^2\omega_5^3 + 12\omega_4^4v_1^4\omega_5^3 + 12cs^4\omega_7^2\omega_5^3 - 6cs^2\omega_7\omega_4^3\omega_5^3 + 12\omega_4^4v_1^4\omega_5^3 - 306cs^2\omega_7^2\omega_4^3v_1^2\omega_5 + 13cs^4\omega_7^2\omega_4^3\omega_5^3 - 90\omega_7^2\omega_4^3v_1^4\omega_5 - 12cs^4\omega_7^2\omega_4^3\omega_5 - 48cs^2\omega_7^2\omega_4^3v_1^2\omega_5^2$$

$$C_{13} = 12\omega_9\omega_7\omega_3^3\omega_8\omega_5^3v_2^2 + 24\omega_9\omega_7^2\omega_4^2\omega_8\omega_5^3v_2^2 - 12cs^2\omega_9\omega_3^3\omega_8v_1^2\omega_5^2 - 12\omega_9\omega_4^3\omega_8v_1^2\omega_5^2v_2^2 + 6cs^2\omega_9\omega_7^3\omega_3^3v_1^2\omega_5^2 - 12cs^2\omega_9\omega_7\omega_4\omega_8v_1^2\omega_5^3 + 12cs^2\omega_7\omega_4^3\omega_8v_1^2\omega_5^2v_2^2 - 36cs^4\omega_7\omega_4^3\omega_8\omega_5^3 - cs^2\omega_9\omega_7^2\omega_4^3\omega_8\omega_5^3 - 12\omega_9\omega_7\omega_4^3\omega_8v_1^2\omega_5^3v_2^2 - 6cs^4\omega_9\omega_7^2\omega_4^3\omega_8\omega_5^2 - 36cs^2\omega_9\omega_7\omega_4^3\omega_8\omega_5v_2^2 + 12cs^2\omega_7^2\omega_4^3v_1^2\omega_5^3 + 12\omega_7\omega_4^3\omega_8v_1^2\omega_5^3v_2^2 - 12cs^2\omega_7\omega_4^3\omega_8\omega_5^2 + 18cs^2\omega_9\omega_7^2\omega_4^3\omega_8\omega_5^2v_2^2 + 12cs^2\omega_7^2\omega_4^3\omega_8\omega_5^3 + 6\omega_9\omega_7^3\omega_3^3\omega_8v_1^2\omega_5^2v_2^2 - 12\omega_9\omega_3^3\omega_8\omega_5^3v_2^2 + 6cs^2\omega_7^2\omega_4^3\omega_8\omega_5^3 + 12cs^2\omega_9\omega_4^3\omega_8v_1^2\omega_5^3 - 36cs^2\omega_9\omega_4^3\omega_8\omega_5^2v_2^2 - 54cs^2\omega_9\omega_7^2\omega_4^3\omega_8\omega_5v_2^2 + 12cs^2\omega_9\omega_7^2\omega_4^3\omega_8v_1^2 - 36cs^4\omega_7^2\omega_4^3\omega_8\omega_5^3 - 18cs^2\omega_9\omega_7\omega_4^3\omega_8\omega_5^2 + 5cs^4\omega_9\omega_7^3\omega_3^3\omega_8\omega_5^3 + 36cs^4\omega_7\omega_4^3\omega_8\omega_5^2 + 2cs^2\omega_9\omega_7^3\omega_3^3\omega_8\omega_5^2 + 12\omega_9\omega_7\omega_4^3\omega_8\omega_5^2v_2^2 + 12cs^2\omega_9\omega_7^2\omega_4\omega_8v_1^2\omega_5^3 + 72cs^2\omega_9\omega_7^2\omega_4\omega_8\omega_5^2v_2^2 + 36cs^2\omega_7^2\omega_4^3\omega_8^3v_2^2 + 24\omega_9\omega_7\omega_3^3\omega_8v_1^2\omega_5^2v_2^2 + 36cs^2\omega_9\omega_7^2\omega_4^3\omega_8\omega_5v_2^2 - 12\omega_9\omega_7^2\omega_4^3\omega_8v_1^2\omega_5^3 + 12cs^2\omega_9\omega_7^2\omega_4^3\omega_8\omega_5^2 + 12cs^2\omega_9\omega_7^2\omega_4^3\omega_8\omega_5 - 12cs^2\omega_7\omega_4^3\omega_8v_1^2\omega_5^3 + 12\omega_9\omega_4^3\omega_8\omega_5^3v_2^2 + 180cs^4\omega_9\omega_7^2\omega_4\omega_8\omega_5^3 + 12cs^2\omega_9\omega_7^2\omega_4\omega_8\omega_5^2 + 12\omega_9\omega_4^3\omega_8v_1^2\omega_5^3v_2^2 + 12cs^4\omega_9\omega_4^3\omega_8\omega_5^2 - 72cs^2\omega_9\omega_7\omega_4\omega_8\omega_5^3v_2^2 - 96cs^4\omega_9\omega_7^2\omega_8\omega_5^3 - 24\omega_9\omega_7\omega_4^3\omega_8\omega_5^2v_2^2 + 6\omega_7^3\omega_4^3\omega_5^3v_2^2 - 12cs^2\omega_9\omega_7^2\omega_4\omega_8\omega_5^3 - 84cs^4\omega_9\omega_7^2\omega_4\omega_8\omega_5^2 + 12\omega_9\omega_7^3\omega_4^3\omega_8v_1^2v_2^2 - 36cs^4\omega_9\omega_7^3\omega_3^3\omega_8\omega_5 - 6\omega_9\omega_7^3\omega_4^3\omega_8\omega_5^2v_2^2 - 12cs^2\omega_9\omega_4^3\omega_8\omega_5^3 - 36\omega_9\omega_7\omega_4^3\omega_8\omega_5^3v_2^2 - 6\omega_9\omega_7^3\omega_4^3\omega_8v_1^2\omega_5^2v_2^2 - 12cs^2\omega_7^2\omega_4^3\omega_5^3 + 36cs^2\omega_9\omega_4^3\omega_8\omega_5^3v_2^2 - 12\omega_9\omega_4^3\omega_8v_1^2\omega_5^3v_2^2 - 12\omega_9\omega_7^2\omega_4^3v_1^2\omega_5^2v_2^2 - 12cs^4\omega_9\omega_4^3\omega_8\omega_5^3 - 88cs^4\omega_9\omega_7^2\omega_4^3\omega_8\omega_5^3 - 18cs^2\omega_9\omega_7^2\omega_4^3\omega_8\omega_5^2 - 42cs^4\omega_9\omega_7\omega_4^3\omega_8\omega_5^2 + 6cs^2\omega_9\omega_7\omega_4^3\omega_8\omega_5^3 + 18cs^4\omega_7^2\omega_3^3\omega_8\omega_5^3 - 12\omega_7\omega_4^3\omega_8\omega_5^3v_2^2 + 6\omega_7^3\omega_4^3\omega_8\omega_5^2v_2^2 - 12\omega_9\omega_4^3\omega_8v_1^2\omega_5^3v_2^2 + 12cs^2\omega_7\omega_4^3\omega_8v_1^2\omega_5^3 + 18cs^2\omega_7^2\omega_4^3\omega_8\omega_5^3v_2^2 + 18cs^2\omega_9\omega_7^2\omega_4^3\omega_8v_1^2\omega_5^2 + 18\omega_9\omega_7^2\omega_4^3\omega_8\omega_5v_2^2 - 36cs^4\omega_9\omega_7^2\omega_4^3\omega_5^2 + 18cs^2\omega_9\omega_7\omega_4^3\omega_8v_1^2\omega_5^3 - 12cs^2\omega_9\omega_7^3\omega_3^3\omega_8v_1^2\omega_5 - 36cs^2\omega_9\omega_7^2\omega_4^3\omega_5^2v_2^2 - 6cs^2\omega_7^2\omega_4^3\omega_8\omega_5^3 - 36cs^2\omega_9\omega_7\omega_4^3\omega_8\omega_5^2v_2^2 + 12cs^2\omega_9\omega_7^2\omega_4^3\omega_5^2 - 12\omega_9\omega_7\omega_4^3\omega_8v_1^2\omega_5^2v_2^2 - 24\omega_9\omega_7^2\omega_4\omega_8\omega_5^2v_2^2 + 12\omega_7\omega_4^3\omega_8v_1^2\omega_5^2v_2^2 - 12cs^2\omega_9\omega_7^2\omega_4^3\omega_8v_1^2\omega_5^3 - 36cs^2\omega_9\omega_7\omega_4^3\omega_8\omega_5^3v_2^2 - 12cs^2\omega_9\omega_7\omega_4^3\omega_8v_1^2\omega_5^3 + 6\omega_7^3\omega_4^3\omega_8v_1^2\omega_5^3v_2^2 - 18cs^4\omega_7^2\omega_4^3\omega_5^3 - 72cs^2\omega_9\omega_7^2\omega_4^3\omega_8\omega_5^2v_2^2 - 6\omega_7^3\omega_4^3\omega_5^3v_2^2 - 12\omega_9\omega_7\omega_4^3\omega_8v_1^2\omega_5v_2^2 - 6cs^2\omega_7^2\omega_4^3v_1^2\omega_5^2 - 6cs^2\omega_7^2\omega_4^3v_1^4\omega_5^2 + 12cs^4\omega_7\omega_4^3\omega_8\omega_5^3 + 36cs^4\omega_7\omega_4^3\omega_8\omega_5^2 + 150cs^4\omega_9\omega_7^2\omega_4^3\omega_8\omega_5^2 - 18cs^2\omega_9\omega_7\omega_4^3\omega_8\omega_5^2 - 18cs^4\omega_7^2\omega_4^3\omega_8\omega_5^2 + 30cs^4\omega_9\omega_7\omega_4^3\omega_8\omega_5^3 + 12\omega_7^2\omega_4^3\omega_8\omega_5^3v_2^2 - 12\omega_7\omega_4^3\omega_8\omega_5^2v_2^2 + 6cs^2\omega_7^3\omega_3^3\omega_8\omega_5^2 - 12cs^2\omega_7\omega_4^3\omega_8\omega_5^3 + 12\omega_9\omega_7\omega_4^3\omega_8\omega_5v_2^2 - 12cs^2\omega_9\omega_7^2\omega_4^3v_1^2\omega_5^2 - 12cs^2\omega_7^2\omega_4^3\omega_8v_1^2\omega_5^3 - 36cs^2\omega_7\omega_4^3\omega_8\omega_5^3v_2^2 + 36\omega_9\omega_7\omega_4^3\omega_8v_1^2\omega_5^2v_2^2 - 18\omega_9\omega_7^2\omega_4^3\omega_8v_1^2\omega_5v_2^2 - 6cs^2\omega_9\omega_7^2\omega_4^3\omega_5^2 - 12\omega_7\omega_4^3\omega_8v_1^2\omega_5^3v_2^2 + 12\omega_9\omega_7^2\omega_4^3\omega_5^2v_2^2 + 12\omega_7^2\omega_4^3v_1^2\omega_5^3v_2^2 + 12cs^4\omega_9\omega_7^2\omega_4^3\omega_8\omega_5^2v_2^2 + 12cs^2\omega_7\omega_4^3\omega_8v_1^2\omega_5^2 + 36cs^4\omega_9\omega_7^2\omega_4^3\omega_8 +$$

$$\begin{aligned}
& 24\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^2\omega_2 + c^2\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 + 108c^2\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 + 18c^2\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 + 6c^2\omega_7^2\omega_4\omega_8\omega_1^2\omega_5^3 + 36c^2\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 + \\
& 18c^2\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 + 12c^2\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 - 18c^2\omega_7^2\omega_4\omega_8\omega_1^2\omega_5^3 - 12c^2\omega_9\omega_7\omega_4\omega_8 - 6\omega_7^2\omega_4\omega_8\omega_1^2\omega_5^3 + 12c^2\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 - 48c^2\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 + \\
& 12c^2\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 - 2c^2\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 - 12c^2\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 - 36c^2\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 - 12\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 + 36c^2\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 - 12\omega_7^2\omega_4\omega_8\omega_1^2\omega_5^3 + \\
& 12\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 + 12c^2\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 - 24\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 - 24\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 - 18c^2\omega_7^2\omega_4\omega_8\omega_1^2\omega_5^3 + 36c^2\omega_7^2\omega_4\omega_8\omega_1^2\omega_5^3 + 12c^2\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 - \\
& 6\omega_7^2\omega_4\omega_8\omega_1^2\omega_5^3 + 72c^2\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 - 6c^2\omega_7^2\omega_4\omega_8\omega_1^2\omega_5^3 - 6c^2\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 - 24\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 - 12c^2\omega_9\omega_7\omega_4\omega_8\omega_1^2\omega_5^3 -
\end{aligned}$$

[illegible]

$$\begin{aligned}
& 12c^2\omega_6^2\omega_7\omega_4^2\omega_8\omega_5\omega_2^2 + 5c^2\omega_9\omega_6\omega_7\omega_4^2\omega_8^2\omega_5^2 - 12\omega_9\omega_6^2\omega_7\omega_4^2\omega_8\omega_1^2 + 18c^4\omega_6^2\omega_7\omega_4^2\omega_8^2\omega_5^2 + 6\omega_9\omega_7\omega_4^2\omega_8^2\omega_1^2\omega_5 - 12\omega_6^2\omega_4^2\omega_8^2\omega_1^2\omega_5\omega_2^2 - \\
& 9\omega_9\omega_6\omega_7\omega_4^2\omega_8^2\omega_1^2\omega_5 - 12\omega_9\omega_6\omega_7\omega_4^2\omega_1^2\omega_5\omega_2^2 - 72c^2\omega_9\omega_6^2\omega_7\omega_4\omega_8\omega_1^2\omega_5 + 24\omega_9\omega_6\omega_7\omega_4\omega_8^2\omega_1^2\omega_5\omega_2^2 + 18c^2\omega_6^2\omega_7\omega_4^2\omega_8^2\omega_1^2\omega_5 - 12\omega_6^2\omega_7\omega_4^2\omega_8^2\omega_1^2\omega_5 - \\
& 6\omega_6^2\omega_7\omega_4^2\omega_8^2\omega_1^2\omega_5\omega_2^2 - 12c^2\omega_6^2\omega_4^2\omega_8^2\omega_1^2\omega_5\omega_2^2 + 156c^4\omega_9\omega_6\omega_7\omega_4\omega_8\omega_1^2\omega_5 - 72c^2\omega_9\omega_6^2\omega_7\omega_4\omega_8\omega_1^2\omega_5 - 36\omega_9\omega_6\omega_7\omega_4\omega_8^2\omega_1^2\omega_5\omega_2^2 - 108c^2\omega_9\omega_6\omega_7\omega_4\omega_8^2\omega_1^2\omega_5 - \\
& 12\omega_6^2\omega_4^2\omega_8^2\omega_1^2 + 12\omega_9\omega_7\omega_4^2\omega_8^2\omega_1^2\omega_5\omega_2^2 - 6c^2\omega_6^2\omega_7\omega_4^2\omega_8^2\omega_1^2\omega_5 - 12c^2\omega_6^2\omega_4^2\omega_8^2 - 36c^4\omega_9\omega_6\omega_7\omega_4\omega_8\omega_1^2\omega_5 - 12\omega_9\omega_6\omega_7\omega_4\omega_8^2\omega_1^2\omega_5 + 12c^2\omega_9\omega_6\omega_7\omega_4\omega_8 - \\
& 42c^4\omega_9\omega_6^2\omega_7\omega_4^2\omega_8\omega_5 + 18c^2\omega_9\omega_6^2\omega_7\omega_4^2\omega_8\omega_5\omega_2^2 + 24\omega_9\omega_6\omega_7\omega_4^2\omega_8^2\omega_1^2\omega_2^2 + 12c^2\omega_9\omega_6\omega_7\omega_4^2\omega_8\omega_5\omega_2^2 - c^2\omega_9\omega_6^2\omega_7\omega_4^2\omega_8^2\omega_5 - 36c^4\omega_9\omega_6\omega_4^2\omega_8^2
\end{aligned}$$

$$\begin{aligned}
C_{18} = & 12cs^2\omega g w_7 w_4 w_8 v_1^2 w_5 + 6cs^2\omega g w_7 w_4 w_8 w_5 - 6\omega g w_6 w_7 w_3 w_8 v_1^2 w_5 + 12cs^4\omega g w_6 w_4 w_8 w_5 - 36cs^2\omega g w_7 w_4 w_8 w_5 v_1^2 + 12cs^2\omega g w_4 w_8 v_1^2 w_5 + \\
& 24cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 - 48\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 + 12\omega g w_6 w_4 w_8 v_1^2 - 48cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 + 36\omega g w_7 w_4 w_8 v_1^2 w_5 v_2^2 - 5cs^2\omega g w_6 w_7 w_4 w_8 w_5 + \\
& 18\omega g w_7 w_4 w_8 v_1^2 w_5 v_2^2 - 12cs^2\omega g w_7 w_4 w_8 v_1^2 w_5 + 12\omega g w_7 w_4 w_8 v_1^2 w_5 + 6\omega g w_6 w_7 w_4 w_8 v_1^2 - 6cs^2\omega g w_6 w_7 w_4 w_8 w_5 + 6cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 - \\
& 16\omega g w_7 w_4 w_8 v_1^2 v_2^2 - 12cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 + 18cs^4\omega g w_6 w_7 w_4 w_8 w_5 - 15cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 + 24cs^2\omega g w_7 w_4 w_8 v_1^2 w_5 v_2^2 - 12cs^2\omega g w_6 w_7 w_4 w_8 w_5 - \\
& 12cs^4\omega g w_4 w_8 w_5 - 12cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 - 6cs^4\omega g w_7 w_4 w_8 v_1^2 w_5 - 15cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 + 36\omega g w_6 w_7 w_4 w_8 v_1^2 v_2^2 + 12\omega g w_6 w_7 w_4 v_1^2 w_5 + \\
& 12\omega g w_4 w_8 v_1^2 w_5 - 15cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 v_2^2 + 27\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 v_2^2 - 24\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 + 12cs^2\omega g w_6 w_7 w_4 w_8 w_5 - 18\omega g w_7 w_4 w_8 v_1^2 w_5 v_2^2 + \\
& 12cs^2\omega g w_4 w_8 w_5 + 6cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 + 6cs^4\omega g w_7 w_4 w_8 v_1^2 w_5 + 48cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 + 12cs^2\omega g w_6 w_4 w_8 v_1^2 w_5 + 12cs^2\omega g w_3 w_8 v_1^2 + \\
& 18\omega g w_7 w_4 w_8 v_1^2 w_5 v_2^2 + cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 + 12cs^2\omega g w_7 w_4 w_8 w_5 - 12\omega g w_4 w_8 v_1^2 w_5 v_2^2 - 72\omega g w_6 w_7 w_4 w_8 v_1^2 v_2^2 - 12cs^2\omega g w_6 w_4 w_8 v_1^2 w_5 + \\
& 54cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 v_2^2 + 36\omega g w_4 w_8 v_1^2 w_5 v_2^2 - 12cs^2\omega g w_6 w_7 w_4 w_8 w_5 - 12cs^2\omega g w_4 w_8 v_1^2 w_5 + 15\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 + 18cs^2\omega g w_7 w_4 w_8 w_5 v_2^2 - \\
& 45\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 v_2^2 + 36cs^2\omega g w_4 w_8 v_1^2 w_5 v_2^2 + 12cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 + 12cs^4\omega g w_4 w_8 v_1^2 w_5 + 60cs^2\omega g w_6 w_7 w_4 w_8 w_5 v_2^2 - 6cs^2\omega g w_7 w_4 w_8 v_1^2 w_5 + \\
& 12\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 - 6\omega g w_7 w_4 w_8 v_1^2 w_5 + 12cs^2\omega g w_7 w_4 w_8 v_1^2 w_5 + 36\omega g w_6 w_4 w_8 v_1^2 w_5 v_2^2 - 36\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 v_2^2 + 24cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 - \\
& 12cs^4\omega g w_6 w_7 w_4 w_8 w_5 - 12cs^4\omega g w_7 w_4 w_8 v_1^2 w_5 + 24\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 - 18cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 - 36cs^2\omega g w_6 w_4 w_8 v_1^2 w_5 - 72\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 v_2^2 - \\
& 6\omega g w_6 w_7 w_4 v_1^2 w_5 + 12\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 - 12cs^2\omega g w_4 w_8 v_1^2 w_5 + 6cs^2\omega g w_7 w_4 w_8 v_1^2 w_5 + 9cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 - 5cs^4\omega g w_6 w_7 w_4 w_8 w_5 - \\
& 6cs^2\omega g w_7 w_4 w_8 v_1^2 + 24\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 - 12cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 - 36\omega g w_7 w_4 w_8 v_1^2 w_5 v_2^2 - 36cs^2\omega g w_6 w_4 w_8 v_1^2 w_5 v_2^2 - 18\omega g w_6 w_7 w_4 w_8 v_1^2 v_2^2 + \\
& 30cs^2\omega g w_6 w_7 w_4 w_8 w_5 v_2^2 + 12cs^4\omega g w_7 w_4 w_8 w_5 - 6cs^4\omega g w_7 w_4 w_8 v_1^2 w_5 - 6cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 + 18cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 + 12cs^2\omega g w_6 w_4 w_8 v_1^2 w_5 + \\
& 12cs^2\omega g w_6 w_4 w_8 w_5 - 12cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 v_2^2 - cs^4\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 - 12\omega g w_7 w_4 w_8 v_1^2 w_5 + 60cs^2\omega g w_6 w_7 w_4 w_8 w_5 v_2^2 + 18\omega g w_6 w_7 w_4 v_1^2 w_5 v_2^2 + \\
& 36\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 + 18\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 v_2^2 - 18cs^2\omega g w_7 w_4 w_8 w_5 v_2^2 + 18cs^2\omega g w_6 w_7 w_4 w_8 w_5 - 24\omega g w_6 w_7 w_4 w_8 v_1^2 - 36\omega g w_6 w_4 w_8 v_1^2 w_5 v_2^2 + \\
& 12cs^2\omega g w_6 w_7 w_4 w_8 w_5 + 6cs^4\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 - 18cs^2\omega g w_7 w_4 w_8 v_1^2 w_5 + 144\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 v_2^2 - 12cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 + 36cs^2\omega g w_4 w_8 w_5 v_2^2 - \\
& 12cs^2\omega g w_6 w_7 w_4 w_8 w_5 + 6\omega g w_7 w_4 w_8 v_1^2 w_5 - 12cs^2\omega g w_7 w_4 w_8 v_1^2 w_5 - 12cs^2\omega g w_7 w_4 w_8 v_1^2 w_5 - 6cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 + 6\omega g w_7 w_4 w_8 v_1^2 w_5 - \\
& 36cs^2\omega g w_6 w_7 w_4 w_8 w_5 v_2^2 + 36cs^2\omega g w_7 w_4 w_8 w_5 v_2^2 + 5cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 - 12\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 + 6cs^4\omega g w_7 w_4 w_8 w_5 + 6\omega g w_7 w_4 w_8 v_1^2 w_5 - \\
& 36\omega g w_4 w_8 v_1^2 w_5 v_2^2 - 9\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 - 36\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 v_2^2 - 24cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 + 72\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 v_2^2 + 6cs^2\omega g w_7 w_4 w_8 v_1^2 w_5 - \\
& 12\omega g w_7 w_4 w_8 v_1^2 w_5 - 18\omega g w_7 w_4 w_8 v_1^2 w_5 v_2^2 - 36cs^2\omega g w_4 w_8 v_1^2 w_5 v_2^2 + 18cs^4\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 - 24cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 - 108\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 v_2^2 + \\
& 36cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 - 12\omega g w_7 w_4 w_8 v_1^2 w_5 v_2^2 + 36\omega g w_7 w_4 w_8 v_1^2 w_5 v_2^2 - 6cs^2\omega g w_7 w_4 w_8 w_5 - 12cs^2\omega g w_4 w_8 v_1^2 w_5 - 12cs^2\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 - 12\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 + \\
& 12cs^2\omega g w_6 w_4 w_8 v_1^2 w_5 - 18cs^4\omega g w_7 w_4 w_8 w_5 - 102cs^2\omega g w_6 w_7 w_4 w_8 w_5 v_2^2 + 72\omega g w_6 w_7 w_4 w_8 v_1^2 w_5 v_2^2 + 36cs^2\omega g w_6 w_4 w_8 v_1^2 w_5 v_2^2 - 12cs^4\omega g w_6 w_4 w_8 v_1^2 w_5 +
\end{aligned}$$

$$C_{20} = 48\omega_4\omega_8v_2^2 - 48cs^4\omega_4\omega_8^2 + 24\omega_1^2v_2^2 + 48cs^2\omega_4^2\omega_8v_2^2 + 72\omega_4^2\omega_8v_4^2 + 24\omega_4^2\omega_8^2v_2^2 + 156cs^2\omega_4\omega_8^2v_2^2 - 24cs^2\omega_4\omega_8 + 24\omega_4\omega_8^2v_4^2 + 6cs^2\omega_4^3\omega_8^2v_2^2 + 3\omega_3^3\omega_8^2v_4^2 + 12cs^2\omega_4\omega_8^2 - 12\omega_3^2v_2^2 + 18\omega_4^3\omega_8v_2^2 + 24cs^4\omega_4\omega_8 - 6cs^2\omega_4^3\omega_8 + 12\omega_4^3v_4^2 + 24cs^4\omega_8^2 - 3\omega_3^3\omega_8^2v_2^2 - 24cs^4\omega_4^2\omega_8 - 18\omega_3^3\omega_8v_4^2 - 12cs^2\omega_3^3\omega_8v_2^2 - 8cs^2\omega_4^2\omega_8^2 - 3cs^4\omega_3^3\omega_8^2 + 12cs^2\omega_3^3v_2^2 - 72\omega_4^2\omega_8v_2^2 + 6cs^4\omega_4^3\omega_8 - 48\omega_4\omega_8v_4^2 + 24cs^2\omega_4^2\omega_8 - 24cs^2\omega_4\omega_8v_2^2 - 24\omega_4^2v_4^2 - 24cs^2\omega_4^2v_2^2 + 24cs^4\omega_4^2\omega_8^2 - 24\omega_4\omega_8^2v_2^2 + cs^2\omega_4^3\omega_8^2 - 24\omega_4^2\omega_8^2v_4^2 - 96cs^2\omega_8^2v_2^2 - 72cs^2\omega_4^2\omega_8^2v_2^2$$

$$60cs^2\omega_6\omega_8^2 - 33cs^2\omega_6^2\omega_4\omega_8 - 84\omega_6^2\omega_8v_2^2 - 12\omega_6\omega_4^2\omega_8 - 48cs^2\omega_6^2\omega_4 - 24cs^2\omega_6\omega_4\omega_8 + 120\omega_4\omega_8^2v_2^2 + 61\omega_6\omega_4^2\omega_8^2v_2^2 + 84\omega_6\omega_8^2v_2^2 - 72\omega_6^2\omega_4\omega_8$$

2.3.4 Conservation of momentum: ρv_2



attached text file: output_d2q9_nse_mrt2_symbolic_pde_02.txt

$$\begin{aligned} & v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{\delta_l v_1 v_2}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\delta_l \rho v_2}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_l \rho v_1}{\delta_t} \frac{\partial v_2}{\partial x_1} + (v_2^2 + cs^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{2\delta_l \rho v_2}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_4) \frac{\delta_l^2 cs^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\ & (-2 + \omega_4) \frac{\delta_l^2 cs^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (-2 + \omega_6 - 3\omega_6 v_2^2 + 6v_2^2 - 2\omega_6 cs^2 + 4cs^2) \frac{\delta_l^2}{\omega_6 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (2 - \omega_6) \frac{3\delta_l^2 \rho v_2}{\omega_6 \delta_t} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + \\ & (-2 + \omega_4) \frac{\delta_l^2 \rho cs^2}{2\delta_t \omega_4} \frac{\partial^2 v_2}{\partial x_1^2} + (-2 + \omega_4) \frac{\delta_l^2 \rho cs^2}{2\delta_t \omega_4} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + (-2 + \omega_6 - \omega_6 v_2^2 + 2v_2^2 - 3\omega_6 cs^2 + 6cs^2) \frac{\delta_l^2 v_2}{2\omega_6 \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + \\ & (-2 + \omega_6 - 3\omega_6 v_2^2 + 6v_2^2 - \omega_6 cs^2 + 2cs^2) \frac{\delta_l^2 \rho}{2\omega_6 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + C_1 \frac{\delta_l^3 v_1 v_2}{12\delta_t \omega_7 \omega_4 \omega_5} \frac{\partial^3 \rho}{\partial x_1^3} + C_2 \frac{\delta_l^3 \rho v_2}{12\delta_t \omega_7 \omega_4 \omega_5} \frac{\partial^3 v_1}{\partial x_1^3} + C_3 \frac{\delta_l^3 \rho v_1}{6\delta_t \omega_7 \omega_4^2} \frac{\partial^3 v_2}{\partial x_1^3} + \\ & (-12 + 12\omega_4 - \omega_4^2) \frac{\delta_l^3 cs^4}{6\delta_t \omega_4^2} \frac{\partial^3 \rho}{\partial x_2^3} + (\omega_7 \omega_4 - 2\omega_7 + 2\omega_4 - \omega_4^2) \frac{\delta_l^3 \rho cs^2 v_1}{\delta_t \omega_7 \omega_4^2} \frac{\partial^3 v_1}{\partial x_1^3 \partial x_2} + \\ & (12\omega_6 \omega_4 \omega_8 - 12\omega_6 \omega_8 - 12\omega_6 \omega_4^2 + 12\omega_4^2 + 12\omega_6 \omega_4 - 12\omega_4 \omega_8 - \omega_6 \omega_4^2 \omega_8) \frac{\delta_l^3 \rho cs^2 v_2}{6\omega_6 \delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_2}{\partial x_1^3 \partial x_2} + C_4 \frac{\delta_l^3 v_1 v_2}{\omega_6^2 \delta_t \omega_4 \omega_8} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + \\ & C_5 \frac{\delta_l^3 \rho v_2}{12\omega_6^2 \delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_6 \frac{\delta_l^3 \rho v_1}{\omega_6^2 \delta_t \omega_4 \omega_8} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_7 \frac{\delta_l^3 \rho}{12\omega_6^2 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\ & (-24 + 5\omega_6^2 cs^2 + 24\omega_6 - 60\omega_6 v_2^2 + 60v_2^2 + 11\omega_6^2 v_2^2 - 4\omega_6^2 - 36\omega_6 cs^2 + 36cs^2) \frac{\delta_l^3 \rho v_2}{6\omega_6^2 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_8 \frac{\delta_l^4 v_2}{24\delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 \rho}{\partial x_1^4} + \\ & C_9 \frac{\delta_l^4 \rho v_1 v_2}{12\delta_t \omega_7^2 \omega_4^2 \omega_5^2} \frac{\partial^4 v_1}{\partial x_1^4} + C_{10} \frac{\delta_l^4 \rho}{24\delta_t \omega_7^2 \omega_4^2} \frac{\partial^4 v_2}{\partial x_1^4} + C_{11} \frac{\delta_l^4 v_1}{12\omega_9 \omega_6 \delta_t \omega_7^2 \omega_4^2 \omega_8^2} \frac{\partial^4 \rho}{\partial x_1^4 \partial x_2} + C_{12} \frac{\delta_l^4 \rho}{12\omega_9 \omega_6 \delta_t \omega_7^2 \omega_4^2 \omega_8^2} \frac{\partial^4 v_1}{\partial x_1^4 \partial x_2} + \\ & C_{13} \frac{\delta_l^4 \rho v_1 v_2}{12\omega_9 \omega_6^2 \delta_t \omega_7^2 \omega_4^2 \omega_8^2} \frac{\partial^4 v_2}{\partial x_1^4 \partial x_2} + C_{14} \frac{\delta_l^4 v_2}{12\omega_9 \omega_6^2 \delta_t \omega_7^2 \omega_4^2 \omega_8^2} \frac{\partial^4 \rho}{\partial x_1^4 \partial x_2^2} + C_{15} \frac{\delta_l^4 \rho v_1 v_2}{2\omega_9 \omega_6^2 \delta_t \omega_7^2 \omega_4^2 \omega_8^2} \frac{\partial^4 v_1}{\partial x_1^4 \partial x_2^2} + \\ & C_{16} \frac{\delta_l^4 \rho}{12\omega_9 \omega_6^2 \delta_t \omega_7^2 \omega_4^2 \omega_8^2} \frac{\partial^4 v_2}{\partial x_1^4 \partial x_2^2} + C_{17} \frac{\delta_l^4 v_1}{4\omega_6^2 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{18} \frac{\delta_l^4 \rho}{12\omega_6^2 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{19} \frac{\delta_l^4 \rho v_1 v_2}{4\omega_6^2 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\ & C_{20} \frac{\delta_l^4 v_2}{12\omega_6^2 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{21} \frac{\delta_l^4 \rho}{12\omega_6^2 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:

$$C_1 = 18\omega_4 cs^2 \omega_5 + 6\omega_7 \omega_4 - 18\omega_7 \omega_4 cs^2 + 36\omega_7 cs^2 + 6\omega_4 v_1^2 \omega_5 + 12\omega_7 v_1^2 - 12\omega_7 - 12v_1^2 \omega_5 - 6\omega_7 \omega_4 v_1^2 + \omega_7 \omega_4 v_1^2 \omega_5 - 6\omega_4 \omega_5 - \omega_7 \omega_4 \omega_5 + 3\omega_7 \omega_4 cs^2 \omega_5 + 12\omega_5 - 36cs^2 \omega_5$$

$$C_2 = 6\omega_4 cs^2 \omega_5 + 6\omega_7 \omega_4 - 6\omega_7 \omega_4 cs^2 + 12\omega_7 cs^2 + 18\omega_4 v_1^2 \omega_5 + 36\omega_7 v_1^2 - 12\omega_7 - 36v_1^2 \omega_5 - 18\omega_7 \omega_4 v_1^2 + 3\omega_7 \omega_4 v_1^2 \omega_5 - 6\omega_4 \omega_5 - \omega_7 \omega_4 \omega_5 + \omega_7 \omega_4 cs^2 \omega_5 + 12\omega_5 - 12cs^2 \omega_5$$

$$C_3 = -3\omega_7 \omega_4 + 15\omega_7 \omega_4 cs^2 + 3\omega_4^2 v_1^2 - 12\omega_7 cs^2 - \omega_7 \omega_4^2 v_1^2 + \omega_7 \omega_4^2 - 6\omega_4 cs^2 + 3\omega_4^2 cs^2 + 6\omega_4 + 3\omega_7 \omega_4 v_1^2 - 3\omega_4^2 - 6\omega_4 v_1^2 - 3\omega_7 \omega_4^2 cs^2$$

$$C_4 = \omega_4 \omega_8 v_2^2 - \omega_6 \omega_4 v_2^2 - 3\omega_6^2 cs^2 + \omega_6 \omega_8 v_2^2 + \omega_6 \omega_4 \omega_8 - \omega_6^2 \omega_4 + 3\omega_6^2 \omega_4 cs^2 - \omega_6 \omega_8 + 3\omega_4 cs^2 \omega_8 + 3\omega_6 cs^2 \omega_8 - \omega_6^2 v_2^2 - 3\omega_6 \omega_4 cs^2 - 3\omega_6 \omega_4 cs^2 \omega_8 + \omega_6^2 \omega_4 v_2^2 - \omega_6 \omega_4 \omega_8 v_2^2 + \omega_6 \omega_4 - \omega_4 \omega_8 + \omega_6^2$$

$$C_5 = -6\omega_6 \omega_4^2 \omega_8 v_2^2 + 3\omega_6^2 \omega_4^2 \omega_8 + 12\omega_6^2 \omega_4^2 v_2^2 - 12\omega_4^2 \omega_8 + 12\omega_6^2 \omega_4 + 42\omega_6^2 \omega_4 cs^2 \omega_8 - 12\omega_6^2 \omega_4^2 - 12\omega_6^2 \omega_4 cs^2 + 6\omega_6^2 \omega_4 \omega_8 v_2^2 - 18\omega_6 \omega_4^2 cs^2 \omega_8 - 12\omega_6 \omega_4^2 cs^2 - 3\omega_6^2 \omega_4^2 \omega_8 v_2^2 - 24\omega_6 \omega_4 cs^2 \omega_8 + 36\omega_4^2 cs^2 \omega_8 + 12\omega_6 \omega_4^2 - 24\omega_6^2 cs^2 \omega_8 + 12\omega_6^2 \omega_4^2 cs^2 - 12\omega_6^2 \omega_4 v_2^2 + 12\omega_4^2 \omega_8 v_2^2 + 6\omega_6 \omega_4^2 \omega_8 - 12\omega_6 \omega_4^2 v_2^2 - 11\omega_6^2 \omega_4^2 cs^2 \omega_8 - 6\omega_6^2 \omega_4 \omega_8$$

$$C_6 = 3\omega_4 \omega_8 v_2^2 - 3\omega_6 \omega_4 v_2^2 - \omega_6^2 cs^2 + 3\omega_6 \omega_8 v_2^2 + \omega_6 \omega_4 \omega_8 - \omega_6^2 \omega_4 + \omega_6^2 \omega_4 cs^2 - \omega_6 \omega_8 + \omega_4 cs^2 \omega_8 + \omega_6 cs^2 \omega_8 - 3\omega_6^2 v_2^2 - \omega_6 \omega_4 cs^2 - \omega_6 \omega_4 cs^2 \omega_8 + 3\omega_6^2 \omega_4 v_2^2 - 3\omega_6 \omega_4 \omega_8 v_2^2 + \omega_6 \omega_4 - \omega_4 \omega_8 + \omega_6^2$$

$$C_7 = -\omega_6^2 cs^2 - 12\omega_6 cs^4 + 12cs^4 - 14\omega_6 cs^2 v_2^2 + 7\omega_6^2 v_2^4 + 36\omega_6 v_2^2 - 36v_2^2 - 7\omega_6^2 v_2^2 + 24\omega_6^2 cs^2 v_2^2 + 144cs^2 v_2^2 - 36\omega_6 v_2^4 + 36v_2^4 + \omega_6^2 cs^4 + 12\omega_6 cs^2 - 12cs^2$$

$$C_8 = 30\omega_7 \omega_4^2 v_1^2 \omega_5^2 + 24\omega_7^2 \omega_4 cs^4 + 48\omega_7 \omega_4 cs^4 \omega_5^2 + 150\omega_7^2 \omega_4^2 cs^2 v_1^2 \omega_5 + 24\omega_4 v_1^2 \omega_5^2 + 12\omega_4^2 cs^4 \omega_5^2 - 48\omega_7 \omega_4 v_1^4 \omega_5 + 96\omega_7^2 \omega_4 v_1^2 \omega_5 + 14\omega_7^2 \omega_4^2 cs^4 \omega_5 - 48\omega_7 v_1^4 \omega_5^2 - 48\omega_7^2 v_1^2 \omega_5 + 288\omega_7^2 \omega_4 cs^2 v_1^2 - 3\omega_7^2 \omega_4^2 v_1^4 \omega_5^2 + 12\omega_7^2 \omega_4^2 cs^2 + 72\omega_7 \omega_4^2 cs^2 v_1^2 \omega_5 + 48\omega_7^2 \omega_4 cs^2 \omega_5 + 36\omega_7^2 \omega_4^2 v_1^4 \omega_5 - 126\omega_7 \omega_4^2 cs^2 v_1^2 \omega_5^2 - 24\omega_7 cs^4 \omega_5^2 - 24\omega_7^2 cs^2 \omega_5 - 216\omega_7 cs^2 v_1^2 \omega_5^2 - \omega_7^2 \omega_4^2 cs^4 \omega_5^2 - 36\omega_7^2 \omega_4^2 v_1^4 + 12\omega_7 \omega_4^2 cs^2 \omega_5^2 - 144\omega_4 cs^2 v_1^2 \omega_5^2 - 72\omega_7^2 \omega_4 v_1^2 + 96\omega_7 \omega_4 v_1^4 \omega_5^2 - 24\omega_7 \omega_4^2 v_1^2 \omega_5 + 24\omega_4 cs^2 \omega_5^2 - 12\omega_7^2 \omega_4^2 cs^2 v_1^2 \omega_5^2 + 12\omega_4^2 v_1^4 \omega_5^2 - 96\omega_7^2 \omega_4 v_1^4 \omega_5 - 14\omega_7^2 \omega_4^2 cs^2 \omega_5 + 36\omega_7^2 \omega_4^2 v_1^2 + 48\omega_7 v_1^2 \omega_5^2 + 48\omega_7^2 v_1^4 \omega_5 - 432\omega_7^2 \omega_4 cs^2 v_1^2 \omega_5 + 3\omega_7^2 \omega_4^2 v_1^2 \omega_5^2 - 30\omega_7 \omega_4^2 v_1^4 \omega_5^2 - 144\omega_7 \omega_4 cs^2 v_1^2 \omega_5 - 48\omega_7 \omega_4 cs^2 \omega_5^2 - 24\omega_4 v_1^4 \omega_5^2 + 72\omega_7^2 \omega_4 v_1^4 + 48\omega_7 \omega_4 v_1^2 \omega_5^2 + 216\omega_7^2 cs^2 v_1^2 \omega_5 - 12\omega_4^2 cs^2 \omega_5^2 - 12\omega_7 \omega_4^2 cs^4 \omega_5^2 - 96\omega_7 \omega_4 v_1^2 \omega_5^2 - 24\omega_4 cs^4 \omega_5^2 + 24\omega_7 \omega_4^2 v_1^4 \omega_5 + 432\omega_7 \omega_4 cs^2 v_1^2 \omega_5^2 - 24\omega_7^2 \omega_4 cs^2 - 12\omega_4^2 v_1^2 \omega_5^2 - 48\omega_7^2 \omega_4 cs^4 \omega_5 - 36\omega_7^2 \omega_4^2 v_1^2 \omega_5 - 12\omega_7^2 \omega_4^2 cs^4 + 24\omega_7 cs^2 \omega_5^2 + 24\omega_7^2 cs^4 \omega_5 + 72\omega_4^2 cs^2 v_1^2 \omega_5^2 + \omega_7^2 \omega_4^2 cs^2 \omega_5^2 - 144\omega_7^2 \omega_4^2 cs^2 v_1^2$$

$$C_9 = -51\omega_7 \omega_4^2 v_1^2 \omega_5^2 - 48\omega_4 v_1^2 \omega_5^2 + 12\omega_7 \omega_4^2 cs^2 \omega_5 - 36\omega_7^2 \omega_5 + 72\omega_7^2 \omega_4 \omega_5 - 168\omega_7^2 \omega_4 v_1^2 \omega_5 - 12\omega_4^2 \omega_5^2 + 84\omega_7^2 v_1^2 \omega_5 - 12\omega_7 \omega_4^2 \omega_5 - 36\omega_7^2 \omega_4^2 cs^2 + 21\omega_7 \omega_4^2 \omega_5^2 - 120\omega_7^2 \omega_4 cs^2 \omega_5 + 60\omega_7 cs^2 \omega_5 - 33\omega_7 \omega_4^2 cs^2 \omega_5^2 + 120\omega_7^2 \omega_4 v_1^2 + 36\omega_7 \omega_4^2 v_1^2 \omega_5 - 48\omega_4 cs^2 \omega_5^2 + 36\omega_7 \omega_5^2 + 24\omega_7^2 \omega_4^2 + 2\omega_7^2 \omega_4^2 \omega_5^2 +$$

$$\begin{aligned}
& 36\omega_6\omega_6^3\omega_7\omega_4^3\omega_8v_1^2v_2^2 + 12\omega_9\omega_6^3\omega_7\omega_4cs^4\omega_8 - 18\omega_6^2\omega_7\omega_4^3cs^2\omega_8^2v_2^2 + 12\omega_9\omega_6^2\omega_7\omega_4^3cs^2\omega_8 + 18\omega_9\omega_6^2\omega_4^3\omega_8^2v_1^2v_2^2 - 42\omega_9\omega_6^3\omega_7\omega_4^3cs^2\omega_8^2v_2^2 + \\
& 6\omega_6^3\omega_4^3\omega_8^2v_1^2 - 72\omega_9\omega_6^2\omega_7\omega_4^3\omega_8^2v_1^2v_2^2 + 24\omega_9\omega_6^2\omega_7\omega_4^3cs^2\omega_8v_1^2 - 12\omega_9\omega_6^2\omega_7\omega_4^3cs^2v_1^2 + 18\omega_6^3\omega_7\omega_4^3\omega_8^2v_1^2v_2^2 + 12\omega_6^3\omega_7\omega_4^3cs^2\omega_8v_1^2 - 6\omega_9\omega_6^2\omega_4^3\omega_8^2v_1^2 - \\
& 54\omega_9\omega_6\omega_7\omega_4^3\omega_8^2v_1^2v_2^2 + 36\omega_9\omega_7\omega_4^3\omega_8^2v_1^2v_2^2 + 24\omega_9\omega_6^2\omega_7\omega_4^3\omega_8^2v_1^2 + 12\omega_9\omega_6^2\omega_7\omega_4^3cs^4\omega_8 - 84\omega_9\omega_6^2\omega_7\omega_4^3cs^2\omega_8^2v_2^2 - 36\omega_9\omega_6^3\omega_7\omega_4^3\omega_8v_1^2 - \\
& 12\omega_9\omega_6\omega_7\omega_4^3\omega_8^2v_1^2 - 6\omega_6^2\omega_7\omega_4^3cs^2\omega_8^2v_1^2 + 12\omega_6^3\omega_7\omega_4^3cs^4\omega_8 + 12\omega_6^3\omega_7\omega_4^3\omega_8^2v_1^2 - 144\omega_9\omega_6^2\omega_7\omega_4^3cs^2\omega_8v_2^2 - 6\omega_6^2\omega_7\omega_4^3cs^4\omega_8^2 + 36\omega_6^3\omega_7\omega_4^3cs^2\omega_8v_2^2 + \\
& 24\omega_9\omega_6^2\omega_7\omega_4^3cs^2v_2^2 - 18\omega_6^3\omega_4^3\omega_8^2v_1^2v_2^2 - 36\omega_9\omega_6^3\omega_7\omega_4^3v_1^2v_2^2 + 12\omega_6^2\omega_7\omega_4^3cs^2\omega_8 + 6\omega_6^2\omega_7\omega_4^3\omega_8^2v_1^2 + 12\omega_6^2\omega_7\omega_4^3cs^4\omega_8 - 6\omega_9\omega_6\omega_7\omega_4^3cs^2\omega_8^2 - \\
& 24\omega_9\omega_6^2\omega_7\omega_4^3\omega_8v_1^2 - 6\omega_6^3\omega_7\omega_4^3cs^2\omega_8^2 - 24\omega_9\omega_6^3\omega_7\omega_4^3cs^2\omega_8v_1^2 - 36\omega_9\omega_6^2\omega_7\omega_4^3v_1^2v_2^2 + 36\omega_6^3\omega_4^3\omega_8^2v_1^2v_2^2 + 72\omega_9\omega_6\omega_7\omega_4^3cs^2\omega_8v_2^2 - 12\omega_6^3\omega_7\omega_4^3cs^4\omega_8^2 + \\
& 24\omega_9\omega_6^2\omega_7\omega_4^3cs^2v_2^2 - 36\omega_6^3\omega_7\omega_4^3cs^2\omega_8v_2^2 + 60\omega_9\omega_6^2\omega_7\omega_4^3cs^2\omega_8v_2^2 - 12\omega_9\omega_6\omega_7\omega_4^3cs^4\omega_8^2 - 36\omega_9\omega_6^2\omega_4^3\omega_8^2v_1^2v_2^2 + 6\omega_9\omega_6^2\omega_4^3cs^4\omega_8^2 + \\
& 24\omega_9\omega_6^3\omega_7\omega_4^3cs^2\omega_8 + 108\omega_9\omega_6^3\omega_7\omega_4^3\omega_8v_1^2v_2^2 - 12\omega_6^3\omega_4^3cs^2\omega_8^2 + 24\omega_9\omega_6^2\omega_7\omega_4^3cs^4\omega_8^2 + 36\omega_6^2\omega_7\omega_4^3\omega_8v_1^2v_2^2 + 60\omega_9\omega_6^3\omega_7\omega_4^3cs^2\omega_8v_2^2 + \\
& 12\omega_9\omega_6^2\omega_7\omega_4^3cs^4\omega_8 - 12\omega_9\omega_6\omega_7\omega_4^3cs^2\omega_8v_1^2 + 36\omega_9\omega_6\omega_7\omega_4^3\omega_8^2v_1^2v_2^2 + 12\omega_9\omega_6^2\omega_4^3cs^2\omega_8^2 - 12\omega_6^3\omega_7\omega_4^3cs^2\omega_8v_1^2 + 12\omega_9\omega_6\omega_7\omega_4^3\omega_8v_1^2 - 6\omega_6^3\omega_4^3cs^4\omega_8^2 - \\
& 12\omega_9\omega_6^2\omega_7\omega_4^3cs^2v_1^2 - 36\omega_6^3\omega_7\omega_4^3\omega_8^2v_1^2v_2^2 + 6\omega_9\omega_6^2\omega_7\omega_4^3cs^2\omega_8^2 + 18\omega_9\omega_6^2\omega_7\omega_4^3cs^2\omega_8v_1^2 + 18\omega_9\omega_6^2\omega_7\omega_4^3\omega_8^2v_1^2v_2^2 + 12\omega_6^3\omega_7\omega_4^3\omega_8v_1^2v_2^2
\end{aligned}$$

$$\begin{aligned}
C_{17} = & -4\omega_6^3\omega_4^3cs^2 + 84\omega_6^3\omega_4cs^2\omega_8v_2^2 + 4\omega_6^3cs^2\omega_8 + 20\omega_6\omega_4^3\omega_8v_2^2 + 72\omega_6\omega_4cs^2\omega_8^2v_2^2 + 96\omega_4^2cs^2\omega_8^2v_2^2 + 12\omega_6\omega_4cs^2\omega_8^2 + 36\omega_6^2cs^2\omega_8^2v_2^2 + \\
& 4\omega_6^2\omega_4^2cs^4\omega_8 - 24\omega_4^2\omega_8^2v_2^2 - 20\omega_6\omega_4\omega_8^2v_2^2 + 4\omega_6^2\omega_4^2v_2^2 - 13\omega_6^2\omega_4^2\omega_8v_2^4 - 20\omega_6^2\omega_4\omega_8v_2^2 - 8\omega_6^2\omega_8^2v_2^2 - 4\omega_6^3\omega_4cs^4 - 4\omega_6^3\omega_4^2cs^4\omega_8 - \\
& 84\omega_6^2\omega_4cs^2\omega_8^2v_2^2 - 36\omega_6\omega_4^2\omega_8^2v_2^4 + 32\omega_6^2\omega_4^2\omega_8^2v_2^4 + 4\omega_6^3\omega_4^2v_2^4 + 120\omega_6^2\omega_4^2cs^2\omega_8v_2^2 + 8\omega_6^2\omega_4cs^2\omega_8^2 + 8\omega_6^2\omega_4cs^2\omega_8 + 16\omega_6^2\omega_4\omega_8v_2^2 + \\
& 4\omega_6^2cs^4\omega_8^2 + 8\omega_6^3\omega_8v_2^2 - 4\omega_6^2\omega_4cs^4 - 20\omega_6^2\omega_4\omega_8^2v_2^4 + 4\omega_6\omega_4^2cs^2\omega_8 - 24\omega_6^3\omega_4cs^2v_2^2 - 72\omega_6\omega_4^2cs^2\omega_8v_2^2 + 4\omega_6^3\omega_4v_2^2 + 8\omega_4^2cs^4\omega_8^2 - 13\omega_6^2\omega_4^2\omega_8^2v_2^2 + \\
& 4\omega_6\omega_4cs^4\omega_8 - 4\omega_6^2\omega_4^2cs^2\omega_8^2 - 16\omega_6^2\omega_4\omega_8v_2^4 - 4\omega_6^3\omega_4^2v_2^2 - 32\omega_6^2\omega_4^2\omega_8v_2^2 - 144\omega_6\omega_4^2cs^2\omega_8^2v_2^2 - 51\omega_6^3\omega_4^2cs^2\omega_8v_2^2 + 4\omega_6^3\omega_4^2cs^2\omega_8 + 24\omega_6^3\omega_4^2cs^2v_2^2 - \\
& 4\omega_6^3cs^4\omega_8 + 13\omega_6^2\omega_4^2\omega_8^2v_2^4 - 4\omega_6^3\omega_4v_2^2 + 51\omega_6^2\omega_4cs^2\omega_8^2v_2^2 - 36\omega_6^3cs^2\omega_8v_2^2 + 4\omega_6^2\omega_4^2cs^2 - 8\omega_6^3\omega_8v_2^2 + 20\omega_6^2\omega_4\omega_8^2v_2^2 - 12\omega_6\omega_4^2cs^4\omega_8^2 - \\
& 4\omega_6^2cs^2\omega_8^2 - 4\omega_6\omega_4^2cs^4\omega_8 - 24\omega_6^2\omega_4^2cs^2v_2^2 + 4\omega_6^3\omega_4^2cs^4 - 4\omega_6\omega_4cs^2\omega_8^2 - 8\omega_4^2cs^2\omega_8^2 - 48\omega_6^2\omega_4cs^2\omega_8v_2^2 - 20\omega_6\omega_4^2\omega_8v_2^4 + 20\omega_6^3\omega_4\omega_8v_2^4 + \\
& 4\omega_6^3\omega_4cs^2 - 8\omega_6^2\omega_8v_2^2 - 8\omega_6^2\omega_4cs^4\omega_8^2 + 36\omega_6\omega_4^2\omega_8^2v_2^2 + 8\omega_6^2\omega_4cs^4\omega_8 + 20\omega_6\omega_4\omega_8^2v_2^2 + 24\omega_4^2\omega_8^2v_2^2 - 8\omega_6^2\omega_4^2cs^2\omega_8 - 4\omega_6^2\omega_4^2v_2^2 + 13\omega_6^2\omega_4^2\omega_8v_2^2
\end{aligned}$$

$$\begin{aligned}
C_{18} = & -12\omega_6^3\omega_4cs^2\omega_8v_2^2 + 12\omega_6^3\omega_4\omega_8^2v_2^4 - 12\omega_6\omega_3cs^4\omega_8^2 + 19\omega_6^2\omega_4^3\omega_8^2v_2^4 + 12\omega_6^3\omega_4^3cs^2\omega_8^2v_2^2 + 12\omega_6^3\omega_4^3cs^2v_2^2 + 12\omega_6\omega_4^3cs^2\omega_8^2 + 18\omega_6^3\omega_4^2\omega_8^2v_2^2 + \\
& 12\omega_6^3\omega_4^2v_2^4 - 36\omega_6\omega_4^3cs^2\omega_8v_2^2 + 6\omega_6^3\omega_4cs^2\omega_8^2 + 6\omega_6^2\omega_4^3cs^4\omega_8^2 - 60\omega_6^2\omega_4^3\omega_8v_2^2 - 6\omega_6^3\omega_4^3cs^2\omega_8 + 54\omega_6^2\omega_4^3cs^2\omega_8v_2^2 + 48\omega_6^3\omega_4^2\omega_8v_2^4 + 24\omega_6^3\omega_4\omega_8v_2^2 - \\
& \omega_6^3\omega_4^3cs^2\omega_8^2 - 18\omega_6^2\omega_4^3cs^4\omega_8 - 48\omega_6^2\omega_4cs^2\omega_8^2v_2^2 - 24\omega_6^2\omega_4^2\omega_8v_2^4 - 12\omega_6^3\omega_4^2v_2^4 - 12\omega_6^2\omega_4^3cs^2\omega_8v_2^2 + 6\omega_6^2\omega_4^3cs^2\omega_8 + 72\omega_4^3\omega_8^2v_2^4 + 27\omega_6^3\omega_4^3\omega_8v_2^2 - \\
& 48\omega_6^3cs^2\omega_8^2v_2^2 + 13\omega_6^3\omega_4^2cs^4\omega_8^2 + 12\omega_6^2\omega_4^2cs^4\omega_8 - 12\omega_6^3\omega_4cs^2\omega_8 + 90\omega_6\omega_4^3\omega_8^2v_2^2 + 252\omega_3^3cs^2\omega_8^2v_2^2 - 12\omega_4^3cs^2\omega_8^2 - 36\omega_6\omega_4^3\omega_8v_2^4 - \\
& 12\omega_6^2\omega_4^3\omega_8^2v_2^2 - 12\omega_6^2\omega_4^3cs^2v_2^2 - 81\omega_6^3\omega_4^3cs^2\omega_8^2v_2^2 + 12\omega_6^2\omega_4^3v_2^2 + 4\omega_6^3\omega_4^3\omega_8^2v_2^2 - 6\omega_6^2\omega_4^3cs^2\omega_8^2 - 24\omega_6^3\omega_4cs^4\omega_8^2 - 90\omega_6\omega_4^3\omega_8^2v_2^2 + 6\omega_6^3\omega_4^3cs^4\omega_8 + \\
& 12\omega_6^3\omega_4^2v_2^2 + 24\omega_6^2\omega_4^3\omega_8v_2^2 - 72\omega_4^3\omega_8^2v_2^2 + \omega_6^2\omega_4^3cs^4\omega_8^2 - 108\omega_6\omega_4^3cs^2\omega_8^2v_2^2 + 30\omega_6^3\omega_4^2cs^2\omega_8v_2^2 - 27\omega_6^3\omega_4^3\omega_8v_2^4 + 18\omega_6^3\omega_4^2cs^2\omega_8 - 12\omega_6^3\omega_4^2cs^2v_2^2 + \\
& 12\omega_6^2\omega_4^3\omega_8^2v_2^2 + 162\omega_6^2\omega_4^3cs^2\omega_8^2v_2^2 - 12\omega_6^2\omega_4^3v_2^4 - 4\omega_6^3\omega_4^3\omega_8^2v_2^2 + 12\omega_6^3\omega_4^3cs^2\omega_8^2 - 12\omega_6\omega_4^3cs^4\omega_8^2 + 36\omega_6\omega_4^3\omega_8v_2^2 - 19\omega_6^2\omega_4^3\omega_8^2v_2^2 + 12\omega_6^3cs^4\omega_8^2 + \\
& 60\omega_6^2\omega_4^3cs^2\omega_8^2v_2^2 - 18\omega_6^3\omega_4^2\omega_8^2v_2^4 - 12\omega_6^3\omega_4^2v_2^2 - 12\omega_6^3\omega_4\omega_8^2v_2^2 + 12\omega_6^3cs^4\omega_8^2 - 24\omega_6^3\omega_4\omega_8v_2^4 + 102\omega_3^3\omega_4cs^2\omega_8^2v_2^2 - 6\omega_6^2\omega_4^3cs^4\omega_8 - 5\omega_6^3\omega_4^2cs^2\omega_8^2 + \\
& 12\omega_6^3\omega_4cs^4\omega_8 - 12\omega_6^2\omega_4^3cs^2\omega_8 + 60\omega_6^2\omega_4^3\omega_8v_2^4 - 21\omega_6^3\omega_4^3cs^2\omega_8v_2^2 - 306\omega_6\omega_4^3cs^2\omega_8^2v_2^2 - \omega_6^3\omega_4^3cs^4\omega_8^2 - 48\omega_6^3\omega_4^3\omega_8v_2^2
\end{aligned}$$

$$\begin{aligned}
C_{19} = & 16\omega_6^3\omega_4^2cs^2 - 20\omega_6^3cs^2\omega_8 - 64\omega_6\omega_4^2\omega_8v_2^2 - 72\omega_6\omega_4^2cs^2\omega_8^2 - 8\omega_6^3\omega_4^2 - 40\omega_6^2\omega_4^2\omega_8 + 80\omega_4^2\omega_8^2v_2^2 + 64\omega_6\omega_4\omega_8^2v_2^2 - 16\omega_6^2\omega_4^2v_2^2 + \\
& 68\omega_6^3\omega_4\omega_8v_2^2 + 28\omega_6^2\omega_8^2v_2^2 - 16\omega_6^2\omega_4cs^2\omega_8 + 8\omega_6^2\omega_4^2 - 24\omega_6\omega_4\omega_8^2 - 32\omega_4^2\omega_8^2 - 28\omega_6^3\omega_4\omega_8 - 44\omega_6^2\omega_4cs^2\omega_8^2 + 44\omega_6^3\omega_4cs^2\omega_8 - 48\omega_6^2\omega_4\omega_8v_2^2 - \\
& 28\omega_6^3\omega_8v_2^2 + 8\omega_6^3\omega_4 - 17\omega_6^2\omega_4^2\omega_8^2 - 32\omega_6\omega_4^2cs^2\omega_8 - 16\omega_6^3\omega_4v_2^2 + 43\omega_6^2\omega_4^2\omega_8^2v_2^2 + 25\omega_6^2\omega_4^2cs^2\omega_8^2 + 28\omega_6^2\omega_4\omega_8^2 + 16\omega_6^3\omega_4^2v_2^2 + 104\omega_6^2\omega_4^2\omega_8v_2^2 - \\
& 25\omega_6^3\omega_4^2cs^2\omega_8 + 48\omega_6\omega_4^2\omega_8^2 + 17\omega_6^3\omega_4^2\omega_8 - 16\omega_6^2\omega_4^2cs^2 - 68\omega_6^2\omega_4\omega_8^2v_2^2 + 20\omega_6^2cs^2\omega_8^2 - 12\omega_6^2\omega_8^2 + 32\omega_6\omega_4cs^2\omega_8^2 + 48\omega_4^2cs^2\omega_8^2 + 24\omega_6\omega_4^2\omega_8 + \\
& 12\omega_6^3\omega_8 - 16\omega_6^2\omega_4cs^2 - 120\omega_6\omega_4^2\omega_8^2v_2^2 + 56\omega_6^2\omega_4^2cs^2\omega_8 - 43\omega_6^2\omega_4\omega_8v_2^2 + 16\omega_6^2\omega_4\omega_8
\end{aligned}$$

$$\begin{aligned}
C_{20} = & 12 - 9\omega_6^3v_2^4 - 78\omega_6^2cs^2 - 216\omega_6cs^4 + 144cs^4 - 34\omega_3^3cs^2v_2^2 - 18\omega_6 - 1008\omega_6cs^2v_2^2 + 6\omega_6^3cs^2 + 90\omega_6^2v_2^4 + 234\omega_6v_2^2 - 156v_2^2 - 98\omega_6^2v_2^2 + \\
& 404\omega_6^2cs^2v_2^2 - 5\omega_6^3cs^4 + 672cs^2v_2^2 - 216\omega_6v_2^4 + 144v_2^4 + 82\omega_6^2cs^4 - \omega_6^3 + 10\omega_6^3v_2^2 + 8\omega_6^2 + 198\omega_6cs^2 - 132cs^2
\end{aligned}$$

$$\begin{aligned}
C_{21} = & 12 - 29\omega_6^3v_2^4 - 22\omega_6^2cs^2 - 36\omega_6cs^4 + 24cs^4 - 18\omega_6^3cs^2v_2^2 - 18\omega_6 - 648\omega_6cs^2v_2^2 + 2\omega_6^3cs^2 + 310\omega_6^2v_2^4 + 378\omega_6v_2^2 - 252v_2^2 - 154\omega_6^2v_2^2 + \\
& 252\omega_6^2cs^2v_2^2 - \omega_6^3cs^4 + 432cs^2v_2^2 - 756\omega_6v_2^4 + 504v_2^4 + 14\omega_6^2cs^4 - \omega_6^3 + 14\omega_6^3v_2^2 + 8\omega_6^2 + 54\omega_6cs^2 - 36cs^2
\end{aligned}$$

2.4 CLBM1

2.4.1 Definitions

Collision operator C :

$$C(f) = \mathbf{K}^{-1} \mathbf{S} \left(\kappa^{(eq)} - \mathbf{K}f \right),$$

where

$$\mathbf{S} = \text{diag}(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9),$$

$$\omega_1, \omega_2, \dots, \omega_9 \in (0, 2).$$

Matrix \mathbf{K} corresponds to the transformation matrix to the central moment basis defined by

$$\kappa = \left(k_{(0,0)}, k_{(1,0)}, k_{(0,1)}, k_{(1,1)}, k_{(2,0)}, k_{(0,2)}, k_{(2,1)}, k_{(1,2)}, k_{(2,2)} \right)^T,$$

and is given by

$$\begin{aligned}
\mathbf{K}_{1,i} &= (\mathbf{c}_i - \mathbf{v})^{(0,0)} \\
\mathbf{K}_{2,i} &= (\mathbf{c}_i - \mathbf{v})^{(1,0)} \\
\mathbf{K}_{3,i} &= (\mathbf{c}_i - \mathbf{v})^{(0,1)} \\
\mathbf{K}_{4,i} &= (\mathbf{c}_i - \mathbf{v})^{(1,1)} \\
\mathbf{K}_{5,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,0)} \\
\mathbf{K}_{6,i} &= (\mathbf{c}_i - \mathbf{v})^{(0,2)} \\
\mathbf{K}_{7,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,1)} \\
\mathbf{K}_{8,i} &= (\mathbf{c}_i - \mathbf{v})^{(1,2)} \\
\mathbf{K}_{9,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,2)},
\end{aligned}$$

$\forall i \in \{1, 2, \dots, 9\}$.

The equilibrium central moments are defined by

$$\boldsymbol{\kappa}^{(eq)} = \mathbf{KM}^{-1} \boldsymbol{\mu}^{(eq)},$$

i.e.,

$$\boldsymbol{\kappa}^{(eq)} = \left(\rho, 0, 0, 0, \rho c_s^2, \rho c_s^2, 0, 0, \rho c_s^4 \right)^T.$$

2.4.2 Conservation of mass: ρ



attached text file: output_d2q9_nse_clbm1_symbolic_pde_00.txt

$$\begin{aligned}
& \frac{\partial \rho}{\partial t} + \frac{v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_l v_2}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + v_1^2 + 3cs^2) \frac{v_1 \delta_l^3}{12 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + 3v_1^2 + cs^2) \frac{\rho \delta_l^3}{12 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\
& \frac{\rho \delta_l^3 cs^2}{6 \delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{\rho \delta_l^3 cs^2}{6 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3cs^2 + v_2^2) \frac{\delta_l^3 v_2}{12 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + cs^2 + 3v_2^2) \frac{\rho \delta_l^3}{12 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& (-6v_1^2 + 3\omega_5 v_1^2 + \omega_5 cs^2 + 24v_1^2 cs^2 - 2cs^2 - \omega_5 cs^4 + 2cs^4 - 12\omega_5 v_1^2 cs^2 + 6v_1^4 - 3\omega_5 v_1^4) \frac{\delta_l^4}{24 \omega_5 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& (-4 + 10v_1^2 + 2\omega_5 - 5\omega_5 v_1^2 - 3\omega_5 cs^2 + 6cs^2) \frac{\rho v_1 \delta_l^4}{12 \omega_5 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + \\
& (3\omega_5 - 3\omega_5 v_1^2 - 9\omega_5 cs^2 + 3\omega_5 \omega_7 cs^2 - \omega_5 \omega_7 + v_1^2 \omega_7 - \omega_7 + 3\omega_7 cs^2 + \omega_5 v_1^2 \omega_7) \frac{\rho v_1 \delta_l^4}{12 \omega_5 \delta_t \omega_7} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& (-2 + \omega_4) \frac{\delta_l^4 cs^4}{6 \delta_t \omega_4} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\
& (-3\omega_6 v_2^2 - \omega_8 + \omega_8 v_2^2 - \omega_8 \omega_6 + 3\omega_8 \omega_6 cs^2 + 3\omega_6 - 9\omega_6 cs^2 + 3\omega_8 cs^2 + \omega_8 \omega_6 v_2^2) \frac{\rho \delta_l^4 v_2}{12 \omega_8 \omega_6 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& (3\omega_6 v_2^2 + 6v_2^4 - \omega_6 cs^4 - 2cs^2 - 12\omega_6 v_2^2 cs^2 + 24v_2^2 cs^2 + \omega_6 cs^2 + 2cs^4 - 3\omega_6 v_2^4 - 6v_2^2) \frac{\delta_l^4}{24 \omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\
& (-4 - 5\omega_6 v_2^2 + 6cs^2 + 2\omega_6 - 3\omega_6 cs^2 + 10v_2^2) \frac{\rho \delta_l^4 v_2}{12 \omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0.
\end{aligned}$$

2.4.3 Conservation of momentum: ρv_1



attached text file: output_d2q9_nse_clbm1_symbolic_pde_01.txt

$$\begin{aligned}
& v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (v_1^2 + cs^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2\rho v_1 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_1 \delta_l v_2}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho \delta_l v_2}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{\rho v_1 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\
& (-2 + 6v_1^2 + \omega_5 - 3\omega_5 v_1^2 + 4cs^2 - 2\omega_5 cs^2) \frac{\delta_l^2}{\omega_5 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + (2 - \omega_5) \frac{3\rho v_1 \delta_l^2}{\omega_5 \delta_t} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + (-2 + \omega_4) \frac{\delta_l^2 cs^2}{2 \delta_t \omega_4} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} +
\end{aligned}$$

$$\begin{aligned}
& (-2 + \omega_4) \frac{\delta_t^2 cs^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (-2 + 2v_1^2 + \omega_5 - \omega_5 v_1^2 + 6cs^2 - 3\omega_5 cs^2) \frac{v_1 \delta_t^2}{2\omega_5 \delta_t} \frac{\partial^2 \rho}{\partial x_1^2} + \\
& (-2 + 6v_1^2 + \omega_5 - 3\omega_5 v_1^2 + 2cs^2 - \omega_5 cs^2) \frac{\rho \delta_t^2}{2\omega_5 \delta_t} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega_4) \frac{\rho \delta_t^2 cs^2}{2\delta_t \omega_4} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_4) \frac{\rho \delta_t^2 cs^2}{2\delta_t \omega_4} \frac{\partial^2 v_1}{\partial x_2^2} + \\
& C_1 \frac{\delta_t^3}{12\omega_5^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-24 + 60v_1^2 + 24\omega_5 - 60\omega_5 v_1^2 + 5\omega_5^2 cs^2 + 36cs^2 - 36\omega_5 cs^2 + 11\omega_5^2 v_1^2 - 4\omega_5^2) \frac{\rho v_1 \delta_t^3}{6\omega_5^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + \\
& C_2 \frac{\rho v_1 \delta_t^3}{12\omega_5^2 \delta_t \omega_7 \omega_4} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + (-12 - \omega_4^2 + 12\omega_4) \frac{\delta_t^3 cs^4}{6\delta_t \omega_4^2} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} - \frac{\rho v_1 \delta_t^3 cs^2}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3cs^2 + v_2^2) \frac{v_1 \delta_t^3 v_2}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + \\
& C_3 \frac{\rho \delta_t^3 v_2}{6\omega_8 \delta_t \omega_4} \frac{\partial^3 v_1}{\partial x_2^3} + (-1 + cs^2 + 3v_2^2) \frac{\rho v_1 \delta_t^3}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_4 \frac{v_1 \delta_t^4}{12\omega_5^3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + C_5 \frac{\rho \delta_t^4}{12\omega_5^3 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_6 \frac{\rho \delta_t^4}{12\omega_5^3 \delta_t \omega_7 \omega_4^3} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& C_7 \frac{v_1 \delta_t^4 cs^2}{12\omega_8 \omega_5^3 \omega_9 \delta_t \omega_7 \omega_4^2} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_8 \frac{\rho \delta_t^4 cs^2}{12\omega_8 \omega_5^3 \omega_9 \delta_t \omega_7 \omega_4^3} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_9 \frac{\delta_t^4 cs^2 v_2}{12\omega_8^2 \omega_5 \omega_9 \omega_6 \delta_t \omega_7 \omega_4^2} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + \\
& (3\omega_8 cs^2 - 3\omega_6 v_2^2 - \omega_8 + \omega_8 v_2^2 - \omega_8 \omega_6 - 9\omega_6 cs^2 + 3\omega_6 + 3\omega_8 \omega_6 cs^2 + \omega_8 \omega_6 v_2^2) \frac{\rho v_1 \delta_t^4 v_2}{12\omega_8 \omega_6 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& C_{10} \frac{\rho \delta_t^4 cs^2}{12\omega_8 \omega_5 \omega_9 \omega_6 \delta_t \omega_7 \omega_4^3} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\
& (3\omega_6 v_2^2 + 6v_2^2 + 2cs^4 + \omega_6 cs^2 - 2cs^2 - \omega_6 cs^4 - 3\omega_6 v_2^4 - 12\omega_6 cs^2 v_2^2 + 24cs^2 v_2^2 - 6v_2^2) \frac{v_1 \delta_t^4}{24\omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{11} \frac{\rho \delta_t^4}{24\omega_5^3 \delta_t \omega_4^3} \frac{\partial^4 v_1}{\partial x_2^4} \\
& + (-4 - 5\omega_6 v_2^2 - 3\omega_6 cs^2 + 2\omega_6 + 6cs^2 + 10v_2^2) \frac{\rho v_1 \delta_t^4 v_2}{12\omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= 7\omega_5^2 v_1^4 - 36v_1^2 - 144\omega_5 v_1^2 cs^2 + 36\omega_5 v_1^2 + 12cs^4 - \omega_5^2 cs^2 - 12\omega_5 cs^4 - 12cs^2 + \omega_5^2 cs^4 + 12\omega_5 cs^2 + 144v_1^2 cs^2 - 7\omega_5^2 v_1^2 + 36v_1^4 - 36\omega_5 v_1^4 + 24\omega_5^2 v_1^2 cs^2 \\
C_2 &= -6\omega_5 v_1^2 \omega_7 \omega_4 + 36cs^2 \omega_7 \omega_4 + 12v_1^2 \omega_7 \omega_4 + 12\omega_5 \omega_4 + 6\omega_5 \omega_7 \omega_4 - 18\omega_5 cs^2 \omega_7 \omega_4 + 18\omega_5^2 cs^2 \omega_7 + 6\omega_5^2 v_1^2 \omega_7 + 36\omega_5^2 cs^2 \omega_4 + 12\omega_5^2 v_1^2 \omega_4 - \\
& 36\omega_5^2 cs^2 - 11\omega_5^2 cs^2 \omega_7 \omega_4 - 12\omega_5 v_1^2 \omega_4 - 12\omega_7 \omega_4 - 3\omega_5^2 v_1^2 \omega_7 \omega_4 - 6\omega_5^2 \omega_7 - 36\omega_5 cs^2 \omega_4 + 3\omega_5^2 \omega_7 \omega_4 - 12\omega_5^2 v_1^2 - 12\omega_5^2 \omega_4 + 12\omega_5^2 \\
C_3 &= 6 + 9\omega_8 cs^2 + 9cs^2 \omega_4 - 3\omega_8 - \omega_8 v_2^2 \omega_4 + 3\omega_8 v_2^2 - 18cs^2 + \omega_8 \omega_4 + 3v_2^2 \omega_4 - 3\omega_4 - 6v_2^2 - 3\omega_8 cs^2 \omega_4 \\
C_4 &= 12 + 6\omega_5^3 cs^2 + 90\omega_5^2 v_1^4 - 156v_1^2 - 18\omega_5 - 1008\omega_5 v_1^2 cs^2 + 234\omega_5 v_1^2 - 34\omega_5^3 v_1^2 cs^2 - 9\omega_5^3 v_1^4 + 144cs^4 - 78\omega_5^2 cs^2 - 216\omega_5 cs^4 + 10\omega_5^3 v_1^2 - \\
& 132cs^2 + 82\omega_5^2 cs^4 + 198\omega_5 cs^2 + 672v_1^2 cs^2 - \omega_5^3 - 5\omega_5^3 cs^4 - 98\omega_5^2 v_1^2 + 144v_1^4 - 216\omega_5 v_1^4 + 404\omega_5^2 v_1^2 cs^2 + 8\omega_5^2 \\
C_5 &= 12 + 2\omega_5^3 cs^2 + 310\omega_5^2 v_1^4 - 252v_1^2 - 18\omega_5 - 648\omega_5 v_1^2 cs^2 + 378\omega_5 v_1^2 - 18\omega_5^3 v_1^2 cs^2 - 29\omega_5^3 v_1^4 + 24cs^4 - 22\omega_5^2 cs^2 - 36\omega_5 cs^4 + 14\omega_5^3 v_1^2 - \\
& 36cs^2 + 14\omega_5^2 cs^4 + 54\omega_5 cs^2 + 432v_1^2 cs^2 - \omega_5^3 - \omega_5^3 cs^4 - 154\omega_5^2 v_1^2 + 504v_1^4 - 756\omega_5 v_1^4 + 252\omega_5^2 v_1^2 cs^2 + 8\omega_5^2 \\
C_6 &= 36\omega_5^2 v_1^2 \omega_4^3 - 108\omega_5^3 v_1^2 cs^2 \omega_4^2 + 6\omega_5^3 cs^2 \omega_7 \omega_4 + 4\omega_5^3 v_1^4 \omega_7 \omega_4^3 - 6\omega_5^2 cs^2 \omega_7 \omega_4^2 + 36\omega_5^3 v_1^4 \omega_4^3 - \omega_5^3 cs^4 \omega_7 \omega_4^3 - 36\omega_5 v_1^4 \omega_7 \omega_4^3 - 6\omega_5^3 v_1^4 \omega_7 \omega_4^2 - \\
& 18\omega_5^3 v_1^2 cs^2 \omega_7 \omega_4 - 19\omega_5^3 v_1^2 \omega_7 \omega_4^3 + 108\omega_5^3 v_1^2 cs^2 \omega_4^3 - 36\omega_5^3 v_1^4 \omega_4^2 + 13\omega_5^3 cs^4 \omega_7 \omega_4^2 - \omega_5^2 cs^2 \omega_7 \omega_4^3 + 198\omega_5^2 v_1^2 cs^2 \omega_7 \omega_4^3 - 108\omega_5^2 v_1^2 cs^2 \omega_4^3 - \\
& 4\omega_5^3 v_1^2 \omega_7 \omega_4^3 - 24\omega_5^3 cs^4 \omega_7 \omega_4 + 12\omega_5^3 cs^4 \omega_7^2 - 3\omega_5^3 v_1^2 cs^2 \omega_7 \omega_4^2 + 6\omega_5^2 cs^4 \omega_7 \omega_4^2 + 36\omega_5 v_1^2 \omega_7 \omega_4^3 + 19\omega_5^2 v_1^4 \omega_7 \omega_4^3 + 6\omega_5^3 v_1^2 \omega_7 \omega_4^2 + 36\omega_5^2 v_1^2 cs^2 \omega_7 \omega_4^2 - \\
& 108\omega_5 v_1^2 cs^2 \omega_7 \omega_4^3 + \omega_5^2 cs^4 \omega_7 \omega_4^3 + 12\omega_5^3 v_1^2 cs^2 \omega_7 \omega_4^3 - 5\omega_5^3 cs^2 \omega_7 \omega_4^2 - 36\omega_5^3 v_1^2 \omega_7 \omega_4^2 + 72\omega_5^2 v_1^4 \omega_7 \omega_4^3 + 18\omega_5^2 v_1^2 cs^2 \omega_7 \omega_4^2 + 12\omega_5 cs^2 \omega_7 \omega_4^2 - \\
& 306\omega_5 v_1^2 cs^2 \omega_7 \omega_4^3 + 18\omega_5^3 cs^2 \omega_7 \omega_4^2 + 36\omega_5^3 v_1^2 \omega_4^2 - 99\omega_5^3 v_1^2 cs^2 \omega_7 \omega_4^3 - 6\omega_5^2 cs^4 \omega_7 \omega_4^3 - 36\omega_5^2 v_1^4 \omega_4^3 + 12\omega_5 cs^2 \omega_7 \omega_4^3 + 60\omega_5^2 v_1^2 cs^2 \omega_7 \omega_4^3 - \\
& 72v_1^2 \omega_7 \omega_4^3 + 39\omega_5^3 v_1^2 \omega_7 \omega_4 + 12\omega_5^3 cs^4 \omega_7 \omega_4 + 12\omega_5^2 cs^4 \omega_7 \omega_4 - 36\omega_5^2 v_1^4 \omega_4^3 - 6\omega_5^2 cs^2 \omega_7 \omega_4^3 - 12cs^2 \omega_7 \omega_4^3 + 252v_1^2 cs^2 \omega_7 \omega_4^3 + 54\omega_5^3 v_1^2 cs^2 \omega_7 \omega_4^2 - \\
& 36\omega_5 v_1^2 cs^2 \omega_7 \omega_4^2 + 90\omega_5 v_1^2 \omega_7 \omega_4^3 - 72\omega_5^2 v_1^2 \omega_7 \omega_4^3 + 36\omega_5^3 v_1^2 cs^2 \omega_7 \omega_4 + 36\omega_5^3 v_1^4 \omega_7 \omega_4^2 - 12\omega_5 cs^4 \omega_7 \omega_4^2 + 6\omega_5^2 cs^2 \omega_7 \omega_4^3 - 18\omega_5^3 cs^4 \omega_7 \omega_4^2 - \\
& 12\omega_5 cs^4 \omega_7 \omega_4^3 + 72v_1^4 \omega_7 \omega_4^3 - 12\omega_5^2 cs^2 \omega_7 \omega_4 - 39\omega_5^3 v_1^4 \omega_7 \omega_4^3 + 6\omega_5^3 cs^4 \omega_7 \omega_4^3 + 12cs^4 \omega_7 \omega_4^3 - 12\omega_5^2 cs^2 \omega_7 \omega_4^2 - 90\omega_5 v_1^2 \omega_7 \omega_4^3 \\
C_7 &= -12\omega_5^2 v_1^2 \omega_9 \omega_7^2 \omega_4 - 36\omega_8 \omega_5^2 \omega_9 cs^2 \omega_4^2 + 36\omega_8 \omega_5^2 \omega_9 cs^2 \omega_7^2 + 36\omega_5^3 cs^2 \omega_7^2 \omega_4 - \omega_8 \omega_5^2 \omega_9 \omega_7^2 \omega_4^2 - 12\omega_8 \omega_5^3 v_1^2 \omega_7 \omega_4^2 - 18\omega_8 \omega_5^2 \omega_9 \omega_7^2 \omega_4 - \\
& 36\omega_8 \omega_5^3 cs^2 \omega_7 \omega_4^2 + 54\omega_8 \omega_5^3 \omega_9 cs^2 \omega_7 \omega_4 + 12\omega_5^3 v_1^2 \omega_7^2 \omega_4 - 36\omega_8 \omega_5^3 \omega_9 cs^2 \omega_4 + 12\omega_8 \omega_5 \omega_9 \omega_7 \omega_4^2 + 18\omega_8 \omega_5^2 v_1^2 \omega_9 \omega_7^2 \omega_4 + 18\omega_8 \omega_5^3 v_1^2 \omega_9 \omega_7 \omega_4 + \\
& 54\omega_8 \omega_5^2 \omega_9 cs^2 \omega_7^2 \omega_4 - 12\omega_8 \omega_5^3 \omega_9 \omega_4^2 - 36\omega_5^2 \omega_9 cs^2 \omega_7^2 \omega_4 - 6\omega_8 \omega_5^3 \omega_7^2 \omega_4 + 12\omega_8 \omega_5^3 \omega_9 \omega_7 + 18\omega_5^2 \omega_9 cs^2 \omega_7^2 \omega_4 + 12\omega_8 \omega_5^3 \omega_7^2 \omega_4 - 12\omega_8 \omega_5^3 \omega_9 \omega_7^2 - \\
& 6\omega_8 \omega_5^3 v_1^2 \omega_9 \omega_7 \omega_4^2 - 6\omega_8 \omega_5^2 \omega_9 cs^2 \omega_7^2 \omega_4^2 - 12\omega_8 \omega_5 v_1^2 \omega_9 \omega_7^2 \omega_4 + 12\omega_8 \omega_5^3 \omega_9 \omega_4 + 2\omega_8 \omega_5^2 \omega_9 \omega_7^2 \omega_4^2 - 36\omega_8 \omega_5^2 \omega_9 cs^2 \omega_7^2 + 12\omega_8 \omega_5^2 v_1^2 \omega_7 \omega_4^2 - \\
& 12\omega_8 \omega_5^2 \omega_7 \omega_4^2 - 6\omega_5^2 v_1^2 \omega_7^2 \omega_4^2 - 18\omega_8 \omega_5^3 \omega_9 cs^2 \omega_7 \omega_4^2 + 36\omega_8 \omega_5^3 cs^2 \omega_7 \omega_4 - 36\omega_8 \omega_5 \omega_9 cs^2 \omega_7^2 \omega_4 + 2\omega_8 \omega_5^2 v_1^2 \omega_9 \omega_7^2 \omega_4 + \\
& 6\omega_5^2 v_1^2 \omega_9 \omega_7^2 \omega_4 + 36\omega_8 \omega_5^2 cs^2 \omega_7 \omega_4^2 + 12\omega_8 \omega_5^3 v_1^2 \omega_7 \omega_4 - 18\omega_5^3 cs^2 \omega_7^2 \omega_4 + 12\omega_8 \omega_5^3 \omega_9 \omega_7^2 \omega_4 - 36\omega_8 \omega_5^3 \omega_9 cs^2 \omega_7 - 12\omega_5^3 \omega_7^2 \omega_4 + 5\omega_8 \omega_5^3 \omega_9 cs^2 \omega_7^2 \omega_4 - \\
& 36\omega_8 \omega_5^3 cs^2 \omega_7^2 \omega_4 + 6\omega_8 \omega_5^2 \omega_7^2 \omega_4 - 12\omega_8 \omega_5^3 v_1^2 \omega_9 \omega_4 - 6\omega_8 \omega_5^2 v_1^2 \omega_7^2 \omega_4 - 18\omega_8 \omega_5^2 \omega_9 \omega_7 \omega_4^2 + 18\omega_8 \omega_5^2 v_1^2 \omega_9 \omega_7 \omega_4^2 - 36\omega_8 \omega_5 \omega_9 cs^2 \omega_7 \omega_4^2 + \\
& 12\omega_8 \omega_5^3 v_1^2 \omega_9 \omega_7^2 - 18\omega_8 \omega_5^3 \omega_9 \omega_7 \omega_4 - 12\omega_8 \omega_5^3 v_1^2 \omega_7 \omega_4 + 12\omega_8 \omega_5^2 \omega_9 \omega_4^2 - 12\omega_8 \omega_5^2 v_1^2 \omega_9 \omega_4^2 - 18\omega_8 \omega_5^2 cs^2 \omega_7^2 \omega_4 + 12\omega_5^2 \omega_9 \omega_7^2 \omega_4 - 12\omega_8 \omega_5^3 \omega_7 \omega_4 - \\
& 12\omega_8 \omega_9 \omega_7^2 \omega_4 + \omega_8 \omega_5^3 v_1^2 \omega_9 \omega_7^2 \omega_4 - 12\omega_8 \omega_5 v_1^2 \omega_9 \omega_7 \omega_4^2 + 54\omega_8 \omega_5^2 \omega_9 cs^2 \omega_7 \omega_4^2 - 12\omega_8 \omega_5^3 v_1^2 \omega_9 \omega_7^2 \omega_4 + 12\omega_8 \omega_5^3 \omega_7 \omega_4^2 + 36\omega_8 \omega_9 cs^2 \omega_7^2 \omega_4^2 - \\
& 12\omega_8 \omega_5^3 v_1^2 \omega_9 \omega_7 + 6\omega_8 \omega_5^3 v_1^2 \omega_7^2 \omega_4 + 12\omega_8 v_1^2 \omega_9 \omega_7 \omega_4^2 + 6\omega_8 \omega_5^3 \omega_9 \omega_7 \omega_4^2 - 6\omega_5^2 \omega_9 \omega_7^2 \omega_4 + 12\omega_8 \omega_5^3 v_1^2 \omega_9 \omega_4^2 + 6\omega_5^3 \omega_7^2 \omega_4 + 18\omega_8 \omega_5^3 cs^2 \omega_7^2 \omega_4^2 - \\
& 40\omega_8 \omega_5^3 \omega_9 cs^2 \omega_7^2 \omega_4 + 12\omega_8 \omega_5 \omega_9 \omega_7^2 \omega_4 - 12\omega_8 \omega_5^2 v_1^2 \omega_9 \omega_7^2 + 12\omega_8 \omega_5^2 \omega_9 \omega_7^2 \\
C_8 &= 36\omega_8 \omega_5 v_1^2 \omega_9 \omega_4^2 + 12\omega_8 \omega_5 cs^2 \omega_4^3 - 24\omega_8 \omega_5^2 \omega_9 cs^2 \omega_4^2 + 36\omega_8 \omega_5 v_1^2 \omega_4^3 - 36\omega_8 v_1^2 \omega_9 \omega_7 \omega_4^2 - 12\omega_8 \omega_5^2 \omega_9 cs^2 \omega_7 + 6\omega_8 \omega_9 cs^2 \omega_7 \omega_4^3 - \\
& 24\omega_8 \omega_5 \omega_9 \omega_7 \omega_4^2 + 18\omega_8 v_1^2 \omega_9 \omega_7 \omega_4^3 - 36\omega_8 \omega_5 v_1^2 \omega_9 \omega_4^3 + 12\omega_8 \omega_5^2 \omega_9 cs^2 \omega_4^3 + 6\omega_8 \omega_5 \omega_9 \omega_7 \omega_4^3 - 12\omega_8 \omega_9 cs^2 \omega_7 \omega_4^2 + 12\omega_8 \omega_9 \omega_7 \omega_4^2 + 12\omega_8 \omega_5^2 \omega_4^3 - \\
& 6\omega_8 \omega_5^2 \omega_7 \omega_4^3 + 18\omega_8 \omega_5^2 v_1^2 \omega_7 \omega_4^3 + 6\omega_8 \omega_5^2 cs^2 \omega_7 \omega_4^3 - 12\omega_8 \omega_5 \omega_9 \omega_4^2 - 36\omega_8 \omega_5^2 v_1^2 \omega_7 \omega_4^2 + 12\omega_8 \omega_5^2 \omega_7 \omega_4^2 - 6\omega_8 \omega_9 \omega_7 \omega_4^3 - 12\omega_8 \omega_5^2 \omega_4^2 + 12\omega_8 \omega_5 \omega_9 \omega_7 \omega_4 + \\
& 12\omega_8 \omega_5^2 \omega_9 cs^2 \omega_4^2 - 12\omega_8 \omega_5^2 cs^2 \omega_7 \omega_4^2 + 12\omega_8 \omega_5 \omega_9 \omega_4^2 + 6\omega_8 \omega_5 \omega_7 \omega_4^3 + 18\omega_5 v_1^2 \omega_9 \omega_7 \omega_4^3 + 24\omega_8 \omega_5 \omega_9 cs^2 \omega_7 \omega_4^2 + 6\omega_5^2 \omega_7 \omega_4^3 - \omega_8 \omega_5^2 \omega_9 cs^2 \omega_7 \omega_4^3 + \\
& 12\omega_5 \omega_9 \omega_7^2 \omega_4 - 18\omega_8 \omega_5 v_1^2 \omega_9 \omega_7 \omega_4^3 - 12\omega_5 \omega_9 cs^2 \omega_7 \omega_4^2 + 24\omega_8 \omega_5^2 \omega_9 \omega_4^2 - 72\omega_8 \omega_5^2 v_1^2 \omega_9 \omega_4^2 + 12\omega_8 \omega_5 \omega_9 cs^2 \omega_4^2 - 12\omega_8 \omega_5 \omega_4^3 - 36\omega_5 v_1^2 \omega_9 \omega_7 \omega_4^2 - \\
& 12\omega_5 \omega_7 \omega_4^2 - 6\omega_8 \omega_5 \omega_9 cs^2 \omega_7 \omega_4^3 + 36\omega_8 \omega_5^2 v_1^2 \omega_9 \omega_4^3 - 12\omega_8 \omega_5 \omega_9 cs^2 \omega_7 \omega_4^2 - 12\omega_8 \omega_5^2 \omega_9 \omega_4^2 + 6\omega_5 \omega_9 cs^2 \omega_7 \omega_4^3 + 4\omega_8 \omega_5^2 \omega_9 cs^2 \omega_7 \omega_4^2 - \\
& 6\omega_5 \omega_9 \omega_7 \omega_4^3 - 18\omega_5^2 v_1^2 \omega_7 \omega_4^3 - 36\omega_8 \omega_5^2 v_1^2 \omega_4^3 - 12\omega_8 \omega_5^2 cs^2 \omega_4^3 - 36\omega_8 \omega_5 v_1^2 \omega_9 \omega_7 \omega_4 - 6\omega_8 \omega_5 cs^2 \omega_7 \omega_4^3 + 18\omega_8 \omega_5^2 \omega_9 cs^2 \omega_7 \omega_4 - 18\omega_8 \omega_5 v_1^2 \omega_7 \omega_4^3 - \\
& 6\omega_5^2 cs^2 \omega_7 \omega_4^3 + 12\omega_8 \omega_5^2 cs^2 \omega_4^2 - 12\omega_8 \omega_5^2 \omega_9 \omega_4 + 36\omega_8 \omega_5^2 v_1^2 \omega_4^3 + 36\omega_5^2 v_1^2 \omega_7 \omega_4^2 + 36\omega_8 \omega_5^2 v_1^2 \omega_9 \omega_4 + 12\omega_5^2 cs^2 \omega_7 \omega_4^2 - 12\omega_8 \omega_5 \omega_9 cs^2 \omega_7 \omega_4
\end{aligned}$$

$$C_9 = 18\omega_8^2\omega_5\omega_6cs^2\omega_7\omega_4^2 - 36\omega_8^2\omega_5\omega_9\omega_6cs^2\omega_7\omega_4 + 3\omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^2 - 12\omega_8^2\omega_5\omega_6\omega_4 + 18\omega_8^2\omega_9cs^2\omega_7\omega_4^2 + 18\omega_8^2\omega_5\omega_9v_2^2\omega_7\omega_4 - 12\omega_8^2\omega_5\omega_6v_2^2\omega_4^2 + 12\omega_8^2\omega_5\omega_9\omega_6\omega_7\omega_4 - 12\omega_8^2\omega_5\omega_9v_2^2\omega_4 - 36\omega_8\omega_5\omega_6\omega_7\omega_4^2 + 6\omega_5\omega_9\omega_6v_2^2\omega_7\omega_4^2 + 36\omega_8^2\omega_6cs^2\omega_4^2 + 12\omega_8^2\omega_5\omega_6\omega_7\omega_4 - 3\omega_8\omega_5\omega_9\omega_6v_2^2\omega_7\omega_4^2 + 5\omega_8^2\omega_5\omega_9\omega_7\omega_4^2 + 12\omega_8^2\omega_9\omega_4^2 + 12\omega_8^2\omega_5\omega_9\omega_6v_2^2\omega_7 + 12\omega_8\omega_5\omega_9\omega_6\omega_7 - 6\omega_8^2\omega_5\omega_6\omega_7\omega_4^2 - 12\omega_8\omega_5\omega_9\omega_6v_2^2\omega_7 - 36\omega_8^2\omega_9cs^2\omega_4^2 - 18\omega_8^2\omega_5\omega_9\omega_7\omega_4 + 18\omega_8\omega_5\omega_9\omega_6v_2^2\omega_7\omega_4 + 12\omega_8^2\omega_5\omega_6v_2^2\omega_4 - \omega_8^2\omega_5\omega_9\omega_6\omega_7\omega_4^2 - 18\omega_8\omega_5\omega_6cs^2\omega_7\omega_4^2 + 12\omega_8^2\omega_5\omega_9v_2^2\omega_4^2 + 6\omega_8\omega_5\omega_6\omega_7\omega_4^2 - 12\omega_8\omega_9\omega_6v_2^2\omega_7\omega_4 + 12\omega_5\omega_9\omega_6\omega_7\omega_4 - 36\omega_8^2\omega_5\omega_6cs^2\omega_7\omega_4 + 3\omega_8^2\omega_5\omega_9\omega_6cs^2\omega_7\omega_4^2 - 12\omega_8^2\omega_5\omega_9v_2^2\omega_7 + 12\omega_8^2\omega_5\omega_6\omega_4^2 - 6\omega_8^2\omega_6v_2^2\omega_7\omega_4^2 - 18\omega_8\omega_5\omega_9\omega_6\omega_7\omega_4 - 12\omega_8^2\omega_5\omega_9\omega_6\omega_7 - 5\omega_8^2\omega_5\omega_9v_2^2\omega_7\omega_4^2 + 12\omega_8\omega_5\omega_6v_2^2\omega_7\omega_4 + 18\omega_8\omega_9\omega_6cs^2\omega_7\omega_4^2 - 12\omega_8^2\omega_6\omega_4^2 - 36\omega_8\omega_5\omega_9\omega_6cs^2\omega_7 + 6\omega_8^2\omega_5\omega_6v_2^2\omega_7\omega_4^2 - 12\omega_8^2\omega_5\omega_9\omega_6v_2^2\omega_7\omega_4 - 12\omega_8^2\omega_9v_2^2\omega_4^2 + 54\omega_8^2\omega_5\omega_9cs^2\omega_7\omega_4 + 6\omega_8^2\omega_9v_2^2\omega_7\omega_4^2 + 6\omega_8^2\omega_6\omega_7\omega_4^2 + 36\omega_8^2\omega_5\omega_6cs^2\omega_4 + 36\omega_8^2\omega_5\omega_9cs^2\omega_4^2 - 9\omega_8\omega_5\omega_9\omega_6cs^2\omega_7\omega_4^2 - 36\omega_8^2\omega_5\omega_9cs^2\omega_7 + 12\omega_8^2\omega_5\omega_9\omega_4 + 54\omega_8\omega_5\omega_9\omega_6cs^2\omega_7\omega_4 - 12\omega_8^2\omega_5\omega_9\omega_4^2 - 36\omega_8^2\omega_5\omega_6cs^2\omega_4^2 + 12\omega_8^2\omega_5\omega_9\omega_7 - 6\omega_8^2\omega_9\omega_7\omega_4^2 - 36\omega_8^2\omega_5\omega_9cs^2\omega_4 - 18\omega_8^2\omega_6cs^2\omega_7\omega_4^2 + 12\omega_8^2\omega_6v_2^2\omega_4^2 - 12\omega_8^2\omega_5\omega_6v_2^2\omega_7\omega_4 + \omega_8^2\omega_5\omega_9\omega_6v_2^2\omega_7\omega_4^2 - 15\omega_8^2\omega_5\omega_9cs^2\omega_7\omega_4^2 - 6\omega_8\omega_5\omega_6v_2^2\omega_7\omega_4^2 + 36\omega_8^2\omega_5\omega_9\omega_6cs^2\omega_7 - 36\omega_5\omega_9\omega_6cs^2\omega_7\omega_4$$

$$C_{10} = \omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^2 - 36\omega_8\omega_5\omega_6v_2^2\omega_4^3 + 6\omega_8\omega_9cs^2\omega_7\omega_4^3 + 54\omega_8\omega_5\omega_9v_2^2\omega_7\omega_4^2 + 6\omega_8\omega_6\omega_7\omega_4^3 + 18\omega_5\omega_9\omega_6\omega_7\omega_4^2 - 54\omega_5\omega_9\omega_6v_2^2\omega_7\omega_4^2 - 18\omega_8\omega_5\omega_9\omega_7\omega_4^2 + 36\omega_8\omega_5\omega_6v_2^2\omega_4^2 - 36\omega_8\omega_9v_2^2\omega_4^3 - 3\omega_8\omega_5\omega_9\omega_6v_2^2\omega_7\omega_4^2 - 6\omega_5\omega_9\omega_6\omega_7\omega_4^3 + 5\omega_8\omega_5\omega_9\omega_7\omega_4^3 + 18\omega_5\omega_9\omega_6v_2^2\omega_7\omega_4^3 - 15\omega_8\omega_5\omega_9v_2^2\omega_7\omega_4^2 - 6\omega_8\omega_5\omega_6cs^2\omega_7\omega_4^3 + 6\omega_8\omega_6cs^2\omega_7\omega_4^3 - 6\omega_5\omega_6cs^2\omega_7\omega_4^3 - 36\omega_8\omega_5\omega_9v_2^2\omega_4^3 + 36\omega_8\omega_6v_2^2\omega_4^3 + 12\omega_8\omega_5\omega_9\omega_4^2 - 12\omega_8\omega_5\omega_9\omega_6cs^2\omega_7\omega_4^2 - 36\omega_8\omega_6\omega_7\omega_4^3 + 12\omega_8\omega_6\omega_4^3 + 36\omega_5\omega_9\omega_6v_2^2\omega_7\omega_4^2 - 6\omega_8\omega_9\omega_7\omega_4^3 - 18\omega_8\omega_5\omega_6v_2^2\omega_7\omega_4^3 + 12\omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^2 + 12\omega_8\omega_5\omega_9\omega_7\omega_4^2 - 12\omega_8\omega_6\omega_4^3 + 36\omega_5\omega_9\omega_6v_2^2\omega_7\omega_4^2 - 12\omega_5\omega_9\omega_6\omega_7\omega_4^2 + 36\omega_8\omega_5\omega_9v_2^2\omega_4^3 - 12\omega_8\omega_5\omega_9\omega_4^2 + 12\omega_5\omega_6cs^2\omega_7\omega_4^2 + 18\omega_8\omega_9v_2^2\omega_7\omega_4^3 - 18\omega_5\omega_9\omega_6cs^2\omega_7\omega_4^2 + 18\omega_8\omega_5\omega_9cs^2\omega_7\omega_4^2 - 12\omega_8\omega_5\omega_9\omega_6cs^2\omega_7 - \omega_8\omega_5\omega_9\omega_6cs^2\omega_7\omega_4^2 + 12\omega_8\omega_6cs^2\omega_4^3 - 12\omega_8\omega_5\omega_9cs^2\omega_4^2 - 5\omega_8\omega_5\omega_9cs^2\omega_7\omega_4^3 + 6\omega_5\omega_9\omega_6cs^2\omega_7\omega_4^3 + 12\omega_8\omega_5\omega_9cs^2\omega_4^3 - 5\omega_8\omega_5\omega_9\omega_6cs^2\omega_7\omega_4^2 + 12\omega_8\omega_5\omega_6\omega_4^3 - 18\omega_5\omega_6v_2^2\omega_7\omega_4^3 - 12\omega_8\omega_5\omega_6cs^2\omega_4^3 + 18\omega_8\omega_5\omega_9\omega_6cs^2\omega_7\omega_4 + 6\omega_5\omega_6\omega_7\omega_4^3 + 12\omega_8\omega_9\omega_4^3 + 18\omega_8\omega_5\omega_6v_2^2\omega_7\omega_4^3 - 12\omega_5\omega_6\omega_7\omega_4^2 + 36\omega_5\omega_6v_2^2\omega_7\omega_4^2 - 12\omega_8\omega_9cs^2\omega_4^2 + 12\omega_8\omega_5\omega_6cs^2\omega_4^2 - 12\omega_8\omega_5\omega_6\omega_4^2 - 36\omega_8\omega_5\omega_6v_2^2\omega_7\omega_4^2 + 12\omega_5\omega_9\omega_6cs^2\omega_7\omega_4 - 12\omega_8\omega_5\omega_9cs^2\omega_7\omega_4 - 6\omega_8\omega_6cs^2\omega_7\omega_4^3$$

$$C_{11} = 6\omega_8cs^4\omega_4^3 + 3\omega_8^2v_2^4\omega_4^3 - 72v_1^4\omega_4^3 + 108cs^2v_2^2\omega_4^3 - 36\omega_8^2cs^2v_2^2\omega_4 - 12\omega_8^2v_1^4\omega_4^3 + 36v_2^4\omega_4^3 - 24\omega_8cs^4\omega_4^2 - 216cs^2v_2^2\omega_4^2 + 24\omega_8^2cs^4 + 12\omega_8^2cs^2\omega_4 + 6\omega_8^2cs^2v_2^2\omega_4^2 - 8\omega_8^2cs^2\omega_4^2 - 72\omega_8v_2^2\omega_4^2 + 24\omega_8cs^4\omega_4 + 30\omega_8v_2^2\omega_4^3 + \omega_8^2cs^2\omega_4^3 - 12\omega_8^2cs^2v_2^2\omega_4^2 - 36v_2^2\omega_4^3 + 72\omega_8cs^2v_2^2\omega_4 + 12\omega_8^2v_2^2\omega_4^2 + 24\omega_8cs^2\omega_4^2 - 48\omega_8^2cs^4\omega_4 - 6\omega_8cs^2\omega_4^2 + 72v_2^2\omega_4^2 - 3\omega_8^2v_2^2\omega_4^3 - 30\omega_8v_1^4\omega_4^3 - 3\omega_8^2cs^4\omega_4^3 - 72\omega_8cs^2v_2^2\omega_4^3 + 24\omega_8^2cs^4\omega_4^2 + 72\omega_8v_1^4\omega_4^2 + 144\omega_8cs^2v_2^2\omega_4^2 - 24\omega_8cs^2\omega_4^2$$

2.4.4 Conservation of momentum: ρv_2



attached text file: output_d2q9_nse_clbm1_symbolic_pde_02.txt

$$\begin{aligned} & v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{v_1 \delta_1 v_2}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho \delta_1 v_2}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\rho v_1 \delta_1}{\delta_t} \frac{\partial v_2}{\partial x_1} + (cs^2 + v_2^2) \frac{\delta_1}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{2\rho \delta_1 v_2}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_4) \frac{\delta_1^2 cs^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\ & (-2 + \omega_4) \frac{\delta_1^2 cs^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (-2 - 3\omega_6 v_2^2 - 2cs^2 \omega_6 + 4cs^2 + \omega_6 + 6v_2^2) \frac{\delta_1^2}{\omega_6 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (2 - \omega_6) \frac{3\rho \delta_1^2 v_2}{\omega_6 \delta_t} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + \\ & (-2 + \omega_4) \frac{\rho \delta_1^2 cs^2}{2\delta_t \omega_4} \frac{\partial^2 v_2}{\partial x_1^2} + (-2 + \omega_4) \frac{\rho \delta_1^2 cs^2}{2\delta_t \omega_4} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + (-2 - \omega_6 v_2^2 - 3cs^2 \omega_6 + 6cs^2 + \omega_6 + 2v_2^2) \frac{\delta_1^2 v_2}{2\omega_6 \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + \\ & (-2 - 3\omega_6 v_2^2 - cs^2 \omega_6 + 2cs^2 + \omega_6 + 6v_2^2) \frac{\rho \delta_1^2}{2\omega_6 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + (-1 + v_1^2 + 3cs^2) \frac{v_1 \delta_1^3 v_2}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + 3v_1^2 + cs^2) \frac{\rho \delta_1^3 v_2}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + \\ & C_1 \frac{\rho v_1 \delta_1^3}{6\delta_t \omega_7 \omega_4} \frac{\partial^3 v_2}{\partial x_1^3} + (-12 - \omega_4^2 + 12\omega_4) \frac{\delta_1^3 cs^4}{6\delta_t \omega_4} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} - \frac{\rho \delta_1^3 cs^2 v_2}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + C_2 \frac{\rho \delta_1^3 v_2}{12\omega_8 \omega_6^2 \delta_t \omega_4} \frac{\partial^3 v_1}{\partial x_1 \partial x_2} + C_3 \frac{\delta_1^3}{12\omega_8^2 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\ & (-24 - 60\omega_6 v_2^2 - 36cs^2 \omega_6 + 5cs^2 \omega_6^2 + 36cs^2 + 24\omega_6 - 4\omega_6^2 + 60v_2^2 + 11\omega_6^2 v_2^2) \frac{\rho \delta_1^3 v_2}{6\omega_6^2 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & (-6v_1^2 + 3\omega_5 v_1^2 + \omega_5 cs^2 + 24v_1^2 cs^2 - 2cs^2 - \omega_5 cs^4 + 2cs^4 - 12\omega_5 v_1^2 cs^2 + 6v_1^4 - 3\omega_5 v_1^4) \frac{\delta_1^4 v_2}{24\omega_5 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\ & (-4 + 10v_1^2 + 2\omega_5 - 5\omega_5 v_1^2 - 3\omega_5 cs^2 + 6cs^2) \frac{\rho v_1 \delta_1^4 v_2}{12\omega_5 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_4 \frac{\rho \delta_1^4}{24\delta_t \omega_7^2 \omega_4^3} \frac{\partial^4 v_2}{\partial x_1^4} + C_5 \frac{v_1 \delta_1^4 cs^2}{12\omega_8 \omega_5 \omega_9 \omega_6 \delta_t \omega_7^2 \omega_4^2} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + \\ & C_6 \frac{\rho \delta_1^4 cs^2}{12\omega_8 \omega_5 \omega_9 \omega_6 \delta_t \omega_7 \omega_4^3} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + \\ & (3\omega_5 cs^2 \omega_7 + 3\omega_5 - 3\omega_5 v_1^2 - 9\omega_5 cs^2 - \omega_5 \omega_7 + v_1^2 \omega_7 + 3cs^2 \omega_7 - \omega_7 + \omega_5 v_1^2 \omega_7) \frac{\rho v_1 \delta_1^4 v_2}{12\omega_5 \delta_t \omega_7} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\ & C_7 \frac{\delta_1^4 cs^2 v_2}{12\omega_8^2 \omega_9 \omega_6^2 \delta_t \omega_7 \omega_4^2} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_8 \frac{\rho \delta_1^4 cs^2}{12\omega_8 \omega_9 \omega_6^2 \delta_t \omega_7 \omega_4^3} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_9 \frac{\rho \delta_1^4}{12\omega_8^2 \omega_6^2 \delta_t \omega_4^3} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{10} \frac{\delta_1^4 v_2}{12\omega_8^2 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{11} \frac{\rho \delta_1^4}{12\omega_8^2 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} \\ & = 0, \end{aligned}$$

where:

$$\begin{aligned} C_1 &= 6 - 6v_1^2 - v_1^2 \omega_7 \omega_4 + 3v_1^2 \omega_4 - 18cs^2 + 3v_1^2 \omega_7 + 9cs^2 \omega_7 + \omega_7 \omega_4 - 3\omega_7 - 3cs^2 \omega_7 \omega_4 + 9cs^2 \omega_4 - 3\omega_4 \\ C_2 &= 36\omega_8 cs^2 \omega_4 + 12\omega_6^2 v_2^2 \omega_4 - 6\omega_8 \omega_6 v_2^2 \omega_4 + 6\omega_8 \omega_6^2 v_2^2 - 6\omega_8 \omega_6^2 + 6\omega_8 \omega_6 \omega_4 - 12\omega_6^2 \omega_4 + 12\omega_8 v_2^2 \omega_4 + 36cs^2 \omega_6^2 \omega_4 - 36cs^2 \omega_6^2 - \\ & 18\omega_8 cs^2 \omega_6 \omega_4 - 11\omega_8 cs^2 \omega_6^2 \omega_4 + 12\omega_6^2 - 36cs^2 \omega_6 \omega_4 - 12\omega_8 \omega_4 + 18\omega_8 cs^2 \omega_6^2 - 3\omega_8 \omega_6^2 v_2^2 \omega_4 - 12\omega_6 v_2^2 \omega_4 + 12\omega_6 \omega_4 + 3\omega_8 \omega_6^2 \omega_4 - 12\omega_6^2 v_2^2 \\ C_3 &= 36\omega_6 v_2^2 + 144cs^2 v_2^2 + 7\omega_6^2 v_4^2 + 12cs^2 \omega_6 + 36v_2^4 - cs^2 \omega_6^2 - 144cs^2 \omega_6 v_2^2 - 12cs^2 + 24cs^2 \omega_6^2 v_2^2 - 12cs^4 \omega_6 + 12cs^4 - 36\omega_6 v_2^4 - 36v_2^2 - 7\omega_6^2 v_2^2 + cs^4 \omega_6^2 \end{aligned}$$

$$C_4 = 12cs^2\omega_7^2\omega_4 - 36v_1^2cs^2\omega_7^2\omega_4 - 30v_1^4\omega_7\omega_4^3 - 216v_1^2cs^2\omega_4^3 - 3cs^4\omega_7^2\omega_4^3 + 72v_1^4\omega_7\omega_4^2 + 24cs^4\omega_7^2\omega_4^2 + 108v_1^2cs^2\omega_4^3 - 48cs^4\omega_7^2\omega_4 - 36v_1^2\omega_4^3 + 30v_1^4\omega_7\omega_4^2 + 24cs^4\omega_7^2 + cs^2\omega_4^2\omega_4^3 + 6v_1^2cs^2\omega_7^2\omega_4^3 - 72v_1^2\omega_7\omega_4^2 + 72v_1^2\omega_4^2 - 12v_1^2cs^2\omega_7^2\omega_4^2 - 8cs^2\omega_7^2\omega_4^2 + 12v_1^2\omega_7^2\omega_4^2 + 144v_1^2cs^2\omega_7\omega_4^2 + 24cs^2\omega_7\omega_4^2 + 24cs^4\omega_7\omega_4 - 3v_1^2\omega_7^2\omega_4^3 - 6cs^2\omega_7\omega_4^3 - 72v_1^2cs^2\omega_7\omega_4^3 - 12v_1^4\omega_7^2\omega_4^3 - 72v_1^4\omega_4^3 - 24cs^4\omega_7\omega_4^2 - 24cs^2\omega_7\omega_4^2 + 72v_1^2cs^2\omega_7\omega_4^2 + 36v_1^4\omega_4^3 + 3v_1^4\omega_7^2\omega_4^3 + 6cs^4\omega_7\omega_4^3$$

$$C_5 = 12\omega_5\omega_6\omega_7^2\omega_4^2 + 54\omega_8\omega_9cs^2\omega_6\omega_7^2\omega_4 + 3\omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^2 - 12\omega_8\omega_5\omega_6\omega_7\omega_4 - 36\omega_9cs^2\omega_6\omega_7^2\omega_4 + 3\omega_8\omega_5\omega_9cs^2\omega_6\omega_7^2\omega_4 - 6\omega_8\omega_5v_1^2\omega_7^2\omega_4^2 + 18\omega_8\omega_5\omega_9cs^2\omega_6\omega_7^2\omega_4 - 18\omega_8\omega_5\omega_6\omega_7^2\omega_4 - 18\omega_8\omega_5cs^2\omega_6\omega_7\omega_4^2 + 12\omega_8\omega_5\omega_9\omega_6\omega_4 + 6\omega_8\omega_5v_1^2\omega_6\omega_7^2\omega_4 - 12\omega_8v_1^2\omega_9\omega_6\omega_7^2 - 36\omega_8\omega_5\omega_9cs^2\omega_6\omega_7 + 18\omega_8\omega_9cs^2\omega_7^2\omega_4^2 - 12\omega_5v_1^2\omega_6\omega_7^2\omega_4^2 + 18\omega_8\omega_5v_1^2\omega_9\omega_6\omega_7\omega_4 - 12\omega_8\omega_5\omega_9\omega_6\omega_7^2 + 12\omega_5v_1^2\omega_6\omega_7^2\omega_4 + 36\omega_8\omega_5\omega_9cs^2\omega_6\omega_7^2 - 3\omega_8\omega_5v_1^2\omega_9\omega_6\omega_7\omega_4^2 + 6\omega_8\omega_5\omega_7^2\omega_4^2 + 12\omega_8\omega_5\omega_9\omega_6\omega_7 + 5\omega_8\omega_9\omega_6\omega_7^2\omega_4^2 + 36\omega_8\omega_5cs^2\omega_6\omega_7\omega_4 - 36\omega_8\omega_5\omega_9cs^2\omega_6\omega_4 - 12\omega_8\omega_5v_1^2\omega_6\omega_7^2\omega_4 - 6\omega_8\omega_5\omega_9\omega_6\omega_4^2 + 12\omega_9\omega_7^2\omega_4^2 + 36\omega_5cs^2\omega_7^2\omega_4^2 - 12v_1^2\omega_9\omega_7^2\omega_4^2 + 12\omega_8\omega_9\omega_6\omega_7^2 + 6\omega_8\omega_5\omega_6\omega_7\omega_4^2 + 36\omega_9cs^2\omega_6\omega_7^2\omega_4^2 - 36\omega_8\omega_5\omega_9cs^2\omega_6\omega_7^2\omega_4 - 12\omega_5\omega_6\omega_7^2\omega_4 - 15\omega_8\omega_9cs^2\omega_6\omega_7^2\omega_4^2 - 18\omega_8\omega_5\omega_9\omega_6\omega_7\omega_4 + 54\omega_8\omega_5\omega_9cs^2\omega_6\omega_7\omega_4 - 12\omega_5\omega_7^2\omega_4^2 - 6\omega_8\omega_5\omega_6\omega_7^2\omega_4^2 - 12\omega_8\omega_5v_1^2\omega_9\omega_6\omega_4 + 12\omega_8\omega_5\omega_9\omega_6\omega_7^2\omega_4 + 12\omega_5v_1^2\omega_7^2\omega_4^2 + 12\omega_8\omega_5v_1^2\omega_9\omega_6\omega_7^2 - 6\omega_8\omega_9\omega_7^2\omega_4^2 - 12v_1^2\omega_9\omega_6\omega_7^2\omega_4 + 36\omega_5cs^2\omega_6\omega_7^2\omega_4 + 12\omega_9\omega_6\omega_7^2\omega_4 + \omega_8\omega_5v_1^2\omega_9\omega_6\omega_7^2\omega_4^2 + 18\omega_8v_1^2\omega_9\omega_6\omega_7^2\omega_4 - 36\omega_8\omega_5cs^2\omega_6\omega_7^2\omega_4 - 36\omega_9cs^2\omega_7^2\omega_4^2 + 12\omega_8\omega_5v_1^2\omega_6\omega_7\omega_4 + 18\omega_8\omega_5cs^2\omega_6\omega_7^2\omega_4^2 - 5\omega_8v_1^2\omega_9\omega_6\omega_7^2\omega_4^2 - 6\omega_8\omega_5v_1^2\omega_6\omega_7\omega_4^2 - 36\omega_5cs^2\omega_6\omega_7^2\omega_4^2 - 12\omega_9\omega_6\omega_7^2\omega_4^2 + 12v_1^2\omega_9\omega_6\omega_7^2\omega_4^2 - 12\omega_8\omega_5v_1^2\omega_9\omega_6\omega_7^2\omega_4 - \omega_8\omega_5\omega_9\omega_6\omega_7^2\omega_4^2 - 18\omega_8\omega_5cs^2\omega_7^2\omega_4^2 + 6\omega_8v_1^2\omega_9\omega_7^2\omega_4^2 - 12\omega_8\omega_5v_1^2\omega_9\omega_6\omega_7 - 36\omega_8\omega_9cs^2\omega_6\omega_7^2 - 9\omega_8\omega_5\omega_9cs^2\omega_6\omega_7\omega_4^2 + 6\omega_8\omega_5v_1^2\omega_9\omega_6\omega_4^2 + 12\omega_8\omega_5\omega_6\omega_7^2\omega_4$$

$$C_6 = \omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^2 + 6\omega_8\omega_5\omega_9cs^2\omega_6\omega_4^3 - 15\omega_8v_1^2\omega_9\omega_6\omega_7\omega_4^3 + 6\omega_8\omega_5cs^2\omega_6\omega_7\omega_4^3 + 36v_1^2\omega_9\omega_6\omega_7\omega_4^3 - 12\omega_9\omega_6\omega_7\omega_4^3 - 12\omega_5cs^2\omega_6\omega_7\omega_4^3 + 18\omega_8v_1^2\omega_9\omega_7\omega_4^3 - 6\omega_8\omega_5cs^2\omega_7\omega_4^3 + 54\omega_8v_1^2\omega_9\omega_6\omega_7\omega_4^2 - 18\omega_8\omega_5\omega_9cs^2\omega_6\omega_4^2 - 12\omega_8\omega_5cs^2\omega_6\omega_7\omega_4^2 - 12\omega_8\omega_5\omega_9\omega_6\omega_4 - 12\omega_8\omega_5\omega_9cs^2\omega_6\omega_7 - 36v_1^2\omega_9\omega_6\omega_7\omega_4^2 + 12\omega_5cs^2\omega_6\omega_7\omega_4^2 + 12\omega_9\omega_6\omega_7\omega_4^2 - 12\omega_5\omega_6\omega_7\omega_4^2 - 6\omega_8\omega_5v_1^2\omega_9\omega_6\omega_7\omega_4^2 + 12\omega_8\omega_5cs^2\omega_6\omega_4^2 + 36\omega_5v_1^2\omega_7\omega_4^3 + 18\omega_8\omega_5\omega_9\omega_6\omega_4^2 - 6\omega_8\omega_5cs^2\omega_6\omega_4^3 - 6\omega_8\omega_9\omega_7\omega_4^3 + 12\omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^2 - 12\omega_9cs^2\omega_7\omega_4^3 - 6\omega_8\omega_5\omega_9\omega_6\omega_4^3 + 6\omega_8\omega_5\omega_7\omega_4^3 + 18\omega_8\omega_5\omega_9cs^2\omega_6\omega_7\omega_4 - 12\omega_9cs^2\omega_6\omega_7\omega_4^2 + 36\omega_8\omega_5v_1^2\omega_9\omega_6\omega_4 - 18\omega_8\omega_5v_1^2\omega_6\omega_4^3 + 18\omega_8\omega_9cs^2\omega_6\omega_7\omega_4^2 + 5\omega_8\omega_9\omega_6\omega_7\omega_4^2 + 12\omega_9cs^2\omega_6\omega_7\omega_4^3 - 36v_1^2\omega_9\omega_7\omega_4^3 + 12\omega_5cs^2\omega_7\omega_4^3 + 12\omega_9\omega_7\omega_4^3 - 18\omega_8\omega_9\omega_6\omega_7\omega_4^2 - 5\omega_8\omega_9cs^2\omega_6\omega_7\omega_4^3 + 36\omega_8\omega_5v_1^2\omega_6\omega_4^2 + 12\omega_8\omega_9\omega_6\omega_7\omega_4 + 6\omega_8\omega_5\omega_6\omega_4^3 + 12\omega_5\omega_6\omega_7\omega_4^3 - 36\omega_8\omega_5v_1^2\omega_6\omega_7\omega_4^2 + 18\omega_8\omega_5v_1^2\omega_9\omega_6\omega_4^3 + 36\omega_5v_1^2\omega_6\omega_7\omega_4^2 - 18\omega_8\omega_5v_1^2\omega_7\omega_4^3 - \omega_8\omega_5\omega_9cs^2\omega_6\omega_7\omega_4^3 + 18\omega_8\omega_5v_1^2\omega_6\omega_7\omega_4^3 - 12\omega_8\omega_9cs^2\omega_6\omega_7\omega_4 - 12\omega_5\omega_6\omega_7\omega_4^2 - 12\omega_8\omega_5\omega_6\omega_4^2 - 36\omega_5v_1^2\omega_6\omega_7\omega_4^3 + 6\omega_8\omega_9cs^2\omega_7\omega_4^3 - 5\omega_8\omega_5\omega_9cs^2\omega_6\omega_7\omega_4^2 - 54\omega_8\omega_5v_1^2\omega_9\omega_6\omega_4^2$$

$$C_7 = 6\omega_8^2\omega_9\omega_6^2v_2^2\omega_4^2 + 36\omega_8^2\omega_9cs^2\omega_6^3\omega_7 + 12\omega_8^2\omega_9\omega_6^2\omega_4 - 12\omega_8^2\omega_9\omega_6^2v_2^2\omega_7 - 12\omega_8^2\omega_9\omega_6v_2^2\omega_7\omega_4^2 - 12\omega_8^2\omega_9\omega_6^3\omega_7 + 6\omega_8^2\omega_6^3\omega_4^2 - 18\omega_8^2cs^2\omega_6^3\omega_4^2 - 36\omega_8^2\omega_9cs^2\omega_6^2\omega_4 + 12\omega_9\omega_6^3\omega_7\omega_4 - 12\omega_8\omega_6^3\omega_7\omega_4 - 36\omega_8\omega_9cs^2\omega_6^3\omega_7 - 6\omega_8^2\omega_9cs^2\omega_6^2\omega_7\omega_4^2 - 18\omega_8^2\omega_9\omega_6^2\omega_7\omega_4 - 18\omega_8^2\omega_9\omega_6^2\omega_7\omega_4^2 - 12\omega_8\omega_9\omega_6v_2^2\omega_7\omega_4^2 - 12\omega_9\omega_6^3\omega_7\omega_4^2 + 18\omega_8^2\omega_9cs^2\omega_6^2\omega_4^2 + 36\omega_8^2cs^2\omega_6^3\omega_4 + 12\omega_8\omega_6^3\omega_7\omega_4^2 + 2\omega_8^2\omega_9\omega_6^2\omega_7\omega_4^2 + 54\omega_8^2\omega_9cs^2\omega_6^2\omega_7\omega_4 - 36\omega_8^2\omega_9cs^2\omega_6^2\omega_7 + 54\omega_8\omega_9cs^2\omega_6^2\omega_7\omega_4^2 + 36\omega_8cs^2\omega_6^2\omega_7\omega_4^2 + 6\omega_8^2\omega_6^2\omega_7\omega_4^2 + 12\omega_8^2\omega_9\omega_6^2\omega_7 - 18\omega_8^2cs^2\omega_6^2\omega_7\omega_4^2 - 12\omega_8^2\omega_9\omega_6^2v_2^2\omega_4 - 6\omega_8^2\omega_9\omega_6^2\omega_4^2 - 18\omega_8\omega_9\omega_6^2\omega_7\omega_4^2 + 6\omega_8^2\omega_6^2v_2^2\omega_7\omega_4^2 + 12\omega_8^2\omega_9\omega_6\omega_7\omega_4^2 - 36\omega_9cs^2\omega_6^3\omega_7\omega_4 + 12\omega_8^2\omega_9\omega_6^2\omega_7\omega_4 + 5\omega_8^2\omega_9cs^2\omega_6^3\omega_7\omega_4^2 - 36\omega_8\omega_9cs^2\omega_6\omega_7\omega_4^2 - 12\omega_8\omega_9\omega_6^3v_2^2\omega_7 - 6\omega_8^2\omega_6^3v_2^2\omega_4 + 36\omega_8cs^2\omega_6^3\omega_7\omega_4 + 54\omega_8\omega_9cs^2\omega_6^2\omega_7\omega_4 + 12\omega_8^2\omega_9v_2^2\omega_7\omega_4^2 + 12\omega_8^2\omega_6^2\omega_7\omega_4 - 12\omega_8^2\omega_9\omega_6^2v_2^2\omega_7 + 12\omega_8\omega_9\omega_6^3\omega_7 - 36\omega_6^2cs^2\omega_6^3\omega_7\omega_4 + 12\omega_9\omega_6^3v_2^2\omega_7\omega_4^2 + 12\omega_8\omega_9\omega_6\omega_7\omega_4^2 - 12\omega_8\omega_6^3v_2^2\omega_7\omega_4^2 - 6\omega_8\omega_9\omega_6^3v_2^2\omega_7\omega_4^2 - 18\omega_8\omega_9\omega_6^3\omega_7\omega_4 - 2\omega_8^2\omega_9\omega_6^3v_2^2\omega_7\omega_4^2 + 12\omega_8\omega_6^3v_2^2\omega_7\omega_4 - 12\omega_9\omega_6^3v_2^2\omega_7\omega_4^2 + 18\omega_8\omega_9\omega_6^3v_2^2\omega_7\omega_4 + 6\omega_8\omega_9\omega_6^3\omega_7\omega_4^2 + 18\omega_8^2\omega_9\omega_6^3v_2^2\omega_7\omega_4^2 + 18\omega_8\omega_9\omega_6^3v_2^2\omega_7\omega_4^2 - 12\omega_8^2\omega_9\omega_7\omega_4^2 - 12\omega_9\omega_6^3v_2^2\omega_7\omega_4 + 12\omega_8\omega_6^3v_2^2\omega_7\omega_4^2 + 36\omega_6^2\omega_9cs^2\omega_7\omega_4^2 + 12\omega_8^2\omega_6^3v_2^2\omega_4 - 36\omega_8cs^2\omega_6^3\omega_7\omega_4^2 - 6\omega_8^2\omega_6^3v_2^2\omega_7\omega_4^2 - 18\omega_8\omega_9cs^2\omega_6^2\omega_7\omega_4^2 - 36\omega_8^2\omega_9cs^2\omega_6\omega_7\omega_4^2 - 12\omega_8^2\omega_6^3v_2^2\omega_7\omega_4 + 12\omega_9\omega_6^2\omega_7\omega_4^2 + 36\omega_9cs^2\omega_6^2\omega_7\omega_4^2 - 40\omega_8^2\omega_9cs^2\omega_6^3\omega_7\omega_4 - \omega_8^2\omega_9\omega_6^2\omega_7\omega_4^2 - 12\omega_8\omega_6^2\omega_7\omega_4^2$$

$$C_8 = 12\omega_8\omega_9\omega_6\omega_4^2 + 12\omega_9\omega_6\omega_7\omega_4^3 + 12\omega_6^2\omega_7\omega_4^3 + 6\omega_8\omega_6\omega_7\omega_4^3 + 18\omega_8\omega_9cs^2\omega_6^2\omega_7\omega_4 - 6\omega_8\omega_9\omega_6\omega_4^3 + 12cs^2\omega_6\omega_7\omega_4^3 - 36\omega_8\omega_9\omega_6v_2^2\omega_7\omega_4 + 12\omega_9cs^2\omega_6^2\omega_7\omega_4 - 12\omega_6^2\omega_7\omega_4^2 - 12\omega_9\omega_6\omega_7\omega_4^2 - 24\omega_9cs^2\omega_6^2\omega_7\omega_4^2 + 72\omega_8\omega_9\omega_6v_2^2\omega_7\omega_4^2 + 12\omega_8\omega_9\omega_7\omega_4^2 + 36\omega_6^2v_2^2\omega_7\omega_4^2 - 4\omega_8\omega_9cs^2\omega_6^2\omega_7\omega_4^2 + 36\omega_9\omega_6v_2^2\omega_7\omega_4^2 - 12\omega_8cs^2\omega_6^2\omega_7\omega_4^2 - 12\omega_8\omega_9cs^2\omega_6^2\omega_7 - 6\omega_8\omega_9\omega_7\omega_4^3 - 18\omega_8\omega_6v_2^2\omega_7\omega_4^3 + 12\omega_9cs^2\omega_6^2\omega_7\omega_4^3 - 18\omega_8\omega_9\omega_6v_2^2\omega_7\omega_4^3 + 6\omega_8\omega_6^2\omega_4^3 + 12\omega_9cs^2\omega_6\omega_7\omega_4^3 - 12\omega_8\omega_9cs^2\omega_6\omega_4^3 + 24\omega_8\omega_9cs^2\omega_6\omega_7\omega_4^3 - 18\omega_8\omega_6^2v_2^2\omega_4^3 + 6\omega_8\omega_9\omega_6\omega_7\omega_4^3 - 12\omega_9cs^2\omega_6\omega_7\omega_4^3 - 12\omega_8\omega_6^2\omega_4^3 - 36\omega_8\omega_9v_2^2\omega_7\omega_4^3 + 6\omega_8\omega_6^2\omega_4^3 + 12\omega_9cs^2\omega_6\omega_7\omega_4^3 - 12\omega_8\omega_9cs^2\omega_6\omega_4^3 + 36\omega_8\omega_6^2v_2^2\omega_4^2 - 6\omega_8\omega_9cs^2\omega_6\omega_7\omega_4^3 - 6\omega_8cs^2\omega_6\omega_7\omega_4^3 + 6\omega_8\omega_9cs^2\omega_6\omega_4^3 + 12\omega_8\omega_9\omega_6\omega_7\omega_4 - 72\omega_9\omega_6^2v_2^2\omega_7\omega_4^2 - 6\omega_8\omega_6^2\omega_7\omega_4^3 + 12cs^2\omega_6^2\omega_7\omega_4^2 + 18\omega_8\omega_9\omega_6v_2^2\omega_4^3 - 12\omega_8\omega_9cs^2\omega_7\omega_4^2 - 12\omega_6\omega_7\omega_4^3 - 6\omega_8cs^2\omega_6^2\omega_4^3 - 36\omega_8\omega_6^2v_2^2\omega_7\omega_4^2 - 12\omega_9\omega_6^2\omega_7\omega_4^3 - 12\omega_8\omega_9cs^2\omega_6\omega_7\omega_4 + 36\omega_6v_2^2\omega_7\omega_4^3 + 36\omega_9\omega_6^2v_2^2\omega_7\omega_4^3 + 24\omega_9\omega_6^2\omega_7\omega_4^2 + 18\omega_8\omega_6^2v_2^2\omega_7\omega_4^3 + 12\omega_8cs^2\omega_6^2\omega_4^3 - 36\omega_8\omega_9\omega_6v_2^2\omega_4^2 + 6\omega_8\omega_9cs^2\omega_7\omega_4^3 - 12cs^2\omega_6^2\omega_7\omega_4^3 + 12\omega_8\omega_6^2\omega_7\omega_4^3$$

$$C_9 = 108cs^2\omega_6^3v_2^2\omega_4^3 - 36\omega_8\omega_6^3v_2^2\omega_4^2 - 24\omega_8^2cs^4\omega_6^3\omega_4 - 19\omega_8^2\omega_6^3v_2^2\omega_4^3 + \omega_8^2cs^4\omega_6^2\omega_4^3 + 60\omega_8^2cs^2\omega_6^2v_2^2\omega_4^3 + 72\omega_8^2v_1^4\omega_4^3 + 36\omega_8cs^2\omega_6^2v_2^2\omega_4^2 + 4\omega_8^2\omega_6^3v_2^2\omega_4^3 + 18\omega_8^2cs^2\omega_6^2v_2^2\omega_4^2 + 6\omega_8^2cs^4\omega_6^2\omega_4^2 + 39\omega_8\omega_6^3v_2^2\omega_4^3 - 108cs^2\omega_6^3v_2^2\omega_4^3 + 12\omega_8^2cs^4\omega_4^3 - 5\omega_8^2cs^2\omega_6^3\omega_4^3 + 72\omega_8\omega_6^3v_2^2\omega_4^3 - 6\omega_8^2\omega_6^3v_2^2\omega_4^2 + 198\omega_8cs^2\omega_6^2v_2^2\omega_4^3 - \omega_8^2cs^2\omega_6^2\omega_4^3 + 6\omega_8^2cs^2\omega_6^3\omega_4 + 36\omega_6^3v_2^2\omega_4^3 + 36\omega_8\omega_6v_2^2\omega_4^3 - \omega_8^2cs^4\omega_6^3\omega_4^3 + 36\omega_6^2v_2^2\omega_4^3 - 36\omega_6^3v_2^2\omega_4^2 - 90\omega_8\omega_6v_2^2\omega_4^3 - 6\omega_8^2cs^2\omega_6^2\omega_4^2 + 13\omega_8^2cs^4\omega_6^2\omega_4^2 - 6\omega_8cs^2\omega_6^3\omega_4^3 + 12\omega_8^2cs^2\omega_6\omega_4^3 - 99\omega_8cs^2\omega_6^3v_2^2\omega_4^3 - 39\omega_8\omega_6^3v_2^2\omega_4^3 - 12\omega_8^2cs^2\omega_4^3 - 3\omega_8^2cs^2\omega_6^3v_2^2\omega_4^2 + 6\omega_8^2\omega_6^3v_2^2\omega_4^2 - 72\omega_8\omega_6^2v_2^2\omega_4^3 + 12\omega_8cs^4\omega_6^3\omega_4 - 6\omega_8cs^4\omega_6^2\omega_4^3 + 54\omega_8cs^2\omega_6^3v_2^2\omega_4^2 + 19\omega_8^2\omega_6^2v_2^2\omega_4^3 + 36\omega_8\omega_6^3v_2^2\omega_4^2 + 12\omega_6^2cs^2\omega_6\omega_4^3 + 252\omega_8cs^2v_2^2\omega_4^3 + 18\omega_8cs^2\omega_6^2\omega_4^3 - 72\omega_8^2v_2^2\omega_4^3 + 12\omega_8cs^4\omega_6^2\omega_4^2 - 4\omega_8^2\omega_6^3v_2^2\omega_4^3 - 108cs^2\omega_6^2v_2^2\omega_4^3 + 12\omega_8^2cs^2\omega_6^3v_2^2\omega_4^3 - 306\omega_8^2cs^2\omega_6v_2^2\omega_4^3 + 6\omega_8cs^4\omega_6^3\omega_4^3 + 36\omega_6^3v_2^2\omega_4^2 - 12\omega_8^2cs^4\omega_6\omega_4^3 + 90\omega_8\omega_6v_2^2\omega_4^3 + 36\omega_8cs^2\omega_6^3v_2^2\omega_4 + 6\omega_8cs^2\omega_6^2\omega_4^3 - 12\omega_8cs^2\omega_6^3\omega_4 - 18\omega_8cs^2\omega_6^3v_2^2\omega_4 - 12\omega_8^2cs^4\omega_6\omega_4^3 - 18\omega_8cs^4\omega_6^2\omega_4^2 - 36\omega_8^2cs^2\omega_6v_2^2\omega_4^2 - 36\omega_6^3v_2^2\omega_4^3 + 12\omega_8^2cs^4\omega_6^3 - 36\omega_8\omega_6v_2^2\omega_4^3 - 12\omega_8cs^2\omega_6^2\omega_4^3 - 108\omega_8cs^2\omega_6v_2^2\omega_4^3 - 36\omega_6^2v_2^4\omega_4^3$$

$$C_{10} = 12 + 234\omega_6v_2^2 + 672cs^2v_2^2 + 90\omega_6^3v_2^4 + 198cs^2\omega_6 + 144v_2^4 - 78cs^2\omega_6^2 - 1008cs^2\omega_6v_2^2 - 132cs^2 + 6cs^2\omega_6^3 - 9\omega_6^3v_2^4 + 404cs^2\omega_6^2v_2^2 - 18\omega_6 - 216cs^4\omega_6 + 8\omega_6^2 + 144cs^4 + 10\omega_6^3v_2^2 - \omega_6^3 - 216\omega_6v_2^4 - 34cs^2\omega_6^3v_2^2 - 5cs^4\omega_6^3 - 156v_2^2 - 98\omega_6^2v_2^2 + 82cs^4\omega_6^2$$

$$C_{11} = 12 + 378\omega_6v_2^2 + 432cs^2v_2^2 + 310\omega_6^2v_2^4 + 54cs^2\omega_6 + 504v_2^4 - 22cs^2\omega_6^2 - 648cs^2\omega_6v_2^2 - 36cs^2 + 2cs^2\omega_6^3 - 29\omega_6^3v_2^4 + 252cs^2\omega_6^2v_2^2 - 18\omega_6 - 36cs^4\omega_6 + 8\omega_6^2 + 24cs^4 + 14\omega_6^3v_2^2 - \omega_6^3 - 756\omega_6v_2^4 - 18cs^2\omega_6^3v_2^2 - cs^4\omega_6^3 - 252v_2^2 - 154\omega_6^2v_2^2 + 14cs^4\omega_6^2$$

2.5 CLBM2

2.5.1 Definitions

Collision operator \mathbf{C} :

$$\mathbf{C}(f) = \mathbf{K}^{-1} \mathbf{S} \left(\boldsymbol{\kappa}^{(eq)} - \mathbf{K} f \right),$$

where

$$\mathbf{S} = \text{diag}(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9),$$

$$\omega_1, \omega_2, \dots, \omega_9 \in (0, 2).$$

Matrix \mathbf{K} corresponds to the transformation matrix to the central moment basis defined by

$$\boldsymbol{\kappa} = \begin{pmatrix} k_{(0,0)} \\ k_{(1,0)} \\ k_{(0,1)} \\ k_{(1,1)} \\ k_{(2,0)} + k_{(0,2)} \\ k_{(2,0)} - k_{(0,2)} \\ k_{(2,1)} \\ k_{(1,2)} \\ k_{(2,2)} \end{pmatrix},$$

thus, the transformation matrix \mathbf{K} satisfies

$$\begin{aligned} \mathbf{K}_{1,i} &= (\mathbf{c}_i - \mathbf{v})^{(0,0)} \\ \mathbf{K}_{2,i} &= (\mathbf{c}_i - \mathbf{v})^{(1,0)} \\ \mathbf{K}_{3,i} &= (\mathbf{c}_i - \mathbf{v})^{(0,1)} \\ \mathbf{K}_{4,i} &= (\mathbf{c}_i - \mathbf{v})^{(1,1)} \\ \mathbf{K}_{5,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,0)} + (\mathbf{c}_i - \mathbf{v})^{(0,2)} \\ \mathbf{K}_{6,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,0)} - (\mathbf{c}_i - \mathbf{v})^{(0,2)} \\ \mathbf{K}_{7,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,1)} \\ \mathbf{K}_{8,i} &= (\mathbf{c}_i - \mathbf{v})^{(1,2)} \\ \mathbf{K}_{9,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,2)}, \end{aligned}$$

$$\forall i \in \{1, 2, \dots, 9\}.$$

The equilibrium central moments are defined by

$$\boldsymbol{\kappa}^{(eq)} = \mathbf{K} \mathbf{M}^{-1} \boldsymbol{\mu}^{(eq)},$$

i.e.,

$$\boldsymbol{\kappa}^{(eq)} = \left(\rho, 0, 0, 0, 2\rho c_s^2, 0, 0, 0, \rho c_s^4 \right)^T.$$

2.5.2 Conservation of mass: ρ



attached text file: output_d2q9_nse_clbm2_symbolic_pde_00.txt

$$\begin{aligned}
& \frac{\partial \rho}{\partial t} + \frac{v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + 3cs^2 + v_1^2) \frac{v_1 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + cs^2 + 3v_1^2) \frac{\rho \delta_l^3}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\
& \frac{cs^2 \rho \delta_l^3}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{cs^2 \rho \delta_l^3}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3cs^2 + v_2^2) \frac{v_2 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + cs^2 + 3v_2^2) \frac{\rho \delta_l^3}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& (-2cs^2 - cs^4 \omega_5 - 3v_1^4 \omega_5 + 6v_1^4 - 12cs^2 v_1^2 \omega_5 + cs^2 \omega_5 - 6v_1^2 + 3v_1^2 \omega_5 + 2cs^4 + 24cs^2 v_1^2) \frac{\delta_l^4}{24\delta_t \omega_5} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& (-4 + 6cs^2 - 3cs^2 \omega_5 + 10v_1^2 - 5v_1^2 \omega_5 + 2\omega_5) \frac{\rho v_1 \delta_l^4}{12\delta_t \omega_5} \frac{\partial^4 v_1}{\partial x_1^4} + \\
& (\omega_7 v_1^2 \omega_5 + 3\omega_7 cs^2 \omega_5 - \omega_7 + 3\omega_7 cs^2 - 9cs^2 \omega_5 - \omega_7 \omega_5 - 3v_1^2 \omega_5 + 3\omega_5 + \omega_7 v_1^2) \frac{\rho v_1 \delta_l^4}{12\delta_t \omega_7 \omega_5} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& (-2 + \omega_4) \frac{cs^4 \delta_l^4}{6\delta_t \omega_4} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\
& (3\omega_6 + 3\omega_6 cs^2 \omega_8 - 3\omega_6 v_2^2 + 3cs^2 \omega_8 - \omega_6 \omega_8 + \omega_8 v_2^2 - \omega_8 - 9\omega_6 cs^2 + \omega_6 \omega_8 v_2^2) \frac{\rho v_2 \delta_l^4}{12\omega_6 \delta_t \omega_8} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& (-2cs^2 + 3\omega_6 v_2^2 + 6v_2^4 - \omega_6 cs^4 - 6v_2^2 - 12\omega_6 cs^2 v_2^2 + \omega_6 cs^2 + 24cs^2 v_2^2 + 2cs^4 - 3\omega_6 v_2^4) \frac{\delta_l^4}{24\omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\
& (-4 + 2\omega_6 + 6cs^2 - 5\omega_6 v_2^2 + 10v_2^2 - 3\omega_6 cs^2) \frac{\rho v_2 \delta_l^4}{12\omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0.
\end{aligned}$$

2.5.3 Conservation of momentum: ρv_1



attached text file: output_d2q9_nse_clbm2_symbolic_pde_01.txt

$$\begin{aligned}
& v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (cs^2 + v_1^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2\rho v_1 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_1 v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho v_2 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{\rho v_1 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\
& (-2 + 4cs^2 - 2cs^2 \omega_5 + 6v_1^2 - 3v_1^2 \omega_5 + \omega_5) \frac{\delta_l^2}{\delta_t \omega_5} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + (2 - \omega_5) \frac{3\rho v_1 \delta_l^2}{\delta_t \omega_5} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + (-2 + \omega_4) \frac{cs^2 \delta_l^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\
& (-2 + \omega_4) \frac{cs^2 \delta_l^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (-2 + 6cs^2 - 3cs^2 \omega_5 + 2v_1^2 - v_1^2 \omega_5 + \omega_5) \frac{v_1 \delta_l^2}{2\delta_t \omega_5} \frac{\partial^2 \rho}{\partial x_1^2} + \\
& (-2 + 2cs^2 - cs^2 \omega_5 + 6v_1^2 - 3v_1^2 \omega_5 + \omega_5) \frac{\rho \delta_l^2}{2\delta_t \omega_5} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega_4) \frac{cs^2 \rho \delta_l^2}{2\delta_t \omega_4} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_4) \frac{cs^2 \rho \delta_l^2}{2\delta_t \omega_4} \frac{\partial^2 v_1}{\partial x_2^2} + \\
& C_1 \frac{\delta_l^3}{12\delta_t \omega_5^2} \frac{\partial^3 \rho}{\partial x_1^3} + (-24 + 36cs^2 - 36cs^2 \omega_5 + 11v_1^2 \omega_5^2 - 4\omega_5^2 + 60v_1^2 - 60v_1^2 \omega_5 + 24\omega_5 + 5cs^2 \omega_5^2) \frac{\rho v_1 \delta_l^3}{6\delta_t \omega_5^2} \frac{\partial^3 v_1}{\partial x_1^3} + \\
& C_2 \frac{\rho v_1 \delta_l^3}{12\delta_t \omega_7 \omega_4 \omega_5^2} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + (-12 + 12\omega_4 - \omega_4^2) \frac{cs^4 \delta_l^3}{6\delta_t \omega_4^3} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} - \frac{cs^2 \rho v_1 \delta_l^3}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3cs^2 + v_2^2) \frac{v_1 v_2 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\
& C_3 \frac{\rho v_2 \delta_l^3}{6\delta_t \omega_4 \omega_8} \frac{\partial^3 v_1}{\partial x_2^3} + (-1 + cs^2 + 3v_2^2) \frac{\rho v_1 \delta_l^3}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_4 \frac{v_1 \delta_l^4}{12\delta_t \omega_5^3} \frac{\partial^4 \rho}{\partial x_1^4} + C_5 \frac{\rho \delta_l^4}{12\delta_t \omega_5^3} \frac{\partial^4 v_1}{\partial x_1^4} + C_6 \frac{\rho \delta_l^4}{12\delta_t \omega_7^2 \omega_4^3 \omega_5^3} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& C_7 \frac{cs^2 v_1 \delta_l^4}{12\omega_9 \delta_t \omega_7^2 \omega_4^2 \omega_8 \omega_5^3} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_8 \frac{cs^2 \rho \delta_l^4}{12\omega_9 \delta_t \omega_7 \omega_4^3 \omega_8 \omega_5^2} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_9 \frac{cs^2 v_2 \delta_l^4}{12\omega_9 \omega_6 \delta_t \omega_7 \omega_4^2 \omega_8 \omega_5} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + \\
& (3\omega_6 cs^2 \omega_8 + 3\omega_6 - 3\omega_6 v_2^2 + 3cs^2 \omega_8 - \omega_6 \omega_8 + \omega_8 v_2^2 - 9\omega_6 cs^2 - \omega_8 + \omega_6 \omega_8 v_2^2) \frac{\rho v_1 v_2 \delta_l^4}{12\omega_6 \delta_t \omega_8} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& C_{10} \frac{cs^2 \rho \delta_l^4}{12\omega_9 \omega_6 \delta_t \omega_7 \omega_4^3 \omega_8 \omega_5} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\
& (-2cs^2 + 3\omega_6 v_2^2 + 6v_2^4 - \omega_6 cs^4 - 6v_2^2 - 12\omega_6 cs^2 v_2^2 + \omega_6 cs^2 + 2cs^4 - 3\omega_6 v_2^4 + 24cs^2 v_2^2) \frac{v_1 \delta_l^4}{24\omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\
& C_{11} \frac{\rho \delta_l^4}{24\delta_t \omega_4^3 \omega_8^2} \frac{\partial^4 v_1}{\partial x_2^4} + (-4 + 6cs^2 + 2\omega_6 - 5\omega_6 v_2^2 + 10v_2^2 - 3\omega_6 cs^2) \frac{\rho v_1 v_2 \delta_l^4}{12\omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= -12cs^2 - 12cs^4 \omega_5 + 7v_1^4 \omega_5^2 - 144cs^2 v_1^2 \omega_5 - 36v_1^4 \omega_5 + 36v_1^4 + 24cs^2 v_1^2 \omega_5^2 + cs^4 \omega_5^2 + 12cs^2 \omega_5 - 7v_1^2 \omega_5^2 - 36v_1^2 + 36v_1^2 \omega_5 + 12cs^4 - cs^2 \omega_5^2 + 144cs^2 v_1^2 \\
C_2 &= 18cs^2 \omega_7 \omega_5^2 + 36cs^2 \omega_7 \omega_4 - 36cs^2 \omega_4 \omega_5 - 6\omega_7 \omega_4 v_1^2 \omega_5 + 6\omega_7 \omega_4 \omega_5 - 3\omega_7 \omega_4 v_1^2 \omega_5^2 + 36cs^2 \omega_4 \omega_5^2 + 3\omega_7 \omega_4 \omega_5^2 + 12\omega_7 \omega_4 v_1^2 + 6\omega_7 v_1^2 \omega_5^2 - \\
& 12\omega_4 v_1^2 \omega_5 - 18cs^2 \omega_7 \omega_4 \omega_5 - 12\omega_4 \omega_5^2 - 12v_1^2 \omega_5^2 + 12\omega_5^2 - 12\omega_7 \omega_4 - 36cs^2 \omega_5^2 - 6\omega_7 \omega_5^2 + 12\omega_4 \omega_5 + 12\omega_4 v_1^2 \omega_5^2 - 11cs^2 \omega_7 \omega_4 \omega_5^2 \\
C_3 &= 6 - 18cs^2 + \omega_4 \omega_8 + 9cs^2 \omega_8 - 3\omega_4 - 6v_2^2 + 3\omega_8 v_2^2 + 3\omega_4 v_2^2 - 3\omega_8 + 9cs^2 \omega_4 - 3cs^2 \omega_4 \omega_8 - \omega_4 \omega_8 v_2^2 \\
C_4 &= 12 - 132cs^2 - 216cs^4 \omega_5 + 90v_1^4 \omega_5^2 - 1008cs^2 v_1^2 \omega_5 - 9v_1^4 \omega_5^3 - 34cs^2 v_1^2 \omega_5^3 - 5cs^4 \omega_5^3 - 216v_1^4 \omega_5 + 144v_1^4 + 404cs^2 v_1^2 \omega_5^2 + 82cs^4 \omega_5^2 - \omega_5^3 + \\
& 10v_1^2 \omega_5^3 + 198cs^2 \omega_5 - 98v_1^2 \omega_5^2 + 8\omega_5^2 - 156v_1^2 + 234v_1^2 \omega_5 - 18\omega_5 + 144cs^4 - 78cs^2 \omega_5^2 + 672cs^2 v_1^2 + 6cs^2 \omega_5^3 \\
C_5 &= 12 - 36cs^2 - 36cs^4 \omega_5 + 310v_1^4 \omega_5^2 - 648cs^2 v_1^2 \omega_5 - 29v_1^4 \omega_5^3 - 18cs^2 v_1^2 \omega_5^3 - cs^4 \omega_5^3 - 756v_1^4 \omega_5 + 504v_1^4 + 252cs^2 v_1^2 \omega_5^2 + 14cs^4 \omega_5^2 - \omega_5^3 + \\
& 14v_1^2 \omega_5^3 + 54cs^2 \omega_5 - 154v_1^2 \omega_5^2 + 8\omega_5^2 - 252v_1^2 + 378v_1^2 \omega_5 - 18\omega_5 + 24cs^4 - 22cs^2 \omega_5^2 + 432cs^2 v_1^2 + 2cs^2 \omega_5^3
\end{aligned}$$

$$C_6 = 36\omega_4^3v_1^4\omega_5^3 + 36cs^2\omega_7\omega_4v_1^2\omega_5^3 + cs^4\omega_2^2\omega_4^3\omega_5^2 - 36cs^2\omega_7^2\omega_2^2v_1^2\omega_5 + 19\omega_7^2\omega_3^4v_1^4\omega_5^2 + 36\omega_7\omega_2^2v_1^4\omega_5^3 + 108cs^2\omega_4^3v_1^2\omega_5^3 + 36\omega_4^2v_1^2\omega_5^3 - 12cs^2\omega_7\omega_4\omega_5^2 + 39\omega_7\omega_4^3v_1^2\omega_5^3 + 6cs^2\omega_2^2\omega_4\omega_5^3 + 4\omega_2^2\omega_3^4v_1^4\omega_5^3 - 108cs^2\omega_7\omega_4^3v_1^2\omega_5 - cs^4\omega_2^2\omega_3^4\omega_5^3 - 12cs^4\omega_2^2\omega_4^2\omega_5 - 36\omega_4^3v_1^4\omega_5^2 + 6\omega_2^2\omega_4^2v_1^2\omega_5^3 - 72\omega_7\omega_4^3v_1^2\omega_5^2 + 18cs^2\omega_7\omega_4^3\omega_5^3 - 72\omega_7^2\omega_4^3v_1^2 - 108cs^2\omega_4^3v_1^2\omega_5^2 + 36\omega_7\omega_4^3v_1^2\omega_5 + 6cs^2\omega_7\omega_4^3\omega_5^2 + 198cs^2\omega_7\omega_4^3v_1^2\omega_5^2 - 3cs^2\omega_2^2\omega_4^2v_1^2\omega_5^3 + 12cs^4\omega_7\omega_4\omega_5^3 + 6cs^4\omega_2^2\omega_4^2\omega_5^2 - 6cs^2\omega_7\omega_4^3\omega_5^3 + 13cs^4\omega_2^2\omega_4^2\omega_5^3 - 12cs^4\omega_2^2\omega_4^3\omega_5 - 99cs^2\omega_7\omega_4^3v_1^2\omega_5^3 - 90\omega_7^2\omega_4^3v_1^4\omega_5 + 18cs^2\omega_7^2\omega_4^3v_1^2\omega_5^2 - 4\omega_7^2\omega_4^3v_1^2\omega_5^3 + 12cs^4\omega_2^2\omega_5^3 + 36\omega_4^3v_1^2\omega_5^2 + 252cs^2\omega_2^2\omega_4^3v_1^2 - cs^2\omega_2^2\omega_4^3\omega_5^2 + 72\omega_7\omega_4^3v_1^4\omega_5^2 - 6\omega_2^2\omega_4^3v_1^4\omega_5^3 + 12cs^4\omega_7\omega_4\omega_5^2 - 24cs^4\omega_2^2\omega_4\omega_5^3 - 18cs^2\omega_7\omega_4^3v_1^2\omega_5^3 - 36\omega_3^4v_1^2\omega_5^3 - 108cs^2\omega_4^2v_1^2\omega_5^3 + 12cs^2\omega_2^2\omega_4^2\omega_5 - 36\omega_7\omega_2^2v_1^2\omega_5^3 - 19\omega_2^2\omega_3^4v_1^2\omega_5^2 - 12cs^2\omega_2^2\omega_4^3 - 36\omega_4^3v_1^4\omega_5^3 - 39\omega_7\omega_4^3v_1^4\omega_5^3 - 18cs^4\omega_7\omega_4\omega_5^3 - 306cs^2\omega_2^2\omega_4^3v_1^2\omega_5 - 6cs^4\omega_7\omega_4\omega_5^2 + 60cs^2\omega_7^2\omega_3^4v_1^2\omega_5^2 + 54cs^2\omega_7\omega_4^2v_1^2\omega_5^3 - 12cs^2\omega_7\omega_4\omega_5^3 - 6cs^2\omega_2^2\omega_4^2\omega_5^2 + 90\omega_7^2\omega_4^3v_1^2\omega_5 + 12cs^2\omega_2^2\omega_4^3v_1^2\omega_5^3 + 36cs^2\omega_7\omega_4^3v_1^2\omega_5^2 - 36\omega_7\omega_4^3v_1^4\omega_5 + 6cs^4\omega_7\omega_3^4\omega_5^3 + 12cs^4\omega_2^2\omega_4^3 + 72\omega_2^2\omega_4^3v_1^4 - 5cs^2\omega_2^2\omega_4^2\omega_5^3 + 12cs^2\omega_2^2\omega_4^3\omega_5$$

$$C_7 = -6\omega_2^2\omega_4^2\omega_8\omega_5^3 - 6\omega_9cs^2\omega_2^2\omega_4^2\omega_8\omega_5^2 + 12\omega_7\omega_4\omega_8v_1^2\omega_5^3 - 18\omega_9\omega_7\omega_2^2\omega_8\omega_5^2 - 36\omega_9cs^2\omega_4^2\omega_8\omega_5^2 + 36cs^2\omega_2^2\omega_4\omega_5^3 + 5\omega_9cs^2\omega_2^2\omega_4^2\omega_8\omega_5^3 + 6\omega_2^2\omega_4^2\omega_8\omega_5^2 - 12\omega_9\omega_2^2\omega_4\omega_8 - 6\omega_2^2\omega_4^2v_1^2\omega_5^3 + 36cs^2\omega_7\omega_4\omega_8\omega_5^3 + 36\omega_9cs^2\omega_2^2\omega_8\omega_5^3 + 12\omega_9\omega_4\omega_8\omega_5^3 + 6\omega_9\omega_7\omega_2^2\omega_8\omega_5^3 - 12\omega_2^2\omega_4\omega_8v_1^2\omega_5^3 - 18\omega_9\omega_2^2\omega_4\omega_8\omega_5^2 + 12\omega_9\omega_2^2\omega_8\omega_5^2 + 18\omega_9\omega_7\omega_4\omega_8v_1^2\omega_5^3 + 6\omega_2^2\omega_4\omega_5^3 - 18cs^2\omega_2^2\omega_4^2\omega_8\omega_5^2 + 12\omega_9\omega_4^2\omega_8v_1^2\omega_5^3 + 12\omega_9\omega_2^2\omega_4\omega_5^2 - 12\omega_7\omega_4\omega_8\omega_5^3 + 18\omega_9\omega_2^2\omega_4\omega_8v_1^2\omega_5^2 - 12\omega_9\omega_2^2\omega_8v_1^2\omega_5^2 - 12\omega_9\omega_4^2\omega_8v_1^2\omega_5^2 - 36\omega_9cs^2\omega_7\omega_8\omega_5^3 + 12\omega_9\omega_7\omega_4\omega_8\omega_5 + 18cs^2\omega_2^2\omega_4^2\omega_8\omega_5^3 + 12\omega_9\omega_2^2\omega_4\omega_8\omega_5^3 - 12\omega_9\omega_2^2\omega_8\omega_5^2 - 12\omega_9\omega_2^2\omega_4v_1^2\omega_5^3 + 54\omega_9cs^2\omega_7\omega_4\omega_8\omega_5^3 + 18\omega_9cs^2\omega_2^2\omega_4^2\omega_5^2 + 12\omega_9\omega_2^2\omega_8v_1^2\omega_5^3 - 12\omega_9\omega_2^2\omega_4\omega_8v_1^2\omega_5^3 - 36\omega_9cs^2\omega_2^2\omega_4^2\omega_8\omega_5^2 - 36\omega_9cs^2\omega_2^2\omega_4\omega_8\omega_5^2 - 6\omega_2^2\omega_4^2\omega_8v_1^2\omega_5^2 + 12\omega_9\omega_2^2\omega_4^2\omega_8v_1^2 + 2\omega_9\omega_2^2\omega_4^2\omega_8v_1^2\omega_5^2 - 12\omega_9\omega_2^2\omega_4\omega_8\omega_5^3 + 6\omega_2^2\omega_4\omega_8v_1^2\omega_5^3 - 36cs^2\omega_2^2\omega_4\omega_8\omega_5^3 - 12\omega_2^2\omega_4\omega_5^3 + 36\omega_9cs^2\omega_2^2\omega_8\omega_5^3 - 12\omega_2^2\omega_4\omega_5^3 + 36\omega_9cs^2\omega_2^2\omega_8\omega_5^3 - 12\omega_7\omega_4^2\omega_8\omega_5^3 - \omega_9\omega_2^2\omega_4^2\omega_8\omega_5^3 + 12\omega_9\omega_7\omega_8\omega_5^3 + 6\omega_2^2\omega_4\omega_8v_1^2\omega_5^3 - 36cs^2\omega_2^2\omega_4\omega_8\omega_5^3 - 12\omega_2^2\omega_4\omega_5^3 + 36\omega_9cs^2\omega_2^2\omega_8\omega_5^3 - 6\omega_9\omega_2^2\omega_4\omega_5^2 - 18\omega_9cs^2\omega_7\omega_4^2\omega_8\omega_5^3 + 12\omega_7\omega_4^2\omega_8v_1^2\omega_5^2 - 12\omega_9\omega_2^2\omega_4^2\omega_8v_1^2\omega_5 + 6\omega_9\omega_2^2\omega_4^2v_1^2\omega_5^2 + 54\omega_9cs^2\omega_2^2\omega_4\omega_8\omega_5^3 + 12\omega_2^2\omega_4\omega_8\omega_5^2 - 2\omega_9\omega_2^2\omega_4^2\omega_8v_1^2\omega_5^2 - 6\omega_9\omega_7\omega_4^2\omega_8v_1^2\omega_5^3 + 36cs^2\omega_7\omega_2^2\omega_8\omega_5^2 - 12\omega_9\omega_4\omega_8v_1^2\omega_5^3 + 12\omega_9\omega_4^2\omega_8\omega_5^2 - 12\omega_9\omega_7\omega_8v_1^2\omega_5^3 + \omega_9\omega_2^2\omega_4\omega_8v_1^2\omega_5^3 - 36\omega_9cs^2\omega_7\omega_4^2\omega_8\omega_5 - 40\omega_9cs^2\omega_2^2\omega_4\omega_8\omega_5^3 + 36\omega_9cs^2\omega_2^2\omega_4\omega_8 - 12\omega_9\omega_4^2\omega_8\omega_5^3 - 18\omega_9\omega_7\omega_4\omega_8\omega_5^3 - 36\omega_9cs^2\omega_4\omega_8\omega_5^3 - 18cs^2\omega_2^2\omega_4^2\omega_5^3 + 12\omega_2^2\omega_4v_1^2\omega_5^3 + 12\omega_9\omega_2^2\omega_4^2\omega_8\omega_5 + 18\omega_9\omega_7\omega_4^2\omega_8v_1^2\omega_5^2 - 36cs^2\omega_7\omega_4^2\omega_8\omega_5^3$$

$$C_8 = -6\omega_7\omega_3^4\omega_8\omega_5^2 - 12\omega_4^2\omega_8\omega_5^2 + 12\omega_9\omega_7\omega_2^2\omega_5 + 36\omega_9\omega_4^2\omega_8v_1^2\omega_5^2 - 12cs^2\omega_3^4\omega_8\omega_5^2 + 36\omega_9\omega_4^2\omega_8v_1^2\omega_5 - 12\omega_9\omega_4\omega_8\omega_5^2 - 36\omega_9\omega_7\omega_4^2\omega_8v_1^2 - 6cs^2\omega_7\omega_4^2\omega_8\omega_5 - 24\omega_9cs^2\omega_4^2\omega_8\omega_5^2 + 12cs^2\omega_7\omega_4^2\omega_5^2 + 12\omega_9\omega_3^4\omega_8\omega_5 - 12\omega_9cs^2\omega_7\omega_4^2\omega_8\omega_5 - 12\omega_9cs^2\omega_7\omega_4^2\omega_5 - \omega_9cs^2\omega_7\omega_4^2\omega_8\omega_5^2 - 12\omega_9cs^2\omega_7\omega_4\omega_8\omega_5 - 18\omega_7\omega_4^3v_1^2\omega_5^2 - 6cs^2\omega_7\omega_4^3\omega_5^2 - 12\omega_9cs^2\omega_7\omega_8\omega_5^2 - 6\omega_9cs^2\omega_7\omega_4^3\omega_8\omega_5 - 6\omega_9\omega_7\omega_3^4\omega_5 - 36\omega_9\omega_7\omega_2^2v_1^2\omega_5 + 18\omega_9cs^2\omega_7\omega_4\omega_8\omega_5^2 - 72\omega_9\omega_4^2\omega_8v_1^2\omega_5^2 - 24\omega_9\omega_7\omega_4^2\omega_8\omega_5 + 6cs^2\omega_7\omega_4^3\omega_8\omega_5^2 + 12\omega_9cs^2\omega_4^2\omega_8\omega_5 - 12\omega_9\omega_4^2\omega_8\omega_5^2 + 6\omega_7\omega_3^4\omega_8\omega_5 + 6\omega_9cs^2\omega_7\omega_4^3\omega_8 + 12cs^2\omega_3^4\omega_8\omega_5 - 36\omega_9\omega_4^2\omega_8v_1^2\omega_5^2 + 36\omega_7\omega_4^2v_1^2\omega_5^2 + 72\omega_9\omega_7\omega_4^2\omega_8v_1^2\omega_5 + 6\omega_9cs^2\omega_7\omega_3^4\omega_5 - 4\omega_9cs^2\omega_7\omega_4^2\omega_8\omega_5^2 + 18\omega_9\omega_7\omega_4^3\omega_8v_1^2 - 18\omega_7\omega_4^3\omega_8v_1^2\omega_5 + 12\omega_9\omega_7\omega_4\omega_8\omega_5 - 12\omega_9\omega_4^2\omega_8\omega_5 - 12\omega_7\omega_4^2\omega_5^2 + 12\omega_9cs^2\omega_2^2\omega_8\omega_5^2 - 36\omega_3^4\omega_8v_1^2\omega_5^2 + 12\omega_7\omega_4^2\omega_8\omega_5^2 + 12\omega_7\omega_4^2\omega_8\omega_5^2 - 36\omega_7\omega_4^2\omega_8v_1^2\omega_5^2 + 12cs^2\omega_4^2\omega_8\omega_5^2 + 12\omega_4^2\omega_8\omega_5^2 + 36\omega_2^2\omega_8v_1^2\omega_5^2 - 12\omega_9cs^2\omega_7\omega_4^2\omega_5 - 12\omega_4^3\omega_8\omega_5 - 12cs^2\omega_7\omega_4^2\omega_8\omega_5^2 + 6\omega_9\omega_7\omega_4^3\omega_8\omega_5 + 12\omega_9cs^2\omega_4\omega_8\omega_5^2 + 18\omega_7\omega_4^3\omega_8v_1^2\omega_5^2 + 24\omega_9\omega_4^2\omega_8\omega_5^2 + 36\omega_4^2\omega_8v_1^2\omega_5 - 12\omega_9cs^2\omega_4^3\omega_8\omega_5 + 12\omega_9\omega_7\omega_4^2\omega_8 + 18\omega_9\omega_7\omega_4^3v_1^2\omega_5 - 18\omega_9\omega_7\omega_4^2\omega_8v_1^2\omega_5 + 24\omega_9cs^2\omega_7\omega_4^2\omega_8\omega_5 + 6\omega_7\omega_4^2\omega_5^2 + 36\omega_9\omega_4\omega_8v_1^2\omega_5^2$$

$$C_9 = -15\omega_9cs^2\omega_7\omega_4^2\omega_8^2\omega_5 + 6\omega_6\omega_7\omega_2^2\omega_8^2\omega_5v_2^2 + 12\omega_6\omega_4\omega_2^2\omega_5v_2^2 - 18\omega_9\omega_6\omega_7\omega_4\omega_8\omega_5 - 18\omega_6cs^2\omega_7\omega_4^2\omega_8\omega_5 + 12\omega_9\omega_7\omega_8^2\omega_5 - 12\omega_9\omega_4^2\omega_8^2v_2^2 - 12\omega_9\omega_6\omega_7\omega_4\omega_2^2\omega_5v_2^2 + 12\omega_9\omega_6\omega_7\omega_8\omega_5 - 6\omega_6\omega_7\omega_4^2\omega_8^2v_2^2 - 36\omega_6cs^2\omega_7\omega_4\omega_8^2\omega_5 + 12\omega_9\omega_6\omega_7\omega_4\omega_5 + 12\omega_6\omega_4^2\omega_8^2\omega_5 - \omega_9\omega_6\omega_7\omega_4\omega_2^2\omega_5 - 36\omega_9cs^2\omega_4\omega_8^2\omega_5 - 3\omega_9\omega_6\omega_7\omega_4^2\omega_8\omega_5v_2^2 - 12\omega_9\omega_4^2\omega_8^2\omega_5 - 18\omega_9\omega_7\omega_4\omega_8^2\omega_5 + 18\omega_9cs^2\omega_7\omega_4^2\omega_8^2 - 12\omega_6\omega_7\omega_4\omega_8^2\omega_5v_2^2 + 12\omega_9\omega_7\omega_4\omega_8\omega_5v_2^2 + 36\omega_9\omega_6cs^2\omega_7\omega_8^2\omega_5 + 12\omega_6\omega_4^2\omega_8^2v_2^2 + 12\omega_9\omega_4^2\omega_8^2\omega_5v_2^2 + 18\omega_9\omega_7\omega_4\omega_8^2\omega_5v_2^2 + 18\omega_9\omega_6cs^2\omega_7\omega_4^2\omega_5 + 3\omega_9\omega_6cs^2\omega_7\omega_4\omega_8^2\omega_5 - 36\omega_6cs^2\omega_4^2\omega_8^2\omega_5 + 12\omega_9\omega_2^2\omega_8^2\omega_5 - 6\omega_6\omega_7\omega_2^2\omega_8^2\omega_5 + 12\omega_9\omega_6\omega_7\omega_8^2\omega_5v_2^2 + 12\omega_9\omega_4\omega_8^2\omega_5 + 5\omega_9\omega_7\omega_4^2\omega_8^2\omega_5 - 36\omega_9\omega_6cs^2\omega_7\omega_8\omega_5 + \omega_9\omega_6\omega_7\omega_4^2\omega_8^2\omega_5v_2^2 + 36\omega_9cs^2\omega_4^2\omega_8^2\omega_5 - 6\omega_9\omega_7\omega_4^2\omega_8^2 - 12\omega_6\omega_7\omega_4\omega_8^2\omega_5v_2^2 - 9\omega_9\omega_6cs^2\omega_7\omega_4^2\omega_8\omega_5 - 36\omega_9cs^2\omega_2^2\omega_8^2 - 36\omega_9\omega_6cs^2\omega_7\omega_4\omega_5 - 12\omega_6\omega_4^2\omega_8^2\omega_5v_2^2 - 12\omega_9\omega_6\omega_7\omega_4\omega_5v_2^2 + 6\omega_6\omega_7\omega_4^2\omega_8\omega_5 - 18\omega_6cs^2\omega_7\omega_4^2\omega_8^2 + 12\omega_6\omega_7\omega_4\omega_8^2\omega_5 - 12\omega_9\omega_6\omega_7\omega_8\omega_5v_2^2 - 36\omega_9\omega_6cs^2\omega_7\omega_4\omega_8^2\omega_5 + 36\omega_6cs^2\omega_4\omega_8^2\omega_5 - 6\omega_6\omega_7\omega_2^2\omega_8\omega_5v_2^2 - 12\omega_9\omega_7\omega_2^2\omega_5v_2^2 - 12\omega_9\omega_6\omega_7\omega_8^2\omega_5 - 12\omega_6\omega_4^2\omega_8^2 + 54\omega_9cs^2\omega_7\omega_4\omega_8^2\omega_5 + 6\omega_9\omega_7\omega_4^2\omega_8^2v_2^2 + 18\omega_9\omega_6\omega_7\omega_4\omega_8\omega_5v_2^2 - 6\omega_9\omega_6\omega_7\omega_4^2\omega_5 - 36\omega_6cs^2\omega_7\omega_4\omega_8\omega_5 + 36\omega_6cs^2\omega_7\omega_4\omega_8\omega_5 - 36\omega_9cs^2\omega_7\omega_4\omega_8\omega_5 + 3\omega_9\omega_6\omega_7\omega_4^2\omega_8\omega_5 + 36\omega_6cs^2\omega_4^2\omega_8^2 + 6\omega_6\omega_7\omega_4^2\omega_8^2 - 12\omega_9\omega_4\omega_8^2\omega_5v_2^2 - 5\omega_9\omega_7\omega_4^2\omega_8^2\omega_5v_2^2 + 12\omega_9\omega_6\omega_7\omega_4\omega_8^2\omega_5 + 18\omega_6cs^2\omega_7\omega_4^2\omega_8^2\omega_5 - 12\omega_6\omega_4\omega_8^2\omega_5$$

$$C_{10} = -12\omega_6cs^2\omega_7\omega_4^2\omega_8\omega_5 - 12\omega_9\omega_4^3\omega_8\omega_5 + 36\omega_6\omega_7\omega_4^2\omega_5v_2^2 - \omega_9\omega_6cs^2\omega_7\omega_4^3\omega_8\omega_5 - 12\omega_6cs^2\omega_4^3\omega_8\omega_5 + 6\omega_9\omega_6cs^2\omega_7\omega_4^3\omega_5 - 12\omega_9cs^2\omega_7\omega_4\omega_8\omega_5 - 6\omega_6\omega_7\omega_4^2\omega_8\omega_5 + 6\omega_6\omega_7\omega_4^2\omega_5 - 12\omega_9\omega_6\omega_7\omega_4\omega_5 + 36\omega_6\omega_3^4\omega_8v_2^2 + 12\omega_6cs^2\omega_7\omega_4^2\omega_5 - 36\omega_6\omega_4^3\omega_8\omega_5v_2^2 + 54\omega_9\omega_7\omega_4^2\omega_8\omega_5v_2^2 - 3\omega_9\omega_6\omega_7\omega_4^2\omega_8\omega_5v_2^2 - 36\omega_9\omega_4^3\omega_8v_2^2 - 5\omega_9cs^2\omega_7\omega_3^4\omega_8\omega_5 - 15\omega_9\omega_7\omega_4^3\omega_8\omega_5v_2^2 + 18\omega_9\omega_6cs^2\omega_7\omega_4\omega_8\omega_5 - 54\omega_9\omega_6\omega_7\omega_4^2\omega_5v_2^2 + 36\omega_6\omega_4^2\omega_8\omega_5v_2^2 + 12\omega_9\omega_3^4\omega_8 + 12\omega_6\omega_4^3\omega_8\omega_5 - 18\omega_9\omega_7\omega_4^2\omega_8\omega_5 - 12\omega_9cs^2\omega_4^2\omega_8\omega_5 - 6\omega_6cs^2\omega_7\omega_4^3\omega_5 - 18\omega_9\omega_6cs^2\omega_7\omega_4^2\omega_5 - 12\omega_6\omega_7\omega_4^2\omega_5 + 6\omega_9cs^2\omega_7\omega_4^3\omega_8 - 18\omega_6\omega_7\omega_4^3\omega_8v_2^2 + 18\omega_9\omega_7\omega_4^3\omega_8v_2^2 - 12\omega_9\omega_6cs^2\omega_7\omega_8\omega_5 - 6\omega_6cs^2\omega_7\omega_3^4\omega_8 - 6\omega_9\omega_6\omega_7\omega_4^3\omega_5 - 5\omega_9\omega_6cs^2\omega_7\omega_4^2\omega_8\omega_5 + 12\omega_6cs^2\omega_4^2\omega_8\omega_5 + 12\omega_9\omega_6cs^2\omega_7\omega_4\omega_5 + 36\omega_9\omega_6\omega_7\omega_4\omega_5v_2^2 + 12\omega_6\omega_7\omega_4^2\omega_8\omega_5 - 36\omega_9\omega_7\omega_4^2\omega_8\omega_5v_2^2 - 36\omega_9\omega_4^2\omega_8\omega_5v_2^2 + 18\omega_9\omega_7\omega_4^3\omega_8\omega_5 + 18\omega_6\omega_7\omega_4^3\omega_8\omega_5v_2^2 + 12\omega_9\omega_4^2\omega_8\omega_5 - 18\omega_6\omega_7\omega_4^3\omega_8v_2^2 + 6\omega_6cs^2\omega_7\omega_4^3\omega_8\omega_5 - 6\omega_9\omega_7\omega_4^3\omega_8 - 12\omega_9cs^2\omega_4^3\omega_8 + 12\omega_6cs^2\omega_3^4\omega_8 - 36\omega_6\omega_7\omega_4^2\omega_8\omega_5v_2^2 + 6\omega_6\omega_7\omega_4^3\omega_8 + 36\omega_9\omega_3^4\omega_8\omega_5v_2^2 + 5\omega_9\omega_7\omega_4^3\omega_8\omega_5 - 12\omega_6\omega_4^2\omega_8\omega_5 + 18\omega_9\omega_6\omega_7\omega_4^2\omega_5 + 12\omega_9cs^2\omega_4^3\omega_8\omega_5 + \omega_9\omega_6\omega_7\omega_4^2\omega_8\omega_5 - 12\omega_6\omega_4^3\omega_8 + 18\omega_9cs^2\omega_7\omega_4^2\omega_8\omega_5 + 18\omega_9\omega_6\omega_7\omega_4^3\omega_5v_2^2$$

$$C_{11} = 24cs^4\omega_1^2\omega_8^2 - 36\omega_4^3v_2^2 + cs^2\omega_3^4\omega_8^2 - 12\omega_4^2\omega_8^2v_2^4 + 6cs^4\omega_3^4\omega_8 - 72\omega_2^2\omega_8\omega_5^2 - 72cs^2\omega_3^4\omega_8v_2^2 + 24cs^2\omega_4^2\omega_8 - 8cs^2\omega_4^2\omega_8^2 - 30\omega_4^3\omega_8v_2^4 + 72cs^2\omega_4\omega_8v_2^2 + 72\omega_2^2v_2^2 - 3cs^4\omega_3^4\omega_8 - 6cs^2\omega_3^4\omega_8 - 24cs^4\omega_4^2\omega_8 - 3\omega_4^3\omega_8v_2^2 - 12cs^2\omega_4^2\omega_8v_2^2 - 72\omega_4^2v_2^4 - 216cs^2\omega_4^2v_2^2 + 24cs^4\omega_8^2 + 24cs^4\omega_4\omega_8 + 144cs^2\omega_4^2\omega_8v_2^2 + 30\omega_4^3\omega_8v_2^2 + 12cs^2\omega_4\omega_8^2 + 3\omega_4^3\omega_8v_2^4 - 36cs^2\omega_4\omega_8^2v_2^2 + 6cs^2\omega_4^3\omega_8^2v_2^2 + 12\omega_4^2\omega_8^2v_2^2 - 24cs^2\omega_4\omega_8 + 108cs^2\omega_4^3v_2^2 + 36\omega_4^3v_2^4 - 48cs^4\omega_4\omega_8^2 + 72\omega_4^2\omega_8v_2^4$$

2.5.4 Conservation of momentum: ρv_2



attached text file: output_d2q9_nse_clbm2_symbolic_pde_02.txt

$$v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{v_1 v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho v_2 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\rho v_1 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_1} + (cs^2 + v_2^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{2\rho v_2 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_4) \frac{cs^2 \delta_l^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + (-2 + \omega_4) \frac{cs^2 \delta_l^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (-2 + \omega_6 + 4cs^2 - 3\omega_6 v_2^2 - 2cs^2 \omega_6 + 6v_2^2) \frac{\delta_l^2}{\omega_6 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (2 - \omega_6) \frac{3\rho v_2 \delta_l^2}{\omega_6 \delta_t} \left(\frac{\partial v_2}{\partial x_2} \right)^2 +$$

$$\begin{aligned}
& (-2 + \omega_4) \frac{cs^2 \rho \delta_l^2}{2\delta_t \omega_4} \frac{\partial^2 v_2}{\partial x_1^2} + (-2 + \omega_4) \frac{cs^2 \rho \delta_l^2}{2\delta_t \omega_4} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + (-2 + \omega_6 + 6cs^2 - \omega_6 v_2^2 - 3cs^2 \omega_6 + 2v_2^2) \frac{v_2 \delta_l^2}{2\omega_6 \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + \\
& (-2 + \omega_6 + 2cs^2 - 3\omega_6 v_2^2 - cs^2 \omega_6 + 6v_2^2) \frac{\rho \delta_l^2}{2\omega_6 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + (-1 + 3cs^2 + v_1^2) \frac{v_1 v_2 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + cs^2 + 3v_1^2) \frac{\rho v_2 \delta_l^3}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + \\
& C_1 \frac{\rho v_1 \delta_l^3}{6\delta_t \omega_7 \omega_4} \frac{\partial^3 v_2}{\partial x_1^3} + (-12 + 12\omega_4 - \omega_4^2) \frac{cs^4 \delta_l^3}{6\delta_t \omega_4^2} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} - \frac{cs^2 \rho v_2 \delta_l^3}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + C_2 \frac{\rho v_2 \delta_l^3}{12\omega_6^2 \delta_t \omega_4 \omega_8} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_3 \frac{\delta_l^3}{12\omega_6^2 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\
& (-24 - 4\omega_6^2 + 24\omega_6 + 36cs^2 + 5cs^2 \omega_6^2 - 60\omega_6 v_2^2 - 36cs^2 \omega_6 + 60v_2^2 + 11\omega_6^2 v_2^2) \frac{\rho v_2 \delta_l^3}{6\omega_6^2 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& (-2cs^2 - cs^4 \omega_5 - 3v_1^4 \omega_5 + 6v_1^4 + 24cs^2 v_1^2 - 12cs^2 v_1^2 \omega_5 - 6v_1^2 + 3v_1^2 \omega_5 + 2cs^4 + cs^2 \omega_5) \frac{v_2 \delta_l^4}{24\delta_t \omega_5} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& (-4 + 6cs^2 + 10v_1^2 - 5v_1^2 \omega_5 + 2\omega_5 - 3cs^2 \omega_5) \frac{\rho v_1 v_2 \delta_l^4}{12\delta_t \omega_5} \frac{\partial^4 v_1}{\partial x_1^4} + C_4 \frac{\rho \delta_l^4}{24\delta_t \omega_7^2 \omega_4^3} \frac{\partial^4 v_2}{\partial x_1^4} + C_5 \frac{cs^2 v_1 \delta_l^4}{12\omega_9 \omega_6 \delta_t \omega_7^2 \omega_4^3 \omega_8 \omega_5} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + \\
& C_6 \frac{cs^2 \rho \delta_l^4}{12\omega_9 \omega_6 \delta_t \omega_7 \omega_4^3 \omega_8 \omega_5} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + \\
& (\omega_7 v_1^2 \omega_5 - \omega_7 - \omega_7 \omega_5 + 3cs^2 \omega_7 - 3v_1^2 \omega_5 + 3\omega_5 + \omega_7 v_1^2 + 3cs^2 \omega_7 \omega_5 - 9cs^2 \omega_5) \frac{\rho v_1 v_2 \delta_l^4}{12\delta_t \omega_7 \omega_5} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& C_7 \frac{cs^2 v_2 \delta_l^4}{12\omega_9 \omega_6^3 \delta_t \omega_7 \omega_4^3 \omega_8^2} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_8 \frac{cs^2 \rho \delta_l^4}{12\omega_9 \omega_6^2 \delta_t \omega_7 \omega_4^3 \omega_8} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_9 \frac{\rho \delta_l^4}{12\omega_6^3 \delta_t \omega_4^3 \omega_8^2} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^2} + C_{10} \frac{v_2 \delta_l^4}{12\omega_6^3 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{11} \frac{\rho \delta_l^4}{12\omega_6^3 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} \\
& = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= 6 - 18cs^2 - \omega_7 \omega_4 v_1^2 - 3\omega_7 - 3\omega_4 + 3\omega_4 v_1^2 + 9cs^2 \omega_4 - 6v_1^2 - 3cs^2 \omega_7 \omega_4 + 9cs^2 \omega_7 + \omega_7 \omega_4 + 3\omega_7 v_1^2 \\
C_2 &= -12\omega_4 \omega_8 + 12\omega_6 \omega_4 + 12\omega_6^2 + 18cs^2 \omega_6^2 \omega_8 + 12\omega_6^2 \omega_4 v_2^2 - 36cs^2 \omega_6^2 + 6\omega_6^2 \omega_8 v_2^2 - 6\omega_6 \omega_4 \omega_8 v_2^2 + 6\omega_6 \omega_4 \omega_8 - 6\omega_6^2 \omega_8 - 11cs^2 \omega_6^2 \omega_4 \omega_8 - \\
& 36cs^2 \omega_6 \omega_4 + 36cs^2 \omega_4 \omega_8 + 3\omega_6^2 \omega_4 \omega_8 + 36cs^2 \omega_6^2 \omega_4 - 18cs^2 \omega_6 \omega_4 \omega_8 - 3\omega_6^2 \omega_4 \omega_8 v_2^2 - 12\omega_6^2 v_2^2 + 12\omega_4 \omega_8 v_2^2 - 12\omega_6 \omega_4 v_2^2 - 12\omega_6^2 \omega_4 \\
C_3 &= 7\omega_6^2 v_2^4 - 12cs^2 - cs^2 \omega_6^2 + 36\omega_6 v_2^2 + 36v_2^4 - 144cs^2 \omega_6 v_2^2 + 12cs^2 \omega_6 + cs^4 \omega_6^2 - 36v_2^2 + 144cs^2 v_2^2 + 24cs^2 \omega_6^2 v_2^2 - 7\omega_6^2 v_2^2 - 36\omega_6 v_2^4 + 12cs^4 - 12cs^4 \omega_6 \\
C_4 &= \\
& cs^2 \omega_7^2 \omega_4^3 - 216cs^2 \omega_4^2 v_1^2 - 72cs^2 \omega_7 \omega_4^3 v_1^2 - 48cs^4 \omega_7^2 \omega_4 - 30\omega_7 \omega_4^3 v_1^4 - 8cs^2 \omega_7^2 \omega_4^2 - 36\omega_4^3 v_1^2 - 3\omega_7^2 \omega_4^3 v_1^2 - 3cs^4 \omega_7^2 \omega_4^3 + 144cs^2 \omega_7 \omega_4^2 v_1^2 + 12cs^2 \omega_7^2 \omega_4 + \\
& 108cs^2 \omega_4^3 v_1^2 - 36cs^2 \omega_7^2 \omega_4 v_1^2 + 12\omega_7^2 \omega_4^3 v_1^2 + 24cs^4 \omega_7^2 \omega_4^2 + 24cs^4 \omega_7^2 + 72\omega_4^2 v_1^2 + 72\omega_7 \omega_4^2 v_1^4 - 12cs^2 \omega_7^2 \omega_4^2 v_1^2 - 24cs^4 \omega_7 \omega_4^2 - 72\omega_4^2 v_1^4 + 6cs^4 \omega_7 \omega_4^3 + \\
& 72cs^2 \omega_7 \omega_4 v_1^2 - 72\omega_7 \omega_4^2 v_1^2 - 12\omega_7^2 \omega_4^3 v_1^4 - 24cs^2 \omega_7 \omega_4 + 24cs^2 \omega_7 \omega_4^2 + 6cs^2 \omega_7^2 \omega_4^3 v_1^2 - 6cs^2 \omega_7 \omega_4^3 + 3\omega_7^2 \omega_4^3 v_1^4 + 30\omega_7 \omega_4^3 v_1^2 + 24cs^4 \omega_7 \omega_4 + 36\omega_4^3 v_1^4 \\
C_5 &= -6\omega_6 \omega_7^2 \omega_4^2 \omega_8 \omega_5 - 12\omega_9 \omega_6 \omega_7 \omega_8 v_1^2 \omega_5 - 18cs^2 \omega_6 \omega_7 \omega_4^2 \omega_8 \omega_5 - 36\omega_9 cs^2 \omega_6 \omega_7 \omega_8 \omega_5 + 3\omega_9 cs^2 \omega_6 \omega_7^2 \omega_4^2 \omega_8 \omega_5 - 18\omega_9 \omega_6 \omega_7 \omega_4 \omega_8 \omega_5 - \\
& 12\omega_7^2 \omega_4^2 \omega_5 - 18\omega_9 \omega_6 \omega_7 \omega_4 \omega_8 + 12\omega_9 \omega_6 \omega_7 \omega_8 \omega_5 + 12\omega_9 \omega_6 \omega_7^2 \omega_8 - 6\omega_9 \omega_7^2 \omega_4^2 \omega_8 - 12\omega_9 \omega_6 \omega_4 \omega_8 v_1^2 \omega_5 - 12\omega_6 \omega_7^2 \omega_4^2 v_1^2 \omega_5 - 5\omega_9 \omega_6 \omega_7^2 \omega_4^2 \omega_8 v_1^2 - \\
& 36\omega_9 cs^2 \omega_6 \omega_7^2 \omega_4 + 54\omega_9 cs^2 \omega_6 \omega_7^2 \omega_4 \omega_8 + 6\omega_6 \omega_7^2 \omega_4^2 \omega_8 v_1^2 \omega_5 - \omega_9 \omega_6 \omega_7^2 \omega_4^2 \omega_8 \omega_5 + 54\omega_9 cs^2 \omega_6 \omega_7 \omega_4 \omega_8 \omega_5 + 36\omega_9 cs^2 \omega_6 \omega_7^2 \omega_4^2 \omega_8 \omega_5 - \\
& 36cs^2 \omega_6 \omega_7^2 \omega_4 \omega_8 \omega_5 - 12\omega_6 \omega_7 \omega_4 \omega_8 \omega_5 - 12\omega_9 \omega_6 \omega_7^2 \omega_4 \omega_8 v_1^2 \omega_5 + 18\omega_9 cs^2 \omega_6 \omega_7^2 \omega_8 \omega_5 - 12\omega_6 \omega_7^2 \omega_4 \omega_5 + 6\omega_7^2 \omega_4^2 \omega_8 \omega_5 + 18\omega_9 \omega_6 \omega_7 \omega_4 \omega_8 v_1^2 \omega_5 + \\
& 12\omega_9 \omega_6 \omega_7^2 \omega_4^2 v_1^2 + 12\omega_7^2 \omega_4^2 v_1^2 \omega_5 - 6\omega_6 \omega_7 \omega_4^2 \omega_8 v_1^2 \omega_5 - 36cs^2 \omega_6 \omega_7^2 \omega_4^2 \omega_5 - 6\omega_9 \omega_6 \omega_7^2 \omega_8 \omega_5 + 12\omega_9 \omega_6 \omega_7^2 \omega_4 \omega_8 \omega_5 - 9\omega_9 cs^2 \omega_6 \omega_7 \omega_4^2 \omega_8 \omega_5 - \\
& 12\omega_9 \omega_6 \omega_7^2 \omega_4 v_1^2 + 6\omega_9 \omega_7^2 \omega_4^2 \omega_8 v_1^2 - 18cs^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 - 36\omega_9 cs^2 \omega_6 \omega_4 \omega_8 \omega_5 + 36cs^2 \omega_6 \omega_7^2 \omega_4 \omega_5 + 6\omega_6 \omega_7^2 \omega_4 \omega_8 \omega_5 + 12\omega_9 \omega_6 \omega_7^2 \omega_8 v_1^2 \omega_5 + \\
& 18cs^2 \omega_6 \omega_7^2 \omega_4^2 \omega_8 \omega_5 - 12\omega_9 \omega_6 \omega_7^2 \omega_8 v_1^2 + 6\omega_9 \omega_6 \omega_7^2 \omega_8 v_1^2 \omega_5 + 12\omega_9 \omega_6 \omega_7^2 \omega_4 + 12\omega_9 \omega_7^2 \omega_4^2 - 36\omega_9 cs^2 \omega_6 \omega_7^2 \omega_8 - 12\omega_9 \omega_7^2 \omega_4^2 v_1^2 + 12\omega_9 \omega_6 \omega_4 \omega_8 \omega_5 + \\
& 12\omega_6 \omega_7^2 \omega_4^2 \omega_5 + 18\omega_9 cs^2 \omega_7^2 \omega_8 + 36\omega_9 cs^2 \omega_6 \omega_7^2 \omega_8 \omega_5 + 36cs^2 \omega_6 \omega_7 \omega_4 \omega_8 \omega_5 + \omega_9 \omega_6 \omega_7^2 \omega_4^2 \omega_8 v_1^2 \omega_5 + 12\omega_6 \omega_7^2 \omega_4 \omega_8 \omega_5 - 15\omega_9 cs^2 \omega_6 \omega_7^2 \omega_4 \omega_8 - \\
& 12\omega_6 \omega_7^2 \omega_4 \omega_8 v_1^2 \omega_5 - 36\omega_9 cs^2 \omega_6 \omega_7^2 \omega_4 \omega_8 \omega_5 - 12\omega_9 \omega_6 \omega_7^2 \omega_4 + 3\omega_9 \omega_6 \omega_7 \omega_4^2 \omega_8 \omega_5 + 12\omega_6 \omega_7 \omega_4 \omega_8 v_1^2 \omega_5 - 12\omega_9 \omega_6 \omega_7^2 \omega_8 \omega_5 + 5\omega_9 \omega_6 \omega_7^2 \omega_4 \omega_8 + \\
& 36cs^2 \omega_7^2 \omega_4^2 \omega_5 - 36\omega_9 cs^2 \omega_7^2 \omega_4^2 - 6\omega_7^2 \omega_4^2 \omega_8 v_1^2 \omega_5 + 18\omega_9 \omega_6 \omega_7^2 \omega_4 \omega_8 v_1^2 - 3\omega_9 \omega_6 \omega_7 \omega_4^2 \omega_8 v_1^2 \omega_5 + 12\omega_6 \omega_7^2 \omega_4 v_1^2 \omega_5 \\
C_6 &= -12cs^2 \omega_6 \omega_7 \omega_4^2 \omega_8 \omega_5 - 12\omega_9 cs^2 \omega_6 \omega_7 \omega_8 \omega_5 + 18\omega_9 cs^2 \omega_6 \omega_7 \omega_4^2 \omega_8 - 12\omega_9 \omega_6 \omega_7 \omega_4^3 - 15\omega_9 \omega_6 \omega_7 \omega_4^3 \omega_8 v_1^2 + 12cs^2 \omega_6 \omega_7^2 \omega_8 \omega_5 - 36\omega_6 \omega_7 \omega_4^3 v_1^2 \omega_5 + \\
& 18\omega_6 \omega_7 \omega_4^3 \omega_8 v_1^2 \omega_5 - 36\omega_9 \omega_7 \omega_4^3 v_1^2 + 6\omega_9 cs^2 \omega_7 \omega_4^3 \omega_8 - 6\omega_6 \omega_7 \omega_4^3 \omega_8 \omega_5 + 12\omega_6 \omega_7 \omega_4^3 \omega_5 + 36\omega_9 \omega_6 \omega_4 \omega_8 v_1^2 \omega_5 - 12\omega_9 cs^2 \omega_7 \omega_4^3 + 12\omega_9 \omega_6 \omega_7 \omega_4^2 - \\
& 18\omega_9 \omega_6 \omega_7 \omega_4^2 \omega_8 - \omega_9 cs^2 \omega_6 \omega_7 \omega_4^2 \omega_8 \omega_5 + 36\omega_7 \omega_4^3 v_1^2 \omega_5 + 18\omega_9 cs^2 \omega_6 \omega_7 \omega_4 \omega_8 \omega_5 - 5\omega_9 cs^2 \omega_6 \omega_7 \omega_4^3 \omega_8 - 18\omega_9 cs^2 \omega_6 \omega_7^2 \omega_8 \omega_5 - 36\omega_9 \omega_6 \omega_7 \omega_4 \omega_8 v_1^2 + \\
& 6\omega_6 \omega_4^3 \omega_8 \omega_5 + 12cs^2 \omega_7 \omega_4^3 \omega_5 + 5\omega_9 \omega_6 \omega_7 \omega_4^3 \omega_8 + 6\omega_7 \omega_4^3 \omega_8 \omega_5 - 36\omega_6 \omega_7 \omega_4^2 \omega_8 v_1^2 \omega_5 + 18\omega_9 \omega_6 \omega_4^2 \omega_8 \omega_5 - 12\omega_6 \omega_7 \omega_4^2 \omega_5 + 12\omega_9 \omega_7 \omega_4^3 - 18\omega_6 \omega_4^3 \omega_8 v_1^2 \omega_5 + \\
& 12\omega_9 cs^2 \omega_6 \omega_7 \omega_4^3 - 12\omega_7 \omega_4^3 \omega_5 - 5\omega_9 cs^2 \omega_6 \omega_7 \omega_4^2 \omega_8 \omega_5 + 12\omega_9 cs^2 \omega_6 \omega_4 \omega_8 \omega_5 + 12\omega_6 \omega_7 \omega_4^2 \omega_8 \omega_5 + 18\omega_9 \omega_7 \omega_4^3 \omega_8 v_1^2 - 18\omega_7 \omega_4^3 \omega_8 v_1^2 \omega_5 - \\
& 36\omega_9 \omega_6 \omega_7 \omega_4^2 v_1^2 - 54\omega_9 \omega_6 \omega_7^2 \omega_8 v_1^2 \omega_5 + 36\omega_6 \omega_7 \omega_4^2 v_1^2 \omega_5 - 6cs^2 \omega_6 \omega_4^3 \omega_8 \omega_5 + 54\omega_9 \omega_6 \omega_7 \omega_4^2 \omega_8 v_1^2 - 12\omega_9 cs^2 \omega_6 \omega_7 \omega_4^2 + 6cs^2 \omega_6 \omega_7 \omega_4^3 \omega_8 \omega_5 + \\
& 12cs^2 \omega_6 \omega_7 \omega_4^2 \omega_5 - 6\omega_9 \omega_7 \omega_4^3 \omega_8 - 12\omega_9 \omega_6 \omega_4 \omega_8 \omega_5 - 6cs^2 \omega_7 \omega_4^3 \omega_8 \omega_5 - 6\omega_9 \omega_6 \omega_4^3 \omega_8 \omega_5 + 18\omega_9 \omega_6 \omega_4^3 \omega_8 v_1^2 \omega_5 - 12\omega_6 \omega_7^2 \omega_8 \omega_5 + 12\omega_9 \omega_6 \omega_7 \omega_4 \omega_8 + \\
& \omega_9 \omega_6 \omega_7 \omega_4^2 \omega_8 \omega_5 - 12cs^2 \omega_6 \omega_7 \omega_4^3 \omega_5 + 6\omega_9 cs^2 \omega_6 \omega_4^3 \omega_8 \omega_5 + 36\omega_6 \omega_4^2 \omega_8 v_1^2 \omega_5 - 12\omega_9 cs^2 \omega_6 \omega_7 \omega_4 \omega_8 - 3\omega_9 \omega_6 \omega_7 \omega_4^2 \omega_8 v_1^2 \omega_5 + 36\omega_9 \omega_6 \omega_7 \omega_4^3 v_1^2 \\
C_7 &= 36\omega_9 cs^2 \omega_7 \omega_4^2 \omega_8^2 - 12\omega_6^2 \omega_7 \omega_4^2 \omega_8 + 54\omega_9 cs^2 \omega_6^3 \omega_7 \omega_4 \omega_8 - 12\omega_6^3 \omega_7 \omega_4 \omega_8^2 v_2^2 - 12\omega_9 \omega_6^2 \omega_7 \omega_8^2 v_2^2 - 36\omega_9 cs^2 \omega_6 \omega_7 \omega_4^2 \omega_8 + 54\omega_9 cs^2 \omega_6^2 \omega_7 \omega_4 \omega_8^2 - \\
& 6\omega_6^3 \omega_7 \omega_4 \omega_8^2 - 12\omega_9 \omega_6^3 \omega_7 \omega_4^2 v_2^2 - 6\omega_6^3 \omega_7 \omega_4^2 \omega_8^2 v_2^2 - 12\omega_6^3 \omega_7 \omega_4 \omega_8 v_2^2 - 18\omega_9 \omega_6^3 \omega_7 \omega_4 \omega_8 - 36\omega_9 cs^2 \omega_6^3 \omega_7 \omega_8 - 36cs^2 \omega_6^3 \omega_7 \omega_4 \omega_8^2 - 18\omega_9 \omega_6^3 \omega_7 \omega_4 \omega_8^2 + \\
& 12\omega_9 \omega_6^3 \omega_4 \omega_8^2 + 12\omega_9 \omega_6^3 \omega_7 \omega_4 + 12\omega_9 \omega_6 \omega_7 \omega_4^2 \omega_8 - 12\omega_9 \omega_6^3 \omega_7 \omega_4^2 + 36cs^2 \omega_6^3 \omega_7 \omega_4 \omega_8 + 18\omega_9 cs^2 \omega_6^2 \omega_4 \omega_8^2 + 12\omega_9 \omega_6 \omega_7 \omega_4^2 \omega_8^2 - 12\omega_9 \omega_6^3 \omega_7 \omega_4 \omega_8^2 v_2^2 - \\
& 18cs^2 \omega_6^3 \omega_4 \omega_8^2 - 12\omega_6^3 \omega_4 \omega_8^2 - 2\omega_9 \omega_6^3 \omega_7 \omega_4^2 \omega_8^2 v_2^2 + 6\omega_9 \omega_6^3 \omega_4^2 \omega_8^2 v_2^2 + 12\omega_6^3 \omega_4 \omega_8^2 v_2^2 + 36\omega_9 cs^2 \omega_6^3 \omega_7 \omega_8^2 + 12\omega_9 \omega_6^3 \omega_7 \omega_4 \omega_8^2 - 36\omega_9 cs^2 \omega_6^2 \omega_7 \omega_4^2 + \\
& 12\omega_9 \omega_6^3 \omega_7 \omega_8^2 v_2^2 - 6\omega_9 \omega_6^3 \omega_7 \omega_4^2 \omega_8 v_2^2 - 36\omega_9 cs^2 \omega_6 \omega_7 \omega_4^2 \omega_8^2 + 12\omega_6^3 \omega_7 \omega_4^2 \omega_8 - 12\omega_9 \omega_6 \omega_7 \omega_4^2 \omega_8^2 v_2^2 + 12\omega_9 \omega_6^3 \omega_7 \omega_4^2 v_2^2 - 40\omega_9 cs^2 \omega_6^3 \omega_7 \omega_4 \omega_8^2 + \\
& 6\omega_6^3 \omega_7 \omega_4^2 \omega_8^2 + 12\omega_9 \omega_6^3 \omega_7 \omega_8^2 + \omega_9 \omega_6^3 \omega_7 \omega_4^2 \omega_8^2 v_2^2 - 12\omega_9 \omega_6^3 \omega_7 \omega_8 v_2^2 + 54\omega_9 cs^2 \omega_6^2 \omega_7 \omega_4^2 \omega_8 - 12\omega_9 \omega_6 \omega_7 \omega_4^2 \omega_8 v_2^2 - 12\omega_6^3 \omega_7 \omega_4 \omega_8 - 12\omega_9 \omega_7 \omega_4^2 \omega_8^2 + \\
& 5\omega_9 cs^2 \omega_6^3 \omega_7 \omega_4 \omega_8^2 - 12\omega_9 \omega_6^3 \omega_4 \omega_8^2 v_2^2 + 18\omega_9 \omega_6^3 \omega_7 \omega_4 \omega_8^2 v_2^2 - 12\omega_9 \omega_6^3 \omega_7 \omega_4 v_2^2 - 6\omega_6^3 \omega_7^2 \omega_8^2 v_2^2 - 36\omega_9 cs^2 \omega_6^3 \omega_7 \omega_4 - 36cs^2 \omega_6^3 \omega_7 \omega_4^2 \omega_8 - 36\omega_9 cs^2 \omega_6^2 \omega_4 \omega_8^2 - \\
& 18\omega_9 \omega_6^2 \omega_7 \omega_4 \omega_8 + 18\omega_9 \omega_6^3 \omega_7 \omega_4 \omega_8 v_2^2 + 36cs^2 \omega_6^3 \omega_4 \omega_8^2 + 6\omega_6^3 \omega_4 \omega_8^2 - \omega_9 \omega_6^3 \omega_7 \omega_4^2 \omega_8^2 - 18cs^2 \omega_6^2 \omega_7 \omega_4^2 \omega_8^2 + 12\omega_9 \omega_6^3 \omega_7 \omega_4 \omega_8 + 18\omega_9 \omega_6^2 \omega_7 \omega_4 \omega_8 v_2^2 + \\
& 6\omega_6^3 \omega_7 \omega_4^2 \omega_8^2 v_2^2 - 12\omega_9 \omega_6^3 \omega_7 \omega_8^2 + 6\omega_9 \omega_6^3 \omega_7 \omega_4 \omega_8 + 36cs^2 \omega_6^2 \omega_7 \omega_4^2 \omega_8 + 12\omega_9 \omega_7 \omega_4^2 \omega_8^2 v_2^2 + 18cs^2 \omega_6^3 \omega_7 \omega_4^2 \omega_8^2 - 6\omega_9 \omega_6^2 \omega_4 \omega_8^2 + 2\omega_9 \omega_6^2 \omega_7 \omega_4 \omega_8^2 + \\
& 36\omega_9 cs^2 \omega_6^3 \omega_7 \omega_4^2 - 18\omega_9 cs^2 \omega_6^3 \omega_7 \omega_4^2 \omega_8 - 36\omega_9 cs^2 \omega_6^2 \omega_7 \omega_8^2 + 12\omega_6^3 \omega_7 \omega_4 \omega_8 v_2^2 + 12\omega_9 \omega_6^2 \omega_7 \omega_4^2 + 12\omega_6^3 \omega_7 \omega_4 \omega_8 v_2^2 + 12\omega_6^3 \omega_7 \omega_4 \omega_8^2 - 6\omega_9 cs^2 \omega_6^2 \omega_7 \omega_4^2 \omega_8^2
\end{aligned}$$

$$\begin{aligned}
C_8 = & 12\omega_6^2\omega_7\omega_4^2\omega_8 - 6cs^2\omega_6\omega_7\omega_4^3\omega_8 + 12\omega_6^2\omega_7\omega_4^3 + 12\omega_9cs^2\omega_6^2\omega_7\omega_4 + 24\omega_9cs^2\omega_6\omega_7\omega_4^2\omega_8 + 12\omega_9\omega_6\omega_7\omega_4^3 - 72\omega_9\omega_6^2\omega_7\omega_4^2v_2^2 + 36\omega_6^2\omega_4^2\omega_8v_2^2 + \\
& 12cs^2\omega_6\omega_7\omega_4^3 - 36\omega_9\omega_6\omega_4^2\omega_8v_2^2 - 12\omega_6^2\omega_7\omega_4^2 + 6\omega_9cs^2\omega_7\omega_4^3\omega_8 + 6\omega_6^2\omega_4^3\omega_8 - 6\omega_9\omega_6\omega_4^3\omega_8 - 36\omega_9\omega_7\omega_4^2\omega_8v_2^2 - 12\omega_9\omega_6\omega_7\omega_4^2 - 18\omega_9\omega_6\omega_7\omega_4^3\omega_8v_2^2 - \\
& 24\omega_9\omega_6\omega_7\omega_4^2\omega_8 - 36\omega_9\omega_6\omega_7\omega_4^3\omega_8v_2^2 + 36\omega_9\omega_6^2\omega_7\omega_4^3v_2^2 + 18\omega_6^2\omega_7\omega_4^3\omega_8v_2^2 + 12\omega_9cs^2\omega_6^2\omega_7\omega_4^3 - 6\omega_9cs^2\omega_6\omega_7\omega_4^3\omega_8 - 6\omega_6^2\omega_7\omega_4^3\omega_8 - \\
& 24\omega_9cs^2\omega_6^2\omega_7\omega_4^2 + 6\omega_9\omega_6\omega_7\omega_4^3\omega_8 + 12\omega_9\omega_6\omega_4^2\omega_8 + 18\omega_9cs^2\omega_6^2\omega_7\omega_4\omega_8 - 12\omega_6^2\omega_4^2\omega_8 - 12\omega_9cs^2\omega_7\omega_4^2\omega_8 - 18\omega_6\omega_7\omega_4^3\omega_8v_2^2 + 36\omega_6\omega_7\omega_4^3v_2^2 + \\
& 18\omega_9\omega_7\omega_4^3\omega_8v_2^2 - 12\omega_9cs^2\omega_6\omega_7\omega_4^3 - 12\omega_9\omega_6^2\omega_7\omega_4 + 36\omega_9\omega_6\omega_7\omega_4^2v_2^2 - 4\omega_9cs^2\omega_6^2\omega_7\omega_4^2\omega_8 + 72\omega_9\omega_6\omega_7\omega_4^2\omega_8v_2^2 + 18\omega_9\omega_6\omega_4^3\omega_8v_2^2 + 12cs^2\omega_6^2\omega_4^2\omega_8 + \\
& 6cs^2\omega_6^2\omega_7\omega_4^3\omega_8 - 12\omega_9cs^2\omega_6^2\omega_7\omega_8 + 6\omega_9cs^2\omega_6\omega_4^3\omega_8 - 18\omega_6^2\omega_4^3\omega_8v_2^2 + 12\omega_9cs^2\omega_6\omega_7\omega_4^2 - 6\omega_9\omega_7\omega_4^3\omega_8 + 36\omega_6^2\omega_7\omega_4^2v_2^2 + 12cs^2\omega_6^2\omega_7\omega_4^2 + \\
& 6\omega_6\omega_7\omega_4^3\omega_8 - 12\omega_6\omega_7\omega_4^3 - 6cs^2\omega_6^2\omega_4^3\omega_8 - 12cs^2\omega_6^2\omega_7\omega_4^3\omega_8 - \omega_9cs^2\omega_6^2\omega_7\omega_4^3\omega_8 + 12\omega_9\omega_6\omega_7\omega_4\omega_8 - 12\omega_9\omega_6^2\omega_7\omega_4^3 - 36\omega_9\omega_6\omega_7\omega_4^3v_2^2 + \\
& 12\omega_9\omega_7\omega_4^2\omega_8 - 36\omega_6^2\omega_7\omega_4^3v_2^2 - 12cs^2\omega_6^2\omega_7\omega_4^3 - 12\omega_9cs^2\omega_6\omega_7\omega_4\omega_8 + 24\omega_9\omega_6^2\omega_7\omega_4^2 - 36\omega_6^2\omega_7\omega_4^2\omega_8v_2^2 + 36\omega_9\omega_6^2\omega_7\omega_4v_2^2 - 12\omega_9cs^2\omega_6\omega_4^2\omega_8
\end{aligned}$$

$$\begin{aligned}
C_9 = & -90\omega_6\omega_4^3\omega_8^2v_2^4 - 99cs^2\omega_6^3\omega_4^3\omega_8v_2^2 - 39\omega_6^3\omega_4^3\omega_8v_2^4 + 12cs^4\omega_6^2\omega_4^2\omega_8 - 306cs^2\omega_6\omega_4^3\omega_8^2v_2^2 - 36\omega_6^3\omega_4^3v_2^2 - 4\omega_6^3\omega_4^3\omega_8^2v_2^2 + 18cs^2\omega_6^3\omega_4^2\omega_8 + \\
& 12cs^2\omega_6\omega_4^2\omega_8^2 + cs^4\omega_6^2\omega_4^2\omega_8^2 + 36\omega_6\omega_4^3\omega_8v_2^2 + 18cs^2\omega_6^2\omega_4^2\omega_8^2v_2^2 - 24cs^4\omega_6^3\omega_4\omega_8^2 - 6cs^4\omega_6^2\omega_4^3\omega_8 - 6\omega_6^3\omega_4^2\omega_8^2v_2^4 - 36\omega_6^3\omega_4^3v_2^4 + 12cs^4\omega_6^3\omega_4\omega_8 - \\
& 19\omega_6^2\omega_4^3\omega_8^2v_2^2 - 5cs^2\omega_6^3\omega_4^2\omega_8^2 - 3cs^2\omega_6^3\omega_4^2\omega_8^2v_2^2 + 198cs^2\omega_6^2\omega_4^3\omega_8v_2^2 + 6cs^4\omega_6^2\omega_4^2\omega_8^2 - 108cs^2\omega_6^2\omega_4^3v_2^2 - 36\omega_6^3\omega_4^2\omega_8v_2^2 + 36cs^2\omega_6^3\omega_4\omega_8v_2^2 + \\
& 12cs^2\omega_6\omega_4^3\omega_8^2 + 12cs^4\omega_6^3\omega_8^2 + 36\omega_6^3\omega_4^3v_2^2 - 72\omega_4^3\omega_8^2v_2^2 + 72\omega_6^3\omega_4^3\omega_8v_2^4 - 6cs^2\omega_6^3\omega_4^3\omega_8 + 60cs^2\omega_6^2\omega_4^3\omega_8^2v_2^2 - cs^4\omega_6^3\omega_4^3\omega_8^2 - 12cs^2\omega_6^2\omega_4^2\omega_8 - \\
& 18cs^2\omega_6^2\omega_4\omega_8^2v_2^2 + 36\omega_6^2\omega_4^3v_2^2 + 252cs^2\omega_6^3\omega_8^2v_2^2 - 108cs^2\omega_6^3\omega_4^2v_2^2 + 19\omega_6^2\omega_4^3\omega_8^2v_2^4 + 36\omega_6^3\omega_4^2\omega_8v_2^4 - 12cs^4\omega_6\omega_4^2\omega_8^2 - 36\omega_6^3\omega_4^2v_2^4 + \\
& 72\omega_6^3\omega_8^2v_2^4 - 72\omega_6^2\omega_4^3\omega_8v_2^2 - 18cs^4\omega_6^3\omega_4^2\omega_8 - 36cs^2\omega_6\omega_4^2\omega_8^2v_2^2 + 6cs^2\omega_6^3\omega_4\omega_8^2 - 12cs^2\omega_4^3\omega_8^2 - cs^2\omega_6^2\omega_4^3\omega_8^2 + 54cs^2\omega_6^3\omega_4\omega_8v_2^2 - 12cs^2\omega_6^3\omega_4\omega_8 + \\
& 39\omega_6^3\omega_4^3\omega_8v_2^2 + 108cs^2\omega_6^3\omega_4^3v_2^2 + 6cs^2\omega_6^2\omega_4^3\omega_8 + 36cs^2\omega_6^2\omega_4^2\omega_8v_2^2 + 90\omega_6\omega_4^2\omega_8^2v_2^2 + 13cs^4\omega_6^3\omega_4^2\omega_8^2 - 6cs^2\omega_6^2\omega_4^2\omega_8^2 - 36\omega_6\omega_4^3\omega_8v_2^2 + \\
& 12cs^2\omega_6^3\omega_4^3\omega_8^2v_2^2 + 4\omega_6^3\omega_4^3\omega_8^2v_2^2 + 36\omega_6^3\omega_4^3v_2^2 + 12cs^4\omega_4^3\omega_8^2 + 6cs^4\omega_6^3\omega_4^3\omega_8 - 108cs^2\omega_6\omega_4^3\omega_8v_2^2 - 12cs^4\omega_6\omega_4^3\omega_8^2
\end{aligned}$$

$$\begin{aligned}
C_{10} = & 12 + 6cs^2\omega_6^3 + 90\omega_6^2v_2^4 + 8\omega_6^2 - 18\omega_6 - 132cs^2 - 78cs^2\omega_6^2 + 234\omega_6v_2^2 - \omega_6^3 + 144v_2^4 - 1008cs^2\omega_6v_2^2 - 9\omega_6^3v_2^4 + 198cs^2\omega_6 + 10\omega_6^3v_2^2 + \\
& 82cs^4\omega_6^2 - 156v_2^2 - 5cs^4\omega_6^3 + 672cs^2v_2^2 + 404cs^2\omega_6^2v_2^2 - 98\omega_6^2v_2^2 - 216\omega_6v_2^4 - 34cs^2\omega_6^3v_2^2 + 144cs^4 - 216cs^4\omega_6
\end{aligned}$$

$$\begin{aligned}
C_{11} = & 12 + 2cs^2\omega_6^3 + 310\omega_6^2v_2^4 + 8\omega_6^2 - 18\omega_6 - 36cs^2 - 22cs^2\omega_6^2 + 378\omega_6v_2^2 - \omega_6^3 + 504v_2^4 - 648cs^2\omega_6v_2^2 - 29\omega_6^3v_2^4 + 54cs^2\omega_6 + 14\omega_6^3v_2^2 + \\
& 14cs^4\omega_6^2 - 252v_2^2 - cs^4\omega_6^3 + 432cs^2v_2^2 + 252cs^2\omega_6^2v_2^2 - 154\omega_6^2v_2^2 - 756\omega_6v_2^4 - 18cs^2\omega_6^3v_2^2 + 24cs^4 - 36cs^4\omega_6
\end{aligned}$$

2.6 CuLBM1

2.6.1 Definitions

Based on [2], collision operator \mathbf{C} :

$$\mathbf{C}(\mathbf{f}) = \mathbf{M}^{-1}\mathbf{G}^{-1}\left(\mathbf{S}\left(\gamma^{(eq)} - \mathbf{G}(\mathbf{M}\mathbf{f})\right)\right),$$

where

$$\mathbf{S} = \text{diag}(0, 0, 0, \omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6),$$

$\omega_1, \omega_2, \dots, \omega_6 \in (0, 2)$.

The nonlinear operator \mathbf{G} (with its inverse \mathbf{G}^{-1}) transforms the raw moment vector $\boldsymbol{\mu}$ defined by matrix \mathbf{M} to the cumulant vector

$$\boldsymbol{\gamma} = \mathbf{G}(\boldsymbol{\mu}) = \left(\gamma_{(0,0)}, \gamma_{(1,0)}, \gamma_{(0,1)}, \gamma_{(2,0)}, \gamma_{(0,2)}, \gamma_{(1,1)}, \gamma_{(2,1)}, \gamma_{(1,2)}, \gamma_{(2,2)}\right)^T$$

as

$$\begin{aligned}
\gamma_{(0,0)} &= m_{(0,0)}, \\
\gamma_{(1,0)} &= \frac{m_{(1,0)}}{m_{(0,0)}}, \\
\gamma_{(0,1)} &= \frac{m_{(0,1)}}{m_{(0,0)}}, \\
\gamma_{(2,0)} &= -\frac{m_{(1,0)}^2}{m_{(0,0)}^2} + \frac{m_{(2,0)}}{m_{(0,0)}}, \\
\gamma_{(0,2)} &= -\frac{m_{(0,1)}^2}{m_{(0,0)}^2} + \frac{m_{(0,2)}}{m_{(0,0)}}, \\
\gamma_{(1,1)} &= -\frac{m_{(1,0)}m_{(0,1)}}{m_{(0,0)}^2} + \frac{m_{(1,1)}}{m_{(0,0)}}, \\
\gamma_{(2,1)} &= \frac{m_{(2,1)}}{m_{(0,0)}} - \frac{m_{(0,1)}m_{(2,0)}}{m_{(0,0)}^2} - 2\frac{m_{(1,0)}m_{(1,1)}}{m_{(0,0)}^2} + 2\frac{m_{(1,0)}^2m_{(0,1)}}{m_{(0,0)}^3}, \\
\gamma_{(1,2)} &= \frac{m_{(1,2)}}{m_{(0,0)}} - \frac{m_{(1,0)}m_{(0,2)}}{m_{(0,0)}^2} - 2\frac{m_{(0,1)}m_{(1,1)}}{m_{(0,0)}^2} + 2\frac{m_{(0,1)}^2m_{(1,0)}}{m_{(0,0)}^3},
\end{aligned}$$

$$\gamma(2,2) = -6 \frac{m_{(1,0)}^2 m_{(0,1)}^2}{m_{(0,0)}^4} + 2 \frac{m_{(0,1)}^2 m_{(2,0)} + m_{(1,0)}^2 m_{(0,2)}}{m_{(0,0)}^2} + 8 \frac{m_{(1,0)} m_{(0,1)} m_{(1,1)}}{m_{(0,0)}^3} - 2 \frac{m_{(1,0)} m_{(1,2)} + m_{(0,1)} m_{(2,1)}}{m_{(0,0)}^2} + \frac{m_{(2,2)}}{m_{(0,0)}} - \frac{m_{(2,0)} m_{(0,2)} + 2m_{(1,1)}^2}{m_{(0,0)}^2}.$$

The equilibrium cumulant vector $\gamma^{(eq)}$ is defined by

$$\gamma^{(eq)} = \left(\rho, v_1, v_2, c_s^2, c_s^2, 0, 0, 0, 0 \right)^T.$$

2.6.2 Conservation of mass: ρ



attached text file: output_d2q9_nse_culbm1_symbolic_pde_00.txt

$$\begin{aligned} & \frac{\partial \rho}{\partial t} + \frac{\delta_l v_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\delta_l \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_l v_2}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\delta_l \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + v_1^2 + 3cs^2) \frac{\delta_l^3 v_1}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + 3v_1^2 + cs^2) \frac{\delta_l^3 \rho}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\ & \frac{\delta_l^3 \rho cs^2}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{\delta_l^3 \rho cs^2}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3cs^2 + v_2^2) \frac{\delta_l^3 v_2}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + cs^2 + 3v_2^2) \frac{\delta_l^3 \rho}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & (24v_1^2 cs^2 - 12v_1^2 \omega_1 cs^2 - 6v_1^2 + \omega_1 cs^2 + 3v_1^2 \omega_1 + 2cs^4 + 6v_1^4 - \omega_1 cs^4 - 2cs^2 - 3v_1^4 \omega_1) \frac{\delta_l^4}{24\delta_t \omega_1} \frac{\partial^4 \rho}{\partial x_1^4} + \\ & (-4 + 10v_1^2 - 3\omega_1 cs^2 - 5v_1^2 \omega_1 + 6cs^2 + 2\omega_1) \frac{\delta_l^4 v_1 \rho}{12\delta_t \omega_1} \frac{\partial^4 v_1}{\partial x_1^4} + \\ & (v_1^2 \omega_4 + 3\omega_4 \omega_1 cs^2 - 9\omega_1 cs^2 - 3v_1^2 \omega_1 + 3\omega_4 cs^2 + v_1^2 \omega_4 \omega_1 - \omega_4 + 3\omega_1 - \omega_4 \omega_1) \frac{\delta_l^4 v_1 \rho}{12\delta_t \omega_4 \omega_1} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\ & (-2 + \omega_3) \frac{\delta_l^4 cs^4}{6\omega_3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\ & (\omega_6 v_2^2 + 3\omega_6 cs^2 + 3\omega_2 \omega_6 cs^2 + \omega_2 \omega_6 v_2^2 - 9\omega_2 cs^2 - 3\omega_2 v_2^2 + 3\omega_2 - \omega_2 \omega_6 - \omega_6) \frac{\delta_l^4 \rho v_2}{12\omega_2 \omega_6 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\ & (\omega_2 cs^2 + 3\omega_2 v_2^2 + 6v_2^4 + 2cs^4 - 2cs^2 - 6v_2^2 - 12\omega_2 cs^2 v_2^2 - 3\omega_2 v_2^4 - \omega_2 cs^4 + 24cs^2 v_2^2) \frac{\delta_l^4}{24\omega_2 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\ & (-4 - 3\omega_2 cs^2 - 5\omega_2 v_2^2 + 2\omega_2 + 6cs^2 + 10v_2^2) \frac{\delta_l^4 \rho v_2}{12\omega_2 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0. \end{aligned}$$

2.6.3 Conservation of momentum: ρv_1



attached text file: output_d2q9_nse_culbm1_symbolic_pde_01.txt

$$\begin{aligned} & v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (v_1^2 + cs^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2\delta_l v_1 \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_l v_1 v_2}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\delta_l \rho v_2}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{\delta_l v_1 \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\ & (-2 + 6v_1^2 - 3v_1^2 \omega_1 - 2cs^2 \omega_1 + 4cs^2 + \omega_1) \frac{\delta_l^2}{\delta_t \omega_1} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + (2 - \omega_1) \frac{3\delta_l^2 v_1 \rho}{\delta_t \omega_1} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + (-2 + \omega_3) \frac{\delta_l^2 cs^2}{2\omega_3 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\ & (-2 + \omega_3) \frac{\delta_l^2 cs^2}{2\omega_3 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (-2 + 2v_1^2 - v_1^2 \omega_1 - 3cs^2 \omega_1 + 6cs^2 + \omega_1) \frac{\delta_l^2 v_1}{2\delta_t \omega_1} \frac{\partial^2 \rho}{\partial x_1^2} + \\ & (-2 + 6v_1^2 - 3v_1^2 \omega_1 - cs^2 \omega_1 + 2cs^2 + \omega_1) \frac{\delta_l^2 \rho}{2\delta_t \omega_1} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega_3) \frac{\delta_l^2 cs^2 \rho}{2\omega_3 \delta_t} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_3) \frac{\delta_l^2 cs^2 \rho}{2\omega_3 \delta_t} \frac{\partial^2 v_1}{\partial x_2^2} + \\ & C_1 \frac{\delta_l^3}{12\delta_t \omega_1^2} \frac{\partial^3 \rho}{\partial x_1^3} + (-24 + 11v_1^2 \omega_1^2 + 60v_1^2 - 60v_1^2 \omega_1 - 36cs^2 \omega_1 - 4\omega_1^2 + 36cs^2 + 24\omega_1 + 5cs^2 \omega_1^2) \frac{\delta_l^3 v_1 \rho}{6\delta_t \omega_1^2} \frac{\partial^3 v_1}{\partial x_1^3} + \\ & C_2 \frac{\delta_l^3 v_1 \rho}{12\omega_3 \delta_t \omega_4 \omega_1^2} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + (-12 - \omega_3^2 + 12\omega_3) \frac{\delta_l^3 cs^4}{6\omega_3^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} - \frac{\delta_l^3 cs^2 v_1 \rho}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + v_2^2 + 3cs^2) \frac{\delta_l^3 v_1 v_2}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\ & C_3 \frac{\delta_l^3 \rho v_2}{6\omega_6 \omega_3 \delta_t} \frac{\partial^3 v_1}{\partial x_2^3} + (-1 + 3v_2^2 + cs^2) \frac{\delta_l^3 v_1 \rho}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_4 \frac{\delta_l^4 v_1}{12\delta_t \omega_1^3} \frac{\partial^4 \rho}{\partial x_1^4} + C_5 \frac{\delta_l^4 \rho}{12\delta_t \omega_1^3} \frac{\partial^4 v_1}{\partial x_1^4} + C_6 \frac{\delta_l^4 \rho}{12\omega_3^3 \delta_t \omega_4^2 \omega_1^3} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\ & C_7 \frac{\delta_l^4 cs^2 v_1}{12\omega_3^3 \delta_t \omega_4^2 \omega_1^3} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_8 \frac{\delta_l^4 cs^2 \rho}{12\omega_3^3 \delta_t \omega_4 \omega_1^2} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_9 \frac{\delta_l^4 cs^2 v_2}{12\omega_2 \omega_6^2 \omega_3^2 \delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + \\ & (\omega_6 v_2^2 + 3\omega_6 cs^2 + \omega_2 \omega_6 v_2^2 - 9\omega_2 cs^2 - 3\omega_2 v_2^2 + 3\omega_2 + 3\omega_2 \omega_6 cs^2 - \omega_2 \omega_6 - \omega_6) \frac{\delta_l^4 v_1 \rho v_2}{12\omega_2 \omega_6 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\ & C_{10} \frac{\delta_l^4 cs^2 \rho}{12\omega_2 \omega_6 \omega_3^3 \delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\ & (\omega_2 cs^2 + 3\omega_2 v_2^2 + 2cs^4 + 6v_2^4 - 6v_2^2 - 2cs^2 + 24cs^2 v_2^2 - 3\omega_2 v_2^4 - 12\omega_2 cs^2 v_2^2 - \omega_2 cs^4) \frac{\delta_l^4 v_1}{24\omega_2 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{11} \frac{\delta_l^4 \rho}{24\omega_6^2 \omega_3^3 \delta_t} \frac{\partial^4 v_1}{\partial x_2^4} + \\ & + (-4 - 3\omega_2 cs^2 - 5\omega_2 v_2^2 + 2\omega_2 + 10v_2^2 + 6cs^2) \frac{\delta_l^4 v_1 \rho v_2}{12\omega_2 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:

$$\begin{aligned}
C_1 &= -7v_1^2\omega_1^2 - 12cs^4\omega_1 + 144cs^2v_1^2 - 36v_1^2 + cs^4\omega_1^2 + 12cs^4 + 36v_1^2\omega_1 + 12cs^2\omega_1 + 36v_1^4 + 7v_1^4\omega_1^2 - 144cs^2v_1^2\omega_1 - 12cs^2 + 24cs^2v_1^2\omega_1^2 - 36v_1^4\omega_1 - cs^2\omega_1^2 \\
C_2 &= 36cs^2\omega_3\omega_1^2 - 12v_1^2\omega_1^2 + 36cs^2\omega_3\omega_4 - 6v_1^2\omega_3\omega_4\omega_1 + 3\omega_3\omega_4\omega_1^2 + 6\omega_3\omega_4\omega_1 - 3v_1^2\omega_3\omega_4\omega_1^2 + 18cs^2\omega_4\omega_1^2 - 36cs^2\omega_3\omega_1 - 12v_1^2\omega_3\omega_1 - \\
&6\omega_4\omega_1^2 + 12\omega_1^2 - 11cs^2\omega_3\omega_4\omega_1^2 + 6v_1^2\omega_4\omega_1^2 + 12\omega_3\omega_1 - 12\omega_3\omega_1^2 - 18cs^2\omega_3\omega_4\omega_1 - 12\omega_3\omega_4 + 12v_1^2\omega_3\omega_4 + 12v_1^2\omega_3\omega_1^2 - 36cs^2\omega_1^2 \\
C_3 &= 6 + 3\omega_6v_2^2 - 3\omega_6cs^2\omega_3 + 9\omega_6cs^2 + 9cs^2\omega_3 - 3\omega_6 + \omega_6\omega_3 - 3\omega_3 - 6v_2^2 - 18cs^2 - \omega_6\omega_3v_2^2 + 3\omega_3v_2^2 \\
C_4 &= 12 - 98v_1^2\omega_1^2 - 216cs^4\omega_1 + 672cs^2v_1^2 + 10v_1^2\omega_1^3 - 156v_1^2 + 82cs^4\omega_1^2 + 144cs^4 + 234v_1^2\omega_1 - 5cs^4\omega_1^3 + 198cs^2\omega_1 - 9v_1^4\omega_1^3 + 8\omega_1^2 + 144v_1^4 + \\
&90v_1^4\omega_1^2 - 1008cs^2v_1^2\omega_1 - 132cs^2 - \omega_1^3 + 404cs^2v_1^2\omega_1^2 - 216v_1^4\omega_1 + 6cs^2\omega_1^3 - 18\omega_1 - 34cs^2v_1^2\omega_1^3 - 78cs^2\omega_1^2 \\
C_5 &= 12 - 154v_1^2\omega_1^2 - 36cs^4\omega_1 + 432cs^2v_1^2 + 14v_1^2\omega_1^3 - 252v_1^2 + 14cs^4\omega_1^2 + 24cs^4 + 378v_1^2\omega_1 - cs^4\omega_1^3 + 54cs^2\omega_1 - 29v_1^4\omega_1^3 + 8\omega_1^2 + 504v_1^4 + \\
&310v_1^4\omega_1^2 - 648cs^2v_1^2\omega_1 - 36cs^2 - \omega_1^3 + 252cs^2v_1^2\omega_1^2 - 756v_1^4\omega_1 + 2cs^2\omega_1^3 - 18\omega_1 - 18cs^2v_1^2\omega_1^3 - 22cs^2\omega_1^2 \\
C_6 &= 6cs^4\omega_3^2\omega_1^2 + 12cs^4\omega_3\omega_4\omega_1^3 - 36v_1^4\omega_3^2\omega_1^2 + 39v_1^2\omega_3^2\omega_4\omega_1^3 + 198cs^2v_1^2\omega_3^2\omega_4\omega_1^2 + 36v_1^4\omega_3^2\omega_1^3 - 90v_1^4\omega_3^2\omega_4\omega_1^2 + 36v_1^4\omega_3^2\omega_4\omega_1^3 - 72v_1^2\omega_3^2\omega_4^2 + \\
&13cs^4\omega_3^2\omega_4\omega_1^3 - 99cs^2v_1^2\omega_3^2\omega_4\omega_1^3 - 36cs^2v_1^2\omega_3^2\omega_4^2\omega_1 - 72v_1^2\omega_3^2\omega_4\omega_1^2 + 36v_1^2\omega_3^2\omega_1^3 - cs^2\omega_3^2\omega_4\omega_1^2 + 18cs^2\omega_3^2\omega_4\omega_1^3 + 6v_1^2\omega_3^2\omega_4^2\omega_1^3 + \\
&36v_1^2\omega_3^2\omega_4\omega_1^2 + 18cs^2v_1^2\omega_3^2\omega_4\omega_1^2 + 36cs^2v_1^2\omega_3\omega_4\omega_1^3 + 36v_1^2\omega_3^2\omega_1^2 + 12cs^2\omega_3^2\omega_4\omega_1^2 - 6cs^4\omega_3^2\omega_4\omega_1^2 + 19v_1^4\omega_3^2\omega_1^2 - 36v_1^2\omega_3^2\omega_1^3 - \\
&3cs^2v_1^2\omega_3^2\omega_4^2\omega_1^3 - 108cs^2v_1^2\omega_3^2\omega_4\omega_1 + 6cs^2\omega_3\omega_4^2\omega_1^3 - 12cs^2\omega_3^2\omega_4\omega_1^2 + 72v_1^4\omega_3^2\omega_4^2 - 12cs^4\omega_3^2\omega_4\omega_1 + 4v_1^4\omega_3^2\omega_4^2\omega_1^3 + 6cs^4\omega_3^2\omega_4\omega_1^3 - 36v_1^4\omega_3^2\omega_1^3 + \\
&54cs^2v_1^2\omega_3^2\omega_4\omega_1^3 - 6cs^2\omega_3^2\omega_4^2\omega_1^2 - 12cs^2\omega_3\omega_4\omega_1^3 - 306cs^2v_1^2\omega_3^2\omega_4\omega_1 - cs^4\omega_3^2\omega_4^2\omega_1^3 + 12cs^4\omega_3^2\omega_4^2 - 39v_1^4\omega_3^2\omega_4\omega_1^3 - 36v_1^2\omega_3^2\omega_4\omega_1^3 - \\
&5cs^2\omega_3^2\omega_4^2\omega_1^3 - 108cs^2v_1^2\omega_3^2\omega_1^3 - 18cs^2v_1^2\omega_3\omega_4^2\omega_1^3 + 90v_1^2\omega_3^2\omega_4^2\omega_1 + 36cs^2v_1^2\omega_3^2\omega_4\omega_1^2 + 72v_1^4\omega_3^2\omega_4\omega_1^2 + cs^4\omega_3^2\omega_4^2\omega_1^2 + 252cs^2v_1^2\omega_3^2\omega_4^2 - \\
&36v_1^4\omega_3^2\omega_4\omega_1 - 12cs^4\omega_3^2\omega_4\omega_1^2 - 18cs^4\omega_3^2\omega_4\omega_1^3 + 12cs^4\omega_4^2\omega_1^3 - 6v_1^4\omega_3^2\omega_4^2\omega_1^3 + 12cs^2v_1^2\omega_3^2\omega_4\omega_1^2 + 6cs^2\omega_3^2\omega_4\omega_1^2 - 12cs^2\omega_3^2\omega_4^2 - 108cs^2v_1^2\omega_3^2\omega_1^2 - \\
&19v_1^2\omega_3^2\omega_4^2\omega_1^2 - 24cs^4\omega_3\omega_4\omega_1^3 + 12cs^4\omega_3^2\omega_4\omega_1^2 + 108cs^2v_1^2\omega_3^2\omega_1^3 - 4v_1^2\omega_3^2\omega_4^2\omega_1^3 - 6cs^2\omega_3^2\omega_4\omega_1^3 + 12cs^2\omega_3^2\omega_4\omega_1 + 60cs^2v_1^2\omega_3^2\omega_1^2 \\
C_7 &= -12\omega_3^2\omega_1^3 - 12v_1^2\omega_3^2\omega_1^2 - 18v_1^2\omega_3^2\omega_4^2\omega_1 - 4\omega_3^2\omega_4^2\omega_1^2 - 18\omega_3\omega_4\omega_1^3 - 36cs^2\omega_3\omega_1^3 + 12\omega_3^2\omega_1^2 - 12\omega_3\omega_4\omega_1^2 - \omega_3^2\omega_1^3 + 12v_1^2\omega_3^2\omega_1^3 - \\
&18cs^2\omega_3^2\omega_4\omega_1^3 + 12v_1^2\omega_3\omega_4\omega_1^2 + v_1^2\omega_3^2\omega_4^2\omega_1^3 + 18cs^2\omega_3\omega_4^2\omega_1^2 - 12\omega_4^2\omega_1^3 - 12\omega_3^2\omega_1^2 - 12v_1^2\omega_4^2\omega_1^2 + 18\omega_3^2\omega_4\omega_1 - 36cs^2\omega_4\omega_1^3 + 4v_1^2\omega_3^2\omega_4^2\omega_1^2 + \\
&12\omega_4^2\omega_1^2 - 40cs^2\omega_3\omega_4^2\omega_1^3 + 18cs^2\omega_3^2\omega_4\omega_1^2 + 18v_1^2\omega_3\omega_4\omega_1^3 + 12v_1^2\omega_3^2\omega_4^2 + 12v_1^2\omega_4^2\omega_1^3 + 36cs^2\omega_3^2\omega_1^2 - 12v_1^2\omega_3\omega_4^2\omega_1^3 + 12cs^2\omega_3^2\omega_4^2\omega_1^2 + \\
&54cs^2\omega_3\omega_4\omega_1^3 + 36cs^2\omega_4^2\omega_1^3 + 6v_1^2\omega_3^2\omega_4\omega_1^2 - 12v_1^2\omega_4\omega_1^3 + 12\omega_4\omega_1^3 + 36cs^2\omega_3\omega_4\omega_1^2 - 6v_1^2\omega_3^2\omega_4\omega_1^3 - 36cs^2\omega_4^2\omega_1^2 + 6v_1^2\omega_3\omega_4^2\omega_1^2 + 5cs^2\omega_3^2\omega_4^2\omega_1^3 + \\
&36cs^2\omega_3^2\omega_1^3 + 6\omega_3^2\omega_4\omega_1^3 - 6\omega_3\omega_4^2\omega_1^2 - 12v_1^2\omega_3\omega_1^3 + 12\omega_3\omega_1^3 - 36cs^2\omega_3^2\omega_1^2 - 54cs^2\omega_3^2\omega_4\omega_1 + 12\omega_3\omega_4^2\omega_1^3 - 6\omega_3^2\omega_4\omega_1^2 \\
C_8 &= 12cs^2\omega_3\omega_1^2 - 36v_1^2\omega_3^2\omega_4 + 24\omega_3^2\omega_1 - 72v_1^2\omega_3^2\omega_1^2 + 12cs^2\omega_3^3 + 12cs^2\omega_3^2\omega_4\omega_1 + 24\omega_3^2\omega_1^2 - 36v_1^2\omega_3\omega_4\omega_1 + 12\omega_3^2\omega_4 - 72v_1^2\omega_3^2\omega_1 + \\
&12\omega_3\omega_4\omega_1 - 12cs^2\omega_4\omega_1^2 + 36v_1^2\omega_3^2\omega_1^2 - 24\omega_3^2\omega_1 + 72v_1^2\omega_3^2\omega_1 - 4cs^2\omega_3^2\omega_4\omega_1^2 - 12\omega_3^3 - 12\omega_3^2\omega_1^2 - 12\omega_3^2\omega_4\omega_1 + 24cs^2\omega_3^2\omega_1 + 18cs^2\omega_3\omega_4\omega_1^2 + \\
&12cs^2\omega_3^2\omega_1^2 - 12\omega_3\omega_1^2 - 24cs^2\omega_3^2\omega_1 - 12cs^2\omega_3\omega_4\omega_1 - cs^2\omega_3^2\omega_4\omega_1^2 + 36v_1^2\omega_3^3 - 24cs^2\omega_3^2\omega_1^2 + 36v_1^2\omega_3\omega_1^2 - 12cs^2\omega_3^2\omega_4 + 36v_1^2\omega_3^2\omega_4\omega_1 \\
C_9 &= 36\omega_6cs^2\omega_3 + 18\omega_2cs^2\omega_3^2 - \omega_6^2\omega_3^2 - 3\omega_2\omega_6\omega_3^2v_2^2 - 12\omega_2\omega_6v_2^2 - \omega_2\omega_6^2\omega_3^2 - 9\omega_2\omega_6cs^2\omega_3^2 - 36\omega_2\omega_6cs^2 + 6\omega_2\omega_3^2v_2^2 - 12\omega_2\omega_6^2\omega_3v_2^2 - \\
&12\omega_2\omega_6^2 + 12\omega_2\omega_6^2\omega_3 + 54\omega_2\omega_6cs^2\omega_3 + 12\omega_2\omega_6 - 6\omega_6\omega_3^2v_2^2 - 36\omega_2cs^2\omega_3 - 6\omega_6^2\omega_3 + 6\omega_6^2\omega_3v_2^2 - 18\omega_6cs^2\omega_3^2 - 12\omega_6\omega_3 - 36\omega_2\omega_6^2cs^2\omega_3 + \\
&\omega_6^2\omega_3^2v_2^2 + 12\omega_6^2 - 18\omega_2\omega_6\omega_3 + 12\omega_2\omega_3 + 3\omega_6^2cs^2\omega_3^2 + 12\omega_6\omega_3v_2^2 - 6\omega_2\omega_3^2 + 12\omega_2\omega_6^2v_2^2 + 18\omega_6^2cs^2\omega_3 + 3\omega_2\omega_6\omega_3^2 - 12\omega_2\omega_3v_2^2 + \omega_2\omega_6^2\omega_3^2v_2^2 + \\
&36\omega_2\omega_6^2cs^2 - 12\omega_6^2v_2^2 + 18\omega_2\omega_6\omega_3v_2^2 + 6\omega_6\omega_3^2 + 3\omega_2\omega_6^2cs^2\omega_3^2 - 36\omega_6^2cs^2 \\
C_{10} &= 3\omega_6\omega_3^2v_2^2 - 12\omega_6cs^2\omega_3 - 18\omega_2cs^2\omega_3^2 - \omega_2\omega_6cs^2\omega_3^2 - 3\omega_2\omega_6\omega_3^2v_2^2 + 36\omega_3^2v_2^2 - 6cs^2\omega_3^3 + 12cs^2\omega_3^2 - 5\omega_2\omega_6cs^2\omega_3^2 - 12\omega_2\omega_6cs^2 + \\
&6\omega_2cs^2\omega_3^3 - 54\omega_2\omega_3^2v_2^2 - 18\omega_3^3v_2^2 + 18\omega_2\omega_6cs^2\omega_3 + 18\omega_6\omega_3^2v_2^2 + \omega_6cs^2\omega_3^3 - 12\omega_6^2 + 12\omega_2cs^2\omega_3 + 18\omega_2\omega_3^2v_2^2 + 6\omega_3^3 + 6\omega_6cs^2\omega_3^2 + 12\omega_6\omega_3 - \\
&12\omega_2\omega_3 - 36\omega_6\omega_3v_2^2 + 18\omega_2\omega_3^2 + \omega_2\omega_6\omega_3^2 + 36\omega_2\omega_3v_2^2 - \omega_6\omega_3^3 - 6\omega_2\omega_3^3 - 6\omega_6\omega_3^3 \\
C_{11} &= 30\omega_6\omega_3^2v_2^2 - 24\omega_6cs^2\omega_3 - 72\omega_6cs^2\omega_3^2v_2^2 + 72\omega_3^2v_2^2 + 24\omega_6^2cs^4\omega_3^2 + 3\omega_6^2\omega_3^2v_2^4 - 36\omega_6^2cs^2\omega_3v_2^2 + 24\omega_6^2cs^4 - 3\omega_6^2cs^4\omega_3^3 - 36\omega_3^3v_2^2 - 12\omega_6^2\omega_3^2v_2^4 + \\
&144\omega_6cs^2\omega_3^2v_2^2 - 72\omega_6\omega_3^2v_2^2 - 6\omega_6cs^2\omega_3^3 - 48\omega_6^2cs^4\omega_3 + 24\omega_6cs^2\omega_3^2 + 108cs^2\omega_3^2v_2^2 + 72\omega_6\omega_3^2v_2^4 + 36\omega_3^3v_2^2 + 12\omega_6^2\omega_3^2v_2^2 + \omega_6^2cs^2\omega_3^3 + 6\omega_6^2cs^2\omega_3^2v_2^2 + \\
&24\omega_6cs^4\omega_3 + 72\omega_6cs^2\omega_3v_2^2 - 8\omega_6^2cs^2\omega_3^2 + 12\omega_6^2cs^2\omega_3 - 72\omega_3^2v_2^4 - 3\omega_6^2\omega_3^2v_2^2 - 12\omega_6^2cs^2\omega_3^2v_2^2 - 216cs^2\omega_3^2v_2^2 - 30\omega_6\omega_3^2v_2^4 - 24\omega_6cs^4\omega_3^2 + 6\omega_6cs^4\omega_3^3
\end{aligned}$$

2.6.4 Conservation of momentum: ρv_2



attached text file: output_d2q9_nse_culbm1_symbolic_pde_02.txt

$$\begin{aligned}
&v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{\delta_1 v_1 v_2}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\delta_1 \rho v_2}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_1 v_1 \rho}{\delta_t} \frac{\partial v_2}{\partial x_1} + (cs^2 + v_2^2) \frac{\delta_1}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{2\delta_1 \rho v_2}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_3) \frac{\delta_1^2 cs^2}{2\omega_3 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\
&(-2 + \omega_3) \frac{\delta_1^2 cs^2}{2\omega_3 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (-2 - 2\omega_2 cs^2 - 3\omega_2 v_2^2 + \omega_2 + 4cs^2 + 6v_2^2) \frac{\delta_1^2}{\omega_2 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (2 - \omega_2) \frac{3\delta_1^2 \rho v_2}{\omega_2 \delta_t} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + \\
&(-2 + \omega_3) \frac{\delta_1^2 cs^2 \rho}{2\omega_3 \delta_t} \frac{\partial^2 v_2}{\partial x_1^2} + (-2 + \omega_3) \frac{\delta_1^2 cs^2 \rho}{2\omega_3 \delta_t} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + (-2 - 3\omega_2 cs^2 - \omega_2 v_2^2 + \omega_2 + 6cs^2 + 2v_2^2) \frac{\delta_1^2 v_2}{2\omega_2 \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + \\
&(-2 - \omega_2 cs^2 - 3\omega_2 v_2^2 + \omega_2 + 2cs^2 + 6v_2^2) \frac{\delta_1^2 \rho}{2\omega_2 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + (-1 + v_1^2 + 3cs^2) \frac{\delta_1^3 v_1 v_2}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + 3v_1^2 + cs^2) \frac{\delta_1^3 \rho v_2}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + \\
&C_1 \frac{\delta_1^3 v_1 \rho}{6\omega_3 \delta_t \omega_4} \frac{\partial^3 v_2}{\partial x_1^3} + (-12 - \omega_3^2 + 12\omega_3) \frac{\delta_1^3 cs^4}{6\omega_3 \delta_t} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} - \frac{\delta_1^3 cs^2 \rho v_2}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + C_2 \frac{\delta_1^3 \rho v_2}{12\omega_2^2 \omega_6 \omega_3 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_3 \frac{\delta_1^3}{12\omega_2^2 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\
&(-24 - 36\omega_2 cs^2 - 60\omega_2 v_2^2 - 4\omega_2^2 + 24\omega_2 + 36cs^2 + 60v_2^2 + 11\omega_2^2 v_2^2 + 5\omega_2^2 cs^2) \frac{\delta_1^3 \rho v_2}{6\omega_2^2 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} +
\end{aligned}$$

$$\begin{aligned}
& (-12cs^2v_1^2\omega_1 - 6v_1^2 + 2cs^4 + 3v_1^2\omega_1 + 24cs^2v_1^2 - cs^4\omega_1 + 6v_1^4 - 2cs^2 - 3v_1^4\omega_1 + cs^2\omega_1) \frac{\delta_t^4 v_2}{24\delta_t\omega_1} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& (-4 + 10v_1^2 - 5v_1^2\omega_1 + 6cs^2 - 3cs^2\omega_1 + 2\omega_1) \frac{\delta_t^4 v_1 \rho v_2}{12\delta_t\omega_1} \frac{\partial^4 v_1}{\partial x_1^4} + C_4 \frac{\delta_t^4 \rho}{24\omega_3^3 \delta_t \omega_4^2} \frac{\partial^4 v_2}{\partial x_1^4} + C_5 \frac{\delta_t^4 cs^2 v_1}{12\omega_3^3 \delta_t \omega_4^2 \omega_1} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + \\
& C_6 \frac{\delta_t^4 cs^2 \rho}{12\omega_3^3 \delta_t \omega_4 \omega_1} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + (v_1^2\omega_4 - 3v_1^2\omega_1 + 3cs^2\omega_4\omega_1 + 3cs^2\omega_4 + v_1^2\omega_4\omega_1 - \omega_4 - 9cs^2\omega_1 + 3\omega_1 - \omega_4\omega_1) \frac{\delta_t^4 v_1 \rho v_2}{12\delta_t \omega_4 \omega_1} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} \\
& + C_7 \frac{\delta_t^4 cs^2 v_2}{12\omega_3^3 \omega_6^3 \omega_3^3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_8 \frac{\delta_t^4 cs^2 \rho}{12\omega_3^2 \omega_6 \omega_3^3 \delta_t} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_9 \frac{\delta_t^4 \rho}{12\omega_3^3 \omega_6^3 \omega_3^3 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^2} + C_{10} \frac{\delta_t^4 v_2}{12\omega_3^3 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{11} \frac{\delta_t^4 \rho}{12\omega_3^3 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= 6 + 9cs^2\omega_3 + 3v_1^2\omega_4 - 6v_1^2 - 3\omega_3 - 18cs^2 - 3cs^2\omega_3\omega_4 + 9cs^2\omega_4 + 3v_1^2\omega_3 - 3\omega_4 + \omega_3\omega_4 - v_1^2\omega_3\omega_4 \\
C_2 &= 18\omega_2^2cs^2\omega_6 + 12\omega_2^2\omega_3v_2^2 - 11\omega_2^2cs^2\omega_6\omega_3 + 12\omega_2^2 - 36\omega_2cs^2\omega_3 - 12\omega_2^2\omega_3 - 3\omega_2^2\omega_6\omega_3v_2^2 + 3\omega_2^2\omega_6\omega_3 - 12\omega_2\omega_3 + 6\omega_2\omega_6\omega_3 - 6\omega_2^2\omega_6 + \\
& 12\omega_2\omega_3 - 18\omega_2cs^2\omega_6\omega_3 + 12\omega_2\omega_3v_2^2 - 12\omega_2^2v_2^2 - 12\omega_2\omega_3v_2^2 - 36\omega_2^2cs^2 + 6\omega_2^2\omega_6v_2^2 + 36\omega_2^2cs^2\omega_3 - 6\omega_2\omega_6\omega_3v_2^2 + 36cs^2\omega_6\omega_3 \\
C_3 &= 12\omega_2cs^2 + 7\omega_2^2v_2^4 + 36\omega_2v_2^2 + \omega_2^2cs^4 + 12cs^4 + 24\omega_2^2cs^2v_2^2 + 36v_2^4 + 144cs^2v_2^2 - 12cs^2 - 36v_2^2 - 144\omega_2cs^2v_2^2 - 12\omega_2cs^4 - 7\omega_2^2v_2^2 - 36\omega_2v_2^4 - \omega_2^2cs^2 \\
C_4 &= -72v_1^2\omega_3^2\omega_4 + 72cs^2v_1^2\omega_3\omega_4 - 24cs^4\omega_3^2\omega_4 - 6cs^2\omega_3^2\omega_4 - 30v_1^4\omega_3^3\omega_4 - 3cs^4\omega_3^3\omega_4 - 3v_1^2\omega_3^3\omega_4^2 - 12v_1^4\omega_3^2\omega_4^2 - 8cs^2\omega_3^2\omega_4^2 + 36v_1^4\omega_3^3 + \\
& 72v_1^4\omega_3^2\omega_4 + 24cs^2\omega_3^2\omega_4 + 6cs^4\omega_3^3\omega_4 + 30v_1^2\omega_3^3\omega_4 + cs^2\omega_3^2\omega_4^2 + 24cs^4\omega_4^2 - 72v_1^4\omega_3^3 + 3v_1^4\omega_3^3\omega_4^2 + 12v_1^2\omega_3^2\omega_4^2 + 24cs^4\omega_3^2\omega_4^2 - 36cs^2v_1^2\omega_3\omega_4^2 + \\
& 108cs^2v_1^2\omega_3^3 - 48cs^4\omega_3\omega_4^2 - 12cs^2v_1^2\omega_3^2\omega_4^2 - 24cs^2\omega_3\omega_4 - 72cs^2v_1^2\omega_3^3\omega_4 - 216cs^2v_1^2\omega_3^3 + 6cs^2v_1^2\omega_3^3\omega_4^2 + 72v_1^2\omega_3^3 + 12cs^2\omega_3\omega_4^2 - 36v_1^2\omega_3^3 + \\
& 144cs^2v_1^2\omega_3^3\omega_4 + 24cs^4\omega_3\omega_4 \\
C_5 &= -6v_1^2\omega_3^2\omega_4 - 12\omega_2^2\omega_1 + v_1^2\omega_3^2\omega_4\omega_1 + 12v_1^2\omega_3^2\omega_1 + 18cs^2\omega_3^2\omega_1 + 18v_1^2\omega_3\omega_4\omega_1 + 6\omega_3^2\omega_4 + 3cs^2\omega_3^2\omega_4^2 - 18\omega_3\omega_4\omega_1 - 36cs^2\omega_3\omega_4\omega_1 - \\
& \omega_3^2\omega_4^2 - 18cs^2\omega_3^2\omega_4 - 6\omega_3^2\omega_1 - 9cs^2\omega_3^2\omega_4\omega_1 + 6v_1^2\omega_3^2\omega_1 - \omega_3^2\omega_4^2\omega_1 - 12v_1^2\omega_4^2 + 36cs^2\omega_4^2\omega_1 + v_1^2\omega_3^2\omega_4^2 - 12v_1^2\omega_3\omega_1 + 3cs^2\omega_3^2\omega_4\omega_1 - \\
& 36cs^2\omega_4\omega_1 + 3\omega_3^2\omega_4\omega_1 + 6v_1^2\omega_3\omega_4^2 + 12\omega_3\omega_4^2\omega_1 + 36cs^2\omega_3\omega_4 - 6\omega_3\omega_4^2 + 54cs^2\omega_3\omega_4\omega_1 + 12\omega_3\omega_1 - 12v_1^2\omega_4\omega_1 - 36cs^2\omega_4^2 - 36cs^2\omega_3\omega_1 - \\
& 12v_1^2\omega_3\omega_4^2\omega_1 + 18cs^2\omega_3\omega_4^2 - 12\omega_3\omega_4 + 12v_1^2\omega_3\omega_4 - 3v_1^2\omega_3^2\omega_4\omega_1 + 12\omega_4^2 + 12\omega_4\omega_1 \\
C_6 &= 18v_1^2\omega_3^2\omega_4 - 6\omega_3^2\omega_1 + cs^2\omega_3^2\omega_4 - 18cs^2\omega_3^2\omega_1 - 6\omega_3^2\omega_4 + 18v_1^2\omega_3^2\omega_1 - 6cs^2\omega_3^2 + 6cs^2\omega_3^2\omega_4 + 3v_1^2\omega_3^2\omega_4 + 18\omega_3^2\omega_1 - 12\omega_3^2 - \omega_3^2\omega_4 - \\
& 5cs^2\omega_3^2\omega_4\omega_1 - 54v_1^2\omega_3^2\omega_1 + 12cs^2\omega_3^2 + 6cs^2\omega_3^2\omega_1 + 6\omega_3^3 + 36v_1^2\omega_3\omega_1 - 12cs^2\omega_4\omega_1 + \omega_3^2\omega_4\omega_1 - 12cs^2\omega_3\omega_4 + 18cs^2\omega_3\omega_4\omega_1 - 12\omega_3\omega_1 + \\
& 12cs^2\omega_3\omega_1 + 36v_1^2\omega_3^2 - cs^2\omega_3^2\omega_4\omega_1 + 12\omega_3\omega_4 - 18v_1^2\omega_3^2 - 36v_1^2\omega_3\omega_4 - 3v_1^2\omega_3^2\omega_4\omega_1 \\
C_7 &= 18\omega_2^3\omega_6\omega_3v_2^2 + 12\omega_2^3\omega_3 - 12\omega_2^3\omega_6^2v_2^2 - 12\omega_2^3\omega_3^2 - 6\omega_2^3\omega_6\omega_3^2 + 36\omega_2^3cs^2\omega_6\omega_3 + 12\omega_2^3\omega_3^2 - 54\omega_2cs^2\omega_6^2\omega_3^2 + \omega_3^2\omega_6^2\omega_3^2v_2^2 + 18\omega_2\omega_6^2\omega_3^2 + \\
& 36\omega_2^3cs^2\omega_3^2 + 12\omega_2^3\omega_6^2\omega_3 - 18\omega_2^3cs^2\omega_6\omega_3^2 + 4\omega_2^3\omega_6^2\omega_3^2v_2^2 - 36\omega_2^3cs^2\omega_3 + 12\omega_2^3\omega_3^2v_2^2 - \omega_3^2\omega_6^2\omega_3^2 + 54\omega_2^3cs^2\omega_6\omega_3 + 18\omega_2^3cs^2\omega_6\omega_3^2 - 12\omega_2^3\omega_6^2v_2^2 + \\
& 36cs^2\omega_6^2\omega_3^2 + 12\omega_2^3\omega_6\omega_3v_2^2 - 12\omega_2^3\omega_6\omega_3 - 36\omega_2^3cs^2\omega_6^2 - 12\omega_2^3\omega_3^2 - 12\omega_2^3\omega_6^2 - 36\omega_2^3cs^2\omega_3^2 - 6\omega_2^3\omega_6^2\omega_3 + 6\omega_2^3\omega_6\omega_3^2v_2^2 + 12\omega_2^3\omega_3^2v_2^2 - 12\omega_2^3\omega_3v_2^2 + \\
& 6\omega_2^3\omega_6\omega_3v_2^2 - 40\omega_2^3cs^2\omega_6^2\omega_3 + 6\omega_2^3\omega_6\omega_3^2 + 12\omega_2^3cs^2\omega_6^2\omega_3^2 + 12\omega_2^3\omega_6^2v_2^2 - 36\omega_2^3cs^2\omega_6 + 18\omega_2^3cs^2\omega_6^2\omega_3 - 12\omega_2^3\omega_6^2\omega_3v_2^2 + 36\omega_2^3cs^2\omega_6^2 + 12\omega_2^3\omega_6^2 + \\
& 5\omega_2^3cs^2\omega_6^2\omega_3 - 18\omega_2\omega_6^2\omega_3^2v_2^2 - 18\omega_2^3\omega_6\omega_3 - 6\omega_2^3\omega_6\omega_3^2v_2^2 + 12\omega_2^3\omega_6 - 12\omega_2^3\omega_3^2v_2^2 - 4\omega_2^3\omega_6^2\omega_3^2 \\
C_8 &= -12\omega_2^3\omega_3^2 - 12\omega_2^3cs^2\omega_6 + 36\omega_2\omega_6\omega_3^2v_2^2 + 36\omega_2^3\omega_3v_2^2 + 18\omega_2^3cs^2\omega_6\omega_3 + 24\omega_2^3\omega_3^2 + 72\omega_2\omega_3^2v_2^2 + 36\omega_3^3v_2^2 + 12cs^2\omega_3^2 + 24\omega_2cs^2\omega_3^2 - \\
& 4\omega_2^3cs^2\omega_6\omega_3 - 12\omega_2^3\omega_3 - 36\omega_6\omega_3^2v_2^2 - 24\omega_2cs^2\omega_3^2 - 72\omega_2\omega_3^2v_2^2 - 12\omega_3^3 - \omega_2cs^2\omega_6\omega_3^2 - 24\omega_2^3cs^2\omega_3^2 - 12cs^2\omega_6\omega_3^2 + 12\omega_2\omega_6\omega_3 + 36\omega_2^3\omega_3^2v_2^2 + \\
& 12\omega_2^3cs^2\omega_3^2 - 12\omega_2cs^2\omega_6\omega_3 - 24\omega_2\omega_3^2 + 12\omega_2cs^2\omega_6\omega_3^2 - 12\omega_2\omega_6\omega_3^2 + 24\omega_2\omega_3^2 + 12\omega_2^3cs^2\omega_3 - 36\omega_2\omega_6\omega_3v_2^2 - 72\omega_2^3\omega_3^2v_2^2 + 12\omega_6\omega_3^2 \\
C_9 &= 12\omega_2cs^2\omega_6^2\omega_3^2 + 72\omega_2^3\omega_6\omega_3^2v_2^4 + 12\omega_2^3cs^4\omega_6\omega_3 + 12cs^4\omega_6^2\omega_3^2 - 3\omega_2^3cs^2\omega_6^2\omega_3^2v_2^2 - 6\omega_2^3cs^2\omega_6\omega_3^2 + 12\omega_2^3cs^4\omega_6\omega_3^2 + 36\omega_2^3cs^2\omega_6\omega_3^2v_2^2 - \\
& 19\omega_2^3\omega_6^2\omega_3^2v_2^2 - 36\omega_2^3\omega_3^2v_2^2 + 72\omega_2^3\omega_3^2v_2^4 + 252cs^2\omega_6^2\omega_3^2v_2^2 + 36\omega_2^3cs^2\omega_6\omega_3v_2^2 + 12\omega_2cs^2\omega_6^2\omega_3^2 + 6\omega_2^3\omega_6^2\omega_3^2v_2^2 + 36\omega_2^3\omega_6\omega_3^2v_2^2 - 6\omega_2^3cs^4\omega_6\omega_3^2 + \\
& 18\omega_2^3cs^2\omega_6\omega_3^2 + 198\omega_2^3cs^2\omega_6\omega_3^2v_2^2 - 12\omega_2cs^4\omega_6^2\omega_3^2 + 36\omega_2^3\omega_3^2v_2^2 + 36\omega_2\omega_6\omega_3^2v_2^2 - 12\omega_2^3cs^2\omega_6\omega_3 - 12cs^2\omega_6^2\omega_3^2 - 90\omega_2\omega_6^2\omega_3^2v_2^2 - \\
& 12\omega_2^3cs^2\omega_6\omega_3^2 + 12\omega_2^3cs^2\omega_6^2\omega_3^2v_2^2 + 6\omega_2^3cs^4\omega_6\omega_3^2 - 12\omega_2cs^4\omega_6^2\omega_3^2 + 12\omega_2^3cs^4\omega_6^2 - 39\omega_2^3\omega_6\omega_3^2v_2^4 - 108\omega_2cs^2\omega_6\omega_3^2v_2^2 - 4\omega_2^3\omega_6^2\omega_3^2v_2^2 - \\
& 18\omega_2^3cs^4\omega_6\omega_3^2 - 36\omega_2^3\omega_3^2v_2^2 + 6\omega_2^3cs^2\omega_6\omega_3^2 + 90\omega_2\omega_6^2\omega_3^2v_2^2 + 108\omega_2^3cs^2\omega_3^2v_2^2 + 60\omega_2^3cs^2\omega_6^2\omega_3^2v_2^2 - 36\omega_2^3\omega_3^2v_2^4 - 99\omega_2^3cs^2\omega_6\omega_3^2v_2^2 - \omega_2^3cs^2\omega_6^2\omega_3^2 + \\
& 13\omega_2^3cs^4\omega_6\omega_3^2 - 36\omega_2\omega_6\omega_3^2v_2^4 + 4\omega_2^3\omega_6^2\omega_3^2v_2^4 + 36\omega_2^3\omega_3^2v_2^2 + 6\omega_2^3cs^2\omega_6\omega_3^2 - \omega_2^3cs^4\omega_6^2\omega_3^2 - 6\omega_2^3cs^2\omega_6\omega_3^2 + 39\omega_2^3\omega_6\omega_3^2v_2^2 - 306\omega_2cs^2\omega_6^2\omega_3^2v_2^2 + \\
& 54\omega_2^3cs^2\omega_6\omega_3^2v_2^2 + 19\omega_2^3\omega_6^2\omega_3^2v_2^4 + 36\omega_2^3\omega_3^2v_2^4 - 72\omega_2^3\omega_3^2v_2^2 + 18\omega_2^3cs^2\omega_6^2\omega_3^2v_2^2 - 72\omega_2^3\omega_6\omega_3^2v_2^2 - 108\omega_2^3cs^2\omega_3^2v_2^2 - 5\omega_2^3cs^2\omega_6^2\omega_3^2 + \omega_2^3cs^4\omega_6^2\omega_3^2 - \\
& 108\omega_2^3cs^2\omega_3^2v_2^2 - 36\omega_2^3\omega_6\omega_3^2v_2^2 - 36\omega_2cs^2\omega_6^2\omega_3^2v_2^2 - 18\omega_2^3cs^2\omega_6^2\omega_3^2v_2^2 - 24\omega_2^3cs^4\omega_6^2\omega_3^2 + 6\omega_2^3cs^4\omega_6^2\omega_3^2 - 6\omega_2^3\omega_6^2\omega_3^2v_2^4 \\
C_{10} &= 12 + 198\omega_2cs^2 + 90\omega_2^2v_2^4 - \omega_2^3 + 234\omega_2v_2^2 + 82\omega_2^2cs^4 + 8\omega_2^2 - 18\omega_2 + 144cs^4 - 9\omega_2^3v_2^4 - 5\omega_2^3cs^4 + 404\omega_2^2cs^2v_2^2 + 144v_2^4 + 10\omega_2^3v_2^2 + \\
& 672cs^2v_2^2 - 132cs^2 - 156v_2^2 + 6\omega_2^3cs^2 - 1008\omega_2cs^2v_2^2 - 34\omega_2^3cs^2v_2^2 - 216\omega_2cs^4 - 98\omega_2^3v_2^2 - 216\omega_2v_2^4 - 78\omega_2^2cs^2 \\
C_{11} &= 12 + 54\omega_2cs^2 + 310\omega_2^2v_2^4 - \omega_2^3 + 378\omega_2v_2^2 + 14\omega_2^2cs^4 + 8\omega_2^2 - 18\omega_2 + 24cs^4 - 29\omega_2^3v_2^4 - \omega_2^3cs^4 + 252\omega_2^2cs^2v_2^2 + 504v_2^4 + 14\omega_2^3v_2^2 + \\
& 432cs^2v_2^2 - 36cs^2 - 252v_2^2 + 2\omega_2^3cs^2 - 648\omega_2cs^2v_2^2 - 18\omega_2^3cs^2v_2^2 - 36\omega_2cs^4 - 154\omega_2^2v_2^2 - 756\omega_2v_2^4 - 22\omega_2^2cs^2
\end{aligned}$$

2.7 CuLBM2

2.7.1 Definitions

Collision operator C :

$$C(f) = \mathbf{M}^{-1} \mathbf{G}^{-1} \left(\mathbf{N}^{-1} \mathbf{S} \mathbf{N} \left(\gamma^{(eq)} - \mathbf{G}(\mathbf{M}f) \right) \right),$$

where

$$\mathbf{S} = \text{diag}(0, 0, 0, \omega_1, \omega_2, \omega_1, \omega_3, \omega_3, \omega_4),$$

$\omega_1, \omega_2, \omega_3, \omega_4 \in (0, 2)$.

Matrix \mathbf{N} defines the combination of cumulants for the collision as

$$\mathbf{N} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

The nonlinear operator \mathbf{G} is the same as in CuLBM1 in Section ?? and, again, the equilibrium cumulant vector $\gamma^{(eq)}$ is defined by

$$\gamma^{(eq)} = \left(\rho, v_1, v_2, 2c_s^2, 0, 0, 0, 0, 0 \right)^T.$$

2.7.2 Conservation of mass: ρ



attached text file: output_d2q9_nse_culbm2_symbolic_pde_00.txt

$$\begin{aligned} & \frac{\partial \rho}{\partial t} + \frac{\delta_I v_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\delta_I \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_I v_2}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\delta_I \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + 3cs^2 + v_1^2) \frac{\delta_I^3 v_1}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + cs^2 + 3v_1^2) \frac{\delta_I^3 \rho}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\ & \frac{cs^2 \delta_I^3 \rho}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{cs^2 \delta_I^3 \rho}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3cs^2 + v_2^2) \frac{\delta_I^3 v_2}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + cs^2 + 3v_2^2) \frac{\delta_I^3 \rho}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_1 \frac{\delta_I^4}{24\omega_1 \omega_2 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\ & (3\omega_1 cs^2 - 2\omega_1 + 3cs^2 \omega_2 - 5\omega_1 \omega_2 v_1^2 - 2\omega_2 + 5\omega_1 v_1^2 - 3\omega_1 cs^2 \omega_2 + 5\omega_2 v_1^2 + 2\omega_1 \omega_2) \frac{\delta_I^4 \rho v_1}{12\omega_1 \omega_2 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + \\ & (3\omega_1 cs^2 - \omega_1 - 3cs^2 \omega_2 + \omega_2 + \omega_1 v_2^2 - \omega_2 v_2^2) \frac{\delta_I^4 v_1 v_2}{8\omega_1 \omega_2 \delta_t} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + \\ & (3\omega_1 cs^2 - \omega_1 - 3cs^2 \omega_2 + \omega_2 + \omega_1 v_2^2 - \omega_2 v_2^2) \frac{\delta_I^4 \rho v_2}{24\omega_1 \omega_2 \delta_t} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + C_2 \frac{\delta_I^4 \rho v_1}{24\omega_1 \omega_2 \omega_3 \delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\ & (2\omega_1 cs^2 - 2\omega_1 - 10cs^2 \omega_2 + 2\omega_2 + 3\omega_1 v_1^2 + 3\omega_1 v_2^2 + 4\omega_1 cs^2 \omega_2 - 3\omega_2 v_1^2 - 3\omega_2 v_2^2) \frac{cs^2 \delta_I^4}{24\omega_1 \omega_2 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\ & (3\omega_1 cs^2 - \omega_1 - 3cs^2 \omega_2 + \omega_2 + \omega_1 v_1^2 - \omega_2 v_1^2) \frac{\delta_I^4 \rho v_1}{24\omega_1 \omega_2 \delta_t} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + \\ & (3\omega_1 cs^2 - \omega_1 - 3cs^2 \omega_2 + \omega_2 + \omega_1 v_2^2 - \omega_2 v_2^2) \frac{\delta_I^4 \rho v_2}{24\omega_1 \omega_2 \delta_t} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + \\ & (3\omega_1 cs^2 - \omega_1 - 3cs^2 \omega_2 + \omega_2 + \omega_1 v_1^2 - \omega_2 v_1^2) \frac{\delta_I^4 v_1 v_2}{8\omega_1 \omega_2 \delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_3 \frac{\delta_I^4 \rho v_2}{24\omega_1 \omega_2 \omega_3 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\ & (3\omega_1 cs^2 - \omega_1 - 3cs^2 \omega_2 + \omega_2 + \omega_1 v_1^2 - \omega_2 v_1^2) \frac{\delta_I^4 \rho v_1}{24\omega_1 \omega_2 \delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + C_4 \frac{\delta_I^4}{24\omega_1 \omega_2 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\ & (3\omega_1 cs^2 - 5\omega_1 \omega_2 v_2^2 - 2\omega_1 + 3cs^2 \omega_2 - 2\omega_2 + 5\omega_1 v_2^2 - 3\omega_1 cs^2 \omega_2 + 2\omega_1 \omega_2 + 5\omega_2 v_2^2) \frac{\delta_I^4 \rho v_2}{12\omega_1 \omega_2 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:

$$C_1 = 3\omega_2 v_1^4 - 12\omega_1 cs^2 \omega_2 v_1^2 - \omega_1 cs^2 - cs^2 \omega_2 + 3\omega_1 \omega_2 v_1^2 + 3\omega_1 v_1^4 - \omega_1 cs^4 \omega_2 + cs^4 \omega_2 + 12\omega_1 cs^2 v_1^2 - 3\omega_1 v_1^2 + \omega_1 cs^2 \omega_2 + 12cs^2 \omega_2 v_1^2 - 3\omega_2 v_1^2 + \omega_1 cs^4 - 3\omega_1 \omega_2 v_1^4$$

$$C_2 = 2\omega_2 \omega_3 + \omega_1 \omega_3 v_1^2 + 9\omega_1 \omega_3 v_2^2 - 6\omega_1 \omega_2 v_1^2 - 4\omega_1 \omega_3 + 2\omega_1 \omega_2 \omega_3 v_1^2 + 6\omega_1 cs^2 \omega_3 + \omega_2 \omega_3 v_1^2 - 9\omega_2 \omega_3 v_2^2 - 18\omega_1 cs^2 \omega_2 + 6\omega_1 cs^2 \omega_2 \omega_3 + 6\omega_1 \omega_2 - 2\omega_1 \omega_2 \omega_3$$

$$C_3 = 2\omega_2 \omega_3 + 9\omega_1 \omega_3 v_1^2 - 6\omega_1 \omega_2 v_2^2 + \omega_1 \omega_3 v_2^2 - 4\omega_1 \omega_3 + 6\omega_1 cs^2 \omega_3 + 2\omega_1 \omega_2 \omega_3 v_2^2 - 9\omega_2 \omega_3 v_1^2 + \omega_2 \omega_3 v_2^2 - 18\omega_1 cs^2 \omega_2 + 6\omega_1 cs^2 \omega_2 \omega_3 + 6\omega_1 \omega_2 - 2\omega_1 \omega_2 \omega_3$$

$$C_4 = -\omega_1 cs^2 + 3\omega_1 \omega_2 v_2^2 + 3\omega_2 v_2^4 - 12\omega_1 cs^2 \omega_2 v_2^2 - cs^2 \omega_2 - \omega_1 cs^4 \omega_2 + 3\omega_1 v_2^4 + cs^4 \omega_2 + 12\omega_1 cs^2 v_2^2 - 3\omega_1 v_2^2 + \omega_1 cs^2 \omega_2 - 3\omega_1 \omega_2 v_2^4 + \omega_1 cs^4 + 12cs^2 \omega_2 v_2^2 - 3\omega_2 v_2^2$$

2.7.3 Conservation of momentum: ρv_1



attached text file: output_d2q9_nse_culbm2_symbolic_pde_01.txt

$$\begin{aligned} & v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (cs^2 + v_1^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2\delta_l \rho v_1}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_l v_1 v_2}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\delta_l \rho v_2}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{\delta_l \rho v_1}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\ & (-\omega_1 - 2\omega_1 \omega_2 cs^2 - 3\omega_1 \omega_2 v_1^2 - \omega_2 + 2\omega_1 cs^2 + 3\omega_1 v_1^2 + 3\omega_2 v_1^2 + 2\omega_2 cs^2 + \omega_1 \omega_2) \frac{\delta_l^2}{\omega_1 \omega_2 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + \\ & (\omega_1 + \omega_2 - \omega_1 \omega_2) \frac{3\delta_l^2 \rho v_1}{\omega_1 \omega_2 \delta_t} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + (-\omega_1 + \omega_2 + \omega_1 cs^2 + 3\omega_1 v_2^2 - \omega_2 cs^2 - 3\omega_2 v_2^2) \frac{\delta_l^2}{2\omega_1 \omega_2 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_2} + \\ & (\omega_1 - \omega_2) \frac{3\delta_l^2 \rho v_2}{\omega_1 \omega_2 \delta_t} \frac{\partial v_2}{\partial x_1} \frac{\partial v_2}{\partial x_2} + (-\omega_1 + \omega_1 \omega_2 cs^2 + \omega_2 + 3\omega_1 cs^2 + 3\omega_1 v_2^2 - 5\omega_2 cs^2 - 3\omega_2 v_2^2) \frac{\delta_l^2}{2\omega_1 \omega_2 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\ & (-2 + \omega_1) \frac{\delta_l^2 cs^2}{2\omega_1 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (-\omega_1 - 3\omega_1 \omega_2 cs^2 - \omega_1 \omega_2 v_1^2 - \omega_2 + 3\omega_1 cs^2 + \omega_1 v_1^2 + \omega_2 v_1^2 + 3\omega_2 cs^2 + \omega_1 \omega_2) \frac{\delta_l^2 v_1}{2\omega_1 \omega_2 \delta_t} \frac{\partial^2 \rho}{\partial x_1^2} + \\ & + (-\omega_1 - \omega_1 \omega_2 cs^2 - 3\omega_1 \omega_2 v_1^2 - \omega_2 + \omega_1 cs^2 + 3\omega_1 v_1^2 + 3\omega_2 v_1^2 + \omega_2 cs^2 + \omega_1 \omega_2) \frac{\delta_l^2 \rho}{2\omega_1 \omega_2 \delta_t} \frac{\partial^2 v_1}{\partial x_1^2} + \\ & (-\omega_1 + \omega_2 + 3\omega_1 cs^2 + \omega_1 v_2^2 - 3\omega_2 cs^2 - \omega_2 v_2^2) \frac{\delta_l^2 v_2}{2\omega_1 \omega_2 \delta_t} \frac{\partial^2 \rho}{\partial x_1 \partial x_2} + \\ & (-\omega_1 + \omega_1 \omega_2 cs^2 + \omega_2 + \omega_1 cs^2 + 3\omega_1 v_2^2 - 3\omega_2 cs^2 - 3\omega_2 v_2^2) \frac{\delta_l^2 \rho}{2\omega_1 \omega_2 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + (-2 + \omega_1) \frac{\delta_l^2 \rho cs^2}{2\omega_1 \delta_t} \frac{\partial^2 v_1}{\partial x_2^2} + C_1 \frac{\delta_l^3}{12\omega_1^2 \omega_2^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3} - \\ & + C_2 \frac{\delta_l^3 \rho v_1}{12\omega_1^2 \omega_2^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + C_3 \frac{3\delta_l^3 v_1 v_2}{4\omega_1^2 \omega_2^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + C_4 \frac{\delta_l^3 \rho v_2}{4\omega_1^2 \omega_2^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} + C_5 \frac{\delta_l^3 \rho v_1}{12\omega_1^2 \omega_2^2 \delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + C_6 \frac{\delta_l^3}{12\omega_1^2 \omega_2^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} - \\ & \frac{\delta_l^3 \rho cs^2 v_1}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_7 \frac{\delta_l^3 \rho v_2}{4\omega_1^2 \omega_2^2 \delta_t} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + (-1 + v_2^2 + 3cs^2) \frac{\delta_l^3 v_1 v_2}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + C_8 \frac{\delta_l^3 \rho v_2}{6\omega_1 \omega_3 \delta_t} \frac{\partial^3 v_1}{\partial x_2^3} + (-1 + 3v_2^2 + cs^2) \frac{\delta_l^3 \rho v_1}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & + C_9 \frac{\delta_l^4 v_1}{24\omega_1^3 \omega_2^2 \omega_3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + C_{10} \frac{\delta_l^4 \rho}{24\omega_1^3 \omega_2^2 \omega_3 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_{11} \frac{\delta_l^4 v_2}{24\omega_1^3 \omega_2^2 \omega_3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{12} \frac{\delta_l^4 \rho v_1 v_2}{24\omega_1^3 \omega_2^2 \omega_3 \delta_t} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + \\ & C_{13} \frac{\delta_l^4 \rho}{24\omega_1^3 \omega_2^2 \omega_3 \delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_{14} \frac{\delta_l^4 v_1}{24\omega_1^3 \omega_2^2 \omega_3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_{15} \frac{\delta_l^4 \rho}{24\omega_1^3 \omega_2^2 \omega_3 \delta_t} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{16} \frac{\delta_l^4 \rho v_1 v_2}{24\omega_1^3 \omega_2^2 \omega_3 \delta_t} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + \\ & C_{17} \frac{\delta_l^4 v_2}{24\omega_1^3 \omega_2^2 \omega_3 \delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{18} \frac{\delta_l^4 \rho v_1 v_2}{24\omega_1 \omega_2 \omega_3 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{19} \frac{\delta_l^4 \rho}{24\omega_1^3 \omega_2^2 \omega_3 \delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + C_{20} \frac{\delta_l^4 v_1}{24\omega_1 \omega_2 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{21} \frac{\delta_l^4 \rho}{24\omega_1^3 \omega_2^3 \delta_t} \frac{\partial^4 v_1}{\partial x_2^4} + \\ & (-5\omega_1 \omega_2 v_2^2 - 2\omega_1 - 3\omega_1 \omega_2 cs^2 - 2\omega_2 + 3\omega_1 cs^2 + 5\omega_1 v_2^2 + 3\omega_2 cs^2 + 2\omega_1 \omega_2 + 5\omega_2 v_2^2) \frac{\delta_l^4 \rho v_1 v_2}{12\omega_1 \omega_2 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:

$$C_1 = -18\omega_1 \omega_2^2 v_1^4 - 6\omega_1 \omega_2^2 cs^4 - \omega_2^2 \omega_2^2 cs^2 - 7\omega_1^2 \omega_2^3 v_1^2 - 18\omega_2^2 \omega_2 v_1^4 - 72\omega_1 \omega_2^2 cs^2 v_1^2 - 6\omega_2^2 \omega_2 cs^4 - 6\omega_2^2 cs^2 - 18\omega_1 \omega_2 v_1^2 - 9\omega_2^2 v_1^2 + 45\omega_1^2 cs^2 v_1^2 - 9\omega_2^2 v_1^2 - 6\omega_1^2 cs^2 - 72\omega_1^2 \omega_2 cs^2 v_1^2 + 9\omega_1^2 v_1^4 + 6\omega_1^2 cs^4 + 24\omega_1^2 \omega_2^2 cs^2 v_1^2 + \omega_1^2 \omega_2^2 cs^4 + 7\omega_1^2 \omega_2^2 v_1^4 + 18\omega_1 \omega_2^2 v_1^2 + 6\omega_1 \omega_2^2 cs^2 + 45\omega_2^2 cs^2 v_1^2 + 6\omega_2^2 cs^4 + 18\omega_1 \omega_2 v_1^4 + 9\omega_2^2 v_1^4 + 54\omega_1 \omega_2 cs^2 v_1^2 + 18\omega_2^2 \omega_2 v_1^2 + 6\omega_1^2 \omega_2 cs^2$$

$$C_2 = 10\omega_1^2 \omega_2^2 cs^2 + 22\omega_1^2 \omega_2^2 v_1^2 + 27\omega_2^2 cs^2 + 18\omega_1 \omega_2 cs^2 + 54\omega_1 \omega_2 v_1^2 + 33\omega_2^2 v_1^2 + 24\omega_1^2 \omega_2 - 8\omega_1^2 \omega_2^2 - 15\omega_1^2 + 33\omega_1^2 v_1^2 + 27\omega_1^2 cs^2 + 24\omega_1 \omega_2^2 - 15\omega_2^2 - 60\omega_1 \omega_2^2 v_1^2 - 36\omega_1 \omega_2^2 cs^2 - 18\omega_1 \omega_2 - 60\omega_1^2 \omega_2 v_1^2 - 36\omega_1^2 \omega_2 cs^2$$

$$C_3 = -\omega_2^2 v_2^2 - 6\omega_1 \omega_2 cs^2 - 2\omega_1 \omega_2 v_1^2 + \omega_2^2 v_1^2 + \omega_1^2 \omega_2 + \omega_1^2 v_2^2 - 2\omega_2^2 + \omega_1^2 v_1^2 + 6\omega_1^2 cs^2 - \omega_1 \omega_2^2 + 3\omega_1 \omega_2^2 cs^2 + 2\omega_1 \omega_2 - \omega_1^2 \omega_2 v_2^2 + \omega_1 \omega_2^2 v_2^2 - 3\omega_1^2 \omega_2 cs^2$$

$$C_4 = -\omega_1 \omega_2^2 \omega_3 + \omega_1^2 \omega_2 \omega_3 + 2\omega_2^2 \omega_2 - 3\omega_2^2 \omega_3 cs^2 - 18\omega_1 \omega_2 \omega_3 v_1^2 + 9\omega_2^2 \omega_3 v_1^2 - 6\omega_1 \omega_2 \omega_3 cs^2 - 2\omega_2^2 \omega_3 v_2^2 + 3\omega_1 \omega_2^2 \omega_3 cs^2 - 5\omega_1^2 \omega_3 + \omega_1 \omega_2^2 \omega_3 v_2^2 - 2\omega_1 \omega_2^2 + 9\omega_1^2 \omega_3 cs^2 + 9\omega_1^2 \omega_3 v_1^2 + 6\omega_1 \omega_2^2 cs^2 - 3\omega_1^2 \omega_2 \omega_3 cs^2 - 2\omega_1^2 \omega_2 v_2^2 + 2\omega_1^2 \omega_3 v_2^2 + 6\omega_1 \omega_2 \omega_3 - \omega_2^2 \omega_3 + 2\omega_1 \omega_2^2 v_2^2 - \omega_1^2 \omega_2 \omega_3 v_2^2 - 6\omega_1^2 \omega_2 cs^2$$

$$C_5 = -12\omega_1 \omega_2^2 \omega_3 + 36\omega_1^2 \omega_2^2 cs^2 - 11\omega_1^2 \omega_2^2 \omega_3 cs^2 - 3\omega_1^2 \omega_2^2 \omega_3 v_1^2 + 12\omega_1^2 \omega_2^2 v_1^2 + 12\omega_1^2 \omega_2 \omega_3 + 6\omega_1^2 \omega_2 + 9\omega_2^2 \omega_3 cs^2 - 12\omega_1^2 \omega_2^2 + 6\omega_2^2 \omega_3 v_1^2 - 27\omega_2^2 \omega_3 v_2^2 + 18\omega_1 \omega_2^2 \omega_3 cs^2 + 3\omega_1 \omega_2^2 \omega_3 v_1^2 - 15\omega_1^2 \omega_3 + 27\omega_1 \omega_2^2 \omega_3 v_2^2 + 18\omega_1 \omega_2^2 + 27\omega_1^2 \omega_3 cs^2 + 6\omega_1^2 \omega_3 v_1^2 + 3\omega_1^2 \omega_2 \omega_3 - 3\omega_1^2 \omega_2 \omega_3 v_1^2 - 18\omega_1 \omega_2^2 v_1^2 - 54\omega_1 \omega_2^2 cs^2 - 18\omega_1^2 \omega_2 \omega_3 cs^2 + 27\omega_1^2 \omega_3 v_2^2 + 3\omega_2^2 \omega_3 - 27\omega_1^2 \omega_2 \omega_3 v_2^2 - 6\omega_1^2 \omega_2 v_1^2 - 18\omega_1^2 \omega_2 cs^2$$

$$C_6 = 30\omega_1 \omega_2^2 cs^4 + 45\omega_1 \omega_2^2 cs^2 v_2^2 - 9\omega_2^2 \omega_2 v_2^4 + 9\omega_2^2 v_2^2 + 9\omega_1 \omega_2^2 v_2^4 - 6\omega_1^2 \omega_2 cs^4 + 6\omega_2^2 cs^2 + 45\omega_2^2 cs^2 v_2^2 - 9\omega_1^2 v_2^2 - 45\omega_1^2 \omega_2 cs^2 v_2^2 - 6\omega_1^2 cs^2 + 9\omega_1^2 v_2^4 + 6\omega_1^2 cs^4 - 45\omega_2^2 cs^2 v_2^2 - 9\omega_2^2 v_2^4 - 2\omega_1^2 \omega_2^2 cs^4 - 6\omega_1 \omega_2^2 cs^2 + 9\omega_1^2 \omega_2 v_2^2 - 30\omega_2^2 cs^4 - 9\omega_1 \omega_2^2 v_2^2 + 6\omega_1^2 \omega_2 cs^2$$

$$C_7 = -11\omega_2^2 v_2^2 - 9\omega_2^2 cs^2 + 5\omega_1^2 \omega_2 + 11\omega_1^2 v_2^2 - 5\omega_1^2 + 9\omega_1^2 cs^2 - 5\omega_1 \omega_2^2 + 5\omega_2^2 + 9\omega_1 \omega_2^2 cs^2 - 11\omega_1^2 \omega_2 v_2^2 + 11\omega_1 \omega_2^2 v_2^2 - 9\omega_1^2 \omega_2 cs^2$$

$$C_8 = 6 - 3\omega_1 \omega_3 cs^2 - 3\omega_1 - \omega_1 \omega_3 v_2^2 + \omega_1 \omega_3 + 9\omega_1 cs^2 + 3\omega_1 v_2^2 - 3\omega_3 + 3\omega_3 v_2^2 - 6v_2^2 + 9\omega_3 cs^2 - 18cs^2$$

$$C_9 = 99\omega_1^2 \omega_2 \omega_3 v_1^4 - 68\omega_1^3 \omega_2^2 \omega_3 cs^2 v_1^2 + 6\omega_1 \omega_2^2 \omega_3 + 54\omega_1^2 \omega_2 \omega_3 cs^4 - 10\omega_1^3 \omega_2^2 \omega_3 cs^4 + 6\omega_1^2 \omega_2 cs^2 v_1^2 - 18\omega_1^3 \omega_2^2 \omega_3 v_1^4 + 12\omega_1^2 \omega_2^2 cs^2 + 114\omega_1^2 \omega_2^2 \omega_3 cs^2 + 210\omega_1^2 \omega_2^2 \omega_3 v_1^2 - 2\omega_1^3 \omega_2^2 \omega_3 + 45\omega_1^2 \omega_2^2 \omega_3 v_1^4 + 6\omega_1^2 \omega_2 \omega_3 + 411\omega_1^3 \omega_2 \omega_3 cs^2 v_1^2 + 90\omega_1^3 \omega_2^2 \omega_3 cs^4 + 129\omega_1 \omega_2^2 \omega_3 v_1^2 + 141\omega_1 \omega_2^2 \omega_3 cs^2 - 6\omega_1^3 \omega_2 cs^2 + 54\omega_1 \omega_2^2 \omega_3 cs^4 + 99\omega_1 \omega_2^2 \omega_3 v_1^4 - 12\omega_1 \omega_2^2 \omega_3 + 18\omega_1 \omega_2^2 cs^4 - 171\omega_1^3 \omega_2 \omega_3 cs^4 + 404\omega_1^3 \omega_2^2 \omega_3 cs^2 v_1^2 + 261\omega_1^3 \omega_2 \omega_3 cs^2 v_1^2 - 117\omega_1^3 \omega_2 \omega_3 v_1^4 -$$

$$\begin{aligned}
& 51\omega_1^3\omega_3v_1^2 - 98\omega_1^3\omega_2^2\omega_3v_1^2 - 72\omega_1^3\omega_3cs^2 - 78\omega_1^3\omega_2^2\omega_3cs^2 + 8\omega_1^3\omega_2^2\omega_3 + 90\omega_1^2\omega_2^3\omega_3v_1^4 + 82\omega_1^2\omega_2^3\omega_3cs^4 + 261\omega_2^3\omega_3cs^2v_1^2 - 60\omega_1\omega_2^3\omega_3cs^2 + \\
& 411\omega_1\omega_2^3\omega_3cs^2v_1^2 - 105\omega_1\omega_2^2\omega_3v_1^2 - 6\omega_1\omega_2^3cs^2 + 8\omega_1^2\omega_2^3\omega_3 + 18\omega_1^2\omega_2cs^4 + 6\omega_1\omega_2^3cs^2v_1^2 - 12\omega_1^3\omega_2\omega_3 + 90\omega_1^2\omega_2^3\omega_3v_1^4 + 45\omega_1^2\omega_3v_1^4 + \\
& 82\omega_1^2\omega_2^3\omega_3cs^4 + 90\omega_1^3\omega_3cs^4 - 98\omega_1^2\omega_2^3\omega_3v_1^2 - 78\omega_1^2\omega_2^3\omega_3cs^2 - 12\omega_1^2\omega_2^3cs^2v_1^2 - 816\omega_1^2\omega_2^3\omega_3cs^2v_1^2 + 141\omega_1^3\omega_2\omega_3cs^2 + 6\omega_2^3\omega_3 + 129\omega_1^3\omega_2\omega_3v_1^2 + \\
& 12\omega_1^3\omega_2^3\omega_3cs^2 + 20\omega_1^3\omega_2^3\omega_3v_1^2 - 36\omega_1^2\omega_2^3cs^4 - 90\omega_1^2\omega_2^3\omega_3cs^4 - 198\omega_1^2\omega_2^3\omega_3v_1^4 - 12\omega_1^2\omega_2^3\omega_3 - 105\omega_1^2\omega_2\omega_3v_1^2 - 60\omega_1^2\omega_2\omega_3cs^2 + \\
& 404\omega_1^2\omega_2^3\omega_3cs^2v_1^2 - 117\omega_1\omega_2^3\omega_3v_1^4 - 171\omega_1\omega_2^3\omega_3cs^4 - 600\omega_1\omega_2^3\omega_3cs^2v_1^2 - 600\omega_1^3\omega_2\omega_3cs^2v_1^2 - 51\omega_2^3\omega_3v_1^2 + 6\omega_1^3\omega_3 - 72\omega_2^3\omega_3cs^2
\end{aligned}$$

$$\begin{aligned}
C_{10} = & 333\omega_1^2\omega_2\omega_3v_1^4 - 36\omega_1^3\omega_2^3\omega_3cs^2v_1^2 + 6\omega_1\omega_2^3\omega_3 + 6\omega_1^2\omega_2\omega_3cs^4 - 2\omega_1^3\omega_2^3\omega_3cs^4 + 18\omega_1^3\omega_2cs^2v_1^2 - 58\omega_1^3\omega_2^3\omega_3v_1^4 + 12\omega_1^2\omega_2cs^2 + \\
& 18\omega_1^2\omega_2^3\omega_3cs^2 + 306\omega_1^2\omega_2^3\omega_3v_1^2 - 2\omega_1^3\omega_2^3\omega_3 + 171\omega_2^3\omega_3v_1^4 + 6\omega_1^2\omega_2\omega_3 + 225\omega_1^2\omega_2\omega_3cs^2v_1^2 + 18\omega_2^3\omega_3cs^4 + 225\omega_1\omega_2^3\omega_3v_1^2 + 45\omega_1\omega_2^3\omega_3cs^2 - \\
& 6\omega_1^3\omega_2cs^2 + 6\omega_1\omega_2^3\omega_3cs^4 + 333\omega_1\omega_2^3\omega_3v_1^4 - 12\omega_1\omega_2^3\omega_3 + 6\omega_1\omega_2^3cs^4 - 33\omega_1^3\omega_2\omega_3cs^4 + 252\omega_1^3\omega_2^3\omega_3cs^2v_1^2 + 207\omega_1^3\omega_3cs^2v_1^2 - 423\omega_1^3\omega_2\omega_3v_1^4 - \\
& 99\omega_1^3\omega_3v_1^2 - 154\omega_1^3\omega_2^2\omega_3v_1^2 - 24\omega_1^3\omega_3cs^2 - 22\omega_1^3\omega_2^3\omega_3cs^2 + 8\omega_1^3\omega_2^2\omega_3 + 310\omega_1^2\omega_2^3\omega_3v_1^4 + 14\omega_1^2\omega_2^3\omega_3cs^4 + 207\omega_2^3\omega_3cs^2v_1^2 - 12\omega_1\omega_2^3\omega_3cs^2 + \\
& 225\omega_1\omega_2^3\omega_3cs^2v_1^2 - 153\omega_1\omega_2^3\omega_3v_1^2 - 6\omega_1\omega_2^3cs^2 + 8\omega_1^2\omega_2^3\omega_3 + 6\omega_1^2\omega_2cs^4 + 18\omega_1\omega_2^3cs^2v_1^2 - 12\omega_1^3\omega_2\omega_3 + 310\omega_1^3\omega_2^3\omega_3v_1^4 + 171\omega_1^3\omega_3v_1^4 + \\
& 14\omega_1^3\omega_2^3\omega_3cs^4 + 18\omega_1^3\omega_3cs^4 - 154\omega_1^3\omega_2^2\omega_3v_1^2 - 22\omega_1^3\omega_2^3\omega_3cs^2 - 36\omega_1^2\omega_2^3cs^2v_1^2 - 432\omega_1^2\omega_2^3\omega_3cs^2v_1^2 + 45\omega_1^3\omega_2\omega_3cs^2 + 6\omega_2^3\omega_3 + 225\omega_1^3\omega_2\omega_3v_1^2 + \\
& 4\omega_1^3\omega_2^3\omega_3cs^2 + 28\omega_1^3\omega_2^3\omega_3v_1^2 - 12\omega_1^2\omega_2^3cs^4 - 6\omega_1^2\omega_2^3\omega_3cs^4 - 666\omega_1^2\omega_2^3\omega_3v_1^4 - 12\omega_1^2\omega_2^3\omega_3 - 153\omega_1^2\omega_2\omega_3v_1^2 - 12\omega_1^2\omega_2\omega_3cs^2 + 252\omega_1^2\omega_2^3\omega_3cs^2v_1^2 - \\
& 423\omega_1\omega_2^3\omega_3v_1^4 - 33\omega_1\omega_2^3\omega_3cs^4 - 432\omega_1\omega_2^3\omega_3cs^2v_1^2 - 432\omega_1^3\omega_2\omega_3cs^2v_1^2 - 99\omega_2^3\omega_3v_1^2 + 6\omega_1^3\omega_3 - 24\omega_2^3\omega_3cs^2
\end{aligned}$$

$$\begin{aligned}
C_{11} = & 27\omega_1\omega_2^3\omega_3cs^4 - 54\omega_1\omega_2^3\omega_3v_1^4 + 21\omega_1\omega_2^3\omega_3cs^2v_1^2 + \omega_1^3\omega_2^3\omega_3^2 + 72\omega_1\omega_2^3\omega_3v_1^2v_2^2 + 6\omega_1^2\omega_2\omega_3^2v_2^2 - 72\omega_1^3\omega_2\omega_3^2v_1^2v_2^2 - 72\omega_1^2\omega_2^3\omega_3cs^2v_1^2 - \\
& 21\omega_1^2\omega_2\omega_3^2cs^2v_2^2 - 9\omega_2^3\omega_3v_1^2 - 54\omega_1\omega_2^3\omega_3cs^2v_1^2 + 12\omega_1^2\omega_3cs^2v_2^2 + 6\omega_1\omega_2^3\omega_3 + 108\omega_1^2\omega_2^3\omega_3v_1^4 + 72\omega_1^2\omega_2^3\omega_3cs^4 + 60\omega_1^2\omega_2\omega_3^2cs^2 - 2\omega_1^3\omega_2^3\omega_3cs^2v_2^2 - \\
& 6\omega_1\omega_2^3\omega_3cs^2 + 45\omega_1^2\omega_2\omega_3^2v_1^2 - 24\omega_1^2\omega_2^3\omega_3v_1^2v_2^2 - 486\omega_1^2\omega_2\omega_3^2cs^2v_1^2 + 6\omega_2^3\omega_3v_2^2 + 6\omega_1^3\omega_2^3\omega_3cs^4 + 90\omega_1^3\omega_2^3cs^2 - 6\omega_1\omega_2^3\omega_3cs^2 + 54\omega_1^2\omega_3v_1^4 - \\
& 297\omega_1\omega_2^3\omega_3cs^2v_1^2 + 5\omega_1^2\omega_2^3\omega_3cs^2 + 24\omega_1^2\omega_2^3\omega_3v_1^2v_2^2 + 18\omega_1^2\omega_2^3\omega_3cs^2v_2^2 - 6\omega_1^2\omega_2\omega_3^2 + 126\omega_1^2\omega_2\omega_3^2v_1^2 + 75\omega_1^2\omega_2\omega_3^2cs^2 + 63\omega_1\omega_2^3\omega_3v_1^2 + 12\omega_1\omega_2^3\omega_3cs^2 - \\
& 9\omega_1\omega_2^3\omega_3v_1^2v_2^2 + 6\omega_1\omega_2^3\omega_3cs^2v_2^2 - 18\omega_1^3\omega_2\omega_3cs^4 + \omega_1^2\omega_2^3\omega_3^2v_2^2 + 6\omega_1\omega_2^3\omega_3 - 18\omega_1^2\omega_2\omega_3cs^2 + 6\omega_1^3\omega_2\omega_3v_2^2 + 540\omega_1^2\omega_2^3\omega_3cs^2v_1^2 - 54\omega_1^2\omega_2^3\omega_3cs^4 - \\
& 36\omega_1^2\omega_2^3cs^4 + 2\omega_1^2\omega_2^3\omega_3cs^2v_2^2 + 18\omega_1^2\omega_2^3cs^2v_2^2 + 6\omega_1^3\omega_2 - 54\omega_1^2\omega_2\omega_3v_1^4 - 99\omega_1^2\omega_2\omega_3cs^4 + 24\omega_1^2\omega_2^3\omega_3v_1^2v_2^2 + 45\omega_1^3\omega_2^3v_1^2v_2^2 - 72\omega_1^3\omega_2^3cs^2 - 5\omega_1^3\omega_2^3\omega_3cs^2 - \\
& 18\omega_2^3\omega_3cs^2v_2^2 + 36\omega_2^3\omega_3cs^2 - 99\omega_1^3\omega_2v_1^2 - 24\omega_1^2\omega_2^3\omega_3v_1^2 - 6\omega_1^2\omega_2^3\omega_3cs^4 - 45\omega_2^3\omega_3v_1^2v_2^2 + 72\omega_1^3\omega_2^3\omega_3cs^2v_1^2 + 405\omega_1^3\omega_2^3cs^2v_1^2 + 54\omega_1^2\omega_2\omega_3cs^4 + \\
& 18\omega_1^2\omega_2^3\omega_3cs^2 + 12\omega_1^2\omega_2^3cs^2 + 135\omega_2^3\omega_3cs^2v_1^2 - 54\omega_1\omega_2^3\omega_3v_1^4 - 18\omega_1\omega_2^3\omega_3cs^4 - 6\omega_1^2\omega_2^3\omega_3^2 - \omega_1^3\omega_2^3\omega_3^2v_2^2 + 6\omega_1^3\omega_2\omega_3cs^2 - 6\omega_1^2\omega_2\omega_3cs^2v_2^2 - 18\omega_2^3\omega_3cs^4 + \\
& 6\omega_1\omega_2^3\omega_3cs^2v_2^2 + 9\omega_1^2\omega_2\omega_3^2v_1^2v_2^2 - 6\omega_2^3\omega_3 + 54\omega_2^3\omega_3v_1^4 - 3\omega_1\omega_2^3\omega_3cs^2 - 18\omega_1\omega_2^3\omega_3v_1^2 - 6\omega_1^3\omega_2\omega_3cs^2v_2^2 - 54\omega_1^2\omega_2\omega_3cs^4 - 243\omega_1^2\omega_2\omega_3cs^2v_1^2 + \\
& 18\omega_1\omega_2^3\omega_3cs^4 - \omega_1^2\omega_2^3\omega_3 - 54\omega_1^2\omega_2\omega_3v_1^4 - 6\omega_1^2\omega_2\omega_3 - 6\omega_1\omega_2^3\omega_3v_2^2 - 18\omega_1^2\omega_2^3\omega_3cs^2v_2^2 - 108\omega_1^2\omega_2^3\omega_3v_1^2 - 12\omega_1^2\omega_2^3cs^2v_2^2 - 72\omega_1^2\omega_2^3\omega_3cs^2
\end{aligned}$$

$$\begin{aligned}
C_{12} = & 90\omega_1\omega_2^3\omega_3 + 45\omega_1\omega_2^3\omega_3v_2^2 - 18\omega_1^3\omega_2^2 + 324\omega_1^2\omega_2^3\omega_3cs^2 + 396\omega_1^2\omega_2^3\omega_3v_1^2 + 90\omega_1^2\omega_2\omega_3 - 198\omega_1\omega_2^3\omega_3v_1^2 - 27\omega_1\omega_2^3\omega_3cs^2 - 54\omega_1^2\omega_2^3cs^2 - \\
& 54\omega_1^2\omega_2cs^2 + 36\omega_1^3\omega_3v_2^2 + 10\omega_1^3\omega_2^3\omega_3v_2^2 + 45\omega_1\omega_2^3\omega_3 + 18\omega_1^2\omega_2^3 + 18\omega_1^3\omega_2 - 18\omega_1^2\omega_2^3v_2^2 - 18\omega_1^3\omega_2v_2^2 + 198\omega_1^3\omega_3v_1^2 + 270\omega_1^3\omega_3cs^2 + \\
& 30\omega_1^3\omega_2^3\omega_3cs^2 - 10\omega_1^3\omega_2^3\omega_3 - 162\omega_1\omega_2^3\omega_3cs^2 - 198\omega_1\omega_2^3\omega_3v_1^2 - 18\omega_1\omega_2^3 - 10\omega_1^2\omega_2^3\omega_3v_2^2 + 18\omega_1^3\omega_2^3v_2^2 + 54\omega_1\omega_2^3cs^2 + 10\omega_1^2\omega_2^3\omega_3 - \\
& 45\omega_1^2\omega_2\omega_3v_2^2 + 135\omega_1^2\omega_2\omega_3 + 18\omega_1\omega_2^3v_2^2 - 30\omega_1^2\omega_2^3\omega_3cs^2 + 54\omega_1^2\omega_2^3cs^2 - 297\omega_1^2\omega_2\omega_3cs^2 - 54\omega_2^3\omega_3 - 198\omega_1^3\omega_2\omega_3v_1^2 - 180\omega_1^2\omega_2^3\omega_3 - \\
& 198\omega_1^2\omega_2\omega_3v_1^2 - 162\omega_1^2\omega_2\omega_3cs^2 - 36\omega_2^3\omega_3v_2^2 + 198\omega_2^3\omega_3v_1^2 - 126\omega_1^3\omega_3 + 54\omega_2^3\omega_3cs^2
\end{aligned}$$

$$\begin{aligned}
C_{13} = & -198\omega_1^3\omega_2^3\omega_3cs^2v_1^2 - 57\omega_1\omega_2^3\omega_3cs^4 - 63\omega_1\omega_2^3\omega_3v_1^4 - 324\omega_1^2\omega_2^3cs^2v_1^2 - 9\omega_1\omega_2^3\omega_2^3cs^2v_2^2 + \omega_1^3\omega_2^3\omega_3^2 + 216\omega_1\omega_2^3\omega_3^2v_1^2v_2^2 + 12\omega_1^3\omega_2^3\omega_3cs^4 + \\
& 18\omega_1^2\omega_2\omega_3^2v_2^2 - 216\omega_1^3\omega_2\omega_3^2v_1^2v_2^2 - 78\omega_1^3\omega_2^3\omega_3v_1^4 + 30\omega_1^2\omega_2^3\omega_3cs^2v_1^2 - 9\omega_1^2\omega_2\omega_3^2cs^2v_2^2 - 24\omega_1^2\omega_2^3\omega_3cs^2 + 36\omega_2^3\omega_3v_1^2 + 9\omega_2^3\omega_3v_1^2 - 8\omega_1^2\omega_2^3\omega_3v_1^2 + \\
& 72\omega_1^3\omega_2^3v_1^4 - 171\omega_1\omega_2^3\omega_3cs^4v_1^2 + 6\omega_1\omega_2^3\omega_3 - 54\omega_1^2\omega_2^3\omega_3v_1^4 + 12\omega_1^2\omega_2^3\omega_3cs^4 + 36\omega_1^2\omega_2\omega_3^2cs^2 + 18\omega_1\omega_2^3\omega_3v_1^2 + 6\omega_1^2\omega_2^3\omega_3cs^2v_2^2 - 18\omega_1\omega_2^3\omega_3cs^2 - \\
& 45\omega_1^2\omega_2\omega_3^2v_1^2 - 72\omega_1^2\omega_2^3\omega_3^2v_1^2v_2^2 - 297\omega_1^2\omega_2\omega_3^2cs^2v_1^2 + 18\omega_2^3\omega_3v_2^2 - \omega_1^3\omega_2^3\omega_3^2cs^4 + 18\omega_1^3\omega_2^3cs^4 - 18\omega_1\omega_2^3\omega_2^3v_2^2 + 108\omega_1^2\omega_2^3v_1^2 + 19\omega_1^3\omega_2^3\omega_3v_1^4 + \\
& 36\omega_1^2\omega_2^3v_1^4 + 99\omega_1\omega_2^3\omega_3cs^4v_1^2 - 12\omega_1^2\omega_2^3\omega_3cs^2 + 17\omega_1^2\omega_2^3\omega_3v_1^2 - 36\omega_1^2\omega_2^3\omega_3cs^2v_2^2 + 18\omega_1^2\omega_2^3\omega_3cs^2v_2^2 - 6\omega_1^2\omega_2\omega_3^2 + 135\omega_1^2\omega_2\omega_3^2v_1^2 + 21\omega_1^2\omega_2\omega_3^2cs^2 - \\
& 27\omega_1\omega_2^3\omega_3v_1^2 - 12\omega_1\omega_2^3\omega_3cs^2 - 27\omega_1\omega_2^3\omega_3v_1^2v_2^2 + 216\omega_1^2\omega_2^3cs^2v_1^2 + 18\omega_1\omega_2^3\omega_2^3cs^2v_2^2 + 6\omega_1^3\omega_2\omega_3cs^4 + 198\omega_1^2\omega_2\omega_3cs^2v_1^2 - 18\omega_1^3\omega_2\omega_3v_1^4 + \\
& 3\omega_1^2\omega_2^3\omega_3v_2^2 + 6\omega_1\omega_2^3\omega_3 - 72\omega_1^2\omega_2^3\omega_3v_1^4 + 18\omega_1^2\omega_2^3\omega_3cs^2 + 18\omega_1^3\omega_2\omega_3^2v_1^2 - 36\omega_1^3\omega_2v_1^4 - 144\omega_1^2\omega_2^3\omega_3cs^2v_1^2 + 144\omega_1^2\omega_2^3\omega_3v_1^4 - 30\omega_1^2\omega_2^3\omega_3cs^4 - \\
& 6\omega_1^3\omega_2^3\omega_3cs^2v_2^2 + 18\omega_1^3\omega_2^3cs^2v_2^2 + 6\omega_1^3\omega_2^3 - 63\omega_1^2\omega_2\omega_3^2v_1^4 - 15\omega_1^3\omega_2\omega_3^2cs^4 + 72\omega_1^3\omega_2^3\omega_3v_1^2v_2^2 + 135\omega_1^3\omega_2^3v_1^2v_2^2 - 24\omega_1^3\omega_2^3cs^2 + 18\omega_2^3\omega_3cs^2v_2^2 + \\
& 36\omega_1^2\omega_2^3\omega_3cs^2v_2^2 - 108\omega_1^2\omega_2^3v_1^4 - 81\omega_1^2\omega_2^3v_1^2 - 43\omega_1^2\omega_2^3\omega_3v_1^2 + 29\omega_1^2\omega_2^3\omega_3cs^4 + 7\omega_1^2\omega_2^3\omega_3v_1^4 - 135\omega_2^3\omega_3v_1^2v_2^2 + 72\omega_1^2\omega_2^3\omega_3v_1^4 + 84\omega_1^2\omega_2^3\omega_3cs^2v_1^2 + \\
& 189\omega_1^2\omega_2^3cs^2v_1^2 - 18\omega_1^2\omega_2^3\omega_3cs^4 + 36\omega_1^2\omega_2^3v_1^2 - 144\omega_1^2\omega_2^3\omega_3v_1^2 + 30\omega_1^2\omega_2^3\omega_3cs^2 + 63\omega_2^3\omega_3cs^2v_1^2 + 36\omega_1\omega_2^3\omega_3v_1^4 + 6\omega_1\omega_2^3\omega_3cs^4 - 18\omega_1^2\omega_2^3v_2^2 - \\
& 3\omega_1^2\omega_2^3\omega_3v_2^2 - 72\omega_1^2\omega_2^3\omega_3cs^2v_1^2 - 6\omega_1^2\omega_2\omega_3cs^2 + 18\omega_1^2\omega_2^3\omega_3cs^2 + 18\omega_1^2\omega_2^3\omega_3cs^2v_1^2 + 12\omega_1^2\omega_2^3\omega_3cs^2 - 54\omega_1^2\omega_2\omega_3^2v_1^2 + 78\omega_1^2\omega_2\omega_3^2v_1^2 + 30\omega_2^3\omega_3cs^2v_2^2 + \\
& 24\omega_1^2\omega_2^3\omega_3cs^4 + 24\omega_1^3\omega_2^3\omega_3cs^2v_1^2 - 36\omega_1^2\omega_2\omega_3v_1^4 + 27\omega_1^2\omega_2\omega_3^2v_1^2v_2^2 - 6\omega_2^3\omega_3 + 36\omega_2^3\omega_3v_1^4 + 15\omega_1\omega_2^3\omega_2^3cs^2 - 9\omega_1\omega_2^3\omega_2^3v_1^2 + 18\omega_1^3\omega_2\omega_3cs^2v_2^2 + \\
& 306\omega_1^2\omega_2^3\omega_3cs^2v_1^2 - 108\omega_1^3\omega_2^3cs^2v_1^2 - 30\omega_1^2\omega_2\omega_3^2cs^4 - 18\omega_1\omega_2^3\omega_3v_1^4 + 81\omega_1^2\omega_2\omega_3^2cs^2v_1^2 + 18\omega_1\omega_2^3\omega_3cs^4 - \omega_1^2\omega_2^3\omega_3 + 36\omega_1^2\omega_2\omega_3^2v_1^4 + 54\omega_1\omega_2^3\omega_3cs^2v_1^2 - \\
& 6\omega_1^3\omega_2\omega_3^2 + 8\omega_1^3\omega_2^3\omega_3v_1^4 - 72\omega_1^3\omega_2^3v_1^2 - 54\omega_1^2\omega_2\omega_3cs^2v_1^2 - 18\omega_1\omega_2^3\omega_2^3v_2^2 - 2\omega_1^2\omega_2^3\omega_3cs^4 + 36\omega_1^2\omega_2^3\omega_3cs^2v_2^2 + 54\omega_1^2\omega_2^3\omega_2^3v_1^2 - 12\omega_1^2\omega_2^3\omega_2^3cs^2
\end{aligned}$$

$$\begin{aligned}
C_{14} = & -12\omega_1^3\omega_2^3\omega_3cs^2v_1^2 + 63\omega_1\omega_2^3\omega_2^3cs^4 - 36\omega_1^2\omega_2^3cs^2v_1^2 + 189\omega_1\omega_2^3\omega_2^3cs^2v_2^2 - 45\omega_1\omega_2^3\omega_2^3v_1^2v_2^2 - 36\omega_1^3\omega_2^3\omega_3cs^4 + 99\omega_1^2\omega_2\omega_3^2v_2^2 - \\
& 45\omega_1^3\omega_2\omega_3^2v_1^2v_2^2 - 24\omega_1^2\omega_2^3\omega_2^3cs^2v_1^2 - 459\omega_1^2\omega_2\omega_3^2cs^2v_2^2 - 36\omega_1^2\omega_2^3\omega_3cs^2 - 6\omega_2^3\omega_3v_1^2 + 3\omega_1\omega_2^3\omega_2^3cs^2v_1^2 + 72\omega_1^2\omega_2^3cs^4 - 2\omega_1^3\omega_2^3\omega_2^3cs^2 + \\
& 54\omega_1\omega_2^3\omega_2^3v_2^2 - 6\omega_1\omega_2^3\omega_2^3 + 54\omega_1^2\omega_2\omega_3^2cs^4 + 24\omega_1^2\omega_2\omega_3^2cs^2 - 138\omega_1^2\omega_2^3\omega_2^3cs^2v_2^2 + 30\omega_1\omega_2^3\omega_2^3cs^2 + 6\omega_1^2\omega_2\omega_3^2v_1^2 - 21\omega_1^2\omega_2\omega_3^2cs^2v_1^2 + 9\omega_2^3\omega_3v_2^2 + \\
& 35\omega_1^2\omega_2^3\omega_2^3cs^4 + 90\omega_1^2\omega_2^3cs^4 - 9\omega_1\omega_2^3\omega_2^3v_2^2 + 36\omega_1^2\omega_2^3cs^2 - 30\omega_1\omega_2^3\omega_2^3cs^2v_1^2 + 41\omega_1^2\omega_2^3\omega_2^3cs^2 + 90\omega_1^2\omega_2^3\omega_2^3v_1^2v_2^2 + 270\omega_1^2\omega_2^3\omega_2^3cs^2v_2^2 - \\
& 6\omega_1^2\omega_2\omega_3^2 + 6\omega_1^3\omega_2\omega_3^2v_1^2 + 93\omega_1^3\omega_2\omega_3^2cs^2 + 6\omega_1\omega_2^3\omega_2^3v_1^2 + 27\omega_1^2\omega_2\omega_3^2v_2^2 + 54\omega_1^2\omega_3^2v_2^2 + 48\omega_1\omega_2^3\omega_2^3cs^2 - 45\omega_1\omega_2^3\omega_2^3v_1^2v_2^2 + 24\omega_1^2\omega_2^3cs^2v_1^2 + \\
& 27\omega_1\omega_2^3\omega_2^3cs^2v_2^2 - 18\omega_1^3\omega_2\omega_3cs^4 + 27\omega_1^2\omega_2^3\omega_2^3v_2^2 - 6\omega_1\omega_2^3\omega_2^3 + 99\omega_1^3\omega_2\omega_3^2v_2^2 + 18\omega_1^2\omega_2^3\omega_2^3cs^2v_1^2 + 144\omega_1^2\omega_2^3\omega_3cs^4 - 36\omega_1^3\omega_2^3cs^4 + \\
& 138\omega_1^2\omega_2^3\omega_2^3cs^2v_2^2 + 405\omega_1^3\omega_2^3cs^2v_2^2 + 6\omega_1^3\omega_2^3 - 117\omega_1^3\omega_2\omega_3^2cs^4 + 45\omega_1^3\omega_2^3v_1^2v_2^2 + 54\omega_1\omega_2^3\omega_2^3v_2^2 - 72\omega_1^3\omega_2^3cs^2 - 25\omega_1^3\omega_2^3\omega_2^3cs^2 - 135\omega_2^3\omega_3cs^2v_2^2 - \\
& 108\omega_1^2\omega_2^3cs^4 - 6\omega_1^3\omega_2^3v_2^2 - 91\omega_1^2\omega_2^3\omega_2^3cs^4 + 45\omega_2^3\omega_3^2v_1^2v_2^2 + 8\omega_1^2\omega_2^3\omega_2^3cs^2v_1^2 + 18\omega_1^2\omega_2^3cs^2v_1^2 - 54\omega_1^3\omega_2\omega_3^2v_2^4 - 48\omega_1^2\omega_2^3\omega_3cs^2 + 12\omega_1^2\omega_2^3cs^2 + \\
& 18\omega_2^3\omega_3cs^2v_1^2 - 90\omega_1\omega_2^3\omega_2^3cs^4 - 99\omega_1^2\omega_2^3v_2^2 - 27\omega_1^2\omega_2^3\omega_2^3v_2^2 + 36\omega_1^2\omega_2^3\omega_2^3cs^2v_1^2 + 6\omega_1^3\omega_2\omega_3cs^2 + 12\omega_1^2\omega_2^3\omega_2^3 - 27\omega_1^2\omega_2^3\omega_2^3v_2^2 + 12\omega_1^3\omega_2^3\omega_3cs^2 - \\
& 297\omega_1^2\omega_2\omega_3^2cs^2v_2^2 - 54\omega_1^2\omega_2\omega_3^2v_2^4 + 18\omega_2^3\omega_3cs^4 + 108\omega_1^2\omega_2^3\omega_3cs^4 + 2\omega_1^3\omega_2^3\omega_2^3cs^2v_1^2 - 45\omega_1^2\omega_2\omega_3^2v_1^2v_2^2 + 6\omega_2^3\omega_3 - 39\omega_1\omega_2^3\omega_2^3cs^2 + 6\omega_1\omega_2^3\omega_2^3v_1^2 - \\
& 90\omega_1^2\omega_2^3\omega_2^3v_2^2 + 48\omega_1^2\omega_2^3\omega_3cs^2v_1^2 - 12\omega_1^3\omega_2^3cs^2v_1^2 - 18\omega_1^2\omega_2\omega_3^2cs^4 - 6\omega_1^2\omega_2\omega_3^2cs^2v_1^2 - 90\omega_1\omega_2^3\omega_3cs^4 - 30\omega_1\omega_2^3\omega_2^3cs^2v_1^2 - 6\omega_1^3\omega_2\omega_3^2 - \\
& 54\omega_2^3\omega_3v_2^2 - 6\omega_1^3\omega_2\omega_3cs^2v_1^2 - 24\omega_1^3\omega_2^3cs^2 - 9\omega_1\omega_2^3\omega_2^3v_2^2 + 10\omega_1^3\omega_2^3\omega_2^3cs^4 - 12\omega_1^2\omega_2^3\omega_2^3v_1^2 - 54\omega_1^2\omega_2^3\omega_2^3cs^2
\end{aligned}$$

$$\begin{aligned}
C_{15} = & 72\omega_1^3\omega_2^3\omega_3cs^2v_2^2 + 45\omega_1\omega_2^3\omega_2^3cs^4 + 108\omega_1\omega_2^3\omega_2^3cs^2v_2^2 - 135\omega_1\omega_2^3\omega_2^3v_1^2v_2^2 + 24\omega_1^2\omega_2^3\omega_3cs^4 - 18\omega_1\omega_2^3\omega_3v_2^2 + 45\omega_1^2\omega_2\omega_3^2v_2^2 - \\
& 135\omega_1^2\omega_2\omega_3^2v_1^2v_2^2 - 6\omega_1^2\omega_2^3\omega_2^3cs^2v_1^2 - 216\omega_1^2\omega_2\omega_3^2cs^2v_2^2 - 60\omega_1^2\omega_2^3\omega_3cs^2 - 18\omega_2^3\omega_3v_1^2 + 9\omega_1\omega_2^3\omega_2^3cs^2v_1^2 + 108\omega_1^2\omega_2^3cs^2v_2^2 + 45\omega_1\omega_2^3\omega_2^3v_2^4 - \\
& 6\omega_1\omega_2^3\omega_2^3 + 30\omega_1^2\omega_2^3\omega_2^3cs^4 + 36\omega_1^2\omega_2\omega_3^2cs^2 - 30\omega_1^2\omega_2^3\omega_2^3cs^2v_2^2 - 30\omega_1\omega_2^3\omega_3cs^2 + 18\omega_1^2\omega_2\omega_3^2v_1^2 - 9\omega_2^3\omega_3v_2^2 + 2\omega_1^3\omega_2^3\omega_2^3cs^4 + \\
& 18\omega_1^2\omega_2^3cs^4 + 45\omega_1\omega_2^3\omega_2^3v_2^2 + \omega_1^3\omega_2^3\omega_3^2v_1^4 - 90\omega_1\omega_2^3\omega_2^3cs^2v_1^2 + 2\omega_1^2\omega_2^3\omega_2^3cs^2 + \omega_1^2\omega_2^3\omega_3^2v_1^2 - 18\omega_1^3\omega_2\omega_3v_2^2 + 144\omega_1^3\omega_2\omega_3cs^2v_2^2 - 54\omega_1^2\omega_2\omega_3v_2^2 +
\end{aligned}$$

$$270\omega_1^2\omega_2^2\omega_3^2v_1^2v_2^2 + 108\omega_1^2\omega_2^2\omega_3^2cs^2v_2^2 - 54\omega_1^2\omega_3^2\omega_3v_2^4 - 36\omega_1^3\omega_2^2v_2^4 - 6\omega_1^2\omega_2\omega_3^2 + 18\omega_1^3\omega_2\omega_3v_1^2 + 21\omega_1^3\omega_2\omega_3cs^2 + 18\omega_1\omega_2^2\omega_3^2v_1^2 + 9\omega_1^3\omega_2^2\omega_3^2v_2^4 + 36\omega_1^2\omega_2^2\omega_3^2cs^2 - 135\omega_1\omega_2^2\omega_3^2v_1^2v_2^2 - 36\omega_1^2\omega_2^2\omega_3^2v_2^2 - 45\omega_1\omega_2^2\omega_3^2cs^2v_2^2 + 6\omega_1^3\omega_2\omega_3cs^4 - 90\omega_1^3\omega_2^2\omega_3cs^2v_1^2 + 9\omega_1^2\omega_2^2\omega_3^2v_2^2 - 6\omega_1\omega_2^2\omega_3^2 + 36\omega_1^3\omega_2^2\omega_3cs^2 + 90\omega_1^3\omega_2\omega_3^2v_2^2 + 72\omega_1^2\omega_2^2\omega_3^2cs^2v_1^2 - 60\omega_1^2\omega_2^2\omega_3cs^4 + 30\omega_1^3\omega_2^2\omega_3cs^2v_2^2 + 189\omega_1^3\omega_2^2cs^2v_2^2 + 54\omega_1^3\omega_2^2\omega_3v_2^4 + 54\omega_1^2\omega_2^2\omega_3^2v_2^2 + 6\omega_1^3\omega_2^2 + 36\omega_1^3\omega_2^2v_2^2 - 15\omega_1^3\omega_2\omega_3cs^4 + 135\omega_1^3\omega_2^2v_1^2v_2^2 - 24\omega_1^3\omega_2^2cs^2 - 2\omega_1^3\omega_2^2\omega_3cs^2 - 63\omega_1^3\omega_2^2cs^2v_2^2 + 36\omega_1^2\omega_2^2\omega_3cs^2v_2^2 - 18\omega_1^3\omega_2^2v_1^2 - \omega_1^3\omega_2^2\omega_3^2v_1^2 - 10\omega_1^2\omega_2^2\omega_3^2cs^4 - \omega_1^2\omega_2^2\omega_3^2v_1^4 + 135\omega_1^2\omega_2^2v_1^2v_2^2 + 18\omega_1^3\omega_2\omega_3v_2^2 + 6\omega_1^3\omega_2^2\omega_3cs^2v_1^2 + 18\omega_1^3\omega_2^2cs^2v_1^2 - 36\omega_1^3\omega_2^2\omega_3cs^4 - 45\omega_1^3\omega_2\omega_3^2v_2^4 + 60\omega_1^2\omega_2^2\omega_3cs^2 - 18\omega_1^2\omega_2^2cs^2v_1^2 - 30\omega_1\omega_2^2\omega_3cs^4 - 81\omega_1^3\omega_2^2v_2^2 - 9\omega_1^3\omega_2^2\omega_3^2v_2^2 + 36\omega_1^2\omega_2^2v_2^4 + 144\omega_1^2\omega_2^2\omega_3cs^2v_1^2 - 6\omega_1^3\omega_2\omega_3cs^2 + 12\omega_1^2\omega_2^2\omega_3^2 - 9\omega_1^2\omega_2^2\omega_3^2v_2^4 + 18\omega_1\omega_2^2\omega_3v_2^4 - 24\omega_1^3\omega_2^2\omega_3cs^2 - 81\omega_1^2\omega_2\omega_3^2cs^2v_2^2 - 30\omega_1^2\omega_2^2cs^4 + 18\omega_1\omega_2^2\omega_3cs^2v_2^2 + 60\omega_1^2\omega_2^2\omega_3cs^4 - 135\omega_1^2\omega_2\omega_3^2v_1^2v_2^2 + 6\omega_1^2\omega_3^2 - 3\omega_1\omega_2^2\omega_3^2cs^2 + 18\omega_1\omega_2^2\omega_3^2v_1^2 - 54\omega_1^3\omega_2\omega_3cs^2v_2^2 - 90\omega_1^2\omega_2^2\omega_3^2v_2^2 - 198\omega_1^2\omega_2\omega_3cs^2v_1^2 - 30\omega_1^2\omega_2\omega_3cs^4 - 54\omega_1^2\omega_2\omega_3^2cs^2v_1^2 + 30\omega_1\omega_2^2\omega_3cs^4 + 126\omega_1\omega_2^2\omega_3cs^2v_1^2 - 6\omega_1^3\omega_2\omega_3^2 - 36\omega_1^2\omega_3^2v_2^4 + 18\omega_1^2\omega_2\omega_3cs^2v_1^2 - 2\omega_1^3\omega_2^2\omega_3cs^4 - 144\omega_1^2\omega_2^2\omega_3cs^2v_2^2 - 36\omega_1^2\omega_2^2\omega_3^2v_1^2 - 108\omega_1^2\omega_2^2cs^2v_2^2 - 42\omega_1^2\omega_2^2\omega_3cs^2$$

$$C_{16} = -18\omega_1\omega_2^2\omega_3 + 216\omega_1\omega_2^2\omega_3v_2^2 + 108\omega_1^2\omega_2^2cs^2 + 162\omega_1^2\omega_2^2\omega_3cs^2 + 54\omega_1^2\omega_2^2\omega_3v_1^2 + 36\omega_1^2\omega_2^2v_1^2 + 90\omega_1^2\omega_2\omega_3 - 27\omega_1\omega_2^2\omega_3v_1^2 + 135\omega_1\omega_2^2\omega_3cs^2 - 54\omega_1^3\omega_2cs^2 - 36\omega_1^2\omega_2^2 - 18\omega_1^3\omega_2v_1^2 + 198\omega_1^3\omega_2v_2^2 + 100\omega_1^3\omega_2^2\omega_3v_2^2 - 81\omega_1\omega_2^2\omega_3 + 18\omega_1^3\omega_2 + 36\omega_1^3\omega_3v_1^2 + 270\omega_1^3\omega_3cs^2 + 84\omega_1^3\omega_2^2\omega_3cs^2 - 46\omega_1^3\omega_2^2\omega_3 - 54\omega_1\omega_2^2\omega_3cs^2 - 36\omega_1\omega_2^2\omega_3v_1^2 + 18\omega_1\omega_2^2 - 100\omega_1^2\omega_2^2\omega_3v_2^2 - 18\omega_1\omega_2^2v_1^2 - 54\omega_1\omega_2^2cs^2 + 46\omega_1^2\omega_2^2\omega_3 - 216\omega_1^2\omega_2\omega_3v_2^2 + 135\omega_1^2\omega_2\omega_3 + 162\omega_1^2\omega_2\omega_3v_2^2 - 84\omega_1^2\omega_2\omega_3cs^2 - 297\omega_1^2\omega_2\omega_3cs^2 + 54\omega_1^2\omega_3 - 27\omega_1^2\omega_2\omega_3v_1^2 - 54\omega_1^2\omega_2\omega_3 - 36\omega_1^2\omega_2\omega_3v_1^2 - 162\omega_1^2\omega_2\omega_3cs^2 - 198\omega_1^2\omega_3v_2^2 - 162\omega_1^2\omega_2\omega_3v_2^2 + 36\omega_1^2\omega_3v_1^2 - 126\omega_1^2\omega_3 - 54\omega_1^2\omega_3cs^2$$

$$C_{17} = 225\omega_1\omega_2^2\omega_3^2cs^4 + 489\omega_1\omega_2^2\omega_3^2cs^2v_2^2 + 7\omega_1^3\omega_2^2\omega_3^2 - 18\omega_1^3\omega_2^2\omega_3cs^4 - 51\omega_1^2\omega_2\omega_3^2v_2^2 - 9\omega_1^2\omega_2^2\omega_3^2cs^2v_1^2 + 72\omega_1^3\omega_2^2\omega_3^2cs^2 - 465\omega_1^3\omega_2\omega_3^2cs^2v_2^2 - 24\omega_1^2\omega_2^2\omega_3cs^2 - 6\omega_1^3\omega_2^2\omega_3cs^2v_2^2 + 36\omega_1^3\omega_2^2cs^4 - 24\omega_1^2\omega_2^2\omega_3^2v_1^4 + 12\omega_1\omega_2^2\omega_3^2 + 12\omega_1\omega_2^2\omega_3^2cs^2 + 30\omega_1\omega_2^2\omega_3cs^2 + 51\omega_1^3\omega_2^2v_2^2 + 72\omega_1^2\omega_2^2\omega_3^2cs^4 + 90\omega_1^3\omega_2^2cs^4 + 51\omega_1\omega_2^2\omega_3^2v_2^2 + 24\omega_1^2\omega_3^2cs^2 + 3\omega_1^3\omega_2^2\omega_3^2v_1^4 + 81\omega_1^2\omega_2^2\omega_3^2cs^2 + 12\omega_1^3\omega_2^2cs^2v_2^2 + 3\omega_1^2\omega_2^2\omega_3^2v_1^2 - 18\omega_1^3\omega_2^2\omega_3cs^2v_2^2 + 12\omega_1^2\omega_2^2\omega_3^2cs^2v_2^2 + 6\omega_1^2\omega_2\omega_3^2 + 123\omega_1^3\omega_2\omega_3^2cs^2 + 39\omega_1^3\omega_2^2\omega_3^2v_2^4 + 45\omega_1^3\omega_2^2v_2^4 + 48\omega_1\omega_2^2\omega_3^2cs^2 - 165\omega_1\omega_2^2\omega_3^2cs^2v_2^2 + 18\omega_1^2\omega_2\omega_3cs^4 + 46\omega_1^2\omega_2^2\omega_3^2v_2^2 - 6\omega_1\omega_2^2\omega_3^2 + 18\omega_1^2\omega_2\omega_3cs^2 + 102\omega_1^3\omega_2\omega_3^2v_2^2 + 126\omega_1^2\omega_2^2\omega_3^2cs^4 + 197\omega_1^3\omega_2^2\omega_3^2cs^2v_2^2 + 261\omega_1^3\omega_2^2cs^2v_2^2 + 6\omega_1^3\omega_2^2 - 153\omega_1^2\omega_2\omega_3^2cs^4 - 45\omega_1\omega_2^2\omega_3^2v_1^4 - 72\omega_1^3\omega_2^2cs^2 - 59\omega_1^2\omega_2^2\omega_3^2cs^2 - 261\omega_1^2\omega_2^2\omega_3^2cs^2v_2^2 - 72\omega_1^2\omega_3^2cs^4 + 24\omega_1^2\omega_2\omega_3cs^2v_2^2 - 3\omega_1^3\omega_2^2\omega_3^2v_1^2 - 138\omega_1^2\omega_2^2\omega_3^2cs^4 - 3\omega_1^2\omega_2^2\omega_3^2v_1^4 + 9\omega_1^3\omega_2^2\omega_3^2cs^2v_1^2 - 54\omega_1^3\omega_2^2\omega_3cs^4 - 90\omega_1^2\omega_2\omega_3^2v_2^4 - 42\omega_1^2\omega_2^2\omega_3cs^2 - 90\omega_1\omega_2^2\omega_3^2cs^4 - 51\omega_1^3\omega_2^2v_2^2 - 46\omega_1^2\omega_2^2\omega_3^2v_2^2 - 6\omega_1^2\omega_2\omega_3cs^2 - 39\omega_1^2\omega_2^2\omega_3^2v_2^4 + 6\omega_1^3\omega_2^2\omega_3cs^2 + 141\omega_1^2\omega_2\omega_3^2cs^2v_2^2 + 45\omega_1^2\omega_2\omega_3^2v_2^4 - 90\omega_1^2\omega_2^2\omega_3^2cs^4 - 30\omega_1\omega_2^2\omega_3cs^2v_2^2 + 72\omega_1^2\omega_2\omega_3cs^4 - 6\omega_1^2\omega_3^2 - 147\omega_1\omega_2^2\omega_3^2cs^2 + 6\omega_1^3\omega_2\omega_3cs^2v_2^2 + 18\omega_1^2\omega_2\omega_3cs^4 - 90\omega_1\omega_2^2\omega_3cs^4 - 7\omega_1^2\omega_2^2\omega_3^2 - 12\omega_1^2\omega_2\omega_3^2 - 45\omega_1^2\omega_3^2v_2^4 + 2\omega_1^3\omega_2^2\omega_3^2cs^2v_2^2 - 12\omega_1^3\omega_2^2cs^2 - 102\omega_1\omega_2^2\omega_3^2v_2^2 + 6\omega_1^3\omega_2^2\omega_3cs^4 + 42\omega_1^2\omega_2^2\omega_3cs^2v_2^2 - 12\omega_1^2\omega_2^2\omega_3^2cs^2$$

$$C_{18} = 2\omega_2\omega_3 + 9\omega_1\omega_3v_1^2 + 6\omega_1\omega_3cs^2 - 6\omega_1\omega_2v_2^2 + \omega_1\omega_3v_2^2 - 18\omega_1\omega_2cs^2 - 4\omega_1\omega_3 + 6\omega_1\omega_2\omega_3cs^2 + 2\omega_1\omega_2\omega_3v_2^2 - 9\omega_2\omega_3v_1^2 + \omega_2\omega_3v_2^2 + 6\omega_1\omega_2 - 2\omega_1\omega_2\omega_3$$

$$C_{19} = -6\omega_1\omega_2^2\omega_3 + 6\omega_1^2\omega_2\omega_3cs^4 - 171\omega_1^3\omega_2v_2^4 + 63\omega_1^2\omega_2\omega_3cs^2v_2^2 - 2\omega_1^3\omega_2^2\omega_3cs^4 - 180\omega_1\omega_2^2\omega_3v_2^2 - 24\omega_1^2\omega_2^2cs^2 - 12\omega_1^2\omega_2^2\omega_3cs^2 + 135\omega_1^2\omega_2\omega_3v_2^4 + 12\omega_1^3\omega_2^2cs^4 - 90\omega_1^2\omega_2^2cs^2v_2^2 + 6\omega_1^2\omega_2\omega_3 - 42\omega_2^2\omega_3cs^4 + 18\omega_1^3\omega_2cs^2v_2^2 - 45\omega_1\omega_2^2\omega_3cs^2 + 30\omega_1^2\omega_2^2cs^2 - 6\omega_1^3\omega_2cs^2 + 36\omega_1^2\omega_2^2cs^2v_2^2 - 324\omega_1^2\omega_2\omega_3^2v_2^4 + 153\omega_1^3\omega_2^2\omega_3cs^2v_2^2 + 207\omega_1^3\omega_2cs^2v_2^2 - 99\omega_1^3\omega_2v_2^2 - 81\omega_1^3\omega_2^2\omega_3v_2^2 - 30\omega_1\omega_2^2\omega_3cs^4 + 12\omega_1\omega_2^2\omega_3 - 138\omega_1^2\omega_2^2\omega_3v_2^4 - 207\omega_1^2\omega_3cs^2v_2^2 + 18\omega_1\omega_2^2cs^4 - 33\omega_1^3\omega_2\omega_3cs^4 + 3\omega_1^3\omega_2^2\omega_3cs^2v_1^2 - \omega_1^3\omega_2^2\omega_3v_1^2 - 24\omega_1^3\omega_2cs^2 - 24\omega_1^3\omega_2\omega_3cs^2 - 135\omega_1\omega_2^2\omega_3v_2^4 + 7\omega_1^3\omega_2^2\omega_3 - \omega_1^2\omega_2^2\omega_3v_1^4 - 25\omega_1^2\omega_2^2\omega_3cs^4 - 18\omega_1^3\omega_2^2cs^4 + 36\omega_1\omega_2^2\omega_3cs^2 + 138\omega_1^2\omega_2^2\omega_3v_2^4 + 171\omega_1^3\omega_2v_2^4 + 81\omega_1^2\omega_2^2\omega_3v_2^2 - 18\omega_1\omega_2^2cs^2 + 72\omega_1^2\omega_2^2cs^2v_2^2 - 30\omega_1^2\omega_2^2cs^4 + 36\omega_1^2\omega_2^2\omega_3cs^2v_2^2 - 7\omega_1^2\omega_2^2\omega_3 + 6\omega_1^3\omega_2cs^4 + 180\omega_1^2\omega_2\omega_3v_2^2 - 12\omega_1^2\omega_2\omega_3 + \omega_1^2\omega_2^2\omega_3v_1^4 + 63\omega_1\omega_2^2\omega_3v_2^2 + 17\omega_1^3\omega_2^2\omega_3cs^4 + 18\omega_1^3\omega_2cs^4 - 135\omega_1\omega_2^2\omega_3cs^2v_2^2 + \omega_1^2\omega_2^2\omega_3v_1^2 + 24\omega_1^2\omega_2^2\omega_3cs^2 + 18\omega_1^3\omega_2^2cs^2 + 45\omega_1^3\omega_2\omega_3cs^2 - 6\omega_1^2\omega_3 + 54\omega_1\omega_2^2cs^2v_2^2 + 324\omega_1\omega_2^2\omega_3v_1^4 + 24\omega_1^2\omega_2^2cs^4 + 351\omega_1\omega_2^2\omega_3cs^2v_2^2 + 12\omega_1^2\omega_2^2\omega_3cs^4 - 12\omega_1^2\omega_2\omega_3cs^2v_2^2 - 3\omega_1^2\omega_2^2\omega_3cs^2v_1^2 + 99\omega_1^2\omega_2^2v_2^2 + 69\omega_1\omega_2^2\omega_3cs^4 - 63\omega_1^2\omega_2\omega_3v_2^2 - 12\omega_1^3\omega_2^2cs^2 + 6\omega_1^3\omega_2 - 153\omega_1^2\omega_2^2\omega_3cs^2v_2^2 - 54\omega_1^3\omega_2^2cs^2v_2^2 + 24\omega_2^2\omega_3cs^2$$

$$C_{20} = \omega_2cs^4 + 12\omega_2cs^2v_2^2 + 3\omega_1\omega_2v_2^2 + 3\omega_2v_2^4 + \omega_1\omega_2cs^2 + \omega_1cs^4 + 3\omega_1v_2^4 + 12\omega_1cs^2v_2^2 - \omega_1cs^2 - 3\omega_1v_2^2 - 12\omega_1\omega_2cs^2v_2^2 - 3\omega_1\omega_2v_2^4 - \omega_2cs^2 - \omega_1\omega_2cs^4 - 3\omega_2v_2^2$$

$$C_{21} = 12\omega_1^2\omega_3^2v_2^2 - 36\omega_1^3v_2^2 - 24\omega_1\omega_3cs^2 - 24\omega_1^2\omega_3cs^4 + 144\omega_1^2\omega_3cs^2v_2^2 - 8\omega_1^2\omega_3^2cs^2 + 72\omega_1^2\omega_3v_2^4 - 48\omega_1\omega_3^2cs^4 - 3\omega_1^3\omega_3cs^4 - 216\omega_1cs^2v_2^2 - 72\omega_1^3\omega_3cs^2v_2^2 + 72\omega_1^2v_2^2 + 30\omega_1^3\omega_3v_2^2 + 3\omega_1^3\omega_3^2v_2^4 - 6\omega_1^3\omega_3cs^2 - 36\omega_1\omega_3^2cs^2v_2^2 - 72\omega_1^2v_2^4 + 6\omega_1^3\omega_3^2cs^2 + 6\omega_1^3\omega_3cs^4 - 3\omega_1^3\omega_3^2v_2^2 + 72\omega_1\omega_3cs^2v_2^2 + 24\omega_1^2\omega_3cs^2 - 12\omega_1^2\omega_3^2v_2^4 + 36\omega_1^3v_2^4 + 24\omega_1^2\omega_3cs^4 + 24\omega_3^2cs^4 - 72\omega_1^2\omega_3v_2^2 + 12\omega_1\omega_3cs^2 - 12\omega_1^2\omega_3^2cs^2v_2^2 + 24\omega_1^2\omega_3^2cs^4 + 108\omega_1^3cs^2v_2^2$$

2.7.4 Conservation of momentum: ρv_2



attached text file: output_d2q9_nse_culbm2_symbolic_pde_02.txt

$$v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{\delta_1 v_1 v_2}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\delta_1 \rho v_2}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_1 \rho v_1}{\delta_t} \frac{\partial v_2}{\partial x_1} + (cs^2 + v_2^2) \frac{\delta_1}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{2\delta_1 \rho v_2}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_1) \frac{cs^2 \delta_1^2}{2\omega_1 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + (-\omega_1 + 3\omega_1 cs^2 - 5cs^2 \omega_2 + \omega_2 + 3\omega_1 v_1^2 + \omega_1 cs^2 \omega_2 - 3\omega_2 v_1^2) \frac{\delta_1^2}{2\omega_1 \omega_2 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (\omega_1 - \omega_2) \frac{3\delta_1^2 \rho v_1}{\omega_1 \omega_2 \delta_t} \frac{\partial v_1}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (-\omega_1 + \omega_1 cs^2 - cs^2 \omega_2 + \omega_2 + 3\omega_1 v_1^2 - 3\omega_2 v_1^2) \frac{\delta_1^2}{2\omega_1 \omega_2 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_1} + (-3\omega_1 \omega_2 v_2^2 - \omega_1 + 2\omega_1 cs^2 + 2cs^2 \omega_2 - \omega_2 - 2\omega_1 cs^2 \omega_2 + 3\omega_1 v_2^2 + \omega_1 \omega_2 + 3\omega_2 v_2^2) \frac{\delta_1^2}{\omega_1 \omega_2 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (\omega_1 + \omega_2 - \omega_1 \omega_2) \frac{3\delta_1^2 \rho v_2}{\omega_1 \omega_2 \delta_t} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + (-2 + \omega_1) \frac{cs^2 \delta_1^2 \rho}{2\omega_1 \delta_t} \frac{\partial^2 v_2}{\partial x_1^2} + (-\omega_1 + 3\omega_1 cs^2 - 3cs^2 \omega_2 + \omega_2 + \omega_1 v_1^2 - \omega_2 v_1^2) \frac{\delta_1^2 v_1}{2\omega_1 \omega_2 \delta_t} \frac{\partial^2 \rho}{\partial x_1 \partial x_2} +$$

$$\begin{aligned}
& (-\omega_1 + \omega_1 cs^2 - 3cs^2\omega_2 + \omega_2 + 3\omega_1 v_1^2 + \omega_1 cs^2\omega_2 - 3\omega_2 v_1^2) \frac{\delta_t^2 \rho}{2\omega_1 \omega_2 \delta_t} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + \\
& (-\omega_1 \omega_2 v_2^2 - \omega_1 + 3\omega_1 cs^2 + 3cs^2\omega_2 - \omega_2 - 3\omega_1 cs^2\omega_2 + \omega_1 v_2^2 + \omega_1 \omega_2 + \omega_2 v_2^2) \frac{\delta_t^2 v_2}{2\omega_1 \omega_2 \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + \\
& (-3\omega_1 \omega_2 v_2^2 - \omega_1 + \omega_1 cs^2 + cs^2\omega_2 - \omega_2 - \omega_1 cs^2\omega_2 + 3\omega_1 v_2^2 + \omega_1 \omega_2 + 3\omega_2 v_2^2) \frac{\delta_t^2 \rho}{2\omega_1 \omega_2 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + \\
& (-1 + 3cs^2 + v_1^2) \frac{\delta_t^3 v_1 v_2}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + cs^2 + 3v_1^2) \frac{\delta_t^3 \rho v_2}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + C_1 \frac{\delta_t^3 \rho v_1}{6\omega_1 \omega_3 \delta_t} \frac{\partial^3 v_2}{\partial x_1^3} + C_2 \frac{\delta_t^3}{12\omega_1^2 \omega_2^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + \\
& C_3 \frac{\delta_t^3 \rho v_1}{4\omega_1^2 \omega_2^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} - \frac{cs^2 \delta_t^3 \rho v_2}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + C_4 \frac{3\delta_t^3 v_1 v_2}{4\omega_1^2 \omega_2^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + C_5 \frac{\delta_t^3 \rho v_2}{12\omega_1^2 \omega_2^2 \omega_3 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_6 \frac{\delta_t^3 \rho v_1}{4\omega_1^2 \omega_2^2 \omega_3 \delta_t} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + \\
& C_7 \frac{\delta_t^3}{12\omega_1^2 \omega_2^2 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + C_8 \frac{\delta_t^3 \rho v_2}{12\omega_1^2 \omega_2^2 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_9 \frac{\delta_t^4 v_2}{24\omega_1 \omega_2 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& (-2\omega_1 + 3\omega_1 cs^2 - 5\omega_1 \omega_2 v_1^2 + 3cs^2\omega_2 - 2\omega_2 + 5\omega_1 v_1^2 - 3\omega_1 cs^2\omega_2 + 5\omega_2 v_1^2 + 2\omega_1 \omega_2) \frac{\delta_t^4 \rho v_1 v_2}{12\omega_1 \omega_2 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + \\
& C_{10} \frac{\delta_t^4 \rho}{24\omega_1^3 \omega_2^3 \delta_t} \frac{\partial^4 v_2}{\partial x_1^4} + C_{11} \frac{\delta_t^4 v_1}{24\omega_1^3 \omega_2^3 \omega_3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{12} \frac{\delta_t^4 \rho}{24\omega_1^3 \omega_2^3 \omega_3 \delta_t} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + C_{13} \frac{\delta_t^4 \rho v_1 v_2}{24\omega_1 \omega_2 \omega_3 \delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& C_{14} \frac{\delta_t^4 v_2}{24\omega_1^3 \omega_2^3 \omega_3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_{15} \frac{\delta_t^4 \rho v_1 v_2}{24\omega_1^3 \omega_2^3 \omega_3 \delta_t} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{16} \frac{\delta_t^4 \rho}{24\omega_1^3 \omega_2^3 \omega_3 \delta_t} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_{17} \frac{\delta_t^4 v_1}{24\omega_1^3 \omega_2^3 \omega_3 \delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + \\
& C_{18} \frac{\delta_t^4 \rho}{24\omega_1^3 \omega_2^3 \omega_3 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{19} \frac{\delta_t^4 \rho v_1 v_2}{24\omega_1^3 \omega_2^3 \omega_3 \delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + C_{20} \frac{\delta_t^4 v_2}{24\omega_1^3 \omega_2^3 \omega_3 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{21} \frac{\delta_t^4 \rho}{24\omega_1^3 \omega_2^3 \omega_3 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= 6 - \omega_1 \omega_3 v_1^2 - 3\omega_1 + 9\omega_1 cs^2 - 3\omega_1 cs^2 \omega_3 + \omega_1 \omega_3 - 18cs^2 + 3\omega_1 v_1^2 - 3\omega_3 + 9cs^2 \omega_3 - 6v_1^2 + 3\omega_3 v_1^2 \\
C_2 &= 6\omega_1^2 cs^4 + 9\omega_1 \omega_2^2 v_1^4 + 45\omega_1^2 cs^2 v_1^2 - 9\omega_1^2 \omega_2 v_1^4 + 45\omega_1 cs^2 \omega_2^2 v_1^2 + 6cs^2 \omega_2^2 + 9\omega_2^2 v_1^2 - 45\omega_1^2 cs^2 \omega_2 v_1^2 + 6\omega_2^2 cs^2 \omega_2 - 9\omega_1^2 v_1^2 + 30\omega_1 cs^4 \omega_2^2 - \\
& 30cs^4 \omega_2^2 + 9\omega_1^2 v_1^4 - 2\omega_1^2 cs^4 \omega_2^2 - 6\omega_1^2 cs^4 \omega_2 - 6\omega_1^2 cs^2 - 9\omega_1 \omega_2^2 v_1^2 - 6\omega_1 cs^2 \omega_2^2 - 9\omega_2^2 v_1^4 - 45cs^2 \omega_2^2 v_1^2 + 9\omega_1^2 \omega_2 v_1^2 \\
C_3 &= -9cs^2 \omega_2^2 - 11\omega_2^2 v_1^2 + 5\omega_1^2 \omega_2 - 9\omega_1^2 cs^2 \omega_2 - 5\omega_1^2 + 11\omega_1^2 v_1^2 - 5\omega_1 \omega_2^2 + 5\omega_2^2 + 9\omega_1^2 cs^2 + 11\omega_1 \omega_2^2 v_1^2 + 9\omega_1 cs^2 \omega_2^2 - 11\omega_1^2 \omega_2 v_1^2 \\
C_4 &= \omega_2^2 v_2^2 - 2\omega_1 \omega_2 v_2^2 - \omega_2^2 v_1^2 + \omega_1^2 \omega_2 + \omega_1^2 v_2^2 - 3\omega_1^2 cs^2 \omega_2 - 2\omega_1^2 + \omega_1^2 v_1^2 - 6\omega_1 cs^2 \omega_2 - \omega_1 \omega_2^2 + 6\omega_1^2 cs^2 + \omega_1 \omega_2^2 v_1^2 + 3\omega_1 cs^2 \omega_2^2 + 2\omega_1 \omega_2 - \omega_1^2 \omega_2 v_1^2 \\
C_5 &= -12\omega_1 \omega_2^2 \omega_3 + 36\omega_1^2 cs^2 \omega_2^2 + 12\omega_1^2 \omega_2 \omega_3 + 9cs^2 \omega_2^2 \omega_3 + 6\omega_1^2 \omega_2 + 12\omega_1^2 \omega_2^2 v_2^2 - 3\omega_1^2 \omega_2^2 \omega_3 v_2^2 - 12\omega_1^2 \omega_2^2 - 27\omega_2^2 \omega_3 v_1^2 - 11\omega_1^2 cs^2 \omega_2^2 \omega_3 - \\
& 18\omega_1^2 cs^2 \omega_2 + 6\omega_2^2 \omega_3 v_2^2 - 18\omega_1^2 cs^2 \omega_2 \omega_3 + 27\omega_1 \omega_2^2 \omega_3 v_1^2 - 15\omega_1^2 \omega_3 + 3\omega_1 \omega_2^2 \omega_3 v_2^2 + 18\omega_1 \omega_2^2 + 27\omega_1^2 \omega_3 v_1^2 + 3\omega_1^2 \omega_2^2 \omega_3 - 27\omega_1^2 \omega_2 \omega_3 v_1^2 - \\
& 54\omega_1 cs^2 \omega_2^2 - 6\omega_1^2 \omega_2 v_2^2 + 6\omega_1^2 \omega_3 v_2^2 + 3\omega_2^2 \omega_3 - 18\omega_1 \omega_2^2 v_2^2 - 3\omega_1^2 \omega_2 \omega_3 v_2^2 + 27\omega_1^2 cs^2 \omega_3 + 18\omega_1 cs^2 \omega_2^2 \omega_3 \\
C_6 &= -\omega_1 \omega_2^2 \omega_3 - 6\omega_1 cs^2 \omega_2 \omega_3 + \omega_1^2 \omega_2 \omega_3 - 3cs^2 \omega_2^2 \omega_3 + 2\omega_2^2 \omega_2 - 2\omega_2^2 \omega_3 v_1^2 - 6\omega_1^2 cs^2 \omega_2 - 18\omega_1 \omega_2 \omega_3 v_2^2 + 9\omega_2^2 \omega_3 v_2^2 - 3\omega_1^2 cs^2 \omega_2 \omega_3 + \omega_1 \omega_2^2 \omega_3 v_1^2 - \\
& 5\omega_1^2 \omega_3 - 2\omega_1 \omega_2^2 + 2\omega_1^2 \omega_3 v_1^2 - \omega_1^2 \omega_2 \omega_3 v_1^2 + 2\omega_1 \omega_2^2 v_1^2 + 6\omega_1 cs^2 \omega_2^2 + 9\omega_1^2 \omega_3 v_2^2 + 6\omega_1 \omega_2 \omega_3 - \omega_2^2 \omega_3 + 9\omega_1^2 cs^2 \omega_3 - 2\omega_1^2 \omega_2 v_1^2 + 3\omega_1 cs^2 \omega_2^2 \omega_3 \\
C_7 &= 6\omega_1^2 cs^4 + 45\omega_2^2 cs^2 v_2^2 - 18\omega_1^2 \omega_2 v_2^4 - 72\omega_1 cs^2 \omega_2^2 v_2^2 - 9\omega_2^2 v_2^2 - 18\omega_1 \omega_2 v_2^2 - \omega_1^2 cs^2 \omega_2^2 - 18\omega_1 \omega_2^2 v_2^4 - 6cs^2 \omega_2^2 - 7\omega_2^2 \omega_2^2 v_2^2 - 9\omega_2^2 v_2^2 + \\
& 6\omega_1^2 cs^2 \omega_2 - 72\omega_1^2 cs^2 \omega_2 v_2^2 - 6\omega_1 cs^4 \omega_2^2 + 9\omega_2^2 v_2^4 + 6cs^4 \omega_2^2 + 24\omega_1^2 cs^2 \omega_2^2 v_2^2 + \omega_1^2 cs^4 \omega_2^2 + 45cs^2 \omega_2^2 v_2^2 + 9\omega_2^2 v_2^4 + 18\omega_1 \omega_2 v_2^4 - 6\omega_1^2 cs^4 \omega_2 - \\
& 6\omega_1^2 cs^2 + 6\omega_1 cs^4 \omega_2^2 + 18\omega_1^2 \omega_2 v_2^2 + 54\omega_1 cs^2 \omega_2 v_2^2 + 7\omega_1^2 \omega_2^2 v_2^2 + 18\omega_1 \omega_2^2 v_2^2 \\
C_8 &= 33\omega_2^2 v_2^2 + 54\omega_1 \omega_2 v_2^2 + 10\omega_1^2 cs^2 \omega_2^2 + 27cs^2 \omega_2^2 + 24\omega_1^2 \omega_2 + 22\omega_1^2 \omega_2^2 v_2^2 - 8\omega_1^2 \omega_2^2 + 33\omega_1^2 v_2^2 - 36\omega_1^2 cs^2 \omega_2 - 15\omega_1^2 + 18\omega_1 cs^2 \omega_2 + 24\omega_1 \omega_2^2 - \\
& 15\omega_2^2 + 27\omega_1^2 cs^2 - 36\omega_1 cs^2 \omega_2^2 - 18\omega_1 \omega_2 - 60\omega_1^2 \omega_2 v_2^2 - 60\omega_1 \omega_2^2 v_2^2 \\
C_9 &= 3\omega_2 v_1^4 + 12cs^2 \omega_2 v_1^2 - \omega_1 cs^4 \omega_2 - \omega_1 cs^2 + 3\omega_1 \omega_2 v_1^2 - cs^2 \omega_2 + 3\omega_1 v_1^4 - 3\omega_1 v_1^2 + \omega_1 cs^2 \omega_2 - 12\omega_1 cs^2 \omega_2 v_1^2 - 3\omega_2 v_1^2 + cs^4 \omega_2 + \omega_1 cs^4 - \\
& 3\omega_1 \omega_2 v_1^4 + 12\omega_1 cs^2 v_1^2 \\
C_{10} &= -72\omega_1^3 cs^2 \omega_3 v_1^2 + 24cs^4 \omega_3^2 - 36\omega_1 cs^2 \omega_3^2 v_1^2 + 72\omega_1^2 \omega_3 v_1^4 + 12\omega_1^2 \omega_3^2 v_1^2 + 24\omega_1^2 cs^4 \omega_3^2 - 216\omega_1^2 cs^2 v_1^2 - 36\omega_1^3 v_1^2 + \omega_1^3 cs^2 \omega_3^2 - 24\omega_1 cs^2 \omega_3 - 6\omega_1^3 cs^2 \omega_3 + \\
& 3\omega_1^3 \omega_3^2 v_1^4 + 12\omega_1 cs^2 \omega_3^2 - 24\omega_1^2 cs^4 \omega_3 + 144\omega_1^2 cs^2 \omega_3 v_1^2 + 30\omega_1^3 \omega_3 v_1^2 + 72\omega_2^2 v_1^2 - 8\omega_1^2 cs^2 \omega_3^2 - 12\omega_1^2 cs^2 \omega_3^2 v_1^2 + 108\omega_1^3 cs^2 v_1^2 - 3\omega_1^3 cs^4 \omega_3^2 - 3\omega_1^3 \omega_3^2 v_1^2 + \\
& 24\omega_1 cs^4 \omega_3 - 30\omega_1^3 \omega_3 v_1^4 - 72\omega_1^2 v_1^4 - 72\omega_1^2 \omega_3 v_1^2 + 6\omega_1^3 cs^4 \omega_3 + 6\omega_1^3 cs^2 \omega_3^2 v_1^2 - 48\omega_1 cs^4 \omega_3^2 - 12\omega_1^2 \omega_3^2 v_1^4 + 72\omega_1 cs^2 \omega_3 v_1^2 + 24\omega_1^2 cs^2 \omega_3 + 36\omega_1^3 v_1^4 \\
C_{11} &= 90\omega_1 \omega_2^2 \omega_3^2 v_1^4 - 138\omega_1^2 cs^4 \omega_3^2 \omega_3 + 18\omega_1^3 cs^2 \omega_2^2 \omega_3 + 7\omega_1^3 \omega_2^2 \omega_3^2 - 12\omega_1^2 cs^2 \omega_2^2 \omega_3^2 - 18\omega_1^3 cs^4 \omega_2^2 \omega_3 - 219\omega_1^2 cs^2 \omega_2^2 \omega_3^2 v_1^2 + 51\omega_2^3 \omega_3^2 v_1^2 + \\
& 72cs^2 \omega_2^3 \omega_3^2 - 2\omega_1^3 cs^2 \omega_2^3 \omega_3^2 + 72\omega_1^2 cs^4 \omega_2^2 \omega_3 + 36\omega_1^3 cs^4 \omega_2^2 - 6\omega_1^3 cs^2 \omega_2^2 \omega_3 v_1^2 + 489\omega_1 cs^2 \omega_2^2 \omega_3^2 v_1^2 + 12\omega_1 \omega_2^2 \omega_3^2 - 72\omega_1^3 cs^2 \omega_2^2 - 51\omega_1^2 \omega_2 \omega_3^2 v_1^2 + \\
& 24\omega_1^2 cs^2 \omega_2^2 - 9\omega_1^2 cs^2 \omega_2^2 \omega_3^2 v_2^2 + 72\omega_1^3 cs^4 \omega_2^2 \omega_3^2 - 42\omega_1^2 cs^2 \omega_2^2 \omega_3 + 39\omega_1^3 \omega_2^2 \omega_3^2 v_1^4 + 45\omega_1^3 \omega_2^2 \omega_3^2 v_1^2 + 24\omega_1^2 cs^2 \omega_2^2 \omega_3 v_1^2 + 46\omega_1^2 \omega_2^2 \omega_3^2 v_1^2 + \\
& 81\omega_1^2 cs^2 \omega_2^2 \omega_3^2 - 54\omega_1^3 cs^4 \omega_2^2 \omega_3 + 36\omega_1^2 cs^4 \omega_2^2 \omega_3^2 + 6\omega_1^3 cs^2 \omega_2^2 \omega_3 + 9\omega_1^3 cs^2 \omega_2^2 \omega_3^2 v_2^2 - 90cs^4 \omega_2^2 \omega_3^2 + 6\omega_2^2 \omega_2 \omega_3^2 + 102\omega_1^3 \omega_2 \omega_3^2 v_1^2 + 6\omega_1^3 cs^2 \omega_2 \omega_3 v_1^2 + \\
& 141\omega_1^2 cs^2 \omega_2 \omega_3^2 v_1^2 + 51\omega_1 \omega_2^2 \omega_3^2 v_1^2 + 6\omega_1^3 cs^4 \omega_2^2 \omega_3^2 - 24\omega_1^2 cs^2 \omega_2^2 \omega_3 + 3\omega_1^3 \omega_2^2 \omega_3^2 v_2^2 + 3\omega_2^2 \omega_2^2 \omega_3^2 v_2^2 - 6\omega_1 \omega_2^2 \omega_3^2 + 197\omega_1^3 cs^2 \omega_2^2 \omega_3^2 v_1^2 - \\
& 59\omega_1^3 cs^2 \omega_2^2 \omega_3^2 + 126\omega_2^2 cs^4 \omega_2^2 \omega_3 - 12\omega_1^3 cs^2 \omega_2^2 \omega_3^2 + 12\omega_1^3 cs^2 \omega_2^2 \omega_3^2 v_1^2 + 6\omega_2^3 \omega_3^2 - 90\omega_1^3 \omega_2 \omega_3^2 v_1^4 + 48\omega_1 cs^2 \omega_2^2 \omega_3^2 + 90\omega_1^3 cs^4 \omega_2^2 + 225\omega_1 cs^4 \omega_2^2 \omega_3^2 - \\
& 18\omega_1^3 cs^2 \omega_2^2 \omega_3 v_1^2 - 72\omega_1^2 cs^4 \omega_2^2 - 51\omega_1^3 \omega_2^2 v_1^2 - 46\omega_1^3 \omega_2^2 \omega_3^2 v_1^2 - 39\omega_2^2 \omega_2^2 \omega_3^2 v_1^4 + 123\omega_1^3 cs^2 \omega_2 \omega_3^2 - 165\omega_1 cs^2 \omega_2^2 \omega_3^2 v_1^2 + 30\omega_1 cs^2 \omega_2^2 \omega_3 + \\
& 18\omega_1^3 cs^4 \omega_2 \omega_3 + 12\omega_1^2 cs^2 \omega_2^2 \omega_3^2 v_1^2 - 45\omega_1 \omega_2^2 \omega_3^2 v_1^4 - 3\omega_1^3 \omega_2^2 \omega_3^2 v_2^2 + 18\omega_1^2 cs^4 \omega_2 \omega_3^2 - 465\omega_1^3 cs^2 \omega_2 \omega_3^2 v_1^2 - 261cs^2 \omega_2^2 \omega_3^2 v_1^2 - 3\omega_1^2 \omega_2^2 \omega_3^2 v_2^2 + \\
& 2\omega_1^3 cs^2 \omega_2^2 \omega_3^2 v_1^2 - 90\omega_1 cs^4 \omega_2^2 \omega_3^2 - 30\omega_1 cs^2 \omega_2^2 \omega_3^2 v_1^2 - 6\omega_2^3 \omega_3^2 - 45\omega_2^2 \omega_3^2 v_1^4 - 147\omega_1 cs^2 \omega_2^2 \omega_3^2 - 102\omega_1 \omega_2^2 \omega_3^2 v_1^2 - 153\omega_1^3 cs^4 \omega_2 \omega_3^2 - \\
& 90\omega_1 cs^4 \omega_2^2 \omega_3 + 261\omega_1^3 cs^2 \omega_2^2 \omega_3^2 v_1^2 - 7\omega_1^2 \omega_2^2 \omega_3^2 + 45\omega_1^2 \omega_2 \omega_3^2 v_1^4 - 12\omega_1^3 \omega_2 \omega_3^2 - 6\omega_1^3 cs^2 \omega_2 \omega_3 - 24\omega_1^2 cs^2 \omega_2^2 \omega_3^2 v_1^2 - 24\omega_1^2 cs^2 \omega_2 \omega_3^2 + 42\omega_1^2 cs^2 \omega_2^2 \omega_3 v_1^2
\end{aligned}$$

$$C_{12} = 207\omega_1^3cs^2\omega_3v_1^2 + 135\omega_1^2\omega_2\omega_3v_1^4 - 6\omega_1\omega_2^3\omega_3 - 24\omega_1^3cs^2\omega_2\omega_3 - 18\omega_1^3cs^4\omega_2^2 - 2\omega_1^3cs^4\omega_2^3\omega_3 - 24\omega_1^2cs^2\omega_2^2\omega_3 + 12\omega_1^2cs^4\omega_2^2\omega_3 + 12\omega_1^3cs^4\omega_2^3 - 42cs^4\omega_2^3\omega_3 - 171\omega_1^3\omega_3v_1^4 + 6\omega_1^2\omega_2\omega_3 - 180\omega_1\omega_2^3\omega_3v_1^2 + 30\omega_1^2cs^2\omega_2^3 + 24\omega_1^2cs^2\omega_2^3\omega_3 + 18\omega_1^3cs^2\omega_2v_1^2 - 24\omega_1^3cs^2\omega_2\omega_3 + 36\omega_1^2cs^2\omega_2^3\omega_3v_1^2 + 17\omega_1^3cs^4\omega_2^2\omega_3 - \omega_1^3\omega_2^3\omega_3v_2^2 - 135\omega_1\omega_2^3\omega_3v_1^4 + 12\omega_1\omega_2^3\omega_3 + 18\omega_1cs^4\omega_2^3 - \omega_1^2\omega_2^3\omega_3v_2^4 - 207cs^2\omega_2^3\omega_3v_1^2 - 351\omega_1^3cs^2\omega_2\omega_3v_1^2 - 12\omega_1^2cs^2\omega_2^3\omega_3 + 24cs^2\omega_2^3\omega_3 - 324\omega_1^3\omega_2\omega_3v_1^4 - 99\omega_1^3\omega_3v_1^2 - 81\omega_1^3\omega_2^3\omega_3v_1^2 + 54\omega_1cs^2\omega_2^3v_1^2 + 7\omega_1^3\omega_2^3\omega_3 - 25\omega_1^2cs^4\omega_2^3\omega_3 - 138\omega_1^2\omega_2^3\omega_3v_1^4 - 135\omega_1cs^2\omega_2^3\omega_3v_1^2 + 6\omega_1^3cs^4\omega_2 - 12\omega_1^3cs^2\omega_2^3 + 36\omega_1^3cs^2\omega_2^3v_1^2 + \omega_1^3\omega_2^3\omega_3v_2^4 - 12\omega_1^2cs^2\omega_2\omega_3 + 63\omega_1\omega_2^3\omega_3v_1^2 + 63\omega_1^2cs^2\omega_2\omega_3v_1^2 + \omega_1^2\omega_2^3\omega_3v_2^2 + 153\omega_1^3cs^2\omega_2^3\omega_3v_1^2 - 7\omega_1^2\omega_2^3\omega_3 - 30\omega_1^2cs^4\omega_2^3 - 12\omega_1^3\omega_2\omega_3 + 138\omega_1^3\omega_2^3\omega_3v_1^4 + 171\omega_1^3\omega_3v_1^4 - 45\omega_1cs^2\omega_2^3\omega_3 + 81\omega_1^2\omega_2^3\omega_3v_1^2 - 33\omega_1^3cs^4\omega_2\omega_3 + 18\omega_1^3cs^2\omega_2^3 + 3\omega_1^3cs^2\omega_2^3\omega_3v_2^2 + 72\omega_1^2cs^2\omega_2^3v_1^2 - 30\omega_1cs^4\omega_2^3\omega_3 + 24\omega_1^2cs^4\omega_2^2 - 6\omega_2^3\omega_3 + 180\omega_1^2\omega_2\omega_3v_1^2 + 6\omega_1^2cs^4\omega_2\omega_3 + 351\omega_1cs^2\omega_2^3\omega_3v_1^2 - 63\omega_1^2\omega_2\omega_3v_1^2 - 54\omega_1^3cs^2\omega_2^3v_1^2 - 3\omega_1^2cs^2\omega_2^3\omega_3v_2^2 - 6\omega_1^3cs^2\omega_2 + 324\omega_1\omega_2^3\omega_3v_1^4 + 69\omega_1cs^4\omega_2^3\omega_3 + 18\omega_1^3cs^4\omega_3 + 45\omega_1^3cs^2\omega_2\omega_3 - 90\omega_1^2cs^2\omega_2^3v_1^2 + 99\omega_2^3\omega_3v_1^2 + 6\omega_1^3\omega_3 - 18\omega_1cs^2\omega_2^3 + 36\omega_1^2cs^2\omega_2^3\omega_3 - 153\omega_1^2cs^2\omega_2^3\omega_3v_1^2$$

$$C_{13} = 2\omega_2\omega_3 + \omega_1\omega_3v_1^2 + 6\omega_1cs^2\omega_2\omega_3 + 9\omega_1\omega_3v_2^2 - 6\omega_1\omega_2v_1^2 + 6\omega_1cs^2\omega_3 - 4\omega_1\omega_3 + 2\omega_1\omega_2\omega_3v_1^2 + \omega_2\omega_3v_1^2 - 9\omega_2\omega_3v_2^2 - 18\omega_1cs^2\omega_2 + 6\omega_1\omega_2 - 2\omega_1\omega_2\omega_3$$

$$C_{14} = 54\omega_1\omega_2^3\omega_3v_1^4 - 12\omega_1^3cs^2\omega_2^3\omega_3v_2^2 - 91\omega_1^2cs^4\omega_2^3\omega_3 + 3\omega_1cs^2\omega_2^3\omega_3v_2^2 - 45\omega_1\omega_2^3\omega_3v_1^2v_2^2 - 36\omega_1^3cs^4\omega_2^2 + 6\omega_1^2\omega_2\omega_3v_2^2 - 45\omega_1^3\omega_2\omega_3v_1^2v_2^2 - 54\omega_1^2cs^2\omega_2^3\omega_3 - 36\omega_1^3cs^4\omega_2^3\omega_3 - 138\omega_1^2cs^2\omega_2^3\omega_3v_1^2 + 9\omega_2^3\omega_3v_1^2 - 2\omega_1^2cs^2\omega_2^3\omega_3 + 108\omega_1^2cs^4\omega_2^3\omega_3 + 72\omega_1^3cs^4\omega_2^3 + 189\omega_1cs^2\omega_2^3\omega_3v_1^2 - 6\omega_1\omega_2^3\omega_3 - 72\omega_1^3cs^2\omega_2^3 + 99\omega_1^2\omega_2\omega_3v_1^2 + 36\omega_1^2cs^2\omega_2^3 - 24\omega_1^2cs^2\omega_2^3\omega_3v_2^2 - 6\omega_2^3\omega_3v_2^2 + 35\omega_1^3cs^4\omega_2^3\omega_3 - 48\omega_1^2cs^2\omega_2^3\omega_3 - 6\omega_1^2cs^2\omega_2\omega_3v_2^2 + 6\omega_1\omega_2^3\omega_3v_2^2 + 27\omega_1^3\omega_2^3\omega_3v_1^4 + 54\omega_1^3\omega_2^3v_1^4 + 27\omega_1^2\omega_2^3\omega_3v_2^2 + 41\omega_1^2cs^2\omega_2^3\omega_3 + 54\omega_1^2cs^4\omega_2^3\omega_3 + 12\omega_1^3cs^2\omega_2^3\omega_3 + 90\omega_1^2cs^2\omega_2^3\omega_3v_1^2v_2^2 + 8\omega_1^3cs^2\omega_2^3\omega_3v_2^2 + 18cs^4\omega_2^3\omega_3 - 6\omega_1^2\omega_2\omega_3 + 99\omega_1^3\omega_2\omega_3v_1^2 - 297\omega_1^2cs^2\omega_2\omega_3v_1^2 - 9\omega_1\omega_2^3\omega_3v_1^2 + 10\omega_1^3cs^4\omega_2^3\omega_3 - 36\omega_1^2cs^2\omega_2^3\omega_3 - 45\omega_1\omega_2^3\omega_3v_1^2v_2^2 + 36\omega_1^2cs^2\omega_2^3\omega_3v_2^2 - 6\omega_1\omega_2^3\omega_3 + 138\omega_1^3cs^2\omega_2^3\omega_3v_2^2 + 6\omega_1^2\omega_2\omega_3v_2^2 - 25\omega_1^3cs^2\omega_2^3\omega_3 + 144\omega_1^2cs^4\omega_2^3\omega_3 - 6\omega_1^3cs^2\omega_2\omega_3v_2^2 - 24\omega_1^3cs^4\omega_2^3 + 6\omega_2^3\omega_3 - 54\omega_1^2\omega_2\omega_3v_1^4 + 18\omega_1^2cs^2\omega_2^3\omega_3v_2^2 + 45\omega_1^2\omega_2^3v_1^2v_2^2 + 48\omega_1cs^2\omega_2^3\omega_3 + 90\omega_1^3cs^4\omega_2^3 + 63\omega_1cs^4\omega_2^3\omega_3 - 108\omega_1^2cs^4\omega_2^3 - 99\omega_1^2\omega_2^3v_1^2 - 27\omega_1^2\omega_2^3\omega_3v_1^2 - 21\omega_1^3cs^2\omega_2\omega_3v_2^2 - 27\omega_1^2\omega_2^3\omega_3v_1^4 + 93\omega_1^2cs^2\omega_2\omega_3 + 45\omega_2^3\omega_3v_1^2v_2^2 + 27\omega_1cs^2\omega_2^3\omega_3v_1^2 + 18cs^2\omega_2^3\omega_3v_2^2 + 24\omega_1^2cs^2\omega_2^3\omega_3v_2^2 + 30\omega_1cs^4\omega_2\omega_3 + 12\omega_1^2cs^2\omega_2^3\omega_3 + 27\omega_1^2cs^2\omega_2^3\omega_3v_1^2 + 54\omega_1\omega_2^3\omega_3v_1^2 - 6\omega_1^2\omega_2^3v_2^2 - 18\omega_1^2cs^4\omega_2\omega_3 - 459\omega_1^3cs^2\omega_2\omega_3v_2^2 + 12\omega_1^2\omega_2^3\omega_3 - 135cs^2\omega_2^3\omega_3v_1^2 - 30\omega_1cs^2\omega_2^3\omega_3v_2^2 + 18\omega_1^3cs^2\omega_2^3\omega_3 - 90\omega_1cs^4\omega_2^3\omega_3 - 36\omega_1^2cs^2\omega_2^3v_2^2 - 45\omega_1^2\omega_2\omega_3v_1^2v_2^2 + 6\omega_2^3\omega_3 - 54\omega_2^3\omega_3v_1^4 - 39\omega_1cs^2\omega_2^3\omega_3 - 9\omega_1\omega_2^3\omega_3v_1^2 - 117\omega_1^2cs^4\omega_2\omega_3 - 12\omega_1^2\omega_2^3\omega_3v_2^2 + 48\omega_1^2cs^2\omega_2^3\omega_3v_2^2 + 2\omega_1^2cs^2\omega_2^3\omega_3v_2^2 - 90\omega_1cs^4\omega_2^3\omega_3 + 405\omega_1^3cs^2\omega_2^3v_1^2 - 54\omega_1^2\omega_2\omega_3v_1^4 - 6\omega_1^3\omega_2\omega_3 + 6\omega_2^3cs^2\omega_2\omega_3 - 30\omega_1cs^2\omega_2^3\omega_3v_2^2 + 24\omega_1^2cs^2\omega_2\omega_3 + 6\omega_1\omega_2^3\omega_3v_2^2 - 90\omega_1^2\omega_2^3\omega_3v_1^2 - 12\omega_1^3cs^2\omega_2^3v_2^2$$

$$C_{15} = -18\omega_1\omega_2^3\omega_3 + 84\omega_1^3cs^2\omega_2^3\omega_3 - 27\omega_1\omega_2^3\omega_3v_2^2 + 108\omega_1^2cs^2\omega_2^2 + 90\omega_1^2\omega_2\omega_3 + 216\omega_1\omega_2^3\omega_3v_1^2 + 36\omega_1^2\omega_2^3v_2^2 - 84\omega_1^2cs^2\omega_2^3\omega_3 + 54\omega_1^2\omega_2^3\omega_3v_2^2 + 270\omega_1^3cs^2\omega_3 - 36\omega_1^2\omega_2^3 + 36\omega_1^2\omega_3v_2^2 - 81\omega_1\omega_2^3\omega_3 + 162\omega_1^2cs^2\omega_2^3\omega_3 + 18\omega_1^2\omega_2 - 54cs^2\omega_2^3\omega_3 - 18\omega_1^2\omega_2v_2^2 + 198\omega_1^3\omega_3v_1^2 + 100\omega_1^2\omega_2^3\omega_3v_1^2 - 46\omega_1^3\omega_2^3\omega_3 - 162\omega_1^2cs^2\omega_2\omega_3 + 162\omega_1\omega_2^3\omega_3v_1^2 + 18\omega_1\omega_2^3 + 46\omega_1^2\omega_2^3\omega_3 - 27\omega_1^2\omega_2\omega_3v_2^2 + 135\omega_1^2\omega_2\omega_3 - 36\omega_1\omega_2^3\omega_3v_2^2 + 135\omega_1cs^2\omega_2^3\omega_3 - 100\omega_1^2\omega_2^3\omega_3v_1^2 - 18\omega_1\omega_2^3v_2^2 + 54\omega_2^3\omega_3 - 216\omega_1^2cs^2\omega_2\omega_3v_1^2 - 54\omega_1^2\omega_2\omega_3 - 162\omega_1^2\omega_2\omega_3v_1^2 + 36\omega_2^3\omega_3v_2^2 - 54\omega_1^2cs^2\omega_2 - 297\omega_1^3cs^2\omega_2\omega_3 - 36\omega_1^2\omega_2\omega_3v_2^2 - 198\omega_2^3\omega_3v_1^2 - 126\omega_1^3\omega_3 - 54\omega_1cs^2\omega_2^3 - 54\omega_1cs^2\omega_2^3\omega_3$$

$$C_{16} = 45\omega_1\omega_2^3\omega_3v_1^4 + 72\omega_1^3cs^2\omega_2^3\omega_3v_2^2 - 10\omega_1^2cs^4\omega_2^3\omega_3 + 36\omega_1^3cs^2\omega_2^3\omega_3 + 9\omega_1cs^2\omega_2^3\omega_3v_2^2 - 135\omega_1\omega_2^3\omega_3v_1^2v_2^2 + 18\omega_1^2\omega_2\omega_3v_2^2 - 135\omega_1^2\omega_2\omega_3v_1^2v_2^2 - 42\omega_1^2cs^2\omega_2^3\omega_3 + 24\omega_1^3cs^4\omega_2^3\omega_3 - 30\omega_1^2cs^2\omega_2^3\omega_3v_1^2 - 9\omega_2^3\omega_3v_1^2 + 60\omega_1^2cs^4\omega_2^3\omega_3 + 108\omega_1cs^2\omega_2^3\omega_3v_1^2 - 6\omega_1\omega_2^3\omega_3 - 24\omega_1^3cs^2\omega_2^3 - 18\omega_1\omega_2^3\omega_3v_1^2 + 45\omega_1^2\omega_2\omega_3v_1^2 - 6\omega_1^2cs^2\omega_2^3\omega_3v_2^2 - 18\omega_2^3\omega_3v_2^2 + 2\omega_1^2cs^4\omega_2^3\omega_3 + 60\omega_1^2cs^2\omega_2^3\omega_3 - 54\omega_1^2cs^2\omega_2\omega_3v_2^2 + 18\omega_1\omega_2^3\omega_3v_2^2 - 36\omega_1^2\omega_2^3v_1^2 + 9\omega_1^2\omega_2^3\omega_3v_1^2 + 36\omega_1^2\omega_2^3v_1^2 + 36\omega_1^2cs^2\omega_2^3\omega_3v_1^2 + 9\omega_1^2\omega_2^3\omega_3v_1^2 + 2\omega_1^2cs^2\omega_2^3\omega_3 - 36\omega_1^3cs^4\omega_2^3\omega_3 + 30\omega_1^2cs^4\omega_2^3\omega_3 - 24\omega_1^3cs^2\omega_2^3\omega_3 + 270\omega_1^2\omega_2^3\omega_3v_1^2v_2^2 + 6\omega_1^3cs^2\omega_2^3\omega_3v_2^2 - 30cs^4\omega_2^3\omega_3 - 6\omega_1^2\omega_2\omega_3 + 90\omega_1^3\omega_2\omega_3v_1^2 - 54\omega_1^3cs^2\omega_2\omega_3v_1^2 - 81\omega_1^2cs^2\omega_2\omega_3v_1^2 + 45\omega_1\omega_2^3\omega_3v_1^2 - 2\omega_1^2cs^4\omega_2^3\omega_3 - 60\omega_1^2cs^2\omega_2^3\omega_3 + \omega_1^2\omega_2^3\omega_3v_2^4 - 135\omega_1\omega_2^3\omega_3v_1^2v_2^2 + 144\omega_1^2cs^2\omega_2^3\omega_3v_2^2 - 18\omega_1^2\omega_2\omega_3v_2^2 + \omega_1^2\omega_2^3\omega_3v_2^2 - 6\omega_1\omega_2^3\omega_3v_2^2 - 30\omega_1^2cs^2\omega_2^3\omega_3v_1^2 + 30\omega_1^2cs^2\omega_2^3\omega_3v_1^2 + 18\omega_1^2\omega_2\omega_3v_2^2 - 36\omega_1^2\omega_2^3v_1^4 - 2\omega_1^2cs^2\omega_2^3\omega_3 - 60\omega_1^2cs^4\omega_2^3\omega_3 - 54\omega_1^2\omega_3\omega_3v_1^4 + 18\omega_1^3cs^2\omega_2\omega_3v_2^2 + 6\omega_1^3\omega_3 - 45\omega_1^2\omega_2\omega_3v_1^4 + 72\omega_1^2cs^2\omega_2^3\omega_3v_2^2 + 135\omega_1^3\omega_2^3v_1^2v_2^2 + 36\omega_1cs^2\omega_2^3\omega_3 + 18\omega_1^3cs^4\omega_2^3 + 45\omega_1cs^4\omega_2^3\omega_3 + 144\omega_1^3cs^2\omega_2^3\omega_3v_1^2 + 36\omega_1^2\omega_2^3v_1^4 - 81\omega_1^3\omega_2^3v_1^2 - 9\omega_1^2\omega_2^3\omega_3v_2^2 - 9\omega_1^2cs^2\omega_2\omega_3v_2^2 - 9\omega_1^2\omega_2^3\omega_3v_1^4 + 21\omega_1^3cs^2\omega_2\omega_3 + 135\omega_1^3\omega_2^3v_1^2v_2^2 - 45\omega_1cs^2\omega_2^3\omega_3v_1^2 - 18cs^2\omega_2^3\omega_3v_2^2 + 54\omega_1^2\omega_2\omega_3v_1^4 - 30\omega_1cs^2\omega_2^3\omega_3 + 36\omega_1^3\omega_2^3v_1^2 + 54\omega_1^2\omega_2^3\omega_3v_1^2 + 6\omega_1^3cs^4\omega_2\omega_3 + 18\omega_1^2cs^2\omega_2^3v_2^2 - 90\omega_1^3cs^2\omega_2^3\omega_3v_2^2 - 18\omega_1^3\omega_2^3v_2^2 - 30\omega_1^2cs^2\omega_2^3\omega_3 - 216\omega_1^2cs^2\omega_2\omega_3v_1^2 + 12\omega_1^2\omega_2^3\omega_3 - 63cs^2\omega_2^3\omega_3v_1^2 + 18\omega_1^3\omega_2\omega_3v_1^2 - 90\omega_1cs^2\omega_2^3\omega_3v_2^2 - \omega_1^2\omega_2^3\omega_3v_2^2 + 18\omega_1^3cs^2\omega_2^3\omega_3 - 30\omega_1cs^4\omega_2^3\omega_3 + 18\omega_1cs^2\omega_2^3\omega_3v_1^2 - 135\omega_1^2\omega_2\omega_3v_1^2v_2^2 + 6\omega_2^3\omega_3 - 36\omega_2^3\omega_3v_1^4 - 3\omega_1cs^2\omega_2^3\omega_3 - 15\omega_1^3cs^4\omega_2\omega_3 - 108\omega_1^3cs^2\omega_2^3v_1^2 - 36\omega_1^2\omega_2^3\omega_3v_2^2 - 198\omega_1^2cs^2\omega_2^3\omega_3v_2^2 + 18\omega_1\omega_2^3\omega_3v_1^4 + 30\omega_1cs^4\omega_2^3\omega_3 + 189\omega_1^3cs^2\omega_2^3v_1^2 - 6\omega_1^2\omega_2\omega_3 - 6\omega_1^3cs^2\omega_2\omega_3 + 126\omega_1cs^2\omega_2^3\omega_3v_2^2 + 108\omega_1^2cs^2\omega_2^3v_1^2 + 36\omega_1^2cs^2\omega_2\omega_3 + 18\omega_1\omega_2^3\omega_3v_2^2 - 90\omega_1^2\omega_2^3\omega_3v_1^2 - 144\omega_1^2cs^2\omega_2^3\omega_3v_1^2$$

$$C_{17} = -6\omega_1^2cs^4\omega_2^3\omega_3 - 18\omega_1^3cs^2\omega_2^3\omega_3 + 108\omega_1^2\omega_2^3\omega_3v_1^4 - 54\omega_1cs^2\omega_2^3\omega_3v_2^2 + \omega_1^2\omega_2^3\omega_3 + 72\omega_1\omega_2^3\omega_3v_1^2v_2^2 - 36\omega_1^3cs^4\omega_2^2 + 45\omega_1^2\omega_2\omega_3v_2^2 - 72\omega_1^3\omega_2\omega_3v_1^2v_2^2 - 72\omega_1^2cs^2\omega_2^3\omega_3 - 2\omega_1^2cs^2\omega_2^3\omega_3v_1^2 + 6\omega_2^3\omega_3v_1^2 - 54\omega_1\omega_2^3\omega_3v_2^2 + 21\omega_1cs^2\omega_2^3\omega_3v_1^2 + 6\omega_1\omega_2^3\omega_3 - 72\omega_1^3cs^2\omega_2^3 + 6\omega_1^2\omega_2\omega_3v_2^2 - 12\omega_1^2cs^2\omega_2^3 - 24\omega_1^2\omega_2^3\omega_3v_1^2v_2^2 - 72\omega_1^2cs^2\omega_2^3\omega_3v_2^2 - 9\omega_2^3\omega_3v_2^2 + 6\omega_1^3cs^4\omega_2^3\omega_3 + 18\omega_1^2cs^2\omega_2^3\omega_3 - 243\omega_1^2cs^2\omega_2\omega_3v_2^2 + 63\omega_1\omega_2^3\omega_3v_2^2 + \omega_1^2\omega_2^3\omega_3v_1^2 + 5\omega_1^2cs^2\omega_2^3\omega_3 + 54\omega_1^3cs^4\omega_2^3\omega_3 + 72\omega_1^2cs^4\omega_2^3\omega_3 + 72\omega_1^3cs^2\omega_2^3\omega_3v_2^2 - 18cs^4\omega_2^3\omega_3 - 6\omega_1^2\omega_2\omega_3 + 6\omega_1^2\omega_2\omega_3v_1^2 - 6\omega_1^3cs^2\omega_2\omega_3v_1^2 - 6\omega_1\omega_2^3\omega_3v_1^2 + 54\omega_1^3\omega_2^3v_2^2 - 9\omega_1\omega_2^3\omega_3v_1^2v_2^2 + 24\omega_1^2\omega_2^3\omega_3v_2^2 + 6\omega_1\omega_2^3\omega_3 + 2\omega_1^3cs^2\omega_2^3\omega_3v_1^2 + 126\omega_1^3\omega_2\omega_3v_2^2 - 5\omega_1^3cs^2\omega_2^3\omega_3 - 54\omega_1^2cs^4\omega_2^3\omega_3 + 6\omega_1^3\omega_3 + 540\omega_1^2cs^2\omega_2^3\omega_3v_2^2 + 24\omega_1^3\omega_2^3\omega_3v_1^2v_2^2 + 45\omega_1^3\omega_2^3v_1^2v_2^2 + 12\omega_1cs^2\omega_2^3\omega_3 + 90\omega_1^3cs^4\omega_2^3 + 27\omega_1cs^4\omega_2^3\omega_3 - 54\omega_1\omega_2^3\omega_3v_2^4 + 18\omega_1^3cs^2\omega_2^3\omega_3v_1^2 + 36\omega_1^2cs^4\omega_2^3 - 6\omega_1^3\omega_2^3v_1^2 - \omega_1^3\omega_2^3\omega_3v_1^2 - 486\omega_1^3cs^2\omega_2\omega_3v_2^2 + 75\omega_1^3cs^2\omega_2\omega_3 - 45\omega_2^3\omega_3v_1^2v_2^2 + 6\omega_1cs^2\omega_2^3\omega_3v_1^2 + 135cs^2\omega_2^3\omega_3v_2^2 - 6\omega_1cs^2\omega_2^3\omega_3 - 54\omega_1^2\omega_2\omega_3v_2^4 - 18\omega_1^3cs^4\omega_2\omega_3 + 12\omega_1^3cs^4\omega_2\omega_3 - 99\omega_1^3\omega_2^3v_2^2 - 24\omega_1^3\omega_2^3\omega_3 - 54\omega_1^2cs^4\omega_2\omega_3 - 21\omega_1^3cs^2\omega_2\omega_3v_1^2 - 18cs^2\omega_2^3\omega_3v_1^2 - 297\omega_1cs^2\omega_2^3\omega_3v_2^2 - 54\omega_1^2\omega_2\omega_3v_2^4 + 405\omega_1^3cs^2\omega_2^3v_2^2 - 18\omega_1cs^4\omega_2^3\omega_3 + 6\omega_1cs^2\omega_2^3\omega_3v_1^2 + 9\omega_1^2\omega_2\omega_3v_1^2v_2^2 - 6\omega_2^3\omega_3 - 3\omega_1cs^2\omega_2^3\omega_3 - 6\omega_1\omega_2^3\omega_3v_1^2 - 99\omega_1^3cs^4\omega_2\omega_3 - 12\omega_1^3cs^2\omega_2^3v_1^2 - 108\omega_1^2\omega_2^3\omega_3v_2^2 + 18\omega_1cs^4\omega_2^3\omega_3 + 18\omega_1^3cs^2\omega_2^3v_1^2 - \omega_1^2\omega_2^3\omega_3 - 6\omega_1^3\omega_2\omega_3 + 54\omega_2^3\omega_3v_2^2 + 6\omega_1^3cs^2\omega_2\omega_3 + 12\omega_1^2cs^2\omega_2^3v_1^2 + 60\omega_1^2cs^2\omega_2\omega_3 - 18\omega_1\omega_2^3\omega_3v_2^2 - 18\omega_1^2cs^2\omega_2^3\omega_3v_1^2$$

$$C_{18} = 72\omega_1^3\omega_2^3v_2^4 - 8\omega_1^3\omega_2^3\omega_3v_2^2 - 198\omega_1^3cs^2\omega_2^3\omega_3v_2^2 + 29\omega_1^2cs^4\omega_2^3\omega_3 + 18\omega_1^3cs^2\omega_2^3\omega_3 - 54\omega_1^2\omega_2^3\omega_3v_2^4 - 171\omega_1cs^2\omega_2^3\omega_3v_2^2 + \omega_1^3\omega_2^3\omega_3 + 216\omega_1\omega_2^3\omega_3v_1^2v_2^2 + 18\omega_1\omega_2^3\omega_3v_2^2 - 45\omega_1^2\omega_2\omega_3v_2^2 - 216\omega_1^2\omega_2\omega_3v_1^2v_2^2 - 12\omega_1^2cs^2\omega_2^3\omega_3 + 12\omega_1^3cs^4\omega_2^3\omega_3 + 6\omega_1^2cs^2\omega_2^3\omega_3v_1^2 + 18\omega_2^3\omega_3v_1^2 + 24\omega_1^2cs^4\omega_2^3\omega_3 - 63\omega_1\omega_2^3\omega_3v_2^4 - 9\omega_1cs^2\omega_2^3\omega_3v_1^2 + 6\omega_1\omega_2^3\omega_3 - 24\omega_1^3cs^2\omega_2^3 - 78\omega_1^3\omega_2^3\omega_3v_2^4 + 18\omega_1^2\omega_2\omega_3v_1^2 - 72\omega_1^2\omega_2^3\omega_3v_1^2v_2^2 + 30\omega_1^2cs^2\omega_2^3\omega_3v_2^2 + 9\omega_2^3\omega_3v_2^2 - \omega_1^3cs^4\omega_2^3\omega_3 + 30\omega_1^2cs^2\omega_2^3\omega_3 + 36\omega_1^2\omega_2\omega_3v_2^2 + 81\omega_1^2cs^2\omega_2\omega_3v_1^2 - 27\omega_1\omega_2^3\omega_3v_2^2 + 36\omega_1^2cs^2\omega_2^3\omega_3v_1^2 + 3\omega_1^2\omega_2^3\omega_3v_1^2 - 12\omega_1^2cs^2\omega_2^3\omega_3 - 18\omega_1^3cs^4\omega_2^3\omega_3 - 18\omega_1^3cs^2\omega_2^3\omega_3v_2^4 - 72\omega_1^3\omega_2^3\omega_3v_2^2 + 12\omega_1^2cs^4\omega_2^3\omega_3 - 12\omega_1^3cs^2\omega_2^3\omega_3 + 84\omega_1^3cs^2\omega_2^3\omega_3v_2^2 + 144\omega_1^2\omega_2^3\omega_3v_2^4 + 30cs^4\omega_2^3\omega_3 - 36\omega_2^3\omega_3v_2^2 - 6\omega_1^2\omega_2\omega_3 + 18\omega_1^2\omega_2\omega_3v_1^2 + 18\omega_1^3cs^2\omega_2\omega_3v_1^2 - 54\omega_1^2cs^2\omega_2\omega_3v_1^2 - 18\omega_1\omega_2^3\omega_3v_1^2 - 2\omega_1^2cs^4\omega_2^3\omega_3 - 24\omega_1^2cs^2\omega_2^3\omega_3 + 19\omega_1^2\omega_2^3\omega_3v_2^4 + 36\omega_1^3\omega_2^3v_2^4 - 27\omega_1\omega_2^3\omega_3v_1^2v_2^2 + 108\omega_1^2\omega_2^3v_2^2 - 72\omega_1^2cs^2\omega_2^3\omega_3v_2^2 + 17\omega_1^2\omega_2^3\omega_3v_2^2 + 6\omega_1\omega_2^3\omega_3 - 6\omega_1^3cs^2\omega_2^3\omega_3v_1^2 + 135\omega_1^3\omega_2\omega_3v_2^2 - 30\omega_1^2cs^4\omega_2^3\omega_3 - 54\omega_1^3cs^2\omega_2\omega_3v_2^2 + 72\omega_1^2\omega_2^3\omega_3v_2^2 -$$

$$144\omega_1^2\omega_3^2\omega_3v_2^2 + 6\omega_1^3\omega_3^2 + 36\omega_1^3\omega_2^2v_2^2 - 144\omega_1^2cs^2\omega_2^2\omega_3^2v_2^2 + 72\omega_1^3\omega_2^2\omega_3^2v_1^2v_2^2 + 135\omega_1^3\omega_3^2v_1^2v_2^2 - 12\omega_1cs^2\omega_2^2\omega_3^2 + 18\omega_1^3cs^4\omega_3^2 - 57\omega_1cs^4\omega_2^2\omega_3^2 + 36\omega_1\omega_2^2\omega_3^2v_1^4 - 36\omega_1^3cs^2\omega_2^2\omega_3v_1^2 - 18\omega_1^3\omega_3^2v_1^2 - 3\omega_1^3\omega_2^2\omega_3^2v_1^2 - 297\omega_1^3cs^2\omega_2\omega_3^2v_2^2 + 21\omega_1^3cs^2\omega_2\omega_3^2 - 135\omega_1^3\omega_3^2v_1^2v_2^2 + 18\omega_1cs^2\omega_2^2\omega_3^2v_1^2 + 18\omega_1^3\omega_2\omega_3^2v_2^2 + 63cs^2\omega_2^2\omega_3^2v_2^2 + 216\omega_1^3cs^2\omega_2^2v_2^2 - 18\omega_1cs^2\omega_2^2\omega_3 - 63\omega_1^3\omega_2\omega_3^2v_2^4 + 6\omega_1^3cs^4\omega_2\omega_3 + 18\omega_1^2cs^2\omega_2^2\omega_3^2v_1^2 + 198\omega_1^3cs^2\omega_2^2\omega_3^2v_2^2 - 81\omega_1^3\omega_3^2v_2^2 - 43\omega_1^3\omega_2^2\omega_3^2v_2^2 - 30\omega_1^3cs^4\omega_2\omega_3 - 108\omega_1^3\omega_3^2v_1^4 - 9\omega_1^3cs^2\omega_2\omega_3^2v_1^2 + 18cs^2\omega_2^2\omega_3^2v_1^2 + 99\omega_1cs^2\omega_2^2\omega_3^2v_2^2 + 7\omega_1^3\omega_2^2\omega_3^2v_1^4 - 18\omega_1\omega_3^2\omega_3v_2^4 + 36\omega_1^3\omega_2\omega_3^2v_2^4 + 189\omega_1^3cs^2\omega_2^2v_2^2 + 6\omega_1cs^4\omega_2^2\omega_3 - 324\omega_1^3cs^2\omega_2^2v_2^2 - 54\omega_1cs^2\omega_2^2\omega_3v_1^2 + 27\omega_1^3\omega_2\omega_3^2v_1^2v_2^2 - 6\omega_1^3\omega_3^2 - 72\omega_1^3\omega_2^2v_2^2 + 8\omega_1^3\omega_2^2\omega_3^2v_2^4 + 15\omega_1cs^2\omega_2^2\omega_3 - 18\omega_1\omega_2^2\omega_3^2v_1^2 - 15\omega_1^3cs^4\omega_2\omega_3 + 54\omega_1^3\omega_2^2\omega_3^2v_2^2 + 306\omega_1^2cs^2\omega_2^2\omega_3v_2^2 + 24\omega_1^3cs^2\omega_2^2\omega_3^2v_2^2 + 18\omega_1cs^4\omega_2^2\omega_3 + 78\omega_1^3\omega_2^2\omega_3v_2^2 + 18\omega_1^3cs^2\omega_2^2v_1^2 - \omega_1^2\omega_2^2\omega_3^2 - 6\omega_1^3\omega_2\omega_3^2 + 36\omega_2^2\omega_3^2v_2^4 - 36\omega_1^3\omega_2^2\omega_3v_2^4 - 6\omega_1^3cs^2\omega_2\omega_3 + 54\omega_1cs^2\omega_2^2\omega_3v_2^2 + 36\omega_1^3cs^2\omega_2\omega_3^2 - 9\omega_1\omega_2^2\omega_3^2v_2^2 - 108\omega_1^3cs^2\omega_2^2v_2^2 + 36\omega_1^3cs^2\omega_2^2\omega_3v_1^2$$

$$C_{19} = 90\omega_1\omega_2^2\omega_3 + 30\omega_1^3cs^2\omega_2^2\omega_3 - 198\omega_1\omega_3^2\omega_3v_2^2 - 18\omega_1^3\omega_2^2 + 90\omega_1^2\omega_2\omega_3 + 45\omega_1\omega_3^2\omega_3v_1^2 - 54\omega_1^2cs^2\omega_2^2 - 30\omega_1^2cs^2\omega_2^2\omega_3 + 396\omega_1^2\omega_2^2\omega_3v_2^2 + 270\omega_1^3cs^2\omega_2^2\omega_3 - 18\omega_1^3\omega_2^2v_1^2 - 18\omega_1^3\omega_2v_1^2 + 198\omega_1^3\omega_3v_2^2 + 45\omega_1\omega_2^2\omega_3 + 18\omega_1^2\omega_2^2 + 324\omega_1^2cs^2\omega_2^2\omega_3 + 18\omega_1^3\omega_2 + 54cs^2\omega_2^2\omega_3 + 36\omega_1^3\omega_3v_1^2 + 10\omega_1^3\omega_2^2\omega_3v_1^2 - 10\omega_1^3\omega_2^2\omega_3 - 162\omega_1^2cs^2\omega_2\omega_3 - 18\omega_1\omega_2^2 + 18\omega_1\omega_3^2v_1^2 + 10\omega_1^3\omega_2^2\omega_3 - 198\omega_1^3\omega_2\omega_3v_2^2 + 135\omega_1^3\omega_2\omega_3 - 198\omega_1\omega_2^2\omega_3v_2^2 - 27\omega_1cs^2\omega_2^2\omega_3 + 18\omega_1^3\omega_2^2v_1^2 - 10\omega_1^2\omega_2^2\omega_3v_1^2 + 54\omega_1^3cs^2\omega_2^2 - 54\omega_2^2\omega_3 - 45\omega_1^3\omega_2\omega_3v_1^2 - 180\omega_1^2\omega_2^2\omega_3 + 198\omega_2^2\omega_3v_2^2 - 54\omega_1^3cs^2\omega_2 - 297\omega_1^3cs^2\omega_2\omega_3 - 198\omega_1^2\omega_2\omega_3v_2^2 - 36\omega_2^2\omega_3v_1^2 - 126\omega_1^3\omega_3 + 54\omega_1cs^2\omega_2^2 - 162\omega_1cs^2\omega_2^2\omega_3$$

$$C_{20} = 6\omega_1\omega_2^2\omega_3 - 68\omega_1^3cs^2\omega_2^2\omega_3v_2^2 - 78\omega_1^3cs^2\omega_2^2\omega_3 + 45\omega_2^2\omega_3v_1^4 + 129\omega_1\omega_3^2\omega_3v_2^2 - 10\omega_1^3cs^4\omega_2^2\omega_3 + 12\omega_1^2cs^2\omega_2^2 + 6\omega_1^3cs^2\omega_2^2v_2^2 - 90\omega_1^3cs^4\omega_2^2\omega_3 + 99\omega_1^2\omega_2\omega_3v_1^4 - 2\omega_1^3\omega_2^2\omega_3 + 261\omega_1^3cs^2\omega_2^2\omega_3v_2^2 + 90cs^4\omega_2^2\omega_3 + 6\omega_1^3\omega_2\omega_3 - 18\omega_1^3\omega_2^2\omega_3v_1^4 - 78\omega_1^3cs^2\omega_2^2\omega_3 + 210\omega_1^2\omega_2^2\omega_3v_2^2 - 72\omega_1^3cs^2\omega_3 + 82\omega_1^3cs^4\omega_2^2\omega_3 - 117\omega_1^3\omega_2\omega_3v_2^4 - 51\omega_1^3\omega_3v_2^2 - 98\omega_1^3\omega_2^2\omega_3v_2^2 + 12\omega_1^3cs^2\omega_2^2\omega_3 + 6\omega_1cs^2\omega_2^2v_2^2 + 411\omega_1cs^2\omega_2^2\omega_3v_2^2 - 12\omega_1\omega_3^2\omega_3 + 18\omega_1cs^4\omega_2^2 + 90\omega_1^2\omega_2^2\omega_3v_1^4 + 114\omega_1^2cs^2\omega_2^2\omega_3 - 816\omega_1^2cs^2\omega_2^2\omega_3v_2^2 - 72cs^2\omega_2^2\omega_3 + 99\omega_1\omega_2^2\omega_3v_2^4 + 261cs^2\omega_2^2\omega_3v_2^2 + 8\omega_1^3\omega_2^2\omega_3 + 82\omega_1^3cs^4\omega_2^2\omega_3 + 18\omega_1^3cs^4\omega_2 - 600\omega_1^3cs^2\omega_2\omega_3v_2^2 + 90\omega_1^3\omega_2^2\omega_3v_2^4 - 60\omega_1^2cs^2\omega_2\omega_3 + 45\omega_1^3\omega_3v_2^4 - 98\omega_1^2\omega_3^2\omega_3v_2^2 + 8\omega_1^2\omega_2^2\omega_3 - 12\omega_1^2cs^2\omega_2^2v_2^2 + 129\omega_1^2\omega_2\omega_3v_2^2 - 12\omega_1^3\omega_2\omega_3 - 105\omega_1\omega_2^2\omega_3v_2^2 + 411\omega_1^2cs^2\omega_2\omega_3v_2^2 + 141\omega_1cs^2\omega_2^2\omega_3 - 171\omega_1^3cs^4\omega_2\omega_3 + 404\omega_1^3cs^2\omega_2^2\omega_3v_2^2 + 54\omega_1cs^4\omega_2^2\omega_3 - 36\omega_1^2cs^4\omega_2^2 + 6\omega_2^2\omega_3 - 117\omega_1\omega_3^2\omega_3v_2^4 + 54\omega_1^2cs^4\omega_2\omega_3 - 12\omega_1^2\omega_2^2\omega_3 - 51\omega_2^2\omega_3v_2^2 + 404\omega_1^2cs^2\omega_2^2\omega_3v_2^2 - 6\omega_1^3cs^2\omega_2 - 171\omega_1cs^4\omega_2^2\omega_3 + 90\omega_1^3cs^4\omega_3 + 20\omega_1^3\omega_2^2\omega_3v_2^2 - 198\omega_1^2\omega_2^2\omega_3v_2^4 + 141\omega_1^3cs^2\omega_2\omega_3 - 600\omega_1cs^2\omega_2^2\omega_3v_2^2 - 105\omega_1^2\omega_2\omega_3v_2^2 + 6\omega_1^3\omega_3 - 6\omega_1cs^2\omega_2^2 - 60\omega_1cs^2\omega_2^2\omega_3$$

$$C_{21} = 6\omega_1\omega_2^2\omega_3 - 36\omega_1^3cs^2\omega_2^2\omega_3v_2^2 - 22\omega_1^3cs^2\omega_2^2\omega_3 + 171\omega_2^2\omega_3v_1^4 + 225\omega_1\omega_3^2\omega_3v_2^2 - 2\omega_1^3cs^4\omega_2^2\omega_3 + 12\omega_1^2cs^2\omega_2^2 + 18\omega_1^3cs^2\omega_2^2v_2^2 - 6\omega_1^2cs^4\omega_2^2\omega_3 + 333\omega_1^2\omega_2\omega_3v_1^4 - 2\omega_1^3\omega_3^2\omega_3 + 207\omega_1^3cs^2\omega_2^2\omega_3v_2^2 + 18cs^4\omega_2^2\omega_3 + 6\omega_1^3\omega_2\omega_3 - 58\omega_1^3\omega_2^2\omega_3v_2^4 - 22\omega_1^3cs^2\omega_2^2\omega_3 + 306\omega_1^2\omega_2^2\omega_3v_2^2 - 24\omega_1^3cs^2\omega_3 + 14\omega_1^3cs^4\omega_2^2\omega_3 - 423\omega_1^3\omega_2\omega_3v_2^4 - 99\omega_1^3\omega_3v_2^2 - 154\omega_1^2\omega_2^2\omega_3v_2^2 + 4\omega_1^3cs^2\omega_2^2\omega_3 + 18\omega_1cs^2\omega_2^2v_2^2 + 225\omega_1cs^2\omega_2^2\omega_3v_2^2 - 12\omega_1\omega_3^2\omega_3 + 6\omega_1cs^4\omega_2^2 + 310\omega_1^2\omega_3^2\omega_3v_2^4 + 18\omega_1^2cs^2\omega_2^2\omega_3 - 432\omega_1^2cs^2\omega_2^2\omega_3v_2^2 - 24cs^2\omega_2^2\omega_3 + 333\omega_1\omega_2^2\omega_3v_2^4 + 207cs^2\omega_2^2\omega_3v_2^2 + 8\omega_1^3\omega_2^2\omega_3 + 14\omega_1^2cs^4\omega_2^2\omega_3 + 6\omega_1^3cs^4\omega_2 - 432\omega_1^3cs^2\omega_2\omega_3v_2^2 + 310\omega_1^3\omega_2^2\omega_3v_2^4 - 12\omega_1^2cs^2\omega_2\omega_3 + 171\omega_1^3\omega_3v_2^4 - 154\omega_1^2\omega_3^2\omega_3v_2^2 + 8\omega_1^2\omega_2^2\omega_3 - 36\omega_1^2cs^2\omega_2^2v_2^2 + 225\omega_1^2\omega_2\omega_3v_2^2 - 12\omega_1^3\omega_2\omega_3 - 153\omega_1\omega_2^2\omega_3v_2^2 + 225\omega_1^2cs^2\omega_2\omega_3v_2^2 + 45\omega_1cs^2\omega_2^2\omega_3 - 33\omega_1^3cs^4\omega_2\omega_3 + 252\omega_1^3cs^2\omega_2^2\omega_3v_2^2 + 6\omega_1cs^4\omega_2^2\omega_3 - 12\omega_1^2cs^4\omega_2^2 + 6\omega_2^2\omega_3 - 423\omega_1\omega_3^2\omega_3v_2^4 + 6\omega_1^2cs^4\omega_2\omega_3 - 12\omega_1^2\omega_2^2\omega_3 - 99\omega_2^2\omega_3v_2^2 + 252\omega_1^2cs^2\omega_2^2\omega_3v_2^2 - 6\omega_1^3cs^2\omega_2 - 33\omega_1cs^4\omega_2^2\omega_3 + 18\omega_1^3cs^4\omega_3 + 28\omega_1^3\omega_2^2\omega_3v_2^2 - 666\omega_1^2\omega_2^2\omega_3v_2^2 + 45\omega_1^3cs^2\omega_2\omega_3 - 432\omega_1cs^2\omega_2^2\omega_3v_2^2 - 153\omega_1^2\omega_2\omega_3v_2^2 + 6\omega_1^3\omega_3 - 6\omega_1cs^2\omega_2^2 - 12\omega_1cs^2\omega_2^2\omega_3$$

3 Comparison of SRT, MRT, CLBM, and CuLBM

3.1 Conservation of mass: ρ

$$\frac{\partial \rho}{\partial t} + v_1 \frac{\delta_l}{\delta t} \frac{\partial \rho}{\partial x_1} + \rho \frac{\delta_l}{\delta t} \frac{\partial v_1}{\partial x_1} + v_2 \frac{\delta_l}{\delta t} \frac{\partial \rho}{\partial x_2} + \rho \frac{\delta_l}{\delta t} \frac{\partial v_2}{\partial x_2} + C_{D_x^3 \rho}^{(0)} \frac{\delta_l^3}{\delta t} \frac{\partial^3 \rho}{\partial x_1^3} + C_{D_x^3 v_1}^{(0)} \frac{\delta_l^3}{\delta t} \frac{\partial^3 v_1}{\partial x_1^3} + C_{D_x^2 D_y v_2}^{(0)} \frac{\delta_l^3}{\delta t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + C_{D_x D_y^2 v_1}^{(0)} \frac{\delta_l^3}{\delta t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_{D_y^3 \rho}^{(0)} \frac{\delta_l^3}{\delta t} \frac{\partial^3 \rho}{\partial x_2^3} + C_{D_y^3 v_2}^{(0)} \frac{\delta_l^3}{\delta t} \frac{\partial^3 v_2}{\partial x_2^3} + C_{D_x \rho}^{(0)} \frac{\delta_l^4}{\delta t} \frac{\partial^4 \rho}{\partial x_1^4} + C_{D_x v_1}^{(0)} \frac{\delta_l^4}{\delta t} \frac{\partial^4 v_1}{\partial x_1^4} + C_{D_x D_y \rho}^{(0)} \frac{\delta_l^4}{\delta t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2} + C_{D_x^2 D_y v_1}^{(0)} \frac{\delta_l^4}{\delta t} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2} + C_{D_x^2 D_y^2 \rho}^{(0)} \frac{\delta_l^4}{\delta t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_{D_x D_y^2 v_1}^{(0)} \frac{\delta_l^4}{\delta t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^2} + C_{D_x D_y^3 \rho}^{(0)} \frac{\delta_l^4}{\delta t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{D_x D_y v_1}^{(0)} \frac{\delta_l^4}{\delta t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{D_x D_y^2 v_2}^{(0)} \frac{\delta_l^4}{\delta t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^2} + C_{D_y^4 \rho}^{(0)} \frac{\delta_l^4}{\delta t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{D_y^4 v_2}^{(0)} \frac{\delta_l^4}{\delta t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,$$

where:

coefficient $C_{D_x^3 \rho}^{(0)}$ at $\frac{\partial^3 \rho}{\partial x_1^3}$:

$$C_{D_x^3 \rho}^{(0), \text{SRT}} = (-1 + v_1^2 + 3cs^2) \frac{v_1}{12}$$

$$C_{D_x^3 \rho}^{(0), \text{MRT}^1} = (-1 + 3cs^2 + v_1^2) \frac{v_1}{12}$$

$$C_{D_x^3 \rho}^{(0), \text{MRT}^2} = (-1 + v_1^2 + 3cs^2) \frac{v_1}{12}$$

$$C_{D_x^3 \rho}^{(0), \text{CLBM}^1} = (-1 + v_1^2 + 3cs^2) \frac{v_1}{12}$$

$$C_{D_x^3 \rho}^{(0), \text{CLBM}^2} = (-1 + v_1^2 + 3cs^2) \frac{v_1}{12}$$

$$C_{D_x^3 \rho}^{(0), \text{CuLBM1}} = (-1 + v_1^2 + 3cs^2) \frac{v_1}{12}$$

$$C_{D_x^3 \rho}^{(0), \text{CuLBM2}} = (-1 + v_1^2 + 3cs^2) \frac{v_1}{12}$$

coefficient $C_{D_x^3 v_1}^{(0)}$ **at** $\frac{\partial^3 v_1}{\partial x_1^3}$:

$$C_{D_x^3 v_1}^{(0), \text{SRT}} = (-1 + 3v_1^2 + cs^2) \frac{\rho}{12}$$

$$C_{D_x^3 v_1}^{(0), \text{MRT1}} = (-1 + cs^2 + 3v_1^2) \frac{\rho}{12}$$

$$C_{D_x^3 v_1}^{(0), \text{MRT2}} = (-1 + 3v_1^2 + cs^2) \frac{\rho}{12}$$

$$C_{D_x^3 v_1}^{(0), \text{CLBM1}} = (-1 + 3v_1^2 + cs^2) \frac{\rho}{12}$$

$$C_{D_x^3 v_1}^{(0), \text{CLBM2}} = (-1 + 3v_1^2 + cs^2) \frac{\rho}{12}$$

$$C_{D_x^3 v_1}^{(0), \text{CuLBM1}} = (-1 + 3v_1^2 + cs^2) \frac{\rho}{12}$$

$$C_{D_x^3 v_1}^{(0), \text{CuLBM2}} = (-1 + 3v_1^2 + cs^2) \frac{\rho}{12}$$

coefficient $C_{D_x^2 D_y v_2}^{(0)}$ **at** $\frac{\partial^3 v_2}{\partial x_1^2 \partial x_2}$:

$$C_{D_x^2 D_y v_2}^{(0), \text{SRT}} = \frac{-\rho cs^2}{6}$$

$$C_{D_x^2 D_y v_2}^{(0), \text{MRT1}} = \frac{-\rho cs^2}{6}$$

$$C_{D_x^2 D_y v_2}^{(0), \text{MRT2}} = \frac{-\rho cs^2}{6}$$

$$C_{D_x^2 D_y v_2}^{(0), \text{CLBM1}} = \frac{-\rho cs^2}{6}$$

$$C_{D_x^2 D_y v_2}^{(0), \text{CLBM2}} = \frac{-cs^2 \rho}{6}$$

$$C_{D_x^2 D_y v_2}^{(0), \text{CuLBM1}} = \frac{-\rho cs^2}{6}$$

$$C_{D_x^2 D_y v_2}^{(0), \text{CuLBM2}} = \frac{-\rho cs^2}{6}$$

coefficient $C_{D_x D_y^2 v_1}^{(0)}$ **at** $\frac{\partial^3 v_1}{\partial x_1 \partial x_2^2}$:

$$C_{D_x D_y^2 v_1}^{(0), \text{SRT}} = \frac{-\rho cs^2}{6}$$

$$C_{D_x D_y^2 v_1}^{(0), \text{MRT1}} = \frac{-\rho cs^2}{6}$$

$$C_{D_x D_y^2 v_1}^{(0), \text{MRT2}} = \frac{-\rho cs^2}{6}$$

$$C_{D_x D_y^2 v_1}^{(0), \text{CLBM1}} = \frac{-\rho cs^2}{6}$$

$$C_{D_x D_y^2 v_1}^{(0), \text{CLBM2}} = \frac{-cs^2 \rho}{6}$$

$$C_{D_x D_y^2 v_1}^{(0), \text{CuLBM1}} = \frac{-\rho cs^2}{6}$$

$$C_{D_x D_y^2 v_1}^{(0), \text{CuLBM2}} = \frac{-\rho cs^2}{6}$$

coefficient $C_{D_y^3 \rho}^{(0)}$ **at** $\frac{\partial^3 \rho}{\partial x_2^3}$:

$$C_{D_y^3 \rho}^{(0), \text{SRT}} = (-1 + 3cs^2 + v_2^2) \frac{v_2}{12}$$

$$C_{D_y^3 \rho}^{(0), \text{MRT1}} = (-1 + 3cs^2 + v_2^2) \frac{v_2}{12}$$

$$C_{D_y^3 \rho}^{(0), \text{MRT2}} = (-1 + v_2^2 + 3cs^2) \frac{v_2}{12}$$

$$C_{D_y^3 \rho}^{(0), \text{CLBM1}} = (-1 + 3cs^2 + v_2^2) \frac{v_2}{12}$$

$$C_{D_y^3 \rho}^{(0), \text{CLBM2}} = (-1 + 3cs^2 + v_2^2) \frac{v_2}{12}$$

$$C_{D_y^3 \rho}^{(0), \text{CuLBM1}} = (-1 + 3cs^2 + v_2^2) \frac{v_2}{12}$$

$$C_{D_y^3 \rho}^{(0), \text{CuLBM2}} = (-1 + 3cs^2 + v_2^2) \frac{v_2}{12}$$

coefficient $C_{D_y^3 v_2}^{(0)}$ **at** $\frac{\partial^3 v_2}{\partial x_2^3}$:

$$C_{D_y^3 v_2}^{(0), \text{SRT}} = (-1 + cs^2 + 3v_2^2) \frac{\rho}{12}$$

$$C_{D_y^3 v_2}^{(0), \text{MRT1}} = (-1 + cs^2 + 3v_2^2) \frac{\rho}{12}$$

$$C_{D_y^3 v_2}^{(0), \text{MRT2}} = (-1 + 3v_2^2 + cs^2) \frac{\rho}{12}$$

$$C_{D_y^3 v_2}^{(0), \text{CLBM1}} = (-1 + cs^2 + 3v_2^2) \frac{\rho}{12}$$

$$C_{D_y^3 v_2}^{(0), \text{CLBM2}} = (-1 + cs^2 + 3v_2^2) \frac{\rho}{12}$$

$$C_{D_y^3 v_2}^{(0), \text{CuLBM1}} = (-1 + cs^2 + 3v_2^2) \frac{\rho}{12}$$

$$C_{D_y^3 v_2}^{(0), \text{CuLBM2}} = (-1 + cs^2 + 3v_2^2) \frac{\rho}{12}$$

coefficient $C_{D_x^4 \rho}^{(0)}$ **at** $\frac{\partial^4 \rho}{\partial x_1^4}$:

$$C_{D_x^4 \rho}^{(0), \text{SRT}} = (-12v_1^2 \omega cs^2 - 6v_1^2 + 3v_1^2 \omega - \omega cs^4 + 24v_1^2 cs^2 - 2cs^2 + 2cs^4 + \omega cs^2 - 3v_1^4 \omega + 6v_1^4) \frac{1}{24\omega}$$

$$C_{D_x^4 \rho}^{(0), \text{MRT1}} = (24v_1^2 cs^2 - 2cs^2 + cs^2 \omega_5 - 6v_1^2 + 3v_1^2 \omega_5 - cs^4 \omega_5 - 12v_1^2 cs^2 \omega_5 + 2cs^4 - 3v_1^4 \omega_5 + 6v_1^4) \frac{1}{24\omega_5}$$

$$C_{D_x^4 \rho}^{(0), \text{MRT2}} = (2cs^4 - 6v_1^2 + 3v_1^2 \omega_5 - 12v_1^2 cs^2 \omega_5 - cs^4 \omega_5 - 3v_1^4 \omega_5 + cs^2 \omega_5 - 2cs^2 + 24v_1^2 cs^2 + 6v_1^4) \frac{1}{24\omega_5}$$

$$C_{D_x^4 \rho}^{(0), \text{CLBM1}} = (-\omega_5 cs^4 - 6v_1^2 + 24v_1^2 cs^2 - 2cs^2 + 3v_1^2 \omega_5 - 12v_1^2 \omega_5 cs^2 + 2cs^4 + \omega_5 cs^2 - 3v_1^4 \omega_5 + 6v_1^4) \frac{1}{24\omega_5}$$

$$C_{D_x^4 \rho}^{(0), \text{CLBM2}} = (-6v_1^2 - 2cs^2 + 24v_1^2 cs^2 + 3v_1^2 \omega_5 + cs^2 \omega_5 - cs^4 \omega_5 - 3v_1^4 \omega_5 - 12v_1^2 cs^2 \omega_5 + 2cs^4 + 6v_1^4) \frac{1}{24\omega_5}$$

$$C_{D_x^4 \rho}^{(0), \text{CuLBM1}} = (-12v_1^2 cs^2 \omega_1 - cs^4 \omega_1 - 6v_1^2 - 3v_1^4 \omega_1 + 2cs^4 - 2cs^2 + 24v_1^2 cs^2 + cs^2 \omega_1 + 3v_1^2 \omega_1 + 6v_1^4) \frac{1}{24\omega_1}$$

$$C_{D_x^4 \rho}^{(0), \text{CuLBM2}} = (-3v_1^2 \omega_2 + 3v_1^2 \omega_1 \omega_2 + 12v_1^2 cs^2 \omega_2 + \omega_1 cs^2 \omega_2 + cs^4 \omega_2 + \omega_1 cs^4 + 12v_1^2 \omega_1 cs^2 + 3v_1^4 \omega_1 - 3v_1^4 \omega_1 \omega_2 - 12v_1^2 \omega_1 cs^2 \omega_2 - \omega_1 cs^4 \omega_2 - cs^2 \omega_2 + 3v_1^4 \omega_2 - \omega_1 cs^2 - 3v_1^2 \omega_1) \frac{1}{24\omega_1 \omega_2}$$

coefficient $C_{D_x^4 v_1}^{(0)}$ **at** $\frac{\partial^4 v_1}{\partial x_1^4}$:

$$C_{D_x^4 v_1}^{(0), \text{SRT}} = (-4 + 10v_1^2 - 5v_1^2\omega + 6cs^2 - 3\omega cs^2 + 2\omega) \frac{v_1 \rho}{12\omega}$$

$$C_{D_x^4 v_1}^{(0), \text{MRT}^1} = (-4 + 6cs^2 - 3cs^2\omega_5 + 10v_1^2 - 5v_1^2\omega_5 + 2\omega_5) \frac{v_1 \rho}{12\omega_5}$$

$$C_{D_x^4 v_1}^{(0), \text{MRT}^2} = (-4 + 10v_1^2 - 5v_1^2\omega_5 - 3cs^2\omega_5 + 2\omega_5 + 6cs^2) \frac{v_1 \rho}{12\omega_5}$$

$$C_{D_x^4 v_1}^{(0), \text{CLBM}^1} = (-4 + 10v_1^2 + 6cs^2 - 5v_1^2\omega_5 - 3\omega_5 cs^2 + 2\omega_5) \frac{v_1 \rho}{12\omega_5}$$

$$C_{D_x^4 v_1}^{(0), \text{CLBM}^2} = (-4 + 10v_1^2 + 6cs^2 - 5v_1^2\omega_5 - 3cs^2\omega_5 + 2\omega_5) \frac{v_1 \rho}{12\omega_5}$$

$$C_{D_x^4 v_1}^{(0), \text{CuLBM}^1} = (-4 + 10v_1^2 + 6cs^2 + 2\omega_1 - 3cs^2\omega_1 - 5v_1^2\omega_1) \frac{v_1 \rho}{12\omega_1}$$

$$C_{D_x^4 v_1}^{(0), \text{CuLBM}^2} = (5v_1^2\omega_2 - 5v_1^2\omega_1\omega_2 - 3\omega_1 cs^2\omega_2 + 3cs^2\omega_2 + 2\omega_1\omega_2 - 2\omega_1 + 3\omega_1 cs^2 + 5v_1^2\omega_1 - 2\omega_2) \frac{v_1 \rho}{12\omega_1\omega_2}$$

coefficient $C_{D_x^3 D_y \rho}^{(0)}$ **at** $\frac{\partial^4 \rho}{\partial x_1^3 \partial x_2}$:

$$C_{D_x^3 D_y \rho}^{(0), \text{SRT}} = 0$$

$$C_{D_x^3 D_y \rho}^{(0), \text{MRT}^1} = (-3cs^2\omega_5 + v_1^2\omega_7 + 3cs^2\omega_7 - v_1^2\omega_5 - \omega_7 + \omega_5) \frac{v_1 v_2}{4\omega_7\omega_5}$$

$$C_{D_x^3 D_y \rho}^{(0), \text{MRT}^2} = (v_1^2\omega_7 - v_1^2\omega_5 - \omega_7 - 3cs^2\omega_5 + \omega_5 + 3\omega_7 cs^2) \frac{v_1 v_2}{4\omega_7\omega_5}$$

$$C_{D_x^3 D_y \rho}^{(0), \text{CLBM}^1} = 0$$

$$C_{D_x^3 D_y \rho}^{(0), \text{CLBM}^2} = 0$$

$$C_{D_x^3 D_y \rho}^{(0), \text{CuLBM}^1} = 0$$

$$C_{D_x^3 D_y \rho}^{(0), \text{CuLBM}^2} = (\omega_1 v_2^2 - 3cs^2\omega_2 - \omega_1 - v_2^2\omega_2 + 3\omega_1 cs^2 + \omega_2) \frac{v_1 v_2}{8\omega_1\omega_2}$$

coefficient $C_{D_x^3 D_y v_1}^{(0)}$ **at** $\frac{\partial^4 v_1}{\partial x_1^3 \partial x_2}$:

$$C_{D_x^3 D_y v_1}^{(0), \text{SRT}} = 0$$

$$C_{D_x^3 D_y v_1}^{(0), \text{MRT}^1} = (-cs^2\omega_5 + 3v_1^2\omega_7 + cs^2\omega_7 - 3v_1^2\omega_5 - \omega_7 + \omega_5) \frac{\rho v_2}{4\omega_7\omega_5}$$

$$C_{D_x^3 D_y v_1}^{(0), \text{MRT}^2} = (3v_1^2\omega_7 - 3v_1^2\omega_5 - \omega_7 - cs^2\omega_5 + \omega_5 + \omega_7 cs^2) \frac{\rho v_2}{4\omega_7\omega_5}$$

$$C_{D_x^3 D_y v_1}^{(0), \text{CLBM}^1} = 0$$

$$C_{D_x^3 D_y v_1}^{(0), \text{CLBM}^2} = 0$$

$$C_{D_x^3 D_y v_1}^{(0), \text{CuLBM}^1} = 0$$

$$C_{D_x^3 D_y v_1}^{(0), \text{CuLBM}^2} = (\omega_1 v_2^2 - 3cs^2\omega_2 - \omega_1 - v_2^2\omega_2 + 3\omega_1 cs^2 + \omega_2) \frac{\rho v_2}{24\omega_1\omega_2}$$

coefficient $C_{D_x^3 D_y v_2}^{(0)}$ **at** $\frac{\partial^4 v_2}{\partial x_1^3 \partial x_2}$:

$$C_{D_x^3 D_y v_2}^{(0), \text{SRT}} = (2 - 2v_1^2 + v_1^2\omega - 6cs^2 + 3\omega cs^2 - \omega) \frac{v_1 \rho}{12\omega}$$

$$C_{D_x^3 D_y v_2}^{(0), \text{MRT}^1} = (-3v_1^2\omega_4\omega_5 - \omega_7\omega_4\omega_5 - 6cs^2\omega_7\omega_5 + v_1^2\omega_7\omega_4\omega_5 + v_1^2\omega_7\omega_4 + 3cs^2\omega_7\omega_4 + 3\omega_4\omega_5 + 3cs^2\omega_7\omega_4\omega_5 - \omega_7\omega_4 - 3cs^2\omega_4\omega_5) \frac{v_1 \rho}{12\omega_7\omega_4\omega_5}$$

$$C_{D_x^3 D_y v_2}^{(0), \text{MRT}^2} = (-3v_1^2\omega_4\omega_5 - \omega_7\omega_4\omega_5 - 6\omega_7 cs^2\omega_5 + v_1^2\omega_7\omega_4\omega_5 + v_1^2\omega_7\omega_4 + 3\omega_4\omega_5 + 3\omega_7\omega_4 cs^2 + 3\omega_7\omega_4 cs^2\omega_5 - \omega_7\omega_4 - 3\omega_4 cs^2\omega_5) \frac{v_1 \rho}{12\omega_7\omega_4\omega_5}$$

$$C_{D_x^3 D_y v_2}^{(0), \text{CLBM1}} = (v_1^2 \omega_7 + 3\omega_7 c s^2 + 3\omega_7 \omega_5 c s^2 - 3v_1^2 \omega_5 - \omega_7 - \omega_7 \omega_5 - 9\omega_5 c s^2 + v_1^2 \omega_7 \omega_5 + 3\omega_5) \frac{v_1 \rho}{12\omega_7 \omega_5}$$

$$C_{D_x^3 D_y v_2}^{(0), \text{CLBM2}} = (v_1^2 \omega_7 + 3c s^2 \omega_7 - 3v_1^2 \omega_5 - 9c s^2 \omega_5 - \omega_7 - \omega_7 \omega_5 + v_1^2 \omega_7 \omega_5 + 3c s^2 \omega_7 \omega_5 + 3\omega_5) \frac{v_1 \rho}{12\omega_7 \omega_5}$$

$$C_{D_x^3 D_y v_2}^{(0), \text{CuLBM1}} = (-\omega_4 \omega_1 + 3\omega_4 c s^2 - \omega_4 + v_1^2 \omega_4 \omega_1 + 3\omega_4 c s^2 \omega_1 + 3\omega_1 - 9c s^2 \omega_1 + v_1^2 \omega_4 - 3v_1^2 \omega_1) \frac{v_1 \rho}{12\omega_4 \omega_1}$$

$$C_{D_x^3 D_y v_2}^{(0), \text{CuLBM2}} = (2\omega_3 \omega_2 - 6v_1^2 \omega_1 \omega_2 - 18\omega_1 c s^2 \omega_2 - 2\omega_3 \omega_1 \omega_2 + 2\omega_3 v_1^2 \omega_1 \omega_2 + 6\omega_3 \omega_1 c s^2 \omega_2 + 6\omega_3 \omega_1 c s^2 + \omega_3 v_1^2 \omega_1 - 9\omega_3 v_2^2 \omega_2 + 6\omega_1 \omega_2 + \omega_3 v_1^2 \omega_2 - 4\omega_3 \omega_1 + 9\omega_3 \omega_1 v_2^2) \frac{v_1 \rho}{24\omega_3 \omega_1 \omega_2}$$

coefficient $C_{D_x^2 D_y^2 \rho}^{(0)}$ **at** $\frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2}$:

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{SRT}} = (-2 + \omega) \frac{c s^4}{6\omega}$$

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{MRT1}} = (-2 + \omega_4) \frac{c s^4}{6\omega_4}$$

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{MRT2}} = (-2 + \omega_4) \frac{c s^4}{6\omega_4}$$

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{CLBM1}} = (-2 + \omega_4) \frac{c s^4}{6\omega_4}$$

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{CLBM2}} = (-2 + \omega_4) \frac{c s^4}{6\omega_4}$$

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{CuLBM1}} = (-2 + \omega_3) \frac{c s^4}{6\omega_3}$$

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{CuLBM2}} = (-3v_1^2 \omega_2 + 4\omega_1 c s^2 \omega_2 + 3\omega_1 v_2^2 - 10c s^2 \omega_2 - 2\omega_1 - 3v_2^2 \omega_2 + 2\omega_1 c s^2 + 3v_1^2 \omega_1 + 2\omega_2) \frac{c s^2}{24\omega_1 \omega_2}$$

coefficient $C_{D_x^2 D_y^2 v_1}^{(0)}$ **at** $\frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2}$:

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{SRT}} = 0$$

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{MRT1}} = (-\omega_7 + \omega_4) \frac{v_1 \rho c s^2}{2\omega_7 \omega_4}$$

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{MRT2}} = (-\omega_7 + \omega_4) \frac{v_1 \rho c s^2}{2\omega_7 \omega_4}$$

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{CLBM1}} = 0$$

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{CLBM2}} = 0$$

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{CuLBM1}} = 0$$

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{CuLBM2}} = (-v_1^2 \omega_2 - 3c s^2 \omega_2 - \omega_1 + 3\omega_1 c s^2 + v_1^2 \omega_1 + \omega_2) \frac{v_1 \rho}{24\omega_1 \omega_2}$$

coefficient $C_{D_x^2 D_y^2 v_2}^{(0)}$ **at** $\frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2}$:

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{SRT}} = 0$$

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{MRT1}} = (\omega_4 - \omega_8) \frac{\rho c s^2 v_2}{2\omega_4 \omega_8}$$

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{MRT2}} = (\omega_4 - \omega_8) \frac{\rho c s^2 v_2}{2\omega_4 \omega_8}$$

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{CLBM1}} = 0$$

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{CLBM2}} = 0$$

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{CuLBM1}} = 0$$

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{CuLBM2}} = (\omega_1 v_2^2 - 3cs^2 \omega_2 - \omega_1 - v_2^2 \omega_2 + 3\omega_1 cs^2 + \omega_2) \frac{\rho v_2}{24\omega_1 \omega_2}$$

coefficient $C_{D_x D_y^3 \rho}^{(0)}$ **at** $\frac{\partial^4 \rho}{\partial x_1 \partial x_2^3}$:

$$C_{D_x D_y^3 \rho}^{(0), \text{SRT}} = 0$$

$$C_{D_x D_y^3 \rho}^{(0), \text{MRT1}} = (\omega_6 + 3cs^2 \omega_8 - 3\omega_6 cs^2 + v_2^2 \omega_8 - \omega_6 v_2^2 - \omega_8) \frac{v_1 v_2}{4\omega_6 \omega_8}$$

$$C_{D_x D_y^3 \rho}^{(0), \text{MRT2}} = (\omega_6 + v_2^2 \omega_8 - \omega_6 v_2^2 - 3\omega_6 cs^2 - \omega_8 + 3cs^2 \omega_8) \frac{v_1 v_2}{4\omega_6 \omega_8}$$

$$C_{D_x D_y^3 \rho}^{(0), \text{CLBM1}} = 0$$

$$C_{D_x D_y^3 \rho}^{(0), \text{CLBM2}} = 0$$

$$C_{D_x D_y^3 \rho}^{(0), \text{CuLBM1}} = 0$$

$$C_{D_x D_y^3 \rho}^{(0), \text{CuLBM2}} = (-v_1^2 \omega_2 - 3cs^2 \omega_2 - \omega_1 + 3\omega_1 cs^2 + v_1^2 \omega_1 + \omega_2) \frac{v_1 v_2}{8\omega_1 \omega_2}$$

coefficient $C_{D_x D_y^3 v_1}^{(0)}$ **at** $\frac{\partial^4 v_1}{\partial x_1 \partial x_2^3}$:

$$C_{D_x D_y^3 v_1}^{(0), \text{SRT}} = (2 - 6cs^2 + v_2^2 \omega + 3\omega cs^2 - 2v_2^2 - \omega) \frac{\rho v_2}{12\omega}$$

$$C_{D_x D_y^3 v_1}^{(0), \text{MRT1}} = (-3\omega_6 \omega_4 v_2^2 - 6\omega_6 cs^2 \omega_8 - \omega_6 \omega_4 \omega_8 - \omega_4 \omega_8 + 3\omega_6 \omega_4 + \omega_6 \omega_4 v_2^2 \omega_8 - 3\omega_6 cs^2 \omega_4 + \omega_4 v_2^2 \omega_8 + 3cs^2 \omega_4 \omega_8 + 3\omega_6 cs^2 \omega_4 \omega_8) \frac{\rho v_2}{12\omega_6 \omega_4 \omega_8}$$

$$C_{D_x D_y^3 v_1}^{(0), \text{MRT2}} = (-3\omega_6 \omega_4 v_2^2 - 3\omega_6 \omega_4 cs^2 - \omega_6 \omega_4 \omega_8 - \omega_4 \omega_8 + 3\omega_6 \omega_4 + \omega_6 \omega_4 v_2^2 \omega_8 + \omega_4 v_2^2 \omega_8 + 3\omega_4 cs^2 \omega_8 + 3\omega_6 \omega_4 cs^2 \omega_8 - 6\omega_6 cs^2 \omega_8) \frac{\rho v_2}{12\omega_6 \omega_4 \omega_8}$$

$$C_{D_x D_y^3 v_1}^{(0), \text{CLBM1}} = (-9\omega_6 cs^2 - \omega_6 \omega_8 + 3\omega_6 + v_2^2 \omega_8 + 3\omega_6 \omega_8 cs^2 + \omega_6 v_2^2 \omega_8 + 3\omega_8 cs^2 - 3\omega_6 v_2^2 - \omega_8) \frac{\rho v_2}{12\omega_6 \omega_8}$$

$$C_{D_x D_y^3 v_1}^{(0), \text{CLBM2}} = (-\omega_6 \omega_8 + 3\omega_6 + 3cs^2 \omega_8 + 3\omega_6 cs^2 \omega_8 - 9\omega_6 cs^2 + v_2^2 \omega_8 + \omega_6 v_2^2 \omega_8 - 3\omega_6 v_2^2 - \omega_8) \frac{\rho v_2}{12\omega_6 \omega_8}$$

$$C_{D_x D_y^3 v_1}^{(0), \text{CuLBM1}} = (-\omega_6 + 3\omega_6 cs^2 \omega_2 - 9cs^2 \omega_2 - \omega_6 \omega_2 + \omega_6 v_2^2 + \omega_6 v_2^2 \omega_2 - 3v_2^2 \omega_2 + 3\omega_6 cs^2 + 3\omega_2) \frac{\rho v_2}{12\omega_6 \omega_2}$$

$$C_{D_x D_y^3 v_1}^{(0), \text{CuLBM2}} = (2\omega_3 \omega_2 + 2\omega_3 \omega_1 v_2^2 \omega_2 - 18\omega_1 cs^2 \omega_2 - 2\omega_3 \omega_1 \omega_2 + 6\omega_3 \omega_1 cs^2 \omega_2 + 6\omega_3 \omega_1 cs^2 + 9\omega_3 v_1^2 \omega_1 - 6\omega_1 v_2^2 \omega_2 + \omega_3 v_2^2 \omega_2 + 6\omega_1 \omega_2 - 9\omega_3 v_1^2 \omega_2 - 4\omega_3 \omega_1 + \omega_3 \omega_1 v_2^2) \frac{\rho v_2}{24\omega_3 \omega_1 \omega_2}$$

coefficient $C_{D_x D_y^3 v_2}^{(0)}$ **at** $\frac{\partial^4 v_2}{\partial x_1 \partial x_2^3}$:

$$C_{D_x D_y^3 v_2}^{(0), \text{SRT}} = 0$$

$$C_{D_x D_y^3 v_2}^{(0), \text{MRT1}} = (\omega_6 + cs^2 \omega_8 - \omega_6 cs^2 + 3v_2^2 \omega_8 - 3\omega_6 v_2^2 - \omega_8) \frac{v_1 \rho}{4\omega_6 \omega_8}$$

$$C_{D_x D_y^3 v_2}^{(0), \text{MRT2}} = (\omega_6 + 3v_2^2 \omega_8 - 3\omega_6 v_2^2 - \omega_6 cs^2 - \omega_8 + cs^2 \omega_8) \frac{v_1 \rho}{4\omega_6 \omega_8}$$

$$C_{D_x D_y^3 v_2}^{(0), \text{CLBM1}} = 0$$

$$C_{D_x D_y^3 v_2}^{(0), \text{CLBM2}} = 0$$

$$C_{D_x D_y^3 v_2}^{(0), \text{CuLBM1}} = 0$$

$$C_{D_x D_y^3 v_2}^{(0), \text{CuLBM2}} = (-v_1^2 \omega_2 - 3cs^2 \omega_2 - \omega_1 + 3\omega_1 cs^2 + v_1^2 \omega_1 + \omega_2) \frac{v_1 \rho}{24\omega_1 \omega_2}$$

coefficient $C_{D_y^4 \rho}^{(0)}$ **at** $\frac{\partial^4 \rho}{\partial x_2^4}$:

$$C_{D_y^4 \rho}^{(0), \text{SRT}} = (6v_2^4 - 3v_2^4 \omega - \omega cs^4 - 2cs^2 + 3v_2^2 \omega - 12v_2^2 \omega cs^2 + 2cs^4 + \omega cs^2 + 24v_2^2 cs^2 - 6v_2^2) \frac{1}{24\omega}$$

$$C_{D_y^4 \rho}^{(0), \text{MRT1}} = (-2cs^2 + 6v_2^4 + \omega_6 cs^2 - 3\omega_6 v_2^4 - \omega_6 cs^4 + 3\omega_6 v_2^2 - 12\omega_6 cs^2 v_2^2 + 24cs^2 v_2^2 + 2cs^4 - 6v_2^2) \frac{1}{24\omega_6}$$

$$C_{D_y^4 \rho}^{(0), \text{MRT2}} = (6v_2^4 + 2cs^4 - 3\omega_6 v_2^4 + 24cs^2 v_2^2 - \omega_6 cs^4 - 12\omega_6 cs^2 v_2^2 + 3\omega_6 v_2^2 + \omega_6 cs^2 - 6v_2^2 - 2cs^2) \frac{1}{24\omega_6}$$

$$C_{D_y^4 \rho}^{(0), \text{CLBM1}} = (\omega_6 cs^2 + 6v_2^4 - 3\omega_6 v_2^4 - 2cs^2 + 2cs^4 + 24v_2^2 cs^2 - 12\omega_6 v_2^2 cs^2 + 3\omega_6 v_2^2 - 6v_2^2 - \omega_6 cs^4) \frac{1}{24\omega_6}$$

$$C_{D_y^4 \rho}^{(0), \text{CLBM2}} = (6v_2^4 - 2cs^2 - 3\omega_6 v_2^4 + \omega_6 cs^2 + 3\omega_6 v_2^2 - \omega_6 cs^4 - 12\omega_6 cs^2 v_2^2 - 6v_2^2 + 24cs^2 v_2^2 + 2cs^4) \frac{1}{24\omega_6}$$

$$C_{D_y^4 \rho}^{(0), \text{CuLBM1}} = (6v_2^4 + cs^2 \omega_2 + 2cs^4 - 3v_2^4 \omega_2 + 24cs^2 v_2^2 - 2cs^2 + 3v_2^2 \omega_2 - 6v_2^2 - 12cs^2 v_2^2 \omega_2 - cs^4 \omega_2) \frac{1}{24\omega_2}$$

$$C_{D_y^4 \rho}^{(0), \text{CuLBM2}} = (12\omega_1 v_2^2 cs^2 + \omega_1 cs^2 \omega_2 + cs^4 \omega_2 + \omega_1 cs^4 - 3\omega_1 v_2^2 + 3\omega_1 v_2^2 \omega_2 + 3v_2^4 \omega_2 - \omega_1 cs^4 \omega_2 - cs^2 \omega_2 + 3\omega_1 v_2^4 + 12v_2^2 cs^2 \omega_2 - 3\omega_1 v_2^4 \omega_2 - 3v_2^2 \omega_2 - \omega_1 cs^2 - 12\omega_1 v_2^2 cs^2 \omega_2) \frac{1}{24\omega_1 \omega_2}$$

coefficient $C_{D_y^4 v_2}^{(0)}$ **at** $\frac{\partial^4 v_2}{\partial x_2^4}$:

$$C_{D_y^4 v_2}^{(0), \text{SRT}} = (-4 + 6cs^2 - 5v_2^2 \omega - 3\omega cs^2 + 10v_2^2 + 2\omega) \frac{\rho v_2}{12\omega}$$

$$C_{D_y^4 v_2}^{(0), \text{MRT1}} = (-4 + 6cs^2 + 2\omega_6 - 3\omega_6 cs^2 - 5\omega_6 v_2^2 + 10v_2^2) \frac{\rho v_2}{12\omega_6}$$

$$C_{D_y^4 v_2}^{(0), \text{MRT2}} = (-4 + 2\omega_6 - 5\omega_6 v_2^2 - 3\omega_6 cs^2 + 10v_2^2 + 6cs^2) \frac{\rho v_2}{12\omega_6}$$

$$C_{D_y^4 v_2}^{(0), \text{CLBM1}} = (-4 - 3\omega_6 cs^2 + 2\omega_6 + 6cs^2 - 5\omega_6 v_2^2 + 10v_2^2) \frac{\rho v_2}{12\omega_6}$$

$$C_{D_y^4 v_2}^{(0), \text{CLBM2}} = (-4 + 6cs^2 + 2\omega_6 - 3\omega_6 cs^2 - 5\omega_6 v_2^2 + 10v_2^2) \frac{\rho v_2}{12\omega_6}$$

$$C_{D_y^4 v_2}^{(0), \text{CuLBM1}} = (-4 - 3cs^2 \omega_2 + 6cs^2 - 5v_2^2 \omega_2 + 10v_2^2 + 2\omega_2) \frac{\rho v_2}{12\omega_2}$$

$$C_{D_y^4 v_2}^{(0), \text{CuLBM2}} = (-3\omega_1 cs^2 \omega_2 + 5\omega_1 v_2^2 - 5\omega_1 v_2^2 \omega_2 + 3cs^2 \omega_2 + 2\omega_1 \omega_2 - 2\omega_1 + 5v_2^2 \omega_2 + 3\omega_1 cs^2 - 2\omega_2) \frac{\rho v_2}{12\omega_1 \omega_2}$$

3.2 Conservation of momentum: ρv_1

$$\begin{aligned} & v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + C_{D_x \rho}^{(1)} \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + 2v_1 \rho \frac{\delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + v_1 v_2 \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \rho v_2 \frac{\delta_l}{\delta_t} \frac{\partial v_1}{\partial x_2} + v_1 \rho \frac{\delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + C_{D_x \rho, D_x v_1}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + \\ & C_{D_x v_1, D_x v_1}^{(1)} \frac{\delta_l^2}{\delta_t} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + C_{D_x \rho, D_y v_2}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_2} + C_{D_x v_2, D_y v_2}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial v_2}{\partial x_1} \frac{\partial v_2}{\partial x_2} + C_{D_y \rho, D_x v_2}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\ & C_{D_y \rho, D_y v_1}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + C_{D_x^2 \rho}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 \rho}{\partial x_1^2} + C_{D_x^2 v_1}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 v_1}{\partial x_1^2} + C_{D_x D_y \rho}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 \rho}{\partial x_1 \partial x_2} + C_{D_x D_y v_2}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + C_{D_y^2 v_1}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 v_1}{\partial x_2^2} + \\ & + C_{D_x^3 \rho}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + C_{D_x^3 v_1}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + C_{D_x^2 D_y \rho}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + C_{D_x^2 D_y v_1}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} + C_{D_x^2 D_y v_2}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + \\ & C_{D_x D_y^2 \rho}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + C_{D_x D_y^2 v_1}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_{D_x D_y^2 v_2}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_{D_y^3 \rho}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + C_{D_y^3 v_1}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_2^3} + C_{D_y^3 v_2}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & C_{D_x^4 \rho}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + C_{D_x^4 v_1}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_{D_x^3 D_y \rho}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{D_x^3 D_y v_1}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + C_{D_x^3 D_y v_2}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_{D_x^2 D_y^2 \rho}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\ & + C_{D_x^2 D_y^2 v_1}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{D_x^2 D_y^2 v_2}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_{D_x D_y^3 \rho}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{D_x D_y^3 v_1}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{D_x D_y^3 v_2}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \end{aligned}$$

$$C_{D_y^4 \rho}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{D_y^4 v_1}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_2^4} + C_{D_y^4 v_2}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,$$

where:

coefficient $C_{D_x \rho}^{(1)}$ **at** $\frac{\partial \rho}{\partial x_1}$:

$$C_{D_x \rho}^{(1), \text{SRT}} = (v_1^2 + cs^2)$$

$$C_{D_x \rho}^{(1), \text{MRT1}} = (v_1^2 + cs^2)$$

$$C_{D_x \rho}^{(1), \text{MRT2}} = (v_1^2 + cs^2)$$

$$C_{D_x \rho}^{(1), \text{CLBM1}} = (v_1^2 + cs^2)$$

$$C_{D_x \rho}^{(1), \text{CLBM2}} = (v_1^2 + cs^2)$$

$$C_{D_x \rho}^{(1), \text{CuLBM1}} = (v_1^2 + cs^2)$$

$$C_{D_x \rho}^{(1), \text{CuLBM2}} = (v_1^2 + cs^2)$$

coefficient $C_{D_x \rho, D_x v_1}^{(1)}$ **at** $\frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1}$:

$$C_{D_x \rho, D_x v_1}^{(1), \text{SRT}} = (-2 - 2cs^2\omega + 6v_1^2 - 3v_1^2\omega + 4cs^2 + \omega) \frac{1}{\omega}$$

$$C_{D_x \rho, D_x v_1}^{(1), \text{MRT1}} = (-2 + 6v_1^2 - 3v_1^2\omega_5 + 4cs^2 + \omega_5 - 2cs^2\omega_5) \frac{1}{\omega_5}$$

$$C_{D_x \rho, D_x v_1}^{(1), \text{MRT2}} = (-2 + 6v_1^2 + 4cs^2 - 2cs^2\omega_5 - 3v_1^2\omega_5 + \omega_5) \frac{1}{\omega_5}$$

$$C_{D_x \rho, D_x v_1}^{(1), \text{CLBM1}} = (-2 - 2cs^2\omega_5 + 6v_1^2 - 3v_1^2\omega_5 + 4cs^2 + \omega_5) \frac{1}{\omega_5}$$

$$C_{D_x \rho, D_x v_1}^{(1), \text{CLBM2}} = (-2 + 6v_1^2 - 3v_1^2\omega_5 + 4cs^2 - 2cs^2\omega_5 + \omega_5) \frac{1}{\omega_5}$$

$$C_{D_x \rho, D_x v_1}^{(1), \text{CuLBM1}} = (-2 + 6v_1^2 - 2\omega_1 cs^2 + \omega_1 + 4cs^2 - 3v_1^2\omega_1) \frac{1}{\omega_1}$$

$$C_{D_x \rho, D_x v_1}^{(1), \text{CuLBM2}} = (3v_1^2\omega_2 - 3v_1^2\omega_1\omega_2 + 2cs^2\omega_2 - 2cs^2\omega_1\omega_2 + \omega_1\omega_2 - \omega_1 + 2cs^2\omega_1 + 3v_1^2\omega_1 - \omega_2) \frac{1}{\omega_1\omega_2}$$

coefficient $C_{D_x v_1, D_x v_1}^{(1)}$ **at** $\left(\frac{\partial v_1}{\partial x_1}\right)^2$:

$$C_{D_x v_1, D_x v_1}^{(1), \text{SRT}} = (2 - \omega) \frac{3v_1\rho}{\omega}$$

$$C_{D_x v_1, D_x v_1}^{(1), \text{MRT1}} = (2 - \omega_5) \frac{3v_1\rho}{\omega_5}$$

$$C_{D_x v_1, D_x v_1}^{(1), \text{MRT2}} = C_{D_x v_1, D_x v_1}^{(1), \text{MRT1}}$$

$$C_{D_x v_1, D_x v_1}^{(1), \text{CLBM1}} = C_{D_x v_1, D_x v_1}^{(1), \text{MRT1}}$$

$$C_{D_x v_1, D_x v_1}^{(1), \text{CLBM2}} = C_{D_x v_1, D_x v_1}^{(1), \text{MRT1}}$$

$$C_{D_x v_1, D_x v_1}^{(1), \text{CuLBM1}} = (2 - \omega_1) \frac{3v_1\rho}{\omega_1}$$

$$C_{D_x v_1, D_x v_1}^{(1), \text{CuLBM2}} = (-\omega_1\omega_2 + \omega_1 + \omega_2) \frac{3v_1\rho}{\omega_1\omega_2}$$

coefficient $C_{D_x \rho, D_y v_2}^{(1)}$ **at** $\frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_2}$:

$$C_{D_x \rho, D_y v_2}^{(1), \text{SRT}} = 0$$

$$C_{D_x \rho, D_y v_2}^{(1), \text{MRT1}} = 0$$

$$C_{D_x \rho, D_y v_2}^{(1), \text{MRT2}} = 0$$

$$C_{D_x \rho, D_y v_2}^{(1), \text{CLBM1}} = 0$$

$$C_{D_x \rho, D_y v_2}^{(1), \text{CLBM2}} = 0$$

$$C_{D_x \rho, D_y v_2}^{(1), \text{CuLBM1}} = 0$$

$$C_{D_x \rho, D_y v_2}^{(1), \text{CuLBM2}} = (-cs^2\omega_2 + 3\omega_1 v_2^2 - \omega_1 - 3v_2^2\omega_2 + cs^2\omega_1 + \omega_2) \frac{1}{2\omega_1\omega_2}$$

coefficient $C_{D_x v_2, D_y v_2}^{(1)}$ **at** $\frac{\partial v_2}{\partial x_1} \frac{\partial v_2}{\partial x_2}$:

$$C_{D_x v_2, D_y v_2}^{(1), \text{SRT}} = 0$$

$$C_{D_x v_2, D_y v_2}^{(1), \text{MRT1}} = 0$$

$$C_{D_x v_2, D_y v_2}^{(1), \text{MRT2}} = 0$$

$$C_{D_x v_2, D_y v_2}^{(1), \text{CLBM1}} = 0$$

$$C_{D_x v_2, D_y v_2}^{(1), \text{CLBM2}} = 0$$

$$C_{D_x v_2, D_y v_2}^{(1), \text{CuLBM1}} = 0$$

$$C_{D_x v_2, D_y v_2}^{(1), \text{CuLBM2}} = (\omega_1 - \omega_2) \frac{3\rho v_2}{\omega_1\omega_2}$$

coefficient $C_{D_y \rho, D_x v_2}^{(1)}$ **at** $\frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1}$:

$$C_{D_y \rho, D_x v_2}^{(1), \text{SRT}} = (-2 + \omega) \frac{cs^2}{2\omega}$$

$$C_{D_y \rho, D_x v_2}^{(1), \text{MRT1}} = (-2 + \omega_4) \frac{cs^2}{2\omega_4}$$

$$C_{D_y \rho, D_x v_2}^{(1), \text{MRT2}} = (-2 + \omega_4) \frac{cs^2}{2\omega_4}$$

$$C_{D_y \rho, D_x v_2}^{(1), \text{CLBM1}} = (-2 + \omega_4) \frac{cs^2}{2\omega_4}$$

$$C_{D_y \rho, D_x v_2}^{(1), \text{CLBM2}} = (-2 + \omega_4) \frac{cs^2}{2\omega_4}$$

$$C_{D_y \rho, D_x v_2}^{(1), \text{CuLBM1}} = (-2 + \omega_3) \frac{cs^2}{2\omega_3}$$

$$C_{D_y \rho, D_x v_2}^{(1), \text{CuLBM2}} = (-5cs^2\omega_2 + 3\omega_1 v_2^2 + cs^2\omega_1\omega_2 - \omega_1 - 3v_2^2\omega_2 + 3cs^2\omega_1 + \omega_2) \frac{1}{2\omega_1\omega_2}$$

coefficient $C_{D_y \rho, D_y v_1}^{(1)}$ **at** $\frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2}$:

$$C_{D_y \rho, D_y v_1}^{(1), \text{SRT}} = (-2 + \omega) \frac{cs^2}{2\omega}$$

$$C_{D_y \rho, D_y v_1}^{(1), \text{MRT1}} = (-2 + \omega_4) \frac{cs^2}{2\omega_4}$$

$$C_{D_y \rho, D_y v_1}^{(1), \text{MRT2}} = (-2 + \omega_4) \frac{cs^2}{2\omega_4}$$

$$C_{D_y \rho, D_y v_1}^{(1), \text{CLBM1}} = (-2 + \omega_4) \frac{cs^2}{2\omega_4}$$

$$C_{D_y \rho, D_y v_1}^{(1), \text{CLBM2}} = (-2 + \omega_4) \frac{cs^2}{2\omega_4}$$

$$C_{D_y \rho, D_y v_1}^{(1), \text{CuLBM1}} = (-2 + \omega_3) \frac{cs^2}{2\omega_3}$$

$$C_{D_y \rho, D_y v_1}^{(1), \text{CuLBM2}} = (-2 + \omega_1) \frac{cs^2}{2\omega_1}$$

coefficient $C_{D_x^2 \rho}^{(1)}$ **at** $\frac{\partial^2 \rho}{\partial x_1^2}$:

$$C_{D_x^2 \rho}^{(1), \text{SRT}} = (-2 - 3cs^2\omega + 2v_1^2 - v_1^2\omega + 6cs^2 + \omega) \frac{v_1}{2\omega}$$

$$C_{D_x^2 \rho}^{(1), \text{MRT1}} = (-2 + 2v_1^2 - v_1^2\omega_5 + 6cs^2 + \omega_5 - 3cs^2\omega_5) \frac{v_1}{2\omega_5}$$

$$C_{D_x^2 \rho}^{(1), \text{MRT2}} = (-2 + 2v_1^2 + 6cs^2 - 3cs^2\omega_5 - v_1^2\omega_5 + \omega_5) \frac{v_1}{2\omega_5}$$

$$C_{D_x^2 \rho}^{(1), \text{CLBM1}} = (-2 - 3cs^2\omega_5 + 2v_1^2 - v_1^2\omega_5 + 6cs^2 + \omega_5) \frac{v_1}{2\omega_5}$$

$$C_{D_x^2 \rho}^{(1), \text{CLBM2}} = (-2 + 2v_1^2 - v_1^2\omega_5 + 6cs^2 - 3cs^2\omega_5 + \omega_5) \frac{v_1}{2\omega_5}$$

$$C_{D_x^2 \rho}^{(1), \text{CuLBM1}} = (-2 + 2v_1^2 - 3\omega_1 cs^2 + \omega_1 + 6cs^2 - v_1^2\omega_1) \frac{v_1}{2\omega_1}$$

$$C_{D_x^2 \rho}^{(1), \text{CuLBM2}} = (v_1^2\omega_2 - v_1^2\omega_1\omega_2 + 3cs^2\omega_2 - 3cs^2\omega_1\omega_2 + \omega_1\omega_2 - \omega_1 + 3cs^2\omega_1 + v_1^2\omega_1 - \omega_2) \frac{v_1}{2\omega_1\omega_2}$$

coefficient $C_{D_x^2 v_1}^{(1)}$ **at** $\frac{\partial^2 v_1}{\partial x_1^2}$:

$$C_{D_x^2 v_1}^{(1), \text{SRT}} = (-2 - cs^2\omega + 6v_1^2 - 3v_1^2\omega + 2cs^2 + \omega) \frac{\rho}{2\omega}$$

$$C_{D_x^2 v_1}^{(1), \text{MRT1}} = (-2 + 6v_1^2 - 3v_1^2\omega_5 + 2cs^2 + \omega_5 - cs^2\omega_5) \frac{\rho}{2\omega_5}$$

$$C_{D_x^2 v_1}^{(1), \text{MRT2}} = (-2 + 6v_1^2 + 2cs^2 - cs^2\omega_5 - 3v_1^2\omega_5 + \omega_5) \frac{\rho}{2\omega_5}$$

$$C_{D_x^2 v_1}^{(1), \text{CLBM1}} = (-2 - cs^2\omega_5 + 6v_1^2 - 3v_1^2\omega_5 + 2cs^2 + \omega_5) \frac{\rho}{2\omega_5}$$

$$C_{D_x^2 v_1}^{(1), \text{CLBM2}} = (-2 + 6v_1^2 - 3v_1^2\omega_5 + 2cs^2 - cs^2\omega_5 + \omega_5) \frac{\rho}{2\omega_5}$$

$$C_{D_x^2 v_1}^{(1), \text{CuLBM1}} = (-2 + 6v_1^2 - \omega_1 cs^2 + \omega_1 + 2cs^2 - 3v_1^2\omega_1) \frac{\rho}{2\omega_1}$$

$$C_{D_x^2 v_1}^{(1), \text{CuLBM2}} = (3v_1^2\omega_2 - 3v_1^2\omega_1\omega_2 + cs^2\omega_2 - cs^2\omega_1\omega_2 + \omega_1\omega_2 - \omega_1 + cs^2\omega_1 + 3v_1^2\omega_1 - \omega_2) \frac{\rho}{2\omega_1\omega_2}$$

coefficient $C_{D_x D_y \rho}^{(1)}$ **at** $\frac{\partial^2 \rho}{\partial x_1 \partial x_2}$:

$$C_{D_x D_y \rho}^{(1), \text{SRT}} = 0$$

$$C_{D_x D_y \rho}^{(1), \text{MRT1}} = 0$$

$$C_{D_x D_y \rho}^{(1), \text{MRT2}} = 0$$

$$C_{D_x D_y \rho}^{(1), \text{CLBM1}} = 0$$

$$C_{D_x D_y \rho}^{(1), \text{CLBM2}} = 0$$

$$C_{D_x D_y \rho}^{(1), \text{CuLBM1}} = 0$$

$$C_{D_x D_y \rho}^{(1), \text{CuLBM2}} = (-3cs^2\omega_2 + \omega_1 v_2^2 - \omega_1 - v_2^2\omega_2 + 3cs^2\omega_1 + \omega_2) \frac{v_2}{2\omega_1\omega_2}$$

coefficient $C_{D_x D_y v_2}^{(1)}$ **at** $\frac{\partial^2 v_2}{\partial x_1 \partial x_2}$:

$$C_{D_x D_y v_2}^{(1), \text{SRT}} = (-2 + \omega) \frac{cs^2 \rho}{2\omega}$$

$$C_{D_x D_y v_2}^{(1), \text{MRT1}} = (-2 + \omega_4) \frac{\rho c s^2}{2\omega_4}$$

$$C_{D_x D_y v_2}^{(1), \text{MRT2}} = (-2 + \omega_4) \frac{\rho c s^2}{2\omega_4}$$

$$C_{D_x D_y v_2}^{(1), \text{CLBM1}} = (-2 + \omega_4) \frac{c s^2 \rho}{2\omega_4}$$

$$C_{D_x D_y v_2}^{(1), \text{CLBM2}} = (-2 + \omega_4) \frac{\rho c s^2}{2\omega_4}$$

$$C_{D_x D_y v_2}^{(1), \text{CuLBM1}} = (-2 + \omega_3) \frac{\rho c s^2}{2\omega_3}$$

$$C_{D_x D_y v_2}^{(1), \text{CuLBM2}} = (-3cs^2\omega_2 + 3\omega_1 v_2^2 + cs^2\omega_1\omega_2 - \omega_1 - 3v_2^2\omega_2 + cs^2\omega_1 + \omega_2) \frac{\rho}{2\omega_1\omega_2}$$

coefficient $C_{D_y^2 v_1}^{(1)}$ **at** $\frac{\partial^2 v_1}{\partial x_2^2}$:

$$C_{D_y^2 v_1}^{(1), \text{SRT}} = (-2 + \omega) \frac{c s^2 \rho}{2\omega}$$

$$C_{D_y^2 v_1}^{(1), \text{MRT1}} = (-2 + \omega_4) \frac{\rho c s^2}{2\omega_4}$$

$$C_{D_y^2 v_1}^{(1), \text{MRT2}} = (-2 + \omega_4) \frac{\rho c s^2}{2\omega_4}$$

$$C_{D_y^2 v_1}^{(1), \text{CLBM1}} = (-2 + \omega_4) \frac{c s^2 \rho}{2\omega_4}$$

$$C_{D_y^2 v_1}^{(1), \text{CLBM2}} = (-2 + \omega_4) \frac{\rho c s^2}{2\omega_4}$$

$$C_{D_y^2 v_1}^{(1), \text{CuLBM1}} = (-2 + \omega_3) \frac{\rho c s^2}{2\omega_3}$$

$$C_{D_y^2 v_1}^{(1), \text{CuLBM2}} = (-2 + \omega_1) \frac{c s^2 \rho}{2\omega_1}$$

coefficient $C_{D_x^3 \rho}^{(1)}$ **at** $\frac{\partial^3 \rho}{\partial x_1^3}$:

$$C_{D_x^3 \rho}^{(1), \text{SRT}} = (12cs^2\omega + 24cs^2v_1^2\omega^2 - 36v_1^2 - 7v_1^2\omega^2 + 36v_1^2\omega + 12cs^4 - 144cs^2v_1^2\omega - cs^2\omega^2 + 144cs^2v_1^2 - 12cs^2 + 7v_1^4\omega^2 - 12cs^4\omega + cs^4\omega^2 - 36v_1^4\omega + 36v_1^4) \frac{1}{12\omega^2}$$

$$C_{D_x^3 \rho}^{(1), \text{MRT1}} = (-7v_1^2\omega_5^2 + 24v_1^2cs^2\omega_5^2 - 36v_1^2 + cs^4\omega_5^2 - 144v_1^2cs^2\omega_5 - 12cs^4\omega_5 + 36v_1^2\omega_5 - 12cs^2 + 144v_1^2cs^2 - cs^2\omega_5^2 + 7v_1^4\omega_5^2 + 12cs^4 - 36v_1^4\omega_5 + 12cs^2\omega_5 + 36v_1^4) \frac{1}{12\omega_5^2}$$

$$C_{D_x^3 \rho}^{(1), \text{MRT2}} = (-7v_1^2\omega_5^2 - 36v_1^2 + 144v_1^2cs^2 - cs^2\omega_5^2 - 12cs^2 + 12cs^2\omega_5 + 36v_1^2\omega_5 + 24v_1^2cs^2\omega_5^2 + cs^4\omega_5^2 + 7v_1^4\omega_5^2 - 36v_1^4\omega_5 - 144v_1^2cs^2\omega_5 - 12cs^4\omega_5 + 12cs^4 + 36v_1^4) \frac{1}{12\omega_5^2}$$

$$C_{D_x^3 \rho}^{(1), \text{CLBM1}} = (-7v_1^2\omega_5^2 + 12cs^2\omega_5 - 36v_1^2 - 144cs^2v_1^2\omega_5 + 12cs^4 + 144cs^2v_1^2 + 24cs^2v_1^2\omega_5^2 + 36v_1^2\omega_5 - cs^2\omega_5^2 - 12cs^2 - 12cs^4\omega_5 + 7v_1^4\omega_5^2 + cs^4\omega_5^2 - 36v_1^4\omega_5 + 36v_1^4) \frac{1}{12\omega_5^2}$$

$$C_{D_x^3 \rho}^{(1), \text{CLBM2}} = (cs^4\omega_5^2 + 24v_1^2cs^2\omega_5^2 - 7v_1^2\omega_5^2 - 36v_1^2 + 36v_1^2\omega_5 + 144v_1^2cs^2 - 12cs^2 - 12cs^4\omega_5 - 144v_1^2cs^2\omega_5 + 12cs^4 - cs^2\omega_5^2 + 7v_1^4\omega_5^2 - 36v_1^4\omega_5 + 12cs^2\omega_5 + 36v_1^4) \frac{1}{12\omega_5^2}$$

$$C_{D_x^3 \rho}^{(1), \text{CuLBM1}} = (12cs^4 + 24v_1^2\omega_1^2cs^2 - 36v_1^2 + 7v_1^4\omega_1^2 - 36v_1^4\omega_1 + 12\omega_1cs^2 + \omega_1^2cs^4 - 12\omega_1cs^4 - 7v_1^2\omega_1^2 - \omega_1^2cs^2 + 144v_1^2cs^2 - 12cs^2 + 36v_1^2\omega_1 + 36v_1^4 - 144v_1^2\omega_1cs^2) \frac{1}{12\omega_1^2}$$

$$C_{D_x^3 \rho}^{(1), \text{CuLBM2}} = (6cs^4\omega_1^2 + 24cs^2v_1^2\omega_1^2\omega_2^2 - 18v_1^4\omega_1\omega_2^2 - 18v_1^2\omega_1\omega_2 + 6cs^2\omega_1\omega_2^2 + 9v_1^4\omega_1^2 + 45cs^2v_1^2\omega_1^2 - 6cs^2\omega_2^2 + 18v_1^2\omega_1\omega_2^2 + 18v_1^4\omega_1\omega_2 - 6cs^4\omega_1\omega_2^2 - 9v_1^2\omega_2^2 - 72cs^2v_1^2\omega_1^2\omega_2 + cs^4\omega_1^2\omega_2^2 - 18v_1^4\omega_1^2\omega_2 + 54cs^2v_1^2\omega_1\omega_2 - 9v_1^2\omega_1^2 - 6cs^2\omega_1^2 + 45cs^2v_1^2\omega_2^2 - 7v_1^2\omega_1^2\omega_2^2 + 6cs^2\omega_1^2\omega_2 - cs^2\omega_1^2\omega_2^2 + 18v_1^2\omega_1^2\omega_2 + 9v_1^4\omega_2^2 + 6cs^4\omega_2^2 - 72cs^2v_1^2\omega_1\omega_2^2 + 7v_1^4\omega_1^2\omega_2^2 - 6cs^4\omega_1^2\omega_2) \frac{1}{12\omega_1^2\omega_2^2}$$

coefficient $C_{D_x^3 v_1}^{(1)}$ **at** $\frac{\partial^3 v_1}{\partial x_1^3}$:

$$C_{D_x^3 v_1}^{(1), \text{SRT}} = (-24 - 36cs^2\omega + 60v_1^2 - 4\omega^2 + 11v_1^2\omega^2 - 60v_1^2\omega + 5cs^2\omega^2 + 36cs^2 + 24\omega) \frac{v_1\rho}{6\omega^2}$$

$$C_{D_x^3 v_1}^{(1), \text{MRT1}} = (-24 - 4\omega_5^2 + 11v_1^2\omega_5^2 + 60v_1^2 - 60v_1^2\omega_5 + 36cs^2 + 5cs^2\omega_5^2 + 24\omega_5 - 36cs^2\omega_5) \frac{v_1\rho}{6\omega_5^2}$$

$$C_{D_x^3 v_1}^{(1), \text{MRT2}} = (-24 - 4\omega_5^2 + 11v_1^2\omega_5^2 + 60v_1^2 + 5cs^2\omega_5^2 + 36cs^2 - 36cs^2\omega_5 - 60v_1^2\omega_5 + 24\omega_5) \frac{v_1\rho}{6\omega_5^2}$$

$$C_{D_x^3 v_1}^{(1), \text{CLBM1}} = (-24 - 4\omega_5^2 + 11v_1^2\omega_5^2 - 36cs^2\omega_5 + 60v_1^2 - 60v_1^2\omega_5 + 5cs^2\omega_5^2 + 36cs^2 + 24\omega_5) \frac{v_1\rho}{6\omega_5^2}$$

$$C_{D_x^3 v_1}^{(1), \text{CLBM2}} = (-24 - 4\omega_5^2 + 11v_1^2\omega_5^2 + 60v_1^2 - 60v_1^2\omega_5 + 36cs^2 + 5cs^2\omega_5^2 - 36cs^2\omega_5 + 24\omega_5) \frac{v_1\rho}{6\omega_5^2}$$

$$C_{D_x^3 v_1}^{(1), \text{CuLBM1}} = (-24 + 60v_1^2 - 36\omega_1cs^2 + 11v_1^2\omega_1^2 - 4\omega_1^2 + 24\omega_1 + 5\omega_1^2cs^2 + 36cs^2 - 60v_1^2\omega_1) \frac{v_1\rho}{6\omega_1^2}$$

$$C_{D_x^3 v_1}^{(1), \text{CuLBM2}} = (-8\omega_1^2\omega_2^2 + 54v_1^2\omega_1\omega_2 - 36cs^2\omega_1\omega_2^2 + 27cs^2\omega_2^2 + 18cs^2\omega_1\omega_2 - 60v_1^2\omega_1\omega_2^2 + 24\omega_1^2\omega_2 - 15\omega_2^2 + 33v_1^2\omega_2^2 - 18\omega_1\omega_2 + 33v_1^2\omega_1^2 - 15\omega_1^2 + 27cs^2\omega_1^2 + 22v_1^2\omega_1^2\omega_2^2 - 36cs^2\omega_1^2\omega_2 + 10cs^2\omega_1^2\omega_2^2 - 60v_1^2\omega_1^2\omega_2 + 24\omega_1\omega_2^2) \frac{v_1\rho}{12\omega_1^2\omega_2^2}$$

coefficient $C_{D_x^2 D_y \rho}^{(1)}$ **at** $\frac{\partial^3 \rho}{\partial x_1^2 \partial x_2}$:

$$C_{D_x^2 D_y \rho}^{(1), \text{SRT}} = 0$$

$$C_{D_x^2 D_y \rho}^{(1), \text{MRT1}} = (-3cs^2\omega_7\omega_4\omega_5 - v_1^2\omega_4\omega_5 + \omega_5^2 - v_1^2\omega_5^2 - 3cs^2\omega_4\omega_5 + \omega_7\omega_4\omega_5 + 3cs^2\omega_4\omega_5^2 + 3cs^2\omega_7\omega_4 - v_1^2\omega_7\omega_4\omega_5 + v_1^2\omega_4\omega_5^2 + v_1^2\omega_7\omega_4 - \omega_7\omega_5 - 3cs^2\omega_5^2 + \omega_4\omega_5 - \omega_7\omega_4 + v_1^2\omega_7\omega_5 - \omega_4\omega_5^2 + 3cs^2\omega_7\omega_5) \frac{v_1v_2}{\omega_7\omega_4\omega_5^2}$$

$$C_{D_x^2 D_y \rho}^{(1), \text{MRT2}} = (3\omega_4cs^2\omega_5^2 - v_1^2\omega_4\omega_5 + \omega_5^2 - v_1^2\omega_5^2 + \omega_7\omega_4\omega_5 - 3cs^2\omega_5^2 - 3\omega_7\omega_4cs^2\omega_5 - v_1^2\omega_7\omega_4\omega_5 + 3\omega_7\omega_4cs^2 + v_1^2\omega_4\omega_5^2 - 3\omega_4cs^2\omega_5 + v_1^2\omega_7\omega_4 - \omega_7\omega_5 + \omega_4\omega_5 - \omega_7\omega_4 + v_1^2\omega_7\omega_5 - \omega_4\omega_5^2 + 3\omega_7cs^2\omega_5) \frac{v_1v_2}{\omega_7\omega_4\omega_5^2}$$

$$C_{D_x^2 D_y \rho}^{(1), \text{CLBM1}} = 0$$

$$C_{D_x^2 D_y \rho}^{(1), \text{CLBM2}} = 0$$

$$C_{D_x^2 D_y \rho}^{(1), \text{CuLBM1}} = 0$$

$$C_{D_x^2 D_y \rho}^{(1), \text{CuLBM2}} = (\omega_1v_2^2\omega_2^2 - 2v_1^2\omega_1\omega_2 + 3cs^2\omega_1\omega_2^2 - 6cs^2\omega_1\omega_2 + \omega_1^2\omega_2 + v_1^2\omega_2^2 + 2\omega_1\omega_2 + v_1^2\omega_1^2 - 2\omega_1^2 + 6cs^2\omega_1^2 - \omega_1^2v_2^2\omega_2 + \omega_1^2v_2^2 - 3cs^2\omega_1^2\omega_2 - v_2^2\omega_2^2 - \omega_1\omega_2^2) \frac{3v_1v_2}{4\omega_1^2\omega_2^2}$$

coefficient $C_{D_x^2 D_y v_1}^{(1)}$ **at** $\frac{\partial^3 v_1}{\partial x_1^2 \partial x_2}$:

$$C_{D_x^2 D_y v_1}^{(1), \text{SRT}} = 0$$

$$C_{D_x^2 D_y v_1}^{(1), \text{MRT1}} = (-cs^2\omega_7\omega_4\omega_5 - 3v_1^2\omega_4\omega_5 + \omega_5^2 - 3v_1^2\omega_5^2 - cs^2\omega_4\omega_5 + \omega_7\omega_4\omega_5 + cs^2\omega_4\omega_5^2 + cs^2\omega_7\omega_4 - 3v_1^2\omega_7\omega_4\omega_5 + 3v_1^2\omega_4\omega_5^2 + 3v_1^2\omega_7\omega_4 - \omega_7\omega_5 - cs^2\omega_5^2 + \omega_4\omega_5 - \omega_7\omega_4 + 3v_1^2\omega_7\omega_5 - \omega_4\omega_5^2 + cs^2\omega_7\omega_5) \frac{\rho v_2}{\omega_7\omega_4\omega_5^2}$$

$$C_{D_x^2 D_y v_1}^{(1), \text{MRT2}} = (\omega_4cs^2\omega_5^2 - 3v_1^2\omega_4\omega_5 + \omega_5^2 - 3v_1^2\omega_5^2 + \omega_7\omega_4\omega_5 - cs^2\omega_5^2 - \omega_7\omega_4cs^2\omega_5 - 3v_1^2\omega_7\omega_4\omega_5 + \omega_7\omega_4cs^2 + 3v_1^2\omega_4\omega_5^2 - \omega_4cs^2\omega_5 + 3v_1^2\omega_7\omega_4 - \omega_7\omega_5 + \omega_4\omega_5 - \omega_7\omega_4 + 3v_1^2\omega_7\omega_5 - \omega_4\omega_5^2 + \omega_7cs^2\omega_5) \frac{\rho v_2}{\omega_7\omega_4\omega_5^2}$$

$$C_{D_x^2 D_y v_1}^{(1), \text{CLBM1}} = 0$$

$$C_{D_x^2 D_y v_1}^{(1), \text{CLBM2}} = 0$$

$$C_{D_x^2 D_y v_1}^{(1), \text{CuLBM1}} = 0$$

$$C_{D_x^2 D_y v_1}^{(1), \text{CuLBM2}} = (-2\omega_3 v_2^2 \omega_2^2 + 2\omega_1 v_2^2 \omega_2^2 + 2\omega_3 \omega_1^2 v_2^2 + 6cs^2 \omega_1 \omega_2^2 + 9\omega_3 v_1^2 \omega_1^2 + 6\omega_3 \omega_1 \omega_2 - 18\omega_3 v_1^2 \omega_1 \omega_2 + 9cs^2 \omega_3 \omega_1^2 - \omega_3 \omega_1 \omega_2^2 - 3cs^2 \omega_3 \omega_1^2 \omega_2 + 2\omega_1^2 \omega_2 - \omega_3 \omega_2^2 + \omega_3 \omega_1 v_2^2 \omega_2^2 - 6cs^2 \omega_3 \omega_1 \omega_2 - 5\omega_3 \omega_1^2 - 2\omega_1^2 v_2^2 \omega_2 - 3cs^2 \omega_3 \omega_2^2 - 6cs^2 \omega_1^2 \omega_2 - \omega_3 \omega_1^2 v_2^2 \omega_2 + 9\omega_3 v_1^2 \omega_2^2 + \omega_3 \omega_1^2 \omega_2 + 3cs^2 \omega_3 \omega_1 \omega_2^2 - 2\omega_1 \omega_2^2) \frac{\rho v_2^2}{4\omega_3 \omega_1^2 \omega_2^2}$$

coefficient $C_{D_x^2 D_y v_2}^{(1)}$ **at** $\frac{\partial^3 v_2}{\partial x_1^2 \partial x_2}$:

$$C_{D_x^2 D_y v_2}^{(1), \text{SRT}} = (12 + 36cs^2 \omega - 12v_1^2 + 3\omega^2 - 3v_1^2 \omega^2 + 12v_1^2 \omega - 11cs^2 \omega^2 - 36cs^2 - 12\omega) \frac{v_1 \rho}{12\omega^2}$$

$$C_{D_x^2 D_y v_2}^{(1), \text{MRT1}} = (-24cs^2 \omega_7 \omega_4 \omega_5 + 12v_1^2 \omega_7 \omega_4^2 + 6v_1^2 \omega_7 \omega_4 \omega_5^2 + 36cs^2 \omega_7 \omega_4^2 - 12\omega_4^2 \omega_5^2 - 12cs^2 \omega_4 \omega_5^2 - 6\omega_7 \omega_4 \omega_5^2 + 12\omega_4^2 \omega_5 + 42cs^2 \omega_7 \omega_4 \omega_5^2 - 12v_1^2 \omega_4 \omega_5^2 - 6v_1^2 \omega_7 \omega_4^2 \omega_5 - 24cs^2 \omega_7 \omega_5^2 + 12v_1^2 \omega_4^2 \omega_5^2 - 11cs^2 \omega_7 \omega_4^2 \omega_5^2 + 12cs^2 \omega_4^2 \omega_5^2 - 12\omega_7 \omega_4^2 + 3\omega_7 \omega_4^2 \omega_5^2 - 12cs^2 \omega_4^2 \omega_5 + 12\omega_4 \omega_5^2 + 6\omega_7 \omega_4^2 \omega_5 - 12v_1^2 \omega_4^2 \omega_5 - 18cs^2 \omega_7 \omega_4^2 \omega_5 - 3v_1^2 \omega_7 \omega_4^2 \omega_5^2) \frac{v_1 \rho}{12\omega_7 \omega_4^2 \omega_5^2}$$

$$C_{D_x^2 D_y v_2}^{(1), \text{MRT2}} = (-12\omega_4 cs^2 \omega_5^2 + 12v_1^2 \omega_7 \omega_4^2 + 6v_1^2 \omega_7 \omega_4 \omega_5^2 + 42\omega_7 \omega_4 cs^2 \omega_5^2 - 12\omega_4^2 \omega_5^2 - 6\omega_7 \omega_4 \omega_5^2 + 12\omega_4^2 \omega_5 - 24\omega_7 \omega_4 cs^2 \omega_5 - 12v_1^2 \omega_4 \omega_5^2 + 36\omega_7 \omega_4^2 cs^2 - 6v_1^2 \omega_7 \omega_4^2 \omega_5 + 12v_1^2 \omega_4^2 \omega_5^2 - 18\omega_7 \omega_4^2 cs^2 \omega_5 - 12\omega_7 \omega_4^2 + 3\omega_7 \omega_4^2 \omega_5^2 - 24\omega_7 cs^2 \omega_5^2 - 12\omega_4^2 cs^2 \omega_5 + 12\omega_4^2 cs^2 \omega_5^2 + 12\omega_4 \omega_5^2 + 6\omega_7 \omega_4^2 \omega_5 - 12v_1^2 \omega_4^2 \omega_5 - 11\omega_7 \omega_4^2 cs^2 \omega_5^2 - 3v_1^2 \omega_7 \omega_4^2 \omega_5^2) \frac{v_1 \rho}{12\omega_7 \omega_4^2 \omega_5^2}$$

$$C_{D_x^2 D_y v_2}^{(1), \text{CLBM1}} = (-12v_1^2 \omega_4 \omega_5 - 6\omega_7 \omega_5^2 + 12\omega_5^2 - 12v_1^2 \omega_5^2 - 3v_1^2 \omega_7 \omega_4 \omega_5^2 + 6\omega_7 \omega_4 \omega_5 + 18cs^2 \omega_7 \omega_5^2 + 3\omega_7 \omega_4 \omega_5^2 - 6v_1^2 \omega_7 \omega_4 \omega_5 - 36cs^2 \omega_5^2 + 12v_1^2 \omega_4 \omega_5^2 + 12v_1^2 \omega_7 \omega_4 + 36cs^2 \omega_4 \omega_5^2 + 36cs^2 \omega_7 \omega_4 + 6v_1^2 \omega_7 \omega_5^2 + 12\omega_4 \omega_5 - 18cs^2 \omega_7 \omega_4 \omega_5 - 12\omega_7 \omega_4 - 12\omega_4 \omega_5^2 - 11cs^2 \omega_7 \omega_4 \omega_5^2 - 36cs^2 \omega_4 \omega_5) \frac{v_1 \rho}{12\omega_7 \omega_4 \omega_5^2}$$

$$C_{D_x^2 D_y v_2}^{(1), \text{CLBM2}} = (-12v_1^2 \omega_4 \omega_5 - 6\omega_7 \omega_5^2 + 12\omega_5^2 - 12v_1^2 \omega_5^2 + 36cs^2 \omega_7 \omega_4 - 3v_1^2 \omega_7 \omega_4 \omega_5^2 + 36cs^2 \omega_4 \omega_5^2 + 6\omega_7 \omega_4 \omega_5 + 3\omega_7 \omega_4 \omega_5^2 - 6v_1^2 \omega_7 \omega_4 \omega_5 - 36cs^2 \omega_4 \omega_5 + 12v_1^2 \omega_4 \omega_5^2 + 12v_1^2 \omega_7 \omega_4 + 6v_1^2 \omega_7 \omega_5^2 - 36cs^2 \omega_5^2 + 12\omega_4 \omega_5 - 18cs^2 \omega_7 \omega_4 \omega_5 + 18cs^2 \omega_7 \omega_5^2 - 11cs^2 \omega_7 \omega_4 \omega_5^2 - 12\omega_7 \omega_4 - 12\omega_4 \omega_5^2) \frac{v_1 \rho}{12\omega_7 \omega_4 \omega_5^2}$$

$$C_{D_x^2 D_y v_2}^{(1), \text{CuLBM1}} = (36\omega_3 \omega_1^2 cs^2 - 18\omega_3 \omega_4 \omega_1 cs^2 + 12\omega_3 v_1^2 \omega_1^2 - 6\omega_4 \omega_1^2 - 12\omega_3 v_1^2 \omega_1 + 12\omega_3 v_1^2 \omega_4 + 18\omega_4 \omega_1^2 cs^2 + 6\omega_3 \omega_4 \omega_1 - 12\omega_3 \omega_1^2 - 12v_1^2 \omega_1^2 + 12\omega_1^2 - 3\omega_3 v_1^2 \omega_4 \omega_1^2 + 36\omega_3 \omega_4 cs^2 - 36v_1^2 cs^2 - 12\omega_3 \omega_4 - 36\omega_3 \omega_1 cs^2 + 3\omega_3 \omega_4 \omega_1^2 + 12\omega_3 \omega_1 - 11\omega_3 \omega_4 \omega_1^2 cs^2 - 6\omega_3 v_1^2 \omega_4 \omega_1 + 6v_1^2 \omega_4 \omega_1^2) \frac{v_1 \rho}{12\omega_3 \omega_4 \omega_1^2}$$

$$C_{D_x^2 D_y v_2}^{(1), \text{CuLBM2}} = (-27\omega_3 v_2^2 \omega_2^2 - 11cs^2 \omega_3 \omega_1^2 \omega_2^2 - 12\omega_1^2 \omega_2^2 + 27\omega_3 \omega_1^2 v_2^2 - 54cs^2 \omega_1 \omega_2^2 + 6\omega_3 v_1^2 \omega_1^2 + 3\omega_3 v_1^2 \omega_1 \omega_2^2 + 27cs^2 \omega_3 \omega_1^2 - 12\omega_3 \omega_1 \omega_2^2 - 18v_1^2 \omega_1 \omega_2^2 - 18cs^2 \omega_3 \omega_1^2 \omega_2 + 6\omega_2^2 \omega_2 + 3\omega_3 \omega_2^2 + 27\omega_3 \omega_1 v_2^2 \omega_2^2 - 15\omega_3 \omega_1^2 - 3\omega_3 v_1^2 \omega_1^2 \omega_2 + 3\omega_3 \omega_1^2 \omega_2^2 + 9cs^2 \omega_3 \omega_2^2 + 12v_1^2 \omega_1^2 \omega_2^2 - 18cs^2 \omega_1^2 \omega_2 + 36cs^2 \omega_1^2 \omega_2^2 - 6v_1^2 \omega_1^2 \omega_2 - 27\omega_3 \omega_1^2 v_2^2 \omega_2 + 6\omega_3 v_1^2 \omega_2^2 - 3\omega_3 v_1^2 \omega_1^2 \omega_2^2 + 12\omega_3 \omega_1^2 \omega_2 + 18cs^2 \omega_3 \omega_1 \omega_2^2 + 18\omega_1 \omega_2^2) \frac{v_1 \rho}{12\omega_3 \omega_1^2 \omega_2^2}$$

coefficient $C_{D_x D_y^2 \rho}^{(1)}$ **at** $\frac{\partial^3 \rho}{\partial x_1 \partial x_2^2}$:

$$C_{D_x D_y^2 \rho}^{(1), \text{SRT}} = (-12 - \omega^2 + 12\omega) \frac{cs^4}{6\omega^2}$$

$$C_{D_x D_y^2 \rho}^{(1), \text{MRT1}} = (-12 + 12\omega_4 - \omega_4^2) \frac{cs^4}{6\omega_4^2}$$

$$C_{D_x D_y^2 \rho}^{(1), \text{MRT2}} = (-12 + 12\omega_4 - \omega_4^2) \frac{cs^4}{6\omega_4^2}$$

$$C_{D_x D_y^2 \rho}^{(1), \text{CLBM1}} = (-12 + 12\omega_4 - \omega_4^2) \frac{cs^4}{6\omega_4^2}$$

$$C_{D_x D_y^2 \rho}^{(1), \text{CLBM2}} = (-12 + 12\omega_4 - \omega_4^2) \frac{cs^4}{6\omega_4^2}$$

$$C_{D_x D_y^2 \rho}^{(1), \text{CuLBM1}} = (-12 + 12\omega_3 - \omega_3^2) \frac{cs^4}{6\omega_3^2}$$

$$C_{D_x D_y^2 \rho}^{(1), \text{CuLBM2}} = (6cs^4 \omega_1^2 - 9\omega_1^2 v_2^4 \omega_2 - 45cs^2 \omega_1^2 v_2^2 \omega_2 - 9\omega_1 v_2^2 \omega_2^2 - 9v_2^4 \omega_2^2 - 6cs^2 \omega_1 \omega_2^2 + 6cs^2 \omega_2^2 - 45cs^2 v_2^2 \omega_2^2 + 30cs^4 \omega_1 \omega_2^2 + 45cs^2 \omega_1^2 v_2^2 + 9\omega_1^2 v_2^4 - 2cs^4 \omega_1^2 \omega_2^2 - 6cs^2 \omega_1^2 + 9\omega_1^2 v_2^2 \omega_2 + 45cs^2 \omega_1 v_2^2 \omega_2^2 - 9\omega_1^2 v_2^2 + 9\omega_1 v_2^4 \omega_2^2 + 6cs^2 \omega_1^2 \omega_2 + 9v_2^2 \omega_2^2 - 30cs^4 \omega_2^2 - 6cs^4 \omega_1^2 \omega_2) \frac{1}{12\omega_1^2 \omega_2^2}$$

coefficient $C_{D_x D_y^2 v_1}^{(1)}$ **at** $\frac{\partial^3 v_1}{\partial x_1 \partial x_2^2}$:

$$C_{D_x D_y^2 v_1}^{(1), \text{SRT}} = \frac{-cs^2 v_1 \rho}{6}$$

$$C_{D_x D_y^2 v_1}^{(1), \text{MRT}^1} = (12\omega_7\omega_4\omega_5 - 12\omega_4^2\omega_5 - 12\omega_7\omega_5 + 12\omega_4\omega_5 + 12\omega_4^2 - 12\omega_7\omega_4 - \omega_7\omega_4^2\omega_5) \frac{v_1 \rho c s^2}{6\omega_7\omega_4^2\omega_5}$$

$$C_{D_x D_y^2 v_1}^{(1), \text{MRT}^2} = (12\omega_7\omega_4\omega_5 - 12\omega_4^2\omega_5 - 12\omega_7\omega_5 + 12\omega_4\omega_5 + 12\omega_4^2 - 12\omega_7\omega_4 - \omega_7\omega_4^2\omega_5) \frac{v_1 \rho c s^2}{6\omega_7\omega_4^2\omega_5}$$

$$C_{D_x D_y^2 v_1}^{(1), \text{CLBM}^1} = \frac{-cs^2 v_1 \rho}{6}$$

$$C_{D_x D_y^2 v_1}^{(1), \text{CLBM}^2} = \frac{-v_1 \rho c s^2}{6}$$

$$C_{D_x D_y^2 v_1}^{(1), \text{CuLBM}^1} = \frac{-v_1 \rho c s^2}{6}$$

$$C_{D_x D_y^2 v_1}^{(1), \text{CuLBM}^2} = \frac{-cs^2 v_1 \rho}{6}$$

coefficient $C_{D_x D_y^2 v_2}^{(1)}$ **at** $\frac{\partial^3 v_2}{\partial x_1 \partial x_2^2}$:

$$C_{D_x D_y^2 v_2}^{(1), \text{SRT}} = 0$$

$$C_{D_x D_y^2 v_2}^{(1), \text{MRT}^1} = (2\omega_4 + \omega_4\omega_8 - \omega_4^2 - 2\omega_8) \frac{\rho c s^2 v_2}{\omega_4^2 \omega_8}$$

$$C_{D_x D_y^2 v_2}^{(1), \text{MRT}^2} = (2\omega_4 + \omega_4\omega_8 - \omega_4^2 - 2\omega_8) \frac{\rho v_2 c s^2}{\omega_4^2 \omega_8}$$

$$C_{D_x D_y^2 v_2}^{(1), \text{CLBM}^1} = 0$$

$$C_{D_x D_y^2 v_2}^{(1), \text{CLBM}^2} = 0$$

$$C_{D_x D_y^2 v_2}^{(1), \text{CuLBM}^1} = 0$$

$$C_{D_x D_y^2 v_2}^{(1), \text{CuLBM}^2} = (11\omega_1 v_2^2 \omega_2^2 + 9cs^2 \omega_1 \omega_2^2 - 9cs^2 \omega_2^2 + 5\omega_1^2 \omega_2 + 5\omega_2^2 - 5\omega_1^2 + 9cs^2 \omega_1^2 - 11\omega_1^2 v_2^2 \omega_2 + 11\omega_1^2 v_2^2 - 9cs^2 \omega_1^2 \omega_2 - 11v_2^2 \omega_2^2 - 5\omega_1 \omega_2^2) \frac{\rho v_2}{4\omega_1^2 \omega_2^2}$$

coefficient $C_{D_y^3 \rho}^{(1)}$ **at** $\frac{\partial^3 \rho}{\partial x_2^3}$:

$$C_{D_y^3 \rho}^{(1), \text{SRT}} = (-1 + 3cs^2 + v_2^2) \frac{v_1 v_2}{12}$$

$$C_{D_y^3 \rho}^{(1), \text{MRT}^1} = (3\omega_6 cs^2 \omega_4 \omega_8 + 6\omega_6 \omega_4 v_2^2 - 18cs^2 \omega_4 \omega_8 + 12\omega_6 - 36\omega_6 cs^2 + 18\omega_6 cs^2 \omega_4 + 12v_2^2 \omega_8 - \omega_6 \omega_4 \omega_8 + 6\omega_4 \omega_8 - 12\omega_6 v_2^2 - 6\omega_6 \omega_4 + \omega_6 \omega_4 v_2^2 \omega_8 - 6\omega_4 v_2^2 \omega_8 - 12\omega_8 + 36cs^2 \omega_8) \frac{v_1 v_2}{12\omega_6 \omega_4 \omega_8}$$

$$C_{D_y^3 \rho}^{(1), \text{MRT}^2} = (6\omega_6 \omega_4 v_2^2 + 12\omega_6 - 36\omega_6 cs^2 + 12v_2^2 \omega_8 - \omega_6 \omega_4 \omega_8 + 6\omega_4 \omega_8 - 12\omega_6 v_2^2 - 6\omega_6 \omega_4 + \omega_6 \omega_4 v_2^2 \omega_8 - 6\omega_4 v_2^2 \omega_8 - 12\omega_8 + 3\omega_6 \omega_4 \omega_8 cs^2 - 18\omega_4 \omega_8 cs^2 + 36\omega_8 cs^2 + 18\omega_6 \omega_4 cs^2) \frac{v_1 v_2}{12\omega_6 \omega_4 \omega_8}$$

$$C_{D_y^3 \rho}^{(1), \text{CLBM}^1} = (-1 + 3cs^2 + v_2^2) \frac{v_1 v_2}{12}$$

$$C_{D_y^3 \rho}^{(1), \text{CLBM}^2} = (-1 + 3cs^2 + v_2^2) \frac{v_1 v_2}{12}$$

$$C_{D_y^3 \rho}^{(1), \text{CuLBM}^1} = (-1 + v_2^2 + 3cs^2) \frac{v_1 v_2}{12}$$

$$C_{D_y^3 \rho}^{(1), \text{CuLBM}^2} = (-1 + 3cs^2 + v_2^2) \frac{v_1 v_2}{12}$$

coefficient $C_{D_y^3 v_1}^{(1)}$ **at** $\frac{\partial^3 v_1}{\partial x_2^3}$:

$$C_{D_y^3 v_1}^{(1), \text{SRT}} = (6 + 18cs^2 \omega + \omega^2 - 3cs^2 \omega^2 - 18cs^2 + 6v_2^2 \omega - 6v_2^2 - 6\omega - v_2^2 \omega^2) \frac{\rho v_2}{6\omega^2}$$

$$C_{D_y^3 v_1}^{(1), \text{MRT}^1} = (-6\omega_4 v_2^2 + 15cs^2 \omega_4 \omega_8 + 3cs^2 \omega_4^2 - 6cs^2 \omega_4 + \omega_4^2 \omega_8 - \omega_4^2 v_2^2 \omega_8 + 6\omega_4 - 3\omega_4 \omega_8 + 3\omega_4 v_2^2 \omega_8 - 3\omega_4^2 - 12cs^2 \omega_8 + 3\omega_4^2 v_2^2 - 3cs^2 \omega_4^2 \omega_8) \frac{\rho v_2}{6\omega_4^2 \omega_8}$$

$$C_{D_y^3 v_1}^{(1), \text{MRT}^2} = (-6\omega_4 v_2^2 + 3\omega_4^2 cs^2 - 3\omega_4^2 \omega_8 cs^2 + \omega_4^2 \omega_8 - \omega_4^2 v_2^2 \omega_8 + 6\omega_4 - 3\omega_4 \omega_8 + 3\omega_4 v_2^2 \omega_8 - 3\omega_4^2 + 15\omega_4 \omega_8 cs^2 + 3\omega_4^2 v_2^2 - 12\omega_8 cs^2 - 6\omega_4 cs^2) \frac{\rho v_2}{6\omega_4^2 \omega_8}$$

$$C_{D_y^3 v_1}^{(1), \text{CLBM}^1} = (6 + 3\omega_4 v_2^2 + 9cs^2 \omega_8 + 3v_2^2 \omega_8 - 3\omega_4 + \omega_4 \omega_8 - 18cs^2 - \omega_4 v_2^2 \omega_8 + 9cs^2 \omega_4 - 3cs^2 \omega_4 \omega_8 - 3\omega_8 - 6v_2^2) \frac{\rho v_2}{6\omega_4 \omega_8}$$

$$C_{D_y^3 v_1}^{(1), \text{CLBM}^2} = (6 + 3\omega_4 v_2^2 - 3cs^2 \omega_4 \omega_8 - 18cs^2 + 9cs^2 \omega_4 + 3v_2^2 \omega_8 - 3\omega_4 + \omega_4 \omega_8 - \omega_4 v_2^2 \omega_8 - 3\omega_8 - 6v_2^2 + 9cs^2 \omega_8) \frac{\rho v_2}{6\omega_4 \omega_8}$$

$$C_{D_y^3 v_1}^{(1), \text{CuLBM}^1} = (6 - 3\omega_6 - 3\omega_3 + 9\omega_6 cs^2 + 3\omega_6 v_2^2 + 3\omega_3 v_2^2 - 6v_2^2 - \omega_6 \omega_3 v_2^2 - 18cs^2 - 3\omega_6 \omega_3 cs^2 + 9\omega_3 cs^2 + \omega_6 \omega_3) \frac{\rho v_2}{6\omega_6 \omega_3}$$

$$C_{D_y^3 v_1}^{(1), \text{CuLBM}^2} = (6 - 3cs^2 \omega_3 \omega_1 - 3\omega_3 + 3\omega_1 v_2^2 + 9cs^2 \omega_3 - 3\omega_1 - 18cs^2 + 3\omega_3 v_2^2 - 6v_2^2 + 9cs^2 \omega_1 + \omega_3 \omega_1 - \omega_3 \omega_1 v_2^2) \frac{\rho v_2}{6\omega_3 \omega_1}$$

coefficient $C_{D_y^3 v_2}^{(1)}$ **at** $\frac{\partial^3 v_2}{\partial x_2^3}$:

$$C_{D_y^3 v_2}^{(1), \text{SRT}} = (-1 + cs^2 + 3v_2^2) \frac{v_1 \rho}{12}$$

$$C_{D_y^3 v_2}^{(1), \text{MRT}^1} = (\omega_6 cs^2 \omega_4 \omega_8 + 18\omega_6 \omega_4 v_2^2 - 6cs^2 \omega_4 \omega_8 + 12\omega_6 - 12\omega_6 cs^2 + 6\omega_6 cs^2 \omega_4 + 36v_2^2 \omega_8 - \omega_6 \omega_4 \omega_8 + 6\omega_4 \omega_8 - 36\omega_6 v_2^2 - 6\omega_6 \omega_4 + 3\omega_6 \omega_4 v_2^2 \omega_8 - 18\omega_4 v_2^2 \omega_8 - 12\omega_8 + 12cs^2 \omega_8) \frac{v_1 \rho}{12\omega_6 \omega_4 \omega_8}$$

$$C_{D_y^3 v_2}^{(1), \text{MRT}^2} = (18\omega_6 \omega_4 v_2^2 + 12\omega_6 - 12\omega_6 cs^2 + 36v_2^2 \omega_8 - \omega_6 \omega_4 \omega_8 + 6\omega_4 \omega_8 - 36\omega_6 v_2^2 - 6\omega_6 \omega_4 + 3\omega_6 \omega_4 v_2^2 \omega_8 - 18\omega_4 v_2^2 \omega_8 - 12\omega_8 + \omega_6 \omega_4 \omega_8 cs^2 - 6\omega_4 \omega_8 cs^2 + 12\omega_8 cs^2 + 6\omega_6 \omega_4 cs^2) \frac{v_1 \rho}{12\omega_6 \omega_4 \omega_8}$$

$$C_{D_y^3 v_2}^{(1), \text{CLBM}^1} = (-1 + cs^2 + 3v_2^2) \frac{v_1 \rho}{12}$$

$$C_{D_y^3 v_2}^{(1), \text{CLBM}^2} = (-1 + cs^2 + 3v_2^2) \frac{v_1 \rho}{12}$$

$$C_{D_y^3 v_2}^{(1), \text{CuLBM}^1} = (-1 + 3v_2^2 + cs^2) \frac{v_1 \rho}{12}$$

$$C_{D_y^3 v_2}^{(1), \text{CuLBM}^2} = (-1 + cs^2 + 3v_2^2) \frac{v_1 \rho}{12}$$

coefficient $C_{D_x^4 \rho}^{(1)}$ **at** $\frac{\partial^4 \rho}{\partial x_1^4}$:

$$C_{D_x^4 \rho}^{(1), \text{SRT}} = (12 - 34cs^2 v_1^2 \omega^3 - \omega^3 + 10v_1^2 \omega^3 + 198cs^2 \omega + 404cs^2 v_1^2 \omega^2 - 156v_1^2 + 8\omega^2 - 98v_1^2 \omega^2 + 6cs^2 \omega^3 + 234v_1^2 \omega + 144cs^4 - 1008cs^2 v_1^2 \omega - 78cs^2 \omega^2 + 672cs^2 v_1^2 - 132cs^2 + 90v_1^4 \omega^2 - 9v_1^4 \omega^3 - 216cs^4 \omega + 82cs^4 \omega^2 - 18\omega - 5cs^4 \omega^3 - 216v_1^4 \omega + 144v_1^4) \frac{v_1}{12\omega^3}$$

$$C_{D_x^4 \rho}^{(1), \text{MRT}^1} = (12 - 5cs^4 \omega_5^3 - 34v_1^2 cs^2 \omega_5^3 + 8\omega_5^2 - 98v_1^2 \omega_5^2 + 404v_1^2 cs^2 \omega_5^2 - 156v_1^2 + 82cs^4 \omega_5^2 - \omega_5^3 + 10v_1^2 \omega_5^3 - 1008v_1^2 cs^2 \omega_5 - 216cs^4 \omega_5 + 234v_1^2 \omega_5 - 132cs^2 + 672v_1^2 cs^2 - 78cs^2 \omega_5^2 - 9v_1^4 \omega_5^3 + 6cs^2 \omega_5^3 + 90v_1^4 \omega_5^2 + 144cs^4 - 216v_1^4 \omega_5 - 18\omega_5 + 198cs^2 \omega_5 + 144v_1^4) \frac{v_1}{12\omega_5^3}$$

$$C_{D_x^4 \rho}^{(1), \text{MRT}^2} = (12 + 8\omega_5^2 + 6cs^2 \omega_5^3 - 98v_1^2 \omega_5^2 - 156v_1^2 + 672v_1^2 cs^2 - 78cs^2 \omega_5^2 - \omega_5^3 + 10v_1^2 \omega_5^3 - 132cs^2 + 198cs^2 \omega_5 + 234v_1^2 \omega_5 + 404v_1^2 cs^2 \omega_5^2 - 9v_1^4 \omega_5^3 + 82cs^4 \omega_5^2 + 90v_1^4 \omega_5^2 - 5cs^4 \omega_5^3 - 34v_1^2 cs^2 \omega_5^3 - 216v_1^4 \omega_5 - 18\omega_5 - 1008v_1^2 cs^2 \omega_5 - 216cs^4 \omega_5 + 144cs^4 + 144v_1^4) \frac{v_1}{12\omega_5^3}$$

$$C_{D_x^4 \rho}^{(1), \text{CLBM}^1} = (12 + 8\omega_5^2 - 98v_1^2 \omega_5^2 + 198cs^2 \omega_5 - 156v_1^2 - 1008cs^2 v_1^2 \omega_5 - \omega_5^3 + 10v_1^2 \omega_5^3 + 144cs^4 + 6cs^2 \omega_5^3 + 672cs^2 v_1^2 + 404cs^2 v_1^2 \omega_5^2 + 234v_1^2 \omega_5 - 78cs^2 \omega_5^2 - 34cs^2 v_1^2 \omega_5^3 - 132cs^2 - 9v_1^4 \omega_5^3 - 216cs^4 \omega_5 + 90v_1^4 \omega_5^2 + 82cs^4 \omega_5^2 - 216v_1^4 \omega_5 - 18\omega_5 - 5cs^4 \omega_5^3 + 144v_1^4) \frac{v_1}{12\omega_5^3}$$

$$C_{D_x^4 \rho}^{(1), \text{CLBM}^2} = (12 + 82cs^4 \omega_5^2 + 404v_1^2 cs^2 \omega_5^2 + 8\omega_5^2 - 98v_1^2 \omega_5^2 - 34v_1^2 cs^2 \omega_5^3 - 156v_1^2 - 5cs^4 \omega_5^3 - \omega_5^3 + 10v_1^2 \omega_5^3 + 234v_1^2 \omega_5 + 672v_1^2 cs^2 - 132cs^2 - 216cs^4 \omega_5 - 1008v_1^2 cs^2 \omega_5 + 6cs^2 \omega_5^3 - 9v_1^4 \omega_5^3 + 144cs^4 - 78cs^2 \omega_5^2 + 90v_1^4 \omega_5^2 - 216v_1^4 \omega_5 + 198cs^2 \omega_5 - 18\omega_5 + 144v_1^4) \frac{v_1}{12\omega_5^3}$$

$$C_{D_x^4 \rho}^{(1), \text{CuLBM}^1} = (12 + 144cs^4 - 9v_1^4 \omega_1^3 + 404v_1^2 \omega_1^2 cs^2 - 156v_1^2 + 90v_1^4 \omega_1^2 - 5\omega_1^3 cs^4 - 34v_1^2 \omega_1^3 cs^2 - 216v_1^4 \omega_1 + 198\omega_1 cs^2 + 82\omega_1^2 cs^4 - 216\omega_1 cs^4 - 98v_1^2 \omega_1^2 + 8\omega_1^2 - 18\omega_1 + 10v_1^2 \omega_1^3 - \omega_1^3 - 78\omega_1^2 cs^2 + 672v_1^2 cs^2 - 132cs^2 + 234v_1^2 \omega_1 + 6\omega_1^3 cs^2 + 144v_1^4 - 1008v_1^2 \omega_1 cs^2) \frac{v_1}{12\omega_1^3}$$

$$C_{D_x^4 \rho}^{(1), \text{CuLBM}^2} = (-51\omega_3 v_1^2 \omega_1^3 - 12cs^2 v_1^2 \omega_1^2 \omega_2^2 - 6cs^2 \omega_1 \omega_2^3 + 114cs^2 \omega_3 \omega_1^2 \omega_2^2 + 129\omega_3 v_1^2 \omega_1 \omega_2^3 + 90cs^4 \omega_3 \omega_1^3 + 6cs^2 v_1^2 \omega_1^3 \omega_2 - 105\omega_3 v_1^2 \omega_1 \omega_2^2 +$$

$$141cs^2\omega_3\omega_1^3\omega_2 - 78cs^2\omega_3\omega_1^2\omega_2^3 + 45\omega_3v_1^4\omega_2^3 + 411cs^2\omega_3v_1^2\omega_1\omega_2 - 78cs^2\omega_3\omega_1^3\omega_2^2 + 18cs^4\omega_1\omega_2^3 + 6\omega_3\omega_1\omega_2^2 - 171cs^4\omega_3\omega_1\omega_2^3 - 117\omega_3v_1^4\omega_1\omega_2^3 + 261cs^2\omega_3v_1^2\omega_1^3\omega_2 + 6\omega_3\omega_2^3 - 12\omega_3\omega_1\omega_2^3 - 60cs^2\omega_3\omega_1^2\omega_2 + 12cs^2\omega_3\omega_1^3\omega_2 - 72cs^2\omega_3\omega_1^3 - 600cs^2\omega_3v_1^2\omega_1\omega_2^3 + 99\omega_3v_1^4\omega_1\omega_2^2 + 54cs^4\omega_3\omega_1\omega_2^2 - 12\omega_3\omega_1^3\omega_2 - 36cs^4\omega_1^3\omega_2^2 - 72cs^2\omega_3\omega_2^3 + 8\omega_3\omega_1^2\omega_2^2 + 404cs^2\omega_3v_1^2\omega_1^3\omega_2 - 98\omega_3v_1^2\omega_1^3\omega_2^2 - 600cs^2\omega_3v_1^2\omega_1^3\omega_2 - 198\omega_3v_1^4\omega_1^3\omega_2^2 - 90cs^4\omega_3\omega_1^2\omega_2^2 + 20\omega_3v_1^2\omega_1^3\omega_2^3 - 105\omega_3v_1^2\omega_1^2\omega_2 + 45\omega_3v_1^4\omega_1^3 - 816cs^2\omega_3v_1^2\omega_1^2\omega_2^2 + 18cs^4\omega_1^3\omega_2 - 12\omega_3\omega_1^2\omega_2^2 - 171cs^4\omega_3\omega_1^3\omega_2 + 82cs^4\omega_3\omega_1^2\omega_2^3 - 117\omega_3v_1^4\omega_1^3\omega_2 + 6\omega_3\omega_1^3 + 261cs^2\omega_3v_1^2\omega_1^3 + 90\omega_3v_1^4\omega_1^2\omega_2^2 + 82cs^4\omega_3\omega_1^3\omega_2^2 + 90cs^4\omega_3\omega_2^3 + 12cs^2\omega_1^2\omega_2^2 + 6cs^2v_1^2\omega_1\omega_2^3 + 90\omega_3v_1^4\omega_1^3\omega_2^2 - 68cs^2\omega_3v_1^2\omega_1^3\omega_2^3 + 411cs^2\omega_3v_1^2\omega_1^2\omega_2 + 210\omega_3v_1^2\omega_1^2\omega_2^2 + 141cs^2\omega_3\omega_1\omega_2^3 + 6\omega_3\omega_1^2\omega_2 - 2\omega_3\omega_1^3\omega_2^3 + 99\omega_3v_1^4\omega_1^2\omega_2 - 51\omega_3v_1^2\omega_2^3 - 18\omega_3v_1^4\omega_1^3\omega_2^3 + 54cs^4\omega_3\omega_1^2\omega_2 - 6cs^2\omega_1^3\omega_2 - 10cs^4\omega_3\omega_1^3\omega_2^3 + 8\omega_3\omega_1^3\omega_2^2 - 60cs^2\omega_3\omega_1\omega_2^2 - 98\omega_3v_1^2\omega_1^2\omega_2^2 + 404cs^2\omega_3v_1^2\omega_1^3\omega_2^2 + 129\omega_3v_1^2\omega_1^3\omega_2^2) \frac{v_1}{24\omega_3\omega_1^3\omega_2^3}$$

coefficient $C_{D_x^4 v_1}^{(1)}$ at $\frac{\partial^4 v_1}{\partial x_1^4}$:

$$C_{D_x^4 v_1}^{(1), \text{SRT}} = (12 - 18cs^2v_1^2\omega^3 - \omega^3 + 14v_1^2\omega^3 + 54cs^2\omega + 252cs^2v_1^2\omega^2 - 252v_1^2 + 8\omega^2 - 154v_1^2\omega^2 + 2cs^2\omega^3 + 378v_1^2\omega + 24cs^4 - 648cs^2v_1^2\omega - 22cs^2\omega^2 + 432cs^2v_1^2 - 36cs^2 + 310v_1^4\omega^2 - 29v_1^4\omega^3 - 36cs^4\omega + 14cs^4\omega^2 - 18\omega - cs^4\omega^3 - 756v_1^4\omega + 504v_1^4) \frac{\rho}{12\omega^3}$$

$$C_{D_x^4 v_1}^{(1), \text{MRT1}} = (12 - cs^4\omega_5^3 - 18v_1^2cs^2\omega_5^3 + 8\omega_5^2 - 154v_1^2\omega_5^2 + 252v_1^2cs^2\omega_5^2 - 252v_1^2 + 14cs^4\omega_5^2 - \omega_5^3 + 14v_1^2\omega_5^3 - 648v_1^2cs^2\omega_5 - 36cs^4\omega_5 + 378v_1^2\omega_5 - 36cs^2 + 432v_1^2cs^2 - 22cs^2\omega_5^2 - 29v_1^4\omega_5^3 + 2cs^2\omega_5^3 + 310v_1^4\omega_5^2 + 24cs^4 - 756v_1^4\omega_5 - 18\omega_5 + 54cs^2\omega_5 + 504v_1^4) \frac{\rho}{12\omega_5^3}$$

$$C_{D_x^4 v_1}^{(1), \text{MRT2}} = (12 + 8\omega_5^2 + 2cs^2\omega_5^3 - 154v_1^2\omega_5^2 - 252v_1^2 + 432v_1^2cs^2 - 22cs^2\omega_5^2 - \omega_5^3 + 14v_1^2\omega_5^3 - 36cs^2 + 54cs^2\omega_5 + 378v_1^2\omega_5 + 252v_1^2cs^2\omega_5^2 - 29v_1^4\omega_5^3 + 14cs^4\omega_5^2 + 310v_1^4\omega_5^2 - cs^4\omega_5^3 - 18v_1^2cs^2\omega_5^3 - 756v_1^4\omega_5 - 18\omega_5 - 648v_1^2cs^2\omega_5 - 36cs^4\omega_5 + 24cs^4 + 504v_1^4) \frac{\rho}{12\omega_5^3}$$

$$C_{D_x^4 v_1}^{(1), \text{CLBM1}} = (12 + 8\omega_5^2 - 154v_1^2\omega_5^2 + 54cs^2\omega_5 - 252v_1^2 - 648cs^2v_1^2\omega_5 - \omega_5^3 + 14v_1^2\omega_5^3 + 24cs^4 + 2cs^2\omega_5^3 + 432cs^2v_1^2 + 252cs^2v_1^2\omega_5^2 + 378v_1^2\omega_5 - 22cs^2\omega_5^2 - 18cs^2v_1^2\omega_5^3 - 36cs^2 - 29v_1^4\omega_5^3 - 36cs^4\omega_5 + 310v_1^4\omega_5^2 + 14cs^4\omega_5^2 - 756v_1^4\omega_5 - 18\omega_5 - cs^4\omega_5^3 + 504v_1^4) \frac{\rho}{12\omega_5^3}$$

$$C_{D_x^4 v_1}^{(1), \text{CLBM2}} = (12 + 14cs^4\omega_5^2 + 252v_1^2cs^2\omega_5^2 + 8\omega_5^2 - 154v_1^2\omega_5^2 - 18v_1^2cs^2\omega_5^3 - 252v_1^2 - cs^4\omega_5^3 - \omega_5^3 + 14v_1^2\omega_5^3 + 378v_1^2\omega_5 + 432v_1^2cs^2 - 36cs^2 - 36cs^4\omega_5 - 648v_1^2cs^2\omega_5 + 2cs^2\omega_5^3 - 29v_1^4\omega_5^3 + 24cs^4 - 22cs^2\omega_5^2 + 310v_1^4\omega_5^2 - 756v_1^4\omega_5 + 54cs^2\omega_5 - 18\omega_5 + 504v_1^4) \frac{\rho}{12\omega_5^3}$$

$$C_{D_x^4 v_1}^{(1), \text{CuLBM1}} = (12 + 24cs^4 - 29v_1^4\omega_1^3 + 252v_1^2\omega_1^2cs^2 - 252v_1^2 + 310v_1^4\omega_1^2 - \omega_1^3cs^4 - 18v_1^2\omega_1^3cs^2 - 756v_1^4\omega_1 + 54\omega_1cs^2 + 14\omega_1^2cs^4 - 36\omega_1cs^4 - 154v_1^2\omega_1^2 + 8\omega_1^2 - 18\omega_1 + 14v_1^2\omega_1^3 - \omega_1^3 - 22v_1^2cs^2 + 432v_1^2cs^2 - 36cs^2 + 378v_1^2\omega_1 + 2\omega_1^3cs^2 + 504v_1^4 - 648v_1^2\omega_1cs^2) \frac{\rho}{12\omega_1^3}$$

$$C_{D_x^4 v_1}^{(1), \text{CuLBM2}} = (-99\omega_3v_1^2\omega_1^3 - 36cs^2v_1^2\omega_1^2\omega_2^2 - 6cs^2\omega_1\omega_2^3 + 18cs^2\omega_3\omega_1^2\omega_2^2 + 225\omega_3v_1^2\omega_1\omega_2^3 + 18cs^4\omega_3\omega_1^3 + 18cs^2v_1^2\omega_1^3\omega_2 - 153\omega_3v_1^2\omega_1\omega_2^2 + 45cs^2\omega_3\omega_1^3\omega_2 - 22cs^2\omega_3\omega_1^2\omega_2^2 + 171\omega_3v_1^4\omega_2^3 + 225cs^2\omega_3v_1^2\omega_1\omega_2^2 - 22cs^2\omega_3\omega_1^3\omega_2^2 + 6cs^4\omega_1\omega_2^3 + 6\omega_3\omega_1\omega_2^2 - 33cs^4\omega_3\omega_1\omega_2^3 - 423\omega_3v_1^4\omega_1\omega_2^3 + 207cs^2\omega_3v_1^2\omega_1^3\omega_2 + 6\omega_3\omega_2^3 - 12\omega_3\omega_1\omega_2^3 - 12cs^2\omega_3\omega_1^2\omega_2 + 4cs^2\omega_3\omega_1^3\omega_2^2 - 24cs^2\omega_3\omega_1^3 - 432cs^2\omega_3v_1^2\omega_1\omega_2^3 + 333\omega_3v_1^4\omega_1\omega_2^2 + 6cs^4\omega_3\omega_1\omega_2^2 - 12\omega_3\omega_1^3\omega_2 - 12cs^4\omega_1^3\omega_2^2 - 24cs^2\omega_3\omega_2^3 + 8\omega_3\omega_1^2\omega_2^2 + 252cs^2\omega_3v_1^2\omega_1^3\omega_2 - 154\omega_3v_1^2\omega_1^3\omega_2^2 - 432cs^2\omega_3v_1^2\omega_1^3\omega_2 - 666\omega_3v_1^4\omega_1^2\omega_2^2 - 6cs^4\omega_3\omega_1^2\omega_2^2 + 28\omega_3v_1^2\omega_1^3\omega_2^2 - 153\omega_3v_1^2\omega_1^2\omega_2 + 171\omega_3v_1^4\omega_1^3 - 432cs^2\omega_3v_1^2\omega_1^2\omega_2^2 + 6cs^4\omega_1^3\omega_2 - 12\omega_3\omega_1^2\omega_2^2 - 33cs^4\omega_3\omega_1^3\omega_2 + 14cs^4\omega_3\omega_1^2\omega_2^3 - 423\omega_3v_1^4\omega_1^3\omega_2 + 6\omega_3\omega_1^3 + 207cs^2\omega_3v_1^2\omega_1^3 + 310\omega_3v_1^4\omega_1^2\omega_2^2 + 14cs^4\omega_3\omega_1^3\omega_2^2 + 18cs^4\omega_3\omega_2^3 + 12cs^2\omega_1^2\omega_2^2 + 18cs^2v_1^2\omega_1\omega_2^3 + 310\omega_3v_1^4\omega_1^3\omega_2^2 - 36cs^2\omega_3v_1^2\omega_1^3\omega_2^3 + 225cs^2\omega_3v_1^2\omega_1^2\omega_2 + 306\omega_3v_1^2\omega_1^2\omega_2^2 + 45cs^2\omega_3\omega_1\omega_2^3 + 6\omega_3\omega_1^2\omega_2 - 2\omega_3\omega_1^3\omega_2^3 + 333\omega_3v_1^4\omega_1^2\omega_2 - 99\omega_3v_1^2\omega_2^3 - 58\omega_3v_1^4\omega_1^3\omega_2^3 + 6cs^4\omega_3\omega_1^2\omega_2 - 6cs^2\omega_1^3\omega_2 - 2cs^4\omega_3\omega_1^3\omega_2^3 + 8\omega_3\omega_1^3\omega_2^2 - 12cs^2\omega_3\omega_1\omega_2^2 - 154\omega_3v_1^2\omega_1^2\omega_2^2 + 252cs^2\omega_3v_1^2\omega_1^3\omega_2^2 + 225\omega_3v_1^2\omega_1^3\omega_2^2) \frac{\rho}{24\omega_3\omega_1^3\omega_2^3}$$

coefficient $C_{D_x^3 D_y \rho}^{(1)}$ at $\frac{\partial^4 \rho}{\partial x_1^3 \partial x_2}$:

$$C_{D_x^3 D_y \rho}^{(1), \text{SRT}} = 0$$

$$C_{D_x^3 D_y \rho}^{(1), \text{MRT1}} = (8v_1^4\omega_7^2\omega_5^2 - 51v_1^2cs^2\omega_7\omega_4^2\omega_5^3 - 84v_1^2cs^2\omega_7^2\omega_4\omega_5^2 - 20v_1^4\omega_7\omega_4^2\omega_5 + 16v_1^2\omega_7\omega_4\omega_5^2 + 8cs^4\omega_7\omega_4^2\omega_5^2 - 4v_1^4\omega_4\omega_5^3 + 120v_1^2cs^2\omega_7\omega_4^2\omega_5^2 + 20v_1^4\omega_7^2\omega_4\omega_5 + 12cs^2\omega_7^2\omega_4^2\omega_5 - 4cs^4\omega_4\omega_5^3 + 36v_1^2cs^2\omega_7^2\omega_5^2 - 4cs^4\omega_7\omega_4^2\omega_5^3 - 24v_1^2cs^2\omega_4\omega_5^3 + 4cs^4\omega_7^2\omega_5^2 - 8cs^4\omega_7^2\omega_4\omega_5^2 - 13v_1^2\omega_7^2\omega_4^2\omega_5^2 - 20v_1^2\omega_7\omega_4\omega_5^3 + 4cs^4\omega_7^2\omega_4\omega_5 - 8v_1^2\omega_7^2\omega_5^2 + 36v_1^2\omega_7^2\omega_4^2\omega_5 - 13v_1^4\omega_7\omega_4^2\omega_5^3 - 20v_1^4\omega_7\omega_4\omega_5^2 - 72v_1^2cs^2\omega_7\omega_4^2\omega_5 + 4v_1^2\omega_4\omega_5^3 - 4cs^2\omega_7^2\omega_4^2\omega_5^2 - 8cs^2\omega_7\omega_4\omega_5^3 + 4cs^2\omega_4\omega_5^3 - 4cs^4\omega_7\omega_4^2\omega_5 - 4cs^2\omega_7^2\omega_5^2 + 32v_1^4\omega_7\omega_4^2\omega_5^2 + 72v_1^2cs^2\omega_7^2\omega_4\omega_5 + 96v_1^2cs^2\omega_7^2\omega_4^2 + 20v_1^2\omega_7\omega_4^2\omega_5 + 8v_1^2\omega_7\omega_5^3 - 4cs^2\omega_4^2\omega_5^3 + 4v_1^2\omega_4^2\omega_5^2 - 8cs^2\omega_7\omega_4^2\omega_5^2 - 16v_1^4\omega_7\omega_4\omega_5^2 - 144v_1^2cs^2\omega_7^2\omega_4^2\omega_5 + 8cs^4\omega_7^2\omega_4^2 + 4cs^2\omega_4^2\omega_5^2 - 12cs^4\omega_7^2\omega_4^2\omega_5 - 20v_1^2\omega_7^2\omega_4\omega_5 + 24v_1^4\omega_7^2\omega_4^2 + 13v_1^4\omega_7^2\omega_4^2\omega_5^2 + 20v_1^4\omega_7\omega_4\omega_5^3 + 4cs^2\omega_7\omega_4^2\omega_5^3 + 4cs^2\omega_7\omega_5^3 + 8cs^2\omega_7^2\omega_4\omega_5^2 - 4v_1^2\omega_4^2\omega_5^3 - 48v_1^2cs^2\omega_7\omega_4\omega_5^2 - 36v_1^4\omega_7^2\omega_4^2\omega_5 - 8v_1^4\omega_7\omega_5^3 + 4cs^4\omega_4^2\omega_5^3 - 4cs^2\omega_7^2\omega_4\omega_5 + 4cs^4\omega_7^2\omega_4^2\omega_5^2 - 4v_1^4\omega_4^2\omega_5^2 + 8cs^4\omega_7\omega_4\omega_5^3 + 24v_1^2cs^2\omega_4^2\omega_5^3 + 13v_1^2\omega_7\omega_4^2\omega_5^3 - 8cs^2\omega_7^2\omega_4^2 + 20v_1^2\omega_7^2\omega_4\omega_5^2 - 36v_1^2cs^2\omega_7\omega_5^3 + 4cs^2\omega_7\omega_4^2\omega_5 - 4cs^4\omega_4^2\omega_5^2 - 24v_1^2\omega_7^2\omega_4^2 + 51v_1^2cs^2\omega_7^2\omega_4^2\omega_5^2 + 84v_1^2cs^2\omega_7\omega_4\omega_5^3 - 32v_1^2\omega_7\omega_4^2\omega_5^2 - 4cs^4\omega_7\omega_5^3 - 24v_1^2cs^2\omega_4^2\omega_5^2 + 4v_1^4\omega_4^2\omega_5^3) \frac{v_2}{4\omega_7^2\omega_4^2\omega_5^3}$$

$$C_{D_x^3 D_y \rho}^{(1), \text{MRT2}} = (8v_1^4\omega_7^2\omega_5^2 + 12\omega_7^2\omega_4^2cs^2\omega_5 - 84v_1^2\omega_7^2\omega_4cs^2\omega_5^2 - 8\omega_7\omega_4cs^2\omega_5^3 - 20v_1^4\omega_7\omega_4^2\omega_5 + 16v_1^2\omega_7\omega_4\omega_5^2 + 96v_1^2\omega_7^2\omega_4^2cs^2 - 8\omega_7^2\omega_4cs^4\omega_5^2 - 4v_1^4\omega_4\omega_5^3 - 4\omega_7cs^4\omega_5^3 + 20v_1^4\omega_7^2\omega_4\omega_5 + 4\omega_4cs^2\omega_5^3 - 72v_1^2\omega_7\omega_4^2cs^2\omega_5 - 13v_1^2\omega_7^2\omega_4^2\omega_5^2 + 36v_1^2\omega_7^2cs^2\omega_5^2 - 4\omega_7\omega_4^2cs^4\omega_5 - 20v_1^2\omega_7\omega_4\omega_5^3 - 8v_1^2\omega_7^2\omega_5^2 + 4\omega_2^2cs^4\omega_5^3 + 36v_1^2\omega_7^2\omega_4^2\omega_5 - 8\omega_7^2\omega_4^2cs^2 - 8\omega_7\omega_4^2cs^4\omega_5^2 - 13v_1^4\omega_7\omega_4^2\omega_5^3 - 20v_1^4\omega_7\omega_4\omega_5^2 + 4v_1^2\omega_4\omega_5^3 + 120v_1^2\omega_7\omega_4^2cs^2\omega_5^2 - 4\omega_7\omega_4^2cs^4\omega_5^3 - 4\omega_7^2cs^2\omega_5^2 + 4\omega_7^2\omega_4cs^4\omega_5 - 4\omega_4^2cs^4\omega_5^2 - 51v_1^2\omega_7\omega_4^2cs^2\omega_5^3 - 24v_1^2\omega_4cs^2\omega_5^3 - 4\omega_7^2\omega_4^2cs^2\omega_5^2 + 72v_1^2\omega_7^2\omega_4cs^2\omega_5 + 32v_1^4\omega_7\omega_4^2\omega_5^2 + 4\omega_7cs^2\omega_5^3 + 20v_1^2\omega_7\omega_4^2\omega_5 - 4\omega_4cs^4\omega_5^3 + 8v_1^2\omega_7\omega_5^3 - 48v_1^2\omega_7\omega_4cs^2\omega_5^2 - 24v_1^2\omega_4^2cs^2\omega_5^2 + 4v_1^2\omega_4^2\omega_5^2 - 16v_1^2\omega_7\omega_4\omega_5^2 + 8\omega_7^2\omega_4cs^4 + 4\omega_7\omega_4^2cs^2\omega_5 -$$

$$12\omega_7^2\omega_4cs^4\omega_5 - 20v_1^2\omega_7^2\omega_4\omega_5 + 24v_1^4\omega_7^2\omega_4^2 + 8\omega_7\omega_4cs^4\omega_5^3 + 13v_1^4\omega_7^2\omega_4^2\omega_5^2 + 20v_1^4\omega_7\omega_4\omega_5^3 - 144v_1^2\omega_7^2\omega_4cs^2\omega_5 + 8\omega_7^2\omega_4cs^2\omega_5^2 + 24v_1^2\omega_4cs^2\omega_5^3 + 84v_1^2\omega_7\omega_4cs^2\omega_5^3 - 4v_1^2\omega_4^2\omega_5^3 + 4\omega_7^2cs^4\omega_5^2 - 36v_1^4\omega_7^2\omega_4^2\omega_5 + 4\omega_7\omega_4^2cs^2\omega_5^3 + 51v_1^2\omega_7^2\omega_4^2cs^2\omega_5^2 - 8v_1^4\omega_7\omega_5^3 + 4\omega_4^2cs^2\omega_5^2 - 4\omega_7^2\omega_4cs^2\omega_5 - 4v_1^4\omega_4^2\omega_5^2 + 4\omega_7^2\omega_4^2cs^4\omega_5^2 + 13v_1^2\omega_7\omega_4^2\omega_5^3 - 36v_1^2\omega_7cs^2\omega_5^3 + 20v_1^2\omega_7^2\omega_4\omega_5^2 - 4\omega_4^2cs^2\omega_5^3 - 24v_1^2\omega_7^2\omega_4^2 - 8\omega_7\omega_4^2cs^2\omega_5^2 - 32v_1^2\omega_7\omega_4^2\omega_5^2 + 4v_1^4\omega_4^2\omega_5^3) \frac{v_2}{4\omega_7^2\omega_4^2\omega_5^3}$$

$$C_{D_x^3 D_y \rho}^{(1), \text{CLBM1}} = 0$$

$$C_{D_x^3 D_y \rho}^{(1), \text{CLBM2}} = 0$$

$$C_{D_x^3 D_y \rho}^{(1), \text{CuLBM1}} = 0$$

$$C_{D_x^3 D_y \rho}^{(1), \text{CuLBM2}} = (9\omega_3^2v_1^2\omega_2^2v_2\omega_2 + 108\omega_3^2v_1^4\omega_1^2\omega_2^2 + 6cs^4\omega_3^2\omega_1^3\omega_2^2 - 6cs^2\omega_3\omega_1^3v_2^2\omega_2 + \omega_3^2\omega_1^2v_2^2\omega_2^3 - 72cs^2\omega_3^2v_1^2\omega_1^2\omega_2^2 - 24\omega_3^2v_1^2\omega_1^3\omega_2^2 - 486cs^2\omega_3^2v_1^2\omega_1^3\omega_2 - 3cs^2\omega_3^2\omega_1\omega_2^3 - 6\omega_3^2\omega_1^2\omega_2^2 - 54\omega_3^2v_1^4\omega_1^3\omega_2 - 6cs^2\omega_3^2\omega_1^2v_2^2\omega_2 - 18cs^4\omega_3^2\omega_2^3 - 54cs^4\omega_3^2\omega_1^2\omega_2 + 6cs^2\omega_3\omega_1v_2^2\omega_2^3 - 9\omega_3^2v_1^2\omega_2^3 + 18cs^2\omega_3^2\omega_1^3v_2^2 + 6cs^2\omega_3\omega_1^3\omega_2 + 12cs^2\omega_3^2\omega_1\omega_2^2 + 45\omega_3^2v_1^2\omega_1^2\omega_2 + 18cs^2\omega_3\omega_1^2\omega_2^2 + 540cs^2\omega_3^2\omega_1^2\omega_2^2 + \omega_3^2\omega_1^3\omega_2^2 - \omega_3^2\omega_1^2\omega_2^3 - 18cs^2\omega_3\omega_1^3\omega_2^2 - 6\omega_3^2\omega_1^3\omega_2 - 243cs^2\omega_3^2v_1^2\omega_1^2\omega_2 + 12cs^2\omega_1^2v_2^2\omega_2^3 - 108\omega_3^2v_1^2\omega_1^2\omega_2^2 + 18cs^4\omega_3\omega_1\omega_2^3 - 18cs^2\omega_3^2v_2^2\omega_2^3 + 6\omega_3^2\omega_1^2v_2^2\omega_2 + 72cs^4\omega_3^2\omega_1^2\omega_2^2 - 24\omega_3^2v_1^2\omega_1^2v_2^2\omega_2^3 + 6\omega_3^2\omega_1^3 + 24\omega_3^2v_1^2\omega_1^2\omega_2^3 + 72cs^2\omega_3^2v_1^2\omega_1^3\omega_2^2 + 126\omega_3^2v_1^2\omega_1^3\omega_2 + 54\omega_3^2v_1^4\omega_1^3 + 405cs^2\omega_3^2v_1^2\omega_1^3 - 54\omega_3^2v_1^4\omega_1^2\omega_2 - 6cs^4\omega_3^2\omega_1^2\omega_2^3 - 2cs^2\omega_3^2\omega_1^2v_2^2\omega_2^3 + 18cs^2\omega_3\omega_1^3v_2^2\omega_2^2 - 99cs^4\omega_3^2\omega_1^3\omega_2 - 72cs^2\omega_3^2\omega_1^3 + 6\omega_3^2\omega_1\omega_2^2 - 18\omega_3^2v_1^2\omega_1\omega_2^3 - 5cs^2\omega_3^2\omega_1^3\omega_2^2 - 6\omega_3^2\omega_1^3v_2^2 + 54\omega_3^2v_1^4\omega_2^3 - 21cs^2\omega_3^2\omega_1^3v_2^2\omega_2 + 72\omega_3^2v_1^2\omega_1v_2^2\omega_2^3 - \omega_3^2\omega_1^3v_2^2\omega_2^2 + 135cs^2\omega_3^2v_1^2\omega_2^3 + 27cs^4\omega_3^2\omega_1\omega_2^3 + 6cs^2\omega_3^2\omega_1v_2^2\omega_2^2 + 12cs^2\omega_1^3\omega_2^2 + 45\omega_3^2v_1^2\omega_1^3v_2^2 + 63\omega_3^2v_1^4\omega_1\omega_2^2 + 36cs^4\omega_1^2\omega_2^3 + 6\omega_3^2\omega_1\omega_2^3 + 60cs^2\omega_3^2\omega_1^2\omega_2 + 21cs^2\omega_3^2\omega_1v_2^2\omega_2^3 - 18cs^4\omega_3\omega_1\omega_2^3 - 72\omega_3^2v_1^2\omega_1^3v_2^2\omega_2 - 18cs^4\omega_3\omega_1^3\omega_2 - 54cs^4\omega_3\omega_1^2\omega_2^3 - 6\omega_3^2\omega_2^3 - 9\omega_3^2v_1^2\omega_1v_2^2\omega_2^2 + 54cs^4\omega_3\omega_1^3\omega_2^2 - 6\omega_3^2\omega_1v_2^2\omega_2^3 + 24\omega_3^2v_1^2\omega_1^3v_2^2\omega_2^2 - 54\omega_3^2v_1^4\omega_1\omega_2^3 - 45\omega_3^2v_1^2v_2^2\omega_2^3 + 90cs^4\omega_3^2\omega_1^3 - 99\omega_3^2v_1^4\omega_2^3 - 12cs^2\omega_1^3v_2^2\omega_2^2 - 297cs^2\omega_3^2v_1^2\omega_1\omega_2^3 - 36cs^4\omega_1^3\omega_2^3 - 6cs^2\omega_3\omega_1\omega_2^3 - 72cs^2\omega_3^2\omega_1^2\omega_2^3 - 18cs^2\omega_3\omega_1^2v_2^2\omega_2^3 + 6\omega_3^2\omega_1^3v_2^2\omega_2 + 2cs^2\omega_3^2\omega_1^3v_2^2\omega_2^2 - 12cs^2\omega_1^2\omega_2^3 + 6\omega_3^2v_2^2\omega_2^3 - 54\omega_3^2v_1^4\omega_1\omega_2^2 - 6\omega_3^2\omega_1v_2^2\omega_2^2 + 5cs^2\omega_3^2\omega_1^2\omega_2^3 + 75cs^2\omega_3^2\omega_1^3\omega_2 - 54cs^2\omega_3^2v_1^2\omega_1\omega_2^3) \frac{v_2}{24\omega_3^2\omega_1^3\omega_2^3}$$

$$\text{coefficient } C_{D_x^3 D_y v_1}^{(1)} \text{ at } \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} :$$

$$C_{D_x^3 D_y v_1}^{(1), \text{SRT}} = 0$$

$$C_{D_x^3 D_y v_1}^{(1), \text{MRT1}} = (48\omega_7^2\omega_2^2\omega_5 - 8\omega_4^2\omega_5^3 - 48v_1^2\omega_7\omega_4\omega_5^2 + 12\omega_7\omega_5^3 - 72cs^2\omega_7^2\omega_4^2\omega_5 + 43v_1^2\omega_7^2\omega_4^2\omega_5^2 + 68v_1^2\omega_7\omega_4\omega_5^3 + 8\omega_4^2\omega_5^2 - 32\omega_7^2\omega_4^2 + 28v_1^2\omega_7^2\omega_5^2 - 120v_1^2\omega_7^2\omega_4^2\omega_5 + 16\omega_7\omega_4\omega_5^2 - 16v_1^2\omega_4\omega_5^3 + 25cs^2\omega_7^2\omega_4^2\omega_5^2 + 44cs^2\omega_7\omega_4\omega_5^3 - 17\omega_7^2\omega_4^2\omega_5^2 - 28\omega_7\omega_4\omega_5^3 - 16cs^2\omega_4\omega_5^3 - 16cs^2\omega_7\omega_4\omega_5^2 + 20cs^2\omega_7^2\omega_5^2 + 17\omega_7\omega_4^2\omega_5^3 - 64v_1^2\omega_7\omega_4^2\omega_5 + 28\omega_7^2\omega_4\omega_5^2 - 28v_1^2\omega_7\omega_5^3 + 16cs^2\omega_4^2\omega_5^3 - 16v_1^2\omega_4^2\omega_5^2 + 56cs^2\omega_7\omega_4^2\omega_5^2 - 16cs^2\omega_7^2\omega_5^2 + 64v_1^2\omega_7^2\omega_4\omega_5 - 40\omega_7\omega_4^2\omega_5^2 - 25cs^2\omega_7\omega_4^2\omega_5^3 - 20cs^2\omega_7\omega_5^3 - 44cs^2\omega_7^2\omega_4\omega_5^2 + 16v_1^2\omega_4^2\omega_5^3 + 32cs^2\omega_7^2\omega_4\omega_5 - 43v_1^2\omega_7\omega_4^2\omega_5^3 + 48cs^2\omega_7^2\omega_4^2 + 24\omega_7\omega_4^2\omega_5 - 68v_1^2\omega_7^2\omega_4\omega_5^2 - 32cs^2\omega_7\omega_4^2\omega_5 + 80v_1^2\omega_7^2\omega_4^2 - 12\omega_7^2\omega_5^2 + 8\omega_4\omega_5^3 - 24\omega_7^2\omega_4\omega_5 + 104v_1^2\omega_7\omega_4^2\omega_5^2) \frac{v_1 \rho v_2}{4\omega_7^2\omega_4^2\omega_5^3}$$

$$C_{D_x^3 D_y v_1}^{(1), \text{MRT2}} = (-72\omega_7^2\omega_4^2cs^2\omega_5 + 44\omega_7\omega_4cs^2\omega_5^3 + 48\omega_7^2\omega_4^2\omega_5 - 8\omega_4^2\omega_5^3 - 48v_1^2\omega_7\omega_4\omega_5^2 - 16\omega_7\omega_4cs^2\omega_5^2 + 12\omega_7\omega_5^3 - 16\omega_4cs^2\omega_5^3 + 43v_1^2\omega_7^2\omega_4^2\omega_5^2 + 68v_1^2\omega_7\omega_4\omega_5^3 + 8\omega_4^2\omega_5^2 - 32\omega_7^2\omega_4^2 + 28v_1^2\omega_7^2\omega_5^2 - 120v_1^2\omega_7^2\omega_4^2\omega_5 + 16\omega_7\omega_4\omega_5^2 - 16v_1^2\omega_4\omega_5^3 + 25cs^2\omega_7^2\omega_4^2\omega_5^2 + 44cs^2\omega_7\omega_4\omega_5^3 - 17\omega_7^2\omega_4^2\omega_5^2 - 28\omega_7\omega_4\omega_5^3 - 16cs^2\omega_4\omega_5^3 - 16cs^2\omega_7\omega_4\omega_5^2 + 20cs^2\omega_7^2\omega_5^2 + 17\omega_7\omega_4^2\omega_5^3 - 64v_1^2\omega_7\omega_4^2\omega_5 + 28\omega_7^2\omega_4\omega_5^2 - 28v_1^2\omega_7\omega_5^3 + 16cs^2\omega_4^2\omega_5^3 - 16v_1^2\omega_4^2\omega_5^2 + 56cs^2\omega_7\omega_4^2\omega_5^2 - 16cs^2\omega_7^2\omega_5^2 + 64v_1^2\omega_7^2\omega_4\omega_5 - 40\omega_7\omega_4^2\omega_5^2 - 44\omega_7^2\omega_4cs^2\omega_5^2 + 16v_1^2\omega_4^2\omega_5^3 - 25\omega_7\omega_4^2cs^2\omega_5^3 - 16\omega_4^2cs^2\omega_5^2 + 32\omega_7\omega_4cs^2\omega_5 - 43v_1^2\omega_7\omega_4^2\omega_5^3 + 24\omega_7\omega_4^2\omega_5 - 68v_1^2\omega_7^2\omega_4\omega_5^2 + 16\omega_4^2cs^2\omega_5^3 + 80v_1^2\omega_7^2\omega_4^2 - 12\omega_7^2\omega_5^2 + 56\omega_7\omega_4^2cs^2\omega_5^2 + 8\omega_4\omega_5^3 - 24\omega_7^2\omega_4\omega_5 + 104v_1^2\omega_7\omega_4^2\omega_5^2) \frac{v_1 \rho v_2}{4\omega_7^2\omega_4^2\omega_5^3}$$

$$C_{D_x^3 D_y v_1}^{(1), \text{CLBM1}} = 0$$

$$C_{D_x^3 D_y v_1}^{(1), \text{CLBM2}} = 0$$

$$C_{D_x^3 D_y v_1}^{(1), \text{CuLBM1}} = 0$$

$$C_{D_x^3 D_y v_1}^{(1), \text{CuLBM2}} = (198\omega_3v_1^2\omega_1^3 + 54cs^2\omega_1\omega_2^3 + 10\omega_3\omega_1^3v_2^2\omega_2^2 + 324cs^2\omega_3\omega_1^2\omega_2^2 - 198\omega_3v_1^2\omega_1\omega_2^3 - 18\omega_1^3v_2^2\omega_2 - 36\omega_3v_2^2\omega_2^3 - 198\omega_3v_1^2\omega_1\omega_2^2 + 18\omega_1^3\omega_2 + 18\omega_1^2\omega_2^3 - 297cs^2\omega_3\omega_1^3\omega_2 + 18\omega_1v_2^2\omega_2^3 - 30cs^2\omega_3\omega_1^2\omega_2^3 - 18\omega_1^3\omega_2^2 + 36\omega_3\omega_1^3v_2^2 + 30cs^2\omega_3\omega_1^2\omega_2^2 + 90\omega_3\omega_1\omega_2^2 + 45\omega_3\omega_1v_2^2\omega_2^3 - 54\omega_3\omega_2^3 + 45\omega_3\omega_1\omega_2^3 - 162cs^2\omega_3\omega_1^2\omega_2 + 270cs^2\omega_3\omega_1^3 + 18\omega_1^3v_2^2\omega_2^2 - 45\omega_3\omega_1^3v_2^2\omega_2 + 135\omega_3\omega_1^3\omega_2 + 54cs^2\omega_3\omega_2^3 + 10\omega_3\omega_1^2\omega_2^3 - 10\omega_3\omega_1^2v_2^2\omega_2^3 + 54cs^2\omega_1^3\omega_2^2 - 198\omega_3v_1^2\omega_1^2\omega_2 - 180\omega_3\omega_1^2\omega_2^2 - 126\omega_3\omega_1^3 - 18\omega_1\omega_2^3 + 396\omega_3v_1^2\omega_1^2\omega_2^2 - 27cs^2\omega_3\omega_1\omega_2^3 + 90\omega_3\omega_1^2\omega_2 + 198\omega_3v_1^2\omega_2^3 - 54cs^2\omega_1^2\omega_2^3 - 54cs^2\omega_1^3\omega_2 - 10\omega_3\omega_1^3\omega_2^2 - 162cs^2\omega_3\omega_1\omega_2^2 - 18\omega_1^2v_2^2\omega_2^3 - 198\omega_3v_1^2\omega_1^3\omega_2) \frac{v_1 \rho v_2}{24\omega_3\omega_1^3\omega_2^3}$$

$$\text{coefficient } C_{D_x^3 D_y v_2}^{(1)} \text{ at } \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} :$$

$$C_{D_x^3 D_y v_2}^{(1), \text{SRT}} = (12cs^2v_1^2\omega^3 - 4v_1^2\omega^3 + 36cs^2\omega - 42cs^2v_1^2\omega^2 + 36v_1^4 + 26v_1^2\omega^2 - 54v_1^2\omega + 36cs^4 + 54cs^2v_1^2\omega - 12cs^2\omega^2 - 36cs^2v_1^2 - 24cs^2 - 26v_1^4\omega^2 + 4v_1^4\omega^3 - 54cs^4\omega + 20cs^4\omega^2 - cs^4\omega^3 + 54v_1^4\omega - 36v_1^4) \frac{\rho}{12\omega^3}$$

$$\begin{aligned}
C_{D_3^2 D_y v_2}^{(1), \text{MRT1}} = & (-cs^2\omega_7^2\omega_4^2\omega_5^2 - 48v_1^2cs^2\omega_7^2\omega_5^3 - 27v_1^4\omega_7\omega_4^3\omega_5^3 + 30v_1^2cs^2\omega_7\omega_4^2\omega_5^3 - 48v_1^2cs^2\omega_7^2\omega_4\omega_5^2 - 36v_1^2cs^2\omega_7\omega_4^3\omega_5 + 90v_1^2\omega_7^2\omega_3^3\omega_5 + \\
& 18v_1^4\omega_7^2\omega_4^2\omega_5^3 - 24cs^4\omega_7^2\omega_4\omega_5^3 + 12cs^4\omega_7\omega_4^2\omega_5^2 + 12cs^4\omega_7^2\omega_5^3 + 102v_1^2cs^2\omega_7^2\omega_4\omega_5^3 + 60v_1^4\omega_7\omega_4^3\omega_5^2 - 12v_1^2cs^2\omega_7\omega_4^2\omega_5^2 + 12cs^2\omega_7^2\omega_4^2\omega_5 - \\
& 18cs^4\omega_7\omega_4^2\omega_5^3 - 12v_1^2\omega_7^2\omega_4^2\omega_5^2 + 24v_1^4\omega_7\omega_4\omega_5^3 - 6cs^4\omega_7\omega_4^3\omega_5^2 - 4v_1^2\omega_7^2\omega_4^3\omega_5^3 - 21v_1^2cs^2\omega_7\omega_4^3\omega_5^3 + 48v_1^4\omega_7\omega_4^2\omega_5^3 - 36v_1^4\omega_7\omega_4^3\omega_5 - 6cs^2\omega_7^2\omega_3^2\omega_5^2 - \\
& 12cs^2\omega_7\omega_4\omega_5^3 - 19v_1^2\omega_7^2\omega_4^3\omega_5^2 + 6cs^4\omega_7\omega_4^3\omega_5^2 + 12cs^2\omega_7^2\omega_4^3\omega_5 - 5cs^2\omega_7^2\omega_4^3\omega_5^3 + 12v_1^4\omega_7^2\omega_4\omega_5^3 + 54v_1^2cs^2\omega_7\omega_4^3\omega_5^2 - 24v_1^4\omega_7\omega_4^2\omega_5^2 + 72v_1^4\omega_7^2\omega_4^3\omega_5 + \\
& 27v_1^2\omega_7\omega_4^3\omega_5^3 + cs^4\omega_7^2\omega_4^3\omega_5^2 + 12v_1^2cs^2\omega_4^3\omega_5^3 - 12v_1^4\omega_7^2\omega_5^2 + 6cs^2\omega_7^2\omega_4\omega_5^3 - 12cs^2\omega_7\omega_4^2\omega_5^2 - 108v_1^2cs^2\omega_7^2\omega_4^2\omega_5 - 90v_1^4\omega_7^2\omega_4^3\omega_5 + 12v_1^2cs^2\omega_7^2\omega_4^3\omega_5^3 - \\
& 18v_1^4\omega_7^2\omega_4^2\omega_5^3 + 12v_1^4\omega_4^3\omega_5^3 - 12v_1^2cs^2\omega_4^3\omega_5^2 - 12cs^4\omega_7^2\omega_4^2\omega_5 - cs^4\omega_7^2\omega_4^3\omega_5^3 + 252v_1^2cs^2\omega_7^2\omega_4^3\omega_5 - 60v_1^2\omega_7\omega_4^3\omega_5^2 + 60v_1^2cs^2\omega_7^2\omega_4^3\omega_5^2 + 12cs^4\omega_7^2\omega_4^3\omega_5 + \\
& 12v_1^4\omega_7^2\omega_4^2\omega_5^2 - 24v_1^4\omega_7\omega_4\omega_5^3 + 18cs^2\omega_7\omega_4^2\omega_5^3 + 12v_1^2\omega_4^3\omega_5^3 - 306v_1^2cs^2\omega_7^2\omega_4^3\omega_5 + 4v_1^4\omega_7^2\omega_4^3\omega_5^3 - 72v_1^2\omega_7^2\omega_4^3\omega_5 - 81v_1^2cs^2\omega_7^2\omega_4^2\omega_5^3 + 12v_1^2\omega_7^2\omega_5^2 + \\
& 6cs^2\omega_7\omega_4^3\omega_5^2 + 6cs^4\omega_7^2\omega_4^3\omega_5^2 + 12cs^4\omega_7\omega_4\omega_5^3 - 12v_1^2cs^2\omega_4^3\omega_5^3 - 48v_1^4\omega_7\omega_4^3\omega_5^3 + 36v_1^4\omega_7\omega_4^3\omega_5 - 6cs^2\omega_7\omega_4^3\omega_5^3 - 12v_1^2\omega_4^3\omega_5^3 + 19v_1^4\omega_7^2\omega_4^3\omega_5^2 + \\
& 162v_1^2cs^2\omega_7^2\omega_4^2\omega_5^2 - 12v_1^2cs^2\omega_7\omega_4\omega_5^3 - 12v_1^2\omega_7^2\omega_4\omega_5^3 - 12cs^2\omega_7^2\omega_4^3\omega_5 + 24v_1^2\omega_7\omega_4^2\omega_5^2 - 12cs^4\omega_7^2\omega_4^3\omega_5 - 12v_1^4\omega_4^3\omega_5^3 + 13cs^4\omega_7^2\omega_4^3\omega_5^3) \frac{\rho}{12\omega_7^2\omega_4^3\omega_5^3}
\end{aligned}$$

$$\begin{aligned}
C_{D_3^2 D_y v_2}^{(1), \text{MRT2}} = & (-306v_1^2\omega_7^2\omega_4^3cs^2\omega_5 + 12\omega_7^2\omega_4^2cs^2\omega_5 - 27v_1^4\omega_7\omega_4^3\omega_5^3 - 48v_1^2\omega_7^2\omega_4cs^2\omega_5^2 + 12v_1^2\omega_4^3cs^2\omega_5^3 - 12\omega_7\omega_4cs^2\omega_5^3 + 90v_1^2\omega_7^2\omega_4^3\omega_5 - \\
& 12\omega_7^2\omega_4^3cs^4\omega_5 + 18v_1^2\omega_7^2\omega_4^3\omega_5^3 - 12\omega_7^2\omega_4^3cs^2\omega_5^2 - 48v_1^2v_1^2cs^2\omega_5^3 + 102v_1^2\omega_7^2\omega_4cs^2\omega_5^3 + 60v_1^4\omega_7\omega_4^3\omega_5^2 - 12v_1^2\omega_4^3cs^2\omega_5^2 - 24\omega_7^2\omega_4cs^4\omega_5^3 - 12v_1^2\omega_7^2\omega_4^2\omega_5^2 + \\
& 24v_1^2\omega_7\omega_4\omega_5^3 - \omega_7^2\omega_3^3cs^3\omega_5^3 + 252v_1^2\omega_7^2\omega_4^3cs^2\omega_5^2 + 12\omega_7\omega_4^2cs^4\omega_5^2 - 4v_1^2\omega_7^2\omega_4^3\omega_5^3 + 48v_1^4\omega_7\omega_4^2\omega_5^3 - 36v_1^4\omega_7\omega_4^3\omega_5 - 5\omega_7^2\omega_4^2cs^2\omega_5^3 + 6\omega_7\omega_3^3cs^2\omega_5^2 - \\
& 12v_1^2\omega_7^2\omega_4^3cs^2\omega_5^2 + 12v_1^2\omega_7^2\omega_4^3cs^2\omega_5^3 + \omega_7^2\omega_4^3cs^4\omega_5^2 - 19v_1^2\omega_7^2\omega_4^3\omega_5^2 - 18\omega_7\omega_4^2cs^4\omega_5^3 + 30v_1^2\omega_7\omega_4^2cs^2\omega_5^3 + 60v_1^2\omega_7^2\omega_4^3cs^2\omega_5^2 - 6\omega_7^2\omega_4^2cs^2\omega_5^2 - \\
& 6\omega_7\omega_4^3cs^2\omega_5^3 + 12v_1^4\omega_7^2\omega_4\omega_5^3 - 24v_1^4\omega_7\omega_4^2\omega_5^2 + 27v_1^4\omega_7\omega_4^3\omega_5^2 - 12v_1^4\omega_4^3\omega_5^2 + 6\omega_7^2\omega_4cs^4\omega_5^3 - 36v_1^2\omega_7\omega_4^3cs^2\omega_5 - 90v_1^4\omega_7^2\omega_4^3\omega_5 - \\
& 18v_1^4\omega_7^2\omega_4^3\omega_5^3 + 12v_1^4\omega_3^3\omega_5^3 - 12\omega_7^2\omega_4^2cs^4\omega_5 - 60v_1^2\omega_7\omega_4^3\omega_5^2 + 12\omega_7\omega_4^2cs^4\omega_5^3 + 12\omega_7^2\omega_4^3cs^2\omega_5 + 12v_1^4\omega_7^2\omega_4^2\omega_5^2 - 24v_1^4\omega_7\omega_4\omega_5^3 - 108v_1^2\omega_7^2\omega_4^2cs^2\omega_5 - \\
& 12v_1^2\omega_4^3cs^2\omega_5^3 - 12v_1^2\omega_7\omega_4cs^2\omega_5^3 + 12v_1^2\omega_7^2\omega_5^3 + 18\omega_7\omega_4^2cs^2\omega_5^3 - \omega_7^2\omega_3^3cs^2\omega_5^2 + 12\omega_7^2\omega_4^3cs^4\omega_5 + 4v_1^4\omega_7^2\omega_5^3 - 72v_1^2\omega_7^2\omega_5^3 + 162v_1^2\omega_7^2\omega_4^2cs^2\omega_5^2 - \\
& 21v_1^2\omega_7\omega_4^3cs^2\omega_5^3 + 12v_1^2\omega_3^3\omega_5^2 + 6\omega_7\omega_3^3cs^4\omega_5^3 + 6\omega_7^2\omega_4^2cs^4\omega_5^2 - 48v_1^2\omega_7\omega_4^3\omega_5^3 + 36v_1^2\omega_7\omega_4^3\omega_5 - 12v_1^2\omega_3^3\omega_5^3 - 81v_1^2\omega_7^2\omega_4^2cs^2\omega_5^3 + 19v_1^4\omega_7^2\omega_4^3\omega_5^2 + \\
& 54v_1^4\omega_7\omega_4^2cs^2\omega_5^2 - 12\omega_7\omega_4^2cs^2\omega_5^2 + 12\omega_7^2cs^4\omega_5^3 - 12v_1^2\omega_7^2\omega_4\omega_5^3 + 24v_1^2\omega_7\omega_4^2\omega_5^2 - 6\omega_7\omega_4^2cs^4\omega_5^2 + 13\omega_7^2\omega_4cs^4\omega_5^3 - 12v_1^4\omega_4^3\omega_5^3) \frac{\rho}{12\omega_7^2\omega_4^3\omega_5^3}
\end{aligned}$$

$$\begin{aligned}
C_{D_3^2 D_y v_2}^{(1), \text{CLB M1}} = & (-39v_1^4\omega_7\omega_4^3\omega_5^3 - cs^4\omega_7^2\omega_3^3\omega_5^3 - 12cs^4\omega_7^2\omega_4^2\omega_5 + 90v_1^2\omega_7^2\omega_4^3\omega_5 + 36cs^2v_1^2\omega_7\omega_4\omega_5^3 + 6v_1^2\omega_7^2\omega_4^3\omega_5^3 + 18cs^2v_1^2\omega_7^2\omega_4^2\omega_5^2 + \\
& 18cs^2\omega_7\omega_4^2\omega_5^3 - 12cs^2\omega_7^2\omega_4^3 + 72v_1^4\omega_7\omega_4^3\omega_5^2 + cs^4\omega_7^2\omega_4^3\omega_5^2 - 108cs^2v_1^2\omega_7^2\omega_5^3 - 12cs^2\omega_7\omega_4^2\omega_5^2 - 3cs^2v_1^2\omega_7^2\omega_4^2\omega_5^3 + 6cs^2\omega_7^2\omega_4\omega_5^3 - 306cs^2v_1^2\omega_7^2\omega_4^3\omega_5 - \\
& 6cs^2\omega_7\omega_4^3\omega_5^3 + 60cs^2v_1^2\omega_7^2\omega_4^2\omega_5^2 - 4v_1^2\omega_7^2\omega_4^3\omega_5^3 + 13cs^4\omega_7^2\omega_4^2\omega_5^3 + 252cs^2v_1^2\omega_7^2\omega_4^3\omega_5^2 + 36v_1^4\omega_7\omega_4^3\omega_5^2 - 12cs^4\omega_7^2\omega_4^3\omega_5 - 36v_1^4\omega_7\omega_4^3\omega_5 - \\
& 108cs^2v_1^2\omega_3^3\omega_5^2 + 12cs^2v_1^2\omega_7^2\omega_4^3\omega_5^3 + 6cs^2\omega_7\omega_4^3\omega_5^2 - 19v_1^2\omega_7^2\omega_4^3\omega_5^2 - 36cs^2v_1^2\omega_7^2\omega_4^2\omega_5 - 12cs^4\omega_7^2\omega_4^3\omega_5 + 108cs^2v_1^2\omega_3^3\omega_5^2 + 12cs^4\omega_7\omega_4\omega_5^3 + \\
& 6cs^4\omega_7^2\omega_4^2\omega_5^2 + 72v_1^4\omega_7^2\omega_4^3\omega_5^2 + 198cs^2v_1^2\omega_7\omega_4^3\omega_5^2 + 39v_1^2\omega_7\omega_4^3\omega_5^3 + 12cs^2\omega_7^2\omega_4^3\omega_5 - 36v_1^4\omega_4^3\omega_5^2 - 90v_1^4\omega_7^2\omega_4^3\omega_5 - 18cs^4\omega_7\omega_4^3\omega_5 - 6v_1^4\omega_7^2\omega_4^3\omega_5^3 + \\
& 36v_1^4\omega_7^2\omega_5^3 - 99cs^2v_1^2\omega_7\omega_4^3\omega_5^2 - cs^2\omega_7^2\omega_4^3\omega_5^2 - 72v_1^2\omega_7\omega_4^3\omega_5^2 + 12cs^4\omega_7\omega_4^2\omega_5^2 + 72v_1^4\omega_7^2\omega_4^3\omega_5^2 - 24cs^4\omega_7^2\omega_4\omega_5^3 + 36v_1^2\omega_7^2\omega_5^3 + 4v_1^4\omega_7^2\omega_5^3 - 72v_1^2\omega_7^2\omega_5^3 + \\
& 6cs^4\omega_7\omega_4^3\omega_5^3 + 36v_1^2\omega_3^3\omega_5^2 + 12cs^4\omega_7^2\omega_5^3 - 36v_1^2\omega_7\omega_4^3\omega_5^3 + 36cs^2v_1^2\omega_7\omega_4^2\omega_5^2 - 5cs^2\omega_7^2\omega_4^3\omega_5^3 - 18cs^2v_1^2\omega_7^2\omega_4^3\omega_5^3 + 36v_1^2\omega_7\omega_4^3\omega_5 + 12cs^2\omega_7^2\omega_4^3\omega_5 - \\
& 36v_1^2\omega_4^3\omega_5^3 + 19v_1^4\omega_7^2\omega_4^3\omega_5^2 - 6cs^4\omega_7\omega_4^3\omega_5^2 - 12cs^2\omega_7\omega_4\omega_5^3 - 108cs^2v_1^2\omega_7\omega_4^3\omega_5 - 6cs^2\omega_7^2\omega_4^2\omega_5^2 + 54cs^2v_1^2\omega_7\omega_4^2\omega_5^3 - 36v_1^4\omega_4^3\omega_5^3) \frac{\rho}{12\omega_7^2\omega_4^3\omega_5^3}
\end{aligned}$$

$$\begin{aligned}
C_{D_3^2 D_y v_2}^{(1), \text{CLB M2}} = & (18cs^2\omega_7\omega_4^2\omega_5^3 - 39v_1^4\omega_7\omega_4^3\omega_5^3 + 60v_1^2cs^2\omega_7^2\omega_4^3\omega_5^2 + 90v_1^2\omega_7^2\omega_3^3\omega_5 + 6v_1^2\omega_7^2\omega_4^2\omega_5^3 - cs^4\omega_7^2\omega_4^3\omega_5^3 - 12cs^4\omega_7^2\omega_2\omega_5 + \\
& 12v_1^2cs^2\omega_7^2\omega_4^3\omega_5^3 + 72v_1^4\omega_7\omega_4^2\omega_5^2 - 36v_1^2cs^2\omega_7^2\omega_4^2\omega_5 - 12cs^2\omega_7\omega_4^2\omega_5^2 + 6cs^2\omega_7^2\omega_4\omega_5^3 + cs^4\omega_7^2\omega_4^3\omega_5^2 + 12cs^4\omega_7^2\omega_5^3 + 13cs^4\omega_7^2\omega_4^2\omega_5^3 - 12cs^4\omega_7^2\omega_4^3\omega_5 - \\
& 4v_1^2\omega_7^2\omega_3^3\omega_5^3 + 36v_1^2cs^2\omega_7\omega_4\omega_5^3 + 36v_1^4\omega_7\omega_4^3\omega_5^2 + 18v_1^2cs^2\omega_7^2\omega_4^2\omega_5^2 - 36v_1^4\omega_7\omega_4^3\omega_5 - 6cs^2\omega_7\omega_4^3\omega_5^3 - 19v_1^2\omega_7^2\omega_4^3\omega_5^2 + 12cs^4\omega_7\omega_4\omega_5^3 + \\
& 6cs^4\omega_7^2\omega_4^2\omega_5^2 + 6cs^2\omega_7\omega_4^3\omega_5^2 - 3v_1^2cs^2\omega_7^2\omega_4^3\omega_5^2 - 306v_1^2cs^2\omega_7^2\omega_4^2\omega_5 + 12cs^4\omega_7\omega_4^2\omega_5^2 + 72v_1^4\omega_7^2\omega_4^3\omega_5^2 - 24cs^4\omega_7^2\omega_4\omega_5^3 + 36v_1^2\omega_7^2\omega_5^3 + 4v_1^4\omega_7^2\omega_5^3 - 72v_1^2\omega_7^2\omega_5^3 + \\
& 12cs^2\omega_7^2\omega_4^2\omega_5 + 36v_1^2cs^2\omega_7\omega_4^2\omega_5^2 - 90v_1^4\omega_7^2\omega_4^3\omega_5 - 18v_1^2cs^2\omega_7^2\omega_4\omega_5^3 - 6v_1^4\omega_7^2\omega_5^3 + 36v_1^4\omega_4^3\omega_5^2 + 12cs^4\omega_7\omega_4^2\omega_5^2 - 24cs^4\omega_7^2\omega_4\omega_5^3 - \\
& 12cs^2\omega_7^2\omega_4^3\omega_5 - 72v_1^2\omega_7\omega_4^3\omega_5^2 - 108v_1^2cs^2\omega_7\omega_4^3\omega_5 + 54v_1^2cs^2\omega_7\omega_4^2\omega_5^3 + 36v_1^2\omega_7^2\omega_4^3\omega_5 - cs^2\omega_7^2\omega_4^3\omega_5^2 + 198v_1^2cs^2\omega_7\omega_4^3\omega_5^2 + 4v_1^4\omega_7^2\omega_5^3 - 72v_1^2\omega_7^2\omega_4^3\omega_5 - \\
& 5cs^2\omega_7^2\omega_4^2\omega_5^3 + 36v_1^2\omega_3^3\omega_5^2 + 12cs^2\omega_7^2\omega_4^3\omega_5 + 108v_1^2cs^2\omega_4^3\omega_5^3 + 6cs^4\omega_7\omega_4^3\omega_5^3 - 36v_1^2\omega_7\omega_4^3\omega_5^3 + 36v_1^2\omega_7\omega_4^3\omega_5 - 12cs^2\omega_7\omega_4\omega_5^3 - 36v_1^2\omega_4^3\omega_5^3 - \\
& 6cs^2\omega_7^2\omega_4^2\omega_5^2 + 19v_1^4\omega_7^2\omega_4^3\omega_5^2 + 12cs^4\omega_7^2\omega_4^3\omega_5 - 99v_1^2cs^2\omega_7\omega_4^3\omega_5^3 + 252v_1^2cs^2\omega_7^2\omega_4^3\omega_5^2 - 6cs^4\omega_7\omega_4^3\omega_5^2 - 36v_1^4\omega_4^3\omega_5^3 - 108v_1^2cs^2\omega_7^2\omega_4^3\omega_5^2) \frac{\rho}{12\omega_7^2\omega_4^3\omega_5^3}
\end{aligned}$$

$$\begin{aligned}
C_{D_3^2 D_y v_2}^{(1), \text{CuLBM1}} = & (-39\omega_3^3v_1^4\omega_4\omega_1^3 + \omega_3^3\omega_4^2\omega_1^2cs^4 - 36\omega_3^3v_1^2\omega_4^2\omega_1cs^2 + 6\omega_3^3\omega_4\omega_2^2cs^2 + 36\omega_3^3v_1^2\omega_4\omega_1 - 108\omega_3^3v_1^2\omega_4\omega_1cs^2 + 36\omega_3^3v_1^2\omega_4\omega_1cs^2 - \\
& 36\omega_3^3v_1^2\omega_4\omega_1^3 + 72\omega_3^3v_1^4\omega_4\omega_1^2 + 18\omega_3^3\omega_4\omega_1^3cs^2 + 12\omega_3^3\omega_4^2\omega_1cs^2 + 13\omega_3^3\omega_4^2\omega_1^3cs^4 + 60\omega_3^3v_1^2\omega_4^2\omega_1^2cs^2 - 12\omega_3^3\omega_4^2cs^2 - 6\omega_3^3\omega_4\omega_1^3cs^2 + \\
& 39\omega_3^3v_1^2\omega_4\omega_1^3 + 36\omega_3^3v_1^2\omega_4\omega_1^3cs^2 + 36\omega_3^3v_1^2\omega_1^2 - \omega_3^3\omega_4^2\omega_1^3cs^4 - 36\omega_3^3v_1^4\omega_4\omega_1 + 12\omega_3^3\omega_4^2\omega_1cs^2 + 12\omega_3\omega_4\omega_1^3cs^4 + 6\omega_3^3\omega_4^2\omega_1cs^4 - 72\omega_3^3v_1^2\omega_4\omega_1^2 - \\
& 36\omega_3^3v_1^4\omega_1^3 + 36\omega_3^3v_1^4\omega_4\omega_1^3 + 12\omega_3^3v_1^2\omega_1^2\omega_1^3cs^2 - 12\omega_3^3\omega_4\omega_2^2cs^2 - 72\omega_3^3v_1^2\omega_1^2 + 54\omega_3^3v_1^2\omega_4\omega_1^3cs^2 + 6\omega_3\omega_4^2\omega_1^3cs^2 - 36\omega_3^3v_1^2\omega_1^3 - 18\omega_3v_1^2\omega_4\omega_1^3cs^2 - \\
& 6\omega_3^3v_1^4\omega_1^3 - 108\omega_3^3v_1^2\omega_1^2cs^2 - 12\omega_3^3\omega_4^2\omega_1cs^4 - 19\omega_3^3v_1^2\omega_4^2\omega_1 + 6\omega_3^3\omega_4\omega_1^3cs^4 - 4\omega_3^3v_1^2\omega_4^2\omega_1^3 - 99\omega_3^3v_1^2\omega_4\omega_1^3cs^2 + 12\omega_3^3\omega_4\omega_1^2cs^4 - \\
& 24\omega_3^3\omega_4^2\omega_1^3cs^4 - 3\omega_3^3v_1^2\omega_4^2\omega_1^3cs^2 - 12\omega_3\omega_4\omega_1^3cs^2 - 6\omega_3^3\omega_4^2\omega_1^2cs^2 - 90\omega_3^3v_1^4\omega_4\omega_1 - 108\omega_3^3v_1^2\omega_4^2\omega_1^3cs^2 + 252\omega_3^3v_1^2\omega_4^2cs^2 + 36\omega_3^3v_1^2\omega_1^3 + 12\omega_4^2\omega_1^3cs^4 - \\
& 6\omega_3^3\omega_4\omega_1^3cs^4 + 19\omega_3^3v_1^4\omega_4\omega_1^2 + 6\omega_3^3v_1^2\omega_4\omega_1^3 - 306\omega_3^3v_1^2\omega_4\omega_1cs^2 + 72\omega_3^3v_1^4\omega_4 - \omega_3^3\omega_4^2\omega_1^3cs^2 + 108\omega_3^3v_1^2\omega_1^3cs^2 + 36\omega_3^3v_1^4\omega_1^3 - 5\omega_3^3\omega_4\omega_1^3cs^2 - \\
& 12\omega_3^3\omega_4\omega_1cs^4 + 4\omega_3^3v_1^4\omega_4\omega_1^3 + 12\omega_3^3\omega_4^2cs^4 + 18\omega_3^3v_1^2\omega_4^2\omega_1^2cs^2 - 36\omega_3^3v_1^4\omega_1^2 + 198\omega_3^3v_1^2\omega_4\omega_1^2cs^2 + 90\omega_3^3v_1^2\omega_4\omega_1 - 18\omega_3^3\omega_4\omega_1^3cs^4) \frac{\rho}{12\omega_3^3\omega_4^2\omega_1^3}
\end{aligned}$$

$$\begin{aligned}
C_{D_3^2 D_y v_2}^{(1), \text{CuLBM2}} = & (27\omega_3^3v_1^2v_1^2v_2^2\omega_2 - 54\omega_3^3v_1^4\omega_1^2\omega_2^2 - cs^4\omega_3^3\omega_2^3\omega_2^2 + 18cs^2\omega_3\omega_1^3v_2^2\omega_2 + 3\omega_3^3\omega_1^2v_2^2\omega_2^3 - 24cs^2\omega_3\omega_1^2\omega_2^2 + 30cs^2\omega_3^3v_1^2\omega_2^2\omega_2^3 - 43\omega_3^3v_1^2\omega_1^3\omega_2^3 - \\
& 297cs^2\omega_3^3v_1^2\omega_1^3\omega_2 + 15cs^2\omega_3^3\omega_1\omega_2^3 - 6\omega_3^3\omega_1^2\omega_2 + 18\omega_3v_1^2\omega_1\omega_2^3 - 324cs^2v_1^2\omega_1^2\omega_2^3 - 2cs^4\omega_3^3\omega_1^3\omega_2^3 - 63\omega_3^3v_1^4\omega_1^3\omega_2 - 54cs^2\omega_3^3\omega_1^2v_2^2\omega_2 + 30cs^4\omega_3^3\omega_2^3 - \\
& 30cs^4\omega_3^3\omega_1^2\omega_2 + 7\omega_3^3v_1^4\omega_1^2\omega_2^3 - 54cs^2\omega_3\omega_1v_2^2\omega_2^3 + 9\omega_3^3v_1^2\omega_2^3 + 18cs^2\omega_3^3\omega_1^3v_2^2\omega_2 - 6cs^2\omega_3\omega_1^3\omega_2 - 8\omega_3^3v_1^2\omega_1^3\omega_2^3 - 12cs^2\omega_3^3\omega_1\omega_2^2 - 45\omega_3^3v_1^2\omega_1^2\omega_2 + \\
& 30cs^2\omega_3\omega_1^2\omega_2^3 - 144cs^2\omega_3^3v_1^2\omega_1^2\omega_2^2 + \omega_3^3\omega_2^3\omega_2^2 + 24cs^2\omega_3^3v_1^2\omega_1^3\omega_2^2 - \omega_3^3\omega_1^2\omega_2^3 + 18cs^2\omega_3\omega_1^3\omega_2 - 6\omega_3^3\omega_1^3\omega_2 + 81cs^2\omega_3^3v_1^2\omega_1\omega_2 + 54\omega_3^3v_1^2\omega_1^2\omega_2^2 + \\
& 18cs^4\omega_3\omega_1\omega_2^3 + 19\omega_3^3v_1^4\omega_1^3\omega_2^2 + 18cs^2\omega_3^3\omega_1^2v_2^2\omega_2^2 + 18cs^2\omega_3^3v_2^2\omega_2^3 + 18\omega_3^3\omega_1^2v_2^2\omega_2 - 108cs^2v_1^2\omega_1^3\omega_2^3 + 12cs^4\omega_3^3\omega_1^2\omega_2^3 - 18\omega_3v_1^4\omega_1\omega_2^3 - 72\omega_3^3v_1^2\omega_1^2v_2^2\omega_2^3 + \\
& 6\omega_3^3\omega_1^3 + 17\omega_3^3v_1^2\omega_1^2\omega_2^3 + 84cs^2\omega_3^3v_1^2\omega_1^3\omega_2^2 - 12cs^2\omega_3\omega_1^3\omega_2^3 + 135\omega_3^3v_1^2\omega_1^3\omega_2 + 36\omega_3^3v_1^4\omega_1^3 + 54cs^2\omega_3v_1^2\omega_1\omega_2^3 + 189cs^2\omega_3^3v_1^2\omega_1^3 + 36\omega_3^3v_1^2\omega_1^2\omega_2 + \\
& 29cs^4\omega_3^3\omega_1^2\omega_2^3 + 216cs^2v_1^2\omega_1^3\omega_2^3 + 8\omega_3^3v_1^4\omega_1^3\omega_2^3 + 6cs^2\omega_3^3\omega_1^2v_2^2\omega_2^3 - 36cs^2\omega_3\omega_1^3v_2^2\omega_2^2 - 15cs^4\omega_3^3\omega_2^3\omega_2 - 24cs^2\omega_3^3\omega_1^3 + 6\omega_3^3\omega_1\omega_2^3 - 9\omega_3^3v_1^2\omega_1\omega_2^3 + \\
& 72v_1^4\omega_1^3\omega_2^3 - 18\omega_3^3\omega_1^3v_2^2 + 306cs^2\omega_3v_1^2\omega_1^2\omega_2^3 + 36\omega_3^3v_1^4\omega_2^3 - 72\omega_3v_2^2\omega_1^3\omega_2^2 - 54cs^2\omega_3v_2^2\omega_1^2\omega_2 - 36\omega_3v_1^4\omega_1^2\omega_2^2 - 9cs^2\omega_3^3\omega_1^3v_2^2\omega_2 + 216\omega_3^3v_1^2\omega_1\omega_2^3\omega_2^3 - \\
& 3\omega_3^3v_1^2\omega_2^3 + 63cs^4\omega_3^3v_1^2\omega_2^3 + 108v_1^2\omega_1^2\omega_2^3 + 24cs^4\omega_3\omega_1^2\omega_2^2 - 57cs^4\omega_3^3\omega_1\omega_2^3 + 18cs^2\omega_3^3\omega_1v_2^2\omega_2^2 + 78\omega_3v_1^4\omega_1^3\omega_2^3 + 135\omega_3^3v_1^2\omega_1^3v_2^2\omega_2^2 - 72cs^2\omega_3v_1^2\omega_1^2\omega_2^3 - \\
& 27\omega_3^3v_1^2\omega_1\omega_2^2 - 36v_1^4\omega_1^3\omega_2^2 + 6\omega_3^3\omega_1\omega_2^3 + 36cs^2\omega_3^3\omega_1^2\omega_2^2 - 9cs^2\omega_3^3\omega_1v_2^2\omega_2^3 + 6cs^4\omega_3^3\omega_1\omega_2^2 - 216\omega_3^3v_1^2\omega_1^3v_2^2\omega_2 + 6cs^4\omega_3\omega_2^3\omega_2 - 30cs^4\omega_3\omega_1^2\omega_2^3 - \\
& 18\omega_3v_1^4\omega_1^3\omega_2 - 6\omega_3^3\omega_2^3 - 27\omega_3^3v_1^2\omega_1\omega_2^2\omega_2^3 + 144\omega_3v_1^2\omega_1^2\omega_2^3 - 18cs^4\omega_3\omega_1^3\omega_2^2 - 18\omega_3^3\omega_1v_2^2\omega_2^3 + 72\omega_3^3v_1^2\omega_1^3v_2^2\omega_2^2 - 63\omega_3^3v_1^4\omega_1\omega_2^3 - 135\omega_3^3v_1^2v_2^2\omega_2^3 - \\
& 72v_1^2\omega_1^3\omega_2^3 + 18cs^4\omega_3^3\omega_1^3 - 81\omega_3^3v_1^2\omega_1^3 + 36cs^2\omega_3\omega_1^2v_2^2\omega_2^2 + 72\omega_3v_1^4\omega_1^3\omega_2^2 - 198cs^2\omega_3v_1^2\omega_1^3\omega_2^2 + 36\omega_3v_1^2\omega_1^2\omega_2^2 + 99cs^2\omega_3^3v_1^2\omega_1\omega_2^2 - 18cs^2\omega_3\omega_1\omega_2^3 -
\end{aligned}$$

coefficient $C_{D_x^2 D_y^2 \rho}^{(1)}$ at $\frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2}$:

$$C_{D_x^2 D_y^2}^{(1), \text{MRT}^1} = (-36\omega_9 c s^4 \omega_7^2 \omega_4^3 \omega_8 \omega_5 + 12v_1^2 \omega_7^2 \omega_4^2 v_2^2 \omega_5^3 - 24\omega_9 \omega_7 \omega_4^3 v_2^2 \omega_8 \omega_5^2 + 6\omega_7^2 \omega_4^3 v_2^2 \omega_5^3 + 12v_1^2 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_5^3 + 24\omega_9 v_1^2 \omega_7 \omega_4^3 v_2^2 \omega_8 \omega_5^2 +$$

$$D_{\mathbf{D}_x^1 \mathbf{D}_\rho^2}^{(1), \text{MRT}^2} = (12v_1^2 \omega_7^2 \omega_4^2 v_2^2 \omega_5^3 - 24\omega_9 \omega_7 \omega_4^3 v_2^2 \omega_8 \omega_5^2 - 12\omega_9 v_1^2 \omega_7 \omega_4 \omega_8 c s^2 \omega_5^2 + 6\omega_7^2 \omega_4^3 v_2^2 \omega_5^3 + 12v_1^2 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_5^3 + 24\omega_9 v_1^2 \omega_7 \omega_4^3 v_2^2 \omega_8 \omega_5^2 -$$

$$C_{D_x^2 D_y^2 \rho}^{(1), \text{CLB} \text{M1}} = (2\omega_9 \omega_7^2 \omega_4^2 \omega_8 \omega_5^2 + 54\omega_9 c s^2 \omega_7 \omega_4^2 \omega_8 \omega_5^2 - 2\omega_9 v_1^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5^2 - 12\omega_9 v_1^2 \omega_4 \omega_8 \omega_5^3 - 6v_1^2 \omega_7^2 \omega_4^2 \omega_5^3 + 12\omega_9 \omega_4 \omega_8 \omega_5^3 + 36\omega_9 c s^2 \omega_4^2 \omega_8 \omega_5^3 -$$

$$36cs^2\omega_7\omega_4^2\omega_8\omega_5^3 - 12\omega_7\omega_4^2\omega_8\omega_5^2 + 6v_1^2\omega_7^2\omega_4^2\omega_8\omega_5^3 + 36\omega_9cs^2\omega_7^2\omega_4^2\omega_8 - 6v_1^2\omega_7^2\omega_4^2\omega_8\omega_5^2 + 36cs^2\omega_7\omega_4^2\omega_8\omega_5^2 + 12\omega_7\omega_4^2\omega_8\omega_5^3 + \omega_9v_1^2\omega_7^2\omega_4^2\omega_8\omega_5^3 -$$

$$\begin{aligned}
& 36\omega_9cs^2\omega_4^2\omega_8\omega_5^2 + 36cs^2\omega_7^2\omega_4\omega_5^3 - \omega_9\omega_7\omega_2^2\omega_8\omega_5^3 - 18\omega_9cs^2\omega_7\omega_4^2\omega_8\omega_5^3 + 54\omega_9cs^2\omega_7^2\omega_4\omega_8\omega_5^2 + 12v_1^2\omega_7\omega_4\omega_8\omega_5^3 + 6\omega_2^2\omega_4^2\omega_5^3 - \\
& 12\omega_9v_1^2\omega_7^2\omega_4\omega_5^2 - 36cs^2\omega_7^2\omega_4\omega_8\omega_5^3 + 12\omega_9\omega_7^2\omega_4\omega_5^2 + 12\omega_9\omega_7^2\omega_8\omega_5^2 - 12\omega_9v_1^2\omega_7^2\omega_8\omega_5^2 - 36\omega_9cs^2\omega_7\omega_4^2\omega_8\omega_5 + 12\omega_7^2\omega_4\omega_8\omega_5^3 + 12\omega_9\omega_7^2\omega_4^2\omega_8\omega_5 + \\
& 18\omega_9cs^2\omega_7^2\omega_4^2\omega_5^2 - 12\omega_9v_1^2\omega_7^2\omega_4^2\omega_8\omega_5 - 18\omega_9\omega_7\omega_4\omega_8\omega_5^3 - 40\omega_9cs^2\omega_7^2\omega_4\omega_8\omega_5^3 + 18\omega_9v_1^2\omega_7\omega_4\omega_8\omega_5^3 - 36\omega_9cs^2\omega_7\omega_8\omega_5^3 - 12\omega_9\omega_7^2\omega_8\omega_5^3 + \\
& 12\omega_9v_1^2\omega_7^2\omega_8\omega_5^3 + 18cs^2\omega_7^2\omega_2^2\omega_8\omega_5^3 - 36\omega_9cs^2\omega_7^2\omega_4\omega_5^2 + 6\omega_7^2\omega_4^2\omega_8\omega_5^2 - 12v_1^2\omega_7\omega_4^2\omega_8\omega_5^3 - 18\omega_9\omega_7\omega_4^2\omega_8\omega_5^2 - 6\omega_9cs^2\omega_7^2\omega_4^2\omega_8\omega_5^2 + \\
& 18\omega_9v_1^2\omega_7\omega_4^2\omega_8\omega_5^2 - 36\omega_9cs^2\omega_7^2\omega_8\omega_5^2 - 6\omega_9v_1^2\omega_7\omega_4^2\omega_8\omega_5^3 + 6\omega_9\omega_7\omega_4^2\omega_8\omega_5^3 + 5\omega_9cs^2\omega_7^2\omega_4^2\omega_8\omega_5^3 - 6\omega_9\omega_7^2\omega_4^2\omega_5^2 + 12v_1^2\omega_7\omega_4^2\omega_8\omega_5^2 - \\
& 12\omega_7^2\omega_4\omega_5^3 + 6\omega_9v_1^2\omega_7^2\omega_4^2\omega_5^2 - 18cs^2\omega_7^2\omega_4^2\omega_8\omega_5^2 - 6\omega_7^2\omega_4^2\omega_8\omega_5^3 + 36\omega_9cs^2\omega_7^2\omega_8\omega_5^3 + 12\omega_9\omega_7\omega_8\omega_5^3 - 12\omega_9v_1^2\omega_7\omega_8\omega_5^3 - 12v_1^2\omega_7^2\omega_4\omega_8\omega_5^3 + \\
& 36cs^2\omega_7\omega_4\omega_8\omega_5^3 + 12\omega_9v_1^2\omega_7\omega_4\omega_8\omega_5^3 + 18\omega_9v_1^2\omega_7^2\omega_4\omega_8\omega_5^2 - 12\omega_9\omega_4^2\omega_8\omega_5^3 - 36\omega_9cs^2\omega_4\omega_8\omega_5^3 - 18cs^2\omega_7^2\omega_4^2\omega_5^3 - 18\omega_9\omega_7^2\omega_4\omega_8\omega_5^3 + \\
& 12\omega_9\omega_7^2\omega_4\omega_8\omega_5^3 + 12v_1^2\omega_7^2\omega_4\omega_5^3 + 54\omega_9cs^2\omega_7\omega_4\omega_8\omega_5^3 - 12\omega_9v_1^2\omega_7^2\omega_4\omega_8\omega_5^3 - 12\omega_9v_1^2\omega_4^2\omega_8\omega_5^2 + 12\omega_9\omega_4^2\omega_8\omega_5^2 - 36\omega_9cs^2\omega_7^2\omega_4^2\omega_8\omega_5 + \\
& 12\omega_9\omega_7\omega_4^2\omega_8\omega_5 - 12\omega_7\omega_4\omega_8\omega_5^3 - 12\omega_9\omega_7^2\omega_4^2\omega_8 - 12\omega_9v_1^2\omega_7\omega_4^2\omega_8\omega_5 + 12\omega_9v_1^2\omega_7^2\omega_4^2\omega_8\omega_5) \frac{cs^2v_1}{12\omega_9\omega_7^2\omega_4^2\omega_8\omega_5^3}
\end{aligned}$$

$$\begin{aligned}
C_{D_x^2D_y^2\rho}^{(1),CLBM2} = & (36\omega_9cs^2\omega_4^2\omega_8\omega_5^3 + 2\omega_9\omega_7^2\omega_4^2\omega_8\omega_5^2 + 36cs^2\omega_7\omega_4^2\omega_8\omega_5^2 - 2\omega_9v_1^2\omega_7^2\omega_4^2\omega_8\omega_5^2 - 12\omega_9v_1^2\omega_4\omega_8\omega_5^3 - 6v_1^2\omega_7^2\omega_4^2\omega_5^3 + 12\omega_9\omega_4\omega_8\omega_5^3 - \\
& 18\omega_9cs^2\omega_7\omega_4^2\omega_8\omega_5^3 - 12\omega_7\omega_4^2\omega_8\omega_5^2 + 6v_1^2\omega_7^2\omega_4^2\omega_8\omega_5^3 + 36\omega_9cs^2\omega_7^2\omega_4^2\omega_8 - 36\omega_9cs^2\omega_4^2\omega_8\omega_5^2 + 36cs^2\omega_7^2\omega_4\omega_5^3 - 6v_1^2\omega_7^2\omega_4^2\omega_8\omega_5^2 + \\
& 54\omega_9cs^2\omega_7\omega_4^2\omega_8\omega_5^3 + 12\omega_7\omega_4^2\omega_8\omega_5^2 + \omega_9v_1^2\omega_7^2\omega_4^2\omega_8\omega_5^3 - \omega_9\omega_7^2\omega_4^2\omega_8\omega_5^3 - 36cs^2\omega_7\omega_4^2\omega_8\omega_5^3 - 36\omega_9cs^2\omega_7\omega_4^2\omega_8\omega_5 + 12v_1^2\omega_7\omega_4\omega_8\omega_5^3 + 6\omega_7^2\omega_4^2\omega_5^3 - \\
& 12\omega_9v_1^2\omega_7^2\omega_4\omega_5^2 - 40\omega_9cs^2\omega_7^2\omega_4\omega_8\omega_5^3 + 12\omega_9\omega_7^2\omega_4\omega_5^2 + 12\omega_9\omega_7^2\omega_8\omega_5^2 - 12\omega_9v_1^2\omega_7^2\omega_8\omega_5^2 + 54\omega_9cs^2\omega_7^2\omega_4\omega_8\omega_5^2 + 12\omega_7^2\omega_4\omega_8\omega_5^3 + \\
& 12\omega_9\omega_7^2\omega_4^2\omega_8\omega_5 - 36\omega_9cs^2\omega_7\omega_8\omega_5^3 - 12\omega_9v_1^2\omega_7^2\omega_4^2\omega_8\omega_5 - 18\omega_9\omega_7\omega_4\omega_8\omega_5^3 - 36cs^2\omega_7^2\omega_4\omega_8\omega_5^3 + 18\omega_9v_1^2\omega_7\omega_4\omega_8\omega_5^3 - 12\omega_9\omega_7^2\omega_8\omega_5^3 + \\
& 12\omega_9v_1^2\omega_7^2\omega_8\omega_5^3 + 18\omega_9cs^2\omega_7^2\omega_4^2\omega_5^2 + 5\omega_9cs^2\omega_7^2\omega_4^2\omega_8\omega_5^3 - 36\omega_9cs^2\omega_7^2\omega_8\omega_5^2 + 6\omega_7^2\omega_4^2\omega_8\omega_5^2 - 12v_1^2\omega_7\omega_4^2\omega_8\omega_5^3 - 18\omega_9\omega_7\omega_4^2\omega_8\omega_5^2 - \\
& 18cs^2\omega_7^2\omega_4^2\omega_8\omega_5^2 + 18\omega_9v_1^2\omega_7\omega_4^2\omega_8\omega_5^2 - 36\omega_9cs^2\omega_7^2\omega_4\omega_5^2 - 6\omega_9v_1^2\omega_7\omega_4^2\omega_8\omega_5^2 + 6\omega_9\omega_7\omega_4^2\omega_8\omega_5^2 + 18cs^2\omega_7^2\omega_4^2\omega_8\omega_5^3 - 6\omega_9\omega_7^2\omega_4^2\omega_5^2 + \\
& 12v_1^2\omega_7\omega_4^2\omega_8\omega_5^2 - 12\omega_7^2\omega_4\omega_5^3 + 6\omega_9v_1^2\omega_7^2\omega_4^2\omega_5^2 - 6\omega_9cs^2\omega_7^2\omega_4^2\omega_8\omega_5^2 - 6\omega_7^2\omega_4^2\omega_8\omega_5^3 + 36\omega_9cs^2\omega_7^2\omega_8\omega_5^3 + 12\omega_9\omega_7\omega_8\omega_5^3 - 12\omega_9v_1^2\omega_7\omega_8\omega_5^3 - \\
& 18cs^2\omega_7^2\omega_4^2\omega_5^3 - 36\omega_9cs^2\omega_4\omega_8\omega_5^3 - 12v_1^2\omega_7^2\omega_4\omega_8\omega_5^3 + 54\omega_9cs^2\omega_7\omega_4\omega_8\omega_5^3 + 12\omega_9v_1^2\omega_7^2\omega_4\omega_8\omega_5^3 + 18\omega_9v_1^2\omega_7^2\omega_4\omega_8\omega_5^2 - 12\omega_9\omega_7^2\omega_8\omega_5^3 - \\
& 18\omega_9\omega_7^2\omega_4\omega_8\omega_5^2 - 36\omega_9cs^2\omega_7^2\omega_4^2\omega_8\omega_5 + 12\omega_9\omega_7\omega_4\omega_8\omega_5^3 + 12v_1^2\omega_7^2\omega_4\omega_5^3 + 36cs^2\omega_7\omega_4\omega_8\omega_5^3 - 12\omega_9v_1^2\omega_7^2\omega_4\omega_8\omega_5^3 - 12\omega_9v_1^2\omega_4^2\omega_8\omega_5^2 + \\
& 12\omega_9\omega_4^2\omega_8\omega_5^2 + 12\omega_9\omega_7\omega_4^2\omega_8\omega_5 - 12\omega_7\omega_4\omega_8\omega_5^3 - 12\omega_9\omega_7^2\omega_4^2\omega_8 - 12\omega_9v_1^2\omega_7\omega_4^2\omega_8\omega_5 + 12\omega_9v_1^2\omega_7^2\omega_4^2\omega_8) \frac{v_1cs^2}{12\omega_9\omega_7^2\omega_4^2\omega_8\omega_5^3}
\end{aligned}$$

$$\begin{aligned}
C_{D_x^2D_y^2\rho}^{(1),CuLBM1} = & (6\omega_3^2\omega_4\omega_1^3 - 12\omega_3v_1^2\omega_1^3 + 6\omega_3^2v_1^2\omega_4\omega_1^2 - 6\omega_3\omega_2^2\omega_1^2 + 36\omega_2^2\omega_1^3cs^2 - 12v_1^2\omega_4^2\omega_1^2 + 18\omega_3\omega_2^2\omega_1^3cs^2 + 12\omega_3\omega_4\omega_1^3 - 6\omega_3^2v_1^2\omega_4\omega_1^3 - \\
& 18\omega_3^2\omega_4\omega_1^3cs^2 - 6\omega_3^2\omega_4\omega_1^2 + 12v_1^2\omega_4^2\omega_1^3 - 36\omega_4^2\omega_1^3cs^2 - 12\omega_3^2\omega_4^2 - 12\omega_3v_1^2\omega_4^2\omega_1^3 - 36\omega_3\omega_1^3cs^2 - 12\omega_3^2\omega_1^3 - 54\omega_3^2\omega_4\omega_1cs^2 + 12\omega_3^2\omega_1^2 + \\
& 18\omega_3^2\omega_4\omega_1^2cs^2 + 6\omega_3v_1^2\omega_4^2\omega_1^2 - 40\omega_3\omega_2^2\omega_1^3cs^2 + 12\omega_4\omega_1^3 + 36\omega_3^2\omega_2^2cs^2 + 36\omega_3^2\omega_1^3cs^2 - 18\omega_2^2v_1^2\omega_4^2\omega_1 - 12\omega_4^2\omega_1^3 + 12\omega_3v_1^2\omega_4\omega_1^2 + 18\omega_3v_1^2\omega_4\omega_1^3 + \\
& 12\omega_2^2\omega_1^2 + 54\omega_3\omega_4\omega_1^3cs^2 + 12\omega_3^2\omega_2^2\omega_1^2cs^2 + 18\omega_3^2\omega_4\omega_1 + 12\omega_3\omega_1^3 + 12\omega_3^2v_1^2\omega_4^2 + 12\omega_3^2v_1^2\omega_1^3 - 4\omega_3^2\omega_4^2\omega_1^2 - 36\omega_3^2\omega_1^2cs^2 - 18\omega_3\omega_4\omega_1^3 + \\
& \omega_3^2v_1^2\omega_4^2\omega_1^3 - 12v_1^2\omega_4\omega_1^3 - 36\omega_4\omega_1^3cs^2 + 4\omega_3^2v_1^2\omega_4^2\omega_1^2 + 5\omega_3^2\omega_4^2\omega_1^3cs^2 - 12\omega_3\omega_4\omega_1^2 - \omega_3^2\omega_4^2\omega_1^3 - 12\omega_3^2v_1^2\omega_1^2 + 36\omega_3\omega_4\omega_1^2cs^2) \frac{v_1cs^2}{12\omega_3^2\omega_4^2\omega_1^3}
\end{aligned}$$

$$\begin{aligned}
C_{D_x^2D_y^2\rho}^{(1),CuLBM2} = & (-45\omega_3^2v_1^2\omega_1^2v_2^2\omega_2 + 35cs^4\omega_3^2\omega_1^3\omega_2^2 + 27\omega_3^2\omega_1^2v_2^2\omega_3^2 - 36cs^2\omega_3\omega_1^2\omega_2^2 - 24cs^2\omega_3^2v_1^2\omega_1^2\omega_3^2 - 21cs^2\omega_3^2v_1^2\omega_1^3\omega_2 - 39cs^2\omega_3^2\omega_1\omega_3^2 - \\
& 6\omega_3^2\omega_1^2\omega_2 - 36cs^2v_1^2\omega_1^2\omega_3^2 - 90\omega_3^2\omega_1^2v_2^2\omega_2^2 + 10cs^4\omega_3^2\omega_1^3\omega_2^2 - 297cs^2\omega_3^2\omega_1^2v_2^2\omega_2 + 18cs^4\omega_3^2\omega_2^2 - 18cs^4\omega_3^2\omega_1^2\omega_2 - 6\omega_3^2v_1^2\omega_2^2 + 405cs^2\omega_3^2\omega_1^3v_2^2 + \\
& 27\omega_3^2\omega_1^3v_2^2\omega_2^2 + 6cs^2\omega_3\omega_1^3\omega_2 + 48cs^2\omega_3^2\omega_1\omega_2^2 + 6\omega_3^2v_1^2\omega_1^2\omega_2 - 48cs^2\omega_3\omega_1^2\omega_2^2 + 18cs^2\omega_3^2v_1^2\omega_1^2\omega_2 - 54\omega_3^2\omega_1^3v_2^4\omega_2 + 2cs^2\omega_3v_1^2\omega_1^3\omega_2^2 - \\
& 54\omega_3^2v_2^2\omega_3^2 - 6\omega_3^2\omega_1^3\omega_2 - 6cs^2\omega_3^2v_1^2\omega_1^2\omega_2 + 54\omega_3^2\omega_1v_2^2\omega_2^2 - 12\omega_3^2v_1^2\omega_1^2\omega_2^2 - 90cs^4\omega_3\omega_1\omega_2^2 + 270cs^2\omega_3^2\omega_1^2v_2^2\omega_2^2 - 135cs^2\omega_3^2v_2^2\omega_3^2 + 99\omega_3^2\omega_1^2v_2^2\omega_2 - \\
& 12cs^2v_1^2\omega_3^2\omega_2^2 + 54cs^4\omega_3^2\omega_2^2\omega_2^2 + 54\omega_3^2\omega_1^3v_2^4\omega_2 + 6\omega_3^2\omega_1^3 + 54\omega_3^2\omega_1v_2^4\omega_3^2 + 12\omega_3^2\omega_2^2\omega_2^2 + 8cs^2\omega_3^2v_1^2\omega_1^3\omega_2^2 + 12cs^2\omega_3\omega_1^3\omega_3^2 + 6\omega_3^2v_1^2\omega_1^3\omega_2 - \\
& 30cs^2\omega_3v_1^2\omega_1\omega_3^2 + 18cs^2\omega_3^2v_1^2\omega_1^3 + 90\omega_3^2v_1^2\omega_1^2v_2^2\omega_2^2 - 91cs^4\omega_3^2\omega_1^2\omega_2^2 + 24cs^2v_1^2\omega_1^2\omega_2^2 - 138cs^2\omega_3^2\omega_1^2v_2^2\omega_2^2 - 117cs^4\omega_3^2\omega_1^3\omega_2 - 72cs^2\omega_3^2\omega_1^3 - \\
& 6\omega_3^2\omega_1^2\omega_2 + 6\omega_3^2v_1^2\omega_1\omega_3^2 - 25cs^2\omega_3^2\omega_1^3\omega_2^2 - 99\omega_3^2\omega_1^3v_2^2 + 48cs^2\omega_3v_2^2\omega_1^2\omega_2^2 - 6cs^2\omega_3v_2^2\omega_1^2\omega_2 - 459cs^2\omega_3^2\omega_1^3v_2^2\omega_2 - 45\omega_3^2v_1^2\omega_1^2\omega_2^2 - \\
& 27\omega_3^2\omega_1^3v_2^2\omega_2 + 18cs^2\omega_3^2v_2^2\omega_2^2 + 108cs^4\omega_3\omega_1^2\omega_2^2 + 63cs^4\omega_3^2\omega_1\omega_2^2 + 27cs^2\omega_3^2\omega_1v_2^2\omega_2^2 + 12cs^2\omega_1^3\omega_2^2 + 45\omega_3^2v_1^2\omega_1^3v_2^2 + 36cs^2\omega_3v_1^2\omega_1^2\omega_2^2 + \\
& 6\omega_3^2v_1^2\omega_1\omega_2^2 - 2cs^2\omega_3^2\omega_1^3\omega_2^2 - 108cs^4\omega_1^2\omega_3^2 - 27\omega_3^2\omega_1^2v_2^4\omega_2^2 - 6\omega_3^2\omega_1\omega_2^2 + 24cs^2\omega_3^2\omega_1^2\omega_2 + 189cs^2\omega_3^2\omega_1v_2^2\omega_2^2 - 90cs^4\omega_3^2\omega_1\omega_2^2 - 45\omega_3^2v_1^2\omega_1^3v_2^2\omega_2 - \\
& 18cs^4\omega_3\omega_1^2\omega_2 - 24cs^2\omega_1^2\omega_2^2 + 144cs^4\omega_3\omega_1^2\omega_2^2 + 6\omega_3^2\omega_2^2 - 45\omega_3^2v_1^2\omega_1v_2^2\omega_2^2 - 9\omega_3^2\omega_1v_2^2\omega_2^2 + 45\omega_3^2v_1^2v_2^2\omega_2^2 + 90cs^4\omega_3^2\omega_1^2 - 6\omega_3^2v_1^2\omega_1^2 - \\
& 12cs^2\omega_3v_1^2\omega_1^3\omega_2^2 - 30cs^2\omega_3^2v_2^2\omega_1\omega_2^2 - 36cs^4\omega_1^3\omega_2^2 + 30cs^2\omega_3\omega_1\omega_2^2 - 54cs^2\omega_3^2\omega_1^2\omega_2^2 + 99\omega_3^2\omega_1^3v_2^2\omega_2 + 138cs^2\omega_3^2\omega_1^3v_2^2\omega_2^2 + 36cs^2\omega_1^2\omega_2^2 + \\
& 9\omega_3^2v_2^2\omega_2^2 - 9\omega_3^2\omega_1v_2^2\omega_2^2 - 36cs^4\omega_3\omega_1^3\omega_2^2 + 41cs^2\omega_3^2\omega_1^2\omega_2^2 - 54\omega_3^2\omega_1^2v_2^2\omega_2 + 93cs^2\omega_3^2\omega_1^2\omega_2 + 3cs^2\omega_3^2v_1^2\omega_1\omega_2^2 + 72cs^4\omega_1^3\omega_2^2) \frac{v_1}{24\omega_3^2\omega_1^3\omega_2^2}
\end{aligned}$$

$$\text{coefficient } C_{D_x^2D_y^2v_1}^{(1)} \text{ at } \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} :$$

$$C_{D_x^2D_y^2v_1}^{(1),SRT} = (-24 - 18cs^2\omega + 72v_1^2 - 12\omega^2 + 36v_1^2\omega^2 - cs^2\omega^3 - 108v_1^2\omega + 8cs^2\omega^2 + 12cs^2 + 36\omega) \frac{cs^2\rho}{12\omega^3}$$

$$\begin{aligned}
C_{D_x^2D_y^2v_1}^{(1),MRT1} = & (6\omega_9cs^4\omega_7^2\omega_4^3\omega_8\omega_5 + 36v_1^2\omega_7^2\omega_4^2v_2^2\omega_5^3 - 24\omega_9\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + 6\omega_7^2\omega_4^3v_2^2\omega_5^3 + 36v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^3 + 72\omega_9v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 - \\
& 24\omega_9v_1^2cs^2\omega_4^3\omega_8\omega_5^3 - 12\omega_9cs^4\omega_7^2\omega_4\omega_8\omega_5^2 - 36\omega_9v_1^2\omega_7^2\omega_4^2v_2^2\omega_5^2 + 12\omega_7\omega_4^3v_2^2\omega_8\omega_5^3 + 18v_1^2cs^2\omega_7^2\omega_4^3\omega_8\omega_5^3 + 12\omega_9\omega_7^2\omega_4^2v_2^2\omega_5^2 + 12\omega_9cs^2\omega_4^3v_2^2\omega_8\omega_5^3 + \\
& 6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 48\omega_9v_1^2cs^2\omega_7^2\omega_8\omega_5^3 - 6\omega_9cs^2\omega_7^2\omega_4^3\omega_8\omega_5 - 84\omega_9v_1^2cs^2\omega_7^2\omega_4\omega_8\omega_5^2 + 6\omega_9cs^4\omega_7^2\omega_4^3\omega_5^2 + 18\omega_9v_1^2cs^2\omega_7^2\omega_4^3\omega_8\omega_5 + 12\omega_9cs^2\omega_7^2\omega_4\omega_8\omega_5^2 + \\
& 36v_1^2cs^2\omega_7\omega_4^2\omega_8\omega_5^3 - 18v_1^2cs^2\omega_7^2\omega_4^3\omega_8\omega_5^2 + 18\omega_9cs^4\omega_7^2\omega_4\omega_8\omega_5^3 - 36\omega_9v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^3 + 24\omega_9v_1^2cs^2\omega_4^3\omega_8\omega_5^2 + 36\omega_9v_1^2\omega_7^2\omega_4^2v_2^2\omega_8\omega_5 - \\
& 12\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + 6cs^2\omega_7^2\omega_4^3\omega_5^2 + 12\omega_9\omega_7\omega_4^3v_2^2\omega_8\omega_5^3 - 12\omega_9\omega_7^2\omega_4^2v_2^2\omega_8\omega_5 + 12\omega_9cs^2\omega_7^2\omega_4^3v_2^2\omega_8 - 6\omega_9\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 36v_1^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^3 - \\
& 6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^3 + 84\omega_9v_1^2cs^2\omega_7^2\omega_4\omega_8\omega_5^3 + 18\omega_9v_1^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 12\omega_9cs^2\omega_7^2v_2^2\omega_8\omega_5^2 + 18\omega_9\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - 12cs^2\omega_7\omega_4^2\omega_8\omega_5^3 - 36\omega_9\omega_7\omega_4^2v_2^2\omega_8\omega_5^3 + \\
& 24\omega_9cs^2\omega_7^2\omega_4\omega_8\omega_5^2 + 60\omega_9v_1^2cs^2\omega_7\omega_4^2\omega_8\omega_5^2 - 12\omega_9cs^4\omega_7^2\omega_4^2\omega_5^2 + 6cs^2\omega_7^2\omega_4^3\omega_8\omega_5^2 + 36v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 - 54\omega_9v_1^2\omega_7^2\omega_4^2v_2^2\omega_8\omega_5 + \\
& 108\omega_9v_1^2\omega_7\omega_4^2v_2^2\omega_8\omega_5^3 - 12\omega_9cs^2\omega_7\omega_4^2\omega_8\omega_5^2 - 6cs^4\omega_7^2\omega_4^2\omega_8\omega_5^2 + 12\omega_2^2\omega_4^2v_2^2\omega_8\omega_5^3 - 72\omega_9v_1^2\omega_2^2\omega_4^2v_2^2\omega_8\omega_5^2 + 12cs^4\omega_7\omega_4^2\omega_8\omega_5^3 + 18v_1^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^3 + \\
& 12\omega_9cs^4\omega_7\omega_4^2\omega_8\omega_5^2 + 24\omega_9\omega_7^2\omega_4^2v_2^2\omega_8\omega_5^2 - \omega_9cs^4\omega_7^2\omega_4^3\omega_8\omega_5^3 - 24\omega_9cs^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^3 - 36\omega_9v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + 6\omega_9cs^2\omega_7^2\omega_4^3\omega_8\omega_5^2 - 12\omega_7\omega_4^3v_2^2\omega_8\omega_5^3 + \\
& 24\omega_9cs^2\omega_7\omega_4^2\omega_8\omega_5^3 - 36v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^3 + 12\omega_9\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 - 18\omega_9v_1^2cs^2\omega_2^2\omega_4^2\omega_8\omega_5^2 - 6cs^2\omega_2^2\omega_4^3\omega_8\omega_5^3 - 132\omega_9v_1^2cs^2\omega_7\omega_4^2\omega_8\omega_5^3 - 6cs^2\omega_7^2\omega_4^3v_2^2\omega_5^3 - \\
& 24\omega_9cs^4\omega_7\omega_4^2\omega_8\omega_5^3 + 12\omega_9\omega_7\omega_4^3v_2^2\omega_8\omega_5 - 6\omega_9cs^4\omega_7^2\omega_4^3\omega_8\omega_5^2 - 18v_1^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 12cs^2\omega_7^2\omega_4^3\omega_5^2 + 6cs^4\omega_7^2\omega_4^3\omega_8\omega_5^3 - 36\omega_9v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5 + \\
& 6\omega_9cs^2\omega_7^2\omega_4^3v_2^2\omega_5^2 - 12\omega_9cs^2\omega_2^2v_2^2\omega_8\omega_5^3 + 72\omega_9v_1^2cs^2\omega_7\omega_4^3\omega_8\omega_5 + 24\omega_9cs^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + 12cs^2\omega_7\omega_4^2v_2^2\omega_8\omega_5^3 - 6\omega_9cs^2\omega_7^2\omega_4^3\omega_5^2 + \\
& 18\omega_9v_1^2cs^2\omega_7^2\omega_4^3\omega_5^2 - 36v_1^2cs^2\omega_7\omega_4^3\omega_8\omega_5^3 - 12\omega_9cs^4\omega_7^2\omega_8\omega_5^3 - 18v_1^2cs^2\omega_7^2\omega_4^3\omega_5^3 + 12\omega_9cs^2\omega_7^2\omega_4^2\omega_8\omega_5 + 60\omega_9v_1^2cs^2\omega_7\omega_4\omega_8\omega_5^3 - 6cs^4\omega_7^2\omega_4^3\omega_5^3 -
\end{aligned}$$

$$\begin{aligned}
& 108\omega_9 v_1^2 c^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 - 12\omega_9 c^2 \omega_7 \omega_4^2 \omega_2^2 \omega_8 \omega_3^2 - 12\omega_9 c^2 \omega_7 \omega_4 \omega_8 \omega_3^2 + 12\omega_9 c^2 \omega_7^2 \omega_4^2 \omega_2^2 \omega_8 \omega_5 - 12\omega_7^2 \omega_4^2 v_2^2 \omega_3^2 + 6\omega_9 c^2 \omega_7^2 \omega_4^2 \omega_2^2 \omega_8 \omega_5^2 - \\
& 12\omega_9 c^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 - 18v_1^2 \omega_7^2 \omega_3^2 v_2^2 \omega_3^2 - 36v_1^2 c^2 \omega_7^2 \omega_4^2 \omega_8 \omega_3^2 - 36\omega_9 v_1^2 \omega_7^2 \omega_4^2 \omega_8 \omega_3^2 - 6\omega_9 \omega_7^2 \omega_4^2 v_2^2 \omega_3^2 + 36v_1^2 c^2 \omega_7 \omega_3^2 \omega_8 \omega_5^2 - 12c^2 \omega_7^2 \omega_3^2 v_2^2 \omega_8 \omega_3^2 + \\
& 12\omega_9 \omega_4^2 v_2^2 \omega_8 \omega_3^2 + 18\omega_9 v_1^2 \omega_7^2 \omega_4^2 v_2^2 \omega_3^2 + 12\omega_9 c^2 \omega_7 \omega_4 \omega_8 \omega_3^2 + 24\omega_9 v_1^2 c^2 \omega_7^2 \omega_4^2 \omega_8 \omega_3^2 + 24\omega_9 c^4 \omega_7^2 \omega_4^2 \omega_8 \omega_3^2 - 18\omega_9 \omega_7^2 \omega_4^2 v_2^2 \omega_8 \omega_5 + 12c^2 \omega_7^2 \omega_4^2 v_2^2 \omega_8 \omega_3^2 + \\
& 12\omega_9 c^2 \omega_7^2 \omega_4^2 \omega_8 \omega_3^2 - 24\omega_9 \omega_7^2 \omega_4^2 v_2^2 \omega_8 \omega_5^2 + 36v_1^2 c^2 \omega_7^2 \omega_4^2 \omega_3^2 + 36\omega_9 c^2 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_3^2 + 12c^2 \omega_7^2 \omega_4^2 v_2^2 \omega_8 \omega_3^2 + 12\omega_9 c^2 \omega_7^2 \omega_4^2 v_2^2 \omega_8 \omega_3^2 - \\
& 36\omega_9 v_1^2 c^2 \omega_7^2 \omega_4^2 \omega_3^2 + 12c^4 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_5^2 + 72\omega_9 v_1^2 \omega_7^2 \omega_4^2 v_2^2 \omega_8 \omega_5^2 - 12c^4 \omega_7^2 \omega_4^2 \omega_8 \omega_3^2 - 72\omega_9 v_1^2 \omega_7 \omega_4 v_2^2 \omega_8 \omega_3^2 - 12\omega_9 \omega_4^2 v_2^2 \omega_8 \omega_3^2 - 12\omega_9 c^2 \omega_7 \omega_4^2 \omega_8 \omega_3^2 - \\
& 24\omega_9 c^2 \omega_7^2 \omega_4^2 \omega_8 \omega_3^2 + 36\omega_9 v_1^2 \omega_4^2 v_2^2 \omega_8 \omega_3^2 + 6c^2 \omega_7^2 \omega_4^2 v_2^2 \omega_8 \omega_3^2 + 24\omega_9 \omega_7 \omega_4 v_2^2 \omega_8 \omega_3^2 - 24\omega_9 c^2 \omega_7^2 \omega_4^2 v_2^2 \omega_8 \omega_3^2 + 78\omega_9 v_1^2 c^2 \omega_7 \omega_4^2 \omega_8 \omega_3^2 + \\
& 12c^2 \omega_7^2 \omega_4^2 \omega_8 \omega_3^2 + 180\omega_9 v_1^2 c^2 \omega_7^2 \omega_4^2 \omega_8 \omega_3^2 - 12c^2 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_3^2 - 12c^4 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_3^2 - 12c^2 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_3^2 - 4\omega_9 c^2 \omega_7^2 \omega_4^2 \omega_8 \omega_3^2 + 36\omega_9 v_1^2 \omega_7^2 \omega_4^2 v_2^2 \omega_8 - \\
& 12\omega_9 c^2 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_5^2 - 12\omega_9 \omega_4^2 \omega_7 \omega_4^2 \omega_8 \omega_5^2 - 12\omega_9 \omega_7^2 \omega_3^2 v_2^2 \omega_8 - 12\omega_9 c^2 \omega_7 \omega_4^2 v_2^2 \omega_8 \omega_5 - 14\omega_9 v_1^2 c^2 \omega_7 \omega_3^2 \omega_8 \omega_5^2 + 12c^2 \omega_7 \omega_3^2 \omega_8 \omega_5^2 - \\
& 42\omega_9 v_1^2 c^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5^2 - 36\omega_9 v_1^2 \omega_4^2 v_2^2 \omega_8 \omega_5^2 - 6c^2 \omega_7^2 \omega_4^2 v_2^2 \omega_8 \omega_5^2 + 12c^4 \omega_7^2 \omega_4^2 \omega_3^2 + 12\omega_9 \omega_4^2 v_2^2 \omega_8 \omega_5^2 + 12\omega_9 c^2 \omega_7 \omega_4^2 \omega_8 \omega_5^2 \Big) \frac{\rho}{12\omega_9 \omega_7^2 \omega_4^2 \omega_8 \omega_3^2}
\end{aligned}$$

$$C_{D_2^2 D_3^2 v_1}^{(1), CLMB1} = (-4\omega_9 c s^2 \omega_7 \omega_4^2 \omega_8 \omega_5^2 - 36\omega_9 v_1^2 \omega_7 \omega_4^2 \omega_5 + 12\omega_9 \omega_7 \omega_4^2 \omega_5 + 12\omega_7 \omega_4^2 \omega_8 \omega_5^2 + 18v_1^2 \omega_7 \omega_4^2 \omega_8 \omega_5^2 - 12c s^2 \omega_4^3 \omega_8 \omega_5^2 + 12\omega_9 \omega_7 \omega_4^2 \omega_8 - 36\omega_9 v_1^2 \omega_4^3 \omega_8 \omega_5 - 36\omega_9 v_1^2 \omega_7 \omega_4^2 \omega_8 + 12\omega_9 \omega_4^3 \omega_8 \omega_5 - 36\omega_9 v_1^2 \omega_7 \omega_4 \omega_8 \omega_5 - 12c s^2 \omega_7 \omega_4^2 \omega_8 \omega_5^2 + 12\omega_9 \omega_7 \omega_4 \omega_8 \omega_5 + 36\omega_9 v_1^2 \omega_4 \omega_8 \omega_5^2 - 24\omega_9 c s^2 \omega_4^2 \omega_8 \omega_5^2 + 12c s^2 \omega_7 \omega_4^2 \omega_5^2 - 12\omega_9 \omega_4^2 \omega_8 \omega_5^2 - 12\omega_4^2 \omega_8 \omega_5^2 - 12\omega_9 \omega_4^2 \omega_8 \omega_5^2 + 36\omega_9 v_1^2 \omega_4^3 \omega_8 \omega_5^2 - 18\omega_9 v_1^2 \omega_7 \omega_4^3 \omega_8 \omega_5 + 12\omega_9 c s^2 \omega_4^3 \omega_8 \omega_5 - 6\omega_9 \omega_7 \omega_4^3 \omega_5 + 6\omega_9 \omega_7 \omega_4^2 \omega_8 \omega_5 + 18\omega_9 v_1^2 \omega_7 \omega_4^3 \omega_5 - 12\omega_9 c s^2 \omega_7 \omega_4^2 \omega_8 \omega_5^2 + 24\omega_9 c s^2 \omega_7 \omega_4^2 \omega_8 \omega_5 - 6c s^2 \omega_7 \omega_4^2 \omega_5^2 + 36v_1^2 \omega_4^2 \omega_8 \omega_5^2 + 18\omega_9 v_1^2 \omega_7 \omega_4^3 \omega_8 - 6\omega_9 \omega_7 \omega_4^3 \omega_8 + 12c s^2 \omega_4^3 \omega_8 \omega_5 - 18v_1^2 \omega_7 \omega_4^3 \omega_8 \omega_5 + 6\omega_9 c s^2 \omega_7 \omega_4^3 \omega_5 + 6c s^2 \omega_7 \omega_4^3 \omega_8 \omega_5^2 - 12\omega_9 \omega_4^2 \omega_8 \omega_5 + 36\omega_9 v_1^2 \omega_4^2 \omega_8 \omega_5 + 12\omega_4^2 \omega_8 \omega_5^2 + 12\omega_9 c s^2 \omega_4^2 \omega_8 \omega_5^2 + 36v_1^2 \omega_4^2 \omega_8 \omega_5 - \omega_9 c s^2 \omega_7 \omega_4^2 \omega_8 \omega_5^2 + 6\omega_9 c s^2 \omega_7 \omega_4^2 \omega_8 + 12c s^2 \omega_4^2 \omega_8 \omega_5^2 - 36v_1^2 \omega_7 \omega_4^2 \omega_8 \omega_5^2 - 6\omega_7 \omega_4^2 \omega_8 \omega_5^2 - 12\omega_9 c s^2 \omega_7 \omega_4 \omega_8 \omega_5 - 18v_1^2 \omega_7 \omega_4^2 \omega_8 \omega_5^2 - 36v_1^2 \omega_4^2 \omega_8 \omega_5^2 - 6\omega_9 c s^2 \omega_7 \omega_4^2 \omega_8 \omega_5 - 12\omega_9 c s^2 \omega_7 \omega_4^2 \omega_5 + 18\omega_9 c s^2 \omega_7 \omega_4 \omega_8 \omega_5^2 + 6\omega_7 \omega_4^2 \omega_8 \omega_5 - 6c s^2 \omega_7 \omega_4^2 \omega_8 \omega_5 - 72\omega_9 v_1^2 \omega_4^2 \omega_8 \omega_5^2 + 36v_1^2 \omega_7 \omega_4^2 \omega_5^2 + 12\omega_9 c s^2 \omega_4^2 \omega_8 \omega_5^2 + 6\omega_7 \omega_4^2 \omega_5^2 + 24\omega_9 \omega_4^2 \omega_8 \omega_5^2 - 24\omega_9 \omega_7 \omega_4^2 \omega_8 \omega_5 - 12\omega_4^2 \omega_8 \omega_5 - 12\omega_9 c s^2 \omega_7 \omega_4^2 \omega_8 + 72\omega_9 v_1^2 \omega_7 \omega_4^2 \omega_8 \omega_5 - 12\omega_9 c s^2 \omega_4^3 \omega_8 \omega_5) \frac{c s^2 \rho}{12\omega_9 \omega_7 \omega_4^2 \omega_8 \omega_5^2}$$

$$\begin{aligned} \mathcal{D}_{\mathbf{D}_2^2 \mathbf{D}_2^2 \mathbf{y}_1}^{(1), \text{CuLBM1}} = & (-24\omega_3^2\omega_1 + 12\omega_3\omega_1^2cs^2 - 12\omega_3\omega_4\omega_1cs^2 - \omega_3^3\omega_4\omega_1^2cs^2 - 72\omega_3^3v_1^2\omega_1 + 12\omega_3^2\omega_4 + 36\omega_3v_1^2\omega_1^2 - 12\omega_3^3\omega_1^2 + 24\omega_3^2\omega_1cs^2 + 36\omega_3^3v_1^2\omega_1^2 + \\ & 24\omega_3^3\omega_1 - 12\omega_3^2\omega_4\omega_1 + 12\omega_3^3cs^2 - 12\omega_3^2\omega_4cs^2 + 24\omega_3^2\omega_1^2 - 4\omega_3^2\omega_4\omega_1^2cs^2 + 36\omega_3^2v_1^2\omega_4\omega_1 + 12\omega_3^2\omega_1^2cs^2 - 12\omega_4\omega_1^2cs^2 + 12\omega_3\omega_3\omega_1 + 12\omega_3^2\omega_4\omega_1cs^2 - \\ & 12\omega_3\omega_1^2 + 72\omega_3^2v_1^2\omega_1 + 36\omega_3^3v_1^2 - 24\omega_3^2\omega_1cs^2 - 36\omega_3^2v_1^2\omega_4 - 24\omega_3^2\omega_1^2cs^2 - 72\omega_3^2v_1^2\omega_1^2 + 18\omega_3\omega_4\omega_1^2cs^2 - 36\omega_3v_1^2\omega_4\omega_1 - 12\omega_3^3) \frac{\rho cs^2}{12\omega_3^2\omega_4\omega_1^2} \end{aligned}$$

coefficient $C_{D_x^2 D_y^2 v_2}^{(1)}$ at $\frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2}$:

[illegible]

$$\begin{aligned} C_{D_2^2 D_2^2 v_2}^{(1), \text{CuLBM2}} = & (36\omega_3 v_1^2 \omega_1^3 - 54cs^2 \omega_1 \omega_2^3 + 100\omega_3 \omega_1^3 v_2^2 \omega_2^2 + 162cs^2 \omega_3 \omega_1^2 \omega_2^2 - 36\omega_1^2 \omega_2^2 - 27\omega_3 v_1^2 \omega_1 \omega_2^3 - 198\omega_3 v_2^2 \omega_2^3 - 36\omega_3 v_1^2 \omega_1 \omega_2^2 + \\ & 18\omega_1^3 \omega_2 - 297cs^2 \omega_3 \omega_1^3 \omega_2 - 84cs^2 \omega_3 \omega_1^2 \omega_2^3 + 198\omega_3 \omega_1^3 v_2^2 + 84cs^2 \omega_3 \omega_1^3 \omega_2^2 - 18\omega_3 \omega_1 \omega_2^2 + 216\omega_3 \omega_1 v_2^2 \omega_2^3 + 54\omega_3 \omega_2^3 - 81\omega_3 \omega_1 \omega_2^3 - \\ & 162cs^2 \omega_3 \omega_1^2 \omega_2 + 270cs^2 \omega_3 \omega_2^3 + 162\omega_3 \omega_1 v_2^2 \omega_2^2 - 216\omega_3 \omega_1^2 v_2^2 \omega_2 - 18v_1^2 \omega_1 \omega_2^3 + 135\omega_3 \omega_1^2 \omega_2 - 54cs^2 \omega_2^3 \omega_2^3 + 46\omega_3 \omega_1^2 \omega_2^3 - 100\omega_3 \omega_1^2 v_2^2 \omega_2^2 - \\ & 18v_1^2 \omega_1^2 \omega_2 - 36\omega_3 v_1^2 \omega_2^2 \omega_2 - 54\omega_3 \omega_1^2 \omega_2^2 + 36v_1^2 \omega_2^2 \omega_2^2 - 126\omega_3 \omega_1^3 + 108cs^2 \omega_1^2 \omega_2^2 - 162\omega_3 \omega_1^2 v_2^2 \omega_2 + 18\omega_1 \omega_2^3 + 54\omega_3 v_1^2 \omega_1^2 \omega_2^2 + 135cs^2 \omega_3 \omega_1 \omega_2^3 + \\ & 90\omega_3 \omega_1^2 \omega_2 + 36\omega_3 v_1^2 \omega_2^3 - 54cs^2 \omega_1^3 \omega_2 - 46\omega_3 \omega_1^3 \omega_2^2 - 54cs^2 \omega_3 \omega_1 \omega_2^2 - 27\omega_3 v_1^2 \omega_1^3 \omega_2) \frac{v_1 \rho v_2}{24\omega_3 \omega_1^3 \omega_2^3} \end{aligned}$$

[illegible]

$$\begin{aligned}
& 9\omega_9cs^2\omega_6\omega_7\omega_4^2\omega_8\omega_5 - 6\omega_9\omega_6\omega_7\omega_2^2\omega_5 + 36cs^2\omega_6\omega_4\omega_8^2\omega_5 + 18\omega_9\omega_7\omega_4v_2^2\omega_8^2\omega_5 - 12\omega_9\omega_4^2\omega_8^2\omega_5 - 6\omega_6\omega_7\omega_4^2v_2^2\omega_8\omega_5 + 3\omega_9\omega_6\omega_7\omega_4^2\omega_8\omega_5 - \\
& 36\omega_9cs^2\omega_4\omega_8^2\omega_5 + 12\omega_6\omega_7\omega_4\omega_8^2\omega_5 - 12\omega_6\omega_4\omega_8^2\omega_5 + 6\omega_9\omega_6\omega_7\omega_4v_2^2\omega_5 - 36\omega_9cs^2\omega_6\omega_7\omega_4\omega_8^2\omega_5 + 36cs^2\omega_6\omega_4\omega_8^2\omega_5 - \omega_9\omega_6\omega_7\omega_4^2\omega_8^2\omega_5 + \\
& 6\omega_6\omega_7\omega_4^2v_2^2\omega_8^2\omega_5 - 15\omega_9cs^2\omega_7\omega_4^2\omega_8^2\omega_5 - 12\omega_6\omega_7\omega_4\omega_8\omega_5 + 18\omega_9cs^2\omega_7\omega_4^2\omega_8^2\omega_5 - 36cs^2\omega_6\omega_4\omega_8^2\omega_5 + 54\omega_9cs^2\omega_6\omega_7\omega_4\omega_8\omega_5 - 36\omega_9cs^2\omega_4^2\omega_8^2\omega_5 + \\
& 6\omega_6\omega_7\omega_4^2\omega_8^2\omega_5 - 12\omega_9\omega_4v_2^2\omega_8^2\omega_5 - 6\omega_6\omega_7\omega_4^2\omega_8^2\omega_5 + 12\omega_9\omega_4\omega_8^2\omega_5 + 36\omega_9cs^2\omega_4^2\omega_8^2\omega_5 - 18\omega_9\omega_6\omega_7\omega_4\omega_8\omega_5 + 12\omega_9\omega_6\omega_7\omega_8\omega_5 - 12\omega_9\omega_6\omega_7\omega_4v_2^2\omega_5 + \\
& 6\omega_9\omega_7\omega_4v_2^2\omega_8^2\omega_5 - 18cs^2\omega_6\omega_7\omega_4^2\omega_8^2\omega_5 + 3\omega_9cs^2\omega_6\omega_7\omega_4^2\omega_8^2\omega_5 + 18\omega_9\omega_6\omega_7\omega_4v_2^2\omega_8\omega_5 - 12\omega_9\omega_6\omega_7v_2^2\omega_8\omega_5 - 12\omega_6\omega_4^2v_2^2\omega_8^2\omega_5 + 12\omega_9\omega_6\omega_7\omega_4\omega_5 + \\
& 12\omega_6\omega_4^2\omega_8^2\omega_5 + 18\omega_9cs^2\omega_6\omega_7\omega_4\omega_5 - 36\omega_9cs^2\omega_7\omega_8^2\omega_5 - 12\omega_9\omega_7v_2^2\omega_8^2\omega_5 + \omega_9\omega_6\omega_7\omega_4^2v_2^2\omega_8^2\omega_5 - 12\omega_9\omega_4^2v_2^2\omega_8^2\omega_5 - 18\omega_9\omega_7\omega_4\omega_8^2\omega_5 + \\
& 36\omega_9cs^2\omega_6\omega_7\omega_8^2\omega_5 + 36cs^2\omega_6\omega_7\omega_4\omega_8\omega_5 - 5\omega_9\omega_7\omega_2^2v_2^2\omega_8^2\omega_5 + 12\omega_6\omega_7\omega_4v_2^2\omega_8\omega_5 + 12\omega_9\omega_4^2\omega_8^2\omega_5 + 18cs^2\omega_6\omega_7\omega_4^2\omega_8^2\omega_5) \frac{cs^2v_2}{12\omega_9\omega_6\omega_7\omega_4^2\omega_8^2\omega_5}
\end{aligned}$$

$$\begin{aligned}
C_{D_x D_y^3 \rho}^{(1), \text{CLBM2}} = & (-36\omega_6cs^2\omega_4^2\omega_8^2\omega_5 + 12\omega_9\omega_4^2v_2^2\omega_8^2\omega_5 + 12\omega_9\omega_7\omega_8^2\omega_5 - 6\omega_6\omega_7\omega_4^2v_2^2\omega_8^2\omega_5 + 12\omega_6\omega_4^2v_2^2\omega_8^2\omega_5 - 36\omega_6cs^2\omega_7\omega_4\omega_8^2\omega_5 - 36\omega_9cs^2\omega_4^2\omega_8^2\omega_5 - \\
& 12\omega_6\omega_4^2\omega_8^2\omega_5 - 12\omega_6\omega_7\omega_4v_2^2\omega_8^2\omega_5 + 5\omega_9\omega_7\omega_4^2v_2^2\omega_8^2\omega_5 + 12\omega_6\omega_4v_2^2\omega_8^2\omega_5 - 18\omega_6cs^2\omega_7\omega_4^2\omega_8^2\omega_5 - 3\omega_9\omega_6\omega_7\omega_4^2v_2^2\omega_8\omega_5 - 18\omega_6cs^2\omega_7\omega_2^2\omega_8\omega_5 + \\
& 36\omega_9\omega_6cs^2\omega_7\omega_8^2\omega_5 - 6\omega_9\omega_7\omega_4^2\omega_8^2\omega_5 + 6\omega_6\omega_7\omega_4^2\omega_8\omega_5 + 12\omega_9\omega_6\omega_7\omega_4\omega_8^2\omega_5 - 12\omega_9\omega_6\omega_7\omega_8^2\omega_5 + 18\omega_9\omega_6cs^2\omega_7\omega_4^2\omega_5 + 12\omega_9\omega_6\omega_7v_2^2\omega_8^2\omega_5 - \\
& 12\omega_9\omega_6\omega_7\omega_4v_2^2\omega_8^2\omega_5 - 6\omega_9\omega_6\omega_7\omega_2^2\omega_5 - 36\omega_9\omega_6cs^2\omega_7\omega_4\omega_8^2\omega_5 + 18\omega_9\omega_7\omega_4v_2^2\omega_8^2\omega_5 - 12\omega_9\omega_4^2\omega_8^2\omega_5 - 6\omega_6\omega_7\omega_4^2v_2^2\omega_8\omega_5 + 3\omega_9\omega_6\omega_7\omega_2^2\omega_8\omega_5 + \\
& 54\omega_9cs^2\omega_7\omega_4\omega_8^2\omega_5 + 12\omega_6\omega_7\omega_4\omega_8^2\omega_5 - 36\omega_9cs^2\omega_4\omega_8^2\omega_5 - 12\omega_6\omega_4^2\omega_8^2\omega_5 - 9\omega_9\omega_6cs^2\omega_7\omega_4^2\omega_8\omega_5 + 6\omega_9\omega_6\omega_7\omega_4^2v_2^2\omega_5 - \omega_9\omega_6\omega_7\omega_4^2\omega_8^2\omega_5 + \\
& 6\omega_6\omega_7\omega_4^2v_2^2\omega_8^2\omega_5 - 12\omega_6\omega_7\omega_4\omega_8\omega_5 + 3\omega_9\omega_6cs^2\omega_7\omega_4^2\omega_8^2\omega_5 + 6\omega_6\omega_7\omega_4^2\omega_8^2\omega_5 + 18\omega_9cs^2\omega_7\omega_4^2\omega_8^2\omega_5 - 12\omega_9\omega_4v_2^2\omega_8^2\omega_5 - 15\omega_9cs^2\omega_7\omega_4^2\omega_8^2\omega_5 - \\
& 6\omega_6\omega_7\omega_4^2\omega_8^2\omega_5 - 36\omega_9\omega_6cs^2\omega_7\omega_8\omega_5 + 12\omega_9\omega_4\omega_8^2\omega_5 - 18\omega_9\omega_6\omega_7\omega_4\omega_8\omega_5 + 12\omega_9\omega_6\omega_7\omega_8\omega_5 - 12\omega_9\omega_6\omega_7\omega_4v_2^2\omega_5 + 6\omega_9\omega_7\omega_4^2v_2^2\omega_8^2\omega_5 + \\
& 18\omega_9\omega_6\omega_7\omega_4v_2^2\omega_8\omega_5 - 12\omega_9\omega_6\omega_7v_2^2\omega_8\omega_5 - 36\omega_9\omega_6cs^2\omega_7\omega_4\omega_5 + 54\omega_9\omega_6cs^2\omega_7\omega_4\omega_8\omega_5 - 12\omega_6\omega_4^2v_2^2\omega_8^2\omega_5 + 36\omega_9cs^2\omega_4^2\omega_8^2\omega_5 + \\
& 12\omega_9\omega_6\omega_7\omega_4\omega_5 + 12\omega_6\omega_4^2\omega_8^2\omega_5 - 12\omega_9\omega_7v_2^2\omega_8^2\omega_5 + \omega_9\omega_6\omega_7\omega_4^2v_2^2\omega_8^2\omega_5 + 36\omega_6cs^2\omega_4^2\omega_8^2\omega_5 + 36\omega_6cs^2\omega_4\omega_8^2\omega_5 - 12\omega_9\omega_4^2v_2^2\omega_8^2\omega_5 - 18\omega_9\omega_7\omega_4\omega_8^2\omega_5 - \\
& 36\omega_9cs^2\omega_7\omega_8^2\omega_5 + 18\omega_6cs^2\omega_7\omega_4^2\omega_8^2\omega_5 + 36\omega_6cs^2\omega_7\omega_4\omega_8\omega_5 - 5\omega_9\omega_7\omega_4^2v_2^2\omega_8^2\omega_5 + 12\omega_6\omega_7\omega_4v_2^2\omega_8\omega_5 + 12\omega_9\omega_4^2\omega_8^2\omega_5) \frac{cs^2v_2}{12\omega_9\omega_6\omega_7\omega_4^2\omega_8^2\omega_5}
\end{aligned}$$

$$\begin{aligned}
C_{D_x D_y^3 \rho}^{(1), \text{CuLBM1}} = & (6\omega_3^2\omega_3v_2^2 + 12\omega_3\omega_2 - 18\omega_6\omega_3\omega_2 + 3\omega_3^2\omega_3^2cs^2\omega_2 + 12\omega_3^2v_2^2\omega_2 + 18\omega_3^2\omega_3cs^2 - 6\omega_6^2\omega_3 - \omega_6^2\omega_3\omega_2 - 6\omega_6\omega_3v_2^2 - 18\omega_6\omega_3cs^2 + \\
& \omega_6^2\omega_3^2v_2^2\omega_2 + 36\omega_6^2cs^2\omega_2 - 36\omega_3cs^2\omega_2 + 12\omega_6\omega_2 + 54\omega_6\omega_3cs^2\omega_2 - 36\omega_6^2cs^2 - \omega_6^2\omega_3^2 + 18\omega_6\omega_3v_2^2\omega_2 - 12\omega_3v_2^2\omega_2 - 12\omega_6^2v_2^2 + 18\omega_3^2cs^2\omega_2 + \\
& 6\omega_6\omega_3^2 - 9\omega_6\omega_3^2cs^2\omega_2 - 3\omega_6\omega_3^2v_2^2\omega_2 + 6\omega_3^2v_2^2\omega_2 - 12\omega_6^2\omega_2 - 12\omega_6v_2^2\omega_2 + 12\omega_6^2\omega_3\omega_2 - 36\omega_6^2\omega_3cs^2\omega_2 + 12\omega_6\omega_3v_2^2 + 36\omega_6\omega_3cs^2 - 6\omega_3^2\omega_2 + \\
& \omega_6^2\omega_3^2v_2^2 + 3\omega_6\omega_3^2\omega_2 + 12\omega_6^2 - 36\omega_6cs^2\omega_2 - 12\omega_6^2\omega_3v_2^2\omega_2 + 3\omega_6^2\omega_3^2cs^2 - 12\omega_6\omega_3) \frac{v_2cs^2}{12\omega_6^2\omega_3^2\omega_2}
\end{aligned}$$

$$\begin{aligned}
C_{D_x D_y^3 \rho}^{(1), \text{CuLBM2}} = & (72cs^4\omega_3^2\omega_1^3\omega_2^2 + 6cs^2\omega_3\omega_1^3v_2^2\omega_2 + 46\omega_3^2\omega_1^2v_2^2\omega_2^3 - 24cs^2\omega_3\omega_1^2\omega_2^2 - 9cs^2\omega_3^2v_2^2\omega_1^2\omega_2^3 - 3\omega_3^2v_1^2\omega_1^3\omega_2^2 - 147cs^2\omega_3^2\omega_1\omega_1^3\omega_2^2 + \\
& 6\omega_3^2\omega_1^2\omega_2 + 6cs^4\omega_3^2\omega_1^3\omega_2^3 + 141cs^2\omega_3^2\omega_1^2v_2^2\omega_2 - 90cs^4\omega_3^2\omega_1^3\omega_2^3 + 18cs^4\omega_3^2\omega_1^2\omega_2 - 3\omega_3^2v_1^4\omega_1^2\omega_2^3 - 30cs^2\omega_3\omega_1v_2^2\omega_2^3 + 261cs^2\omega_3^2\omega_1^3v_2^2\omega_2^3 + 39\omega_3^2\omega_1^3v_2^4\omega_2^2 - \\
& 6cs^2\omega_3\omega_1^3\omega_2 + 48cs^2\omega_3^2\omega_1^2\omega_2^3 - 42cs^2\omega_3\omega_1^2\omega_2^3 + 7\omega_3^2\omega_1^2\omega_2^3 - 90\omega_3^2\omega_1^2v_2^2\omega_2 - 7\omega_3^2\omega_1^2\omega_2^3 + 18cs^2\omega_3\omega_1^2\omega_2^3 - 45\omega_3^2v_2^2\omega_2^3 - 12\omega_3^2\omega_1^2\omega_2 - \\
& 45\omega_3^2\omega_1v_2^4\omega_2^2 - 24cs^2\omega_1^2v_2^2\omega_2^3 - 90cs^4\omega_3\omega_1\omega_2^3 + 3\omega_3^2v_1^4\omega_1^3\omega_2^2 + 12cs^2\omega_3^2\omega_1^2v_2^2\omega_2^3 - 261cs^2\omega_3^2v_2^2\omega_2^3 - 6cs^2\omega_3\omega_1^3v_2^2\omega_2^3 - 51\omega_3^2\omega_1^2v_2^2\omega_2 + \\
& 36cs^4\omega_3^2\omega_1^2\omega_2^3 + 45\omega_3^2\omega_1^3v_2^4\omega_2^2 + 6\omega_3^2\omega_1^3\omega_2^3 + 3\omega_3^2v_1^2\omega_1^3\omega_2^3 + 90\omega_3^2\omega_1v_2^4\omega_2^3 + 9cs^2\omega_3^2v_1^2\omega_1^3\omega_2^2 + 6cs^2\omega_3\omega_1^3\omega_2^3 + 72cs^2\omega_3^2\omega_2^3 - 138cs^4\omega_3^2\omega_1^2\omega_2^3 - \\
& 219cs^2\omega_3^2\omega_1^2v_2^2\omega_2^3 - 18cs^2\omega_3\omega_1^3v_2^2\omega_2^3 - 153cs^4\omega_3^2\omega_1^2\omega_2 - 72cs^2\omega_3^2\omega_1^3\omega_2^3 - 6\omega_3^2\omega_1\omega_2^2 - 59cs^2\omega_3^2\omega_1^3\omega_2^2 - 51\omega_3^2\omega_1^3v_2^2\omega_2 - 465cs^2\omega_3^2\omega_1^2v_2^2\omega_2 - \\
& 46\omega_3^2\omega_1^2v_2^2\omega_2 + 72cs^4\omega_3\omega_1^2\omega_2^3 + 225cs^4\omega_3^2\omega_1\omega_2^3 - 165cs^4\omega_3^2\omega_1v_2^2\omega_2^3 - 2cs^2\omega_3^2\omega_1^3\omega_2^3 - 72cs^4\omega_2^2\omega_1^3\omega_2^3 + 12\omega_3^2\omega_1\omega_2^3 - 24cs^2\omega_3^2\omega_1^2\omega_2 + \\
& 489cs^2\omega_3^2\omega_1v_2^2\omega_2^3 - 90cs^4\omega_3^2\omega_1\omega_2^2 + 18cs^4\omega_3\omega_1^2\omega_2 - 12cs^2\omega_3^2\omega_2^3 + 126cs^4\omega_3\omega_1^2\omega_2^3 - 6\omega_3^2\omega_2^3 - 54cs^4\omega_3\omega_1^2\omega_2 - 102\omega_3^2\omega_1v_2^2\omega_2^3 + 90cs^4\omega_3^2\omega_1^3 + \\
& 24cs^2\omega_3\omega_1^2v_2^2\omega_2^2 + 2cs^2\omega_3^2\omega_1^2v_2^2\omega_2^3 + 30cs^2\omega_3\omega_1\omega_2^3 - 12cs^2\omega_3^2\omega_1^2\omega_2^2 + 42cs^2\omega_3\omega_1^2v_2^2\omega_2^3 + 102\omega_3^2\omega_1^2v_2^2\omega_2 + 197cs^2\omega_3^2\omega_1^2v_2^2\omega_2^2 + 24cs^2\omega_1^2\omega_2^3 + \\
& 51\omega_3^2v_2^2\omega_2^3 + 51\omega_3^2\omega_1v_2^2\omega_2^3 - 18cs^4\omega_3\omega_1^2\omega_2^3 + 81cs^2\omega_3^2\omega_1^2\omega_2^3 + 45\omega_3^2\omega_1^2v_2^2\omega_2 + 123cs^2\omega_3^2\omega_1^2\omega_2 + 36cs^4\omega_1^3\omega_2^3 + 12cs^2\omega_1^3v_2^2\omega_2^3) \frac{v_2}{24\omega_3^2\omega_1^3\omega_2^3}
\end{aligned}$$

$$\text{coefficient } C_{D_x D_y^3 v_1}^{(1)} \text{ at } \frac{\partial^4 v_1}{\partial x_1 \partial x_2^2} :$$

$$C_{D_x D_y^3 v_1}^{(1), \text{SRT}} = (2 + 3cs^2\omega - 6cs^2 + v_2^2\omega - 2v_2^2 - \omega) \frac{v_1 \rho v_2}{12\omega}$$

$$\begin{aligned}
C_{D_x D_y^3 v_1}^{(1), \text{MRT1}} = & (12\omega_9\omega_6^2\omega_7\omega_4^3\omega_8^2\omega_5 + 6\omega_9\omega_6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 24\omega_6^2cs^2\omega_7\omega_4^3\omega_8^2\omega_5^2 - 42\omega_9\omega_6cs^2\omega_7^2\omega_4^2\omega_8^2\omega_5^2 + 66\omega_9\omega_6^2\omega_7^2\omega_4^2v_2^2\omega_8\omega_5^2 + \\
& 18\omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8\omega_5^2 + 84\omega_9\omega_6^2cs^2\omega_7^2\omega_4^2\omega_8^2\omega_5 + 12\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_5^2 + 12\omega_9\omega_6\omega_7^2\omega_4v_2^2\omega_8^2\omega_5^2 + 12\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + 24\omega_9\omega_6^2cs^2\omega_4^3\omega_8^2\omega_5 - \\
& 12\omega_9\omega_6^2\omega_7^2\omega_4^3\omega_5^2 - 24\omega_6^2\omega_7\omega_4^3\omega_8^2\omega_5 + 12\omega_6^2\omega_7^2\omega_4^3\omega_8\omega_5^2 - 12\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 24\omega_6^2cs^2\omega_7^2\omega_4^2\omega_8^2\omega_5^2 - 12\omega_9\omega_6^2\omega_7^2\omega_4^2\omega_8^2\omega_5 - 24\omega_9\omega_6^2cs^2\omega_7^2\omega_4^3\omega_8\omega_5^2 + \\
& 36\omega_9cs^2\omega_7^2\omega_4^3\omega_8^2\omega_5^2 + 72\omega_9\omega_6^2cs^2\omega_7\omega_4\omega_8^2\omega_5^2 + 24\omega_9\omega_6^2cs^2\omega_7^2\omega_4^3\omega_5^2 - 84\omega_9\omega_6^2cs^2\omega_7\omega_4^3\omega_8^2\omega_5 - 6\omega_9\omega_6\omega_7^2\omega_4^3\omega_8\omega_5^2 + 6\omega_9\omega_7^2\omega_4^3\omega_8^2\omega_5^2 + \\
& 12\omega_9\omega_6^2cs^2\omega_7^2\omega_4^3\omega_8\omega_5 - 24\omega_9\omega_6^2cs^2\omega_7^2\omega_4^2\omega_8^2\omega_5 + 24\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 36\omega_9\omega_6^2cs^2\omega_7^2\omega_4\omega_8\omega_5^2 + 60\omega_9\omega_6^2cs^2\omega_7\omega_4^3\omega_8^2\omega_5^2 + 24\omega_6^2\omega_7\omega_4^3\omega_8^2\omega_5^2 + \\
& 6\omega_9\omega_6cs^2\omega_7^2\omega_4^3\omega_8\omega_5^2 + 12\omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8^2\omega_5^2 - 48\omega_9\omega_6^2cs^2\omega_7^2\omega_4^3\omega_5^2 - 66\omega_9\omega_6^2cs^2\omega_7^2\omega_4^2\omega_8^2\omega_5^2 - 18\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 12\omega_9\omega_6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - \\
& 24\omega_9\omega_6^2cs^2\omega_4^3\omega_8^2\omega_5^2 + 18\omega_9\omega_6\omega_7^2\omega_4^3\omega_8^2\omega_5^2 - 96\omega_9\omega_6^2cs^2\omega_7^2\omega_8^2\omega_5^2 + 24\omega_6^2cs^2\omega_7\omega_4^3\omega_8^2\omega_5 - 12\omega_9\omega_6^2\omega_7\omega_4^3\omega_8^2\omega_5^2 + 12\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5^2 + \\
& 36\omega_9\omega_6^2\omega_7^2\omega_4\omega_8\omega_5^2 - 24\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + 24\omega_6^2\omega_7^2\omega_4^3\omega_8^2\omega_5^2 - 12\omega_6^2cs^2\omega_7^2\omega_4^3\omega_8\omega_5^2 + 24\omega_9\omega_6^2\omega_7^2\omega_4^3\omega_5^2 - 12\omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8\omega_5 + \omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5^2 - \\
& 18\omega_9cs^2\omega_7^2\omega_4^3\omega_8^2\omega_5^2 + 90\omega_9\omega_6^2cs^2\omega_7^2\omega_4^3\omega_8\omega_5^2 + 48\omega_9\omega_6^2cs^2\omega_7\omega_4^3\omega_8^2\omega_5 + 12\omega_9\omega_6\omega_7^2\omega_4^3\omega_8\omega_5^2 - 18\omega_9\omega_6\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5^2 + 24\omega_6^2\omega_7\omega_4^3v_2^2\omega_8^2\omega_5 - \\
& 36\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 12\omega_9\omega_6^2\omega_7^2\omega_4\omega_8^2\omega_5^2 + 24\omega_6^2\omega_7\omega_4^3v_2^2\omega_8^2\omega_5^2 + 12\omega_6^2cs^2\omega_7^2\omega_4^3\omega_8^2\omega_5^2 - 24\omega_6^2\omega_7^2\omega_4^3\omega_8\omega_5^2 + 12\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - \\
& 24\omega_6^2cs^2\omega_7\omega_4^3\omega_8^2\omega_5^2 + 12\omega_9\omega_6\omega_7^2\omega_4^3\omega_8^2\omega_5^2 - 24\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_5^2 + 156\omega_9\omega_6^2cs^2\omega_7^2\omega_4\omega_8^2\omega_5^2 + 12\omega_9\omega_6^2\omega_7\omega_4^3v_2^2\omega_8^2\omega_5^2 + 12\omega_9\omega_6cs^2\omega_7^2\omega_4^3\omega_8^2\omega_5^2 + \\
& 24\omega_6^2cs^2\omega_7\omega_4^3\omega_8^2\omega_5^2 + 12\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 66\omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8\omega_5^2 + 12\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5 - 12\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5 - 12\omega_9\omega_6^2\omega_7\omega_4^3v_2^2\omega_8^2\omega_5 + \\
& 12\omega_9\omega_6^2\omega_7\omega_4^3\omega_8^2\omega_5^2 + 24\omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8\omega_5 + 24\omega_6^2cs^2\omega_7^2\omega_4^3\omega_8\omega_5^2 + 12\omega_9\omega_6cs^2\omega_7^2\omega_4\omega_8^2\omega_5^2 - 12\omega_6^2\omega_7^2\omega_4^3\omega_8^2\omega_5^2 + 12\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5^2 + \\
& 24\omega_9\omega_6^2cs^2\omega_7\omega_4^3\omega_8^2\omega_5^2 + 3\omega_9\omega_6^2cs^2\omega_7^2\omega_4^3\omega_8^2\omega_5^2 - 6\omega_9\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5^2 - 4\omega_9\omega_6\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5^2 + 24\omega_9\omega_6^2cs^2\omega_4^3\omega_8^2\omega_5^2 - 12\omega_9\omega_6\omega_7\omega_4^3v_2^2\omega_8^2\omega_5^2 - \\
& 72\omega_9\omega_6^2cs^2\omega_7^2\omega_4\omega_8^2\omega_5 - 12\omega_9\omega_6cs^2\omega_7^2\omega_4\omega_8\omega_5^2 - 24\omega_6^2\omega_7\omega_4^3\omega_8^2\omega_5^2 + 4\omega_9\omega_6\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5^2 - 12\omega_6^2cs^2\omega_7^2\omega_4^3\omega_8^2\omega_5 - \omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8^2\omega_5^2 - \\
& 12\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5^2 - 12\omega_9\omega_7^2\omega_4^3\omega_8^2\omega_5^2 - 24\omega_9\omega_6^2cs^2\omega_7^2\omega_4\omega_8\omega_5 - 132\omega_9\omega_6^2cs^2\omega_7\omega_4^3\omega_8^2\omega_5^2 - 24\omega_6^2\omega_7\omega_4^3v_2^2\omega_8^2\omega_5^2) \frac{v_1 \rho v_2}{12\omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8^2\omega_5^2}
\end{aligned}$$

$$C_{D_x D_y^3 v_1}^{(1), \text{MRT2}} = (12\omega_9\omega_6^2\omega_7\omega_4^3\omega_8^2\omega_5 + 6\omega_9\omega_6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 + 12\omega_9\omega_6\omega_7^2\omega_4^3\omega_8^2cs^2\omega_5^2 + 66\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 66\omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8^2cs^2\omega_5^2 +$$

$$C_{\frac{D \times D}{3} v_2}^{(1), \text{MRT}^2} = (-6\omega_6^2 \omega_7 \omega_4^3 \omega_8 c s^4 \omega_5 + 18\omega_9 \omega_6^2 v_1^2 \omega_7 \omega_3^2 v_2^2 \omega_5 + 12\omega_6^2 v_1^2 \omega_7 \omega_4^3 \omega_8^2 \omega_5 + 12\omega_6^2 \omega_7 \omega_4^3 \omega_8^2 c s^2 \omega_5 + 12\omega_6^2 v_1^2 \omega_4^3 \omega_8^2 c s^2 + 12\omega_6^3 \omega_3^2 \omega_8^2 c s^2 \omega_5 + 72\omega_9 \omega_6 v_1^2 \omega_7 \omega_4 v_2^2 \omega_8^2 \omega_5 + 12\omega_9 v_1^2 \omega_7 \omega_4^2 \omega_8^2 c s^2 \omega_5 - 12\omega_9 \omega_6 \omega_4^3 \omega_8^2 c s^2 \omega_5 - 12\omega_9 \omega_6 v_1^2 \omega_3^2 \omega_8^2 c s^2 - 12\omega_9 \omega_6 \omega_7 \omega_4 \omega_3^2 c s^4 \omega_5 - 12\omega_9 \omega_6 v_1^2 \omega_3^2 \omega_8^2 c s^4 - 36\omega_9 \omega_6 v_1^2 v_2^2 \omega_8^2 c s^2 \omega_5 - 12\omega_9 \omega_7^2 \omega_4 \omega_8 c s^2 \omega_5 - 12\omega_9 v_1^2 \omega_7 \omega_4^2 \omega_8^2 \omega_5 + 36\omega_6^2 v_1^2 \omega_4^3 v_2^2 \omega_8^2 \omega_5 - 18\omega_6^2 v_1^2 \omega_7 \omega_4^3 v_2^2 \omega_8^2 + 9\omega_9 \omega_6 v_1^2 \omega_7 \omega_3^2 \omega_8^2 c s^2 \omega_5 -$$

$$\begin{aligned}
& 15\omega_9\omega_6^2v_1^2\omega_7\omega_4^3\omega_8cs^2\omega_5 + 18\omega_9\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5 + 12\omega_6^3\omega_4^3\omega_8^2cs^4 + 15\omega_9\omega_6^2v_1^2\omega_7\omega_4^3\omega_8\omega_5 - 12\omega_6^2v_1^2\omega_7\omega_4^2\omega_8^2cs^2\omega_5 - 12\omega_9\omega_6^2\omega_7\omega_8^2cs^4\omega_5 - \\
& 72\omega_9\omega_6^2v_1^2\omega_7\omega_4v_2^2\omega_8\omega_5 - 18\omega_9\omega_6^2\omega_7\omega_4^2\omega_8cs^4\omega_5 + 36\omega_6^2v_1^2\omega_7\omega_4^2v_2^2\omega_8\omega_5 + 36\omega_9\omega_6^2v_1^2\omega_7\omega_4^3v_2^2\omega_8 - 18\omega_9\omega_6\omega_7\omega_4^2\omega_8^2cs^2\omega_5 + 36\omega_9\omega_6v_1^2\omega_7\omega_4^2\omega_8^2\omega_5 + \\
& 36\omega_6^2\omega_7\omega_4^2v_2^2\omega_8cs^2\omega_5 - 18\omega_9\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8 - 36\omega_9\omega_6v_1^2\omega_4^2v_2^2\omega_8^2\omega_5 + 36\omega_6^2\omega_4^2v_2^2\omega_8^2cs^2\omega_5 + 18\omega_6^2\omega_7\omega_4^3v_2^2\omega_8^2\omega_5 + 12\omega_9\omega_6^2v_1^2\omega_7\omega_4^3\omega_8cs^2 + \\
& 5\omega_9\omega_6\omega_7\omega_4^2\omega_8^2cs^2\omega_5 - 18\omega_6^2v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5 + 60\omega_9\omega_6^2\omega_7\omega_4v_2^2\omega_8^2cs^2\omega_5 + 6\omega_6^2v_1^2\omega_7\omega_4^3\omega_8^2cs^2\omega_5 + \omega_9\omega_6^2\omega_7\omega_4^2\omega_8^2cs^2\omega_5 + 6\omega_9\omega_6^2\omega_7\omega_4^3\omega_8cs^4\omega_5 - \\
& 12\omega_9\omega_6v_1^2\omega_4^3\omega_8^2\omega_5 + 6\omega_6^2v_1^2\omega_7\omega_4^3\omega_8\omega_5 + 12\omega_6^2v_1^2\omega_4^3\omega_8^2\omega_5 - 12\omega_9\omega_6^2v_1^2\omega_7\omega_4^2cs^2\omega_5 + 36\omega_9\omega_6v_1^2\omega_4^3v_2^2\omega_8^2\omega_5 - 12\omega_6^2v_1^2\omega_4^3\omega_8^2 + 24\omega_9\omega_6^2v_1^2\omega_7\omega_4\omega_8\omega_5 + \\
& 30\omega_9\omega_6\omega_7\omega_4^2\omega_8^2cs^2\omega_5 + 12\omega_9\omega_6v_1^2\omega_4^3\omega_8cs^2\omega_5 - 12\omega_6^2\omega_4^2\omega_8^2cs^2\omega_5 + 12\omega_6^2\omega_7\omega_4^2\omega_8cs^4\omega_5 - 15\omega_9\omega_6\omega_7\omega_4^3v_2^2\omega_8^2cs^2\omega_5 - 6\omega_6^2\omega_7\omega_4^3\omega_8^2cs^2\omega_5 - \\
& 6\omega_9\omega_6\omega_7\omega_4^2\omega_8^2cs^2 + 48\omega_9\omega_6^2v_1^2\omega_7\omega_4^3\omega_8cs^2\omega_5 - 36\omega_9\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5 - 36\omega_6^2v_1^2\omega_4^3v_2^2\omega_8^2\omega_5 - 36\omega_9\omega_6v_1^2\omega_7\omega_4^2\omega_8^2cs^2\omega_5 + 24\omega_9\omega_6v_1^2\omega_7\omega_4^2\omega_8^2cs^2 + \\
& 18\omega_9\omega_6^2\omega_7\omega_4\omega_8^2cs^4\omega_5 + 6\omega_6^2\omega_7\omega_4^3\omega_8^2cs^2 - 6\omega_9\omega_6v_1^2\omega_7\omega_4^3\omega_8\omega_5 - 15\omega_9\omega_6\omega_7\omega_4^2v_2^2\omega_8^2cs^2\omega_5 + 12\omega_9\omega_6\omega_4^2\omega_8^2cs^2\omega_5 - 6\omega_9v_1^2\omega_7\omega_4^3\omega_8^2cs^2\omega_5 + \\
& 60\omega_9\omega_6^2\omega_7\omega_4v_2^2\omega_8cs^2\omega_5 + 18\omega_9\omega_6^2\omega_7\omega_4^2\omega_8cs^2\omega_5 - 6\omega_6^2v_1^2\omega_7\omega_4^3\omega_8cs^2\omega_5 - \omega_9\omega_6^2\omega_7\omega_4^3\omega_8^2cs^4\omega_5 + 6\omega_9\omega_6v_1^2\omega_7\omega_4^3\omega_8^2 - 36\omega_9\omega_6\omega_7\omega_4v_2^2\omega_8^2cs^2\omega_5 + \\
& 72\omega_9\omega_6v_1^2\omega_7\omega_4^2v_2^2\omega_8^2 + 18\omega_9\omega_6\omega_7\omega_4^2\omega_8^2cs^4\omega_5 - 72\omega_9\omega_6^2v_1^2\omega_7\omega_4^2v_2^2\omega_8 - 108\omega_9\omega_6v_1^2\omega_7\omega_4^2v_2^2\omega_8^2\omega_5 + 6\omega_6^2v_1^2\omega_7\omega_4^3\omega_8^2 - 12\omega_6^2v_1^2\omega_4^3\omega_8^2cs^2\omega_5 + \\
& 24\omega_9\omega_6^2v_1^2\omega_7\omega_4^2\omega_8 - 18\omega_9v_1^2\omega_7\omega_4^3v_2^2\omega_8^2\omega_5 - 9\omega_9\omega_6v_1^2\omega_7\omega_4^3\omega_8^2\omega_5 - 12\omega_6^2\omega_7\omega_4^2\omega_8^2cs^4\omega_5 + 6\omega_6^2\omega_7\omega_4^3\omega_8cs^2\omega_5 - 12\omega_6^2\omega_4^3\omega_8^2cs^4\omega_5 - \\
& 48\omega_9\omega_6^2v_1^2\omega_7\omega_4^2\omega_8\omega_5 - 6\omega_6^2\omega_7\omega_4^3\omega_8^2cs^4 - 36\omega_9\omega_6\omega_4^3v_2^2\omega_8^2cs^2 + 6\omega_9v_1^2\omega_7\omega_4^3\omega_8^2\omega_5 + 18\omega_6^2v_1^2\omega_7\omega_4^3v_2^2\omega_8^2\omega_5 + 12\omega_9\omega_6\omega_7\omega_4\omega_8^2cs^2\omega_5 + \\
& 12\omega_9\omega_6\omega_4^3\omega_8^2cs^4\omega_5 - 102\omega_9\omega_6^2\omega_7\omega_4^2v_2^2\omega_8cs^2\omega_5 + 54\omega_9\omega_6\omega_7\omega_4^2v_2^2\omega_8^2cs^2\omega_5 - 36\omega_9\omega_6^2v_1^2\omega_7\omega_4^2v_2^2\omega_8 - 6\omega_6^2v_1^2\omega_7\omega_4^3\omega_8^2\omega_5 + 144\omega_9\omega_6^2v_1^2\omega_7\omega_4^2v_2^2\omega_8\omega_5 + \\
& 36\omega_6^2\omega_4^3v_2^2\omega_8^2cs^2 - 12\omega_9\omega_6v_1^2\omega_7\omega_4^2\omega_8cs^2\omega_5 + 12\omega_9\omega_6^2\omega_7\omega_4\omega_8cs^4\omega_5 + 12\omega_9\omega_6v_1^2\omega_4^3\omega_8^2cs^2\omega_5 + 6\omega_9\omega_6\omega_7\omega_4^3\omega_8^2cs^4 + 12\omega_9\omega_6^2v_1^2\omega_7\omega_4^2\omega_5 + \\
& 24\omega_9\omega_6^2\omega_7\omega_4^2v_2^2cs^2\omega_5 - 6\omega_9\omega_6v_1^2\omega_7\omega_4^3\omega_8^2cs^2 - 12\omega_9\omega_6^2v_1^2\omega_7\omega_4^3\omega_8^2cs^2 - 36\omega_9\omega_6\omega_4^3v_2^2\omega_8^2\omega_5 + 36\omega_9\omega_6\omega_4^3v_2^2\omega_8^2cs^2 + \\
& 12\omega_6^2\omega_4^3\omega_8^2cs^4\omega_5 + 6\omega_6^2\omega_7\omega_4^3\omega_8^2cs^4\omega_5 + 12\omega_9\omega_6v_1^2\omega_7\omega_4^2\omega_8\omega_5 - 12\omega_6^2\omega_4^3\omega_8^2cs^2 - 12\omega_6^2\omega_7\omega_4^2\omega_8cs^2\omega_5 - 18\omega_6^2\omega_7\omega_4^3v_2^2\omega_8^2cs^2 - 12\omega_9\omega_6v_1^2\omega_4^3\omega_8^2cs^2\omega_5 - \\
& 12\omega_9\omega_6^2\omega_7\omega_4^3v_2^2cs^2\omega_5 - 45\omega_9\omega_6^2v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5 + 6\omega_9\omega_6v_1^2\omega_7\omega_4^3\omega_8cs^2\omega_5 - 24\omega_9\omega_6v_1^2\omega_7\omega_4^2\omega_8^2 - 12\omega_9\omega_6\omega_4^2\omega_8^2cs^4\omega_5 - 36\omega_9\omega_6v_1^2\omega_4^3v_2^2\omega_8^2 - \\
& 48\omega_9\omega_6^2\omega_7v_2^2\omega_8^2cs^2\omega_5 - 36\omega_6^2\omega_7\omega_4^3v_2^2\omega_8^2cs^2\omega_5 - 6\omega_9\omega_6^2v_1^2\omega_7\omega_4^3\omega_5 - 24\omega_9\omega_6v_1^2\omega_7\omega_4\omega_8^2\omega_5 - 12\omega_6^2v_1^2\omega_7\omega_4^3\omega_8\omega_5 - 5\omega_9\omega_6\omega_7\omega_4^3\omega_8^2cs^4\omega_5 - \\
& 24\omega_9\omega_6^2v_1^2\omega_7\omega_4^2\omega_8cs^2\omega_5 - 36\omega_6^2\omega_4^3v_2^2\omega_8^2cs^2\omega_5 + 6\omega_9\omega_6^2v_1^2\omega_7\omega_4^3cs^2\omega_5 - 6\omega_6^2v_1^2\omega_7\omega_4^3\omega_8^2cs^2 + 12\omega_6^2v_1^2\omega_7\omega_4^2\omega_8cs^2\omega_5 - 12\omega_6^2v_1^2\omega_4^3\omega_8^2\omega_5 - \\
& 6\omega_9\omega_6^2\omega_7\omega_4^3\omega_8cs^2\omega_5 - 5\omega_9\omega_6^2\omega_7\omega_4^2\omega_8^2cs^4\omega_5 + 24\omega_9\omega_6v_1^2\omega_7\omega_4\omega_8^2cs^2\omega_5 + 36\omega_9v_1^2\omega_7\omega_4^2v_2^2\omega_8^2\omega_5 - 18\omega_6^2\omega_7\omega_4^3v_2^2\omega_8cs^2\omega_5 - 24\omega_9\omega_6^2v_1^2\omega_7\omega_4^2\omega_8cs^2 + \\
& 36\omega_6^2v_1^2\omega_4^3v_2^2\omega_8^2 + 27\omega_9\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8^2\omega_5 + 12\omega_9\omega_6v_1^2\omega_4^2\omega_8^2\omega_5 + 18\omega_9\omega_6\omega_7\omega_4^3v_2^2\omega_8^2cs^2 + 12\omega_6^2v_1^2\omega_4^2\omega_8^2cs^2\omega_5) \frac{\rho}{12\omega_9\omega_6^2\omega_7\omega_4^3\omega_8^2\omega_5}
\end{aligned}$$

$$\begin{aligned}
C_{D_x D_y^3 v_2}^{(1), \text{CLBMM1}} = & (-6\omega_9\omega_6\omega_7\omega_4^3\omega_5 - 36\omega_9\omega_4^3v_2^2\omega_8 + 18\omega_6\omega_7\omega_4^3v_2^2\omega_8\omega_5 - 12cs^2\omega_6\omega_7\omega_4^2\omega_8\omega_5 + 54\omega_9\omega_7\omega_4^2v_2^2\omega_8\omega_5 + 12\omega_9cs^2\omega_6\omega_7\omega_4\omega_5 - 3\omega_9\omega_6\omega_7\omega_4^2v_2^2\omega_8\omega_5 - \\
& 36\omega_6\omega_4^3v_2^2\omega_8\omega_5 - 12\omega_9\omega_4^3\omega_8\omega_5 - 12\omega_9cs^2\omega_6\omega_7\omega_8\omega_5 + 12\omega_9\omega_7\omega_4\omega_8\omega_5 - 12\omega_6\omega_4^3\omega_8 + 12\omega_6\omega_7\omega_4^2\omega_8\omega_5 - 12\omega_9cs^2\omega_4^3\omega_8\omega_5 + 5\omega_9\omega_7\omega_4^3\omega_8\omega_5 - \\
& 5\omega_9cs^2\omega_6\omega_7\omega_4^2\omega_8\omega_5 - 12\omega_6\omega_4^2\omega_8\omega_5 + 36\omega_6\omega_7\omega_4^2v_2^2\omega_5 + 18\omega_9\omega_6\omega_7\omega_4^2\omega_5 + 36\omega_6\omega_4^2v_2^2\omega_8\omega_5 + 18\omega_9cs^2\omega_7\omega_4^2\omega_8\omega_5 - 36\omega_6\omega_7\omega_4^2v_2^2\omega_8\omega_5 + \\
& \omega_9\omega_6\omega_7\omega_4^2\omega_8\omega_5 - 6\omega_9\omega_7\omega_4^3\omega_8 - 15\omega_9\omega_7\omega_4^3v_2^2\omega_8\omega_5 + 12cs^2\omega_6\omega_4^2\omega_8\omega_5 + 18\omega_9\omega_7\omega_4^3v_2^2\omega_8 - 54\omega_9\omega_6\omega_7\omega_4^2v_2^2\omega_5 + 12\omega_9\omega_4^2\omega_8\omega_5 - 36\omega_9\omega_7\omega_4v_2^2\omega_8\omega_5 + \\
& 36\omega_9\omega_4^2v_2^2\omega_8\omega_5 + 6\omega_6\omega_7\omega_4^3\omega_5 + 18\omega_9cs^2\omega_6\omega_7\omega_4\omega_8\omega_5 + 6\omega_9cs^2\omega_6\omega_7\omega_4^3\omega_5 - 6cs^2\omega_6\omega_7\omega_4^3\omega_8 + 6\omega_9cs^2\omega_7\omega_4^3\omega_8 + 12cs^2\omega_6\omega_4^3\omega_8 + 6cs^2\omega_6\omega_7\omega_4^3\omega_8\omega_5 - \\
& 12\omega_9cs^2\omega_7\omega_4\omega_8\omega_5 + 36\omega_9\omega_6\omega_7\omega_4v_2^2\omega_5 - 12\omega_9cs^2\omega_4^3\omega_8 + 6\omega_6\omega_7\omega_4^3\omega_8 - 6cs^2\omega_6\omega_7\omega_4^3\omega_5 - 12\omega_9\omega_6\omega_7\omega_4\omega_5 - 18\omega_9cs^2\omega_6\omega_7\omega_4^2\omega_5 - \\
& 12cs^2\omega_6\omega_4^3\omega_8\omega_5 - 12\omega_6\omega_7\omega_4^2\omega_5 + 18\omega_9\omega_6\omega_7\omega_4^3v_2^2\omega_5 - 18\omega_6\omega_7\omega_4^3v_2^2\omega_8 + 12\omega_9\omega_4^3\omega_8 + 36\omega_6\omega_4^3v_2^2\omega_8 - 5\omega_9cs^2\omega_7\omega_4^3\omega_8\omega_5 - 36\omega_9\omega_4^2v_2^2\omega_8\omega_5 - \\
& \omega_9cs^2\omega_6\omega_7\omega_4^3\omega_8\omega_5 + 12cs^2\omega_6\omega_7\omega_4^2\omega_5 - 18\omega_6\omega_7\omega_4^3v_2^2\omega_5 + 12\omega_6\omega_4^3\omega_8\omega_5 - 6\omega_6\omega_7\omega_4^3\omega_8\omega_5 - 18\omega_9\omega_7\omega_4^2\omega_8\omega_5 + 12\omega_9cs^2\omega_4^3\omega_8\omega_5) \frac{cs^2\rho}{12\omega_9\omega_6\omega_7\omega_4^3\omega_8\omega_5}
\end{aligned}$$

$$\begin{aligned}
C_{D_x D_y^3 v_2}^{(1), \text{CLBMM2}} = & (-6\omega_9\omega_6\omega_7\omega_4^3\omega_5 - 36\omega_9\omega_4^3v_2^2\omega_8 - 6\omega_6cs^2\omega_7\omega_4^3\omega_8 + 6\omega_9\omega_6cs^2\omega_7\omega_4^3\omega_5 + 18\omega_6\omega_7\omega_4^3v_2^2\omega_8\omega_5 + 54\omega_9\omega_7\omega_4^2v_2^2\omega_8\omega_5 - \\
& 6\omega_6cs^2\omega_7\omega_4^3\omega_5 - 3\omega_9\omega_6\omega_7\omega_4^2v_2^2\omega_8\omega_5 - 36\omega_6\omega_4^3v_2^2\omega_8\omega_5 - 12\omega_9\omega_4^3\omega_8\omega_5 + 12\omega_9\omega_7\omega_4\omega_8\omega_5 - 12\omega_6cs^2\omega_7\omega_4^2\omega_8\omega_5 - 12\omega_9cs^2\omega_4^3\omega_8 - \\
& 12\omega_6\omega_4^3\omega_8\omega_5 + 12\omega_6\omega_7\omega_4^2\omega_8\omega_5 + 18\omega_9\omega_6\omega_7\omega_4^2\omega_5 + 36\omega_6\omega_4^2v_2^2\omega_8\omega_5 - 36\omega_6\omega_7\omega_4^2v_2^2\omega_8\omega_5 + \omega_9\omega_6\omega_7\omega_4^2\omega_8\omega_5 - 6\omega_9\omega_7\omega_4^3\omega_8 - \\
& 15\omega_9\omega_7\omega_4^3v_2^2\omega_8\omega_5 + 18\omega_9\omega_7\omega_4^3v_2^2\omega_8 - 5\omega_9\omega_6cs^2\omega_7\omega_4^2\omega_8\omega_5 + 12\omega_6cs^2\omega_7\omega_4^2\omega_5 - 54\omega_9\omega_6\omega_7\omega_4^2v_2^2\omega_5 + 12\omega_9\omega_4^2\omega_8\omega_5 - 36\omega_9\omega_7\omega_4v_2^2\omega_8\omega_5 - \\
& 12\omega_9cs^2\omega_7\omega_4\omega_8\omega_5 + 6\omega_6cs^2\omega_7\omega_4^3\omega_8\omega_5 + 36\omega_9\omega_4^3v_2^2\omega_8\omega_5 + 6\omega_6\omega_7\omega_4^3\omega_5 - 12\omega_9\omega_6cs^2\omega_7\omega_8\omega_5 + 36\omega_9\omega_6\omega_7\omega_4v_2^2\omega_5 + 6\omega_6\omega_7\omega_4^3\omega_8 + \\
& 6\omega_9cs^2\omega_7\omega_4^3\omega_8 + 12\omega_9\omega_6cs^2\omega_7\omega_4\omega_5 + 18\omega_9\omega_6cs^2\omega_7\omega_4\omega_8\omega_5 - 12\omega_9\omega_6\omega_7\omega_4\omega_5 - \omega_9\omega_6cs^2\omega_7\omega_4^3\omega_8\omega_5 - 12\omega_6\omega_7\omega_4^2\omega_5 + 18\omega_9\omega_6\omega_7\omega_4^3v_2^2\omega_5 - \\
& 18\omega_6\omega_7\omega_4^3v_2^2\omega_8 + 12\omega_9\omega_4^3\omega_8 + 36\omega_6\omega_4^3v_2^2\omega_8 - 36\omega_9\omega_4^2v_2^2\omega_8\omega_5 + 12\omega_6cs^2\omega_4^3\omega_8 + 12\omega_6cs^2\omega_4^2\omega_8\omega_5 + 12\omega_9cs^2\omega_4^3\omega_8\omega_5 - 18\omega_6\omega_7\omega_4^3v_2^2\omega_5 + \\
& 12\omega_6\omega_4^3\omega_8\omega_5 - 5\omega_9cs^2\omega_7\omega_4^3\omega_8\omega_5 - 6\omega_6\omega_7\omega_4^3\omega_8\omega_5 - 18\omega_9\omega_7\omega_4^2\omega_8\omega_5) \frac{\rho cs^2}{12\omega_9\omega_6\omega_7\omega_4^3\omega_8\omega_5}
\end{aligned}$$

$$\begin{aligned}
C_{D_x D_y^3 v_2}^{(1), \text{CuLBM1}} = & (18\omega_3^3v_2^2\omega_2 - 12\omega_3\omega_2 + 36\omega_3^2v_2^2 - \omega_6\omega_3^3cs^2\omega_2 + 18\omega_6\omega_3^2v_2^2 + 6\omega_3^3cs^2\omega_2 + 6\omega_6\omega_3^2cs^2 + 12\omega_3^2cs^2 + 12\omega_3cs^2\omega_2 + \\
& 18\omega_6\omega_3cs^2\omega_2 - 6\omega_3^3cs^2 + \omega_6\omega_3^3cs^2 + 3\omega_6\omega_3^3v_2^2 + 36\omega_3v_2^2\omega_2 - 18\omega_3^3v_2^2 - 18\omega_3^2cs^2\omega_2 - 6\omega_6\omega_3^3 - 5\omega_6\omega_3^2cs^2\omega_2 - 3\omega_6\omega_3^2v_2^2\omega_2 - \omega_6\omega_3^3 - \\
& 54\omega_3^2v_2^2\omega_2 - 6\omega_3^3\omega_2 - 36\omega_6\omega_3v_2^2 - 12\omega_6\omega_3cs^2 - 12\omega_3^3 + 18\omega_3^2\omega_2 + \omega_6\omega_3^2\omega_2 + 6\omega_3^3 - 12\omega_6cs^2\omega_2 + 12\omega_6\omega_3) \frac{\rho cs^2}{12\omega_6\omega_3^2\omega_2}
\end{aligned}$$

$$\begin{aligned}
C_{D_x D_y^3 v_2}^{(1), \text{CuLBM2}} = & (-135cs^2\omega_3\omega_1v_2^2\omega_2^2 - 351cs^2\omega_3\omega_1^3v_2^2\omega_2 - 18cs^2\omega_1\omega_2^3 - 81\omega_3\omega_1^3v_2^2\omega_2^2 - 12cs^2\omega_3\omega_1^2\omega_2^2 + 18cs^4\omega_3\omega_1^3 + 99\omega_3v_2^2\omega_2^3 + \\
& 351cs^2\omega_3\omega_1v_2^2\omega_2^3 - 138\omega_3\omega_1^3v_2^2\omega_2^3 + 45cs^2\omega_3\omega_1^3\omega_2 + 24cs^2\omega_3\omega_1^2\omega_2^3 - 99\omega_3\omega_1^3v_2^2 - 24cs^2\omega_3\omega_1^2\omega_2^2 + 18cs^4\omega_1\omega_2^3 - 6\omega_3\omega_1\omega_2^3 - 90cs^2\omega_1^2v_2^2\omega_2^3 + \\
& 69cs^4\omega_3\omega_1\omega_2^3 - 180\omega_3\omega_1v_2^2\omega_2^3 - 6\omega_3\omega_2^3 + 72cs^2\omega_1^2v_2^2\omega_2^2 + 12\omega_3\omega_1\omega_2^3 - 12cs^2\omega_3\omega_1^2\omega_2 - 24cs^2\omega_3\omega_1^3 + 135\omega_3\omega_1^2v_2^2\omega_2 + 63\omega_3\omega_1v_2^2\omega_2^2 + \\
& 180\omega_3\omega_1^3v_2^2\omega_2 - 30cs^4\omega_3\omega_1\omega_2^2 + 153cs^2\omega_3\omega_1^3v_2^2\omega_2^2 - 12\omega_3\omega_1^3\omega_2 + 24cs^4\omega_1^2\omega_2^2 + 24cs^2\omega_3\omega_2^3 + 54cs^2\omega_1v_2^2\omega_2^3 - 7\omega_3\omega_1^2\omega_2^3 - 171\omega_3v_2^2\omega_2^3 - \\
& 3cs^2\omega_3v_2^2\omega_2^3 - \omega_3v_2^2\omega_1^3\omega_2^2 + 81\omega_3\omega_1^2v_2^2\omega_2^3 + 171\omega_3\omega_1^3v_2^2 + 12cs^4\omega_3\omega_1^2\omega_2^2 - 207cs^2\omega_3v_2^2\omega_2^3 + 18cs^2\omega_3^3\omega_2^2 + 18cs^2\omega_1^3v_2^2\omega_2 + 6cs^4\omega_1^3\omega_2 + \\
& 138\omega_3\omega_1^3v_2^2\omega_2^2 - 30cs^4\omega_1^2\omega_2^3 - 33cs^4\omega_3\omega_1^3\omega_2 - 12cs^2\omega_1^3\omega_2^3 - 25cs^4\omega_3\omega_1^2\omega_2^3 + 63cs^2\omega_3\omega_1^2v_2^2\omega_2 + 6\omega_3\omega_1^3 - \omega_3v_2^2\omega_1^2\omega_2^3 + 17cs^4\omega_3\omega_1^3\omega_2^2 - \\
& 42cs^4\omega_3\omega_2^3 - 24cs^2\omega_1^2\omega_2^2 + 36cs^2\omega_3\omega_1^2v_2^2\omega_2^2 + \omega_3v_2^2\omega_1^3\omega_2^2 - 63\omega_3\omega_1^2v_2^2\omega_2 - 54cs^2\omega_1^3v_2^2\omega_2^2 - 135\omega_3\omega_1v_2^2\omega_2^2 - 18cs^4\omega_1^3\omega_2^2 - 45cs^2\omega_3\omega_1\omega_2^3 + \\
& 6\omega_3\omega_1^2\omega_2 - 324\omega_3\omega_1^3v_2^2\omega_2 - 153cs^2\omega_3\omega_1^2v_2^2\omega_2^2 + 6cs^4\omega_3\omega_1^2\omega_2 + 30cs^2\omega_1^2\omega_2^3 - 6cs^2\omega_1^3\omega_2 - 2cs^4\omega_3\omega_1^3\omega_2^3 + 7\omega_3\omega_1^3\omega_2^2 + 36cs^2\omega_3\omega_1\omega_2^2 + \\
& 12cs^4\omega_1^3\omega_2^2 + 36cs^2\omega_1^3v_2^2\omega_2^2 + 324\omega_3\omega_1v_2^2\omega_2^3 + \omega_3v_2^2\omega_1^4\omega_2^2 + 3cs^2\omega_3v_2^2\omega_1^3\omega_2^2 + 207cs^2\omega_3\omega_1^3v_2^2) \frac{\rho}{24\omega_3\omega_1^3\omega_2^3}
\end{aligned}$$

$$\text{coefficient } C_{D_y^3 \rho}^{(1)} \text{ at } \frac{\partial^4 \rho}{\partial x^2 \partial y^2}:$$

$$C_{D_y^4 \rho}^{(1), \text{SRT}} = (cs^2\omega + 6v_2^4 - 3v_2^4\omega + 2cs^4 - 2cs^2 + 3v_2^2\omega - 12cs^2v_2^2\omega + 24cs^2v_2^2 - cs^4\omega - 6v_2^2) \frac{v_1}{24\omega}$$

$$C_{D_y^4 \rho}^{(1), \text{MRT}^1} = (24\omega_6cs^4\omega_8^2 + 36\omega_4^2v_2^2\omega_8^2 + \omega_6^2cs^2\omega_4^2\omega_8^2 - 48\omega_6\omega_4v_2^4\omega_8 - 36\omega_6\omega_4^2v_2^2\omega_8^2 - 144\omega_6^2cs^2\omega_4v_2^2 + 150\omega_6cs^2\omega_4^2v_2^2\omega_8^2 - 24\omega_6^2\omega_4v_2^4 + 24\omega_6^2cs^2\omega_8 - 12\omega_6^2cs^2\omega_4^2v_2^2\omega_8^2 - 12\omega_6^2\omega_4^2v_2^2 + 24cs^4\omega_4\omega_8^2 - 3\omega_6^2\omega_4^2v_2^4\omega_8^2 - 48\omega_6cs^4\omega_4\omega_8^2 - 24\omega_6^2cs^4\omega_4 - 96\omega_6^2\omega_4v_2^2\omega_8 - 12\omega_6^2cs^4\omega_4^2\omega_8 - 144cs^2\omega_4^2v_2^2\omega_8^2 + 12\omega_6^2cs^4\omega_4^2 + 48\omega_6^2v_2^2\omega_8 - \omega_6^2cs^4\omega_4^2\omega_8^2 - 126\omega_6^2cs^2\omega_4^2v_2^2\omega_8 - 30\omega_6^2\omega_4^2v_2^4\omega_8 + 48\omega_6v_2^4\omega_8^2 - 24cs^2\omega_4\omega_8^2 - 24\omega_6\omega_4^2v_2^2\omega_8 + 72\omega_6cs^2\omega_4^2v_2^2\omega_8 + 72\omega_4v_2^4\omega_8^2 + 48\omega_6cs^2\omega_4\omega_8^2 + 216\omega_6cs^2v_2^2\omega_8^2 + 12\omega_6^2cs^2\omega_4^2\omega_8 - 96\omega_6\omega_4v_2^4\omega_8^2 + 12cs^2\omega_4^2\omega_8^2 + 3\omega_6^2\omega_4^2v_2^2\omega_8^2 + 24\omega_6^2cs^2\omega_4 - 216\omega_6^2cs^2v_2^2\omega_8 - 14\omega_6cs^2\omega_4^2\omega_8^2 + 96\omega_6^2\omega_4v_2^4\omega_8 + 432\omega_6^2cs^2\omega_4v_2^2\omega_8 - 48\omega_6^2cs^2\omega_4\omega_8 - 36\omega_4^2v_2^4\omega_8^2 - 24\omega_6cs^2\omega_8^2 - 144\omega_6cs^2\omega_4v_2^2\omega_8 + 48\omega_6\omega_4v_2^2\omega_8 + 36\omega_6\omega_4^2v_2^4\omega_8 - 24\omega_6^2cs^4\omega_8 - 12cs^4\omega_4^2\omega_8^2 + 24\omega_6\omega_4^2v_2^2\omega_8 + 24\omega_6^2\omega_4v_2^2 - 72\omega_4v_2^2\omega_8^2 + 14\omega_6cs^4\omega_4^2\omega_8^2 + 48\omega_6^2cs^4\omega_4\omega_8 - 432\omega_6cs^2\omega_4v_2^2\omega_8^2 + 96\omega_6\omega_4v_2^2\omega_8^2 - 12\omega_6^2cs^2\omega_4^2 - 48\omega_6^2v_2^4\omega_8 + 288cs^2\omega_4v_2^2\omega_8^2 + 12\omega_6^2\omega_4^2v_2^4 + 72\omega_6^2cs^2\omega_4^2v_2^2 - 48\omega_6v_2^2\omega_8^2 + 30\omega_6^2\omega_4^2v_2^2\omega_8) \frac{v_1}{24\omega_6^2\omega_4^2\omega_8^2}$$

$$C_{D_y^4 \rho}^{(1), \text{MRT}^2} = (72\omega_6\omega_4^2v_2^2\omega_8cs^2 + 12\omega_6^2\omega_4^2cs^4 + 36\omega_4^2v_2^2\omega_8^2 - 24\omega_6\omega_8^2cs^2 - 48\omega_6\omega_4v_2^4\omega_8 - 36\omega_6\omega_4^2v_2^2\omega_8^2 - 24\omega_6^2\omega_4v_2^4 + 24\omega_6^2\omega_8cs^2 - 12\omega_6^2\omega_4^2v_2^2 - 12\omega_4^2\omega_8^2cs^4 - 3\omega_6^2\omega_4^2v_2^4\omega_8^2 + 48\omega_6\omega_4\omega_8^2cs^2 + 14\omega_6\omega_4^2\omega_8^2cs^4 + 24\omega_6^2\omega_4cs^2 - 126\omega_6^2\omega_4^2v_2^2\omega_8cs^2 + 216\omega_6v_2^2\omega_8^2cs^2 - 96\omega_6^2\omega_4v_2^2\omega_8 - 24\omega_4\omega_8^2cs^2 + 48\omega_6^2v_2^2\omega_8cs^2 - 216\omega_6^2v_2^2\omega_8cs^2 - 144\omega_6^2\omega_4v_2^2cs^2 - 12\omega_6^2\omega_4^2\omega_8cs^4 - 48\omega_6^2\omega_4\omega_8cs^2 + 288\omega_4v_2^2\omega_8^2cs^2 - 30\omega_6^2\omega_4^2v_2^4\omega_8 + 48\omega_6v_2^2\omega_8^2 - 24\omega_6\omega_4^2v_2^2\omega_8 + 72\omega_4v_2^4\omega_8^2 + \omega_6^2\omega_4^2\omega_8^2cs^2 - 96\omega_6\omega_4v_2^4\omega_8^2 - 432\omega_6\omega_4v_2^2\omega_8^2cs^2 + 72\omega_6^2\omega_4^2v_2^2cs^2 + 48\omega_6^2\omega_4\omega_8cs^4 + 3\omega_6^2\omega_4^2v_2^2\omega_8^2 + 96\omega_6^2\omega_4v_2^4\omega_8 - 144\omega_6\omega_4v_2^2\omega_8cs^2 + 12\omega_6^2\omega_4^2\omega_8cs^2 - 36\omega_4^2v_2^2\omega_8^2 - \omega_6^2\omega_4^2\omega_8^2cs^4 + 432\omega_6^2\omega_4v_2^2\omega_8cs^2 + 48\omega_6\omega_4v_2^2\omega_8 + 36\omega_6\omega_4^2v_2^4\omega_8^2 + 24\omega_6\omega_4^2v_2^4\omega_8 + 24\omega_6^2\omega_4v_2^2 - 72\omega_4v_2^2\omega_8^2 - 144\omega_6^2v_2^2\omega_8^2cs^2 - 12\omega_6^2\omega_4^2v_2^2\omega_8^2cs^2 - 12\omega_6^2\omega_4^2cs^2 + 24\omega_6\omega_8^2cs^4 + 96\omega_6\omega_4v_2^2\omega_8^2 - 24\omega_6^2\omega_4cs^4 - 14\omega_6\omega_4^2\omega_8^2cs^2 - 48\omega_6^2v_2^4\omega_8 + 150\omega_6\omega_4^2v_2^2\omega_8^2cs^2 + 24\omega_4\omega_8^2cs^4 - 24\omega_6^2\omega_8cs^4 + 12\omega_6^2\omega_4^2v_2^4 + 12\omega_4^2\omega_8^2cs^2 - 48\omega_6v_2^2\omega_8^2 + 30\omega_6^2\omega_4^2v_2^2\omega_8 - 48\omega_6\omega_4\omega_8^2cs^4) \frac{v_1}{24\omega_6^2\omega_4^2\omega_8^2}$$

$$C_{D_y^4 \rho}^{(1), \text{CLBM}^1} = (6v_2^4 + 2cs^4 - 3\omega_6v_2^4 - cs^4\omega_6 - 2cs^2 + 3\omega_6v_2^2 + 24cs^2v_2^2 - 6v_2^2 - 12cs^2\omega_6v_2^2 + cs^2\omega_6) \frac{v_1}{24\omega_6}$$

$$C_{D_y^4 \rho}^{(1), \text{CLBM}^2} = (6v_2^4 + \omega_6cs^2 - 3\omega_6v_2^4 - 2cs^2 - 12\omega_6cs^2v_2^2 + 24cs^2v_2^2 + 3\omega_6v_2^2 + 2cs^4 - 6v_2^2 - \omega_6cs^4) \frac{v_1}{24\omega_6}$$

$$C_{D_y^4 \rho}^{(1), \text{CuLBM}^1} = (2cs^4 + 24v_2^2cs^2 + 6v_2^4 - cs^4\omega_2 - 3v_2^4\omega_2 + 3v_2^2\omega_2 - 6v_2^2 - 2cs^2 - 12v_2^2cs^2\omega_2 + cs^2\omega_2) \frac{v_1}{24\omega_2}$$

$$C_{D_y^4 \rho}^{(1), \text{CuLBM}^2} = (-cs^4\omega_1\omega_2 + 12cs^2v_2^2\omega_2 - cs^2\omega_2 - 3\omega_1v_2^2 + cs^2\omega_1\omega_2 + 3\omega_1v_2^2\omega_2 + 3v_2^4\omega_2 + cs^4\omega_1 + 12cs^2\omega_1v_2^2 + cs^4\omega_2 + 3\omega_1v_2^4 - 3\omega_1v_2^4\omega_2 - 3v_2^2\omega_2 - cs^2\omega_1 - 12cs^2\omega_1v_2^2\omega_2) \frac{v_1}{24\omega_1\omega_2}$$

$$\text{coefficient } C_{D_y^4 v_1}^{(1)} \text{ at } \frac{\partial^4 v_1}{\partial x_2^4} :$$

$$C_{D_y^4 v_1}^{(1), \text{SRT}} = (36cs^2\omega - 72v_2^4 + 108v_2^4\omega + cs^2\omega^3 + 48cs^4 - 42v_2^4\omega^2 - 14cs^2\omega^2 + 3v_2^4\omega^3 - 24cs^2 - 108v_2^2\omega + 216cs^2v_2^2\omega - 144cs^2v_2^2 - 72cs^4\omega + 30cs^4\omega^2 + 72v_2^2 + 6cs^2v_2^2\omega^3 - 3v_2^2\omega^3 - 84cs^2v_2^2\omega^2 - 3cs^4\omega^3 + 42v_2^2\omega^2) \frac{\rho}{24\omega_3}$$

$$C_{D_y^4 v_1}^{(1), \text{MRT}^1} = (24\omega_4^2v_2^2\omega_8^2 + 24cs^4\omega_8^2 - 24cs^2\omega_4\omega_8 + 3\omega_4^3v_2^4\omega_8^2 - 48\omega_4v_2^4\omega_8 + 12cs^2\omega_4^3v_2^2 - 48cs^4\omega_4\omega_8^2 - 24\omega_4^2v_2^4 - 72cs^2\omega_4^2v_2^2\omega_8^2 + 48cs^2\omega_4^2v_2^2\omega_8 + 24cs^4\omega_4\omega_8 - 18\omega_4^3v_2^4\omega_8^2 + 12cs^2\omega_4\omega_8^2 + 24\omega_4v_2^4\omega_8^2 + 12\omega_4^3v_2^4 - 96cs^2v_2^2\omega_8^2 - 72\omega_4^2v_2^2\omega_8 - 24cs^2\omega_4^2v_2^2 - 24cs^2\omega_4v_2^2\omega_8 - 8cs^2\omega_4^2\omega_8^2 + 6cs^2\omega_4^3v_2^2\omega_8^2 - 3cs^4\omega_4^3\omega_8^2 - 24\omega_4^2v_2^4\omega_8^2 - 6cs^2\omega_4^2\omega_8 - 3\omega_4^3v_2^2\omega_8^2 - 24cs^4\omega_4^2\omega_8 + 48\omega_4v_2^2\omega_8 - 12\omega_4^3v_2^2 - 24cs^4\omega_4^2\omega_8^2 + 18\omega_4^3v_2^2\omega_8 - 24\omega_4v_2^2\omega_8^2 + 72\omega_4^2v_2^2\omega_8 + cs^2\omega_4^3\omega_8^2 + 24\omega_4^2v_2^2 + 6cs^4\omega_4^3\omega_8 + 156cs^2\omega_4v_2^2\omega_8^2 + 24cs^2\omega_4^2\omega_8 - 12cs^2\omega_4^3v_2^2\omega_8) \frac{\rho}{24\omega_3^3\omega_8^2}$$

$$C_{D_y^4 v_1}^{(1), \text{MRT}^2} = (24\omega_4^2v_2^2\omega_8^2 + 24\omega_4\omega_8cs^4 - 24\omega_4^2v_2^2cs^2 + 24\omega_4^2\omega_8cs^2 + 3\omega_4^3v_2^4\omega_8^2 - 48\omega_4v_2^4\omega_8 - 96v_2^2\omega_8^2cs^2 + 24\omega_4^2\omega_8^2cs^4 + 48\omega_4^2v_2^2\omega_8cs^2 - 24\omega_4^2v_2^4 + 12\omega_4\omega_8^2cs^2 - 12\omega_4^3v_2^2\omega_8cs^2 + \omega_4^3\omega_8^2cs^2 + 156\omega_4v_2^2\omega_8^2cs^2 - 18\omega_4^3v_2^4\omega_8 + 24\omega_4v_2^4\omega_8^2 + 12\omega_4^3v_2^4 - 72\omega_4^2v_2^2\omega_8 + 6\omega_4^3\omega_8cs^4 - 3\omega_4^3\omega_8^2cs^4 - 24\omega_4^2v_2^4\omega_8^2 + 6\omega_4^3v_2^2\omega_8cs^2 - 6\omega_4^3\omega_8cs^2 - 3\omega_4^3v_2^2\omega_8^2 - 24\omega_4v_2^2\omega_8cs^2 + 12\omega_4^3v_2^2cs^2 + 48\omega_4v_2^2\omega_8 - 12\omega_4^3v_2^2 - 24\omega_4^2\omega_8cs^4 + 18\omega_4^3v_2^2\omega_8 - 24\omega_4v_2^2\omega_8^2 - 72\omega_4^2v_2^2\omega_8^2cs^2 + 24\omega_8^2cs^4 + 72\omega_4^2v_2^4\omega_8 - 24\omega_4\omega_8cs^2 + 24\omega_4^2v_2^2 - 48\omega_4\omega_8^2cs^4 - 8\omega_4^2\omega_8^2cs^2) \frac{\rho}{24\omega_3^3\omega_8^2}$$

$$C_{D_y^4 v_1}^{(1), \text{CLBM}^1} = (12\omega_4^2v_2^2\omega_8^2 + cs^2\omega_4^3\omega_8^2 + 3\omega_4^3v_2^4\omega_8^2 + 24cs^4\omega_4^2\omega_8^2 + 24cs^2\omega_4^2\omega_8 + 6cs^2\omega_4^3v_2^2\omega_8^2 + 72cs^2\omega_4v_2^2\omega_8 - 72\omega_4^2v_2^4 + 6cs^4\omega_4^3\omega_8 - 3cs^4\omega_4^3\omega_8^2 - 8cs^2\omega_4^2\omega_8^2 - 72cs^2\omega_4^3v_2^2\omega_8 - 36cs^2\omega_4v_2^2\omega_8^2 - 30\omega_4^3v_2^4\omega_8 - 24cs^4\omega_4^2\omega_8 + 36\omega_4^3v_2^4 - 72\omega_4^2v_2^2\omega_8 - 6cs^2\omega_4^3\omega_8 + 24cs^4\omega_4\omega_8 - 12cs^2\omega_4^2v_2^2\omega_8^2 - 12\omega_4^2v_2^4\omega_8^2 - 216cs^2\omega_4^2v_2^2 - 3\omega_4^3v_2^2\omega_8^2 + 12cs^2\omega_4\omega_8^2 - 36\omega_4^3v_2^2 + 30\omega_4^3v_2^2\omega_8 - 24cs^2\omega_4\omega_8 + 72\omega_4^2v_2^4\omega_8 + 144cs^2\omega_4^2v_2^2\omega_8 + 72\omega_4^2v_2^2 + 24cs^4\omega_8^2 - 48cs^4\omega_4\omega_8^2 + 108cs^2\omega_4^3v_2^2) \frac{\rho}{24\omega_3^3\omega_8^2}$$

$$C_{D_y^4 v_1}^{(1), \text{CLBM}^2} = (12\omega_4^2v_2^2\omega_8^2 + 24cs^4\omega_4\omega_8 + 3\omega_4^3v_2^4\omega_8^2 + 24cs^4\omega_8^2 - 216cs^2\omega_4^2v_2^2 + 6cs^2\omega_4^3v_2^2\omega_8^2 - 72\omega_4^2v_2^4 + 12cs^2\omega_4\omega_8^2 + 72cs^2\omega_4v_2^2\omega_8 - 72cs^2\omega_4^3v_2^2\omega_8 - 36cs^2\omega_4v_2^2\omega_8^2 - 24cs^2\omega_4\omega_8 - 30\omega_4^3v_2^4\omega_8 + 36\omega_4^3v_2^4 - 72\omega_4^2v_2^2\omega_8 - 48cs^4\omega_4\omega_8^2 + 108cs^2\omega_4^3v_2^2 + cs^2\omega_4^3\omega_8^2 - 12cs^2\omega_4^2v_2^2\omega_8^2 + 24cs^4\omega_4^2\omega_8^2 - 12\omega_4^2v_2^4\omega_8^2 + 24cs^2\omega_4^2\omega_8 - 3\omega_4^3v_2^2\omega_8^2 + 6cs^4\omega_4^3\omega_8 - 36\omega_4^3v_2^2 + 30\omega_4^3v_2^2\omega_8 - 3cs^4\omega_4^3\omega_8^2 + 72\omega_4^2v_2^4\omega_8 - 8cs^2\omega_4^2\omega_8^2 + 72\omega_4^2v_2^2 - 24cs^4\omega_4^2\omega_8 + 144cs^2\omega_4^2v_2^2\omega_8 - 6cs^2\omega_4^3\omega_8) \frac{\rho}{24\omega_3^3\omega_8^2}$$

$$C_{D_y^4 v_1}^{(1), \text{CuLBM}^1} =$$

$$(24\omega_6\omega_3cs^4 + 12\omega_6^2\omega_3cs^2 + 72\omega_3^2v_2^2 - 36\omega_6^2\omega_3v_2^2cs^2 - 72\omega_6\omega_3^2v_2^2 + 24\omega_6^2\omega_3^2cs^4 + 24\omega_6\omega_3^2cs^2 - 12\omega_6^2\omega_3^2v_2^4 + 6\omega_6^2\omega_3^2v_2^2cs^2 - 6\omega_6\omega_3^3cs^2 - 216\omega_3^2v_2^2cs^2 + 3\omega_6^2\omega_3^3v_2^4 + 30\omega_6\omega_3^3v_2^2 - 3\omega_6^2\omega_3^3cs^4 - 36\omega_3^3v_2^2 + 144\omega_6\omega_3^2v_2^2cs^2 - 30\omega_6\omega_3^3v_2^4 + \omega_6^2\omega_3^3cs^2 + 36\omega_3^3v_2^4 + 72\omega_6\omega_3v_2^2cs^2 + 24\omega_6^2cs^4 + 6\omega_6\omega_3^3cs^4 - 3\omega_6^2\omega_3^3v_2^2 - 48\omega_6\omega_3cs^4 - 72\omega_6\omega_3^2v_2^2cs^2 - 24\omega_6\omega_3cs^2 + 108\omega_3^3v_2^2cs^2 - 24\omega_6\omega_3^2cs^4 - 12\omega_6^2\omega_3^2v_2^2cs^2 + 12\omega_6^2\omega_3^2v_2^2 - 72\omega_3^2v_2^4 + 72\omega_6\omega_3v_2^4 - 8\omega_6^2\omega_3^2cs^2) \frac{\rho}{24\omega_6^2\omega_3^3}$$

$$C_{D_y^4 v_1}^{(1), \text{CuLBM2}} = (-12\omega_3^2\omega_1^2v_2^4 - 24cs^4\omega_3\omega_1^2 - 72\omega_3\omega_1^2v_2^2 + 6cs^4\omega_3\omega_1^3 + 36\omega_1^3v_2^4 + 108cs^2\omega_3^3v_2^2 + 6cs^2\omega_3^2\omega_1^3v_2^2 - 24cs^2\omega_3\omega_1 + 72cs^2\omega_3\omega_1v_2^2 + 30\omega_3\omega_1^3v_2^2 + 24cs^4\omega_3^2 + 24cs^2\omega_3\omega_1^2 + 3\omega_3^2\omega_1^3v_2^4 - 6cs^2\omega_3\omega_1^3 - 12cs^2\omega_3^2\omega_1^2v_2^2 - 216cs^2\omega_1^2v_2^2 + 24cs^4\omega_3\omega_1 - 72\omega_1^2v_2^4 + cs^2\omega_3^2\omega_1^3 - 3\omega_3^2\omega_1^3v_2^2 - 36cs^2\omega_3^2\omega_1v_2^2 - 30\omega_3\omega_1^3v_2^4 - 48cs^4\omega_3^2\omega_1 + 144cs^2\omega_3\omega_1^2v_2^2 - 8cs^2\omega_3^2\omega_1^2 + 72\omega_1^2v_2^2 - 3cs^4\omega_3^2\omega_1^3 + 72\omega_3\omega_1^2v_2^4 + 12\omega_3^2\omega_1^2v_2^2 + 12cs^2\omega_3^2\omega_1 + 24cs^4\omega_3^2\omega_1^2 - 36\omega_1^3v_2^2 - 72cs^2\omega_3\omega_1^3v_2^2) \frac{\rho}{24\omega_3^2\omega_1^3}$$

coefficient $C_{D_y^4 v_2}^{(1)}$ **at** $\frac{\partial^4 v_2}{\partial x_2^4}$:

$$C_{D_y^4 v_2}^{(1), \text{SRT}} = (-4 - 3cs^2\omega + 6cs^2 - 5v_2^2\omega + 10v_2^2 + 2\omega) \frac{v_1\rho v_2}{12\omega}$$

$$C_{D_y^4 v_2}^{(1), \text{MRT1}} = (-24\omega_6cs^2\omega_4\omega_8 - 60\omega_4^2v_2^2\omega_8^2 - 12\omega_6^2\omega_4^2 - 3\omega_6^2cs^2\omega_4^2\omega_8^2 + 61\omega_6\omega_4^2v_2^2\omega_8^2 - 25\omega_6\omega_4^2\omega_8^2 - 60\omega_6^2cs^2\omega_8 + 24\omega_4^2\omega_8^2 + 24\omega_6^2\omega_4^2v_2^2 + 168\omega_6^2\omega_4v_2^2\omega_8 - 84\omega_6^2v_2^2\omega_8 - 72\omega_6^2\omega_4\omega_8 - 12\omega_6\omega_4^2\omega_8 + 72cs^2\omega_4\omega_8^2 + 36\omega_6\omega_4^2v_2^2\omega_8 - 36\omega_6\omega_8^2 - 120\omega_6cs^2\omega_4\omega_8^2 - 33\omega_6^2cs^2\omega_4^2\omega_8 + 24\omega_6^2\omega_4 - 36cs^2\omega_4^2\omega_8^2 + 24\omega_6\omega_4\omega_8 - 5\omega_6^2\omega_4^2v_2^2\omega_8^2 - 48\omega_6^2cs^2\omega_4 + 39\omega_6cs^2\omega_4^2\omega_8^2 + 120\omega_6^2cs^2\omega_4\omega_8 + 60\omega_6cs^2\omega_8^2 + 21\omega_6^2\omega_4^2\omega_8 - 72\omega_6\omega_4v_2^2\omega_8 - 48\omega_6^2\omega_4v_2^2 + 120\omega_4v_2^2\omega_8^2 - 168\omega_6\omega_4v_2^2\omega_8^2 + 2\omega_6^2\omega_4^2\omega_8^2 + 12\omega_6cs^2\omega_4^2\omega_8 + 24\omega_6^2cs^2\omega_4^2 + 36\omega_6^2\omega_8 + 72\omega_6\omega_4\omega_8^2 + 84\omega_6v_2^2\omega_8^2 - 51\omega_6^2\omega_4^2v_2^2\omega_8 - 48\omega_4\omega_8^2) \frac{v_1\rho v_2}{12\omega_6^2\omega_4^2\omega_8^2}$$

$$C_{D_y^4 v_2}^{(1), \text{MRT2}} = (-60\omega_4^2v_2^2\omega_8^2 + 12\omega_6\omega_4^2\omega_8cs^2 - 12\omega_6^2\omega_4^2 + 60\omega_6\omega_8^2cs^2 + 61\omega_6\omega_4^2v_2^2\omega_8^2 - 25\omega_6\omega_4^2\omega_8^2 + 24\omega_4^2\omega_8^2 - 60\omega_6^2\omega_8cs^2 + 24\omega_6^2\omega_4^2v_2^2 - 120\omega_6\omega_4\omega_8^2cs^2 - 48\omega_6^2\omega_4cs^2 + 168\omega_6^2\omega_4v_2^2\omega_8 + 72\omega_4\omega_8^2cs^2 - 84\omega_6^2v_2^2\omega_8 - 72\omega_6^2\omega_4\omega_8 + 120\omega_6^2\omega_4\omega_8cs^2 - 12\omega_6\omega_4^2\omega_8 + 36\omega_6\omega_4^2v_2^2\omega_8 - 36\omega_6\omega_8^2 - 3\omega_6^2\omega_4^2cs^2 + 24\omega_6^2\omega_4 + 24\omega_6\omega_4\omega_8 - 5\omega_6^2\omega_4^2v_2^2\omega_8^2 - 33\omega_6^2\omega_4^2\omega_8cs^2 + 21\omega_6^2\omega_4^2\omega_8 - 72\omega_6\omega_4v_2^2\omega_8 - 48\omega_6^2\omega_4v_2^2 + 120\omega_4v_2^2\omega_8^2 - 24\omega_6\omega_4\omega_8cs^2 + 24\omega_6^2\omega_4^2cs^2 - 168\omega_6\omega_4v_2^2\omega_8^2 + 2\omega_6^2\omega_4^2\omega_8^2 + 39\omega_6\omega_4^2\omega_8^2cs^2 + 36\omega_6^2\omega_8 + 72\omega_6\omega_4\omega_8^2 - 36\omega_4^2\omega_8^2cs^2 + 84\omega_6v_2^2\omega_8^2 - 51\omega_6^2\omega_4^2v_2^2\omega_8 - 48\omega_4\omega_8^2) \frac{v_1\rho v_2}{12\omega_6^2\omega_4^2\omega_8^2}$$

$$C_{D_y^4 v_2}^{(1), \text{CLBM1}} = (-4 + 2\omega_6 + 6cs^2 - 5\omega_6v_2^2 + 10v_2^2 - 3cs^2\omega_6) \frac{v_1\rho v_2}{12\omega_6}$$

$$C_{D_y^4 v_2}^{(1), \text{CLBM2}} = (-4 - 3\omega_6cs^2 + 2\omega_6 + 6cs^2 - 5\omega_6v_2^2 + 10v_2^2) \frac{v_1\rho v_2}{12\omega_6}$$

$$C_{D_y^4 v_2}^{(1), \text{CuLBM1}} = (-4 - 5v_2^2\omega_2 + 10v_2^2 + 6cs^2 + 2\omega_2 - 3cs^2\omega_2) \frac{v_1\rho v_2}{12\omega_2}$$

$$C_{D_y^4 v_2}^{(1), \text{CuLBM2}} = (3cs^2\omega_2 + 5\omega_1v_2^2 - 3cs^2\omega_1\omega_2 - 5\omega_1v_2^2\omega_2 + 2\omega_1\omega_2 - 2\omega_1 + 5v_2^2\omega_2 + 3cs^2\omega_1 - 2\omega_2) \frac{v_1\rho v_2}{12\omega_1\omega_2}$$

3.3 Conservation of momentum: ρv_2

$$\begin{aligned} & v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + v_1 v_2 \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \rho v_2 \frac{\delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + v_1 \rho \frac{\delta_l}{\delta_t} \frac{\partial v_2}{\partial x_1} + C_{D_y \rho, D_x v_2}^{(2)} \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + 2\rho v_2 \frac{\delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + C_{D_x \rho, D_x v_2}^{(2)} \frac{\delta_l^2}{\delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\ & C_{D_x \rho, D_y v_1}^{(2)} \frac{\delta_l^2}{\delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + C_{D_x v_1, D_y v_1}^{(2)} \frac{\delta_l^2}{\delta_t} \frac{\partial v_1}{\partial x_1} \frac{\partial v_1}{\partial x_1} + C_{D_y \rho, D_x v_1}^{(2)} \frac{\delta_l^2}{\delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_1} + C_{D_y \rho, D_y v_2}^{(2)} \frac{\delta_l^2}{\delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + \\ & C_{D_y v_2, D_y v_2}^{(2)} \frac{\delta_l^2}{\delta_t} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + C_{D_x^2 v_2}^{(2)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 v_2}{\partial x_1^2} + C_{D_x D_y \rho}^{(2)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 \rho}{\partial x_1 \partial x_2} + C_{D_x D_y v_1}^{(2)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + C_{D_x^2 \rho}^{(2)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + C_{D_y^2 v_2}^{(2)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + \\ & C_{D_x^3 \rho}^{(2)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + C_{D_x^3 v_1}^{(2)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + C_{D_x^3 v_2}^{(2)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_1^3} + C_{D_x^2 D_y \rho}^{(2)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + C_{D_x^2 D_y v_1}^{(2)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} + C_{D_x^2 D_y v_2}^{(2)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + \\ & C_{D_x D_y^2 \rho}^{(2)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + C_{D_x D_y^2 v_1}^{(2)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_{D_x D_y^2 v_2}^{(2)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_{D_y^3 \rho}^{(2)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + C_{D_y^3 v_2}^{(2)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_{D_x^4 \rho}^{(2)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\ & C_{D_x^4 v_1}^{(2)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_{D_x^4 v_2}^{(2)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1^4} + C_{D_x^3 D_y \rho}^{(2)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{D_x^3 D_y v_1}^{(2)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + C_{D_x^3 D_y v_2}^{(2)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\ & C_{D_x^2 D_y^2 \rho}^{(2)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_{D_x^2 D_y^2 v_1}^{(2)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{D_x^2 D_y^2 v_2}^{(2)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_{D_x D_y^3 \rho}^{(2)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{D_x D_y^3 v_1}^{(2)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\ & C_{D_x D_y^3 v_2}^{(2)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + C_{D_y^4 \rho}^{(2)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{D_y^4 v_2}^{(2)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:

coefficient $C_{D_y \rho}^{(2)}$ **at** $\frac{\partial \rho}{\partial x_2}$:

$$C_{D_y \rho}^{(2), \text{SRT}} = (cs^2 + v_2^2)$$

$$C_{D_y \rho}^{(2), \text{MRT1}} = (v_2^2 + cs^2)$$

$$C_{D_y \rho}^{(2), \text{MRT2}} = (cs^2 + v_2^2)$$

$$C_{D_y \rho}^{(2), \text{CLBM1}} = (cs^2 + v_2^2)$$

$$C_{D_y \rho}^{(2), \text{CLBM2}} = (cs^2 + v_2^2)$$

$$C_{D_y \rho}^{(2), \text{CuLBM1}} = (cs^2 + v_2^2)$$

$$C_{D_y \rho}^{(2), \text{CuLBM2}} = (v_2^2 + cs^2)$$

coefficient $C_{D_x \rho, D_x v_2}^{(2)}$ **at** $\frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1}$:

$$C_{D_x \rho, D_x v_2}^{(2), \text{SRT}} = (-2 + \omega) \frac{cs^2}{2\omega}$$

$$C_{D_x \rho, D_x v_2}^{(2), \text{MRT1}} = (-2 + \omega_4) \frac{cs^2}{2\omega_4}$$

$$C_{D_x \rho, D_x v_2}^{(2), \text{MRT2}} = (-2 + \omega_4) \frac{cs^2}{2\omega_4}$$

$$C_{D_x \rho, D_x v_2}^{(2), \text{CLBM1}} = (-2 + \omega_4) \frac{cs^2}{2\omega_4}$$

$$C_{D_x \rho, D_x v_2}^{(2), \text{CLBM2}} = (-2 + \omega_4) \frac{cs^2}{2\omega_4}$$

$$C_{D_x \rho, D_x v_2}^{(2), \text{CuLBM1}} = (-2 + \omega_3) \frac{cs^2}{2\omega_3}$$

$$C_{D_x \rho, D_x v_2}^{(2), \text{CuLBM2}} = (-2 + \omega_1) \frac{cs^2}{2\omega_1}$$

coefficient $C_{D_x \rho, D_y v_1}^{(2)}$ **at** $\frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2}$:

$$C_{D_x \rho, D_y v_1}^{(2), \text{SRT}} = (-2 + \omega) \frac{cs^2}{2\omega}$$

$$C_{D_x \rho, D_y v_1}^{(2), \text{MRT1}} = (-2 + \omega_4) \frac{cs^2}{2\omega_4}$$

$$C_{D_x \rho, D_y v_1}^{(2), \text{MRT2}} = (-2 + \omega_4) \frac{cs^2}{2\omega_4}$$

$$C_{D_x \rho, D_y v_1}^{(2), \text{CLBM1}} = (-2 + \omega_4) \frac{cs^2}{2\omega_4}$$

$$C_{D_x \rho, D_y v_1}^{(2), \text{CLBM2}} = (-2 + \omega_4) \frac{cs^2}{2\omega_4}$$

$$C_{D_x \rho, D_y v_1}^{(2), \text{CuLBM1}} = (-2 + \omega_3) \frac{cs^2}{2\omega_3}$$

$$C_{D_x \rho, D_y v_1}^{(2), \text{CuLBM2}} = (-3v_1^2\omega_2 + 3\omega_1cs^2 + \omega_1cs^2\omega_2 - \omega_1 + 3v_1^2\omega_1 + \omega_2 - 5cs^2\omega_2) \frac{1}{2\omega_1\omega_2}$$

coefficient $C_{D_x v_1, D_y v_1}^{(2)}$ **at** $\frac{\partial v_1}{\partial x_1} \frac{\partial v_1}{\partial x_2}$:

$$C_{D_x v_1, D_y v_1}^{(2), \text{SRT}} = 0$$

$$C_{D_x v_1, D_y v_1}^{(2), \text{MRT1}} = 0$$

$$C_{D_x v_1, D_y v_1}^{(2), \text{MRT2}} = 0$$

$$C_{D_x v_1, D_y v_1}^{(2), \text{CLBM1}} = 0$$

$$C_{D_x v_1, D_y v_1}^{(2), \text{CLBM2}} = 0$$

$$C_{D_x v_1, D_y v_1}^{(2), \text{CuLBM1}} = 0$$

$$C_{D_x v_1, D_y v_1}^{(2), \text{CuLBM2}} = (\omega_1 - \omega_2) \frac{3v_1 \rho}{\omega_1 \omega_2}$$

coefficient $C_{D_y \rho, D_x v_1}^{(2)}$ **at** $\frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_1}$:

$$C_{D_y \rho, D_x v_1}^{(2), \text{SRT}} = 0$$

$$C_{D_y \rho, D_x v_1}^{(2), \text{MRT1}} = 0$$

$$C_{D_y \rho, D_x v_1}^{(2), \text{MRT2}} = 0$$

$$C_{D_y \rho, D_x v_1}^{(2), \text{CLBM1}} = 0$$

$$C_{D_y \rho, D_x v_1}^{(2), \text{CLBM2}} = 0$$

$$C_{D_y \rho, D_x v_1}^{(2), \text{CuLBM1}} = 0$$

$$C_{D_y \rho, D_x v_1}^{(2), \text{CuLBM2}} = (-3v_1^2 \omega_2 + \omega_1 c s^2 - \omega_1 + 3v_1^2 \omega_1 + \omega_2 - c s^2 \omega_2) \frac{1}{2\omega_1 \omega_2}$$

coefficient $C_{D_y \rho, D_y v_2}^{(2)}$ **at** $\frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2}$:

$$C_{D_y \rho, D_y v_2}^{(2), \text{SRT}} = (-2 + 4cs^2 - 3v_2^2 \omega + 6v_2^2 + \omega - 2\omega cs^2) \frac{1}{\omega}$$

$$C_{D_y \rho, D_y v_2}^{(2), \text{MRT1}} = (-2 + \omega_6 - 3\omega_6 v_2^2 - 2\omega_6 cs^2 + 6v_2^2 + 4cs^2) \frac{1}{\omega_6}$$

$$C_{D_y \rho, D_y v_2}^{(2), \text{MRT2}} = (-2 - 2\omega_6 cs^2 + \omega_6 + 4cs^2 - 3\omega_6 v_2^2 + 6v_2^2) \frac{1}{\omega_6}$$

$$C_{D_y \rho, D_y v_2}^{(2), \text{CLBM1}} = (-2 + \omega_6 + 4cs^2 - 2\omega_6 cs^2 - 3\omega_6 v_2^2 + 6v_2^2) \frac{1}{\omega_6}$$

$$C_{D_y \rho, D_y v_2}^{(2), \text{CLBM2}} = (-2 + \omega_6 + 4cs^2 - 3\omega_6 v_2^2 + 6v_2^2 - 2cs^2 \omega_6) \frac{1}{\omega_6}$$

$$C_{D_y \rho, D_y v_2}^{(2), \text{CuLBM1}} = (-2 + 4cs^2 - 2cs^2 \omega_2 - 3v_2^2 \omega_2 + 6v_2^2 + \omega_2) \frac{1}{\omega_2}$$

$$C_{D_y \rho, D_y v_2}^{(2), \text{CuLBM2}} = (3\omega_1 v_2^2 + 2\omega_1 cs^2 - 2\omega_1 cs^2 \omega_2 - 3\omega_1 v_2^2 \omega_2 + \omega_1 \omega_2 - \omega_1 + 3v_2^2 \omega_2 - \omega_2 + 2cs^2 \omega_2) \frac{1}{\omega_1 \omega_2}$$

coefficient $C_{D_y v_2, D_y v_2}^{(2)}$ **at** $\left(\frac{\partial v_2}{\partial x_2} \right)^2$:

$$C_{D_y v_2, D_y v_2}^{(2), \text{SRT}} = (2 - \omega) \frac{3\rho v_2}{\omega}$$

$$C_{D_y v_2, D_y v_2}^{(2), \text{MRT1}} = (2 - \omega_6) \frac{3\rho v_2}{\omega_6}$$

$$C_{D_y v_2, D_y v_2}^{(2), \text{MRT2}} = C_{D_y v_2, D_y v_2}^{(2), \text{MRT1}}$$

$$C_{D_y v_2, D_y v_2}^{(2), \text{CLBM1}} = C_{D_y v_2, D_y v_2}^{(2), \text{MRT1}}$$

$$C_{D_y v_2, D_y v_2}^{(2), \text{CLBM2}} = C_{D_y v_2, D_y v_2}^{(2), \text{MRT1}}$$

$$C_{D_y v_2, D_y v_2}^{(2), \text{CuLBM1}} = (2 - \omega_2) \frac{3\rho v_2}{\omega_2}$$

$$C_{D_y v_2, D_y v_2}^{(2), \text{CuLBM2}} = (-\omega_1 \omega_2 + \omega_1 + \omega_2) \frac{3\rho v_2}{\omega_1 \omega_2}$$

coefficient $C_{D_x^2 v_2}^{(2)}$ **at** $\frac{\partial^2 v_2}{\partial x_1^2}$:

$$C_{D_x^2 v_2}^{(2), \text{SRT}} = (-2 + \omega) \frac{\rho cs^2}{2\omega}$$

$$C_{D_x^2 v_2}^{(2), \text{MRT1}} = (-2 + \omega_4) \frac{cs^2 \rho}{2\omega_4}$$

$$C_{D_x^2 v_2}^{(2), \text{MRT2}} = (-2 + \omega_4) \frac{\rho cs^2}{2\omega_4}$$

$$C_{D_x^2 v_2}^{(2), \text{CLBM1}} = (-2 + \omega_4) \frac{\rho cs^2}{2\omega_4}$$

$$C_{D_x^2 v_2}^{(2), \text{CLBM2}} = (-2 + \omega_4) \frac{cs^2 \rho}{2\omega_4}$$

$$C_{D_x^2 v_2}^{(2), \text{CuLBM1}} = (-2 + \omega_3) \frac{\rho cs^2}{2\omega_3}$$

$$C_{D_x^2 v_2}^{(2), \text{CuLBM2}} = (-2 + \omega_1) \frac{\rho cs^2}{2\omega_1}$$

coefficient $C_{D_x D_y \rho}^{(2)}$ **at** $\frac{\partial^2 \rho}{\partial x_1 \partial x_2}$:

$$C_{D_x D_y \rho}^{(2), \text{SRT}} = 0$$

$$C_{D_x D_y \rho}^{(2), \text{MRT1}} = 0$$

$$C_{D_x D_y \rho}^{(2), \text{MRT2}} = 0$$

$$C_{D_x D_y \rho}^{(2), \text{CLBM1}} = 0$$

$$C_{D_x D_y \rho}^{(2), \text{CLBM2}} = 0$$

$$C_{D_x D_y \rho}^{(2), \text{CuLBM1}} = 0$$

$$C_{D_x D_y \rho}^{(2), \text{CuLBM2}} = (-v_1^2 \omega_2 + 3\omega_1 cs^2 - \omega_1 + v_1^2 \omega_1 + \omega_2 - 3cs^2 \omega_2) \frac{v_1}{2\omega_1 \omega_2}$$

coefficient $C_{D_x D_y v_1}^{(2)}$ **at** $\frac{\partial^2 v_1}{\partial x_1 \partial x_2}$:

$$C_{D_x D_y v_1}^{(2), \text{SRT}} = (-2 + \omega) \frac{\rho cs^2}{2\omega}$$

$$C_{D_x D_y v_1}^{(2), \text{MRT1}} = (-2 + \omega_4) \frac{cs^2 \rho}{2\omega_4}$$

$$C_{D_x D_y v_1}^{(2), \text{MRT2}} = (-2 + \omega_4) \frac{\rho cs^2}{2\omega_4}$$

$$C_{D_x D_y v_1}^{(2), \text{CLBM1}} = (-2 + \omega_4) \frac{\rho cs^2}{2\omega_4}$$

$$C_{D_x D_y v_1}^{(2), \text{CLBM2}} = (-2 + \omega_4) \frac{cs^2 \rho}{2\omega_4}$$

$$C_{D_x D_y v_1}^{(2), \text{CuLBM1}} = (-2 + \omega_3) \frac{\rho cs^2}{2\omega_3}$$

$$C_{D_x D_y v_1}^{(2), \text{CuLBM2}} = (-3v_1^2 \omega_2 + \omega_1 cs^2 + \omega_1 cs^2 \omega_2 - \omega_1 + 3v_1^2 \omega_1 + \omega_2 - 3cs^2 \omega_2) \frac{\rho}{2\omega_1 \omega_2}$$

coefficient $C_{D_y^2 \rho}^{(2)}$ **at** $\frac{\partial^2 \rho}{\partial x_2^2}$:

$$C_{D_y^2 \rho}^{(2), \text{SRT}} = (-2 + 6cs^2 - v_2^2 \omega + 2v_2^2 + \omega - 3\omega cs^2) \frac{v_2}{2\omega}$$

$$C_{D_y^2 \rho}^{(2), \text{MRT1}} = (-2 + \omega_6 - \omega_6 v_2^2 - 3\omega_6 cs^2 + 2v_2^2 + 6cs^2) \frac{v_2}{2\omega_6}$$

$$C_{D_y^2 \rho}^{(2), \text{MRT2}} = (-2 - 3\omega_6 cs^2 + \omega_6 + 6cs^2 - \omega_6 v_2^2 + 2v_2^2) \frac{v_2}{2\omega_6}$$

$$C_{D_y^2 \rho}^{(2), \text{CLBM1}} = (-2 + \omega_6 + 6cs^2 - 3\omega_6 cs^2 - \omega_6 v_2^2 + 2v_2^2) \frac{v_2}{2\omega_6}$$

$$C_{D_y^2 \rho}^{(2), \text{CLBM2}} = (-2 + \omega_6 + 6cs^2 - \omega_6 v_2^2 + 2v_2^2 - 3cs^2 \omega_6) \frac{v_2}{2\omega_6}$$

$$C_{D_y^2 \rho}^{(2), \text{CuLBM1}} = (-2 + 6cs^2 - 3cs^2 \omega_2 - v_2^2 \omega_2 + 2v_2^2 + \omega_2) \frac{v_2}{2\omega_2}$$

$$C_{D_y^2 \rho}^{(2), \text{CuLBM2}} = (\omega_1 v_2^2 + 3\omega_1 cs^2 - 3\omega_1 cs^2 \omega_2 - \omega_1 v_2^2 \omega_2 + \omega_1 \omega_2 - \omega_1 + v_2^2 \omega_2 - \omega_2 + 3cs^2 \omega_2) \frac{v_2}{2\omega_1 \omega_2}$$

coefficient $C_{D_y^2 v_2}^{(2)}$ **at** $\frac{\partial^2 v_2}{\partial x_2^2}$:

$$C_{D_y^2 v_2}^{(2), \text{SRT}} = (-2 + 2cs^2 - 3v_2^2 \omega + 6v_2^2 + \omega - \omega cs^2) \frac{\rho}{2\omega}$$

$$C_{D_y^2 v_2}^{(2), \text{MRT1}} = (-2 + \omega_6 - 3\omega_6 v_2^2 - \omega_6 cs^2 + 6v_2^2 + 2cs^2) \frac{\rho}{2\omega_6}$$

$$C_{D_y^2 v_2}^{(2), \text{MRT2}} = (-2 - \omega_6 cs^2 + \omega_6 + 2cs^2 - 3\omega_6 v_2^2 + 6v_2^2) \frac{\rho}{2\omega_6}$$

$$C_{D_y^2 v_2}^{(2), \text{CLBM1}} = (-2 + \omega_6 + 2cs^2 - \omega_6 cs^2 - 3\omega_6 v_2^2 + 6v_2^2) \frac{\rho}{2\omega_6}$$

$$C_{D_y^2 v_2}^{(2), \text{CLBM2}} = (-2 + \omega_6 + 2cs^2 - 3\omega_6 v_2^2 + 6v_2^2 - cs^2 \omega_6) \frac{\rho}{2\omega_6}$$

$$C_{D_y^2 v_2}^{(2), \text{CuLBM1}} = (-2 + 2cs^2 - cs^2 \omega_2 - 3v_2^2 \omega_2 + 6v_2^2 + \omega_2) \frac{\rho}{2\omega_2}$$

$$C_{D_y^2 v_2}^{(2), \text{CuLBM2}} = (3\omega_1 v_2^2 + \omega_1 cs^2 - \omega_1 cs^2 \omega_2 - 3\omega_1 v_2^2 \omega_2 + \omega_1 \omega_2 - \omega_1 + 3v_2^2 \omega_2 - \omega_2 + cs^2 \omega_2) \frac{\rho}{2\omega_1 \omega_2}$$

coefficient $C_{D_x^3 \rho}^{(2)}$ **at** $\frac{\partial^3 \rho}{\partial x_1^3}$:

$$C_{D_x^3 \rho}^{(2), \text{SRT}} = (-1 + 3cs^2 + v_1^2) \frac{v_1 v_2}{12}$$

$$C_{D_x^3 \rho}^{(2), \text{MRT1}} = (6v_1^2 \omega_4 \omega_5 + 12v_1^2 \omega_7 - 18cs^2 \omega_7 \omega_4 - \omega_7 \omega_4 \omega_5 + 18cs^2 \omega_4 \omega_5 - 12v_1^2 \omega_5 + v_1^2 \omega_7 \omega_4 \omega_5 - 12\omega_7 - 6v_1^2 \omega_7 \omega_4 + 36cs^2 \omega_7 - 6\omega_4 \omega_5 + 3cs^2 \omega_7 \omega_4 \omega_5 - 36cs^2 \omega_5 + 6\omega_7 \omega_4 + 12\omega_5) \frac{v_1 v_2}{12\omega_7 \omega_4 \omega_5}$$

$$C_{D_x^3 \rho}^{(2), \text{MRT2}} = (6v_1^2 \omega_4 \omega_5 + 12v_1^2 \omega_7 - \omega_7 \omega_4 \omega_5 + 36\omega_7 cs^2 - 12v_1^2 \omega_5 + v_1^2 \omega_7 \omega_4 \omega_5 - 12\omega_7 - 6v_1^2 \omega_7 \omega_4 - 36cs^2 \omega_5 - 6\omega_4 \omega_5 - 18\omega_7 cs^2 \omega_4 + 6\omega_7 \omega_4 + 12\omega_5 + 18cs^2 \omega_4 \omega_5 + 3\omega_7 cs^2 \omega_4 \omega_5) \frac{v_1 v_2}{12\omega_7 \omega_4 \omega_5}$$

$$C_{D_x^3 \rho}^{(2), \text{CLBM1}} = (-1 + v_1^2 + 3cs^2) \frac{v_1 v_2}{12}$$

$$C_{D_x^3 \rho}^{(2), \text{CLBM2}} = (-1 + v_1^2 + 3cs^2) \frac{v_1 v_2}{12}$$

$$C_{D_x^3 \rho}^{(2), \text{CuLBM1}} = (-1 + v_1^2 + 3cs^2) \frac{v_1 v_2}{12}$$

$$C_{D_x^3 \rho}^{(2), \text{CuLBM2}} = (-1 + v_1^2 + 3cs^2) \frac{v_1 v_2}{12}$$

coefficient $C_{D_x^3 v_1}^{(2)}$ **at** $\frac{\partial^3 v_1}{\partial x_1^3}$:

$$C_{D_x^3 v_1}^{(2), \text{SRT}} = (-1 + cs^2 + 3v_1^2) \frac{\rho v_2}{12}$$

$$C_{D_x^3 v_1}^{(2), \text{MRT1}} = (18v_1^2 \omega_4 \omega_5 + 36v_1^2 \omega_7 - 6cs^2 \omega_7 \omega_4 - \omega_7 \omega_4 \omega_5 + 6cs^2 \omega_4 \omega_5 - 36v_1^2 \omega_5 + 3v_1^2 \omega_7 \omega_4 \omega_5 - 12\omega_7 - 18v_1^2 \omega_7 \omega_4 + 12cs^2 \omega_7 - 6\omega_4 \omega_5 + cs^2 \omega_7 \omega_4 \omega_5 - 12cs^2 \omega_5 + 6\omega_7 \omega_4 + 12\omega_5) \frac{\rho v_2}{12\omega_7 \omega_4 \omega_5}$$

$$C_{D_x^3 v_1}^{(2), \text{MRT2}} = (18v_1^2 \omega_4 \omega_5 + 36v_1^2 \omega_7 - \omega_7 \omega_4 \omega_5 + 12\omega_7 cs^2 - 36v_1^2 \omega_5 + 3v_1^2 \omega_7 \omega_4 \omega_5 - 12\omega_7 - 18v_1^2 \omega_7 \omega_4 - 12cs^2 \omega_5 - 6\omega_4 \omega_5 - 6\omega_7 cs^2 \omega_4 + 6\omega_7 \omega_4 + 12\omega_5 + 6cs^2 \omega_4 \omega_5 + \omega_7 cs^2 \omega_4 \omega_5) \frac{\rho v_2}{12\omega_7 \omega_4 \omega_5}$$

$$C_{D_x^3 v_1}^{(2), \text{CLBM1}} = (-1 + 3v_1^2 + cs^2) \frac{\rho v_2}{12}$$

$$C_{D_x^3 v_1}^{(2), \text{CLBM2}} = (-1 + 3v_1^2 + cs^2) \frac{\rho v_2}{12}$$

$$C_{D_x^3 v_1}^{(2), \text{CuLBM1}} = (-1 + 3v_1^2 + cs^2) \frac{\rho v_2}{12}$$

$$C_{D_x^3 v_1}^{(2), \text{CuLBM2}} = (-1 + 3v_1^2 + cs^2) \frac{\rho v_2}{12}$$

coefficient $C_{D_x^3 v_2}^{(2)}$ **at** $\frac{\partial^3 v_2}{\partial x_1^3}$:

$$C_{D_x^3 v_2}^{(2), \text{SRT}} = (6 - 18cs^2 - 6v_1^2 - 3\omega^2 cs^2 + \omega^2 - v_1^2 \omega^2 + 6v_1^2 \omega - 6\omega + 18\omega cs^2) \frac{v_1 \rho}{6\omega^2}$$

$$C_{D_x^3 v_2}^{(2), \text{MRT1}} = (-v_1^2 \omega_7 \omega_4^2 + 3cs^2 \omega_4^2 + 15cs^2 \omega_7 \omega_4 - 3cs^2 \omega_7 \omega_4^2 - 6cs^2 \omega_4 + 3v_1^2 \omega_7 \omega_4 + 6\omega_4 - 12cs^2 \omega_7 + \omega_7 \omega_4^2 + 3v_1^2 \omega_4^2 - 3\omega_4^2 - 6v_1^2 \omega_4 - 3\omega_7 \omega_4) \frac{v_1 \rho}{6\omega_7 \omega_4^2}$$

$$C_{D_x^3 v_2}^{(2), \text{MRT2}} = (-v_1^2 \omega_7 \omega_4^2 - 6cs^2 \omega_4 - 12\omega_7 cs^2 + 3cs^2 \omega_4^2 + 3v_1^2 \omega_7 \omega_4 + 6\omega_4 + \omega_7 \omega_4^2 + 15\omega_7 cs^2 \omega_4 + 3v_1^2 \omega_4^2 - 3\omega_4^2 - 3\omega_7 cs^2 \omega_4^2 - 6v_1^2 \omega_4 - 3\omega_7 \omega_4) \frac{v_1 \rho}{6\omega_7 \omega_4^2}$$

$$C_{D_x^3 v_2}^{(2), \text{CLBM1}} = (6 + 3v_1^2 \omega_7 - 6v_1^2 + 9\omega_7 cs^2 - 18cs^2 - 3\omega_7 \omega_4 cs^2 - 3\omega_7 - v_1^2 \omega_7 \omega_4 - 3\omega_4 + 3v_1^2 \omega_4 + \omega_7 \omega_4 + 9\omega_4 cs^2) \frac{v_1 \rho}{6\omega_7 \omega_4}$$

$$C_{D_x^3 v_2}^{(2), \text{CLBM2}} = (6 + 3v_1^2 \omega_7 - 6v_1^2 + 9cs^2 \omega_7 - 3\omega_7 - v_1^2 \omega_7 \omega_4 - 3\omega_4 - 3cs^2 \omega_7 \omega_4 - 18cs^2 + 9cs^2 \omega_4 + 3v_1^2 \omega_4 + \omega_7 \omega_4) \frac{v_1 \rho}{6\omega_7 \omega_4}$$

$$C_{D_x^3 v_2}^{(2), \text{CuLBM1}} = (6 + 3\omega_3 v_1^2 - 6v_1^2 - 3\omega_3 - 18cs^2 + 9cs^2 \omega_4 + 9\omega_3 cs^2 - \omega_3 v_1^2 \omega_4 - 3\omega_4 + \omega_3 \omega_4 - 3\omega_3 cs^2 \omega_4 + 3v_1^2 \omega_4) \frac{v_1 \rho}{6\omega_3 \omega_4}$$

$$C_{D_x^3 v_2}^{(2), \text{CuLBM2}} = (6 + 3\omega_3 v_1^2 - 6v_1^2 - 3\omega_3 + 9\omega_1 cs^2 - \omega_3 v_1^2 \omega_1 - 3\omega_1 - 18cs^2 + 9\omega_3 cs^2 - 3\omega_3 \omega_1 cs^2 + \omega_3 \omega_1 + 3v_1^2 \omega_1) \frac{v_1 \rho}{6\omega_3 \omega_1}$$

coefficient $C_{D_x^2 D_y \rho}^{(2)}$ **at** $\frac{\partial^3 \rho}{\partial x_1^2 \partial x_2}$:

$$C_{D_x^2 D_y \rho}^{(2), \text{SRT}} = (-12 - \omega^2 + 12\omega) \frac{cs^4}{6\omega^2}$$

$$C_{D_x^2 D_y \rho}^{(2), \text{MRT1}} = (-12 + 12\omega_4 - \omega_4^2) \frac{cs^4}{6\omega_4^2}$$

$$C_{D_x^2 D_y \rho}^{(2), \text{MRT2}} = (-12 + 12\omega_4 - \omega_4^2) \frac{cs^4}{6\omega_4^2}$$

$$C_{D_x^2 D_y \rho}^{(2), \text{CLBM1}} = (-12 + 12\omega_4 - \omega_4^2) \frac{cs^4}{6\omega_4^2}$$

$$C_{D_x^2 D_y \rho}^{(2), \text{CLBM2}} = (-12 + 12\omega_4 - \omega_4^2) \frac{cs^4}{6\omega_4^2}$$

$$C_{D_x^2 D_y \rho}^{(2), \text{CuLBM1}} = (-12 + 12\omega_3 - \omega_3^2) \frac{cs^4}{6\omega_3^2}$$

$$C_{D_x^2 D_y \rho}^{(2), \text{CuLBM2}} = (9v_1^4 \omega_1 \omega_2^2 + 45v_1^2 \omega_1^2 cs^2 - 30cs^4 \omega_2^2 - 45v_1^2 cs^2 \omega_2^2 - 6\omega_1 cs^2 \omega_2^2 - 6\omega_1^2 cs^4 \omega_2 + 9v_1^4 \omega_1^2 - 2\omega_1^2 cs^4 \omega_2^2 - 9v_1^2 \omega_1 \omega_2^2 + 6\omega_1^2 cs^4 + 45v_1^2 \omega_1 cs^2 \omega_2^2 + 9v_1^2 \omega_2^2 + 6cs^2 \omega_2^2 - 9v_1^4 \omega_1^2 \omega_2 + 30\omega_1 cs^4 \omega_2^2 + 6\omega_1^2 cs^2 \omega_2 - 9v_1^2 \omega_1^2 - 6\omega_1^2 cs^2 + 9v_1^2 \omega_1^2 \omega_2 - 45v_1^2 \omega_1^2 cs^2 \omega_2 - 9v_1^4 \omega_2^2) \frac{1}{12\omega_1^2 \omega_2^2}$$

coefficient $C_{D_x^2 D_y v_1}^{(2)}$ **at** $\frac{\partial^3 v_1}{\partial x_1^2 \partial x_2}$:

$$C_{D_x^2 D_y v_1}^{(2), \text{SRT}} = 0$$

$$C_{D_x^2 D_y v_1}^{(2), \text{MRT1}} = (-2\omega_7 + 2\omega_4 - \omega_4^2 + \omega_7 \omega_4) \frac{cs^2 v_1 \rho}{\omega_7 \omega_4^2}$$

$$C_{D_x^2 D_y v_1}^{(2), \text{MRT2}} = (-2\omega_7 + 2\omega_4 - \omega_4^2 + \omega_7 \omega_4) \frac{v_1 \rho cs^2}{\omega_7 \omega_4^2}$$

$$C_{D_x^2 D_y v_1}^{(2), \text{CLBM1}} = 0$$

$$C_{D_x^2 D_y v_1}^{(2), \text{CLBM2}} = 0$$

$$C_{D_x^2 D_y v_1}^{(2), \text{CuLBM1}} = 0$$

$$C_{D_x^2 D_y v_1}^{(2), \text{CuLBM2}} = (9\omega_1 cs^2 \omega_2^2 + 11v_1^2 \omega_1 \omega_2^2 + 5\omega_1^2 \omega_2 + 5\omega_2^2 - 11v_1^2 \omega_2^2 - 9cs^2 \omega_2^2 - 9\omega_1^2 cs^2 \omega_2 + 11v_1^2 \omega_1^2 - 5\omega_1^2 + 9\omega_1^2 cs^2 - 11v_1^2 \omega_1^2 \omega_2 - 5\omega_1 \omega_2^2) \frac{v_1 \rho}{4\omega_1^2 \omega_2^2}$$

coefficient $C_{D_x^2 D_y v_2}^{(2)}$ **at** $\frac{\partial^3 v_2}{\partial x_1^2 \partial x_2}$:

$$C_{D_x^2 D_y v_2}^{(2), \text{SRT}} = \frac{-\rho v_2 cs^2}{6}$$

$$C_{D_x^2 D_y v_2}^{(2), \text{MRT1}} = (-12\omega_6 \omega_8 - \omega_6 \omega_4^2 \omega_8 + 12\omega_6 \omega_4 \omega_8 - 12\omega_4 \omega_8 + 12\omega_6 \omega_4 + 12\omega_4^2 - 12\omega_6 \omega_4^2) \frac{cs^2 \rho v_2}{6\omega_6 \omega_4^2 \omega_8}$$

$$C_{D_x^2 D_y v_2}^{(2), \text{MRT2}} = (-12\omega_6 \omega_8 - \omega_6 \omega_4^2 \omega_8 + 12\omega_6 \omega_4 \omega_8 - 12\omega_4 \omega_8 + 12\omega_6 \omega_4 + 12\omega_4^2 - 12\omega_6 \omega_4^2) \frac{\rho cs^2 v_2}{6\omega_6 \omega_4^2 \omega_8}$$

$$C_{D_x^2 D_y v_2}^{(2), \text{CLBM1}} = \frac{-\rho v_2 cs^2}{6}$$

$$C_{D_x^2 D_y v_2}^{(2), \text{CLBM2}} = \frac{-cs^2 \rho v_2}{6}$$

$$C_{D_x^2 D_y v_2}^{(2), \text{CuLBM1}} = \frac{-\rho cs^2 v_2}{6}$$

$$C_{D_x^2 D_y v_2}^{(2), \text{CuLBM2}} = \frac{-\rho v_2 cs^2}{6}$$

coefficient $C_{D_x D_y^2 \rho}^{(2)}$ **at** $\frac{\partial^3 \rho}{\partial x_1 \partial x_2^2}$:

$$C_{D_x D_y^2 \rho}^{(2), \text{SRT}} = 0$$

$$C_{D_x D_y^2 \rho}^{(2), \text{MRT1}} = (-\omega_6 \omega_4 v_2^2 - \omega_6 \omega_8 - 3\omega_6^2 cs^2 - 3\omega_6 cs^2 \omega_4 \omega_8 - 3\omega_6 cs^2 \omega_4 + 3cs^2 \omega_4 \omega_8 - \omega_6^2 \omega_4 - \omega_6^2 v_2^2 + \omega_6 \omega_4 \omega_8 + \omega_6 v_2^2 \omega_8 - \omega_4 \omega_8 + \omega_6 \omega_4 + 3\omega_6^2 cs^2 \omega_4 - \omega_6 \omega_4 v_2^2 \omega_8 + \omega_4 v_2^2 \omega_8 + \omega_6^2 \omega_4 v_2^2 + 3\omega_6 cs^2 \omega_8 + \omega_6^2) \frac{v_1 v_2}{\omega_6^2 \omega_4 \omega_8}$$

$$C_{D_x D_y^2 \rho}^{(2), \text{MRT2}} = (-\omega_6 \omega_4 v_2^2 - \omega_6 \omega_8 - 3\omega_6 cs^2 \omega_4 - \omega_6^2 \omega_4 - \omega_6^2 v_2^2 + \omega_6 \omega_4 \omega_8 + \omega_6 v_2^2 \omega_8 - \omega_4 \omega_8 + \omega_6 \omega_4 + 3\omega_6 cs^2 \omega_8 - \omega_6 \omega_4 v_2^2 \omega_8 + \omega_4 v_2^2 \omega_8 + \omega_6^2 \omega_4 v_2^2 + 3\omega_6^2 cs^2 \omega_4 - 3\omega_6^2 cs^2 - 3\omega_6 cs^2 \omega_4 \omega_8 + \omega_6^2 + 3cs^2 \omega_4 \omega_8) \frac{v_1 v_2}{\omega_6^2 \omega_4 \omega_8}$$

$$C_{D_x D_y^2 \rho}^{(2), \text{CLBM1}} = 0$$

$$C_{D_x D_y^2 \rho}^{(2), \text{CLBM2}} = 0$$

$$C_{D_x D_y^2 \rho}^{(2), \text{CuLBM1}} = 0$$

$$C_{D_x D_y^2 \rho}^{(2), \text{CuLBM2}} =$$

$$(3\omega_1 cs^2 \omega_2^2 + v_1^2 \omega_1 \omega_2^2 - 6\omega_1 cs^2 \omega_2 - 2\omega_1 v_2^2 \omega_2 + \omega_1^2 \omega_2 - v_1^2 \omega_2^2 + 2\omega_1 \omega_2 - 3\omega_1^2 cs^2 \omega_2 + v_1^2 \omega_1^2 - 2\omega_1^2 + \omega_1^2 v_2^2 + 6\omega_1^2 cs^2 + v_2^2 \omega_2^2 - v_1^2 \omega_1^2 \omega_2 - \omega_1 \omega_2^2) \frac{3v_1 v_2}{4\omega_1^2 \omega_2^2}$$

coefficient $C_{D_x D_y^2 v_1}^{(2)}$ **at** $\frac{\partial^3 v_1}{\partial x_1 \partial x_2^2}$:

$$C_{D_x D_y^2 v_1}^{(2), \text{SRT}} = (12 - 36cs^2 - 11\omega^2 cs^2 + 3\omega^2 + 12v_2^2 \omega - 12v_2^2 - 12\omega + 36\omega cs^2 - 3v_2^2 \omega^2) \frac{\rho v_2}{12\omega^2}$$

$$C_{D_x D_y^2 v_1}^{(2), \text{MRT1}} = (-12\omega_6^2 \omega_4^2 - 11\omega_6^2 cs^2 \omega_4^2 \omega_8 - 12\omega_6 cs^2 \omega_4^2 + 12\omega_6^2 \omega_4^2 v_2^2 - 24\omega_6^2 cs^2 \omega_8 + 6\omega_6^2 \omega_4 v_2^2 \omega_8 - 6\omega_6^2 \omega_4 \omega_8 + 6\omega_6 \omega_4^2 \omega_8 - 6\omega_6 \omega_4^2 v_2^2 \omega_8 - 24\omega_6 cs^2 \omega_4 \omega_8 - 12\omega_4^2 \omega_8 + 12\omega_4^2 v_2^2 \omega_8 + 12\omega_6^2 \omega_4 - 18\omega_6 cs^2 \omega_4^2 \omega_8 + 36cs^2 \omega_4^2 \omega_8 - 12\omega_6^2 cs^2 \omega_4 + 3\omega_6^2 \omega_4^2 \omega_8 - 12\omega_6^2 \omega_4 v_2^2 + 12\omega_6^2 cs^2 \omega_4^2 + 12\omega_6 \omega_4^2 - 12\omega_6 \omega_4^2 v_2^2 - 3\omega_6^2 \omega_4^2 v_2^2 \omega_8 + 42\omega_6^2 cs^2 \omega_4 \omega_8) \frac{\rho v_2}{12\omega_6^2 \omega_4^2 \omega_8}$$

$$C_{D_x D_y^2 v_1}^{(2), \text{MRT2}} = (-12\omega_6^2 \omega_4^2 - 18\omega_6 cs^2 \omega_4^2 \omega_8 + 36cs^2 \omega_4^2 \omega_8 + 12\omega_6^2 \omega_4^2 v_2^2 + 6\omega_6^2 \omega_4 v_2^2 \omega_8 - 6\omega_6^2 \omega_4 \omega_8 - 12\omega_6 cs^2 \omega_4^2 + 6\omega_6 \omega_4^2 \omega_8 - 6\omega_6 \omega_4^2 v_2^2 \omega_8 - 12\omega_4^2 \omega_8 - 24\omega_6^2 cs^2 \omega_8 + 42\omega_6^2 cs^2 \omega_4 \omega_8 + 12\omega_4^2 v_2^2 \omega_8 + 12\omega_6^2 \omega_4 - 11\omega_6^2 cs^2 \omega_4^2 \omega_8 + 12\omega_6^2 cs^2 \omega_4^2 + 3\omega_6^2 \omega_4^2 \omega_8 - 12\omega_6^2 \omega_4 v_2^2 - 12\omega_6^2 cs^2 \omega_4 - 24\omega_6 cs^2 \omega_4 \omega_8 + 12\omega_6 \omega_4^2 - 12\omega_6 \omega_4^2 v_2^2 - 3\omega_6^2 \omega_4^2 v_2^2 \omega_8) \frac{\rho v_2}{12\omega_6^2 \omega_4^2 \omega_8}$$

$$C_{D_x D_y^2 v_1}^{(2), \text{CLBM1}} = (-12\omega_6\omega_4v_2^2 + 18\omega_6^2\omega_8cs^2 - 3\omega_6^2\omega_4v_2^2\omega_8 + 36\omega_6^2\omega_4cs^2 + 6\omega_6^2v_2^2\omega_8 + 3\omega_6^2\omega_4\omega_8 - 11\omega_6^2\omega_4\omega_8cs^2 - 12\omega_6^2\omega_4 - 12\omega_6^2v_2^2 + 6\omega_6\omega_4\omega_8 - 36\omega_6^2cs^2 - 12\omega_4\omega_8 + 12\omega_6\omega_4 - 6\omega_6\omega_4v_2^2\omega_8 + 12\omega_4v_2^2\omega_8 - 18\omega_6\omega_4\omega_8cs^2 + 12\omega_6^2\omega_4v_2^2 + 36\omega_4\omega_8cs^2 + 12\omega_6^2 - 6\omega_6^2\omega_8 - 36\omega_6\omega_4cs^2) \frac{\rho v_2}{12\omega_6^2\omega_4\omega_8}$$

$$C_{D_x D_y^2 v_1}^{(2), \text{CLBM2}} = (-18cs^2\omega_6\omega_4\omega_8 - 12\omega_6\omega_4v_2^2 - 3\omega_6^2\omega_4v_2^2\omega_8 + 36cs^2\omega_6^2\omega_4 + 6\omega_6^2v_2^2\omega_8 + 3\omega_6^2\omega_4\omega_8 - 12\omega_6^2\omega_4 - 12\omega_6^2v_2^2 + 6\omega_6\omega_4\omega_8 - 36cs^2\omega_6^2 - 12\omega_4\omega_8 + 12\omega_6\omega_4 - 6\omega_6\omega_4v_2^2\omega_8 + 18cs^2\omega_6^2\omega_8 + 12\omega_4v_2^2\omega_8 + 12\omega_6^2\omega_4v_2^2 - 11cs^2\omega_6^2\omega_4\omega_8 + 36cs^2\omega_4\omega_8 - 36cs^2\omega_6\omega_4 + 12\omega_6^2 - 6\omega_6^2\omega_8) \frac{\rho v_2}{12\omega_6^2\omega_4\omega_8}$$

$$C_{D_x D_y^2 v_1}^{(2), \text{CuLBM1}} = (12\omega_3\omega_2 - 36\omega_3cs^2\omega_2 + 6\omega_6\omega_3\omega_2 + 12\omega_3v_2^2\omega_2^2 - 18\omega_6\omega_3cs^2\omega_2 - 3\omega_6\omega_3v_2^2\omega_2^2 - 6\omega_6\omega_2^2 - 11\omega_6\omega_3cs^2\omega_2^2 - 6\omega_6\omega_3v_2^2\omega_2 - 12\omega_3\omega_2^2 + 36\omega_6\omega_3cs^2 + 36\omega_3cs^2\omega_2^2 + 3\omega_6\omega_3\omega_2^2 - 12\omega_3v_2^2\omega_2 + 12\omega_2^2 + 6\omega_6v_2^2\omega_2^2 - 12v_2^2\omega_2^2 - 36cs^2\omega_2^2 + 12\omega_6\omega_3v_2^2 + 18\omega_6cs^2\omega_2^2 - 12\omega_6\omega_3) \frac{\rho v_2}{12\omega_6\omega_3\omega_2^2}$$

$$C_{D_x D_y^2 v_1}^{(2), \text{CuLBM2}} = (27\omega_3\omega_1cs^2 + 6\omega_3v_2^2\omega_2^2 - 18\omega_1v_2^2\omega_2^2 - 12\omega_1^2\omega_2^2 + 6\omega_3\omega_1^2v_2^2 + 27\omega_3v_1^2\omega_1^2 - 54\omega_1cs^2\omega_2^2 + 9\omega_3cs^2\omega_2^2 + 27\omega_3v_1^2\omega_1\omega_2^2 + 18\omega_3\omega_1cs^2\omega_2^2 - 12\omega_3\omega_1\omega_2^2 + 6\omega_1^2\omega_2 + 3\omega_3\omega_2^2 + 3\omega_3\omega_1v_2^2\omega_2^2 - 11\omega_3\omega_1^2cs^2\omega_2^2 - 18\omega_1^2cs^2\omega_2 - 15\omega_3\omega_1^2 - 27\omega_3v_1^2\omega_1^2\omega_2 + 3\omega_3\omega_1^2\omega_2^2 - 6\omega_1^2v_2^2\omega_2 - 3\omega_3\omega_1^2v_2^2\omega_2^2 - 3\omega_3\omega_1^2v_2^2\omega_2 - 27\omega_3v_1^2\omega_2^2 + 12\omega_1^2v_2^2\omega_2^2 + 12\omega_3\omega_1^2\omega_2 + 36\omega_1^2cs^2\omega_2^2 - 18\omega_3\omega_1^2cs^2\omega_2 + 18\omega_1\omega_2^2) \frac{\rho v_2}{12\omega_3\omega_1^2\omega_2^2}$$

coefficient $C_{D_x D_y^2 v_2}^{(2)}$ **at** $\frac{\partial^3 v_2}{\partial x_1 \partial x_2^2}$:

$$C_{D_x D_y^2 v_2}^{(2), \text{SRT}} = 0$$

$$C_{D_x D_y^2 v_2}^{(2), \text{MRT1}} = (-3\omega_6\omega_4v_2^2 - \omega_6\omega_8 - \omega_6^2cs^2 - \omega_6cs^2\omega_4\omega_8 - \omega_6cs^2\omega_4 + cs^2\omega_4\omega_8 - \omega_6^2\omega_4 - 3\omega_6^2v_2^2 + \omega_6\omega_4\omega_8 + 3\omega_6v_2^2\omega_8 - \omega_4\omega_8 + \omega_6\omega_4 + \omega_6^2cs^2\omega_4 - 3\omega_6\omega_4v_2^2\omega_8 + 3\omega_4v_2^2\omega_8 + 3\omega_6^2\omega_4v_2^2 + \omega_6cs^2\omega_8 + \omega_6^2) \frac{v_1\rho}{\omega_6^2\omega_4\omega_8}$$

$$C_{D_x D_y^2 v_2}^{(2), \text{MRT2}} = (-3\omega_6\omega_4v_2^2 - \omega_6\omega_8 - \omega_6cs^2\omega_4 - \omega_6^2\omega_4 - 3\omega_6^2v_2^2 + \omega_6\omega_4\omega_8 + 3\omega_6v_2^2\omega_8 - \omega_4\omega_8 + \omega_6\omega_4 + \omega_6cs^2\omega_8 - 3\omega_6\omega_4v_2^2\omega_8 + 3\omega_4v_2^2\omega_8 + 3\omega_6^2\omega_4v_2^2 + \omega_6^2cs^2\omega_4 - \omega_6^2cs^2 - \omega_6cs^2\omega_4\omega_8 + \omega_6^2 + cs^2\omega_4\omega_8) \frac{v_1\rho}{\omega_6^2\omega_4\omega_8}$$

$$C_{D_x D_y^2 v_2}^{(2), \text{CLBM1}} = 0$$

$$C_{D_x D_y^2 v_2}^{(2), \text{CLBM2}} = 0$$

$$C_{D_x D_y^2 v_2}^{(2), \text{CuLBM1}} = 0$$

$$C_{D_x D_y^2 v_2}^{(2), \text{CuLBM2}} = (9\omega_3\omega_1^2cs^2 - 18\omega_3\omega_1v_2^2\omega_2 + 9\omega_3v_2^2\omega_2^2 + 9\omega_3\omega_1^2v_2^2 + 2\omega_3v_1^2\omega_1^2 + 6\omega_1cs^2\omega_2^2 - 3\omega_3cs^2\omega_2^2 + \omega_3v_1^2\omega_1\omega_2^2 + 6\omega_3\omega_1\omega_2 - 6\omega_3\omega_1cs^2\omega_2 + 3\omega_3\omega_1cs^2\omega_2^2 - \omega_3\omega_1\omega_2^2 + 2v_1^2\omega_1\omega_2^2 + 2\omega_1^2\omega_2 - \omega_3\omega_2^2 - 6\omega_1^2cs^2\omega_2 - 5\omega_3\omega_1^2 - \omega_3v_1^2\omega_1^2\omega_2 - 2v_1^2\omega_1^2\omega_2 - 2\omega_3v_1^2\omega_2^2 + \omega_3\omega_1^2\omega_2 - 3\omega_3\omega_1^2cs^2\omega_2 - 2\omega_1\omega_2^2) \frac{v_1\rho}{4\omega_3\omega_1^2\omega_2^2}$$

coefficient $C_{D_y^3 \rho}^{(2)}$ **at** $\frac{\partial^3 \rho}{\partial x_2^3}$:

$$C_{D_y^3 \rho}^{(2), \text{SRT}} = (-12\omega cs^4 + 24v_2^2\omega^2cs^2 + 36v_2^4 - 12cs^2 - 36v_2^4\omega - \omega^2cs^2 + 7v_2^4\omega^2 + 36v_2^2\omega - 36v_2^2 + 12cs^4 + 12\omega cs^2 + 144v_2^2cs^2 + \omega^2cs^4 - 144v_2^2\omega cs^2 - 7v_2^2\omega^2) \frac{1}{12\omega^2}$$

$$C_{D_y^3 \rho}^{(2), \text{MRT1}} = (36v_2^4 + 12cs^4 - \omega_6^2cs^2 - 36\omega_6v_2^4 + 144cs^2v_2^2 - 144\omega_6cs^2v_2^2 - 12\omega_6cs^4 - 7\omega_6^2v_2^2 + 36\omega_6v_2^2 + \omega_6^2cs^4 + 24\omega_6^2cs^2v_2^2 + 7\omega_6^2v_2^4 + 12\omega_6cs^2 - 36v_2^2 - 12cs^2) \frac{1}{12\omega_6^2}$$

$$C_{D_y^3 \rho}^{(2), \text{MRT2}} = (12\omega_6cs^2 + 36v_2^4 + 144cs^2v_2^2 - 144\omega_6cs^2v_2^2 + \omega_6^2cs^4 - 36\omega_6v_2^4 - 12cs^2 - 7\omega_6^2v_2^2 + 12cs^4 + 36\omega_6v_2^2 + 7\omega_6^2v_2^4 - 36v_2^2 - 12\omega_6cs^4 + 24\omega_6^2cs^2v_2^2 - \omega_6^2cs^2) \frac{1}{12\omega_6^2}$$

$$C_{D_y^3 \rho}^{(2), \text{CLBM1}} = (36v_2^4 - 12cs^2 + 24\omega_6^2v_2^2cs^2 - 36\omega_6v_2^4 + \omega_6^2cs^4 + 12\omega_6cs^2 - 7\omega_6^2v_2^2 - \omega_6^2cs^2 + 36\omega_6v_2^2 + 7\omega_6^2v_2^4 - 12\omega_6cs^4 - 36v_2^2 + 12cs^4 + 144v_2^2cs^2 - 144\omega_6v_2^2cs^2) \frac{1}{12\omega_6^2}$$

$$C_{D_y^3 \rho}^{(2), \text{CLBM2}} = (36v_2^4 + cs^4\omega_6^2 + 24cs^2\omega_6^2v_2^2 + 12cs^4 - 36\omega_6v_2^4 - 12cs^4\omega_6 - 7\omega_6^2v_2^2 - cs^2\omega_6^2 - 12cs^2 + 36\omega_6v_2^2 + 7\omega_6^2v_2^4 + 144cs^2v_2^2 - 36v_2^2 - 144cs^2\omega_6v_2^2 + 12cs^2\omega_6) \frac{1}{12\omega_6^2}$$

$$C_{D_y^3 \rho}^{(2), \text{CuLBM1}} = (-144cs^2v_2^2\omega_2 - 12cs^4\omega_2 + 36v_2^4 + 7v_2^4\omega_2^2 + 24cs^2v_2^2\omega_2^2 + cs^4\omega_2^2 - 36v_2^4\omega_2 + 144cs^2v_2^2 - 12cs^2 + 12cs^4 + 12cs^2\omega_2 - 7v_2^2\omega_2^2 - cs^2\omega_2^2 + 36v_2^2\omega_2 - 36v_2^2)\frac{1}{12\omega_2^2}$$

$$C_{D_y^3 \rho}^{(2), \text{CuLBM2}} = (-18\omega_1^2v_2^4\omega_2 - 72\omega_1^2v_2^2cs^2\omega_2 + 18\omega_1v_2^2\omega_2^2 + 9v_2^4\omega_2^2 + 6cs^4\omega_2^2 + 6\omega_1cs^2\omega_2^2 - 6\omega_1^2cs^4\omega_2 + \omega_1^2cs^4\omega_2^2 + 6\omega_1^2cs^4 + 24\omega_1^2v_2^2cs^2\omega_2^2 - 18\omega_1v_2^2\omega_2 + 45\omega_1^2v_2^2cs^2 + 7\omega_1^2v_2^4\omega_2^2 + 9\omega_1^2v_2^4 - 6cs^2\omega_2^2 - 6\omega_1cs^4\omega_2^2 + 54\omega_1v_2^2cs^2\omega_2 + 6\omega_1^2cs^2\omega_2 + 45v_2^2cs^2\omega_2^2 + 18\omega_1^2v_2^2\omega_2 - 9\omega_1^2v_2^2 - 6\omega_1^2cs^2 - 18\omega_1v_2^4\omega_2^2 - 9v_2^2\omega_2^2 + 18\omega_1v_2^4\omega_2 - 7\omega_1^2v_2^2\omega_2^2 - \omega_1^2cs^2\omega_2^2 - 72\omega_1v_2^2cs^2\omega_2^2)\frac{1}{12\omega_1^2\omega_2^2}$$

coefficient $C_{D_y^3 v_2}^{(2)}$ **at** $\frac{\partial^3 v_2}{\partial x_1^3}$:

$$C_{D_y^3 v_2}^{(2), \text{SRT}} = (-24 + 36cs^2 + 5\omega^2cs^2 - 4\omega^2 - 60v_2^2\omega + 60v_2^2 + 24\omega - 36\omega cs^2 + 11v_2^2\omega^2)\frac{\rho v_2}{6\omega^2}$$

$$C_{D_y^3 v_2}^{(2), \text{MRT1}} = (-24 + 24\omega_6 + 5\omega_6^2cs^2 + 11\omega_6^2v_2^2 - 60\omega_6v_2^2 - 36\omega_6cs^2 + 60v_2^2 - 4\omega_6^2 + 36cs^2)\frac{\rho v_2}{6\omega_6^2}$$

$$C_{D_y^3 v_2}^{(2), \text{MRT2}} = (-24 - 36\omega_6cs^2 + 24\omega_6 + 36cs^2 + 11\omega_6^2v_2^2 - 60\omega_6v_2^2 + 60v_2^2 + 5\omega_6^2cs^2 - 4\omega_6^2)\frac{\rho v_2}{6\omega_6^2}$$

$$C_{D_y^3 v_2}^{(2), \text{CLBM1}} = (-24 + 24\omega_6 + 36cs^2 - 36\omega_6cs^2 + 11\omega_6^2v_2^2 + 5\omega_6^2cs^2 - 60\omega_6v_2^2 + 60v_2^2 - 4\omega_6^2)\frac{\rho v_2}{6\omega_6^2}$$

$$C_{D_y^3 v_2}^{(2), \text{CLBM2}} = (-24 + 24\omega_6 + 11\omega_6^2v_2^2 + 5cs^2\omega_6^2 + 36cs^2 - 60\omega_6v_2^2 + 60v_2^2 - 4\omega_6^2 - 36cs^2\omega_6)\frac{\rho v_2}{6\omega_6^2}$$

$$C_{D_y^3 v_2}^{(2), \text{CuLBM1}} = (-24 + 36cs^2 - 4\omega_2^2 - 36cs^2\omega_2 + 11v_2^2\omega_2^2 + 5cs^2\omega_2^2 - 60v_2^2\omega_2 + 60v_2^2 + 24\omega_2)\frac{\rho v_2}{6\omega_2^2}$$

$$C_{D_y^3 v_2}^{(2), \text{CuLBM2}} = (-60\omega_1v_2^2\omega_2^2 - 8\omega_1^2\omega_2^2 - 36\omega_1cs^2\omega_2^2 + 18\omega_1cs^2\omega_2 + 54\omega_1v_2^2\omega_2 + 24\omega_1^2\omega_2 - 15\omega_2^2 + 27cs^2\omega_2^2 - 18\omega_1\omega_2 - 36\omega_1^2cs^2\omega_2 - 15\omega_1^2 - 60\omega_1^2v_2^2\omega_2 + 33\omega_1^2v_2^2 + 27\omega_1^2cs^2 + 33v_2^2\omega_2^2 + 22\omega_1^2v_2^2\omega_2^2 + 10\omega_1^2cs^2\omega_2^2 + 24\omega_1\omega_2^2)\frac{\rho v_2}{12\omega_1^2\omega_2^2}$$

coefficient $C_{D_x^4 \rho}^{(2)}$ **at** $\frac{\partial^4 \rho}{\partial x_1^4}$:

$$C_{D_x^4 \rho}^{(2), \text{SRT}} = (-\omega cs^4 + 24v_1^2cs^2 - 2cs^2 - 6v_1^2 + 3v_1^2\omega - 12v_1^2\omega cs^2 + 2cs^4 + \omega cs^2 - 3v_1^4\omega + 6v_1^4)\frac{v_2}{24\omega}$$

$$C_{D_x^4 \rho}^{(2), \text{MRT1}} = (-126cs^2v_1^2\omega_7\omega_4^2\omega_5^2 + 48cs^2\omega_7^2\omega_4\omega_5 + 24v_1^4\omega_7\omega_4^2\omega_5 - 96v_1^2\omega_7\omega_4\omega_5^2 - cs^4\omega_7^2\omega_4^2\omega_5^2 - 144cs^2v_1^2\omega_7^2\omega_4^2\omega_5 + 24cs^4\omega_7^2\omega_4\omega_5^2 - 96v_1^4\omega_7^2\omega_4\omega_5 - 24v_1^4\omega_4\omega_5^2 - 48v_1^2\omega_7^2\omega_5 + 3v_1^2\omega_7^2\omega_4^2\omega_5^2 + 48cs^4\omega_7\omega_4\omega_5^2 - 36v_1^2\omega_7^2\omega_4^2\omega_5 + 12cs^2\omega_7\omega_4^2\omega_5^2 - 432cs^2v_1^2\omega_7^2\omega_4\omega_5 - 216cs^2v_1^2\omega_7\omega_5^2 + 48v_1^4\omega_7\omega_4\omega_5 - 24cs^4\omega_4\omega_5^2 + 288cs^2v_1^2\omega_7^2\omega_4 - 24cs^2\omega_7^2\omega_5 + 14cs^4\omega_7^2\omega_4^2\omega_5 + 72cs^2v_1^2\omega_4^2\omega_5^2 + 48v_1^4\omega_7^2\omega_5 + 24v_1^2\omega_4\omega_5^2 + 72cs^2v_1^2\omega_7\omega_4^2\omega_5 - 30v_1^4\omega_7\omega_4^2\omega_5^2 - 48cs^4\omega_7^2\omega_4\omega_5 + 12cs^4\omega_4^2\omega_5^2 - 24v_1^2\omega_7\omega_4^2\omega_5 - 144cs^2v_1^2\omega_7\omega_4^2\omega_5 - 144cs^2v_1^2\omega_4\omega_5^2 + 24cs^4\omega_7^2\omega_4 - 12v_1^2\omega_4^2\omega_5^2 + 96v_1^4\omega_7\omega_4\omega_5^2 - 72v_1^2\omega_7^2\omega_4 + cs^2\omega_7^2\omega_4^2\omega_5^2 - 144cs^2v_1^2\omega_7\omega_4\omega_5 + 48v_1^2\omega_7\omega_5^2 + 96v_1^2\omega_7^2\omega_4\omega_5 - 36v_1^4\omega_7^2\omega_4^2 - 3v_1^4\omega_7^2\omega_4^2\omega_5^2 - 48cs^2\omega_7\omega_4\omega_5^2 + 150cs^2v_1^2\omega_7^2\omega_4^2\omega_5 - 24cs^4\omega_7\omega_5^2 + 12cs^2\omega_7^2\omega_4^2 - 12cs^2\omega_4^2\omega_5^2 + 36v_1^4\omega_7^2\omega_4^2\omega_5 - 12cs^2v_1^2\omega_7^2\omega_4^2\omega_5^2 - 24cs^2\omega_7^2\omega_4 + 216cs^2v_1^2\omega_7^2\omega_5 + 12v_1^4\omega_4^2\omega_5^2 - 12cs^4\omega_7\omega_4^2\omega_5^2 + 72v_1^4\omega_7^2\omega_4 - 48v_1^4\omega_7\omega_5^2 + 36v_1^2\omega_7^2\omega_4^2 - 48v_1^4\omega_7\omega_4\omega_5 + 432cs^2v_1^2\omega_7\omega_4\omega_5^2 - 14cs^2\omega_7^2\omega_4^2\omega_5 + 24cs^2\omega_7\omega_5^2 + 30v_1^2\omega_7\omega_4^2\omega_5^2 - 12cs^4\omega_7^2\omega_4^2)\frac{v_2}{24\omega_7^2\omega_4^2\omega_5^2}$$

$$C_{D_x^4 \rho}^{(2), \text{MRT2}} = (12cs^4\omega_4^2\omega_5^2 + 24v_1^4\omega_7\omega_4^2\omega_5 + 12\omega_7^2cs^2\omega_4^2 + 72v_1^2cs^2\omega_4^2\omega_5^2 - 96v_1^2\omega_7\omega_4\omega_5^2 - 48\omega_7^2cs^4\omega_4\omega_5 + 288v_1^2\omega_7cs^2\omega_4 - 96v_1^4\omega_7^2\omega_4\omega_5 - 12\omega_7cs^4\omega_4^2\omega_5^2 - 24v_1^4\omega_4\omega_5^2 - 216v_1^2\omega_7cs^2\omega_5^2 - 48v_1^2\omega_7^2\omega_5 + 150v_1^2\omega_7^2cs^2\omega_4^2\omega_5 + 3v_1^2\omega_7^2\omega_4^2\omega_5^2 + 432v_1^2\omega_7cs^2\omega_4\omega_5^2 - 12v_1^2\omega_7^2cs^2\omega_4^2\omega_5^2 - 144v_1^2\omega_7cs^2\omega_4\omega_5 - 12cs^2\omega_4^2\omega_5^2 - 36v_1^2\omega_7^2\omega_4^2\omega_5 - 144v_1^2\omega_7^2cs^2\omega_4^2 + 48v_1^2\omega_7\omega_4\omega_5 + 48\omega_7^2cs^2\omega_4\omega_5 + 24\omega_7cs^2\omega_5^2 + 12\omega_7cs^2\omega_4^2\omega_5^2 + 48v_1^4\omega_7^2\omega_5 + 24\omega_7^2cs^4\omega_5 + 24v_1^2\omega_4\omega_5^2 - 24\omega_7^2cs^2\omega_4 - 30v_1^4\omega_7\omega_4^2\omega_5^2 - 24v_1^2\omega_7\omega_4^2\omega_5 - 14\omega_7^2cs^2\omega_4^2\omega_5 - 48\omega_7cs^2\omega_4\omega_5^2 + 216v_1^2\omega_7^2cs^2\omega_5 - 12v_1^2\omega_4^2\omega_5^2 + 96v_1^4\omega_7\omega_4\omega_5^2 - 72v_1^2\omega_7^2\omega_4 + 48v_1^2\omega_7\omega_5^2 + 96v_1^2\omega_7^2\omega_4\omega_5 - 12\omega_7^2cs^4\omega_4^2 - 36v_1^4\omega_7^2\omega_4^2 + 72v_1^2\omega_7cs^2\omega_4^2\omega_5 + 24cs^2\omega_4\omega_5^2 - \omega_7^2cs^4\omega_4^2\omega_5^2 - 3v_1^4\omega_7^2\omega_4^2\omega_5^2 + 36v_1^4\omega_7^2\omega_5^2 + 14\omega_7^2cs^4\omega_4^2\omega_5 - 24\omega_7cs^4\omega_5^2 + 48\omega_7cs^4\omega_4\omega_5^2 - 432v_1^2\omega_7^2cs^2\omega_4\omega_5 - 24\omega_7^2cs^2\omega_5 + 12v_1^4\omega_4^2\omega_5^2 - 126v_1^2\omega_7cs^2\omega_4^2\omega_5^2 + 72v_1^4\omega_7^2\omega_4 + 24\omega_7^2cs^4\omega_4 - 48v_1^4\omega_7\omega_5^2 + 36v_1^2\omega_7^2\omega_4^2 - 24cs^4\omega_4\omega_5^2 - 48v_1^4\omega_7\omega_4\omega_5 + \omega_7^2cs^2\omega_4^2\omega_5^2 + 30v_1^2\omega_7\omega_4^2\omega_5^2 - 144v_1^2cs^2\omega_4\omega_5^2)\frac{v_2}{24\omega_7^2\omega_4^2\omega_5^2}$$

$$C_{D_x^4 \rho}^{(2), \text{CLBM1}} = (-6v_1^2 - 2cs^2 + 24v_1^2cs^2 + cs^2\omega_5 + 3v_1^2\omega_5 - 3v_1^4\omega_5 + 2cs^4 - cs^4\omega_5 + 6v_1^4 - 12v_1^2cs^2\omega_5)\frac{v_2}{24\omega_5}$$

$$C_{D_x^4 \rho}^{(2), \text{CLBM2}} = (cs^2\omega_5 - 12cs^2v_1^2\omega_5 - 6v_1^2 + 2cs^4 + 24cs^2v_1^2 + 3v_1^2\omega_5 - 2cs^2 - cs^4\omega_5 - 3v_1^4\omega_5 + 6v_1^4)\frac{v_2}{24\omega_5}$$

$$C_{D_x^4 \rho}^{(2), \text{CuLBM1}} = (-6v_1^2 - 3v_1^4\omega_1 + cs^2\omega_1 - 2cs^2 + 24v_1^2cs^2 + 2cs^4 + 3v_1^2\omega_1 - cs^4\omega_1 + 6v_1^4 - 12v_1^2cs^2\omega_1)\frac{v_2}{24\omega_1}$$

$$C_{D_x^4 \rho}^{(2), \text{CuLBM2}} = (-12v_1^2\omega_1cs^2\omega_2 - 3v_2^2\omega_2 + 3v_1^2\omega_1\omega_2 + 3v_1^4\omega_1 + cs^4\omega_2 - \omega_1cs^2 + \omega_1cs^2\omega_2 + 12v_1^2cs^2\omega_2 - 3v_1^4\omega_1\omega_2 + \omega_1cs^4 + 3v_1^4\omega_2 - 3v_1^4\omega_1 - cs^2\omega_2 + 12v_1^2\omega_1cs^2 - \omega_1cs^4\omega_2)\frac{v_2}{24\omega_1\omega_2}$$

coefficient $C_{D_x^4 v_1}^{(2)}$ at $\frac{\partial^4 v_1}{\partial x_1^4}$:

$$C_{D_x^4 v_1}^{(2), \text{SRT}} = (-4 + 6cs^2 + 10v_1^2 - 5v_1^2\omega + 2\omega - 3\omega cs^2) \frac{v_1 \rho v_2}{12\omega}$$

$$C_{D_x^4 v_1}^{(2), \text{MRT}^1} = (-120cs^2\omega_7\omega_4\omega_5 + 36\omega_7\omega_5^2 - 25\omega_7^2\omega_4^2\omega_5 + 168v_1^2\omega_7\omega_4\omega_5^2 + 12cs^2\omega_7\omega_4^2\omega_5 - 48cs^2\omega_4\omega_5^2 + 84v_1^2\omega_7^2\omega_5 - 48\omega_7^2\omega_4 + 24\omega_7\omega_4\omega_5 - 5v_1^2\omega_7^2\omega_4^2\omega_5^2 - 12\omega_4^2\omega_5^2 + 24\omega_7^2\omega_4^2 + 61v_1^2\omega_7^2\omega_4^2\omega_5 - 72\omega_7\omega_4\omega_5^2 - 33cs^2\omega_7\omega_4^2\omega_5^2 - 72v_1^2\omega_7\omega_4\omega_5 + 2\omega_7^2\omega_4^2\omega_5^2 + 60cs^2\omega_7^2\omega_5 - 48v_1^2\omega_4\omega_5^2 + 36v_1^2\omega_7\omega_4^2\omega_5 + 24v_1^2\omega_4^2\omega_5^2 - 36\omega_7^2\omega_5 + 120v_1^2\omega_7^2\omega_4 - 3cs^2\omega_7^2\omega_4^2\omega_5 - 84v_1^2\omega_7\omega_5^2 - 168v_1^2\omega_7^2\omega_4\omega_5 + 21\omega_7\omega_4^2\omega_5^2 + 120cs^2\omega_7\omega_4\omega_5^2 - 36cs^2\omega_7^2\omega_4^2 + 24cs^2\omega_4^2\omega_5^2 - 24cs^2\omega_7\omega_4\omega_5 + 72cs^2\omega_7^2\omega_4 + 24\omega_4\omega_5^2 - 12\omega_7\omega_4^2\omega_5 - 60v_1^2\omega_7^2\omega_4^2 + 39cs^2\omega_7^2\omega_4^2\omega_5 - 60cs^2\omega_7\omega_5^2 + 72\omega_7^2\omega_4\omega_5 - 51v_1^2\omega_7\omega_4^2\omega_5^2) \frac{v_1 \rho v_2}{12\omega_7^2\omega_4^2\omega_5^2}$$

$$C_{D_x^4 v_1}^{(2), \text{MRT}^2} = (36\omega_7\omega_5^2 - 36\omega_7^2cs^2\omega_4^2 - 25\omega_7^2\omega_4^2\omega_5 + 168v_1^2\omega_7\omega_4\omega_5^2 + 12\omega_7cs^2\omega_4^2\omega_5 + 84v_1^2\omega_7^2\omega_5 - 48\omega_7^2\omega_4 + 24\omega_7\omega_4\omega_5 - 5v_1^2\omega_7^2\omega_4^2\omega_5^2 - 12\omega_4^2\omega_5^2 + 24\omega_7^2\omega_4^2 + 24cs^2\omega_7^2\omega_5^2 + 61v_1^2\omega_7^2\omega_4^2\omega_5 - 72\omega_7\omega_4\omega_5^2 - 72v_1^2\omega_7\omega_4\omega_5 + 2\omega_7^2\omega_4^2\omega_5^2 - 120\omega_7^2cs^2\omega_4\omega_5 - 60\omega_7cs^2\omega_5^2 - 33\omega_7cs^2\omega_4^2\omega_5^2 - 48v_1^2\omega_4\omega_5^2 + 72\omega_7^2cs^2\omega_4 + 36v_1^2\omega_7\omega_4^2\omega_5 + 39\omega_7^2cs^2\omega_4^2\omega_5 + 120\omega_7cs^2\omega_4\omega_5^2 + 24v_1^2\omega_4^2\omega_5^2 - 36\omega_7^2\omega_5 + 120v_1^2\omega_7^2\omega_4 - 84v_1^2\omega_7\omega_5^2 - 168v_1^2\omega_7^2\omega_4\omega_5 + 21\omega_7\omega_4^2\omega_5^2 - 48cs^2\omega_4\omega_5^2 + 60\omega_7^2cs^2\omega_5 + 24\omega_4\omega_5^2 - 12\omega_7\omega_4^2\omega_5 - 60v_1^2\omega_7^2\omega_4^2 - 3\omega_7^2cs^2\omega_4^2\omega_5^2 + 72\omega_7^2\omega_4\omega_5 - 51v_1^2\omega_7\omega_4^2\omega_5^2 - 24\omega_7cs^2\omega_4\omega_5) \frac{v_1 \rho v_2}{12\omega_7^2\omega_4^2\omega_5^2}$$

$$C_{D_x^4 v_1}^{(2), \text{CLBM}^1} = (-4 + 10v_1^2 + 6cs^2 - 3cs^2\omega_5 - 5v_1^2\omega_5 + 2\omega_5) \frac{v_1 \rho v_2}{12\omega_5}$$

$$C_{D_x^4 v_1}^{(2), \text{CLBM}^2} = (-4 - 3cs^2\omega_5 + 10v_1^2 - 5v_1^2\omega_5 + 6cs^2 + 2\omega_5) \frac{v_1 \rho v_2}{12\omega_5}$$

$$C_{D_x^4 v_1}^{(2), \text{CuLBM}^1} = (-4 + 10v_1^2 - 3cs^2\omega_1 + 6cs^2 + 2\omega_1 - 5v_1^2\omega_1) \frac{v_1 \rho v_2}{12\omega_1}$$

$$C_{D_x^4 v_1}^{(2), \text{CuLBM}^2} = (5v_1^2\omega_2 - 5v_1^2\omega_1\omega_2 + 3\omega_1cs^2 - 3\omega_1cs^2\omega_2 + 2\omega_1\omega_2 - 2\omega_1 + 5v_1^2\omega_1 - 2\omega_2 + 3cs^2\omega_2) \frac{v_1 \rho v_2}{12\omega_1\omega_2}$$

coefficient $C_{D_x^4 v_2}^{(2)}$ at $\frac{\partial^4 v_2}{\partial x_1^4}$:

$$C_{D_x^4 v_2}^{(2), \text{SRT}} = (-72\omega cs^4 - 3v_1^2\omega^3 - 144v_1^2cs^2 - 24cs^2 + 72v_1^2 - 14\omega^2cs^2 + 42v_1^2\omega^2 - 108v_1^2\omega + \omega^3cs^2 + 216v_1^2\omega cs^2 - 42v_1^4\omega^2 - 84v_1^2\omega^2cs^2 + 3v_1^4\omega^3 - 3\omega^3cs^4 + 6v_1^3\omega^3cs^2 + 48cs^4 + 36\omega cs^2 + 30\omega^2cs^4 + 108v_1^4\omega - 72v_1^4) \frac{\rho}{24\omega^3}$$

$$C_{D_x^4 v_2}^{(2), \text{MRT}^1} = (-72v_1^2\omega_7\omega_4^2 - 72cs^2v_1^2\omega_7^2\omega_4^2 - 24cs^4\omega_7\omega_4^2 - 24v_1^4\omega_4^2 + 18v_1^2\omega_7\omega_4^3 - 24cs^2\omega_7\omega_4 + 12v_1^4\omega_4^3 + 6cs^4\omega_7\omega_4^3 - 48v_1^4\omega_7\omega_4 + 6cs^2v_1^2\omega_7^2\omega_4^3 - 24cs^2v_1^2\omega_4^2 + 72v_1^4\omega_7\omega_4^2 + 24cs^4\omega_7^2 + 24cs^2\omega_7\omega_4^2 + 156cs^2v_1^2\omega_7^2\omega_4 - 18v_1^4\omega_7\omega_4^3 + 24cs^4\omega_7\omega_4 + 12cs^2v_1^2\omega_4^3 - 6cs^2\omega_7\omega_4^3 + 48v_1^2\omega_7\omega_4 + 3v_1^4\omega_7^2\omega_4^3 - 24cs^2v_1^2\omega_7\omega_4 - 48cs^4\omega_7^2\omega_4 - 12v_1^2\omega_4^3 + cs^2\omega_7^2\omega_4^3 - 24v_1^2\omega_7^2\omega_4 - 96cs^2v_1^2\omega_7^2 - 24v_1^4\omega_7^2\omega_4^2 - 8cs^2\omega_7^2\omega_4^2 + 24v_1^2\omega_4^3 - 3v_1^2\omega_7^2\omega_4^3 + 12cs^2\omega_7^2\omega_4 - 3cs^4\omega_7^2\omega_4^3 - 12cs^2v_1^2\omega_7\omega_4^3 + 24v_1^4\omega_7^2\omega_4 + 24v_1^2\omega_7^2\omega_4^2 + 48cs^2v_1^2\omega_7\omega_4^2 + 24cs^4\omega_7^2\omega_4^2) \frac{\rho}{24\omega_7^2\omega_4^3}$$

$$C_{D_x^4 v_2}^{(2), \text{MRT}^2} = (-96v_1^2\omega_7^2cs^2 - 72v_1^2\omega_7\omega_4^2 - 8\omega_7^2cs^2\omega_4^2 + 24\omega_7cs^4\omega_4 - 24v_1^4\omega_4^2 + \omega_7^2cs^2\omega_4^3 + 18v_1^2\omega_7\omega_4^3 + 156v_1^2\omega_7^2cs^2\omega_4 + 12v_1^4\omega_4^3 - 48v_1^4\omega_7\omega_4 + 72v_1^4\omega_7\omega_4^2 + 6\omega_7cs^4\omega_4^3 - 72v_1^2\omega_7^2cs^2\omega_4^2 - 24\omega_7cs^4\omega_4^2 - 18v_1^4\omega_7\omega_4^3 + 24\omega_7^2cs^4 + 6v_1^2\omega_7^2cs^2\omega_4^3 + 48v_1^2\omega_7\omega_4 + 12\omega_7^2cs^2\omega_4 + 3v_1^4\omega_7^2\omega_4^3 - 3\omega_7^2cs^4\omega_4^3 - 12v_1^2\omega_4^3 + 48v_1^2\omega_7cs^2\omega_4^2 - 24v_1^2\omega_7^2\omega_4 + 24\omega_7^2cs^4\omega_4^2 - 24v_1^4\omega_7^2\omega_4^2 - 24\omega_7cs^2\omega_4 - 12v_1^2\omega_7cs^2\omega_4^3 + 24v_1^2\omega_4^2 + 24\omega_7cs^2\omega_4^2 - 3v_1^2\omega_7^2\omega_4^3 + 12v_1^2cs^2\omega_4^3 + 24v_1^4\omega_7^2\omega_4 - 48\omega_7^2cs^4\omega_4 + 24v_1^2\omega_7^2\omega_4^2 - 24v_1^2\omega_7cs^2\omega_4 - 6\omega_7cs^2\omega_4^3 - 24v_1^2cs^2\omega_4^2) \frac{\rho}{24\omega_7^2\omega_4^3}$$

$$C_{D_x^4 v_2}^{(2), \text{CLBM}^1} = (6\omega_7\omega_4^3cs^4 - 72v_1^2\omega_7\omega_4^2 - 12v_1^2\omega_7^2\omega_4^2cs^2 + 24\omega_7^2cs^4 + \omega_7^2\omega_4^3cs^2 - 72v_1^4\omega_4^2 + 30v_1^2\omega_7\omega_4^3 + 36v_1^4\omega_4^3 + 72v_1^2\omega_7\omega_4cs^2 - 8\omega_7^2\omega_4^3cs^2 + 6v_1^2\omega_7^2\omega_4^3cs^2 + 72v_1^4\omega_7\omega_4^2 - 24\omega_7\omega_4^2cs^4 - 30v_1^4\omega_7\omega_4^3 - 24\omega_7\omega_4cs^2 - 48\omega_7^2\omega_4^3cs^4 + 3v_1^4\omega_7^2\omega_4^3 + 24\omega_7\omega_4^2cs^2 - 72v_1^2\omega_7\omega_4^3cs^2 - 36v_1^2\omega_4^3 + 24\omega_7^2\omega_4^3cs^4 + 12\omega_7^2\omega_4cs^2 - 12v_1^4\omega_7^2\omega_4^2 - 216v_1^2\omega_4^2cs^2 + 24\omega_7\omega_4cs^4 + 72v_1^2\omega_4^2 - 3v_1^2\omega_7^2\omega_4^3 - 3\omega_7^2\omega_4^3cs^4 + 144v_1^2\omega_7\omega_4^2cs^2 - 6\omega_7\omega_4^3cs^2 + 12v_1^2\omega_7^2\omega_4^2 + 108v_1^2\omega_4^3cs^2 - 36v_1^2\omega_7^2\omega_4cs^2) \frac{\rho}{24\omega_7^2\omega_4^3}$$

$$C_{D_x^4 v_2}^{(2), \text{CLBM}^2} = (-8cs^2\omega_7^2\omega_4^2 - 72v_1^2\omega_7\omega_4^2 - 36cs^2v_1^2\omega_7^2\omega_4 - 72v_1^4\omega_4^2 + 30v_1^2\omega_7\omega_4^3 + cs^2\omega_7^2\omega_4^3 + 36v_1^4\omega_4^3 - 48cs^4\omega_7^2\omega_4 + 24cs^4\omega_7^2\omega_4^2 + 6cs^2v_1^2\omega_7^2\omega_4^3 + 72v_1^4\omega_7\omega_4^2 - 30v_1^4\omega_7\omega_4^3 - 12cs^2v_1^2\omega_7^2\omega_4^2 - 3cs^4\omega_7^2\omega_4^3 + 12cs^2\omega_7^2\omega_4 + 3v_1^4\omega_7^2\omega_4^3 + 24cs^4\omega_7^2 + 6cs^4\omega_7\omega_4^3 + 144cs^2v_1^2\omega_7\omega_4^2 - 36v_1^2\omega_4^3 - 24cs^2\omega_7\omega_4 - 72cs^2v_1^2\omega_7\omega_4^3 - 24cs^4\omega_7\omega_4^2 - 12v_1^4\omega_7^2\omega_4^2 + 72v_1^2\omega_4^2 - 3v_1^2\omega_7^2\omega_4^3 - 6cs^2\omega_7\omega_4^3 + 108cs^2v_1^2\omega_4^3 + 24cs^4\omega_7\omega_4 + 24cs^2\omega_7\omega_4^2 + 12v_1^2\omega_7^2\omega_4^2 - 216cs^2v_1^2\omega_4^2 + 72cs^2v_1^2\omega_7\omega_4) \frac{\rho}{24\omega_7^2\omega_4^3}$$

$$C_{D_x^4 v_2}^{(2), \text{CuLBM}^1} = (30\omega_3^3v_1^2\omega_4 + 24\omega_3^2cs^2\omega_4 - 216\omega_3^2v_1^2cs^2 + 72\omega_3^2v_1^4\omega_4 - 72\omega_3^3v_1^4 - 48\omega_3cs^4\omega_4^2 + 6\omega_3^3cs^4\omega_4 + 144\omega_3^3v_1^2cs^2\omega_4 + 6\omega_3^3v_1^2cs^2\omega_4^2 + 36\omega_3^3v_1^4 - 72\omega_3^3v_1^2cs^2\omega_4 - 12\omega_3^3v_1^2cs^2\omega_4^2 - 12\omega_3^3v_1^4\omega_4^2 + 24\omega_3cs^4\omega_4 - 3\omega_3^3cs^4\omega_4^2 - 3\omega_3^3v_1^2\omega_4^2 - 8\omega_3^3cs^2\omega_4^2 - 36\omega_3^3v_1^2 + 24cs^4\omega_4^2 - 30\omega_3^3v_1^4\omega_4 - 24\omega_3^2cs^4\omega_4 - 72\omega_3^2v_1^2\omega_4 + 12\omega_3cs^2\omega_4^2 + 72\omega_3v_1^2cs^2\omega_4 + 108\omega_3^3v_1^2cs^2 - 6\omega_3^3cs^2\omega_4 + 12\omega_3^2v_1^2\omega_4^2 + 72\omega_3^2v_1^2 - 24\omega_3cs^2\omega_4 - 36\omega_3v_1^2cs^2\omega_4^2 + \omega_3^3cs^2\omega_4^2 + 3\omega_3^3v_1^2\omega_4^2 + 24\omega_3^2cs^4\omega_4^2) \frac{\rho}{24\omega_3^3\omega_4^2}$$

$$C_{D_x^4 v_2}^{(2), \text{CuLBM}^2} = (24\omega_3\omega_1^2cs^2 - 36\omega_3^2v_1^2\omega_1cs^2 + 30\omega_3v_1^2\omega_1^3 + 36v_1^4\omega_1^3 + 24\omega_3^2\omega_1^2cs^4 - 216v_1^2\omega_1^2cs^2 - 72\omega_3v_1^2\omega_1^3cs^2 + 12\omega_3^2\omega_1cs^2 - 72\omega_3v_1^2\omega_1^2 + 24\omega_3\omega_1cs^4 - 72v_1^4\omega_1^2 +$$

$$\begin{aligned} & 108v_1^2\omega_1^3cs^2 - 3\omega_2^3\omega_1^3cs^4 - 12\omega_2^3v_1^4\omega_1^2 - 6\omega_3\omega_1^3cs^2 + 3\omega_3^2v_1^4\omega_1^3 + 144\omega_3v_1^2\omega_1^2cs^2 + 6\omega_3\omega_1^3cs^4 + 72\omega_3v_1^2\omega_1^2 + \omega_3^2\omega_1^3cs^2 + 72v_1^2\omega_1^2 + 72\omega_3v_1^2\omega_1cs^2 - \\ & 30\omega_3v_1^4\omega_1^3 - 36v_1^2\omega_1^3 + 6\omega_2^3v_1^2\omega_1^3cs^2 - 3\omega_2^3v_1^2\omega_1^3 - 8\omega_2^3\omega_1^2cs^2 - 24\omega_3\omega_1^2cs^4 - 24\omega_3\omega_1cs^2 + 12\omega_2^3v_1^2\omega_1^2 - 12\omega_2^3v_1^2\omega_1^2cs^2 + 24\omega_2^3cs^4 - 48\omega_2^3\omega_1cs^4) \frac{\rho}{24\omega_3\omega_1^3} \end{aligned}$$

coefficient $C_{D_x^3 D_y \rho}^{(2)}$ at $\frac{\partial^4 \rho}{\partial x_1^3 \partial x_2}$:

$$G_{\mathbf{D}_x^3 \mathbf{D}_y \rho}^{(2), \text{SRT}} = (24 - \omega^3 + v_1^2 \omega^3 - 72cs^2 - 24v_1^2 - 42\omega^2 cs^2 + 14\omega^2 - 14v_1^2 \omega^2 + 36v_1^2 \omega + 3\omega^3 cs^2 - 36\omega + 108\omega cs^2) \frac{v_1 cs^2}{12\omega^3}$$

$$\begin{aligned}
C_{D_3^2, \text{MRT1}}^{(2)} = & (48\omega_6\omega_6v_1^2\omega_7\omega_4^2v_2^2\omega_8\omega_5^2 - 12\omega_6\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + 36\omega_6cs^2\omega_7\omega_4^2v_2^2\omega_8\omega_5^2 - 36\omega_6\omega_6cs^2\omega_7\omega_4^2v_2^2\omega_8\omega_5 + 12\omega_6\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 - \\
& 72\omega_6\omega_6cs^2\omega_7\omega_4^2v_2^2\omega_8\omega_5^2 + 6\omega_6cs^2\omega_7\omega_4^3\omega_8\omega_5^2 + 12\omega_6\omega_6\omega_4^3v_2^2\omega_8\omega_5^2 + 12\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + 54\omega_6\omega_6cs^4\omega_7^2\omega_4^2\omega_8\omega_5 - 12\omega_6\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + \\
& 6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 18\omega_6\omega_6cs^2\omega_7^2\omega_4^2\omega_8\omega_5 - 18\omega_6cs^4\omega_7\omega_4^3\omega_8\omega_5^2 + 6\omega_6\omega_6v_1^2\omega_7\omega_4^2v_2^2\omega_8\omega_5 + 36\omega_6\omega_6\omega_7\omega_4^2v_2^2\omega_8\omega_5 + 36\omega_6\omega_6cs^2\omega_7^2\omega_4^2v_2^2\omega_8\omega_5 + \\
& 12\omega_6\omega_7^2\omega_4^2v_2^2\omega_8\omega_5 - 5\omega_6\omega_6cs^2v_1^2\omega_7^2\omega_4^3\omega_8\omega_5 + 12\omega_6cs^4\omega_7^2\omega_4^3\omega_5 + 24\omega_6v_1^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + 12\omega_6cs^2\omega_7^2\omega_4^3\omega_8\omega_5^2 + 12\omega_6\omega_6cs^4\omega_7\omega_4\omega_8\omega_5^2 - \\
& 12\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 + 12\omega_6\omega_6cs^2v_1^2\omega_7^2\omega_4\omega_8\omega_5 + 9\omega_6\omega_6v_1^2\omega_7^2\omega_4^2v_2^2\omega_8\omega_5 - 24\omega_6\omega_7\omega_4^3v_2^2\omega_8\omega_5 + 12\omega_6\omega_6\omega_7\omega_4^2v_2^2\omega_8\omega_5 - 36\omega_6cs^2\omega_7^2\omega_4^3\omega_8\omega_5^2 - \\
& 36\omega_6cs^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 + 15\omega_6\omega_6\omega_7\omega_4^2v_2^2\omega_8\omega_5 + 36\omega_6\omega_6cs^2\omega_7^2\omega_4^3\omega_5 + 36\omega_6cs^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + 36\omega_6cs^2\omega_7^2\omega_4^3\omega_5^2 + 6\omega_6\omega_6cs^2v_1^2\omega_7^2\omega_8\omega_5^2 + \\
& 12\omega_6\omega_6cs^2\omega_7\omega_4\omega_8\omega_5^2 - 6\omega_6\omega_6\omega_7\omega_4^3v_2^2\omega_8\omega_5 + 12\omega_6\omega_6cs^4\omega_4^3\omega_8\omega_5^2 + 6\omega_6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - 36\omega_6\omega_6v_1^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + 72\omega_6\omega_6cs^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + \\
& 3\omega_6\omega_6cs^2\omega_7\omega_4^3\omega_8\omega_5^2 + 6\omega_6cs^2v_1^2\omega_7^2\omega_4^3\omega_8\omega_5 - 6\omega_6v_1^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - 12\omega_6\omega_6v_1^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + 36cs^4\omega_7^2\omega_4^3\omega_5^2 - 6\omega_6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 + \\
& \omega_6\omega_6cs^2v_1^2\omega_7^2\omega_4^3\omega_8\omega_5^2 + 15\omega_6\omega_6cs^4\omega_7\omega_4^3\omega_8\omega_5^2 - 12\omega_6\omega_6cs^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - 6\omega_6cs^2v_1^2\omega_7^2\omega_4^3\omega_8\omega_5^2 - 12\omega_6\omega_6cs^2v_1^2\omega_7^2\omega_4\omega_8\omega_5^2 + 18\omega_6cs^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - \\
& 48\omega_6\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 - 36\omega_6cs^2\omega_7^2\omega_4^3\omega_5 - 24\omega_6v_1^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 6\omega_6\omega_6\omega_7^2v_2^2\omega_8\omega_5^2 + 12\omega_6\omega_6cs^2\omega_7^2\omega_4^3\omega_8\omega_5^2 - 36\omega_6\omega_6cs^2\omega_7^2\omega_4^3\omega_5 + \\
& 24\omega_6\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 - 15\omega_6\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + 12\omega_6\omega_6v_2^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - 18\omega_6cs^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 - 9\omega_6\omega_6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - 12\omega_6\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5 + \\
& 6\omega_6\omega_6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - 6v_1^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 + 12\omega_6v_1^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 6cs^4v_1^2\omega_7^2\omega_4^3\omega_8\omega_5^2 - 12\omega_6cs^2v_1^2\omega_7^2\omega_4^3\omega_8\omega_5^2 + 6\omega_6\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 - 6\omega_6\omega_6cs^2\omega_7^2\omega_4^3\omega_8\omega_5^2 + \\
& 18\omega_6\omega_6cs^2v_1^2\omega_7^2\omega_4^3\omega_8\omega_5^2 - 60\omega_6\omega_6cs^4\omega_7^2\omega_4^3\omega_8\omega_5^2 + 18\omega_6\omega_6cs^2\omega_7\omega_4^3v_2^2\omega_8\omega_5 - 36\omega_6cs^2\omega_7^2\omega_4^3\omega_8\omega_5^2 - 36\omega_6cs^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + 18\omega_6\omega_6cs^2\omega_4^3v_2^2\omega_8\omega_5^2 - 24\omega_6\omega_6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + \\
& 18\omega_6cs^4\omega_7^2\omega_4^3\omega_8\omega_5^2 + 18\omega_6cs^4\omega_7^2\omega_4^3\omega_8\omega_5 - 12\omega_6\omega_6cs^2v_1^2\omega_7^2\omega_4^3\omega_8\omega_5^2 - 36\omega_6cs^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 6\omega_6cs^2\omega_7^2\omega_4^3\omega_8\omega_5^2 - 18\omega_6\omega_6cs^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + \\
& 18\omega_6\omega_6cs^2v_1^2\omega_7^2\omega_4^3\omega_8\omega_5 + 36\omega_6cs^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 - 24\omega_6\omega_6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 + 12\omega_6cs^2v_1^2\omega_7^2\omega_4^3\omega_8\omega_5^2 + 12\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + \\
& 12cs^2v_1^2\omega_7^2\omega_4^3\omega_5^2 + 144\omega_6\omega_6cs^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 - 6\omega_6cs^2\omega_7^2\omega_4^3\omega_8\omega_5 - 12\omega_6\omega_6cs^2v_1^2\omega_7^2\omega_4^3\omega_5 - 12\omega_6\omega_6cs^2\omega_7^2\omega_4\omega_8\omega_5^2 - 12\omega_6\omega_6v_1^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 + \\
& 36\omega_6cs^4\omega_7\omega_4^3\omega_8\omega_5^2 + 5\omega_6\omega_6cs^2\omega_7^2\omega_4^3\omega_8\omega_5 - 36\omega_6\omega_4^3\omega_7^2\omega_4^3\omega_5 - 12\omega_6v_1^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 12\omega_6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 + 27\omega_6\omega_6cs^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + \\
& 36\omega_6\omega_6cs^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - 12\omega_6cs^2\omega_7^2\omega_4^3\omega_5^2 - 12\omega_6\omega_6cs^2v_1^2\omega_7^2\omega_4\omega_8\omega_5^2 + 156\omega_6\omega_6cs^4\omega_7^2\omega_4\omega_8\omega_5^2 - 15\omega_6\omega_6cs^2\omega_7^2\omega_4\omega_8\omega_5 + 72\omega_6cs^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - \\
& 12\omega_6\omega_6cs^2\omega_7^2\omega_4^3\omega_5 - 12\omega_6cs^2\omega_7\omega_4^3\omega_8\omega_5^2 - 36\omega_6\omega_6cs^4\omega_7^2\omega_4\omega_8\omega_5 - 12\omega_6v_1^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + 6\omega_$$

$$\begin{aligned}
C_{D_3^{\text{D}}\chi\rho}^{(2),\text{MRT2}} = & (48\omega_6\omega_6v_1^2\omega_7\omega_4^2v_2^2\omega_5\omega_5^2 + 36\omega_7^2cs^2\omega_4^3v_2^2\omega_5^2 - 12\omega_6\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + 12\omega_6\omega_6\omega_7^2cs^2\omega_4\omega_8\omega_5 - 42\omega_6\omega_6\omega_7cs^4\omega_2^3\omega_8\omega_5^2 - \\
& 72\omega_6\omega_7cs^2\omega_1^2v_2^2\omega_8\omega_5^2 + 12\omega_6\omega_7\omega_4^3v_2^2\omega_5^2 + 12\omega_6\omega_6\omega_4^3v_2^2\omega_8\omega_5^2 + 12\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 - \omega_6\omega_6\omega_7^2cs^2\omega_3^3\omega_8\omega_5^2 - 36\omega_6\omega_7^2cs^4\omega_4^3\omega_5 - \\
& 45\omega_6\omega_6\omega_7cs^2\omega_1^3v_2^2\omega_8\omega_5^2 - 12\omega_6\omega_7\omega_1^3v_2^2\omega_8\omega_5^2 + 72\omega_6\omega_7^2cs^2\omega_1^3v_2^2\omega_8\omega_5 - 36\omega_6\omega_6\omega_7cs^2\omega_1^3v_2^2\omega_8\omega_5 + 6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 + 27\omega_6\omega_6\omega_7^2cs^2\omega_3^3v_2^2\omega_8\omega_5 - \\
& 12\omega_6\omega_6v_1^2\omega_7cs^2\omega_4\omega_8\omega_5^2 + 6\omega_6\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + 6\omega_7^2cs^2\omega_1^3\omega_8\omega_5^2 + 36\omega_6\omega_6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + 12\omega_6v_1^2\omega_7^2cs^2\omega_1^2\omega_5^2 + 12v_1^2\omega_7^2cs^2\omega_4^3\omega_5 - \\
& 12\omega_6\omega_6v_1^2\omega_7^2cs^2\omega_4^3\omega_5 + 12\omega_6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 + 24\omega_6v_1^2\omega_7\omega_1^2v_2^2\omega_8\omega_5 - 5\omega_6\omega_6v_1^2\omega_7^2cs^2\omega_3^3\omega_8\omega_5 - 12\omega_7^2\omega_1^3v_2^2\omega_5^2 + 9\omega_6\omega_6v_1^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - \\
& 24\omega_6v_1^2\omega_7^2v_2^2\omega_8\omega_5 + 12\omega_6\omega_6\omega_7\omega_1^2v_2^2\omega_8\omega_5 + 12\omega_6\omega_6v_1^2\omega_7^2cs^2\omega_4\omega_8\omega_5^2 + 15\omega_6\omega_6\omega_7\omega_1^3v_2^2\omega_8\omega_5^2 - 60\omega_6\omega_6\omega_7^2cs^4\omega_2^3\omega_8\omega_5^2 + 3\omega_6\omega_6\omega_7cs^2\omega_1^3\omega_8\omega_5^2 - \\
& 6\omega_6\omega_6\omega_7\omega_1^3v_2^2\omega_8\omega_5 - 18\omega_7^2cs^2\omega_1^3v_2^2\omega_8\omega_5^2 + 6\omega_6\omega_7\omega_1^3v_2^2\omega_8\omega_5 - 36\omega_6\omega_6v_1^2\omega_7^2\omega_1^2v_2^2\omega_8\omega_5 - 18\omega_6\omega_6\omega_7^2cs^2\omega_3^3\omega_4^3\omega_5^2 + 12\omega_6\omega_6\omega_7cs^2\omega_4\omega_8\omega_5^2 + \\
& 6\omega_6v_1^2\omega_7^2cs^2\omega_4^3\omega_8\omega_5 - 12\omega_6v_1^2\omega_7^2cs^2\omega_1^2\omega_5^2 + 54\omega_6\omega_6\omega_7^2cs^4\omega_4^3\omega_8\omega_5 - 6\omega_6\omega_7^2cs^2\omega_4^3\omega_8\omega_5^2 - 6\omega_6v_1^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - 12\omega_6\omega_6v_1^2\omega_7^2\omega_4^3v_2^2\omega_5 - \\
& 6\omega_6\omega_7^2cs^2\omega_3^3\omega_8\omega_5 + 36\omega_6\omega_7cs^2\omega_4^3\omega_8\omega_5^2 - 12\omega_6v_1^2\omega_7^2cs^2\omega_3^3\omega_5 + 12\omega_6v_1^2\omega_7\omega_1cs^2\omega_4^3\omega_8\omega_5^2 - 6\omega_6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 96\omega_6\omega_6\omega_7^2cs^4\omega_8\omega_5^2 + \\
& 12\omega_6\omega_7^2cs^2\omega_4^3\omega_5 - 12\omega_6\omega_6\omega_7\omega_4^3v_2^2\omega_5 + 12\omega_6\omega_6cs^4\omega_4\omega_8\omega_5^2 + \omega_6\omega_6v_1^2\omega_7^2cs^2\omega_4^3\omega_8\omega_5^2 - 36\omega_6\omega_7^2cs^2\omega_1^3v_2^2\omega_5 - 36\omega_6\omega_7^2cs^2\omega_4^3v_2^2\omega_5^2 - \\
& 12\omega_6\omega_6v_1^2\omega_7^2cs^2\omega_4\omega_8\omega_5 - 48\omega_6\omega_6\omega_7\omega_1^2v_2^2\omega_8\omega_5^2 + 144\omega_6\omega_6\omega_7cs^2\omega_4^3v_2^2\omega_8\omega_5^2 - 24\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5 - 6\omega_6\omega_6\omega_1^2v_2^2\omega_8\omega_5^2 + \\
& 36\omega_6\omega_6\omega_7^2cs^2\omega_3^3\omega_4^3\omega_5^2 - 3\omega_6\omega_6v_1^2\omega_7cs^2\omega_4^3\omega_8\omega_5^2 + 36\omega_6\omega_7cs^2\omega_3^3v_2^2\omega_8\omega_5^2 + 24\omega_6\omega_7\omega_1^2v_2^2\omega_8\omega_5^2 - 15\omega_6\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + \\
& 12\omega_6\omega_6v_1^2\omega_7^2\omega_1^2v_2^2\omega_8 - 6\omega_6\omega_6cs^2\omega_4^3\omega_8\omega_5^2 - 12\omega_6\omega_6\omega_7^2cs^2\omega_4\omega_8\omega_5^2 - 9\omega_6\omega_6\omega_7^2\omega_1^2v_2^2\omega_8\omega_5 - 12\omega_6\omega_6v_1^2\omega_7\omega_1^2v_2^2\omega_8\omega_5 - \\
& 108\omega_6\omega_6\omega_7^2cs^2\omega_4^3v_2^2\omega_8\omega_5 + 6\omega_6\omega_6\omega_7\omega_4^3v_2^2\omega_8 - 6v_1^2\omega_7^2\omega_1^2v_2^2\omega_8\omega_5^2 + 6\omega_6\omega_7cs^2\omega_4^3\omega_8\omega_5^2 + 12\omega_6v_1^2\omega_7^2\omega_1^2\omega_5^2 - 12\omega_6\omega_6v_1^2cs^2\omega_4^3\omega_8\omega_5^2 + \\
& 5\omega_6\omega_6\omega_7^2cs^2\omega_3^3\omega_4^3\omega_8\omega_5^2 - 36\omega_6\omega_7^2cs^4\omega_1^2\omega_8\omega_5^2 + 12\omega_6\omega_6v_1^2\omega_7^2cs^2\omega_4^3\omega_5^2 + 18\omega_6\omega_6\omega_7^2cs^2\omega_4^3v_2^2\omega_8\omega_5 - 18\omega_6\omega_7^2cs^2\omega_4^3v_2^2\omega_8\omega_5 + 6\omega_6\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 - \\
& 6v_1^2\omega_7^2cs^2\omega_3^3\omega_8\omega_5^2 - 12\omega_6v_1^2\omega_7^2cs^2\omega_4^3\omega_8\omega_5^2 + 3\omega_6\omega_6\omega_7^2cs^4\omega_3^3\omega_8\omega_5^2 + 12\omega_6\omega_7^2cs^2\omega_4^3\omega_5 - 24\omega_6\omega_6\omega_7\omega_4^3v_2^2\omega_8\omega_5 - 36\omega_6\omega_6\omega_7^2cs^4\omega_4\omega_8\omega_5 + \\
& 12\omega_6\omega_6\omega_7^2cs^2\omega_4^3\omega_5 - 18\omega_6\omega_6\omega_7cs^4\omega_4\omega_8\omega_5^2 + 18\omega_6\omega_6v_1^2\omega_7^2cs^2\omega_4^3\omega_8\omega_5 - 18\omega_6\omega_7cs^2\omega_4^3v_2^2\omega_8\omega_5^2 + 36\omega_6\omega_6\omega_7^2cs^4\omega_4^3\omega_5 - 18\omega_7^2cs^4\omega_2^3\omega_8\omega_5^2 - \\
& 24\omega_6\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + 12v_1^2\omega_7^2\omega_4^3v_2^2\omega_5^2 - 36\omega_6\omega_7^2cs^2\omega_4^3v_2^2\omega_8\omega_5^2 + 36\omega_6\omega_7^2cs^4\omega_1^2\omega_5^2 + 12\omega_6v_1^2\omega_7^2v_2^2\omega_8\omega_5^2 + 18\omega_6\omega_6cs^2\omega_1^3v_2^2\omega_8\omega_5^2 - \\
& 12\omega_6\omega_6v_1^2\omega_7^2v_2^2\omega_8\omega_5^2 - 12\omega_7^2cs^2\omega_4^3\omega_5 - 12\omega_6v_1^2\omega_7\omega_1^2v_2^2\omega_8\omega_5^2 - 12\omega_6\omega_7^2\omega_1^3v_2^2\omega_5^2 + 15\omega_6\omega_6\omega_7cs^4\omega_3^3\omega_8\omega_5^2 + 12\omega_6\omega_6\omega_7^2cs^2\omega_4^3\omega_8\omega_5^2 + \\
& 18\omega_6\omega_7^2\omega_1^3\omega_8\omega_5 - 12\omega_6\omega_7cs^2\omega_1^3\omega_8\omega_5^2 + 6\omega_6\omega_7cs^2\omega_1^3\omega_8\omega_5^2 - 12\omega_6v_1^2\omega_7^2\omega_1^2v_2^2\omega$$

$$C_{D_x^3 D_y \rho}^{(2), \text{CLBM1}} = (18\omega_9\omega_6v_1^2\omega_7\omega_4\omega_8\omega_5 + 12\omega_9\omega_6\omega_4\omega_8\omega_5 + 12\omega_6\omega_7^2\omega_5 + 12\omega_6\omega_7^2\omega_4\omega_8\omega_5 - 12\omega_7^2\omega_5 + 36\omega_7^2\omega_4^2\omega_5 + 6\omega_6v_1^2\omega_7^2\omega_5\omega_8\omega_5 - 36\omega_6\omega_7^2\omega_4\omega_8\omega_5 - 12\omega_9\omega_6v_1^2\omega_7^2\omega_4 + 6\omega_9\omega_6v_1^2\omega_4^2\omega_8\omega_5 - 12\omega_9v_1^2\omega_7^2\omega_4 - 36\omega_9\omega_6\omega_7^2\omega_4\omega_8\omega_5 - 6v_1^2\omega_7^2\omega_4^2\omega_8\omega_5 + 6\omega_6\omega_7\omega_4^2\omega_8\omega_5 - 36\omega_9\omega_6\omega_7^2\omega_8\omega_5 + 12\omega_9\omega_7^2\omega_5\omega_8\omega_5 - 36\omega_6\omega_7^2\omega_4\omega_8\omega_5 - 36\omega_6\omega_7^2\omega_4^2\omega_8\omega_5 - 36\omega_9\omega_6\omega_7^2\omega_4\omega_8\omega_5 - 6v_1^2\omega_7^2\omega_4^2\omega_8\omega_5 + 6\omega_6\omega_7\omega_4^2\omega_8\omega_5 + 12v_1^2\omega_7^2\omega_4^2\omega_5 + 12\omega_6v_1^2\omega_7\omega_4\omega_8\omega_5 - 36\omega_9\omega_6\omega_7^2\omega_4\omega_8\omega_5 - 12\omega_9\omega_6\omega_7^2\omega_4\omega_8\omega_5 - 36\omega_9\omega_6\omega_7\omega_8\omega_5 + 54\omega_9\omega_6\omega_7^2\omega_4\omega_8\omega_5 - 5\omega_9\omega_6v_1^2\omega_7^2\omega_4\omega_8\omega_5 - 12\omega_6v_1^2\omega_7^2\omega_4^2\omega_8\omega_5 + 12\omega_9\omega_6v_1^2\omega_7^2\omega_4^2 - 12\omega_9\omega_6v_1^2\omega_7\omega_8\omega_5 + 12\omega_9\omega_6\omega_7^2\omega_8 + 12\omega_6v_1^2\omega_7^2\omega_4\omega_5 + 18\omega_9\omega_6v_1^2\omega_7^2\omega_4\omega_8 - 6\omega_6v_1^2\omega_7\omega_8\omega_5 + 18\omega_6\omega_7^2\omega_4\omega_8\omega_5 - 12\omega_9\omega_6\omega_7^2\omega_4 - 12\omega_6\omega_7\omega_4\omega_8\omega_5 - 36\omega_9\omega_7^2\omega_5\omega_8 - 12\omega_9\omega_6v_1^2\omega_7^2\omega_8 - 12\omega_9\omega_6v_1^2\omega_7^2\omega_4\omega_8\omega_5 + 12\omega_9\omega_7^2\omega_4 - 15\omega_9\omega_6\omega_7^2\omega_4\omega_8\omega_5 - 18\omega_9\omega_6\omega_7^2\omega_4\omega_8\omega_5 + 12\omega_9\omega_6\omega_7^2\omega_4\omega_8\omega_5 + 5\omega_9\omega_6\omega_7^2\omega_4\omega_8 + 12\omega_9\omega_6v_1^2\omega_7^2\omega_8\omega_5 - 18\omega_6\omega_7\omega_4^2\omega_8\omega_5 - 36\omega_9\omega_6\omega_7^2\omega_4^2\omega_8\omega_5 - 3\omega_9\omega_6v_1^2\omega_7\omega_4^2\omega_8\omega_5 - 12\omega_9\omega_6v_1^2\omega_4\omega_8\omega_5 - 12\omega_6v_1^2\omega_7^2\omega_4\omega_8\omega_5 - 6\omega_6\omega_7^2\omega_4\omega_8\omega_5 - 18\omega_7^2\omega_4\omega_8\omega_5 - 12\omega_6\omega_7^2\omega_4\omega_5 + 6\omega_7^2\omega_4\omega_8\omega_5 + 12\omega_9\omega_6\omega_7^2\omega_4 - 9\omega_9\omega_6\omega_7\omega_4^2\omega_8\omega_5 + 36\omega_9\omega_6\omega_7^2\omega_8\omega_5 - \omega_9\omega_6\omega_7^2\omega_4^2\omega_8\omega_5 + 36\omega_6\omega_7^2\omega_4\omega_8\omega_5 - 6\omega_9\omega_7^2\omega_4\omega_8 + 6\omega_9v_1^2\omega_7^2\omega_4\omega_8) \frac{v_1cs^2}{12\omega_9\omega_6\omega_7^2\omega_4^2\omega_8\omega_5}$$

$$C_{D_x^3 D_y \rho}^{(2), \text{CLBM2}} = (18\omega_9\omega_6v_1^2\omega_7\omega_4\omega_8\omega_5 + 12\omega_9\omega_6\omega_4\omega_8\omega_5 + 12\omega_6\omega_7^2\omega_4\omega_5 - 15\omega_9cs^2\omega_6\omega_7^2\omega_4\omega_8 - 36cs^2\omega_6\omega_7^2\omega_4\omega_5 - 9\omega_9cs^2\omega_6\omega_7\omega_4^2\omega_8\omega_5 + 12\omega_6\omega_7^2\omega_4\omega_8\omega_5 - 12\omega_7^2\omega_5 - 36cs^2\omega_6\omega_7^2\omega_4\omega_8\omega_5 - 36\omega_9cs^2\omega_6\omega_7\omega_8\omega_5 + 6\omega_6v_1^2\omega_7^2\omega_4^2\omega_8\omega_5 - 12\omega_9\omega_6v_1^2\omega_7^2\omega_4 + 6\omega_9\omega_6v_1^2\omega_4^2\omega_8\omega_5 - 12\omega_9v_1^2\omega_7^2\omega_4 + 18\omega_9cs^2\omega_7^2\omega_4\omega_8 - 36\omega_9cs^2\omega_6\omega_7^2\omega_4 + 12\omega_9\omega_6\omega_7^2\omega_4\omega_8\omega_5 - 6v_1^2\omega_7^2\omega_4^2\omega_8\omega_5 + 6\omega_6\omega_7\omega_4^2\omega_8\omega_5 - 18cs^2\omega_6\omega_7\omega_4^2\omega_8\omega_5 + 12v_1^2\omega_7^2\omega_4\omega_5 + 12\omega_6v_1^2\omega_7\omega_4\omega_8\omega_5 + \omega_9\omega_6v_1^2\omega_7^2\omega_4\omega_8\omega_5 - 18\omega_9\omega_6\omega_7^2\omega_4\omega_8 + 36\omega_9cs^2\omega_6\omega_7^2\omega_4 - 36\omega_9cs^2\omega_6\omega_7^2\omega_4\omega_8\omega_5 - 12\omega_9\omega_6\omega_7^2\omega_8\omega_5 + 3\omega_9\omega_6\omega_7\omega_4^2\omega_8\omega_5 - 5\omega_9\omega_6v_1^2\omega_7^2\omega_4\omega_8 - 12\omega_6v_1^2\omega_7^2\omega_4\omega_5 + 12\omega_9\omega_6v_1^2\omega_7^2\omega_4^2 - 12\omega_9\omega_6v_1^2\omega_7\omega_8\omega_5 + 12\omega_9\omega_6\omega_7^2\omega_8 + 18\omega_9cs^2\omega_6\omega_7^2\omega_4\omega_5 + 12\omega_6v_1^2\omega_7^2\omega_4\omega_5 + 18\omega_9\omega_6v_1^2\omega_7^2\omega_4\omega_8 - 6\omega_6v_1^2\omega_7\omega_4^2\omega_8\omega_5 - 12\omega_9\omega_6\omega_7^2\omega_4 - 12\omega_6\omega_7\omega_4\omega_8\omega_5 + 36cs^2\omega_6\omega_7\omega_4\omega_8\omega_5 + 3\omega_9cs^2\omega_6\omega_7^2\omega_4\omega_8\omega_5 + 36cs^2\omega_7^2\omega_4\omega_5 - 12\omega_9\omega_6v_1^2\omega_7^2\omega_8 - 12\omega_9\omega_6v_1^2\omega_7^2\omega_4\omega_8\omega_5 + 12\omega_9\omega_7^2\omega_4 - 36\omega_9cs^2\omega_6\omega_7^2\omega_8 - 18\omega_9\omega_6\omega_7\omega_4\omega_8\omega_5 + 12\omega_9\omega_6\omega_7\omega_8\omega_5 + 5\omega_9\omega_6\omega_7^2\omega_4\omega_8 - 36\omega_9cs^2\omega_6\omega_4\omega_8\omega_5 + 12\omega_9\omega_6v_1^2\omega_7^2\omega_8\omega_5 + 54\omega_9cs^2\omega_6\omega_7\omega_4\omega_8\omega_5 - 6\omega_9\omega_6\omega_4^2\omega_8\omega_5 - 3\omega_9\omega_6v_1^2\omega_7\omega_4^2\omega_8\omega_5 - 36\omega_9cs^2\omega_7^2\omega_4 - 12\omega_9\omega_6v_1^2\omega_4\omega_8\omega_5 + 36\omega_9cs^2\omega_6\omega_7^2\omega_8\omega_5 - 12\omega_6v_1^2\omega_7^2\omega_4\omega_8\omega_5 - 6\omega_6\omega_7^2\omega_4\omega_8\omega_5 - 18cs^2\omega_6\omega_7^2\omega_4\omega_8\omega_5 - 12\omega_6\omega_7^2\omega_4\omega_5 + 6\omega_7^2\omega_4\omega_8\omega_5 + 12\omega_9\omega_6\omega_7^2\omega_4 - \omega_9\omega_6\omega_7^2\omega_4\omega_8\omega_5 - 6\omega_9\omega_7^2\omega_4\omega_8 + 6\omega_9v_1^2\omega_7^2\omega_4\omega_8) \frac{cs^2v_1}{12\omega_9\omega_6\omega_7^2\omega_4^2\omega_8\omega_5}$$

$$C_{D_x^3 D_y \rho}^{(2), \text{CuLBM1}} = (-9\omega_3^2cs^2\omega_4\omega_1 - 6\omega_3^2\omega_1 - 36cs^2\omega_4^2 - 12\omega_3v_1^2\omega_1 - 18\omega_3^2cs^2\omega_4 + 6\omega_3v_1^2\omega_4 + 36cs^2\omega_4^2\omega_1 + 12\omega_4\omega_1 + 6\omega_3^2\omega_4 + 18\omega_3^2cs^2\omega_1 - \omega_3^2\omega_4^2 - 12\omega_3v_1^2\omega_1 + 3\omega_3^2\omega_4\omega_1 + 12v_1^2\omega_4^2\omega_1 + 12\omega_3\omega_4^2\omega_1 - 3\omega_3^2v_1^2\omega_4\omega_1 + 3\omega_3^2cs^2\omega_4^2 + 12\omega_3v_1^2\omega_4 - 36\omega_3cs^2\omega_7^2\omega_1 - 12v_1^2\omega_4\omega_1 - 18\omega_3\omega_4\omega_1 + \omega_3^2v_1^2\omega_4^2\omega_1 + 6\omega_3^2v_1^2\omega_1 + 54\omega_3cs^2\omega_4\omega_1 - 6\omega_3^2v_1^2\omega_4 - 6\omega_3\omega_4^2 + 18\omega_3cs^2\omega_4^2 - \omega_3^2\omega_4^2\omega_1 - 12v_1^2\omega_4^2 + 12\omega_4^2 + \omega_3^2v_1^2\omega_4^2 - 12\omega_3\omega_4 + 36\omega_3cs^2\omega_4 - 12\omega_7^2\omega_1 - 36\omega_3cs^2\omega_1 + 12\omega_3\omega_1 + 3\omega_3^2cs^2\omega_4^2\omega_1 + 18\omega_3v_1^2\omega_4\omega_1 - 36cs^2\omega_4\omega_1) \frac{v_1cs^2}{12\omega_3^2\omega_4^2\omega_1}$$

$$C_{D_x^3 D_y \rho}^{(2), \text{CuLBM2}} = (72\omega_3^2\omega_1^3cs^4\omega_2^2 + 3\omega_3^2\omega_1^2v_2^2\omega_3^2 + 126\omega_3\omega_1^2cs^4\omega_2^3 - 46\omega_3^2v_1^2\omega_1^2\omega_2^2 + 6\omega_3^2\omega_1^2\omega_2 - 12\omega_3^2\omega_1^2cs^2\omega_2^2 + 6\omega_3\omega_1^3cs^2\omega_2^3 + 6\omega_3^2\omega_1^3cs^4\omega_2^3 + 12v_1^2\omega_1^3cs^2\omega_2^3 - 90\omega_3^2v_1^4\omega_1^3\omega_2 + 72\omega_3\omega_1^2cs^4\omega_2^2 - 39\omega_3^2v_1^4\omega_1^2\omega_2^3 + 51\omega_3^2v_1^2\omega_2^3 + 3\omega_3^2\omega_1^3v_2^2\omega_2^2 + 81\omega_3^2\omega_1^2cs^2\omega_2^3 + 18\omega_3\omega_1^3cs^2\omega_2^2 + 141\omega_3^2v_1^2\omega_1^2cs^2\omega_2 - 51\omega_3^2v_1^2\omega_1^2\omega_2 + 7\omega_3^2\omega_1^3\omega_2^3 + 90\omega_3^2\omega_1^3cs^4 - 6\omega_3\omega_1^2cs^2\omega_2 - 7\omega_3^2\omega_1^2\omega_2^3 - 12\omega_3^2\omega_1^2\omega_2 + 12\omega_3^2v_1^2\omega_1^2cs^2\omega_2^3 + 42\omega_3v_1^2\omega_1^2cs^2\omega_2^3 + 39\omega_3^2v_1^4\omega_1^2\omega_2^2 + 225\omega_3^2\omega_1^2cs^4\omega_2^3 + 6\omega_3^2\omega_1^3 + 30\omega_3\omega_1^2cs^2\omega_2^3 - 219\omega_3^2v_1^2\omega_1^2cs^2\omega_2^3 + 46\omega_3^2v_1^2\omega_1^2\omega_2^3 + 102\omega_3^2v_1^2\omega_1^2\omega_2 + 45\omega_3^2v_1^4\omega_1^3 - 24\omega_3^2\omega_1^2cs^2\omega_2 - 72\omega_1^4cs^4\omega_2^3 - 90\omega_3^2cs^4\omega_2^3 - 90\omega_3^2\omega_1^2cs^4\omega_2^3 - 12\omega_1^3cs^2\omega_2^3 + 45\omega_3^2v_1^4\omega_1^2\omega_2 + 24\omega_3v_1^2\omega_1^2cs^2\omega_2^3 - 153\omega_3^2\omega_1^3cs^4\omega_2 - 24\omega_3\omega_1^2cs^2\omega_2^3 - 6\omega_3^2\omega_1^2\omega_2^3 - 165\omega_3^2v_1^2\omega_1^2cs^2\omega_2^3 - 2\omega_3^2\omega_1^3cs^2\omega_2^3 - 102\omega_3^2v_1^2\omega_1^2\omega_2^3 - 465\omega_3^2v_1^2\omega_1^2cs^2\omega_2 - 45\omega_3^2v_1^4\omega_2^3 - 72\omega_3^2\omega_1^3cs^2 - 54\omega_3\omega_1^3cs^4\omega_2^3 - 3\omega_3^2\omega_1^3v_2^2\omega_2^3 - 24v_1^2\omega_1^2cs^2\omega_2^3 - 138\omega_3^2\omega_1^2cs^4\omega_2^3 - 30\omega_3v_1^2\omega_1^2cs^2\omega_2^3 + 51\omega_3^2v_1^2\omega_1^2\omega_2^3 - 42\omega_3\omega_1^2cs^2\omega_2^3 - 59\omega_3^2\omega_1^2cs^2\omega_2^3 + 489\omega_3^2v_1^2\omega_1^2cs^2\omega_2^3 - 3\omega_3^2\omega_1^2v_2^2\omega_2^3 + 12\omega_3^2\omega_1^2\omega_2^3 - 18\omega_3\omega_1^3cs^4\omega_2^3 + 6\omega_3v_1^2\omega_1^2cs^2\omega_2^3 + 36\omega_3^2\omega_1^2cs^4\omega_2^3 - 6\omega_3^2\omega_2^3 - 9\omega_3^2v_1^2v_2^2cs^2\omega_2^3 + 261\omega_3^2v_1^2\omega_1^2cs^2 + 90\omega_3^2v_1^4\omega_1\omega_2^3 - 90\omega_3\omega_1^2cs^4\omega_2^3 - 51\omega_3^2v_1^2\omega_1^2 + 18\omega_3^2\omega_1^2cs^4\omega_2 - 18\omega_3v_1^2\omega_1^2cs^2\omega_2^3 + 24v_1^2cs^2\omega_2^3 + 72\omega_3^2cs^2\omega_2^3 + 48\omega_3^2\omega_1^2cs^2\omega_2^3 + 2\omega_3^2v_1^2\omega_1^2cs^2\omega_2^3 + 36v_1^2cs^4\omega_2^3 + 123\omega_3^2\omega_1^2cs^2\omega_2 - 6\omega_3v_1^2\omega_1^2cs^2\omega_2^3 + 18\omega_3\omega_1^2cs^4\omega_2 - 45\omega_3^2v_1^4\omega_1\omega_2^3 + 197\omega_3^2v_1^2\omega_1^2cs^2\omega_2^3 - 147\omega_3\omega_1^2cs^2\omega_2^3 - 261\omega_3^2v_1^2cs^2\omega_2^3 + 9\omega_3^2\omega_1^2v_2^2cs^2\omega_2^3) \frac{v_1}{24\omega_3^2\omega_1^3\omega_2^3}$$

$$\text{coefficient } C_{D_x^3 D_y v_1}^{(2)} \text{ at } \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} :$$

$$C_{D_x^3 D_y v_1}^{(2), \text{SRT}} = (-12 + 36v_1^2 + 2\omega^2cs^2 - 6\omega^2 + 18v_1^2\omega^2 - 54v_1^2\omega - \omega^3cs^2 + 18\omega) \frac{\rho cs^2}{12\omega^3}$$

$$C_{D_x^3 D_y v_1}^{(2), \text{MRT1}} = (144\omega_9\omega_6v_1^2\omega_7\omega_4^2v_2^2\omega_8\omega_5^2 - 12\omega_9\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + 12\omega_6cs^2\omega_7\omega_4^2v_2^2\omega_8\omega_5^2 - 12\omega_9\omega_6cs^2\omega_7^2\omega_4^2v_2^2\omega_5 + 12\omega_6\omega_7^2\omega_4^3v_2^2\omega_5^2 - 24\omega_9\omega_6cs^2\omega_7\omega_4^2v_2^2\omega_8\omega_5^2 + 6\omega_6cs^2\omega_7\omega_4^3\omega_8\omega_5^2 + 12\omega_9\omega_6\omega_4^2v_2^2\omega_8\omega_5^2 + 36\omega_9v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + 18\omega_9\omega_6cs^4\omega_7^2\omega_4^2\omega_8\omega_5 - 12\omega_6\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + 6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 18\omega_9\omega_6cs^2\omega_7^2\omega_4^2\omega_8\omega_5 - 6\omega_6cs^4\omega_7\omega_4^3\omega_8\omega_5^2 + 18\omega_9\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5 + 36\omega_9\omega_6\omega_7^2\omega_4^2v_2^2\omega_8\omega_5 + 12\omega_9\omega_6cs^2\omega_7^2\omega_4^2v_2^2\omega_8 + 12\omega_6\omega_7^2\omega_4^3\omega_8\omega_5^2 - 12\omega_7^2\omega_4^3v_2^2\omega_5^2 + 60\omega_9\omega_6cs^2v_1^2\omega_7^2\omega_4\omega_8\omega_5^2 + 27\omega_9\omega_6v_1^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - 24\omega_9\omega_7^2\omega_4^2v_2^2\omega_8\omega_5 + 12\omega_9\omega_6\omega_7\omega_4^3v_2^2\omega_8\omega_5 - 12\omega_6cs^4\omega_7^2\omega_4^2\omega_8\omega_5^2 - 12\omega_6cs^2\omega_7^2\omega_4^2v_2^2\omega_8\omega_5^2 + 15\omega_9\omega_6\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + 12\omega_9\omega_6cs^4\omega_7^2\omega_4^3\omega_5 + 12\omega_6cs^4\omega_7^2\omega_4^2\omega_5^2 + 12\omega_6cs^2\omega_7^2\omega_4^2v_2^2\omega_5^2 - 12\omega_9\omega_6cs^2v_1^2\omega_4^3\omega_8\omega_5^2 - 12\omega_9\omega_6cs^2\omega_7\omega_4\omega_8\omega_5^2 - 6\omega_9\omega_6\omega_7\omega_4^3v_2^2\omega_8\omega_5 + 6\omega_9\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - 108\omega_9\omega_6v_1^2\omega_7^2\omega_4^2v_2^2\omega_8\omega_5 + 24\omega_9\omega_6cs^2\omega_7^2\omega_4^2v_2^2\omega_8\omega_5 - 6\omega_9\omega_6cs^2\omega_7\omega_4^3\omega_8\omega_5^2 + 18\omega_9cs^2v_1^2\omega_7^2\omega_4^3\omega_8\omega_5 - 18\omega_9v_1^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - 36\omega_9\omega_6v_1^2\omega_7^2\omega_4^2v_2^2\omega_5 + 12cs^4\omega_7^2\omega_4^3\omega_5^2 - 6\omega_6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 + 6\omega_9\omega_6cs^2\omega_7\omega_4^3\omega_8\omega_5^2 - 12\omega_9\omega_6\omega_7^2\omega_4^3v_2^2\omega_5 - 18\omega_6cs^2v_1^2\omega_7\omega_4^3\omega_8\omega_5^2 - 36\omega_9\omega_6cs^2v_1^2\omega_7^2\omega_4\omega_8\omega_5 + 6\omega_6cs^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 48\omega_9\omega_6\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 - 12\omega_6cs^4\omega_7^2\omega_4^3\omega_5^2 - 72\omega_9v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 - 6\omega_9\omega_6\omega_4^3v_2^2\omega_8\omega_5^2 + \omega_9\omega_6cs^2\omega_7^2\omega_4^2\omega_8\omega_5^2 - 12\omega_9\omega_6cs^4\omega_7^2\omega_4^2\omega_5 + 24\omega_9\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 - 45\omega_9\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + 36\omega_9\omega_6v_1^2\omega_7^2\omega_4^2v_2^2\omega_8 - 6\omega_6cs^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 - 9\omega_9\omega_6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - 36\omega_9\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5 + 6\omega_9\omega_6\omega_7^2\omega_4^3v_2^2\omega_8 - 18v_1^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 + 36\omega_6v_1^2\omega_7^2\omega_4^2v_2^2\omega_5 - 18cs^2v_1^2\omega_7^2\omega_4^3\omega_8\omega_5^2 - 36\omega_6cs^2v_1^2\omega_7^2\omega_4^2\omega_8\omega_5^2 + 6\omega_6\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 - 102\omega_9\omega_6cs^2v_1^2\omega_7\omega_4^3\omega_8\omega_5^2 - 5\omega_9\omega_6cs^4\omega_7^2\omega_4\omega_8\omega_5^2 + 6\omega_9\omega_6cs^2\omega_7\omega_4^3v_2^2\omega_8\omega_5 - 12\omega_9cs^2\omega_7^2\omega_4^2v_2^2\omega_5 + 6\omega_9\omega_6cs^2\omega_4^3v_2^2\omega_8\omega_5^2 - 24\omega_9\omega_6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + 6\omega_6cs^4\omega_7^2\omega_4^3\omega_8\omega_5^2 + 6\omega_9cs^4\omega_7^2\omega_4^3\omega_8\omega_5 + 24\omega_9\omega_6cs^2v_1^2\omega_7^2\omega_4\omega_8\omega_5^2 - 12\omega_6cs^2\omega_7^2\omega_4^3v_2^2\omega_5 - 6\omega_6cs^2\omega_7^2\omega_4^3\omega_8\omega_5^2 - 6\omega_9\omega_6cs^2\omega_7^2\omega_4^2v_2^2\omega_8\omega_5^2 + 54\omega_9\omega_6cs^2v_1^2\omega_7^2\omega_4^2\omega_8\omega_5 + 12\omega_9cs^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 - 72\omega_9\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + 36v_1^2\omega_7^2\omega_4^3v_2^2\omega_5^2 + 36\omega_6cs^2v_1^2\omega_7^2\omega_4^2\omega_5^2 + 36\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + 36cs^2v_1^2\omega_7^2\omega_4^3\omega_5^2 + 48\omega_9\omega_6cs^2\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 - 6\omega_9cs^2\omega_7^2\omega_4^3\omega_8\omega_5 - 36\omega_9\omega_6cs^2v_1^2\omega_7^2\omega_4^2\omega_5 - 36\omega_9\omega_6v_1^2\omega_4^3v_2^2\omega_8\omega_5^2 +$$

$$\begin{aligned}
& 12w_6cs^4w_7w_4^2ws_5^2 + 5w_9w_6cs^2w_7^2w_4^2ws_8w_5 - 12w_9cs^4w_7^2w_4^2w_5 - 36w_6v_1^2w_7^2w_4^2v_2^2ws_8^2 - 12w_6w_7^2w_4^2v_2^2w_5^2 + 9w_9w_6cs^2w_7^2w_4^2v_2^2ws_8w_5 + \\
& 12w_9w_6cs^2w_7^2w_4^2v_2^2w_5 - 12w_6cs^2w_7^2w_4^2w_5^2 + 60w_9w_6cs^2v_1^2w_7w_4ws_8w_5^2 + 18w_9w_6cs^4w_7^2w_4ws_8w_5^2 - 5w_9w_6cs^4w_7^2w_4^3ws_8w_5 + 24w_9cs^4w_7^2w_4^2v_2^2ws_8w_5 - \\
& 12w_9w_7^2w_4^2v_2^2w_5 - 12w_6cs^2w_7^2w_4^2ws_8w_5^2 - 12w_9w_6cs^4w_7^2w_4ws_8w_5^2 - 36w_6v_1^2w_7^2w_4^2v_2^2w_5^2 + 18w_6v_1^2w_7^2w_4^2v_2^2ws_8w_5^2 - 12w_9w_6cs^2w_7^2w_4^2v_2^2ws_8 - \\
& 12cs^2w_7^2w_4^2w_5^2 - 36w_6cs^2v_1^2w_7^2w_4^2w_5^2 + 24w_9w_6w_7w_4^2v_2^2ws_8w_5^2 - w_9w_6cs^4w_7^2w_4^3ws_8w_5^2 - 36w_9cs^2v_1^2w_7^2w_4^2w_5^2 + 30w_9w_6cs^2v_1^2w_7w_4^3ws_8w_5 + \\
& 12w_9w_7^2w_4^2v_2^2w_5 + 18w_9w_6v_1^2w_4^2v_2^2ws_8w_5^2 + 6cs^2w_7^2w_4^2ws_8w_5^2 + 18w_6cs^2v_1^2w_7^2w_4^2ws_8w_5^2 - 36w_9v_1^2w_7^2w_4^2v_2^2w_5 + 12cs^2w_7^2w_4^2v_2^2w_5^2 + \\
& 12w_9w_6cs^2w_7^2w_4ws_8w_5 - 6w_9cs^2w_7^2w_4^3v_2^2ws_8w_5 + 72w_9w_6v_1^2w_7^2w_4^2v_2^2ws_8w_5 - 18w_9w_6v_1^2w_7^2w_4^3v_2^2w_5 - 6cs^4w_7^2w_4^3ws_8w_5^2 - 36w_9w_6cs^2w_7^2w_4^2v_2^2ws_8w_5 + \\
& 36w_6cs^2v_1^2w_7w_4^2ws_8w_5^2 + 12w_9w_6cs^2w_7^2w_4^2v_2^2w_5 - 18w_9w_6cs^2v_1^2w_7^2w_4ws_8w_5^2 - 15w_9w_6cs^2v_1^2w_7^2w_4ws_8w_5^2 + 12w_9w_6cs^2w_7^2w_4^2w_5^2 - 12w_9w_6cs^2w_7^2v_2^2ws_8w_5^2 + \\
& 12w_6cs^2w_7^2w_4^2w_5^2 - 6cs^2w_7^2w_4^3v_2^2ws_8w_5^2 - 12w_9w_6cs^4w_7^2ws_8w_5^2 - 12w_9w_6cs^2w_7w_4^2v_2^2ws_8w_5 + 18w_9w_6cs^2w_7w_4^2ws_8w_5^2 - 18w_6v_1^2w_7w_4^3v_2^2ws_8w_5 - \\
& 15w_9w_6cs^4w_7w_4^2v_2^2ws_8w_5^2 - 24w_9cs^2w_7w_4^2v_2^2ws_8w_5^2 + 36w_9w_6cs^2v_1^2w_7^2w_4^2w_5 + 36w_9w_6v_1^2w_7^2w_4^2v_2^2w_5) \frac{\rho}{12w_9w_6w_7^2w_4^3ws_8w_5^2}
\end{aligned}$$

$$C_{D_x \partial_x v_1}^{(2), \text{CLBML1}} = (-12\omega_9\omega_6\omega_4\omega_8\omega_5 - 12\omega_9\omega_6\omega_7\omega_4^3c^2s^2 - 12\omega_6\omega_7\omega_4^3c^2s^2\omega_5 - 12\omega_9\omega_7\omega_3^3c^2s^2 + 6\omega_9\omega_7\omega_4^3\omega_8c^2s^2 - 54\omega_9\omega_6v_1^2\omega_4^3\omega_8\omega_5 + 5\omega_9\omega_6\omega_7\omega_4^3\omega_8 + 18\omega_9\omega_6\omega_7\omega_4^3\omega_8c^2s^2 + 18\omega_6v_1^2\omega_7\omega_4^3\omega_8\omega_5 + 12\omega_7\omega_4^3c^2s^2\omega_5 + 12\omega_9\omega_6\omega_4\omega_8c^2s^2\omega_5 + 36\omega_6v_1^2\omega_4^3\omega_8\omega_5 + 12\omega_6\omega_7\omega_3^3\omega_8\omega_5 - 36\omega_9\omega_6v_1^2\omega_7\omega_4\omega_8 - 12\omega_9\omega_6\omega_7\omega_4^3 + 12\omega_9\omega_6\omega_7\omega_4^3c^2s^2 - 12\omega_6\omega_4^3\omega_8\omega_5 + 12\omega_9\omega_6\omega_7\omega_4^3 - 12\omega_9\omega_6\omega_7\omega_8c^2s^2\omega_5 + 18\omega_9\omega_6\omega_7\omega_4\omega_8c^2s^2\omega_5 + \omega_9\omega_6\omega_7\omega_4^3\omega_8\omega_5 + 18\omega_9v_1^2\omega_7\omega_4^3\omega_8 + 12\omega_9\omega_7\omega_4^3 - 6\omega_9\omega_7\omega_4^3\omega_8 - 18v_1^2\omega_7\omega_4^3\omega_8\omega_5 - 18\omega_9\omega_6\omega_7\omega_4^3\omega_8 - 6\omega_9\omega_6\omega_7^3\omega_8\omega_5 - 6\omega_7\omega_4^3\omega_8c^2s^2\omega_5 - 36\omega_6v_1^2\omega_7^3\omega_8\omega_5 - 12\omega_7\omega_4^3\omega_5 - 5\omega_9\omega_6\omega_7^3\omega_4\omega_8c^2s^2 + 12\omega_6\omega_4^3\omega_8c^2s^2\omega_5 - 36\omega_9\omega_6v_1^2\omega_7\omega_4^3 - 18\omega_6v_1^2\omega_4^3\omega_8\omega_5 + 12\omega_6\omega_7\omega_4^3 - \omega_9\omega_6\omega_7\omega_4^3\omega_8c^2s^2\omega_5 + 12\omega_9\omega_6\omega_7\omega_4\omega_8 + 36\omega_9\omega_6v_1^2\omega_7\omega_4^3 + 18\omega_9\omega_6v_1^2\omega_3^3\omega_8\omega_5 + 36\omega_6v_1^2\omega_7\omega_2^3\omega_5 + 54\omega_9\omega_6v_1^2\omega_7\omega_4^3\omega_8 + 12\omega_6\omega_7\omega_4^3c^2s^2\omega_5 - 18\omega_9\omega_6\omega_4^3\omega_8c^2s^2\omega_5 - 12\omega_9\omega_7\omega_2^3\omega_8c^2s^2\omega_5 + 6\omega_9\omega_6\omega_4^3\omega_8c^2s^2\omega_5 + 18\omega_9\omega_6\omega_4^3\omega_8\omega_5 - 12\omega_6\omega_7\omega_2^3\omega_5 - 3\omega_9\omega_6v_1^2\omega_7\omega_2^3\omega_8\omega_5 + 36\omega_9\omega_6v_1^2\omega_4\omega_8\omega_5 + 6\omega_6\omega_7\omega_3^3\omega_8c^2s^2\omega_5 + 36v_1^2\omega_7\omega_2^3\omega_5 + 6\omega_7\omega_4^3\omega_8\omega_5 - 6\omega_6\omega_4^3\omega_8c^2s^2\omega_5 - 36\omega_9v_1^2\omega_7\omega_4^3 + 6\omega_6\omega_4^3\omega_8\omega_5 - 15\omega_9\omega_6v_1^2\omega_7\omega_4^3\omega_8 - 36\omega_6v_1^2\omega_7\omega_4^3\omega_5 - 6\omega_6\omega_7\omega_4^3\omega_8\omega_5 - 5\omega_9\omega_6\omega_7\omega_4^3\omega_8c^2s^2\omega_5 - 12\omega_9\omega_6\omega_7\omega_4\omega_8c^2s^2\omega_5) \frac{\rho c s^2}{12\omega_9\omega_6\omega_7\omega_4^3\omega_8\omega_5}$$

$$C_{D_3^2 D_2 v_1}^{(2), \text{CuLBMI}} = (-5\omega_3^2 cs^2 \omega_4 \omega_1 + 18\omega_3^2 \omega_1 + 3\omega_3^3 v_1^2 \omega_4 + 6\omega_3^2 cs^2 \omega_4 + 18\omega_3^3 v_1^2 \omega_1 - 6\omega_3^2 \omega_4 - 18\omega_3^2 cs^2 \omega_1 - 6\omega_3^3 \omega_1 + 36\omega_3 v_1^2 \omega_1 + \omega_3^2 \omega_4 \omega_1 - \omega_3^3 cs^2 \omega_4 \omega_1 - 3\omega_3^2 v_1^2 \omega_4 \omega_1 - \omega_3^3 \omega_4 - 36\omega_3 v_1^2 \omega_4 - 54\omega_3^2 v_1^2 \omega_1 - 18\omega_3^3 v_1^2 + 18\omega_3 cs^2 \omega_4 \omega_1 + 6\omega_3^3 cs^2 \omega_1 + 12\omega_3^2 cs^2 + 18\omega_3^2 v_1^2 \omega_4 + \omega_3^3 cs^2 \omega_4 + 12\omega_3 \omega_4 + 36\omega_3^2 v_1^2 - 12\omega_3 cs^2 \omega_4 - 6\omega_3^3 cs^2 - 12\omega_3^2 + 12\omega_3 cs^2 \omega_1 - 12\omega_3 \omega_1 + 6\omega_3^3 - 12cs^2 \omega_4 \omega_1) \frac{\rho cs^2}{12\omega_3^2 \omega_4 \omega_1}$$

coefficient $C_{D_x^2 D_y^2 \rho}^{(2)}$ at $\frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2}$:

[illegible]

$$C_{2D_3D_3\rho}^{(2),MRT2} = (-12\omega_9\omega_6\omega_7cs^2w_4^3v_2^2\omega_8^2 + 36\omega_6^2v_1^2w_7cs^2w_4^2\omega_8 + 12\omega_9v_1^2w_7w_4^3v_2^2\omega_8^2 - 48\omega_9\omega_6w_7cs^4w_4^2\omega_8^2 + 180\omega_9\omega_6^2w_7cs^4w_4\omega_8^2 - 24\omega_9\omega_6^2v_1^2w_7w_4\omega_8 + 6\omega_6^2v_1^2w_4^3\omega_8^2 + 12\omega_9\omega_6^2w_7cs^2w_4^3v_2^2 + 6\omega_6^2w_7cs^2w_4^3\omega_8^2 - 36\omega_9\omega_6^2v_1^2cs^2w_4^2\omega_8^2 + 12\omega_9\omega_6^2w_7cs^4w_4^2 - 36\omega_9\omega_6^2v_1^2w_7cs^2w_4^3\omega_8 + 12\omega_6^2v_1^2w_7w_4^2\omega_8^2 + 18\omega_9\omega_6^2w_7cs^2w_4^3v_2^2\omega_8 + 18\omega_9\omega_6^2w_7cs^2w_4^3\omega_8^2 + 12\omega_9\omega_6^2w_7cs^2w_4^3v_2^2\omega_8^2 + 36\omega_9\omega_6^2v_1^2w_7w_4^3v_2^2\omega_8 + 24\omega_9\omega_6^2v_1^2w_7w_4^3v_2^2\omega_8 + 18\omega_6^2v_1^2w_7cs^2w_4^3\omega_8^2 + 2\omega_9\omega_6^2w_7cs^2w_4^3\omega_8^2 - 6\omega_9\omega_6^2cs^2w_4^3\omega_8^2 - 12\omega_6^2w_7cs^2w_4^3v_2^2\omega_8^2 - 6\omega_6^2w_7cs^2w_4^3v_2^2\omega_8 - 6\omega_6^2v_1^2w_4^3v_2^2\omega_8^2 + 12\omega_9\omega_6^2w_7cs^4w_4^3\omega_8 + 12\omega_6^2cs^2w_4^3v_2^2\omega_8^2 + 24\omega_9\omega_6^2v_1^2w_4^3v_2^2\omega_8^2 - 18\omega_9\omega_6^2v_1^2w_7w_4^3v_2^2\omega_8^2 + 12\omega_6^2w_7cs^2w_4^3v_2^2\omega_8^2 - 36\omega_9\omega_6^2cs^4w_4^2\omega_8^2 + 150\omega_9\omega_6^2w_7cs^4w_4^2\omega_8^2 + 36\omega_6^2w_7cs^4w_4^3\omega_8 + 12\omega_9\omega_6^2w_7cs^2w_4^4\omega_8 - 12\omega_9\omega_6^2w_7cs^4w_4^3 + 12\omega_6^2v_1^2w_7w_4^3\omega_8 + 12\omega_9\omega_6^2v_1^2w_7w_4^3 - 12\omega_9\omega_6^2v_1^2w_7w_4^3v_2^2\omega_8 - 36\omega_9\omega_6^2v_1^2w_7w_4^3\omega_8^2 - 36\omega_9\omega_6^2v_1^2w_7w_4^3v_2^2\omega_8 - 18\omega_6^2w_7cs^4w_4^3\omega_8^2 - 12\omega_9\omega_6^2v_1^2w_7w_4^3v_2^2 - 6\omega_6^2v_1^2w_7w_4^3\omega_8^2 + 108\omega_9\omega_6^2v_1^2w_7cs^2w_4^4\omega_8 - 12\omega_9\omega_6^2w_7cs^2w_4^4\omega_8^2 + 12\omega_9\omega_6^2w_7cs^4w_4^3 - 12\omega_9\omega_6^2v_1^2w_7w_4^3 + 36\omega_9\omega_6^2v_1^2w_7cs^2w_4^3 + 6\omega_9\omega_6^2v_1^2w_7w_4^3v_2^2\omega_8^2 - 12\omega_9\omega_6^2w_7cs^2w_4^3v_2^2 - 12\omega_6^2v_1^2w_4^3v_2^2\omega_8^2 + 12\omega_6^2w_7cs^2w_4^3v_2^2\omega_8 + 12\omega_6^2w_7cs^2w_4^3v_2^2\omega_8 - 18\omega_9\omega_6^2w_7cs^2w_4^3\omega_8 - 36\omega_6^2v_1^2w_7cs^2w_4^3\omega_8^2 - 36\omega_9\omega_6^2w_7cs^4w_4^3\omega_8 + 6\omega_9\omega_6^2cs^2w_4^3v_2^2\omega_8^2 - 12\omega_9\omega_6^2w_7cs^2w_4^3v_2^2\omega_8 - 12\omega_6^2v_1^2w_7w_4^3\omega_8^2 - 12\omega_6^2v_1^2w_7w_4^3v_2^2\omega_8 - 2\omega_9\omega_6^2w_7cs^2w_4^3v_2^2\omega_8^2 - 6\omega_9\omega_6^2w_7cs^4w_4^3\omega_8^2 - 18\omega_9\omega_6^2w_7cs^2w_4^2\omega_8^2 + 12\omega_9\omega_6^2cs^2w_4^2\omega_8^2 - 12\omega_9\omega_6^2w_7cs^2w_4^3v_2^2\omega_8 - 36\omega_6^2v_1^2w_7cs^2w_4^2\omega_8^2 + 12\omega_9\omega_6^2w_7cs^4w_4\omega_8 - 36\omega_9\omega_6^2v_1^2w_7cs^2w_4^2 - 12\omega_6^2w_7cs^2w_4^3\omega_8^2 + 12\omega_9\omega_6^2v_1^2w_7w_4^3 + 6\omega_9\omega_6^2w_7cs^2w_4^3\omega_8 - 12\omega_9\omega_7cs^2w_4^3\omega_8^2 + 18\omega_9\omega_6^2v_1^2w_7w_4^3\omega_8^2 + 36\omega_9v_1^2w_7cs^2w_4^3\omega_8^2 + 12\omega_9\omega_6^2v_1^2w_7w_4^3v_2^2\omega_8 - 72\omega_9\omega_6^2v_1^2w_7cs^2w_4^3\omega_8^2 - 12\omega_6^2w_7cs^2w_4^2\omega_8^2 + 12\omega_6^2w_7cs^2w_4^3\omega_8 - 36\omega_9\omega_6^2v_1^2w_7cs^2w_4^3 + 18\omega_6^2w_7cs^4w_4^3\omega_8^2 - 12\omega_9\omega_6^2v_1^2w_7w_4^3v_2^2\omega_8 - 18\omega_6^2cs^4w_4^3\omega_8^2 - 12\omega_9\omega_6^2v_1^2w_7w_4^3\omega_8 + 12\omega_6^2v_1^2w_4^3v_2^2\omega_8^2 + 6\omega_6^2w_7cs^2w_4^3v_2^2\omega_8^2 - 12\omega_9\omega_6^2w_7cs^4w_4^2\omega_8 - 18\omega_6^2v_1^2w_7cs^2w_4^3\omega_8^2 - 6\omega_9\omega_6^2w_7cs^2w_4^3v_2^2\omega_8 + 12\omega_9\omega_6^2v_1^2w_7cs^2w_4^3\omega_8 + 24\omega_9\omega_6^2v_1^2w_7w_4\omega_8 + 12\omega_6^2w_7cs^2w_4^3\omega_8^2 - 72\omega_9\omega_6^2v_1^2w_7cs^2w_4^4\omega_8 - 12\omega_9\omega_6^2v_1^2w_7w_4\omega_8^2 + 12\omega_9\omega_6^2v_1^2w_4^3\omega_8^2 + 12\omega_9\omega_6^2v_1^2w_7cs^2w_4^3v_2^2\omega_8^2 - 18\omega_6^2v_1^2cs^4w_4^2\omega_8^2 + 12\omega_9\omega_6^2w_7cs^2w_4^3v_2^2\omega_8 + 6\omega_6^2v_1^2w_7w_4^3v_2^2\omega_8^2 - 6\omega_9\omega_6^2v_1^2w_7w_4^3\omega_8 + 30\omega_9\omega_6^2w_7cs^4w_4^3\omega_8 + 36\omega_9w_7cs^4w_4^2\omega_8 - 36\omega_9\omega_6^2v_1^2w_7cs^2w_4^3\omega_8 + 36\omega_6^2cs^4w_4^2\omega_8^2 - 36\omega_6^2w_7cs^4w_4^2\omega_8^2 - 12\omega_9\omega_6^2v_1^2w_7w_4^3v_2^2 + 6\omega_9\omega_6^2v_1^2w_4^3v_2^2\omega_8^2 + 18\omega_9\omega_6^2v_1^2w_7cs^2w_4^2\omega_8^2 - 84\omega_9\omega_6^2w_7cs^4w_4\omega_8^2 - 12\omega_9\omega_6^2cs^2w_4^3v_2^2\omega_8^2 + \omega_9\omega_6^2w_7cs^2w_4^3v_2^2\omega_8^2 + 18\omega_9\omega_6^2w_7cs^2w_4^3v_2^2\omega_8 - 12\omega_9\omega_6^2w_7cs^2w_4^3\omega_8^2 - 6\omega_6^2w_7cs^2w_4^3\omega_8^2 + 6\omega_6^2cs^2w_4^3\omega_8^2 - 12\omega_9\omega_6^2v_1^2w_7w_4^3\omega_8 + 18\omega_9\omega_6^2w_7cs^2w_4^3\omega_8 - 12\omega_9\omega_6^2w_7cs^2w_4^3v_2^2\omega_8 - 18\omega_9\omega_6^2w_7cs^2w_4^3\omega_8^2 - 12\omega_9\omega_6^2w_7cs^2w_4^3\omega_8 - 24\omega_9\omega_6^2w_7cs^2w_4^3v_2^2\omega_8 - 36\omega_6^2w_7cs^2w_4^3\omega_8^2 - 12\omega_6^2v_1^2w_7w_4^3\omega_8 + 24\omega_9\omega_6^2v_1^2w_7w_4^3\omega_8 - 88\omega_9\omega_6^2w_7cs^4w_4^2\omega_8^2 + 36\omega_6^2v_1^2w_7cs^2w_4^3\omega_8 - 12\omega_6^2w_7cs^2w_4^3v_2^2\omega_8 + 36\omega_9\omega_6^2v_1^2w_7cs^2w_4^2\omega_8^2 + 12\omega_9\omega_6^2w_7cs^2w_4^3\omega_8 - 6\omega_9\omega_6^2v_1^2w_4^3\omega_8^2 - 36\omega_9\omega_6^2v_1^2w_7cs^2w_4^2\omega_8 - 12\omega_6^2w_7cs^2w_4^2\omega_8^2) \frac{v_2^2}{12\omega_9\omega_6^2w_7w_4^3\omega_8^2}$$

83

$$40\omega_9\omega_6^3\omega_7\omega_4\omega_8^2cs^2 - \omega_9\omega_6^3\omega_7\omega_4^2\omega_8^2 + 36\omega_9\omega_6^3\omega_7\omega_4^2cs^2 - 12\omega_6^3\omega_7\omega_4\omega_8 - 36\omega_6^3\omega_7\omega_4^2\omega_8cs^2) \frac{v_2cs^2}{12\omega_9\omega_6^3\omega_7\omega_4^2\omega_8^2}$$

$$\begin{aligned} C_{D_x^2D_y^2\rho}^{(2),\text{CLBM2}} = & (-18cs^2\omega_6^2\omega_7\omega_2^2\omega_8^2 + 6\omega_6^2\omega_7\omega_4^2\omega_8^2 + 12\omega_9\omega_6^3\omega_7\omega_2^2v_2^2 - 12\omega_9\omega_6^3\omega_7\omega_4v_2^2\omega_8^2 + 36cs^2\omega_6^3\omega_4\omega_8^2 + 12\omega_9\omega_6\omega_7\omega_4^2\omega_8^2 + \\ & 12\omega_9\omega_6^3\omega_7v_2^2\omega_8^2 - 6\omega_9\omega_6^2\omega_4^2\omega_8^2 - 2\omega_9\omega_6^2\omega_7\omega_4^2v_2^2\omega_8^2 - 12\omega_9\omega_6^3\omega_7\omega_4^2v_2^2\omega_8^2 - 36cs^2\omega_6^3\omega_7\omega_2^2\omega_8^2 + 6\omega_9\omega_6^2\omega_4^2v_2^2\omega_8^2 + 12\omega_6^3\omega_7\omega_2^2\omega_8^2 + 12\omega_9\omega_6^3\omega_7\omega_4\omega_8^2 - \\ & 6\omega_9cs^2\omega_6^2\omega_7\omega_4^2\omega_8^2 + 12\omega_6^3\omega_4v_2^2\omega_8^2 - 12\omega_9\omega_6\omega_7\omega_4^2v_2^2\omega_8^2 - 36\omega_9cs^2\omega_6^3\omega_4\omega_8^2 - 12\omega_6^3\omega_7\omega_4v_2^2\omega_8^2 - 36\omega_9cs^2\omega_6^3\omega_7\omega_8 - \\ & 6\omega_6^2\omega_7\omega_4^2v_2^2\omega_8^2 - 18\omega_9cs^2\omega_6^3\omega_7\omega_4^2\omega_8 - 12\omega_9\omega_6^3\omega_7\omega_4^2\omega_8^2 + 36\omega_9cs^2\omega_6^3\omega_7\omega_8^2 + 12\omega_6^3\omega_7\omega_4^2v_2^2\omega_8^2 + 5\omega_9cs^2\omega_6^3\omega_7\omega_4^2\omega_8^2 + 54\omega_9cs^2\omega_6^2\omega_7\omega_4^2\omega_8 + \\ & 12\omega_6^3\omega_7\omega_4v_2^2\omega_8 - 12\omega_9\omega_6\omega_7\omega_4^2v_2^2\omega_8 - 12\omega_9\omega_6^3\omega_7v_2^2\omega_8 - 6\omega_6^3\omega_7\omega_2^2\omega_8^2 + 18\omega_9\omega_6^2\omega_7\omega_4^2v_2^2\omega_8 + 18cs^2\omega_6^3\omega_7\omega_4^2\omega_8^2 + 6\omega_6^3\omega_4^2\omega_8^2 + 12\omega_9\omega_6^3\omega_7\omega_4 - \\ & 12\omega_9\omega_6^2\omega_7\omega_4^2v_2^2 - 18\omega_9\omega_6^3\omega_7\omega_4\omega_8 - 18\omega_9\omega_6^2\omega_7\omega_4\omega_8^2 - 12\omega_6^2\omega_7\omega_4^2\omega_8 - 36\omega_9cs^2\omega_6^2\omega_7\omega_4^2 + 36cs^2\omega_6^2\omega_7\omega_4^2\omega_8 + 12\omega_9\omega_6\omega_7\omega_4^2\omega_8 - \\ & 12\omega_9\omega_6^2\omega_7v_2^2\omega_8^2 + 18\omega_9\omega_6^3\omega_7\omega_4v_2^2\omega_8 + 6\omega_9\omega_6^3\omega_7\omega_4^2\omega_8 + 12\omega_6^3\omega_7\omega_4\omega_8^2 - 12\omega_6^3\omega_7\omega_4\omega_8^2 - 36cs^2\omega_6^3\omega_7\omega_4\omega_8^2 - 12\omega_9\omega_6^3\omega_7\omega_4^2v_2^2\omega_8 + \\ & 2\omega_9\omega_6^2\omega_7\omega_4^2\omega_8^2 + 36\omega_9cs^2\omega_7\omega_4^2\omega_8^2 - 40\omega_9cs^2\omega_6^3\omega_7\omega_4^2\omega_8 + 12\omega_9\omega_6^2\omega_7\omega_4^2 + 12\omega_9\omega_6^3\omega_7\omega_4^2v_2^2\omega_8^2 - 6\omega_9\omega_6^3\omega_7\omega_4^2v_2^2\omega_8 - 36\omega_9cs^2\omega_6^3\omega_7\omega_4 - \\ & 36\omega_9cs^2\omega_6\omega_7\omega_4^2\omega_8^2 + 36\omega_9cs^2\omega_6^3\omega_7\omega_4^2 + \omega_9\omega_6^3\omega_7\omega_4^2v_2^2\omega_8^2 + 54\omega_9cs^2\omega_6^2\omega_7\omega_4\omega_8^2 - 36\omega_9cs^2\omega_6\omega_7\omega_4^2\omega_8 + 18\omega_9cs^2\omega_6^2\omega_4^2\omega_8^2 - 12\omega_9\omega_6^2\omega_4v_2^2\omega_8^2 + \\ & 18\omega_9\omega_6^2\omega_7\omega_4v_2^2\omega_8^2 + 54\omega_9cs^2\omega_6^2\omega_7\omega_4\omega_8 + 12\omega_9\omega_6^3\omega_7\omega_8 - 12\omega_9\omega_6^3\omega_7\omega_4v_2^2 - 6\omega_6^3\omega_4^2v_2^2\omega_8^2 - 18\omega_9\omega_6^3\omega_7\omega_4^2\omega_8 + 6\omega_6^3\omega_7\omega_4^2v_2^2\omega_8^2 - 18cs^2\omega_6^3\omega_4^2\omega_8^2 + \\ & 12\omega_9\omega_6^2\omega_4\omega_8^2 - \omega_9\omega_6^3\omega_7\omega_4^2\omega_8^2 + 36cs^2\omega_6^3\omega_7\omega_4\omega_8 - 12\omega_6^3\omega_7\omega_4\omega_8 - 36\omega_9cs^2\omega_6^2\omega_7\omega_8) \frac{cs^2v_2}{12\omega_9\omega_6^3\omega_7\omega_4^2\omega_8^2} \end{aligned}$$

$$\begin{aligned} C_{D_x^2D_y^2\rho}^{(2),\text{CuLBM1}} = & (12\omega_6\omega_2^3 - 36\omega_6^2cs^2\omega_2^2 + 5\omega_6^2\omega_3cs^2\omega_2^3 + 12\omega_6\omega_3v_2^2\omega_2^2 + 18\omega_6^2\omega_3\omega_2 - 12\omega_3v_2^2\omega_2^3 + 36\omega_6^2cs^2\omega_2^3 + 18\omega_6\omega_3v_2^2\omega_2^3 + 12\omega_6^2\omega_3cs^2\omega_2^2 - \\ & 18\omega_6^2\omega_3^2v_2^2\omega_2 - 54\omega_6^2\omega_3^2cs^2\omega_2 + 4\omega_6^2\omega_3^2v_2^2\omega_2^2 + 54\omega_6\omega_3cs^2\omega_2^3 - 4\omega_6^2\omega_3^2\omega_2^2 + 12\omega_6^2v_2^2\omega_2^3 - 18\omega_6\omega_3\omega_2^3 - 36\omega_3cs^2\omega_2^3 + 36\omega_6^2\omega_3^2cs^2 + 12\omega_3\omega_2^3 + \\ & 36\omega_6\omega_3cs^2\omega_2^2 + \omega_6^2\omega_3^2v_2^2\omega_2^2 - 12\omega_6^2\omega_3^2 - 12\omega_6\omega_3\omega_2^3 - \omega_6^2\omega_3^2\omega_2^3 - 12\omega_6^2v_2^2\omega_2^2 - 18\omega_6\omega_3^2cs^2\omega_2^3 + 6\omega_6^2\omega_3v_2^2\omega_2^2 + 36\omega_6^2cs^2\omega_2^3 + 12\omega_3^2\omega_2^2 + \\ & 12\omega_6^2\omega_3\omega_2^3 - 6\omega_6\omega_3^2\omega_2^2 - 12\omega_6\omega_3^2\omega_2^3 - 12\omega_6\omega_3v_2^2\omega_2^2 + 18\omega_6\omega_3^2cs^2\omega_2^2 + 6\omega_6\omega_3^2\omega_2 - 36\omega_3^2cs^2\omega_2^2 - 12\omega_3^2\omega_2^3 - 12\omega_3^2v_2^2\omega_2^2 + 12\omega_6^2\omega_2^2 + \\ & 6\omega_6\omega_3^2v_2^2\omega_2^2 - 40\omega_6^2\omega_3cs^2\omega_2^3 - 36\omega_6cs^2\omega_2^3 - 12\omega_6^2\omega_2^3 + 12\omega_6^2\omega_3^2v_2^2 + 12\omega_3^2v_2^2\omega_2^3 + 18\omega_6^2\omega_3cs^2\omega_2^2 - 6\omega_6\omega_3^2v_2^2\omega_2^2) \frac{cs^2v_2}{12\omega_6^2\omega_3^2\omega_2^3} \end{aligned}$$

$$\begin{aligned} C_{D_x^2D_y^2\rho}^{(2),\text{CuLBM2}} = & (-45\omega_3^2v_2^2\omega_1^2v_2^2\omega_2 - 12\omega_3^3v_2^2cs^2\omega_2^2 - 30\omega_3^2\omega_1v_2^2cs^2\omega_2^2 + 35\omega_3^3\omega_1^3cs^4\omega_2^2 + 144\omega_3\omega_1^3cs^4\omega_2^3 - 27\omega_3^2v_2^2\omega_1^3\omega_2^2 - 30\omega_3\omega_1v_2^2cs^2\omega_2^3 - \\ & 6\omega_3^2\omega_1^2\omega_2 - 54\omega_3^2\omega_1^3cs^2\omega_2^2 + 12\omega_3\omega_1^3cs^2\omega_2^3 - 12\omega_3^2\omega_1^2v_2^2\omega_2^2 + 10\omega_3^3\omega_1^3cs^4\omega_2^3 - 54\omega_3^2v_1^4\omega_1^3\omega_2 + 108\omega_3\omega_1^3cs^4\omega_2^2 + 3\omega_3^2\omega_1v_2^2cs^2\omega_2^3 + 24\omega_3^3v_2^2cs^2\omega_2^3 - \\ & 27\omega_3^2v_1^4\omega_1^2\omega_2^3 + 9\omega_3^2v_1^2\omega_2^3 + 41\omega_3^2\omega_1^2cs^2\omega_2^3 - 297\omega_3^2v_1^4\omega_1^2cs^2\omega_2 + 18\omega_3^3v_2^2cs^2 + 99\omega_3^2v_1^4\omega_1^2\omega_2 + 90\omega_3^3\omega_1^3cs^4 + 6\omega_3\omega_1^3cs^2\omega_2 - 36\omega_1^3v_2^2cs^2\omega_2^3 - \\ & 6\omega_3^2\omega_1^3\omega_2 + 270\omega_3^2v_1^4\omega_1^2cs^2\omega_2^3 - 90\omega_3^2v_1^4\omega_1^2\omega_2^3 + 27\omega_3^2v_1^4\omega_1^2\omega_2^3 + 6\omega_3^2\omega_1^2v_2^2\omega_2^2 + 12\omega_3^3cs^2\omega_2^2 + 63\omega_3^2\omega_1cs^4\omega_2^3 + 6\omega_3^2\omega_1^3 + 30\omega_3\omega_1cs^2\omega_2^3 - \\ & 138\omega_3^2v_1^2\omega_1^2cs^2\omega_2^3 + 27\omega_3^2v_1^4\omega_1^2\omega_2^3 + 12\omega_3^2\omega_1^2\omega_2^2 + 99\omega_3^2v_1^4\omega_1^2\omega_2 + 54\omega_3^2v_1^4\omega_1^3 + 24\omega_3^2\omega_1^2cs^2\omega_2 - 108\omega_1^4cs^4\omega_2^3 + 18\omega_3^4cs^4\omega_2^3 + 18\omega_3^2v_2^2cs^2\omega_2^3 - \\ & 90\omega_3^2\omega_1cs^4\omega_2^3 + 90\omega_3^2v_1^2\omega_1^2v_2^2\omega_2^2 - 24\omega_1^3cs^2\omega_2^3 - 54\omega_3^2v_1^4\omega_1^2\omega_2 - 117\omega_3^2\omega_1^3cs^4\omega_2 - 36\omega_3\omega_1^2cs^2\omega_2^3 - 6\omega_3^2\omega_1\omega_2^3 + 27\omega_3^2v_1^4\omega_1cs^2\omega_2^2 - \\ & 2\omega_3^2\omega_1^3cs^2\omega_2^3 - 9\omega_3^2v_1^4\omega_1\omega_2^3 - 6\omega_3^2\omega_1^3v_2^2 - 459\omega_3^2v_1^4\omega_1^3cs^2\omega_2 - 54\omega_3^2v_1^4\omega_2^3 - 21\omega_3^2\omega_1^3v_2^2cs^2\omega_2 + 48\omega_3\omega_1^2v_2^2cs^2\omega_2^3 + 18\omega_3^2\omega_1^2v_2^2cs^2\omega_2^2 - \\ & 72\omega_3^2\omega_1^3cs^2 - 45\omega_3^2v_1^4\omega_1v_2^2\omega_2^3 - 91\omega_3^2v_1^2cs^4\omega_2^3 + 36\omega_3\omega_1^2v_2^2cs^2\omega_2^2 + 45\omega_3^2v_1^4\omega_1^3v_2^2 - 9\omega_3^2v_1^4\omega_1\omega_2^3 - 48\omega_3\omega_1^2cs^2\omega_2^3 - 25\omega_3^2\omega_1^3cs^2\omega_2^2 + \\ & 189\omega_3^2v_1^2\omega_1cs^2\omega_2^3 - 6\omega_3^2\omega_1\omega_2^3 - 45\omega_3^2v_1^4\omega_1^3v_2^2\omega_2 - 36\omega_3\omega_1^3cs^4\omega_2^3 + 54\omega_3^2\omega_1^3cs^4\omega_2^3 + 6\omega_3^2\omega_2^3 - 45\omega_3^2v_1^4\omega_1v_2^2\omega_2^2 - 6\omega_3\omega_1^3v_2^2cs^2\omega_2 - \\ & 24\omega_3^2\omega_1^2v_2^2cs^2\omega_2^2 + 405\omega_3^2v_1^4\omega_1^3cs^2 + 6\omega_3^2\omega_1v_2^2\omega_2^3 + 54\omega_3^2v_1^4\omega_1\omega_2^3 - 90\omega_3\omega_1cs^4\omega_2^3 + 45\omega_3^2v_1^2\omega_2^3 - 99\omega_3^2\omega_1^2\omega_2^3 - 18\omega_3^2\omega_1^3cs^4\omega_2 + 36\omega_1^3cs^2\omega_2^3 + \\ & 2\omega_3^2\omega_1^3v_2^2cs^2\omega_2^3 + 48\omega_3^2\omega_1cs^2\omega_2^2 + 72\omega_1^3cs^4\omega_2^3 + 93\omega_3^2\omega_1^3cs^2\omega_2 - 12\omega_3\omega_1^3v_2^2cs^2\omega_2^2 - 6\omega_3^2\omega_1^2v_2^2cs^2\omega_2 + 6\omega_3^2\omega_1^3v_2^2\omega_2 - 18\omega_3\omega_1^3cs^4\omega_2 - \\ & 6\omega_3^2v_2^2\omega_2^3 + 54\omega_3^2v_1^4\omega_1\omega_2^2 + 6\omega_3^2\omega_1v_2^2\omega_2^2 - 36\omega_1^3cs^4\omega_2^2 + 138\omega_3^2v_1^4\omega_1^3cs^2\omega_2 - 39\omega_3^2\omega_1cs^2\omega_2^3 - 135\omega_3^2v_1^2cs^2\omega_2^3 + 8\omega_3^2\omega_1^3v_2^2cs^2\omega_2^2) \frac{v_2}{24\omega_3^2\omega_1^3\omega_2^3} \end{aligned}$$

$$\text{coefficient } C_{D_x^2D_y^2v_1}^{(2)} \text{ at } \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} :$$

$$C_{D_x^2D_y^2v_1}^{(2),\text{SRT}} = 0$$

$$\begin{aligned} C_{D_x^2D_y^2v_1}^{(2),\text{MRT1}} = & (8\omega_9\omega_6^2cs^2\omega_7\omega_4^2\omega_8^2\omega_5 + 8\omega_9\omega_6^3cs^2\omega_7\omega_4^3\omega_8^2\omega_5 - 2\omega_9\omega_6^3cs^2\omega_7^3\omega_8^2\omega_5 - 6\omega_9\omega_6^3cs^2\omega_7^2\omega_4\omega_8\omega_5 + 2\omega_9\omega_6^2\omega_7^2\omega_4v_2^2\omega_8^2\omega_5 + 2\omega_6^3\omega_2^2\omega_4^3v_2^2\omega_8^2\omega_5 - \\ & 4\omega_9\omega_6^3\omega_7^2\omega_4^2v_2^2\omega_5 - 8\omega_9\omega_6^3cs^2\omega_7^2\omega_4^2\omega_5 - 9\omega_9\omega_6^3\omega_7^2\omega_4^2\omega_8\omega_5 + 7\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - 2\omega_9\omega_6^3\omega_7\omega_4^2v_2^2\omega_8^2\omega_5 - 2\omega_9\omega_6\omega_7^2\omega_4^2\omega_8^2\omega_5 - 16\omega_9\omega_6^3cs^2\omega_7^2\omega_8^2\omega_5 - \\ & \omega_9\omega_6^3\omega_7^2\omega_4^2v_2^2\omega_8^2\omega_5 + 3\omega_9\omega_6^2\omega_7^2\omega_4^2\omega_8^2\omega_5 - 8\omega_9\omega_6^3cs^2\omega_7^2\omega_4^2\omega_8^2\omega_5 + 3\omega_9\omega_6^2cs^2\omega_7^2\omega_4^2\omega_8^2\omega_5 - 4\omega_6^3cs^2\omega_7^2\omega_4^3\omega_8\omega_5 - 4\omega_9\omega_7^2\omega_4^3\omega_8\omega_5 - 4\omega_6^3\omega_7^2\omega_4^2\omega_8\omega_5 - \\ & 4\omega_9\omega_6^3\omega_7^2\omega_4^2\omega_5 - 2\omega_9\omega_6^3\omega_7^2\omega_4\omega_8^2\omega_5 - 2\omega_6^3\omega_7\omega_4^3v_2^2\omega_8^2\omega_5 - 15\omega_9\omega_6cs^2\omega_7^2\omega_4^3\omega_8^2\omega_5 - 4\omega_6^3\omega_7\omega_4^2\omega_8^2\omega_5 + 4\omega_9\omega_6\omega_7^2\omega_4^3\omega_8\omega_5 - 2\omega_6^3cs^2\omega_7\omega_4^3\omega_8^2\omega_5 + \\ & 6\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + 4\omega_9\omega_6^2\omega_7^2\omega_4^3\omega_5 - 6\omega_9\omega_6^2cs^2\omega_7^2\omega_4^3\omega_8^2\omega_5 + 8\omega_9\omega_6^3cs^2\omega_7^2\omega_4^3\omega_5 + 4\omega_6^3\omega_7\omega_4^2v_2^2\omega_8^2\omega_5 - 4\omega_6^2\omega_7^2\omega_4^3\omega_8\omega_5 + 4\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + \\ & 2\omega_9\omega_6^3cs^2\omega_7^2\omega_4^2\omega_8^2\omega_5 - 4\omega_6^3cs^2\omega_7^2\omega_4^2\omega_8^2\omega_5 - 4\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_5 - 2\omega_6^2cs^2\omega_7^2\omega_4^3\omega_8^2\omega_5 + 4\omega_9\omega_6^3\omega_7^2\omega_4^2\omega_8^2\omega_5 - 2\omega_6^3\omega_7^2\omega_4^3\omega_8^2\omega_5 - 4\omega_6^3\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5 + \\ & 2\omega_9\omega_6^3\omega_7^2\omega_4^2\omega_8^2\omega_5 - 4\omega_9\omega_6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + \omega_9\omega_6^3\omega_7\omega_4^3v_2^2\omega_8^2\omega_5 + 12\omega_9cs^2\omega_7^2\omega_4^3\omega_8^2\omega_5 - 2\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + 12\omega_9\omega_6^3cs^2\omega_7\omega_4\omega_8^2\omega_5 - \\ & 7\omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8\omega_5 - 5\omega_9\omega_6^3cs^2\omega_7^2\omega_4^3\omega_8\omega_5 + 2\omega_9\omega_6^2cs^2\omega_7^2\omega_4^3\omega_8\omega_5 + 4\omega_6^3\omega_7^2\omega_4^3\omega_8\omega_5 + 4\omega_6^3\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + 4\omega_6^3cs^2\omega_7^2\omega_4^3\omega_8\omega_5 + 4\omega_6^2cs^2\omega_7^2\omega_4^3\omega_8\omega_5 + 4\omega_6^2cs^2\omega_7^2\omega_4^3\omega_8\omega_5 - \\ & 8\omega_9\omega_6^2cs^2\omega_7^2\omega_4^3\omega_5 - 5\omega_9\omega_6\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5 + 4\omega_9\omega_6^3cs^2\omega_7\omega_4^2\omega_8^2\omega_5 - \omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8^2\omega_5 - \omega_9\omega_6^2cs^2\omega_7^2\omega_4^2\omega_8^2\omega_5 - 3\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5 - \\ & 3\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + 3\omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8\omega_5 + 4\omega_9\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5 + 5\omega_9\omega_6\omega_7^2\omega_4^3\omega_8^2\omega_5 - 8\omega_9\omega_6cs^2\omega_7^2\omega_4^3\omega_8\omega_5 + 2\omega_9\omega_6^3\omega_7\omega_4^3v_2^2\omega_8^2\omega_5 - \\ & 4\omega_9\omega_6^2cs^2\omega_7\omega_4^3\omega_8^2\omega_5 + 4\omega_9\omega_6^3cs^2\omega_7^2\omega_4^3\omega_8^2\omega_5 - 24\omega_9\omega_6^3cs^2\omega_7\omega_4^2\omega_8^2\omega_5 + 4\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_5 - 4\omega_9\omega_6^3cs^2\omega_7^2\omega_4\omega_8^2 + 2\omega_6^2\omega_7^2\omega_4^3\omega_8^2\omega_5 - \\ & 2\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5 + 11\omega_9\omega_6^3cs^2\omega_7^2\omega_4^3\omega_8\omega_5 + 2\omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8\omega_5 + 13\omega_9\omega_6^2cs^2\omega_7^2\omega_4^3\omega_8\omega_5 - 2\omega_9\omega_6^3cs^2\omega_7\omega_4^3\omega_8^2 + 9\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + \\ & 4\omega_6^3\omega_7^2\omega_4^3\omega_8^2\omega_5 + 2\omega_6^3cs^2\omega_7^2\omega_4^3\omega_8^2\omega_5 - \omega_9\omega_6^3\omega_7\omega_4^3\omega_8^2\omega_5 + 6\omega_9\omega_6^2\omega_7^2\omega_4\omega_8\omega_5 - 2\omega_9\omega_6^2\omega_7^2\omega_4\omega_8^2\omega_5 + 26\omega_9\omega_6^3cs^2\omega_7^2\omega_4\omega_8^2\omega_5 - 4\omega_6^3\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + \\ & \omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5 + \omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8^2\omega_5 + 4\omega_6^3cs^2\omega_7\omega_4^3\omega_8^2\omega_5 + 2\omega_9\omega_6\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5 + 2\omega_9\omega_6cs^2\omega_7^2\omega_4^3\omega_8^2\omega_5 + 2\omega_6^3\omega_7\omega_4^3\omega_8^2\omega_5) \frac{v_1v_2}{2\omega_9\omega_6^3\omega_7^2\omega_4^3\omega_8^2\omega_5} \end{aligned}$$

$$\begin{aligned} C_{D_x^2D_y^2v_1}^{(2),\text{MRT2}} = & (2\omega_6^3\omega_7^2cs^2\omega_4^3\omega_8^2\omega_5 + 12\omega_9\omega_6^3\omega_7cs^2\omega_4\omega_8^2\omega_5 + 2\omega_9\omega_6^2\omega_7^2\omega_4v_2^2\omega_8^2\omega_5 + 2\omega_6^3\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5 - 4\omega_9\omega_6^3\omega_7^2\omega_4^2v_2^2\omega_5 - 2\omega_9\omega_6^3cs^2\omega_4^3\omega_8^2\omega_5 - \\ & 9\omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8\omega_5 + 7\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - 2\omega_9\omega_6^3\omega_7\omega_4^2v_2^2\omega_8^2\omega_5 - 2\omega_9\omega_6\omega_7^2\omega_4^3\omega_8^2\omega_5 + 13\omega_9\omega_6^2\omega_7^2cs^2\omega_4^3\omega_8\omega_5 + 11\omega_9\omega_6^3\omega_7^2cs^2\omega_4^3\omega_8\omega_5 + \\ & 2\omega_9\omega_6\omega_7^2cs^2\omega_4^3\omega_8^2\omega_5 - \omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5 + 3\omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8^2\omega_5 + 2\omega_9\omega_6^3\omega_7^2cs^2\omega_4^3\omega_8^2 - 2\omega_6^3\omega_7cs^2\omega_4^3\omega_8^2\omega_5 - 4\omega_9\omega_7^2\omega_4^3\omega_8^2\omega_5 - 4\omega_6^3\omega_7^2\omega_4^3\omega_8\omega_5 + \\ & 26\omega_9\omega_6^3\omega_7^2cs^2\omega_4\omega_8^2\omega_5 - 4\omega_9\omega_6^3\omega_7^2\omega_4^3\omega_5 - 2\omega_9\omega_6^3\omega_7^2\omega_4\omega_8^2\omega_5 - 2\omega_6^3\omega_7\omega_4^3v_2^2\omega_8^2\omega_5 + 8\omega_9\omega_6^3\omega_7^2cs^2\omega_4^3\omega_5 - 4\omega_6^3\omega_7\omega_4^3\omega_8^2\omega_5 - \omega_9\omega_6^2\omega_7^2cs^2\omega_4^3\omega_8^2\omega_5 + \\ & 4\omega_9\omega_6\omega_7^2\omega_4^3\omega_8\omega_5 + 4\omega_6^2\omega_7^2cs^2\omega_4^3\omega_8\omega_5 + 4\omega_6^2\omega_7^2cs^2\omega_4^3\omega_8\omega_5 - 6\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + 4\omega_9\omega_6^2\omega_7^2\omega_4^3\omega_5 + 4\omega_6^2\omega_7\omega_4^3v_2^2\omega_8^2\omega_5 - 4\omega_6^2\omega_7^2\omega_4^3\omega_8\omega_5 + \end{aligned}$$

$$\begin{aligned}
& 4\omega_6^2\omega_7^3\omega_4^3\omega_8\omega_5 - 4\omega_9\omega_6^2\omega_7^3\omega_2^2\omega_5 + 4\omega_9\omega_6^3\omega_7^2\omega_4^2\omega_5 - 2\omega_6^3\omega_7^2\omega_4^3\omega_8\omega_5 - 8\omega_9\omega_6^3\omega_7^2cs^2\omega_4^2\omega_5 - 4\omega_6^3\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + 2\omega_9\omega_6^3\omega_7\omega_4^2\omega_8^2\omega_5 - \\
& 4\omega_9\omega_6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + \omega_9\omega_6^2\omega_7\omega_4^3v_2^2\omega_8^2\omega_5 - 16\omega_9\omega_6^3\omega_7^2cs^2\omega_4^2\omega_8\omega_5 - 8\omega_9\omega_6\omega_7^2cs^2\omega_4^3\omega_8\omega_5 - 2\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - 7\omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8\omega_5 + \\
& 8\omega_9\omega_6^2\omega_7cs^2\omega_4^2\omega_8^2\omega_5 + 8\omega_9\omega_6^3\omega_7cs^2\omega_4^3\omega_8\omega_5 - 6\omega_9\omega_6^2\omega_7^2cs^2\omega_4^2\omega_8^2\omega_5 + 4\omega_6^3\omega_7^2\omega_4^3\omega_8\omega_5 + 4\omega_6^3\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + 4\omega_6^3\omega_7cs^2\omega_4^2\omega_8^2\omega_5 - \\
& 5\omega_9\omega_6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - \omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8\omega_5 - 3\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5 - 15\omega_9\omega_6\omega_7^2cs^2\omega_4^2\omega_8^2\omega_5 - 3\omega_9\omega_6^3\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + 4\omega_9\omega_6^3cs^2\omega_4^2\omega_8^2\omega_5 + \\
& 3\omega_9\omega_6^3\omega_7^2\omega_4^3\omega_8\omega_5 + 4\omega_9\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5 + 4\omega_9\omega_6^2\omega_7cs^2\omega_4^2\omega_8^2\omega_5 + 2\omega_9\omega_6^2\omega_7^2cs^2\omega_4^2\omega_8\omega_5 + 5\omega_9\omega_6\omega_7^2\omega_4^3\omega_8^2\omega_5 - 5\omega_9\omega_6^3\omega_7^2cs^2\omega_4^3\omega_8\omega_5 + 12\omega_9\omega_7^2cs^2\omega_4^3\omega_8^2\omega_5 + \\
& 2\omega_9\omega_6^3\omega_7\omega_4^3v_2^2\omega_8^2\omega_5 - 2\omega_6^2\omega_7^2cs^2\omega_4^3\omega_8^2\omega_5 - 4\omega_6^3\omega_7^2cs^2\omega_4^2\omega_8^2\omega_5 + 4\omega_9\omega_6^3\omega_7^2\omega_4^3v_2^2\omega_5 + 2\omega_6^2\omega_7^2\omega_4^3\omega_8^2\omega_5 - 2\omega_6^2\omega_7\omega_4^3v_2^2\omega_8^2\omega_5 - 4\omega_9\omega_6^2\omega_7cs^2\omega_4^3\omega_8^2\omega_5 + \\
& 2\omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8\omega_5 - 24\omega_9\omega_6^3\omega_7cs^2\omega_4^2\omega_8^2\omega_5 + 9\omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 - 6\omega_9\omega_6^2\omega_7^2cs^2\omega_4^2\omega_8\omega_5 + 4\omega_6^3\omega_7^2\omega_4^3\omega_8^2\omega_5 - 4\omega_9\omega_6^3\omega_7^2cs^2\omega_4^2\omega_8^2\omega_5 - \\
& \omega_9\omega_6^3\omega_7\omega_4^3\omega_8^2\omega_5 + 6\omega_9\omega_6^3\omega_7\omega_4^3\omega_8\omega_5 - 2\omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8^2\omega_5 - 8\omega_9\omega_6^2\omega_7^2cs^2\omega_4^3\omega_5 - 4\omega_6^3\omega_7^2cs^2\omega_4^3\omega_8\omega_5 - 4\omega_6^3\omega_7^2\omega_4^3v_2^2\omega_8\omega_5 + \omega_9\omega_6^2\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5 - \\
& 2\omega_9\omega_6^2\omega_7cs^2\omega_4^3\omega_8^2\omega_5 + \omega_9\omega_6^2\omega_7^2\omega_4^3\omega_8^2\omega_5 + 2\omega_9\omega_6\omega_7^2\omega_4^3v_2^2\omega_8^2\omega_5 + 3\omega_9\omega_6^2\omega_7^2cs^2\omega_4^3\omega_8^2\omega_5 - 8\omega_9\omega_6^2\omega_7^2cs^2\omega_4^2\omega_8^2\omega_5 + 2\omega_6^3\omega_7\omega_4^3\omega_8^2\omega_5) \frac{v_1\rho v_2}{24\omega_3\omega_1^3\omega_2^3}
\end{aligned}$$

$$C_{D_x^2D_y^2v_1}^{(2),CLBM1} = 0$$

$$C_{D_x^2D_y^2v_1}^{(2),CLBM2} = 0$$

$$C_{D_x^2D_y^2v_1}^{(2),CuLBM1} = 0$$

$$\begin{aligned}
C_{D_x^2D_y^2v_1}^{(2),CuLBM2} = & (-54\omega_1cs^2\omega_2^3 + 198\omega_3v_1^3\omega_1^3 - 36\omega_1^2\omega_2^2 - 54\omega_3cs^2\omega_2^3 + 216\omega_3v_1^3\omega_1\omega_2^3 - 18\omega_1^3v_2^2\omega_2 + 36\omega_3v_2^2\omega_2^3 - 54\omega_1cs^2\omega_2 + \\
& 162\omega_3v_1^3\omega_1\omega_2^2 + 18\omega_1^3\omega_2 + 84\omega_3\omega_1^3cs^2\omega_2^2 - 18\omega_1v_2^2\omega_2^3 - 297\omega_3\omega_1^3cs^2\omega_2 + 36\omega_3\omega_1^3v_2^2 - 54\omega_3\omega_1cs^2\omega_2^2 - 18\omega_3\omega_1\omega_2^2 - 27\omega_3\omega_1v_2^2\omega_2^3 + \\
& 270\omega_3\omega_1^3cs^2 + 54\omega_3\omega_2^3 + 135\omega_3\omega_1cs^2\omega_2^3 - 81\omega_3\omega_1\omega_2^3 - 36\omega_3\omega_1v_2^2\omega_2^3 - 27\omega_3\omega_1^3v_2^2\omega_2 + 135\omega_3\omega_1^3\omega_2 + 162\omega_3\omega_1^3cs^2\omega_2^2 + 46\omega_3\omega_1^3\omega_2^3 + \\
& 100\omega_3v_1^3\omega_1\omega_2^2 - 162\omega_3v_1^3\omega_1^2\omega_2 - 54\omega_3\omega_1^2\omega_2^2 - 84\omega_3\omega_1^2cs^2\omega_2^3 + 54\omega_3\omega_1^2v_2^2\omega_2^2 - 126\omega_3\omega_1^3 - 36\omega_3\omega_1^2v_2^2\omega_2 + 18\omega_1\omega_2^3 + 36\omega_1^2v_2^2\omega_2^2 + 90\omega_3\omega_1^2\omega_2 + \\
& 108\omega_1^2cs^2\omega_2^2 - 198\omega_3v_1^3\omega_2^2 - 162\omega_3\omega_1^3cs^2\omega_2 - 46\omega_3\omega_1\omega_2^2 - 100\omega_3v_1^3\omega_1^2\omega_2^2 - 216\omega_3v_1^3\omega_1^3\omega_2) \frac{v_1\rho v_2}{24\omega_3\omega_1^3\omega_2^3}
\end{aligned}$$

$$\text{coefficient } C_{D_x^2D_y^2v_2}^{(2)} \text{ at } \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} :$$

$$C_{D_x^2D_y^2v_2}^{(2),SRT} = (-24 + 12cs^2 + 8\omega^2cs^2 - 12\omega^2 - \omega^3cs^2 - 108v_2^2\omega + 72v_2^2 + 36\omega - 18\omega cs^2 + 36v_2^2\omega^2) \frac{\rho cs^2}{12\omega^3}$$

$$\begin{aligned}
C_{D_x^2D_y^2v_2}^{(2),MRT1} = & (36\omega_9v_1^3\omega_7\omega_4^3v_2^2\omega_8^2 - 12\omega_9\omega_6^3cs^2\omega_7\omega_4\omega_8 - 12\omega_6^3cs^2\omega_4^2\omega_8^2 + 6\omega_9\omega_6^2cs^2v_1^2\omega_7\omega_4^3\omega_8^2 - 24\omega_9\omega_6^2v_1^2\omega_7\omega_4\omega_8^2 + 72\omega_9\omega_6cs^2\omega_7\omega_4^3v_2^2\omega_8 + \\
& 6\omega_6^3v_1^3\omega_4^3\omega_8^2 - 12\omega_9\omega_6cs^2v_1^2\omega_7\omega_4^3\omega_8 - 12\omega_9\omega_6^3cs^2v_1^2\omega_7\omega_4^2 - 12\omega_6^3cs^4\omega_7\omega_4^3\omega_8 + 12\omega_6^3v_1^2\omega_7\omega_4^2\omega_8^2 + 6\omega_9\omega_6^2cs^2\omega_7\omega_4^3\omega_8^2 + 12\omega_9\omega_6^3v_1^2\omega_7\omega_4^3\omega_8 - \\
& 42\omega_9\omega_6^3cs^2\omega_7\omega_4^3v_2^2\omega_8^2 - 12\omega_9v_1^2\omega_7\omega_4^3\omega_8^2 - 18\omega_6^3v_1^2\omega_7\omega_4^3v_2^2\omega_8^2 - 36\omega_6^3v_1^2\omega_7\omega_4^3v_2^2\omega_8^2 - 18\omega_9\omega_6^2cs^2\omega_7\omega_4^3v_2^2\omega_8^2 - 6\omega_6^3cs^4\omega_4^3\omega_8^2 - 4\omega_9\omega_6^3cs^4\omega_7\omega_4^3\omega_8^2 - \\
& 12\omega_9\omega_6^3cs^4\omega_7\omega_4\omega_8^2 - 6\omega_9\omega_6cs^2\omega_7\omega_4^3\omega_8^2 + 36\omega_6^2cs^2\omega_7\omega_4^3v_2^2\omega_8 + 108\omega_9\omega_6^3v_1^2\omega_7\omega_4^3v_2^2\omega_8 + 72\omega_9\omega_6^2v_1^2\omega_7\omega_4^3v_2^2\omega_8 + 36\omega_6^3cs^2\omega_7\omega_4^3v_2^2\omega_8 + \\
& 6\omega_9\omega_6^2cs^2v_1^2\omega_4^3\omega_8^2 - 18\omega_6^3v_1^2\omega_4^3v_2^2\omega_8^2 - 12\omega_9\omega_6^2cs^2v_1^2\omega_7\omega_4^2\omega_8 + 72\omega_9\omega_6^2v_1^2\omega_7\omega_4v_2^2\omega_8^2 - 54\omega_9\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8^2 - 24\omega_9\omega_6^3cs^2\omega_7\omega_4^3v_2^2 + \\
& 6\omega_6^2cs^2\omega_7\omega_4^3\omega_8^2 + 12\omega_9\omega_6^3cs^4\omega_7\omega_4\omega_8 + 12\omega_9\omega_6cs^2v_1^2\omega_7\omega_4^2\omega_8^2 - 12\omega_6^3cs^4\omega_7\omega_4^2\omega_8^2 + 12\omega_6^2cs^2v_1^2\omega_7\omega_4^3\omega_8 - 12\omega_9\omega_6^2cs^2\omega_7\omega_4^2\omega_8 + 12\omega_9\omega_6^2v_1^2\omega_7\omega_4^3 + 12\omega_9\omega_6^3cs^2v_1^2\omega_7\omega_4^3 + 12\omega_6^3cs^4\omega_4^2\omega_8^2 - 36\omega_9\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8 - 36\omega_9\omega_6^3v_1^2\omega_7\omega_4^2\omega_8 - 12\omega_6^2cs^2\omega_7\omega_4^3\omega_8 - \\
& \omega_9\omega_6^3cs^4\omega_7\omega_4^3\omega_8^2 - 6\omega_6^2cs^2v_1^2\omega_7\omega_4^3\omega_8^2 + 12\omega_6^3cs^4\omega_7\omega_4\omega_8 - 36\omega_9\omega_6^2v_1^2\omega_7\omega_4^3v_2^2 - 6\omega_6^3v_1^2\omega_7\omega_4^3\omega_8^2 - 24\omega_9\omega_6^2cs^2\omega_7\omega_4^3\omega_8^2 + 12\omega_9cs^2v_1^2\omega_7\omega_4^3\omega_8^2 - \\
& 12\omega_9\omega_6^3v_1^2\omega_7\omega_4^3 - 36\omega_6^3cs^2\omega_7\omega_4^3v_2^2\omega_8^2 + 18\omega_9\omega_6^2v_1^2\omega_7\omega_4^3v_2^2\omega_8^2 - 18\omega_6^3cs^2\omega_7\omega_4^3\omega_8^2 - 24\omega_9\omega_6^2cs^2v_1^2\omega_7\omega_4^2\omega_8^2 + 6\omega_6^3cs^2\omega_4^3\omega_8^2 + \\
& 6\omega_6^3cs^4\omega_7\omega_4^3\omega_8^2 - 84\omega_9\omega_6^2cs^2\omega_7\omega_4v_2^2\omega_8^2 - 18\omega_9\omega_6cs^2v_1^2\omega_7\omega_4^3\omega_8^2 - 12\omega_6^3v_1^2\omega_7\omega_4^2\omega_8 + 12\omega_9\omega_6^2cs^2\omega_7\omega_4^3\omega_8 - 36\omega_9\omega_6^2v_1^2\omega_4^3\omega_8^2 - 6\omega_9\omega_6^3v_1^2\omega_4^3v_2^2\omega_8^2 - \\
& 144\omega_9\omega_6^2cs^2\omega_7\omega_4^3v_2^2\omega_8 + 36\omega_6^3v_1^2\omega_7\omega_4^3v_2^2\omega_8 + 36\omega_6^2v_1^2\omega_7\omega_4^3v_2^2\omega_8 - 132\omega_9\omega_6^3cs^2\omega_7\omega_4^3v_2^2\omega_8 + 24\omega_9\omega_6^3cs^2\omega_7\omega_4^3v_2^2 - 24\omega_9\omega_6^3cs^4\omega_7\omega_4^3\omega_8 - \\
& 12\omega_9\omega_6^2cs^2v_1^2\omega_4^3\omega_8^2 + 24\omega_9\omega_6^2cs^2v_1^2\omega_7\omega_4^3\omega_8 + 18\omega_9\omega_6^2cs^2v_1^2\omega_7\omega_4\omega_8 + 18\omega_9\omega_6^2v_1^2\omega_7\omega_4^3\omega_8^2 + 12\omega_9\omega_6^3cs^4\omega_7\omega_4\omega_8 - 18\omega_6^3cs^2\omega_4^3v_2^2\omega_8^2 - \\
& 12\omega_6^2cs^2v_1^2\omega_7\omega_4^3\omega_8 + 12\omega_6^3cs^2v_1^2\omega_4^3\omega_8^2 - 36\omega_9\omega_6^3v_1^2\omega_7\omega_4^3v_2^2\omega_8 - 36\omega_9\omega_6^2v_1^2\omega_7\omega_4^2\omega_8^2 - 36\omega_6^3cs^2\omega_7\omega_4^3v_2^2\omega_8 + 36\omega_9\omega_6^3v_1^2\omega_7\omega_4^3v_2^2 + \\
& 6\omega_6^2v_1^2\omega_7\omega_4^3\omega_8^2 + 36\omega_9\omega_6^3cs^2v_1^2\omega_7\omega_4^2\omega_8 + 12\omega_6^3cs^2\omega_7\omega_4^3\omega_8 + 12\omega_9\omega_6^2v_1^2\omega_7\omega_4^3\omega_8 + 36\omega_6^3v_1^2\omega_4^3v_2^2\omega_8^2 - 6\omega_9\omega_6^2cs^4\omega_7\omega_4^3\omega_8^2 - 6\omega_9\omega_6^2cs^2\omega_4^3\omega_8^2 + \\
& 24\omega_9\omega_6^3v_1^2\omega_7\omega_4\omega_8 - 12\omega_6^2cs^2v_1^2\omega_7\omega_4^2\omega_8^2 + 60\omega_9\omega_6^3cs^2\omega_7\omega_4v_2^2\omega_8 + 12\omega_9\omega_6^2v_1^2\omega_4^3\omega_8^2 + 12\omega_9\omega_6^2cs^2\omega_7\omega_4\omega_8^2 + 18\omega_6^3v_1^2\omega_7\omega_4^3v_2^2\omega_8^2 + \\
& 180\omega_9\omega_6^2cs^2\omega_7\omega_4^3v_2^2\omega_8^2 + 6\omega_9\omega_6cs^4\omega_7\omega_4^3\omega_8^2 + 12\omega_6^3cs^2\omega_7\omega_4^2\omega_8^2 - 6\omega_9\omega_6^2v_1^2\omega_7\omega_4^3\omega_8^2 + 12\omega_9\omega_6^3cs^4\omega_7\omega_4^2\omega_8 - 6\omega_6^3cs^4\omega_7\omega_4^3\omega_8^2 + \\
& 24\omega_9\omega_6^2cs^2v_1^2\omega_7\omega_4\omega_8^2 - 12\omega_9\omega_6^3cs^2\omega_7\omega_4^3\omega_8 - 12\omega_9\omega_6^2cs^4\omega_4^3\omega_8^2 - 36\omega_9\omega_6^2cs^2\omega_4^3v_2^2\omega_8^2 - 12\omega_6^3cs^2\omega_7\omega_4^3\omega_8 - 24\omega_9\omega_6^2v_1^2\omega_7\omega_4^3\omega_8 - \\
& 12\omega_9\omega_6^3cs^2v_1^2\omega_7\omega_4^3\omega_8 + 24\omega_9\omega_6^2cs^4\omega_7\omega_4^2\omega_8^2 - 108\omega_9\omega_6cs^2\omega_7\omega_4^3v_2^2\omega_8^2 + 12\omega_6^3cs^4\omega_7\omega_4^3\omega_8 - 36\omega_9\omega_6^3v_1^2\omega_7\omega_4^3v_2^2 + 18\omega_9\omega_6^2v_1^2\omega_4^3v_2^2\omega_8^2 - \\
& 6\omega_6^3cs^2v_1^2\omega_4^3\omega_8^2 + 84\omega_9\omega_6^3cs^2\omega_7\omega_4v_2^2\omega_8^2 + 12\omega_6^3cs^2v_1^2\omega_7\omega_4^2\omega_8 - 12\omega_9\omega_6v_1^2\omega_7\omega_4^3\omega_8^2 + 60\omega_9\omega_6^2cs^2\omega_7\omega_4^3v_2^2\omega_8 - 36\omega_6^3v_1^2\omega_7\omega_4^3v_2^2\omega_8 + \\
& 78\omega_9\omega_6^3cs^2\omega_7\omega_4^3v_2^2\omega_8 - 12\omega_9\omega_6^3cs^4\omega_7\omega_8 + 18\omega_6^3cs^2\omega_7\omega_4^3v_2^2\omega_8^2 - 72\omega_9\omega_6^2v_1^2\omega_7\omega_4^3v_2^2\omega_8^2 - 12\omega_9\omega_6^2cs^2v_1^2\omega_7\omega_4^3 + 6\omega_9\omega_6^2cs^4\omega_4^3\omega_8^2 + \\
& 24\omega_9\omega_6^3cs^2\omega_7\omega_4^2\omega_8 - 12\omega_6^2v_1^2\omega_7\omega_4^3\omega_8 + 24\omega_9\omega_6^2cs^2\omega_7\omega_4^3v_2^2 - 72\omega_9\omega_6^3v_1^2\omega_7\omega_4v_2^2\omega_8 - 6\omega_6^3cs^2\omega_7\omega_4^3\omega_8^2 + 24\omega_9\omega_6^2v_1^2\omega_7\omega_4^3\omega_8^2 - 12\omega_9\omega_6^2cs^4\omega_7\omega_4^3\omega_8 + \\
& 12\omega_9\omega_6v_1^2\omega_7\omega_4^3\omega_8 - 6\omega_9\omega_6^2v_1^2\omega_4^3\omega_8^2 - 12\omega_9\omega_6cs^4\omega_7\omega_4^3\omega_8^2 + 12\omega_9\omega_6^2cs^2\omega_4^3\omega_8^2 + 18\omega_9\omega_6^3cs^4\omega_7\omega_4\omega_8^2 + 6\omega_6^3cs^2v_1^2\omega_7\omega_4^3\omega_8^2) \frac{\rho}{12\omega_9\omega_6^2\omega_7\omega_4^3\omega_8^2}
\end{aligned}$$

$$\begin{aligned}
C_{D_x^2D_y^2v_2}^{(2),MRT2} = & (18\omega_9\omega_6\omega_7cs^2\omega_4^3v_2^2\omega_8^2 + 12\omega_6^3v_1^3\omega_7cs^2\omega_4^3\omega_8 - 12\omega_9\omega_6^2\omega_7cs^2\omega_4^3\omega_8 + 36\omega_9v_1^3\omega_7\omega_4^3v_2^2\omega_8^2 - 12\omega_9\omega_6\omega_7cs^4\omega_4^3\omega_8^2 + \\
& 18\omega_9\omega_6^3\omega_7cs^4\omega_4\omega_8^2 - 24\omega_9\omega_6^2v_1^2\omega_7\omega_4\omega_8^2 + 6\omega_6^3v_1^2\omega_4^3\omega_8^2 - 24\omega_9\omega_6^3\omega_7cs^2\omega_4^3v_2^2 + 6\omega_6^2\omega_7cs^2\omega_4^3\omega_8^2 - 12\omega_9\omega_6^2v_1^2cs^2\omega_4^3\omega_8^2 - 12\omega_9\omega_6^3v_1^2\omega_7cs^2\omega_4^3\omega_8 + \\
& 12\omega_6^3v_1^2\omega_7\omega_4^2\omega_8^2 - 144\omega_9\omega_6^2\omega_7cs^2\omega_4^3v_2^2\omega_8 - 132\omega_9\omega_6^3\omega_7cs^2\omega_4^3v_2^2\omega_8 + 12\omega_9\omega_6^3v_1^2\omega_7\omega_4^3\omega_8 - 12\omega_9v_1^3\omega_7\omega_4^3\omega_8^2 - 18\omega_6^2v_1^2\omega_7\omega_4^3v_2^2\omega_8^2 - \\
& 36\omega_6^3v_1^2\omega_7\omega_4^3v_2^2\omega_8^2 - 6\omega_9\omega_6\omega_7cs^4\omega_4^3\omega_8^2 - 12\omega_9\omega_6^2\omega_7cs^4\omega_4^3\omega_8 - 84\omega_9\omega_6^2\omega_7cs^2\omega_4^3v_2^2\omega_8^2 + 108\omega_9\omega_6^3v_1^2\omega_7\omega_4^3v_2^2\omega_8 + 72\omega_9\omega_6^2v_1^2\omega_7\omega_4^3v_2^2\omega_8 + \\
& 6\omega_6^3v_1^2\omega_7cs^2\omega_4^3\omega_8^2 + 6\omega_9\omega_6^2\omega_7cs^2\omega_4^3\omega_8^2 - 6\omega_9\omega_6^2cs^2\omega_4^3\omega_8^2 - 36\omega_6^3\omega_7cs^2\omega_4^3v_2^2\omega_8^2 - 18\omega_6^2\omega_7cs^2\omega_4^3v_2^2\omega_8^2 - 18\omega_6^3v_1^2\omega_4^3v_2^2\omega_8^2 + 36\omega_6^3cs^2\omega_4^3v_2^2\omega_8^2 + \\
& 72\omega_9\omega_6^2v_1^2\omega_7\omega_4v_2^2\omega_8^2 - 54\omega_9\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8^2 - 12\omega_9\omega_6^2cs^4\omega_4^3\omega_8^2 + 24\omega_9\omega_6^2\omega_7cs^4\omega_4^3\omega_8^2 + 12\omega_6^2\omega_7cs^4\omega_4^3\omega_8 - 12\omega_9\omega_6^3\omega_7cs^2\omega_4\omega_8 + \\
& 12\omega_6^3v_1^2\omega_7\omega_4^3\omega_8 + 12\omega_9\omega_6^2v_1^2\omega_7\omega_4^3 + 12\omega_9\omega_6\omega_7cs^2\omega_4^3\omega_8 - 36\omega_9\omega_6v_1^2\omega_7\omega_4^3v_2^2\omega_8 - 48\omega_9\omega_6^3\omega_7cs^2v_2^2\omega_8^2 - 36\omega_9\omega_6^3v_1^2\omega_7\omega_4^3\omega_8 + 12\omega_9\omega_6^2\omega_7cs^4\omega_4^3\omega_8 - \\
& 6\omega_6^2\omega_7cs^4\omega_4^3\omega_8^2 - 36\omega_9\omega_6^2v_1^2\omega_7\omega_4^3v_2^2 - 6\omega_6^3v_1^2\omega_7\omega_4^3\omega_8^2 + 36\omega_9\omega_6^3v_1^2\omega_7cs^2\omega_4^3\omega_8 - 12\omega_9\omega_6^2v_1^2\omega_7\omega_4^3 + 12\omega_9\omega_6^3v_1^2\omega_7cs^2\omega_4^3 + 18\omega_9\omega_6^2v_1^2\omega_7\omega_4^3v_2^2\omega_8^2 +
\end{aligned}$$

$$\begin{aligned}
& 24\omega_9\omega_6^3\omega_7cs^2\omega_4^2v_2^2 + 6\omega_9\omega_6^2v_1^2cs^2\omega_4^3\omega_8^2 - 12\omega_6^3v_1^2\omega_4^2\omega_8^2 + 36\omega_6^2\omega_7cs^2\omega_4^3v_2^2\omega_8 + 36\omega_6^3\omega_7cs^2\omega_4^2v_2^2\omega_8 + 12\omega_9\omega_6^2\omega_7cs^2\omega_4^3\omega_8 - 12\omega_6^3v_1^2\omega_7cs^2\omega_4^3\omega_8 + \\
& 6\omega_9\omega_6\omega_7cs^4\omega_4^3\omega_8^2 + 18\omega_9\omega_6^2cs^2\omega_4^3v_2^2\omega_8^2 - 42\omega_9\omega_6^3\omega_7cs^2\omega_4^2v_2^2\omega_8^2 - 12\omega_6^3v_1^2\omega_7\omega_4^2\omega_8 - 18\omega_9\omega_6^2\omega_7cs^2\omega_4^3v_2^2\omega_8^2 - 36\omega_9\omega_6^2v_1^2\omega_4^2v_2^2\omega_8^2 + \\
& 36\omega_6^3v_1^2\omega_7\omega_4^2v_2^2\omega_8 + 36\omega_6^2v_1^2\omega_7\omega_4^3v_2^2\omega_8 + 24\omega_9\omega_6^2v_1^2\omega_7cs^2\omega_4^2\omega_8^2 + 6\omega_9\omega_6^2cs^4\omega_4^3\omega_8^2 - 6\omega_9\omega_6^2\omega_7cs^4\omega_4^3\omega_8^2 - 24\omega_9\omega_6^2\omega_7cs^2\omega_4^2\omega_8^2 + 12\omega_9\omega_6^2cs^2\omega_4^2\omega_8^2 + \\
& 72\omega_9\omega_6\omega_7cs^2\omega_4^3v_2^2\omega_8 - 12\omega_6^3v_1^2\omega_7cs^2\omega_4^2\omega_8^2 + 12\omega_9\omega_6^3\omega_7cs^4\omega_4\omega_8 - 12\omega_9\omega_6^3v_1^2\omega_7cs^2\omega_4^2 - 12\omega_6^2\omega_7cs^2\omega_4^3\omega_8 + 12\omega_9\omega_6^3v_1^2\omega_7\omega_4^2 - \\
& 12\omega_9\omega_6^3\omega_7cs^2\omega_4^3\omega_8 + 18\omega_9\omega_6v_1^2\omega_7\omega_4^3\omega_8^2 + 12\omega_9v_1^2\omega_7cs^2\omega_4^3\omega_8^2 + 36\omega_9\omega_6v_1^2\omega_7\omega_4^2v_2^2\omega_8^2 - 24\omega_9\omega_6^2v_1^2\omega_7cs^2\omega_4^2\omega_8^2 - 12\omega_6^3cs^2\omega_4^2\omega_8^2 + \\
& 12\omega_6^3\omega_7cs^2\omega_4^2\omega_8^2 - 12\omega_9\omega_6^2v_1^2\omega_7cs^2\omega_4^3 + 6\omega_6^3\omega_7cs^4\omega_4^3\omega_8^2 - 36\omega_9\omega_6^3v_1^2\omega_7\omega_4^3v_2^2\omega_8 - 36\omega_9\omega_6^2v_1^2\omega_7\omega_4^2v_2^2\omega_8 + 6\omega_6^3cs^4\omega_4^3\omega_8^2 + 12\omega_9\omega_6^2\omega_7cs^2\omega_4\omega_8^2 + \\
& 36\omega_9\omega_6^3v_1^2\omega_7\omega_4^3v_2^2 + 6\omega_6^3v_1^2\omega_7\omega_4^3\omega_8^2 - 12\omega_9\omega_6^3\omega_7cs^4\omega_8^2 - 18\omega_6^3cs^2\omega_4^3v_2^2\omega_8^2 + 12\omega_9\omega_6^2v_1^2\omega_7\omega_4^3\omega_8 + 36\omega_6^3v_1^2\omega_4^3v_2^2\omega_8^2 + 18\omega_6^3\omega_7cs^2\omega_4^3v_2^2\omega_8^2 - \\
& 24\omega_9\omega_6^3\omega_7cs^4\omega_4\omega_8 - 6\omega_6^3v_1^2\omega_7cs^2\omega_4^3\omega_8^2 + 60\omega_9\omega_6^2\omega_7cs^2\omega_4^2v_2^2\omega_8 + 78\omega_9\omega_6^3\omega_7cs^2\omega_4^3v_2^2\omega_8 + 24\omega_9\omega_6^2v_1^2\omega_7cs^2\omega_4^3\omega_8 + 24\omega_9\omega_6^3v_1^2\omega_7\omega_4\omega_8 + \\
& 12\omega_6^3\omega_7cs^2\omega_4^3\omega_8 - 24\omega_9\omega_6^3v_1^2\omega_7cs^2\omega_4\omega_8 + 12\omega_9\omega_6^2v_1^2\omega_4^3\omega_8^2 + 84\omega_9\omega_6^3\omega_7cs^2\omega_4v_2^2\omega_8^2 - 6\omega_6^3v_1^2cs^2\omega_4^3\omega_8^2 + 18\omega_6^3v_1^2\omega_7\omega_4^3v_2^2\omega_8^2 - 6\omega_9\omega_6^2v_1^2\omega_7\omega_4^3\omega_8^2 - \\
& 18\omega_9\omega_6v_1^2\omega_7cs^2\omega_4^3\omega_8^2 - 108\omega_9\omega_6\omega_7cs^2\omega_4^2v_2^2\omega_8^2 - \omega_9\omega_6^3\omega_7cs^4\omega_4^3\omega_8^2 + 12\omega_6^3\omega_7cs^4\omega_4^3\omega_8 - 24\omega_9\omega_6^2v_1^2\omega_7\omega_4^3\omega_8 + 12\omega_9\omega_6^3\omega_7cs^4\omega_4^3\omega_8 - \\
& 12\omega_9\omega_6^2v_1^2\omega_7cs^2\omega_4^3\omega_8 + 12\omega_6^3cs^4\omega_4^3\omega_8^2 - 12\omega_6^3\omega_7cs^4\omega_4^3\omega_8^2 - 36\omega_9\omega_6^3v_1^2\omega_7\omega_4^2v_2^2 + 18\omega_9\omega_6^2v_1^2\omega_4^3v_2^2\omega_8^2 + 6\omega_9\omega_6^2v_1^2\omega_7cs^2\omega_4^3\omega_8^2 - \\
& 12\omega_9\omega_6^2\omega_7cs^4\omega_4\omega_8^2 - 36\omega_9\omega_6^2cs^2\omega_4^2v_2^2\omega_8^2 + 180\omega_9\omega_6^2\omega_7cs^2\omega_4^2v_2^2\omega_8 + 24\omega_9\omega_6^2\omega_7cs^2\omega_4^3v_2^2 - 6\omega_6^3\omega_7cs^2\omega_4^3\omega_8^2 + 6\omega_6^3cs^2\omega_4^3\omega_8^2 - \\
& 12\omega_9\omega_6v_1^2\omega_7\omega_4^3\omega_8^2 + 24\omega_9\omega_6^3\omega_7cs^2\omega_4^3\omega_8 + 60\omega_9\omega_6^3\omega_7cs^2\omega_4v_2^2\omega_8 - 36\omega_6^3v_1^2\omega_7\omega_4^3v_2^2\omega_8 - 72\omega_9\omega_6^3v_1^2\omega_7\omega_4^3v_2^2\omega_8^2 - 12\omega_6^3\omega_7cs^4\omega_4^3\omega_8 - \\
& 12\omega_6^3v_1^2\omega_7\omega_4^3\omega_8 - 72\omega_9\omega_6^3v_1^2\omega_7\omega_4^2v_2^2\omega_8 + 24\omega_9\omega_6^2v_1^2\omega_7\omega_4^2\omega_8^2 - 4\omega_9\omega_6^3\omega_7cs^4\omega_4^3\omega_8^2 + 12\omega_6^3v_1^2\omega_7cs^2\omega_4^3\omega_8 - 36\omega_6^3\omega_7cs^2\omega_4^3v_2^2\omega_8 + \\
& 12\omega_9\omega_6v_1^2\omega_7cs^2\omega_4^3\omega_8^2 + 12\omega_9\omega_6v_1^2\omega_7\omega_4^3\omega_8 + 12\omega_6^3v_1^2cs^2\omega_4^3\omega_8^2 - 6\omega_9\omega_6^3v_1^2\omega_4^3\omega_8^2 - 12\omega_9\omega_6^2v_1^2\omega_7cs^2\omega_4^3\omega_8 - 12\omega_6^3\omega_7cs^2\omega_4^3\omega_8^2) \frac{\rho}{12\omega_9\omega_6^3\omega_7\omega_4^3\omega_8^2}
\end{aligned}$$

$$\begin{aligned}
C_{D_x^2 D_y^2 v_2}^{(2), \text{CLBM1}} &= (12\omega_6^2\omega_7\omega_2^2cs^2 - 4\omega_9\omega_6^2\omega_7\omega_4^2\omega_8cs^2 + 12\omega_9\omega_6\omega_7\omega_4^2cs^2 + 6\omega_9\omega_7\omega_4^3\omega_8cs^2 - 12\omega_6^2\omega_7\omega_4^2\omega_8cs^2 - 6\omega_6^2\omega_4^3\omega_8cs^2 + 6\omega_9\omega_6\omega_7\omega_4^3\omega_8 + \\
& 6\omega_9\omega_6\omega_7^3\omega_8cs^2 - 6\omega_6^2\omega_7\omega_4^3\omega_8 + 12\omega_9\omega_7\omega_4^2\omega_8 + 24\omega_9\omega_6\omega_7\omega_4^2\omega_8cs^2 + 36\omega_9\omega_6^2\omega_7\omega_4^3v_2^2 + 36\omega_6\omega_7\omega_4^3v_2^2 + 18\omega_9\omega_6\omega_4^3v_2^2\omega_8 + 12\omega_6^2\omega_7\omega_4^3 - \\
& 18\omega_6^2\omega_4^3v_2^2\omega_8 + 12\omega_9\omega_6\omega_7\omega_4^3 - 36\omega_6^2\omega_7\omega_4^3v_2^2\omega_8 - 12\omega_9\omega_6\omega_7\omega_4^3cs^2 - 12\omega_6^2\omega_7\omega_4^3cs^2 + 72\omega_9\omega_6\omega_7\omega_4^2v_2^2\omega_8 - 12\omega_9\omega_6\omega_7\omega_4^2 - 12\omega_9\omega_6^2\omega_7\omega_8cs^2 - \\
& 12\omega_6^2\omega_7\omega_4^2 - 72\omega_9\omega_6^2\omega_7\omega_4^3v_2^2 - 6\omega_9\omega_7\omega_4^3\omega_8 + 18\omega_9\omega_7\omega_4^3v_2^2\omega_8 + 12\omega_6^2\omega_7\omega_4^2\omega_8 + 12\omega_9\omega_6^2\omega_7\omega_4cs^2 - 24\omega_9\omega_6\omega_7\omega_4^2\omega_8 - 12\omega_9\omega_6^2\omega_7\omega_4^3 - \\
& 12\omega_6\omega_7\omega_4^3 - 6\omega_9\omega_6\omega_7\omega_4^3\omega_8cs^2 + 36\omega_9\omega_6^2\omega_7\omega_4v_2^2 + 12\omega_6^2\omega_4^2\omega_8cs^2 - 24\omega_9\omega_6^2\omega_7\omega_4cs^2 + 6\omega_6^2\omega_7\omega_4^3\omega_8cs^2 - 36\omega_9\omega_6\omega_7\omega_4v_2^2\omega_8 - 12\omega_9\omega_6\omega_4^2\omega_8cs^2 + \\
& 12\omega_9\omega_6\omega_7\omega_4\omega_8 - 6\omega_6\omega_7\omega_4^3\omega_8cs^2 - 36\omega_6^2\omega_7\omega_4^3v_2^2 - 36\omega_9\omega_6\omega_7\omega_4^3v_2^2 - 12\omega_9\omega_7\omega_4^3\omega_8cs^2 + 12\omega_9\omega_6\omega_4^2\omega_8 - 12\omega_6^2\omega_4^2\omega_8 + 6\omega_6\omega_7\omega_4^3\omega_8 - \\
& \omega_9\omega_6^2\omega_7\omega_4^3\omega_8cs^2 + 12\omega_6\omega_7\omega_4^3cs^2 - 36\omega_9\omega_7\omega_4^2v_2^2\omega_8 - 18\omega_6\omega_7\omega_4^3v_2^2\omega_8 + 12\omega_9\omega_6^2\omega_7\omega_4cs^2 + 18\omega_9\omega_6^2\omega_7\omega_4\omega_8cs^2 - 12\omega_9\omega_6\omega_7\omega_4 - 18\omega_9\omega_6\omega_7\omega_4^3v_2^2\omega_8 - \\
& 36\omega_9\omega_6\omega_4^2v_2^2\omega_8 - 6\omega_9\omega_6\omega_4^3\omega_8 + 36\omega_9\omega_6\omega_7\omega_4v_2^2 + 6\omega_6^2\omega_4^3\omega_8 + 18\omega_6^2\omega_7\omega_4^3v_2^2\omega_8 + 36\omega_6^2\omega_7\omega_4^3v_2^2 + 36\omega_6^2\omega_4^3v_2^2\omega_8 - 12\omega_9\omega_6\omega_7\omega_4\omega_8cs^2) \frac{\rho cs^2}{12\omega_9\omega_6^2\omega_7\omega_4^3\omega_8}
\end{aligned}$$

$$\begin{aligned}
C_{D_x^2 D_y^2 v_2}^{(2), \text{CLBM2}} &= (12cs^2\omega_6\omega_7\omega_4^3 + 12\omega_9cs^2\omega_6^2\omega_7\omega_4 - \omega_9cs^2\omega_6^2\omega_7\omega_4^3\omega_8 + 6\omega_9\omega_6\omega_7\omega_4^3\omega_8 + 6\omega_9cs^2\omega_6\omega_4^3\omega_8 + 6cs^2\omega_6^2\omega_7\omega_4^3\omega_8 + 12cs^2\omega_6^2\omega_2^2\omega_8 - \\
& 6\omega_6^2\omega_7\omega_4^3\omega_8 - 12\omega_9cs^2\omega_6\omega_7\omega_4\omega_8 + 12\omega_9\omega_7\omega_4^2\omega_8 + 36\omega_9\omega_6^2\omega_7\omega_4^3v_2^2 + 36\omega_6\omega_7\omega_4^3v_2^2 + 18\omega_9\omega_6\omega_4^3v_2^2\omega_8 + 12\omega_6^2\omega_7\omega_4^3 - 18\omega_6^2\omega_4^3v_2^2\omega_8 + 12\omega_9\omega_6\omega_7\omega_4^3 - \\
& 36\omega_6^2\omega_7\omega_4^3v_2^2\omega_8 - 4\omega_9cs^2\omega_6^2\omega_7\omega_4^2\omega_8 + 12\omega_9cs^2\omega_6^2\omega_7\omega_4^3 + 72\omega_9\omega_6\omega_7\omega_4^2v_2^2\omega_8 - 12\omega_9\omega_6\omega_7\omega_4^2 - 12\omega_6^2\omega_7\omega_4^2 - 72\omega_9\omega_6^2\omega_7\omega_4^3v_2^2 - 6\omega_9\omega_7\omega_4^3\omega_8 - \\
& 12\omega_9cs^2\omega_6\omega_4^3\omega_8 + 18\omega_9\omega_7\omega_4^3v_2^2\omega_8 + 12\omega_6^2\omega_7\omega_4^2\omega_8 - 24\omega_9cs^2\omega_6^2\omega_7\omega_4^2 - 12cs^2\omega_6^2\omega_7\omega_4^3\omega_8 - 6cs^2\omega_6^2\omega_4^3\omega_8 - 24\omega_9\omega_6\omega_7\omega_4^2\omega_8 - 12\omega_9\omega_6^2\omega_7\omega_4^3 - \\
& 12\omega_6\omega_7\omega_4^3 + 36\omega_9\omega_6^2\omega_7\omega_4v_2^2 - 12\omega_9cs^2\omega_6^2\omega_7\omega_8 - 6\omega_9cs^2\omega_6\omega_7\omega_4^3\omega_8 - 36\omega_9\omega_6\omega_7\omega_4v_2^2\omega_8 - 12\omega_9cs^2\omega_6\omega_7\omega_4^3 + 12\omega_9\omega_6\omega_7\omega_4\omega_8 - 36\omega_6^2\omega_7\omega_4^3v_2^2 - \\
& 36\omega_9\omega_7\omega_4^3v_2^2 + 36\omega_9\omega_6^2\omega_7\omega_4^3\omega_8 - 12\omega_6^2\omega_4^3\omega_8 + 24\omega_9\omega_6^2\omega_7\omega_4^2 + 6\omega_9cs^2\omega_7\omega_4^3\omega_8 + 6\omega_6\omega_7\omega_4^3\omega_8 - 6cs^2\omega_6\omega_7\omega_4^3\omega_8 + 18\omega_9cs^2\omega_6^2\omega_7\omega_4\omega_8 + \\
& 12\omega_9cs^2\omega_6\omega_7\omega_4^2 - 36\omega_9\omega_7\omega_4^2v_2^2\omega_8 - 12cs^2\omega_6^2\omega_7\omega_4^3 + 24\omega_9cs^2\omega_6\omega_7\omega_4^2\omega_8 - 18\omega_6\omega_7\omega_4^3v_2^2\omega_8 - 12\omega_9\omega_6^2\omega_7\omega_4 + 12cs^2\omega_6^2\omega_7\omega_4^2 - 12\omega_9cs^2\omega_7\omega_4^2\omega_8 - \\
& 18\omega_9\omega_6\omega_7\omega_4^2v_2^2\omega_8 - 36\omega_9\omega_6\omega_4^2v_2^2\omega_8 - 6\omega_9\omega_6\omega_4^3\omega_8 + 36\omega_9\omega_6\omega_7\omega_4^2v_2^2 + 6\omega_6^2\omega_4^3\omega_8 + 18\omega_6^2\omega_7\omega_4^3v_2^2\omega_8 + 36\omega_6^2\omega_7\omega_4^3v_2^2 + 36\omega_6^2\omega_4^3v_2^2\omega_8) \frac{cs^2\rho}{12\omega_9\omega_6^2\omega_7\omega_4^3\omega_8}
\end{aligned}$$

$$\begin{aligned}
C_{D_x^2 D_y^2 v_2}^{(2), \text{CuLBM1}} &= (-72\omega_3^3v_2^2\omega_2 + 12\omega_6\omega_3\omega_2 + 36\omega_3v_2^2\omega_2^2 + 12\omega_3^3cs^2\omega_2^2 - 12\omega_6\omega_3cs^2\omega_2 - \omega_6\omega_3^3cs^2\omega_2^2 - 36\omega_6\omega_3^2v_2^2 + 18\omega_6\omega_3cs^2\omega_2^2 - 36\omega_6\omega_3v_2^2\omega_2 - \\
& 12\omega_3\omega_2^2 + 36\omega_3^3v_2^2\omega_2^2 + 12\omega_3cs^2\omega_2^2 + 36\omega_3^3v_2^2 - 24\omega_3^3cs^2\omega_2 + 12\omega_6\omega_3 - 12\omega_6\omega_3cs^2 + 24\omega_3^2\omega_2^2 + 36\omega_6\omega_3^2v_2^2\omega_2 - 4\omega_6\omega_3^3cs^2\omega_2^2 + 72\omega_3^2v_2^2\omega_2 - \\
& 24\omega_3^3cs^2\omega_2^2 + 24\omega_3^3\omega_2 - 72\omega_3^3v_2^2\omega_2^2 + 12\omega_3^3cs^2 + 24\omega_3^3cs^2\omega_2 - 12\omega_3^3\omega_2^2 - 12\omega_6cs^2\omega_2^2 + 12\omega_6\omega_3cs^2\omega_2 - 24\omega_3^2\omega_2 - 12\omega_6\omega_3\omega_2 - 12\omega_3^3) \frac{\rho cs^2}{12\omega_6\omega_3^3\omega_2^2}
\end{aligned}$$

$$\begin{aligned}
C_{D_x^2 D_y^2 v_2}^{(2), \text{CuLBM2}} &= (-135\omega_3^2v_1^2\omega_1^2v_2^2\omega_2 - 90\omega_3^2\omega_1v_2^2cs^2\omega_2^2 + 2\omega_3^3\omega_1^3cs^4\omega_2^2 - 108v_1^2\omega_1^3cs^2\omega_2^2 + \omega_3^2\omega_1^2v_2^2\omega_2^3 - 60\omega_3\omega_1^2cs^4\omega_3^3 - 9\omega_3^2v_1^2\omega_1^3\omega_2^2 + \\
& 126\omega_3\omega_1v_2^2cs^2\omega_2^3 - 6\omega_3^2\omega_1^2\omega_2 - 42\omega_3^2\omega_1^2cs^2\omega_2^3 - 24\omega_3\omega_1^3cs^2\omega_2^3 - 18\omega_3v_1^2\omega_1\omega_2^3 - 36\omega_3^2\omega_1^2v_2^2\omega_2^2 - 2\omega_3^3\omega_1^3cs^4\omega_2^3 - 45\omega_3^2v_1^4\omega_1^3\omega_2 + 60\omega_3\omega_1^2cs^4\omega_2^2 + \\
& 9\omega_3^2\omega_1v_2^2cs^2\omega_2^3 - 9\omega_3^2v_1^4\omega_1^3\omega_2^3 - 9\omega_3^2v_1^2\omega_2^3 + \omega_3^2\omega_1^3v_2^2\omega_2^2 + 2\omega_3^2\omega_1^2cs^2\omega_2^3 + 36\omega_3\omega_1^3cs^2\omega_2^2 - 81\omega_3^2v_1^2\omega_1^2cs^2\omega_2 + 18\omega_3^2\omega_1^3v_2^2cs^2 + 45\omega_3^2v_1^2\omega_1^2\omega_2 + \\
& 18\omega_3^2\omega_1^3cs^4 - 6\omega_3\omega_1^3cs^2\omega_2 - 6\omega_3^2\omega_1^3\omega_2 + 108\omega_3^2v_1^2\omega_1^2cs^2\omega_2^2 - 90\omega_3^2v_1^2\omega_2^2 - 144\omega_3v_1^2\omega_1^2cs^2\omega_2^3 + 9\omega_3^2v_1^4\omega_1^3\omega_2^2 + 18\omega_3\omega_1^2v_2^2\omega_2 + 18\omega_3v_1^4\omega_1\omega_2^3 + \\
& 45\omega_3^2\omega_1cs^4\omega_2^3 + 6\omega_3^2\omega_1^3 - 30\omega_3\omega_1cs^2\omega_2^3 - 30\omega_3^2v_1^2\omega_1^2cs^2\omega_2^3 + 9\omega_3^2v_1^2\omega_1^2\omega_2^3 + 12\omega_3^2\omega_1^2\omega_2^2 + 90\omega_3^2v_1^2\omega_1^3\omega_2 + 36\omega_3^2v_1^4\omega_1^3 + 36\omega_3^2\omega_1^2cs^2\omega_2 - 30\omega_3^2cs^4\omega_2^3 - \\
& 18\omega_3^2v_2^2cs^2\omega_2^3 - 30\omega_3^2\omega_1cs^4\omega_2^2 + 270\omega_3^2v_1^2\omega_1^2v_2^2\omega_2^2 + 36\omega_3v_1^2\omega_1^2cs^2\omega_2^2 - 15\omega_3^2\omega_1^3cs^4\omega_2 - 60\omega_3\omega_1^2cs^2\omega_2^3 - 6\omega_3^2\omega_1\omega_2^2 - 45\omega_3^2v_1^2\omega_1cs^2\omega_2^2 - 18\omega_3^2\omega_1^3v_2^2 - \\
& 216\omega_3^2v_1^2\omega_1^3cs^2\omega_2 - 36\omega_3^2v_1^4\omega_2^3 - 54\omega_3v_1^2\omega_1^2\omega_2^3 - 9\omega_3^2\omega_1^3v_2^2cs^2\omega_2 - 198\omega_3v_1^2v_2^2cs^2\omega_2^3 + 72\omega_3^2v_1^2v_2^2cs^2\omega_2^2 - 24\omega_3^2\omega_1^3cs^2 - 36\omega_3\omega_1^3cs^4\omega_2^2 - \\
& 135\omega_3^2v_1^2\omega_1v_2^2\omega_2^3 - \omega_3^2\omega_1^3v_2^2\omega_2^3 + 108v_1^2\omega_1^2cs^2\omega_2^3 - 10\omega_3^2\omega_1^2cs^4\omega_2^3 - 36v_1^2\omega_1^2\omega_2^3 + 18\omega_3v_1^2\omega_1cs^2\omega_2^3 + 144\omega_3\omega_1^2v_2^2cs^2\omega_2^2 + 135\omega_3^2v_1^2\omega_1^3v_2^2 + \\
& 45\omega_3^2v_1^4\omega_1\omega_2^3 - 36v_1^4\omega_1^3\omega_2^3 + 60\omega_3v_1^2cs^2\omega_2^3 - 2\omega_3^2\omega_1^3cs^2\omega_2^3 + 108\omega_3^2v_1^2\omega_1^3cs^2\omega_2^3 - \omega_3^2\omega_1^3v_2^4\omega_2^3 - 6\omega_3^2\omega_1\omega_2^3 - 135\omega_3^2v_1^2\omega_1^3v_2^2\omega_2 + 24\omega_3\omega_1^3cs^4\omega_2^3 - \\
& 54\omega_3v_1^2\omega_1^3cs^2\omega_2 + 30\omega_3^2\omega_1^3cs^4\omega_2^2 - 18\omega_3v_1^4\omega_1^3\omega_2 + 6\omega_3^2\omega_2^3 - 135\omega_3^2v_1^2\omega_1v_2^2\omega_2^3 + 18\omega_3\omega_1^3v_2^2cs^2\omega_2 - 6\omega_3^2\omega_1^2v_2^2cs^2\omega_2^3 - 54\omega_3v_1^4\omega_1^2\omega_2^3 + \\
& 189\omega_3^2v_1^2\omega_1^3cs^2 + 18\omega_3^2\omega_1v_2^2\omega_2^3 + 45\omega_3^2v_1^4\omega_1\omega_2^3 + 30\omega_3\omega_1cs^4\omega_2^3 + 135\omega_3^2v_1^2v_2^2\omega_2^3 - 81\omega_3^2v_1^2\omega_2^3 + 54\omega_3v_1^4\omega_1^3\omega_2^2 - 30\omega_3^2\omega_1^3cs^4\omega_2 + 144\omega_3v_1^2\omega_1^3cs^2\omega_2^2 - \\
& 90\omega_3\omega_1^3v_2^2cs^2\omega_2^2 + 36\omega_3^2\omega_1cs^2\omega_2^2 + 36v_1^4\omega_1^3\omega_2^3 + 21\omega_3^2\omega_1^3cs^2\omega_2 + 72\omega_3\omega_1^3v_2^2cs^2\omega_2^3 - 54\omega_3^2\omega_1^2v_2^2cs^2\omega_2 + 18\omega_3^2\omega_1^3v_2^2\omega_2 + 6\omega_3\omega_1^3cs^4\omega_2 - 18\omega_3^2v_2^2\omega_2^3 + \\
& 18\omega_3^2\omega_1v_2^2\omega_2^2 + 36v_1^2\omega_1^3\omega_2^2 + 30\omega_3^2v_1^2\omega_1^3cs^2\omega_2^2 - 3\omega_3^2\omega_1cs^2\omega_2^2 - 63\omega_3^2v_1^2cs^2\omega_2^2 + 54\omega_3v_1^2\omega_1^2\omega_2^2 + 6\omega_3^2\omega_1^3v_2^2cs^2\omega_2^2 + 18\omega_3^2v_1^2\omega_1^3\omega_2^2) \frac{\rho}{24\omega_3^3\omega_1^3\omega_2^3}
\end{aligned}$$

$$\text{coefficient } C_{D_x D_y^3 \rho}^{(2)} \text{ at } \frac{\partial^4 \rho}{\partial x_1 \partial x_3^2} :$$

$$C_{D_x D_y^3 \rho}^{(2), \text{SRT}} = 0$$

$$C_{D_x D_y^3 \rho}^{(2), \text{MRT1}} = (-24\omega_4^2v_2^2\omega_8^2 + 36\omega_6\omega_4^2v_2^2\omega_8^2 + 4\omega_6^3cs^2\omega_4 - 4\omega_6cs^2\omega_4\omega_8^2 + 72\omega_6cs^2\omega_4v_2^2\omega_8 - 8\omega_6^3cs^2\omega_4^2\omega_8 - 24\omega_6^3cs^2\omega_4v_2^2 + 4\omega_6^2\omega_4^2v_2^2 -$$

$$84\omega_6^2cs^2\omega_4v_2^2\omega_8^2+13\omega_6^2\omega_1^4v_2^2\omega_8^2+4\omega_6^2cs^4\omega_4^2\omega_8^2-4\omega_6^3cs^4\omega_1^2\omega_8-8\omega_6^2v_2^2\omega_8^2-13\omega_6^3\omega_4^2v_2^4\omega_8+16\omega_6^2\omega_4v_2^2\omega_8-4\omega_6^3cs^4\omega_8+4\omega_6^3\omega_1^2v_2^4+20\omega_6^2\omega_4v_2^2\omega_8^2-51\omega_6^3cs^2\omega_4^2v_2^2\omega_8+4\omega_6cs^2\omega_4\omega_8^2-8\omega_6^3v_2^4\omega_8+32\omega_6^2\omega_4^2v_2^4\omega_8-48\omega_6^2cs^2\omega_4v_2^2\omega_8-4\omega_6^2cs^2\omega_8^2-20\omega_6^3\omega_4v_2^2\omega_8+8\omega_6^2cs^4\omega_4^2\omega_8+20\omega_6\omega_4^2v_2^2\omega_8-4\omega_6^3cs^2\omega_4^2-4\omega_6^2cs^2\omega_1^2\omega_8^2-4\omega_6^2cs^4\omega_4^2+4\omega_6^3\omega_4v_2^2+4\omega_6^3cs^2\omega_4^2\omega_8+20\omega_6\omega_4v_2^4\omega_8^2+84\omega_6^3cs^2\omega_4v_2^2\omega_8+4\omega_6cs^2\omega_1^2\omega_8+36\omega_6^2cs^2v_2^2\omega_8^2-13\omega_6^3\omega_4^2v_2^2\omega_8^2+8\omega_6^2cs^2\omega_4\omega_8^2-8\omega_6^3cs^2\omega_4\omega_8-24\omega_6^2cs^2\omega_1^2v_2^2-16\omega_6^2\omega_4v_2^4\omega_8+120\omega_6^2cs^2\omega_4^2v_2^2\omega_8-4\omega_6^3\omega_4^2v_2^2+8\omega_6^2v_2^4\omega_8^2+13\omega_6^3\omega_4^2v_2^2\omega_8-36\omega_6^3cs^3v_2^2\omega_8+4\omega_6^3cs^2\omega_8-4\omega_6^3\omega_4v_2^4+8cs^4\omega_4^2\omega_8^2+24\omega_4^2v_2^4\omega_8^2-72\omega_6cs^2\omega_4^2v_2^2\omega_8-36\omega_6\omega_4^2v_2^2\omega_8^2-12\omega_6cs^4\omega_4^2\omega_8^2-4\omega_6^3cs^4\omega_4-20\omega_6\omega_4^2v_2^4\omega_8-4\omega_6cs^4\omega_4^2\omega_8+4\omega_6^3cs^4\omega_4^2-8\omega_6^2cs^4\omega_4\omega_8^2+4\omega_6^2cs^2\omega_4^2-144\omega_6cs^2\omega_4^2v_2^2\omega_8^2+8\omega_6^3cs^4\omega_4\omega_8-20\omega_6\omega_4v_2^2\omega_8^2+51\omega_6^2cs^2\omega_4^2v_2^2\omega_8^2-20\omega_6^2\omega_4v_2^4\omega_8^2-8cs^2\omega_4^2\omega_8^2+8\omega_6^3v_2^2\omega_8+12\omega_6cs^2\omega_4^2\omega_8^2-4\omega_6^2\omega_4^2v_2^4+4\omega_6^2cs^4\omega_8^2+96cs^2\omega_4^2v_2^2\omega_8^2+24\omega_6^3cs^2\omega_4^2v_2^2+20\omega_6^3\omega_4v_2^4\omega_8-32\omega_6^2\omega_4^2v_2^2\omega_8^2)\frac{v_1}{4\omega_6^3\omega_4^2\omega_8^2}$$

$$C_{\mathbf{D}_x\mathbf{D}_y^3\rho}^{(2),\text{MRT}2} = (-24\omega_4^2v_2^2\omega_8^2+8\omega_6^2cs^2\omega_4\omega_8^2+36\omega_6^2cs^2v_2^2\omega_8^2+4\omega_6cs^2\omega_4^2\omega_8+72\omega_6cs^2\omega_4v_2^2\omega_8^2+36\omega_6\omega_4^2v_2^2\omega_8^2-36\omega_6^3cs^2v_2^2\omega_8-8\omega_6^3cs^2\omega_4\omega_8-24\omega_6^2cs^2\omega_4^2v_2^2-4\omega_6^2cs^2\omega_8^2-4\omega_6^3cs^2\omega_4^2+4\omega_6^2\omega_4^2v_2^2+8cs^4\omega_4^2\omega_8^2+13\omega_6^2\omega_4^2v_2^2\omega_8^2-8\omega_6^2v_2^2\omega_8^2-4\omega_6^2cs^4\omega_4^2-13\omega_6^3\omega_4^2v_2^2\omega_8+16\omega_6^2\omega_4v_2^2\omega_8-84\omega_6^2cs^2\omega_4v_2^2\omega_8^2-12\omega_6cs^4\omega_4^2\omega_8^2-8\omega_6^2cs^4\omega_4\omega_8^2+4\omega_6^3\omega_4^2v_2^4+20\omega_6^2\omega_4v_2^2\omega_8^2-48\omega_6^2cs^2\omega_4v_2^2\omega_8-4\omega_6cs^4\omega_4^2\omega_8+4\omega_6^3cs^2\omega_4-8\omega_6^3v_2^4\omega_8+32\omega_6^3\omega_4^2v_2^2\omega_8-20\omega_6^3\omega_4v_2^2\omega_8+8\omega_6^3cs^3\omega_4\omega_8-51\omega_6^3cs^2\omega_1^2v_2^2\omega_8+20\omega_6\omega_4^2v_2^2\omega_8-8cs^2\omega_4^2\omega_8^2+24\omega_6^3cs^2\omega_4^2v_2^2+4\omega_6^3\omega_4v_2^4+12\omega_6cs^2\omega_4^2\omega_8^2-4\omega_6^3cs^4\omega_8+20\omega_6\omega_4v_2^4\omega_8^2+4\omega_6^3cs^4\omega_4^2+120\omega_6^2cs^2\omega_4^2v_2^2\omega_8-13\omega_6^2\omega_4^2v_2^2\omega_8^2+84\omega_6^3cs^2\omega_4v_2^2\omega_8-8\omega_6^2cs^2\omega_4^2\omega_8-16\omega_6^2\omega_4v_2^4\omega_8-4\omega_6^3\omega_1^2v_2^2-24\omega_6^3cs^2\omega_4v_2^2+8\omega_6^2v_2^2\omega_8+4\omega_6cs^2\omega_4^2-4\omega_6cs^2\omega_4\omega_8^2-4\omega_6^3\omega_4^2v_2^4+24\omega_4^2v_2^4\omega_8^2+4\omega_6^2cs^4\omega_4^2\omega_8^2+4\omega_6^2cs^2\omega_4^2-144\omega_6cs^2\omega_4^2v_2^2\omega_8^2+8\omega_6^3cs^4\omega_4\omega_8-20\omega_6\omega_4v_2^2\omega_8^2+51\omega_6^2cs^2\omega_4^2v_2^2\omega_8^2-20\omega_6^2\omega_4v_2^4\omega_8^2-8cs^2\omega_4^2\omega_8^2+8\omega_6^3v_2^2\omega_8+12\omega_6cs^2\omega_4^2\omega_8^2-4\omega_6^2\omega_4^2v_2^4+4\omega_6^2cs^4\omega_8^2+96cs^2\omega_4^2v_2^2\omega_8^2+24\omega_6^3cs^2\omega_4^2v_2^2+20\omega_6^3\omega_4v_2^4\omega_8-32\omega_6^2\omega_4^2v_2^2\omega_8^2+4\omega_6^2cs^2\omega_4^2\omega_8^2)\frac{v_1}{4\omega_6^3\omega_4^2\omega_8^2}$$

$$C_{\mathbf{D}_x\mathbf{D}_y^3\rho}^{(2),\text{CLBM}1} = 0$$

$$C_{\mathbf{D}_x\mathbf{D}_y^3\rho}^{(2),\text{CLBM}2} = 0$$

$$C_{\mathbf{D}_x\mathbf{D}_y^3\rho}^{(2),\text{CuLBM}1} = 0$$

$$C_{\mathbf{D}_x\mathbf{D}_y^3\rho}^{(2),\text{CuLBM}2} = (9\omega_3^2v_1^2\omega_1^2v_2^2\omega_2-297\omega_3^2\omega_1v_2^2cs^2\omega_2^2+6\omega_3^2\omega_1cs^4\omega_2^2-12v_1^2\omega_1^3cs^2\omega_2^2+24\omega_3^2\omega_1^2v_2^2\omega_3^2-54\omega_3\omega_1^2cs^4\omega_3^2-\omega_3^2v_1^2\omega_1^3\omega_2^2-6\omega_3^2\omega_1^2\omega_2-72\omega_3^2\omega_1^2cs^2\omega_2^2-108\omega_3^2\omega_1^2v_2^2\omega_2^2-54\omega_3^2\omega_1v_2^2cs^2\omega_2^3+6\omega_3^2v_1^2\omega_2^3+5\omega_3^2\omega_1^2cs^2\omega_3^2-18\omega_3\omega_1^3cs^2\omega_2^2-6\omega_3^2v_1^2\omega_1^2cs^2\omega_2+405\omega_3^2\omega_1^3v_2^2cs^2+6\omega_3^2v_1^2\omega_1^2\omega_2+2\omega_3^2\omega_1^3\omega_2^2+90\omega_3^2\omega_1^3cs^4-6\omega_3\omega_1^3cs^2\omega_2-54\omega_3^2\omega_1^3v_2^2\omega_2-54\omega_3^2\omega_1v_2^2\omega_2^2-18\omega_3v_1^2\omega_1^2cs^2\omega_2^2+45\omega_3^2\omega_1^2v_2^2\omega_2+12\omega_1^3cs^2\omega_2^2+54\omega_3^2\omega_1^3v_2^4-24\omega_3^2v_1^2\omega_1^2v_2^2\omega_3^2+27\omega_3^2\omega_1cs^4\omega_3^2+6\omega_3^2\omega_1^3-6\omega_3\omega_1cs^2\omega_3^2-2\omega_3^2v_1^2\omega_1^2cs^2\omega_3^2+\omega_3^2v_1^2\omega_1^2\omega_3^2-54\omega_3^2\omega_1v_2^2\omega_3^2+6\omega_3^2v_1^2\omega_1^2\omega_2+60\omega_3^2\omega_1^2cs^2\omega_2+36\omega_1^2cs^4\omega_3^2-18\omega_3^2cs^4\omega_3^2+135\omega_3^2v_2^2cs^2\omega_3^2-18\omega_3^2\omega_1cs^4\omega_2^2-99\omega_3^2\omega_1^3cs^4\omega_2+6\omega_3^2\omega_1\omega_2^2+108\omega_3^2v_2^2\omega_2^2+6\omega_3^2v_1^2\omega_1cs^2\omega_2^2-6\omega_3^2v_1^2\omega_1\omega_3^2-99\omega_3^2\omega_1^3v_2^2-21\omega_3^2v_1^2\omega_1^2cs^2\omega_2-486\omega_3^2\omega_1^3v_2^2cs^2\omega_2+540\omega_3^2\omega_1^3v_2^2cs^2\omega_2^2-72\omega_3^2\omega_1^3cs^2+54\omega_3\omega_1^3cs^4\omega_2^2+72\omega_3^2v_1^2\omega_1v_2^2\omega_3^2-24\omega_3^2\omega_1^3v_2^2\omega_2^2+12v_1^2\omega_1^2cs^2\omega_3^2-6\omega_3^2\omega_1^2cs^4\omega_3^2+6\omega_3v_1^2\omega_1cs^2\omega_3^2+45\omega_3^2v_1^2\omega_1^3v_2^2-6\omega_3^2v_1^2\omega_1\omega_2^2+18\omega_3\omega_1^2cs^2\omega_3^2-5\omega_3^2\omega_1^3cs^2\omega_2^2+21\omega_3^2v_1^2\omega_1cs^2\omega_3^2+6\omega_3^2\omega_1\omega_3^2-72\omega_3^2v_1^2\omega_1^3v_2^2\omega_2-6\omega_3v_1^2\omega_1^3cs^2\omega_2+72\omega_3^2\omega_1^3cs^4\omega_2^2-6\omega_3^2\omega_3^2-9\omega_3^2v_1^2\omega_1v_2^2\omega_2^2-72\omega_3^2\omega_1^2v_2^2cs^2\omega_3^2+18\omega_3^2v_1^2\omega_1^3cs^2-18\omega_3^2\omega_1v_2^2\omega_3^2+24\omega_3^2v_1^2\omega_1^3v_2^2\omega_2^2+18\omega_3\omega_1cs^4\omega_3^2-45\omega_3^2v_1^2v_2^2\omega_3^2-6\omega_3^2v_1^2\omega_1^3-54\omega_3^2v_1^2cs^4\omega_2+18\omega_3v_1^2\omega_1^3cs^2\omega_2^2-12\omega_1^3cs^2\omega_2^2+12\omega_3^2\omega_1cs^2\omega_2^2+75\omega_3^2\omega_1^3cs^2\omega_2-243\omega_3^2\omega_1^2v_2^2cs^2\omega_2+126\omega_3^2\omega_1^3v_2^2\omega_2-18\omega_3\omega_1^3cs^4\omega_2-9\omega_3^2v_2^2\omega_3^2+63\omega_3^2\omega_1v_2^2\omega_2^2-54\omega_3^2\omega_1^2v_2^2\omega_2-36\omega_1^3cs^4\omega_2^2+2\omega_3^2v_1^2\omega_1^3cs^2\omega_2^2-3\omega_3^2\omega_1cs^2\omega_3^2-18\omega_3^2v_1^2cs^2\omega_3^2+72\omega_3^2\omega_1^3v_2^2cs^2\omega_2^2)\frac{v_1}{24\omega_3^2\omega_1^3\omega_3^2}$$

$$\text{coefficient } C_{\mathbf{D}_x\mathbf{D}_y^3v_1}^{(2)} \text{ at } \frac{\partial^4 v_1}{\partial x_1 \partial x_2^2} :$$

$$C_{\mathbf{D}_x\mathbf{D}_y^3v_1}^{(2),\text{SRT}} = (-54\omega cs^4-42v_2^2\omega^2cs^2-36v_2^4-24cs^2+54v_2^4\omega-12\omega^2cs^2+12v_2^2\omega^3cs^2-26v_2^4\omega^2+4v_2^4\omega^3-54v_2^2\omega-\omega^3cs^4+36v_2^2+36cs^4+36\omega cs^2-4v_2^2\omega^3-36v_2^2cs^2+20\omega^2cs^4+54v_2^2\omega cs^2+26v_2^2\omega^2)\frac{\rho}{12\omega^3}$$

$$C_{\mathbf{D}_x\mathbf{D}_y^3v_1}^{(2),\text{MRT}1} = (-90\omega_6\omega_3^3v_2^2\omega_8^2-6\omega_6^2cs^4\omega_3^3\omega_8-5\omega_6^3cs^2\omega_4^2\omega_8^2-36\omega_6cs^2\omega_3^4v_2^2\omega_8+72\omega_4^3v_2^4\omega_8^2-12\omega_6^2cs^2\omega_4^2\omega_8-\omega_6^3cs^4\omega_4^3\omega_8+12\omega_6^3\omega_3^4v_2^4+27\omega_6^3\omega_3^3v_2^2\omega_8+54\omega_6^2cs^2\omega_3^3v_2^2\omega_8-12\omega_6^3\omega_4v_2^2\omega_8^2-48\omega_6^2cs^2\omega_4v_2^2\omega_8^2+12\omega_6^2\omega_4^2v_2^4\omega_8^2-6\omega_6^3cs^2\omega_3^3\omega_8+6\omega_6^2cs^4\omega_4^2\omega_8^2-18\omega_6^3cs^4\omega_4^2\omega_8-\omega_6^2cs^2\omega_4^3\omega_8^2+48\omega_6^3\omega_4^2v_2^4\omega_8-81\omega_6^3cs^2\omega_4^2v_2^2\omega_8^2-19\omega_6^2\omega_4^3v_2^2\omega_8^2+6\omega_6^2cs^2\omega_4^3\omega_8+13\omega_6^3cs^4\omega_4^2\omega_8^2-12\omega_6^3\omega_4^2v_2^4-18\omega_6^3\omega_4^2v_2^4\omega_8^2-60\omega_6^2\omega_4^3v_2^2\omega_8+12\omega_6^3cs^4\omega_8^2+30\omega_6^3cs^2\omega_4^2v_2^2\omega_8+252cs^2\omega_3^4v_2^2\omega_8^2+60\omega_6^2cs^2\omega_4^3v_2^2\omega_8^2-4\omega_6^3\omega_4^3v_2^2\omega_8^2-24\omega_6^2\omega_4^2v_2^4\omega_8^2+24\omega_6^3\omega_4v_2^2\omega_8+12\omega_6^2cs^4\omega_4^2\omega_8-306\omega_6cs^2\omega_3^4v_2^2\omega_8^2+6\omega_6^3cs^4\omega_4^3\omega_8-6\omega_6^2cs^2\omega_4^2\omega_8^2+18\omega_6^3cs^2\omega_4^2\omega_8+\omega_6^2cs^4\omega_3^3\omega_8^2-36\omega_6\omega_4^3v_2^4\omega_8+12\omega_6^2\omega_4^3v_2^2-12\omega_6^3cs^2\omega_4v_2^2\omega_8-27\omega_6^3\omega_4^3v_2^4\omega_8-12\omega_6^2\omega_4^3v_2^2\omega_8^2+12\omega_6^3\omega_4v_2^4\omega_8^2+12\omega_6^2cs^2\omega_3^3v_2^2\omega_8^2-12\omega_6^3cs^2\omega_4\omega_8-12\omega_6^2cs^2\omega_4^2v_2^2\omega_8+12\omega_6^3\omega_4^2v_2^2-48\omega_6^3\omega_4^2v_2^2\omega_8+19\omega_6^2\omega_3^3v_2^2\omega_8^2+12\omega_6^3cs^2\omega_4^3v_2^2-24\omega_6^3cs^4\omega_4\omega_8^2+90\omega_6\omega_4^3v_2^2\omega_8^2-12\omega_6^2\omega_4^3v_2^4+12\omega_6cs^2\omega_4^3\omega_8^2-72\omega_4^3v_2^2\omega_8^2-12\omega_6cs^4\omega_4^2\omega_8^2-12cs^2\omega_4^3\omega_8^2-12\omega_6\omega_3^4v_2^2-12\omega_6^2cs^2\omega_4^3v_2^2-108\omega_6cs^2\omega_4^2v_2^2\omega_8^2+36\omega_6\omega_4^3v_2^2\omega_8+12\omega_6^3cs^4\omega_4\omega_8+6\omega_6^3cs^2\omega_4\omega_8^2+18\omega_6^3\omega_4^2v_2^2\omega_8^2+162\omega_6^2cs^2\omega_4^2v_2^2\omega_8^2+60\omega_6^2\omega_3^3v_2^2\omega_8-48\omega_6^3cs^2v_2^2\omega_8^2-12\omega_6cs^4\omega_3^3\omega_8+4\omega_6^3\omega_3^4v_2^2\omega_8^2+102\omega_6^3cs^2\omega_4v_2^2\omega_8^2+12\omega_6cs^2\omega_4^2\omega_8^2-21\omega_6^3cs^2\omega_4^3v_2^2\omega_8+12cs^4\omega_4^3\omega_8^2-12\omega_6^3cs^2\omega_4^2v_2^2-24\omega_6^3\omega_4^3v_2^4\omega_8+24\omega_6^2\omega_4^3v_2^2\omega_8^2)\frac{\rho}{12\omega_6^3\omega_4^3\omega_8^2}$$

$$C_{\mathbf{D}_x\mathbf{D}_y^3v_1}^{(2),\text{MRT}2} = (-36\omega_6cs^2\omega_4^3v_2^2\omega_8-90\omega_6\omega_4^3v_2^4\omega_8^2+12\omega_6^3cs^4\omega_8^2+72\omega_3^3v_2^4\omega_8^2-12\omega_6^3cs^2\omega_4\omega_8+12\omega_6^2\omega_3^3v_2^4+27\omega_6^3\omega_3^3v_2^2\omega_8+12\omega_6cs^2\omega_4^3\omega_8^2-12\omega_6^3\omega_4v_2^2\omega_8^2+12\omega_6^2cs^2\omega_4^3v_2^2+12\omega_6^2\omega_4^3v_2^4\omega_8^2-81\omega_6^3cs^2\omega_4^3v_2^2\omega_8^2-24\omega_6^3cs^4\omega_4\omega_8^2+54\omega_6^2cs^2\omega_4^3v_2^2\omega_8+48\omega_6^3\omega_4^3v_2^4\omega_8-12cs^2\omega_3^3\omega_8^2-48\omega_6^2cs^2\omega_4v_2^2\omega_8^2-12\omega_6cs^4\omega_4^2\omega_8^2-19\omega_6^2\omega_4^3v_2^2\omega_8^2+60\omega_6^2cs^2\omega_3^3v_2^2\omega_8^2-12\omega_6^3\omega_4^2v_2^4-18\omega_6^3\omega_4^2v_2^4\omega_8^2-12\omega_6^2cs^2\omega_4^3v_2^2-60\omega_6^2\omega_4^3v_2^2\omega_8-4\omega_6^3\omega_4^3v_2^2\omega_8^2-24\omega_6^3\omega_4^2v_2^4\omega_8+24\omega_6^3\omega_4v_2^2\omega_8+12\omega_6^3cs^4\omega_4\omega_8+252cs^2\omega_3^4v_2^2\omega_8^2+30\omega_6^3cs^2\omega_4^3v_2^2\omega_8-12\omega_6cs^4\omega_3^4\omega_8^2-48\omega_6^3cs^2v_2^2\omega_8^2+6\omega_6^3cs^2\omega_4\omega_8^2-12\omega_6^3cs^2\omega_4^2v_2^2+12cs^4\omega_3^3\omega_8^2-306\omega_6cs^2\omega_3^3v_2^2\omega_8^2-36\omega_6\omega_4^3v_2^2\omega_8+12\omega_6cs^2\omega_4^3\omega_8^2+12\omega_6^2\omega_4^3v_2^2-27\omega_6^3\omega_4^3v_2^4\omega_8-12\omega_6^2cs^2\omega_4^2v_2^2\omega_8-12\omega_6^2\omega_4^2v_2^2\omega_8^2+12\omega_6^3\omega_4^2v_2^4\omega_8^2-6\omega_6^2cs^4\omega_4^3\omega_8-5\omega_6^3cs^2\omega_4^2\omega_8^2-12\omega_6^3cs^2\omega_4v_2^2\omega_8-12\omega_6^2cs^2\omega_4^2\omega_8-\omega_6^2cs^4\omega_4^3\omega_8^2+12\omega_6^2\omega_4^3v_2^2-48\omega_6^2\omega_4^2v_2^2\omega_8+12\omega_6^3cs^2\omega_3^3v_2^2\omega_8^2+19\omega_6^2\omega_4^3v_2^2\omega_8^2-6\omega_6^3cs^2\omega_4^3\omega_8+6\omega_6^2cs^4\omega_4^2\omega_8^2+90\omega_6\omega_3^3v_2^2\omega_8^2-12\omega_6^2\omega_3^3v_2^4-72\omega_4^3v_2^2\omega_8^2-18\omega_6^3cs^4\omega_4^2\omega_8-\omega_6^2cs^2\omega_4^3\omega_8^2-108\omega_6cs^2\omega_4^2v_2^2\omega_8^2+)$$

$$6\omega_6^2cs^2\omega_4^3\omega_8 + 13\omega_6^3cs^4\omega_2^2\omega_8^2 - 12\omega_6^3\omega_4^3v_2^2 + 36\omega_6\omega_4^3v_2^2\omega_8 + 12\omega_6^2cs^4\omega_4^2\omega_8 + 6\omega_6^3cs^4\omega_4^3\omega_8 - 6\omega_6^2cs^2\omega_2^2\omega_8^2 + 102\omega_6^3cs^2\omega_4v_2^2\omega_8^2 + 18\omega_6^3\omega_4^2v_2^2\omega_8^2 - 21\omega_6^2cs^2\omega_4^3v_2^2\omega_8 + 60\omega_6^3\omega_4^3v_2^4\omega_8 + 4\omega_6^4\omega_4^3v_2^2\omega_8^2 + 162\omega_6^2cs^2\omega_4^2v_2^2\omega_8^2 - 24\omega_6^3\omega_4v_2^4\omega_8 + 24\omega_6^2\omega_4^2v_2^2\omega_8 + 18\omega_6^3cs^2\omega_4^2\omega_8 + \omega_6^2cs^4\omega_4^3\omega_8^2) \frac{\rho}{12\omega_6^3\omega_4^3\omega_8^2}$$

$$C_{D_x D_y^3 v_1}^{(2), \text{CLBM1}} = (-3\omega_6^3\omega_2^2v_2^2\omega_8^2cs^2 - 90\omega_6\omega_4^3v_2^4\omega_8^2 - 24\omega_6^3\omega_4\omega_8^2cs^4 + \omega_6^2\omega_3^3\omega_8^2cs^4 + 72\omega_4^3v_2^4\omega_8^2 - 5\omega_6^3\omega_4^2\omega_8^2cs^2 + 36\omega_6^3\omega_4^3v_2^4 + 39\omega_6^3\omega_4^3v_2^2\omega_8 + 6\omega_6^2\omega_4^3\omega_8cs^2 - 18\omega_6^3\omega_4^2\omega_8cs^4 - 12\omega_6^3\omega_4\omega_8cs^2 + 36\omega_6^3\omega_2^2v_2^4\omega_8 - 108\omega_6^2\omega_4^3v_2^2cs^2 + 36\omega_6^2\omega_4^2v_2^2\omega_8cs^2 - 12\omega_6\omega_4^2\omega_8^2cs^4 - 19\omega_6^2\omega_4^3v_2^2\omega_8^2 + 12\omega_6^3\omega_4^2\omega_8cs^4 - 36\omega_6^3\omega_2^2v_2^4 - 6\omega_6^3\omega_4^3v_2^4\omega_8^2 - 6\omega_6^3\omega_3^3\omega_8cs^2 - 72\omega_6^2\omega_4^3v_2^2\omega_8 - 12\omega_4^3\omega_8^2cs^2 + 198\omega_6^2\omega_4^3v_2^2\omega_8cs^2 - 4\omega_6^3\omega_3^3v_2^2\omega_8^2 + 108\omega_6^3\omega_4^3v_2^2cs^2 + 12\omega_6\omega_4^3\omega_8^2cs^2 - 108\omega_6\omega_4^3v_2^2\omega_8cs^2 + 12\omega_6^3\omega_4^3v_2^2\omega_8^2cs^2 - 6\omega_6^2\omega_4^2\omega_8^2cs^2 - 36\omega_6\omega_4^3v_2^4\omega_8 + 36\omega_6^2\omega_3^3v_2^2 - \omega_6^3\omega_4^3\omega_8^2cs^4 + 36\omega_6^3\omega_4v_2^2\omega_8cs^2 - 39\omega_6^3\omega_4^3v_2^2\omega_8 - 306\omega_6\omega_4^3v_2^2\omega_8^2cs^2 - 12\omega_6\omega_4^3\omega_8^2cs^4 - 99\omega_6^3\omega_4^3v_2^2\omega_8cs^2 - 12\omega_6^2\omega_4^3\omega_8cs^2 + 36\omega_6^3\omega_4^3v_2^2 - 36\omega_6^3\omega_2^2v_2^2\omega_8 - 18\omega_6^3\omega_4v_2^2\omega_8^2cs^2 + 6\omega_6^3\omega_4^3\omega_8cs^4 + 19\omega_6^2\omega_4^3v_2^4\omega_8^2 + 12\omega_4^3\omega_8^2cs^4 + 90\omega_6\omega_4^3v_2^2\omega_8^2 + 6\omega_6^2\omega_4^2\omega_8^2cs^4 + 252\omega_4^3v_2^2\omega_8^2cs^2 - 36\omega_6^2\omega_3^3v_2^4 + 12\omega_6^2\omega_8^2cs^4 - 72\omega_4^3v_2^2\omega_8^2 + 60\omega_6^2\omega_4^3v_2^2\omega_8^2cs^2 - \omega_6^2\omega_4^3\omega_8^2cs^2 + 13\omega_6^3\omega_2^2\omega_8^2cs^4 - 36\omega_6^3\omega_4^3v_2^2 + 36\omega_6\omega_4^3v_2^2\omega_8 + 6\omega_6^3\omega_4\omega_8^2cs^2 + 18\omega_6^2\omega_4^2v_2^2\omega_8^2cs^2 + 12\omega_6^2\omega_4\omega_8cs^4 - 108\omega_6^3\omega_4^3v_2^2cs^2 + 6\omega_6^3\omega_4^3v_2^2\omega_8^2 + 54\omega_6^3\omega_4^3v_2^2\omega_8cs^2 - 36\omega_6\omega_4^3v_2^2\omega_8^2cs^2 + 72\omega_6^2\omega_4^3v_2^4\omega_8 + 12\omega_6\omega_4^2\omega_8^2cs^2 + 4\omega_6^3\omega_4^3v_2^4\omega_8^2 - 6\omega_6^2\omega_4^3\omega_8cs^4 + 18\omega_6^3\omega_4\omega_8cs^2) \frac{\rho}{12\omega_6^3\omega_4^3\omega_8^2}$$

$$C_{D_x D_y^3 v_1}^{(2), \text{CLBM2}} = (-12cs^2\omega_4^3\omega_8^2 - 90\omega_6\omega_4^3v_2^4\omega_8^2 - cs^2\omega_6^2\omega_4^3\omega_8^2 + 6cs^2\omega_6^3\omega_4\omega_8^2 - 18cs^4\omega_6^3\omega_4^2\omega_8 - 36cs^2\omega_6\omega_4^2v_2^2\omega_8^2 + 72\omega_4^3v_2^4\omega_8^2 - 108cs^2\omega_6^3\omega_4^2v_2^2 - 12cs^4\omega_6\omega_4^2\omega_8^2 + 36\omega_6^3\omega_2^2v_2^4 - 99cs^2\omega_6^3\omega_4^3v_2^2\omega_8 + 39\omega_6^3\omega_3^3v_2^2\omega_8 - 18cs^2\omega_6^2\omega_4v_2^2\omega_8^2 - 12cs^2\omega_6^2\omega_4^2\omega_8 + 252cs^2\omega_4^3v_2^2\omega_8^2 + 36\omega_6^3\omega_4^2v_2^4\omega_8 - cs^4\omega_6^3\omega_4^2\omega_8^2 + 60cs^2\omega_6^2\omega_4^3v_2^2\omega_8^2 - 19\omega_6^2\omega_4^3v_2^2\omega_8^2 - 12cs^4\omega_6\omega_4^2\omega_8^2 - 36\omega_6^3\omega_4^2v_2^4 + 12cs^4\omega_4^3\omega_8^2 - 6\omega_6^2\omega_4^3v_2^2\omega_8^2 + 6cs^4\omega_6^3\omega_4^2\omega_8 - 72\omega_6^2\omega_4^3v_2^2\omega_8 + 198cs^2\omega_6^3\omega_4^3v_2^2\omega_8 + 108cs^2\omega_6^3\omega_4^3v_2^2 - 4\omega_6^3\omega_4^3v_2^2\omega_8^2 + 12cs^2\omega_6^3\omega_4^3v_2^2\omega_8^2 + 36cs^2\omega_6^3\omega_4v_2^2\omega_8 - 6cs^2\omega_6^2\omega_4^2\omega_8^2 + 13cs^4\omega_6^3\omega_4^2\omega_8^2 - 36\omega_6\omega_4^3v_2^4\omega_8 + 6cs^2\omega_6^3\omega_4^3v_2^2 - 12cs^2\omega_6^3\omega_4\omega_8 - 39\omega_6^3\omega_4^3v_2^2\omega_8 - 24cs^4\omega_6^2\omega_4\omega_8^2 + 18cs^2\omega_6^2\omega_4^2v_2^2\omega_8^2 + cs^4\omega_6^2\omega_4^2\omega_8^2 + 12cs^2\omega_6\omega_4^2\omega_8^2 + 36\omega_6^3\omega_4^3v_2^2 + 54cs^2\omega_6^2\omega_4^2v_2^2\omega_8 - 36\omega_6^3\omega_4^2v_2^2\omega_8 + 18cs^2\omega_6^3\omega_4^2\omega_8 + 19\omega_6^2\omega_4^3v_2^4\omega_8^2 + 90\omega_6\omega_4^3v_2^2\omega_8^2 - 306cs^2\omega_6\omega_4^3v_2^2\omega_8^2 + 12cs^4\omega_6^2\omega_4^2\omega_8 - 36\omega_6^3\omega_4^3v_2^4 - 72\omega_4^3v_2^2\omega_8^2 - 6cs^2\omega_6^3\omega_4^3v_2^2 - 6cs^2\omega_6^3\omega_4^2\omega_8 + 12cs^2\omega_6\omega_4^2\omega_8^2 - 36\omega_6^3\omega_4^3v_2^2 + 6cs^4\omega_6^2\omega_4^2\omega_8^2 - 108cs^2\omega_6\omega_4^2v_2^2\omega_8 + 36\omega_6\omega_4^3v_2^2\omega_8 + 12cs^4\omega_6^3\omega_4^2\omega_8^2 + 6\omega_6^3\omega_4^2v_2^2\omega_8^2 - 3cs^2\omega_6^3\omega_4^2v_2^2\omega_8^2 - 5cs^2\omega_6^3\omega_4^2\omega_8^2 + 72\omega_6^2\omega_4^3v_2^4\omega_8 + 4\omega_6^3\omega_4^3v_2^4\omega_8^2 + 12cs^4\omega_4^3\omega_8 - 108cs^2\omega_6^2\omega_4^3v_2^2 + 36cs^2\omega_6^2\omega_4^2v_2^2\omega_8 - 6cs^4\omega_6^2\omega_4^2\omega_8^2) \frac{\rho}{12\omega_6^3\omega_4^3\omega_8^2}$$

$$C_{D_x D_y^3 v_1}^{(2), \text{CuLBM1}} = (36\omega_6\omega_3cs^2v_2^2\omega_8^3 - 12\omega_6^2\omega_3^3cs^2 + 12\omega_6\omega_3^2cs^4\omega_2^2 - \omega_6^2\omega_3^3cs^4\omega_2^3 + 36\omega_6\omega_3^3v_2^2\omega_2 + 6\omega_6\omega_3^3cs^2\omega_2^2 - 5\omega_6^2\omega_3^2cs^2\omega_2^3 - 18\omega_6\omega_3^2cs^4\omega_2^3 + \omega_6^2\omega_3^3cs^4\omega_2^2 - 90\omega_6^2\omega_3^3v_2^4\omega_2 - 18\omega_6^2\omega_3cs^2v_2^2\omega_2^3 - 6\omega_6\omega_3^3cs^2\omega_2^3 + 252\omega_6^2\omega_3^3cs^2v_2^2 - 6\omega_6^2\omega_3^3cs^2\omega_2^2 + 12\omega_6^2\omega_3^3cs^2\omega_2 + 39\omega_6\omega_3^3v_2^2\omega_2^3 - 36\omega_3^3v_2^4\omega_2^3 - 12\omega_6\omega_3^2cs^2\omega_2^3 - 12\omega_6^2\omega_3^3cs^4\omega_2 + 36\omega_6\omega_3^2v_2^4\omega_2^3 + 19\omega_6^2\omega_3^3v_2^4\omega_2^2 - 36\omega_3^3v_2^2\omega_2^3 - 72\omega_6\omega_3^3v_2^2\omega_2^2 + 6\omega_6^2\omega_3^3v_2^2\omega_2^2 + 6\omega_6^2\omega_3^3v_2^2\omega_2 + 4\omega_6^2\omega_3^3v_2^4\omega_2^3 - 24\omega_6^2\omega_3cs^4\omega_2^3 - \omega_6^2\omega_3^3cs^2\omega_2^2 + 18\omega_6\omega_3^2cs^2\omega_2^3 + 18\omega_6^2\omega_3^3cs^2v_2^2\omega_2^2 + 12\omega_6^2cs^4\omega_2^3 + 90\omega_6^2\omega_3^3v_2^2\omega_2 + 6\omega_6^2\omega_3^3cs^4\omega_2^2 + 6\omega_6\omega_3^3cs^4\omega_2^3 + 54\omega_6\omega_2^2cs^2v_2^2\omega_2^3 - 306\omega_6^2\omega_3^3cs^2v_2^2\omega_2 - 3\omega_6^2\omega_2^2cs^2v_2^2\omega_2^3 - 108\omega_6\omega_3^3cs^2v_2^2\omega_2 - 12\omega_6\omega_3^3cs^2\omega_2^2 - 36\omega_6\omega_3^3v_2^4\omega_2 - 108\omega_3^3cs^2v_2^2\omega_2^3 + 36\omega_6\omega_3^3cs^2v_2^2\omega_2^2 + 13\omega_6^2\omega_3cs^4\omega_2^3 - 6\omega_6\omega_3^3cs^4\omega_2^2 - 72\omega_6^2\omega_3^3v_2^2 - 6\omega_6^2\omega_3^2v_2^4\omega_2^3 + 72\omega_6\omega_3^3v_2^4\omega_2^2 + 12\omega_6^2\omega_3^3cs^2v_2^2\omega_2^3 - 4\omega_6^2\omega_3^3v_2^2\omega_2^3 - 36\omega_3^3v_2^4\omega_2^2 + 12\omega_6^2\omega_3^3cs^4 - 6\omega_6^2\omega_3cs^4\omega_2^3 + 198\omega_6\omega_3^3cs^2v_2^2\omega_2^3 + 108\omega_3^3cs^2v_2^2\omega_2^3 - 12\omega_6^2\omega_3cs^4\omega_2 + 36\omega_3^3v_2^2\omega_2^3 + 60\omega_6^2\omega_3^3cs^2v_2^2\omega_2^2 - 39\omega_6\omega_3^3v_2^4\omega_2^3 + 12\omega_6\omega_3cs^4\omega_2^3 + 12\omega_6^2\omega_3^3cs^2\omega_2 - 36\omega_6^2\omega_3cs^2v_2^2\omega_2 - 99\omega_6\omega_3^3cs^2v_2^2\omega_2^3 - 108\omega_3^3cs^2v_2^2\omega_2^2 + 36\omega_3^3v_2^4\omega_2^3 - 19\omega_6^2\omega_3^3v_2^2\omega_2^2 - 36\omega_6\omega_3^2v_2^2\omega_2^3) \frac{\rho}{12\omega_6^2\omega_3^3\omega_2^3}$$

$$C_{D_x D_y^3 v_1}^{(2), \text{CuLBM2}} = (27\omega_3^2v_1^2\omega_1^2v_2^2\omega_2 - 108\omega_1^3v_2^2cs^2\omega_2^2 + 99\omega_3^2\omega_1v_2^2cs^2\omega_2^2 - \omega_3^2\omega_1^3cs^4\omega_2^2 + 17\omega_3^2\omega_1^2v_2^2\omega_2^3 - 30\omega_3\omega_1^3cs^4\omega_2^3 - 72\omega_3\omega_1^3v_2^2\omega_2^3 - 3\omega_3^2v_1^2\omega_2^3 + 54\omega_3\omega_1v_2^2cs^2\omega_2^3 - 6\omega_3^2\omega_1^2\omega_2 - 12\omega_3^2\omega_1^2cs^2\omega_2^3 + 8\omega_3^2\omega_1^2v_2^4\omega_2^3 - 12\omega_3\omega_1^3cs^2\omega_2^3 - 36\omega_3\omega_1^2v_2^4\omega_2^3 + 54\omega_3^2\omega_1^2v_2^2\omega_2^3 - 2\omega_3^2\omega_1^3cs^4\omega_2^3 + 78\omega_3\omega_1^3v_2^2\omega_2^3 + 24\omega_3\omega_2^2cs^4\omega_2^3 - 171\omega_3^2\omega_1v_2^2cs^2\omega_2^3 + 216\omega_1^3v_2^2cs^2\omega_2^3 + 18\omega_3^2v_1^2\omega_2^3 + 19\omega_3^2\omega_1^3v_2^4\omega_2^3 - 12\omega_3^2\omega_1^2cs^2\omega_2^3 + 144\omega_3\omega_1^2v_2^4\omega_2^3 + 18\omega_3\omega_1^3cs^2\omega_2^3 - 54\omega_3^2\omega_1^2v_1^2cs^2\omega_2 + 189\omega_3^2\omega_1^2v_2^2cs^2 + 18\omega_3^2v_1^2\omega_2^3 + \omega_3^2\omega_1^3\omega_2^3 - 72\omega_1^3v_2^2\omega_2^3 + 18\omega_3^2\omega_1^3cs^4 - 6\omega_3\omega_1^3cs^2\omega_2 - 63\omega_3^2\omega_1^3v_2^4\omega_2 - 324\omega_1^2v_2^2cs^2\omega_2^3 - \omega_3^2\omega_1^2\omega_2^3 + 36\omega_3^2v_2^4\omega_2^3 - 6\omega_3^2\omega_1^2\omega_2 + 36\omega_3^2\omega_1v_2^4\omega_2^3 + 18\omega_3^2v_1^2\omega_1^2cs^2\omega_2^3 + 36\omega_3v_1^2\omega_1^2cs^2\omega_2^3 - 45\omega_3^2\omega_1^2v_2^2\omega_2 - 108\omega_1^2v_2^4\omega_2^3 + 18\omega_3\omega_1v_2^2\omega_2^3 + 36\omega_3^2\omega_1^2v_2^2\omega_2^3 - 72\omega_3^2v_1^2\omega_1^2v_2^2\omega_2^3 - 57\omega_3^2\omega_1^3cs^4\omega_2^3 + 6\omega_3^2\omega_1^3 - 18\omega_3\omega_1cs^2\omega_2^3 + 6\omega_3^2v_1^2\omega_1^2cs^2\omega_2^3 + 3\omega_3^2v_1^2\omega_1^2\omega_2^3 - 63\omega_3^2\omega_1v_2^2\omega_2^3 + 18\omega_3^2v_1^2\omega_1^3\omega_2 + 36\omega_3^2\omega_1^2cs^2\omega_2 + 36\omega_1^3v_2^2\omega_2^2 + 30\omega_3^2cs^4\omega_2^3 + 63\omega_3^2v_2^2cs^2\omega_2^3 + 6\omega_3^2\omega_1cs^4\omega_2^2 + 18\omega_3\omega_1^3v_2^2\omega_2 + 36\omega_3v_1^2\omega_1^2cs^2\omega_2^3 - 15\omega_3^2\omega_1^3cs^4\omega_2 - 78\omega_3\omega_1^3v_2^2\omega_2^3 - 24\omega_3\omega_1^3cs^2\omega_2^3 + 6\omega_3^2\omega_1\omega_2^3 - 54\omega_3^2\omega_1^2v_2^4\omega_2^3 + 18\omega_3^2v_1^2\omega_1cs^2\omega_2^3 - 18\omega_3^2v_1^2\omega_1\omega_2^3 - 81\omega_3^2\omega_1^3v_2^2 - 9\omega_3^2v_1^2\omega_1^3cs^2\omega_2 - 297\omega_3^2\omega_1^3v_2^2cs^2\omega_2 + 306\omega_3\omega_1^2v_2^2cs^2\omega_2^3 - 144\omega_3^2\omega_1^2v_2^2cs^2\omega_2^3 - 144\omega_3\omega_1^2v_2^2\omega_2^3 - 24\omega_3^2\omega_1^3cs^2 - 18\omega_3\omega_1^3cs^4\omega_2^3 + 216\omega_3^2v_1^2\omega_1v_2^2\omega_2^3 - 43\omega_3^2\omega_1^3v_2^2\omega_2^3 + 29\omega_3^2\omega_1^2cs^4\omega_2^3 - 54\omega_3\omega_1^2v_2^2cs^2\omega_2^3 - 72\omega_3\omega_1^2v_2^2cs^2\omega_2^3 + 135\omega_3^2v_1^2\omega_1^2v_2^2 - 18\omega_3^2v_1\omega_1\omega_2^3 + 30\omega_3\omega_1^2cs^2\omega_2^3 + 72\omega_3\omega_1^2v_2^2\omega_2^3 - 9\omega_3^2v_1^2\omega_1cs^2\omega_2^3 + 7\omega_3^2\omega_1^2v_2^4\omega_2^3 + 6\omega_3^2\omega_1\omega_2^3 - 216\omega_3^2v_1^2\omega_1^3v_2^2\omega_2 + 12\omega_3\omega_1^3cs^4\omega_2^3 + 18\omega_3v_1^2\omega_1^3cs^2\omega_2 + 36\omega_3\omega_1^2v_2^2\omega_2^3 + 12\omega_3^2\omega_1^3cs^4\omega_2^2 - 8\omega_3^2\omega_1^3v_2^2\omega_2^3 - 6\omega_3^2\omega_2^3 - 27\omega_3^2v_1^2\omega_1v_2^2\omega_2^3 - 54\omega_3\omega_1^3v_2^2cs^2\omega_2 + 30\omega_3^2\omega_1^2v_2^2cs^2\omega_2^3 + 18\omega_3^2v_1^2\omega_1^3cs^2 - 9\omega_3^2\omega_1v_2^2\omega_2^3 + 72\omega_3^2v_1^2\omega_1^3v_2^2\omega_2^3 + 18\omega_3\omega_1cs^4\omega_2^3 - 135\omega_3^2v_1^2v_2^2\omega_2^3 - 18\omega_3^2v_1^2\omega_1^3 - 30\omega_3^2\omega_1^2cs^4\omega_2 - 36\omega_3v_1^2\omega_1^3cs^2\omega_2^3 - 36\omega_1^3v_2^4\omega_2^3 + 198\omega_3\omega_1^3v_2^2cs^2\omega_2^3 + 24\omega_3\omega_1^3v_2^2cs^2\omega_2^3 - 12\omega_3^2\omega_1cs^2\omega_2^3 + 21\omega_3^2\omega_1^3cs^2\omega_2 - 18\omega_3\omega_1^3v_2^2\omega_2 - 198\omega_3\omega_1^3v_2^2cs^2\omega_2^3 + 72\omega_1^3v_2^4\omega_2^3 + 81\omega_3^2\omega_1^2v_2^2cs^2\omega_2 + 135\omega_3^2\omega_1^3v_2^2\omega_2 + 6\omega_3\omega_1^3cs^4\omega_2 + 9\omega_3^2v_2^2\omega_2^3 - 27\omega_3^2\omega_1v_2^2\omega_2^3 + 36\omega_3^2\omega_1^2v_2^4\omega_2 + 108\omega_1^2v_2^2\omega_2^3 - 6\omega_3^2v_1^2\omega_1^3cs^2\omega_2^3 + 15\omega_3^2\omega_1cs^2\omega_2^3 + 18\omega_3^2v_1^2cs^2\omega_2^3 - 18\omega_3\omega_1v_2^4\omega_2^3 + 84\omega_3^2\omega_1^3v_2^2cs^2\omega_2^3) \frac{\rho}{24\omega_3^2\omega_1^3\omega_2^3}$$

$$\text{coefficient } C_{D_x D_y^3 v_2}^{(2)} \text{ at } \frac{\partial^4 v_2}{\partial x_1 \partial x_2^2} :$$

$$C_{D_x D_y^3 v_2}^{(2), \text{SRT}} = 0$$

$$C_{D_x D_y^3 v_2}^{(2), \text{MRT1}} = (80\omega_4^2v_2^2\omega_8^2 + 8\omega_6^2\omega_4^2 - 120\omega_6\omega_4^2v_2^2\omega_8^2 + 48\omega_6\omega_2^2\omega_8^2 - 16\omega_6^3cs^2\omega_4 + 32\omega_6cs^2\omega_4\omega_8^2 - 32\omega_4^2\omega_8^2 + 17\omega_6^3\omega_4^2\omega_8 + 56\omega_6^2cs^2\omega_4^2\omega_8 + 8\omega_6^3\omega_4 - 16\omega_6^2\omega_4^2v_2^2 + 28\omega_6^2v_2^2\omega_8^2 - 48\omega_6^2\omega_4v_2^2\omega_8 + 28\omega_6^2\omega_4\omega_8^2 - 68\omega_6^2\omega_4v_2^2\omega_8^2 + 16\omega_6^2\omega_4\omega_8 - 8\omega_6^3\omega_4^2 + 20\omega_6^2cs^2\omega_8^2 + 68\omega_6^3\omega_4v_2^2\omega_8 + 24\omega_6\omega_4^2\omega_8 - 64\omega_6\omega_4^2v_2^2\omega_8 + 16\omega_6^3cs^2\omega_4^2 + 25\omega_6^2cs^2\omega_4^2\omega_8^2 - 16\omega_6^3\omega_4v_2^2 - 25\omega_6^3cs^2\omega_4^2\omega_8 - 12\omega_6^2\omega_8^2 - 32\omega_6cs^2\omega_4^2\omega_8 + 43\omega_6^2\omega_4^2v_2^2\omega_8^2 - 44\omega_6^3cs^2\omega_4\omega_8^2 + 44\omega_6^3cs^2\omega_4\omega_8 + 16\omega_6^3\omega_4^2v_2^2 - 43\omega_6^3\omega_4^2v_2^2\omega_8 - 20\omega_6^3cs^2\omega_8 + 12\omega_6^3\omega_8 - 40\omega_6^2\omega_4^2\omega_8 - 16\omega_6^2cs^2\omega_4^2 + 64\omega_6\omega_4v_2^2\omega_8^2 - 17\omega_6^2\omega_4^2\omega_8^2 + 48cs^2\omega_4^2\omega_8^2 - 28\omega_6^2v_2^2\omega_8 - 72\omega_6cs^2\omega_4^2\omega_8^2 - 24\omega_6\omega_4\omega_8^2 + 104\omega_6^2\omega_4^2v_2^2\omega_8 - 28\omega_6^3\omega_4\omega_8 - 16\omega_6^2cs^2\omega_4\omega_8) \frac{v_1 \rho v_2}{4\omega_6^3\omega_4^2\omega_8^2}$$

$$C_{D_x D_y^3 v_2}^{(2), \text{MRT2}} = (80\omega_4^2v_2^2\omega_8^2 - 44\omega_6^2cs^2\omega_4\omega_8^2 + 8\omega_6^2\omega_4^2 - 32\omega_6cs^2\omega_4^2\omega_8 - 120\omega_6\omega_4^2v_2^2\omega_8^2 + 48\omega_6\omega_4^2\omega_8^2 - 32\omega_4^2\omega_8^2 + 44\omega_6^3cs^2\omega_4\omega_8 + 17\omega_6^3\omega_4^2\omega_8 +$$

$$20\omega_6^2cs^2\omega_8^2 + 8\omega_6^3\omega_4 + 16\omega_6^3cs^2\omega_4^2 - 16\omega_6^2\omega_4^2v_2^2 + 28\omega_6^2v_2^2\omega_8^2 - 48\omega_6^2\omega_4v_2^2\omega_8 + 28\omega_6^2\omega_4\omega_8^2 - 68\omega_6^2\omega_4v_2^2\omega_8^2 + 16\omega_6^2\omega_4\omega_8 - 16\omega_6^3cs^2\omega_4 - 8\omega_6^3\omega_4^2 + 68\omega_6^3\omega_4v_2^2\omega_8 + 24\omega_6\omega_4^2\omega_8 - 64\omega_6\omega_4^2v_2^2\omega_8 + 48cs^2\omega_4^2\omega_8^2 - 16\omega_6^3\omega_4v_2^2 - 16\omega_6^2cs^2\omega_4\omega_8 - 72\omega_6cs^2\omega_4^2\omega_8^2 - 12\omega_6^2\omega_8^2 + 43\omega_6^2\omega_4^2v_2^2\omega_8^2 + 56\omega_6^2cs^2\omega_4^2\omega_8 + 16\omega_6^3\omega_4^2v_2^2 - 43\omega_6^3\omega_4^2v_2^2\omega_8 - 16\omega_6^2cs^2\omega_4^2 + 32\omega_6cs^2\omega_4\omega_8^2 + 12\omega_6^3\omega_8 - 40\omega_6^2\omega_4^2\omega_8 - 20\omega_6^3cs^2\omega_8 + 64\omega_6\omega_4v_2^2\omega_8^2 - 17\omega_6^2\omega_4^2\omega_8 + 25\omega_6^2cs^2\omega_4^2\omega_8^2 - 28\omega_6^3v_2^2\omega_8 - 24\omega_6\omega_4\omega_8^2 + 104\omega_6^2\omega_4^2v_2^2\omega_8 - 28\omega_6^3\omega_4\omega_8 - 25\omega_6^3cs^2\omega_4^2\omega_8) \frac{v_1\rho v_2}{4\omega_8^3\omega_4^2\omega_8^3}$$

$$C_{D_x D_y^3 v_2}^{(2), \text{CLBM1}} = 0$$

$$C_{D_x D_y^3 v_2}^{(2), \text{CLBM2}} = 0$$

$$C_{D_x D_y^3 v_2}^{(2), \text{CuLBM1}} = 0$$

$$C_{D_x D_y^3 v_2}^{(2), \text{CuLBM2}} = (54\omega_1cs^2\omega_2^3 + 36\omega_3v_1^2\omega_1^3 + 54\omega_3cs^2\omega_2^3 + 45\omega_3v_1^2\omega_1\omega_2^3 + 198\omega_3v_2^2\omega_2^3 - 54\omega_1^3cs^2\omega_2 + 18\omega_1^3\omega_2 + 18\omega_1^2\omega_2^3 + 30\omega_3\omega_1^3cs^2\omega_2^2 - 18\omega_1^3\omega_2^2 - 297\omega_3\omega_1^3cs^2\omega_2 + 198\omega_3\omega_1^3v_2^2 - 162\omega_3\omega_1cs^2\omega_2^2 + 90\omega_3\omega_1\omega_2^2 + 54\omega_1^3cs^2\omega_2^2 - 198\omega_3\omega_1v_2^2\omega_2^2 + 270\omega_3\omega_1^3cs^2 - 54\omega_3\omega_2^3 - 27\omega_3\omega_1cs^2\omega_2^3 + 45\omega_3\omega_1\omega_2^3 - 198\omega_3\omega_1v_2^2\omega_2^2 - 198\omega_3\omega_1^3v_2^2\omega_2 + 18v_1^2\omega_1\omega_2^3 + 135\omega_3\omega_1^3\omega_2 + 324\omega_3\omega_1^3cs^2\omega_2^2 + 10\omega_3\omega_1^2\omega_2^3 + 10\omega_3v_1^2\omega_1^3\omega_2^2 - 18v_1^2\omega_1^3\omega_2^3 - 18v_1^2\omega_1^3\omega_2 - 180\omega_3\omega_1^2\omega_2^2 - 30\omega_3\omega_1^2cs^2\omega_2^3 + 396\omega_3\omega_1^2v_2^2\omega_2^2 - 126\omega_3\omega_1^3 - 198\omega_3\omega_1^2v_2^2\omega_2 - 54\omega_1^2cs^2\omega_2^2 - 18\omega_1\omega_2^3 + 90\omega_3\omega_1^2\omega_2 - 36\omega_3v_1^2\omega_2^3 + 18v_1^2\omega_1^3\omega_2^2 - 162\omega_3\omega_1^2cs^2\omega_2 - 10\omega_3\omega_1^2\omega_2^2 - 10\omega_3v_1^2\omega_1^3\omega_2^3 - 45\omega_3v_1^2\omega_1^3\omega_2) \frac{v_1\rho v_2}{24\omega_3\omega_1^3\omega_2^3}$$

$$\text{coefficient } C_{D_y^4 \rho}^{(2)} \text{ at } \frac{\partial^4 \rho}{\partial x_2^4} :$$

$$C_{D_y^4 \rho}^{(2), \text{SRT}} = (12 - 216\omega cs^4 + 404v_2^2\omega^2cs^2 - \omega^3 + 144v_2^4 - 132cs^2 - 216v_2^4\omega - 78\omega^2cs^2 + 8\omega^2 - 34v_2^2\omega^3cs^2 + 90v_2^4\omega^2 + 6\omega^3cs^2 - 9v_2^4\omega^3 + 234v_2^2\omega^2 - 5\omega^3cs^4 - 156v_2^2 + 144cs^4 - 18\omega + 198\omega cs^2 + 10v_2^2\omega^3 + 672v_2^2cs^2 + 82\omega^2cs^4 - 1008v_2^2\omega cs^2 - 98v_2^2\omega^2) \frac{v_2}{12\omega^3}$$

$$C_{D_y^4 \rho}^{(2), \text{MRT1}} = (12 - 34\omega_6^3cs^2v_2^2 + 6\omega_6^3cs^2 + 144v_2^4 + 10\omega_6^3v_2^2 - 18\omega_6 + 144cs^4 - 78\omega_6^2cs^2 - 216\omega_6v_2^4 + 672cs^2v_2^2 - 1008\omega_6cs^2v_2^2 - 216\omega_6cs^4 - 98\omega_6^2v_2^2 + 234\omega_6v_2^2 + 82\omega_6^2cs^4 + 404\omega_6^2cs^2v_2^2 + 90\omega_6^2v_2^4 + 198\omega_6cs^2 - 5\omega_6^3cs^4 - 156v_2^2 - \omega_6^3 + 8\omega_6^2 - 9\omega_6^3v_2^4 - 132cs^2) \frac{v_2}{12\omega_6^3}$$

$$C_{D_y^4 \rho}^{(2), \text{MRT2}} = (12 + 198\omega_6cs^2 + 144v_2^4 + 672cs^2v_2^2 + 10\omega_6^3v_2^2 - 18\omega_6 - 1008\omega_6cs^2v_2^2 + 82\omega_6^2cs^4 - 216\omega_6v_2^4 - 34\omega_6^3cs^2v_2^2 - 132cs^2 - 5\omega_6^3cs^4 - 98\omega_6^2v_2^2 + 144cs^4 + 234\omega_6v_2^2 + 90\omega_6^2v_2^4 + 6\omega_6^3cs^2 - 156v_2^2 - 216\omega_6cs^4 - \omega_6^3 + 404\omega_6^2cs^2v_2^2 - 78\omega_6^2cs^2 + 8\omega_6^2 - 9\omega_6^3v_2^4) \frac{v_2}{12\omega_6^3}$$

$$C_{D_y^4 \rho}^{(2), \text{CLBM1}} = (12 - 5\omega_6^3cs^4 + 144v_2^4 + 10\omega_6^3v_2^2 - 18\omega_6 - 132cs^2 + 404\omega_6^2v_2^2cs^2 - 216\omega_6v_2^4 + 82\omega_6^2cs^4 + 198\omega_6cs^2 - 98\omega_6^2v_2^2 - 34\omega_6^3v_2^2cs^2 - 78\omega_6^2cs^2 + 234\omega_6v_2^2 + 90\omega_6^2v_2^4 - 216\omega_6cs^4 - 156v_2^2 + 6\omega_6^3cs^2 - \omega_6^3 + 144cs^4 + 8\omega_6^2 + 672v_2^2cs^2 - 9\omega_6^3v_2^4 - 1008\omega_6v_2^2cs^2) \frac{v_2}{12\omega_6^3}$$

$$C_{D_y^4 \rho}^{(2), \text{CLBM2}} = (12 - 5cs^4\omega_6^3 + 144v_2^4 + 82cs^4\omega_6^2 + 10\omega_6^3v_2^2 - 18\omega_6 + 404cs^2\omega_6^2v_2^2 + 144cs^4 - 216\omega_6v_2^4 - 216cs^4\omega_6 - 34cs^2\omega_6^3v_2^2 - 98\omega_6^2v_2^2 - 78cs^2\omega_6^2 - 132cs^2 + 234\omega_6v_2^2 + 6cs^2\omega_6^3 + 90\omega_6^2v_2^4 + 672cs^2v_2^2 - 156v_2^2 - 1008cs^2\omega_6v_2^2 - \omega_6^3 + 8\omega_6^2 + 198cs^2\omega_6 - 9\omega_6^3v_2^4) \frac{v_2}{12\omega_6^3}$$

$$C_{D_y^4 \rho}^{(2), \text{CuLBM1}} = (12 - 1008cs^2v_2^2\omega_2 - 216cs^4\omega_2 + 144v_2^4 + 90v_2^4\omega_2^2 - 9v_2^4\omega_2^3 - 5cs^4\omega_2^3 - 34cs^2v_2^2\omega_2^3 - \omega_2^3 + 404cs^2v_2^2\omega_2^2 + 82cs^4\omega_2^2 - 216v_2^4\omega_2 + 672cs^2v_2^2 - 132cs^2 + 8\omega_2^2 + 10v_2^2\omega_2^3 + 144cs^4 + 198cs^2\omega_2 - 98v_2^2\omega_2^2 - 78cs^2\omega_2^2 + 234v_2^2\omega_2 - 156v_2^2 + 6cs^2\omega_2^3 - 18\omega_2) \frac{v_2}{12\omega_2^3}$$

$$C_{D_y^4 \rho}^{(2), \text{CuLBM2}} = (-6\omega_1cs^2\omega_2^3 + 82\omega_3\omega_1^2cs^4\omega_2^3 - 98\omega_3\omega_1^3v_2^2\omega_2^2 - 600\omega_3\omega_1v_2^2cs^2\omega_2^3 + 12\omega_3\omega_1^3cs^2\omega_2^3 - 198\omega_3\omega_1^2v_2^4\omega_2^2 - 72\omega_3cs^2\omega_2^3 + 20\omega_3\omega_1^3v_2^2\omega_2^3 - 90\omega_3\omega_1^2cs^4\omega_2^2 - 51\omega_3v_2^2\omega_2^3 - 6\omega_1^3cs^2\omega_2 + 90\omega_3\omega_1^2v_2^4\omega_2^3 - 78\omega_3\omega_1^3cs^2\omega_2^2 + 411\omega_3\omega_1v_2^2cs^2\omega_2^2 - 36\omega_1^2cs^4\omega_2^2 + 141\omega_3\omega_1^3cs^2\omega_2 - 51\omega_3\omega_1^3v_2^2 - 60\omega_3\omega_1cs^2\omega_2^2 + 6\omega_3\omega_1\omega_2^2 + 54\omega_3\omega_1^2cs^4\omega_2 + 129\omega_3\omega_1v_2^2\omega_2^3 - 72\omega_3\omega_1^3cs^2 + 6\omega_3\omega_2^3 + 141\omega_3\omega_1cs^2\omega_2^3 - 12\omega_3\omega_1\omega_2^3 - 12\omega_1^2v_2^2cs^2\omega_2^2 + 99\omega_3\omega_1^2v_2^4\omega_2 - 105\omega_3\omega_1v_2^2\omega_2^2 + 6\omega_1^3v_2^2cs^2\omega_2 + 129\omega_3\omega_1^3v_2^2\omega_2 - 18\omega_3\omega_1^3v_2^2\omega_2^3 - 12\omega_3\omega_1^3\omega_2 + 114\omega_3\omega_1^2cs^2\omega_2^2 + 18\omega_1^3cs^4\omega_2 + 8\omega_3\omega_1^2\omega_2^3 + 45\omega_3v_2^4\omega_2^3 + 90\omega_3\omega_1^3cs^4 + 404\omega_3\omega_1^2v_2^2cs^2\omega_2^3 - 98\omega_3\omega_1^2v_2^2\omega_2^3 + 45\omega_3\omega_1^3v_2^4 + 82\omega_3\omega_1^3cs^4\omega_2^2 + 18\omega_1cs^4\omega_2^3 - 816\omega_3\omega_1^2v_2^2cs^2\omega_2^2 + 261\omega_3\omega_1^3v_2^2cs^2 - 12\omega_3\omega_1^2\omega_2^2 - 78\omega_3\omega_1^2cs^2\omega_2^3 + 90\omega_3\omega_1^2v_2^4\omega_2^2 - 10\omega_3\omega_1^3cs^4\omega_2^3 + 210\omega_3\omega_1^2v_2^2\omega_2^2 + 6\omega_3\omega_1^3 - 600\omega_3\omega_1^2v_2^2cs^2\omega_2 + 90\omega_3cs^4\omega_2^3 - 171\omega_3\omega_1cs^4\omega_2^3 - 105\omega_3\omega_1^2v_2^2\omega_2 + 404\omega_3\omega_1^2v_2^2cs^2\omega_2^2 + 411\omega_3\omega_1^2v_2^2cs^2\omega_2 + 6\omega_1v_2^2cs^2\omega_2^3 + 99\omega_3\omega_1v_2^4\omega_2^2 + 6\omega_3\omega_1^2\omega_2 - 117\omega_3\omega_1^3v_2^4\omega_2 - 2\omega_3\omega_1^3\omega_2^3 - 68\omega_3\omega_1^2v_2^2cs^2\omega_2^3 + 12\omega_1^2cs^2\omega_2^2 - 171\omega_3\omega_1^3cs^4\omega_2 + 261\omega_3v_2^2cs^2\omega_2^3 + 54\omega_3\omega_1cs^4\omega_2^2 - 60\omega_3\omega_1^2cs^2\omega_2 + 8\omega_3\omega_1^3\omega_2^2 - 117\omega_3\omega_1v_2^4\omega_2^3) \frac{v_2}{24\omega_3\omega_1^3\omega_2^3}$$

$$\text{coefficient } C_{D_y^4 v_2}^{(2)} \text{ at } \frac{\partial^4 v_2}{\partial x_2^4} :$$

$$C_{D_y^4 v_2}^{(2), \text{SRT}} = (12 - 36\omega cs^4 + 252v_2^2\omega^2cs^2 - \omega^3 + 504v_2^4 - 36cs^2 - 756v_2^4\omega - 22\omega^2cs^2 + 8\omega^2 - 18v_2^2\omega^3cs^2 + 310v_2^4\omega^2 + 2\omega^3cs^2 - 29v_2^4\omega^3 + 378v_2^2\omega^2 - \omega^3cs^4 - 252v_2^2 + 24cs^4 - 18\omega + 54\omega cs^2 + 14v_2^2\omega^3 + 432v_2^2cs^2 + 14\omega^2cs^4 - 648v_2^2\omega cs^2 - 154v_2^2\omega^2) \frac{\rho}{12\omega^3}$$

$$C_{D_y^4 v_2}^{(2), \text{MRT1}} = (12 - 18\omega_6^3cs^2v_2^2 + 2\omega_6^3cs^2 + 504v_2^4 + 14\omega_6^3v_2^2 - 18\omega_6 + 24cs^4 - 22\omega_6^2cs^2 - 756\omega_6v_2^4 + 432cs^2v_2^2 - 648\omega_6cs^2v_2^2 - 36\omega_6cs^4 -$$

$$\begin{aligned}
& 154\omega_6^2v_2^2 + 378\omega_6v_2^2 + 14\omega_6^2cs^4 + 252\omega_6^2cs^2v_2^2 + 310\omega_6^2v_2^4 + 54\omega_6cs^2 - \omega_6^3cs^4 - 252v_2^2 - \omega_6^3 + 8\omega_6^2 - 29\omega_6^3v_2^4 - 36cs^2) \frac{\rho}{12\omega_6^3} \\
C_{D_y^4v_2}^{(2),\text{MRT}^2} &= (12 + 54\omega_6cs^2 + 504v_2^4 + 432cs^2v_2^2 + 14\omega_6^3v_2^2 - 18\omega_6 - 648\omega_6cs^2v_2^2 + 14\omega_6^2cs^4 - 756\omega_6v_2^4 - 18\omega_6^3cs^2v_2^2 - 36cs^2 - \omega_6^3cs^4 - \\
& 154\omega_6^2v_2^2 + 24cs^4 + 378\omega_6v_2^2 + 310\omega_6^2v_2^4 + 2\omega_6^3cs^2 - 252v_2^2 - 36\omega_6cs^4 - \omega_6^3 + 252\omega_6^2cs^2v_2^2 - 22\omega_6^2cs^2 + 8\omega_6^2 - 29\omega_6^3v_2^4) \frac{\rho}{12\omega_6^3} \\
C_{D_y^4v_2}^{(2),\text{CLBM1}} &= (12 - \omega_6^3cs^4 + 504v_2^4 + 14\omega_6^3v_2^2 - 18\omega_6 - 36cs^2 + 252\omega_6^2v_2^2cs^2 - 756\omega_6v_2^4 + 14\omega_6^2cs^4 + 54\omega_6cs^2 - 154\omega_6^2v_2^2 - 18\omega_6^3v_2^2cs^2 - \\
& 22\omega_6^2cs^2 + 378\omega_6v_2^2 + 310\omega_6^2v_2^4 - 36\omega_6cs^4 - 252v_2^2 + 2\omega_6^3cs^2 - \omega_6^3 + 24cs^4 + 8\omega_6^2 + 432v_2^2cs^2 - 29\omega_6^3v_2^4 - 648\omega_6v_2^2cs^2) \frac{\rho}{12\omega_6^3} \\
C_{D_y^4v_2}^{(2),\text{CLBM2}} &= (12 - cs^4\omega_6^3 + 504v_2^4 + 14cs^4\omega_6^2 + 14\omega_6^3v_2^2 - 18\omega_6 + 252cs^2\omega_6^2v_2^2 + 24cs^4 - 756\omega_6v_2^4 - 36cs^4\omega_6 - 18cs^2\omega_6^3v_2^2 - 154\omega_6^2v_2^2 - \\
& 22cs^2\omega_6^2 - 36cs^2 + 378\omega_6v_2^2 + 2cs^2\omega_6^3 + 310\omega_6^2v_2^4 + 432cs^2v_2^2 - 252v_2^2 - 648cs^2\omega_6v_2^2 - \omega_6^3 + 8\omega_6^2 + 54cs^2\omega_6 - 29\omega_6^3v_2^4) \frac{\rho}{12\omega_6^3} \\
C_{D_y^4v_2}^{(2),\text{CuLBM1}} &= (12 - 648cs^2v_2^2\omega_2 - 36cs^4\omega_2 + 504v_2^4 + 310v_2^4\omega_2^2 - 29v_2^4\omega_2^3 - cs^4\omega_2^3 - 18cs^2v_2^2\omega_2^3 - \omega_2^3 + 252cs^2v_2^2\omega_2^2 + 14cs^4\omega_2^2 - 756v_2^4\omega_2 + \\
& 432cs^2v_2^2 - 36cs^2 + 8\omega_2^2 + 14v_2^2\omega_2^3 + 24cs^4 + 54cs^2\omega_2 - 154v_2^2\omega_2^2 - 22cs^2\omega_2^2 + 378v_2^2\omega_2 - 252v_2^2 + 2cs^2\omega_2^3 - 18\omega_2) \frac{\rho}{12\omega_2^3} \\
C_{D_y^4v_2}^{(2),\text{CuLBM2}} &= (-6\omega_1cs^2\omega_2^3 + 14\omega_3\omega_1^2cs^4\omega_2^3 - 154\omega_3\omega_1^3v_2^2\omega_2^2 - 432\omega_3\omega_1v_2^2cs^2\omega_2^3 + 4\omega_3\omega_1^3cs^2\omega_2^3 - 666\omega_3\omega_1^2v_2^4\omega_2^2 - 24\omega_3cs^2\omega_2^3 + \\
& 28\omega_3\omega_1^3v_2^2\omega_2^3 - 6\omega_3\omega_1^2cs^4\omega_2^2 - 99\omega_3v_2^2\omega_2^3 - 6\omega_1^3cs^2\omega_2 + 310\omega_3\omega_1^2v_2^4\omega_2^3 - 22\omega_3\omega_1^3cs^2\omega_2^2 + 225\omega_3\omega_1v_2^2cs^2\omega_2^2 - 12\omega_1^2cs^4\omega_2^2 + 45\omega_3\omega_1^3cs^2\omega_2 - \\
& 99\omega_3\omega_1^3v_2^2 - 12\omega_3\omega_1cs^2\omega_2^2 + 6\omega_3\omega_1^2\omega_2^2 + 6\omega_3\omega_1^2cs^4\omega_2 + 225\omega_3\omega_1v_2^2\omega_2^3 - 24\omega_3\omega_1^3cs^2 + 6\omega_3\omega_2^3 + 45\omega_3\omega_1cs^2\omega_2^3 - 12\omega_3\omega_1\omega_2^3 - 36\omega_2^2v_2^2cs^2\omega_2^2 + \\
& 333\omega_3\omega_1^2v_2^4\omega_2 - 153\omega_3\omega_1v_2^2\omega_2^2 + 18\omega_1^3v_2^2cs^2\omega_2 + 225\omega_3\omega_1^3v_2^2\omega_2 - 58\omega_3\omega_1^3v_2^4\omega_2^3 - 12\omega_3\omega_1^3\omega_2 + 18\omega_3\omega_1^2cs^2\omega_2^2 + 6\omega_1^3cs^4\omega_2 + 8\omega_3\omega_1^2\omega_2^3 + \\
& 171\omega_3v_2^4\omega_2^3 + 18\omega_3\omega_1^3cs^4 + 252\omega_3\omega_1^2v_2^2cs^2\omega_2^3 - 154\omega_3\omega_1^2v_2^2\omega_2^3 + 171\omega_3\omega_1^3v_2^4 + 14\omega_3\omega_1^3cs^4\omega_2^2 + 6\omega_1cs^4\omega_2^3 - 432\omega_3\omega_1^2v_2^2cs^2\omega_2^2 + 207\omega_3\omega_1^3v_2^2cs^2 - \\
& 12\omega_3\omega_1^2\omega_2^2 - 22\omega_3\omega_1^2cs^2\omega_2^3 + 310\omega_3\omega_1^3v_2^4\omega_2^2 - 2\omega_3\omega_1^3cs^4\omega_2^3 + 306\omega_3\omega_1^2v_2^2\omega_2^2 + 6\omega_3\omega_1^3 - 432\omega_3\omega_1^3v_2^2cs^2\omega_2 + 18\omega_3cs^4\omega_2^3 - 33\omega_3\omega_1cs^4\omega_2^3 - \\
& 153\omega_3\omega_1^2v_2^2\omega_2 + 252\omega_3\omega_1^2v_2^2cs^2\omega_2^2 + 225\omega_3\omega_1^2v_2^2cs^2\omega_2 + 18\omega_1v_2^2cs^2\omega_2^3 + 333\omega_3\omega_1v_2^4\omega_2^2 + 6\omega_3\omega_1^2\omega_2 - 423\omega_3\omega_1^3v_2^4\omega_2 - 2\omega_3\omega_1^3\omega_2^3 - \\
& 36\omega_3\omega_1^3v_2^2cs^2\omega_2^3 + 12\omega_1^2cs^2\omega_2^2 - 33\omega_3\omega_1^3cs^4\omega_2 + 207\omega_3v_2^2cs^2\omega_2^3 + 6\omega_3\omega_1cs^4\omega_2^2 - 12\omega_3\omega_1^2cs^2\omega_2 + 8\omega_3\omega_1^3\omega_2^2 - 423\omega_3\omega_1v_2^4\omega_2^3) \frac{\rho}{24\omega_3\omega_1^3\omega_2^3}
\end{aligned}$$

References

- [1] T. Krüger, H. Kusumaatmaja, A. Kuzmin, O. Shardt, G. Silva, E. M. Viggien, The lattice Boltzmann method, Springer International Publishing 10 (978-3) (2017) 4–15.
- [2] C. Coreixas, B. Chopard, J. Latt, Comprehensive comparison of collision models in the lattice boltzmann framework: Theoretical investigations, Physical Review E 100 (3) (2019) 033305.