

D2Q5 ADE,

a supplementary material for

Lattice Boltzmann Method Analysis Tool (LBMAT)

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1 Global definitions

In \mathbb{R}^2 , the position and velocity vectors are given by $\mathbf{x} = (x_1, x_2)^T$ and $\mathbf{v} = (v_1, v_2)^T$, respectively.

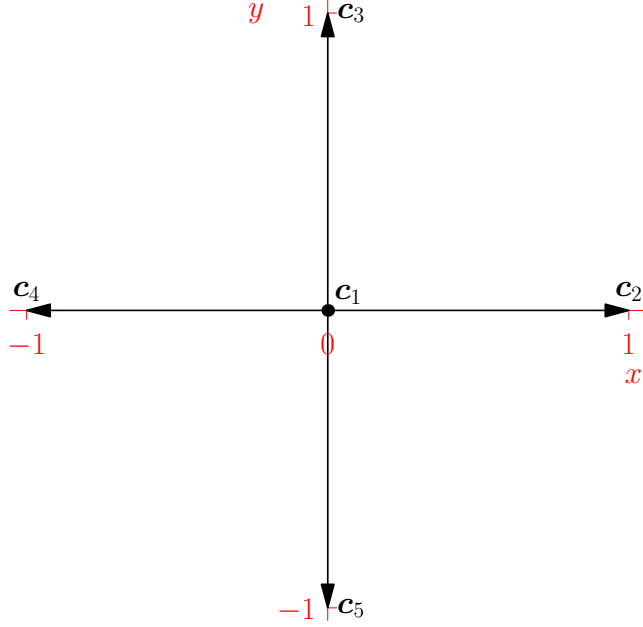
1.1 Discrete velocity vectors

Discrete velocity vectors and the lattice speed of sound are defined by

$$\{\mathbf{c}_i\}_{i=1}^5 = \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix} \right),$$

$$c_s = \frac{1}{\sqrt{3}},$$

respectively [1].



1.2 Raw and central moments

The raw and central moments are defined by

$$m_{\alpha} := \sum_{i=1}^5 f_i \mathbf{c}_i^{\alpha},$$

and

$$k_{\alpha} := \sum_{i=1}^5 f_i (\mathbf{c}_i - \mathbf{v})^{\alpha},$$

respectively, where $\alpha = (\alpha_1, \alpha_2) \in \mathbb{Z}^2$ denotes a multi-index (as a row vector) and $\mathbf{c}_i^{\alpha} := \prod_{j=1}^2 [\mathbf{c}_i]_j^{\alpha_j}$.

1.3 Transformation matrix M

Matrix \mathbf{M} , that defines macroscopic quantities (moments) $\boldsymbol{\mu}$ by

$$\boldsymbol{\mu} = \mathbf{M}\mathbf{f},$$

with $\mathbf{f} = (f_1, f_2, \dots, f_5)^T$, is selected such that

$$\boldsymbol{\mu} = \left(m_{(0,0)}, m_{(1,0)}, m_{(0,1)}, m_{(2,0)}, m_{(0,2)} \right)^T,$$

i.e., \mathbf{M} is given by

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}.$$

1.4 Equilibrium

The corresponding equilibrium raw moments are defined using the continuous Maxwell–Boltzmann distribution function [1]

$$f^{(eq)}(\boldsymbol{\xi}) = \frac{\rho}{2\pi c_s^2} \exp\left(-\frac{\|\boldsymbol{\xi} - \mathbf{v}\|^2}{2c_s^2}\right)$$

as

$$m_{\boldsymbol{\alpha}}^{(eq)} = \int_{\mathbb{R}^2} \boldsymbol{\xi}^{\boldsymbol{\alpha}} f^{(eq)}(\boldsymbol{\xi}) d\boldsymbol{\xi},$$

where $\alpha_i \in \{0, 1, 2\}$, $i = 1, 2$. Hence, the equilibrium moments $\boldsymbol{\mu}^{(eq)}$ satisfy

$$\boldsymbol{\mu}^{(eq)} = \left(\rho, \rho v_1, \rho v_2, \rho(v_1^2 + c_s^2), \rho(v_2^2 + c_s^2) \right)^T.$$

2 Spatial EPDEs

2.1 SRT

2.1.1 Definitions

Collision operator \mathcal{C} :

$$\mathcal{C}(\mathbf{f}) = \omega \left(\mathbf{M}^{-1} \boldsymbol{\mu}^{(eq)} - \mathbf{f} \right),$$

$\omega \in (0, 2)$.

2.1.2 Conservation of mass equation



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$$\begin{aligned} & \frac{\partial \rho}{\partial t} + \frac{v_1 \delta_L}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho \delta_L}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 \delta_L}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho \delta_L}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega) \frac{\delta_L}{2\omega} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial t} + (-2 + \omega) \frac{v_1 \delta_L^2}{2\delta_t \omega} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + \\ & (-2 + \omega) \frac{\rho \delta_L^2}{2\delta_t \omega} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + (2 - \omega) \frac{v_2 \delta_L^2}{2\delta_t \omega} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (2 - \omega) \frac{v_1 \delta_L^2}{\delta_t \omega} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_2} + (2 - \omega) \frac{\rho \delta_L^2}{\delta_t \omega} \frac{\partial v_1}{\partial x_1} \frac{\partial v_2}{\partial x_2} + (-2 + \omega) \frac{\delta_L}{2\omega} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial t} + \\ & (2 - \omega) \frac{v_2 \delta_L^2}{\delta_t \omega} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_1} + (2 - \omega) \frac{v_1 \delta_L^2}{2\delta_t \omega} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + (-2 + \omega) \frac{v_2 \delta_L^2}{2\delta_t \omega} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (-2 + \omega) \frac{\rho \delta_L^2}{2\delta_t \omega} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + (-2 + \omega) \frac{\rho \delta_L}{2\omega} \frac{\partial^2 v_1}{\partial t \partial x_1} + \\ & (-2 + \omega) \frac{c_s^2 \delta_L^2}{2\delta_t \omega} \frac{\partial^2 \rho}{\partial x_1^2} + (-2 + \omega) \frac{v_1 \rho \delta_L^2}{2\delta_t \omega} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega) \frac{\rho \delta_L}{2\omega} \frac{\partial^2 v_2}{\partial t \partial x_2} + (2 - \omega) \frac{v_2 \rho \delta_L^2}{\delta_t \omega} \frac{\partial^2 \rho}{\partial x_1 \partial x_2} + (2 - \omega) \frac{v_2 \rho \delta_L^2}{2\delta_t \omega} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + \\ & (2 - \omega) \frac{v_1 \rho \delta_L^2}{2\delta_t \omega} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega) \frac{c_s^2 \delta_L^2}{2\delta_t \omega} \frac{\partial^2 \rho}{\partial x_2^2} + (-2 + \omega) \frac{v_2 \rho \delta_L^2}{2\delta_t \omega} \frac{\partial^2 v_2}{\partial x_2^2} + (12 + \omega^2 - 12\omega) \frac{\delta_t \rho \delta_L}{12\omega^2} \frac{\partial^3 v_1}{\partial t^2 \partial x_1} + \end{aligned}$$

$$\begin{aligned}
& (12 + \omega^2 - 12\omega) \frac{v_1 \rho \delta_l^2}{6\omega^2} \frac{\partial^3 v_1}{\partial t \partial x_1^2} + (6 - 6v_1^2 + 6v_1^2 \omega + 18c_s^2 \omega + \omega^2 - v_1^2 \omega^2 - 3c_s^2 \omega^2 - 18c_s^2 - 6\omega) \frac{v_1 \delta_l^3}{6\delta_t \omega^2} \frac{\partial^3 \rho}{\partial x_1^3} + \\
& (12 - 24v_1^2 + 24v_1^2 \omega + 24c_s^2 \omega + 2\omega^2 - 5v_1^2 \omega^2 - 3c_s^2 \omega^2 - 24c_s^2 - 12\omega) \frac{\rho \delta_l^3}{12\delta_t \omega^2} \frac{\partial^3 v_1}{\partial x_1^3} + (12 + \omega^2 - 12\omega) \frac{\delta_t \rho \delta_l}{12\omega^2} \frac{\partial^3 v_2}{\partial t^2 \partial x_2} + \\
& (-6 - \omega^2 + 6\omega) \frac{v_2 \rho \delta_l^2}{3\omega^2} \frac{\partial^3 v_1}{\partial t \partial x_1 \partial x_2} + (-6 - \omega^2 + 6\omega) \frac{v_1 \rho \delta_l^2}{3\omega^2} \frac{\partial^3 v_2}{\partial t \partial x_1 \partial x_2} + \\
& (6v_1^2 - 6v_1^2 \omega + 6c_s^2 \omega + v_1^2 \omega^2 - c_s^2 \omega^2 - 6c_s^2) \frac{v_2 \delta_l^3}{2\delta_t \omega^2} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + (12 + \omega^2 - 12\omega) \frac{v_2 v_1 \rho \delta_l^3}{6\delta_t \omega^2} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} + \\
& (24c_s^2 \omega + v_1^2 \omega^2 - 3c_s^2 \omega^2 - 24c_s^2) \frac{\rho \delta_l^3}{12\delta_t \omega^2} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + (12 + \omega^2 - 12\omega) \frac{v_2 \rho \delta_l^2}{6\omega^2} \frac{\partial^3 v_2}{\partial t \partial x_2^2} + \\
& (6c_s^2 \omega + 6v_2^2 + v_2^2 \omega^2 - 6v_2^2 \omega - c_s^2 \omega^2 - 6c_s^2) \frac{v_1 \delta_l^3}{2\delta_t \omega^2} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + (24c_s^2 \omega + v_2^2 \omega^2 - 3c_s^2 \omega^2 - 24c_s^2) \frac{\rho \delta_l^3}{12\delta_t \omega^2} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + \\
& (12 + \omega^2 - 12\omega) \frac{v_2 v_1 \rho \delta_l^3}{6\delta_t \omega^2} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + (6 + 18c_s^2 \omega - 6v_2^2 - v_2^2 \omega^2 + \omega^2 + 6v_2^2 \omega - 3c_s^2 \omega^2 - 18c_s^2 - 6\omega) \frac{v_2 \delta_l^3}{6\delta_t \omega^2} \frac{\partial^3 \rho}{\partial x_2^3} + \\
& (12 + 24c_s^2 \omega - 24v_2^2 - 5v_2^2 \omega^2 + 2\omega^2 + 24v_2^2 \omega - 3c_s^2 \omega^2 - 24c_s^2 - 12\omega) \frac{\rho \delta_l^3}{12\delta_t \omega^2} \frac{\partial^3 v_2}{\partial x_2^3} + (-2 - \omega^2 + 3\omega) \frac{\delta_t^2 \rho \delta_l}{2\omega^3} \frac{\partial^4 v_1}{\partial t^2 \partial x_1} + \\
& (-2 - \omega^2 + 3\omega) \frac{3v_1 \delta_t \rho \delta_l^2}{2\omega^3} \frac{\partial^4 v_1}{\partial t^2 \partial x_1^2} + C_1 \frac{\rho \delta_l^3}{12\omega^3} \frac{\partial^4 v_1}{\partial t \partial x_1^3} + C_2 \frac{\delta_l^4}{24\delta_t \omega^3} \frac{\partial^4 \rho}{\partial x_1^4} + C_3 \frac{v_1 \rho \delta_l^4}{12\delta_t \omega^3} \frac{\partial^4 v_1}{\partial x_1^4} + (-2 - \omega^2 + 3\omega) \frac{\delta_t^2 \rho \delta_l}{2\omega^3} \frac{\partial^4 v_2}{\partial t^2 \partial x_2} + \\
& (36 - \omega^3 + 20\omega^2 - 54\omega) \frac{v_2 \delta_t \rho \delta_l^2}{12\omega^3} \frac{\partial^4 v_1}{\partial t^2 \partial x_1 \partial x_2} + (36 - \omega^3 + 20\omega^2 - 54\omega) \frac{v_1 \delta_t \rho \delta_l^2}{12\omega^3} \frac{\partial^4 v_2}{\partial t^2 \partial x_1 \partial x_2} + \\
& (-24 + \omega^3 - 14\omega^2 + 36\omega) \frac{v_2 v_1 \rho \delta_l^3}{6\omega^3} \frac{\partial^4 v_1}{\partial t \partial x_1^2 \partial x_2} + (-90c_s^2 \omega - 2c_s^2 \omega^3 + v_1^2 \omega^3 - 2v_1^2 \omega^2 + 34c_s^2 \omega^2 + 60c_s^2) \frac{\rho \delta_l^3}{12\omega^3} \frac{\partial^4 v_2}{\partial t \partial x_1^2 \partial x_2} + \\
& (24 - \omega^3 + 180c_s^2 \omega + 14\omega^2 + 6c_s^2 \omega^3 - 72c_s^2 \omega^2 - 120c_s^2 - 36\omega) \frac{v_2 v_1 \delta_l^4}{6\delta_t \omega^3} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_4 \frac{v_2 \rho \delta_l^4}{12\delta_t \omega^3} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + C_5 \frac{v_1 \rho \delta_l^4}{12\delta_t \omega^3} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& (-2 - \omega^2 + 3\omega) \frac{3v_2 \delta_t \rho \delta_l^2}{2\omega^3} \frac{\partial^4 v_2}{\partial t^2 \partial x_2^2} + (v_2^2 \omega^3 - 90c_s^2 \omega - 2v_2^2 \omega^2 - 2c_s^2 \omega^3 + 34c_s^2 \omega^2 + 60c_s^2) \frac{\rho \delta_l^3}{12\omega^3} \frac{\partial^4 v_1}{\partial t \partial x_1 \partial x_2^2} + \\
& (-24 + \omega^3 - 14\omega^2 + 36\omega) \frac{v_2 v_1 \rho \delta_l^3}{6\omega^3} \frac{\partial^4 v_2}{\partial t \partial x_1 \partial x_2^2} + C_6 \frac{\delta_l^4}{4\delta_t \omega^3} \frac{\partial^4 \rho}{\partial x_1^4 \partial x_2^2} + \\
& (-4v_2^2 \omega^3 + 72c_s^2 \omega + 84v_2^2 + 50v_2^2 \omega^2 + c_s^2 \omega^3 - 126v_2^2 \omega - 26c_s^2 \omega^2 - 48c_s^2) \frac{v_1 \rho \delta_l^4}{12\delta_t \omega^3} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + \\
& (84v_1^2 - 126v_1^2 \omega + 72c_s^2 \omega + c_s^2 \omega^3 - 4v_1^2 \omega^3 + 50v_1^2 \omega^2 - 26c_s^2 \omega^2 - 48c_s^2) \frac{v_2 \rho \delta_l^4}{12\delta_t \omega^3} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_7 \frac{\rho \delta_l^3}{12\omega^3} \frac{\partial^4 v_2}{\partial t \partial x_2^3} + \\
& (24 - \omega^3 + 180c_s^2 \omega + 14\omega^2 + 6c_s^2 \omega^3 - 72c_s^2 \omega^2 - 120c_s^2 - 36\omega) \frac{v_2 v_1 \delta_l^4}{6\delta_t \omega^3} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_8 \frac{v_2 \rho \delta_l^4}{12\delta_t \omega^3} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_9 \frac{v_1 \rho \delta_l^4}{12\delta_t \omega^3} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\
& + C_{10} \frac{\delta_l^4}{24\delta_t \omega^3} \frac{\partial^4 \rho}{\partial x_2^4} + C_{11} \frac{v_2 \rho \delta_l^4}{12\delta_t \omega^3} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= -36 + 72v_1^2 + \omega^3 - 108v_1^2 \omega - 90c_s^2 \omega - 20\omega^2 - 2c_s^2 \omega^3 - 3v_1^2 \omega^3 + 42v_1^2 \omega^2 + 34c_s^2 \omega^2 + 60c_s^2 + 54\omega \\
C_2 &= 72v_1^2 - 108v_1^2 \omega + 36c_s^2 \omega + c_s^2 \omega^3 - 3v_1^2 \omega^3 + 48c_s^4 + 42v_1^2 \omega^2 - 14c_s^2 \omega^2 - 24c_s^2 + 216v_1^2 c_s^2 \omega + 108v_1^4 \omega - 72c_s^4 \omega - 42v_1^4 \omega^2 + 30c_s^4 \omega^2 + \\
& 6v_1^2 c_s^2 \omega^3 - 72v_1^4 - 144v_1^2 c_s^2 - 3c_s^4 \omega^3 + 3v_1^4 \omega^3 - 84v_1^2 c_s^2 \omega^2 \\
C_3 &= 24 - 36v_1^2 - \omega^3 + 54v_1^2 \omega + 72c_s^2 \omega + 14\omega^2 + c_s^2 \omega^3 + 2v_1^2 \omega^3 - 22v_1^2 \omega^2 - 26c_s^2 \omega^2 - 48c_s^2 - 36\omega \\
C_4 &= 12 - 12v_1^2 - \omega^3 + 18v_1^2 \omega + 144c_s^2 \omega + 8\omega^2 + 4c_s^2 \omega^3 + 3v_1^2 \omega^3 - 12v_1^2 \omega^2 - 56c_s^2 \omega^2 - 96c_s^2 - 18\omega \\
C_5 &= 36 - 36v_1^2 - \omega^3 + 54v_1^2 \omega + 144c_s^2 \omega + 20\omega^2 + 4c_s^2 \omega^3 + v_1^2 \omega^3 - 20v_1^2 \omega^2 - 56c_s^2 \omega^2 - 96c_s^2 - 54\omega \\
C_6 &= 56v_2^2 v_1^2 - 84v_2^2 v_1^2 \omega + 36v_2^2 c_s^2 \omega + v_2^2 c_s^2 \omega^3 - 3v_2^2 v_1^2 \omega^3 + 16c_s^4 + 34v_2^2 v_1^2 \omega^2 - 14v_2^2 c_s^2 \omega^2 + 36v_1^2 c_s^2 \omega - 24v_2^2 c_s^2 - 24c_s^4 \omega + 10c_s^4 \omega^2 + v_1^2 c_s^2 \omega^3 - \\
& 24v_1^2 c_s^2 - c_s^4 \omega^3 - 14v_1^2 c_s^2 \omega^2 \\
C_7 &= -36 - 3v_2^2 \omega^3 + \omega^3 - 90c_s^2 \omega + 72v_2^2 + 42v_2^2 \omega^2 - 20\omega^2 - 2c_s^2 \omega^3 - 108v_2^2 \omega + 34c_s^2 \omega^2 + 60c_s^2 + 54\omega \\
C_8 &= 36 + v_2^2 \omega^3 - \omega^3 + 144c_s^2 \omega - 36v_2^2 - 20v_2^2 \omega^2 + 20\omega^2 + 4c_s^2 \omega^3 + 54v_2^2 \omega - 56c_s^2 \omega^2 - 96c_s^2 - 54\omega \\
C_9 &= 12 + 3v_2^2 \omega^3 - \omega^3 + 144c_s^2 \omega - 12v_2^2 - 12v_2^2 \omega^2 + 8\omega^2 + 4c_s^2 \omega^3 + 18v_2^2 \omega - 56c_s^2 \omega^2 - 96c_s^2 - 18\omega \\
C_{10} &= -3v_2^2 \omega^3 + 216v_2^2 c_s^2 \omega + 36c_s^2 \omega + 72v_2^2 + 42v_2^2 \omega^2 + c_s^2 \omega^3 + 6v_2^2 c_s^2 \omega^3 + 48c_s^4 - 108v_2^2 \omega - 84v_2^2 c_s^2 \omega^2 - 14c_s^2 \omega^2 - 24c_s^2 - 42v_2^2 \omega^2 - \\
& 144v_2^2 c_s^2 + 3v_2^2 \omega^3 - 72c_s^4 \omega + 30c_s^4 \omega^2 - 3c_s^4 \omega^3 - 72v_2^4 + 108v_2^4 \omega \\
C_{11} &= 24 + 2v_2^2 \omega^3 - \omega^3 + 72c_s^2 \omega - 36v_2^2 - 22v_2^2 \omega^2 + 14\omega^2 + c_s^2 \omega^3 + 54v_2^2 \omega - 26c_s^2 \omega^2 - 48c_s^2 - 36\omega
\end{aligned}$$

2.2 MRT1

2.2.1 Definitions

Collision operator \mathcal{C} :

$$\mathcal{C}(f) = \mathbf{M}^{-1} \mathbf{S} \left(\mu^{(eq)} - \mathbf{M} f \right),$$

where

$$\mathbf{S} = \text{diag}(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5),$$

$$\omega_1, \omega_2, \dots, \omega_5 \in (0, 2).$$

2.2.2 Conservation of mass equation



attached text file: output_d2q5_ade_mrt1_symbolic_pde_00.txt

$$\begin{aligned} & \frac{\partial \rho}{\partial t} + \frac{\delta_l v_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\delta_l \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_l v_2}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\delta_l \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_2) \frac{\delta_l}{2\omega_2} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial t} + (-2 + \omega_2) \frac{\delta_l^2 v_1}{2\omega_2 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + \\ & (-2 + \omega_2) \frac{\delta_l^2 \rho}{2\omega_2 \delta_t} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + (2 - \omega_3) \frac{\delta_l^2 v_2}{2\omega_3 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (-\omega_2 \omega_3 + \omega_2 + \omega_3) \frac{\delta_l^2 v_1}{\omega_2 \omega_3 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_2} + \\ & (-\omega_2 \omega_3 + \omega_2 + \omega_3) \frac{\delta_l^2 \rho}{\omega_2 \omega_3 \delta_t} \frac{\partial v_1}{\partial x_1} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_3) \frac{\delta_l}{2\omega_3} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial t} + (-\omega_2 \omega_3 + \omega_2 + \omega_3) \frac{\delta_l^2 v_2}{\omega_2 \omega_3 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_1} + \\ & (2 - \omega_2) \frac{\delta_l^2 v_1}{2\omega_2 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + (-2 + \omega_3) \frac{\delta_l^2 v_2}{2\omega_3 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_3) \frac{\delta_l^2 \rho}{2\omega_3 \delta_t} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + (-2 + \omega_2) \frac{\delta_l \rho}{2\omega_2} \frac{\partial^2 v_1}{\partial t \partial x_1} + \\ & (-2 + \omega_2) \frac{\delta_l^2 c_s^2}{2\omega_2 \delta_t} \frac{\partial^2 \rho}{\partial x_1^2} + (-2 + \omega_2) \frac{\delta_l^2 \rho v_1}{2\omega_2 \delta_t} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega_3) \frac{\delta_l \rho}{2\omega_3} \frac{\partial^2 v_2}{\partial t \partial x_2} + (-\omega_2 \omega_3 + \omega_2 + \omega_3) \frac{\delta_l^2 v_2 v_1}{\omega_2 \omega_3 \delta_t} \frac{\partial^2 \rho}{\partial x_1 \partial x_2} + \\ & (2 - \omega_3) \frac{\delta_l^2 v_2 \rho}{2\omega_3 \delta_t} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + (2 - \omega_2) \frac{\delta_l^2 \rho v_1}{2\omega_2 \delta_t} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_3) \frac{\delta_l^2 c_s^2}{2\omega_3 \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + (-2 + \omega_3) \frac{\delta_l^2 v_2 \rho}{2\omega_3 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + \\ & (12 - 12\omega_2 + \omega_2^2) \frac{\delta_l \delta_t \rho}{12\omega_2^2} \frac{\partial^3 v_1}{\partial t^2 \partial x_1} + (12 - 6\omega_2 + \omega_2 \omega_4 - 6\omega_4) \frac{\delta_l^2 \rho v_1}{6\omega_2 \omega_4} \frac{\partial^3 v_1}{\partial t \partial x_1^2} + C_1 \frac{\delta_l^3 v_1}{6\omega_2^3 \delta_t \omega_4} \frac{\partial^3 \rho}{\partial x_1^3} + C_2 \frac{\delta_l^3 \rho}{12\omega_2^2 \delta_t \omega_4} \frac{\partial^3 v_1}{\partial x_1^3} + \\ & (12 - 12\omega_3 + \omega_3^2) \frac{\delta_l \delta_t \rho}{12\omega_3^2} \frac{\partial^3 v_2}{\partial t^2 \partial x_2} + (9\omega_2 \omega_3 - 2\omega_2 \omega_3^2 - 6\omega_2 - 6\omega_3 + 3\omega_3^2) \frac{\delta_l^2 v_2 \rho}{6\omega_2 \omega_3^2} \frac{\partial^3 v_1}{\partial t \partial x_1 \partial x_2} + \\ & (9\omega_2 \omega_3 - 6\omega_2 + 3\omega_2^2 - 6\omega_3 - 2\omega_2^2 \omega_3) \frac{\delta_l^2 \rho v_1}{6\omega_2^2 \omega_3} \frac{\partial^3 v_2}{\partial t \partial x_1 \partial x_2} + C_3 \frac{\delta_l^3 v_2}{2\omega_2^2 \omega_3^2 \delta_t \omega_4} \frac{\partial^3 \rho}{\partial x_1^3 \partial x_2} + \\ & (-6\omega_2 \omega_3^2 + 6\omega_2^2 + \omega_2^2 \omega_3^2 + 6\omega_3^2 - 6\omega_2^2 \omega_3) \frac{\delta_l^3 \rho v_1}{6\omega_2^2 \omega_3^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3 \partial x_2} + \\ & (\omega_2^2 v_1^2 \omega_4 + 6\omega_2^2 c_s^2 - 3\omega_2^2 c_s^2 \omega_4 + 6\omega_2^2 v_1^2 - 12\omega_2 v_1^2 + 12v_1^2 \omega_4 + 18\omega_2 c_s^2 \omega_4 - 12\omega_2 c_s^2 - 12c_s^2 \omega_4 - 6\omega_2 v_1^2 \omega_4) \frac{\delta_l^3 \rho}{12\omega_2^2 \delta_t \omega_4} \frac{\partial^3 v_2}{\partial x_1^3 \partial x_2} \\ & + (12 - 6\omega_5 + \omega_5 \omega_3 - 6\omega_3) \frac{\delta_l^2 v_2 \rho}{6\omega_5 \omega_3} \frac{\partial^3 v_2}{\partial t \partial x_2^2} + C_4 \frac{\delta_l^3 v_1}{2\omega_5 \omega_2^2 \omega_3^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + \\ & (-12\omega_5 c_s^2 + 6\omega_3^2 c_s^2 - 3\omega_5 \omega_3^2 c_s^2 + \omega_5 v_2^2 \omega_3^2 + 6v_2^2 \omega_3^2 - 12v_2^2 \omega_3 + 12\omega_5 v_2^2 \omega_3 - 6\omega_5 v_2^2 \omega_3 - 12\omega_3 c_s^2) \frac{\delta_l^3 \rho}{12\omega_5 \omega_3^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} \\ & + (-6\omega_2 \omega_3^2 + 6\omega_2^2 + \omega_2^2 \omega_3^2 + 6\omega_3^2 - 6\omega_2^2 \omega_3) \frac{\delta_l^3 v_2 \rho v_1}{6\omega_2^2 \omega_3^2 \delta_t} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_5 \frac{\delta_l^3 v_2}{6\omega_5 \omega_3^2 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + C_6 \frac{\delta_l^3 \rho}{12\omega_5 \omega_3^2 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & (-2 + 3\omega_2 - \omega_2^2) \frac{\delta_l \delta_t \rho}{2\omega_2^2} \frac{\partial^4 v_1}{\partial t^3 \partial x_1} + (-2\omega_2^3 \omega_4 - \omega_2^2 \omega_4^2 + 2\omega_2^2 + 8\omega_2^2 \omega_4 + 2\omega_4^2 - 4\omega_2^2 - 4\omega_2 \omega_4 - \omega_2 \omega_4^2) \frac{\delta_l^2 \delta_t \rho v_1}{2\omega_2^2 \omega_4^2} \frac{\partial^4 v_1}{\partial t^2 \partial x_1^2} + \\ & C_7 \frac{\delta_l^3 \rho}{12\omega_2^2 \omega_4^2} \frac{\partial^4 v_1}{\partial t \partial x_1^3} + C_8 \frac{\delta_l^4}{24\omega_2^3 \delta_t \omega_4^2} \frac{\partial^4 \rho}{\partial x_1^4} + C_9 \frac{\delta_l^4 \rho v_1}{12\omega_2^3 \delta_t \omega_4^2} \frac{\partial^4 v_1}{\partial x_1^4} + (-2 + 3\omega_3 - \omega_3^2) \frac{\delta_l \delta_t \rho}{2\omega_3^2} \frac{\partial^4 v_2}{\partial t^3 \partial x_2} + \\ & (12\omega_2 \omega_3 - 24\omega_2 \omega_3^2 + 12\omega_2^2 + 7\omega_2 \omega_3^3 - \omega_2^2 \omega_3^3 + 13\omega_2^2 \omega_3^2 + 12\omega_3^2 - 24\omega_2^2 \omega_3 - 6\omega_3^3) \frac{\delta_l^2 v_2 \delta_t \rho}{12\omega_2^2 \omega_3^3} \frac{\partial^4 v_1}{\partial t^2 \partial x_1 \partial x_2} + \\ & (12\omega_2 \omega_3 - 24\omega_2 \omega_3^2 - 6\omega_3^2 + 12\omega_2^2 + 7\omega_2^2 \omega_3 + 13\omega_2^2 \omega_3^2 + 12\omega_3^2 - 24\omega_2^2 \omega_3 - \omega_2^2 \omega_3^2) \frac{\delta_l^2 \delta_t \rho v_1}{12\omega_2^2 \omega_3^3} \frac{\partial^4 v_2}{\partial t^2 \partial x_1 \partial x_2} + \\ & C_{10} \frac{\delta_l^3 v_2 \rho v_1}{6\omega_2^2 \omega_3^3 \omega_4} \frac{\partial^4 v_1}{\partial t \partial x_1^2 \partial x_2} + C_{11} \frac{\delta_l^3 \rho}{12\omega_2^2 \omega_3^3 \omega_4} \frac{\partial^4 v_2}{\partial t \partial x_1^2 \partial x_2} + C_{12} \frac{\delta_l^4 v_2 v_1}{6\omega_2^2 \omega_3^3 \delta_t \omega_4} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{13} \frac{\delta_l^4 v_2 \rho}{12\omega_2^2 \omega_3^3 \delta_t \omega_4} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + \\ & C_{14} \frac{\delta_l^4 \rho v_1}{12\omega_2^2 \delta_t \omega_4^2} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + (-\omega_5^2 \omega_3^2 + 2\omega_5^2 - \omega_5^2 \omega_3 - 4\omega_5 \omega_3 - 4\omega_3^2 - 2\omega_5 \omega_3^2 + 2\omega_3^2 + 8\omega_5 \omega_3^2) \frac{\delta_l^2 v_2 \delta_t \rho}{2\omega_5^2 \omega_3^3} \frac{\partial^4 v_2}{\partial t^2 \partial x_2^2} + \\ & C_{15} \frac{\delta_l^3 \rho}{12\omega_5^2 \omega_2 \omega_3^3} \frac{\partial^4 v_1}{\partial t \partial x_1 \partial x_2^2} + C_{16} \frac{\delta_l^3 v_2 \rho v_1}{6\omega_5 \omega_2 \omega_3^3} \frac{\partial^4 v_2}{\partial t \partial x_1 \partial x_2^2} + C_{17} \frac{\delta_l^4}{4\omega_5^2 \omega_2^2 \omega_3^3 \delta_t \omega_4^2} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_{18} \frac{\delta_l^4 \rho v_1}{12\omega_5^2 \omega_2^2 \omega_3^3 \delta_t} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + \\ & C_{19} \frac{\delta_l^4 v_2 \rho}{12\omega_2^2 \omega_3^3 \delta_t \omega_4^2} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_{20} \frac{\delta_l^4 \rho}{12\omega_2^2 \omega_3^3} \frac{\partial^4 v_2}{\partial t \partial x_2^3} + C_{21} \frac{\delta_l^4 v_2 v_1}{6\omega_5^2 \omega_2^2 \omega_3^3 \delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^2} + C_{22} \frac{\delta_l^4 v_2 \rho}{12\omega_5^2 \omega_3^3 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^2} + C_{23} \frac{\delta_l^4 \rho v_1}{12\omega_5^2 \omega_2^2 \omega_3^3 \delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^2} \\ & + C_{24} \frac{\delta_l^4}{24\omega_5^2 \omega_3^3 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{25} \frac{\delta_l^4 v_2 \rho}{12\omega_5^2 \omega_3^3 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:

$$\begin{aligned}
C_1 &= -\omega_2^2 v_1^2 \omega_4 + \omega_2^2 \omega_4 + 3\omega_2^2 c_s^2 - 3\omega_2^2 c_s^2 \omega_4 + 6\omega_2 - 3\omega_2^2 + 3\omega_2^2 v_1^2 - 6\omega_2 v_1^2 - 3\omega_2 \omega_4 + 15\omega_2 c_s^2 \omega_4 - 6\omega_2 c_s^2 - 12c_s^2 \omega_4 + 3\omega_2 v_1^2 \omega_4 \\
C_2 &= -5\omega_2^2 v_1^2 \omega_4 + 2\omega_2^2 \omega_4 + 6\omega_2^2 c_s^2 - 3\omega_2^2 c_s^2 \omega_4 + 12\omega_2 - 6\omega_2^2 + 6\omega_2^2 v_1^2 - 12\omega_2 v_1^2 - 12v_1^2 \omega_4 - 6\omega_2 \omega_4 + 18\omega_2 c_s^2 \omega_4 - 12\omega_2 c_s^2 - 12c_s^2 \omega_4 + 18\omega_2 v_1^2 \omega_4 \\
C_3 &= \omega_2^2 \omega_3 c_s^2 \omega_4 - 4\omega_2 \omega_3^2 v_1^2 \omega_4 + 2\omega_2^2 v_1^2 \omega_4 - 3\omega_2^2 \omega_3 v_1^2 \omega_4 + 4\omega_2 \omega_3^2 c_s^2 \omega_4 - 2\omega_2 \omega_3^2 c_s^2 - 2\omega_2 \omega_3^2 v_1^2 + \omega_2^2 \omega_3^2 c_s^2 - 2\omega_2 \omega_3 c_s^2 \omega_4 + \omega_2^2 \omega_3^2 v_1^2 \omega_4 + \\
&\omega_2^2 \omega_3^2 v_1^2 + 4\omega_3^2 v_1^2 \omega_4 + 2\omega_2 \omega_3 v_1^2 \omega_4 - \omega_2^2 \omega_3^2 c_s^2 \omega_4 - 2\omega_3^2 c_s^2 \omega_4 \\
C_4 &= \omega_5 \omega_2 \omega_3^2 c_s^2 - 4\omega_5 \omega_2^2 v_2^2 \omega_3 + 4\omega_5 \omega_2^2 \omega_3 c_s^2 + \omega_2^2 v_2^2 \omega_3^2 - 2\omega_2^2 v_2^2 \omega_3 + \omega_5 \omega_2^2 v_2^2 \omega_3^2 - 2\omega_2^2 \omega_3 c_s^2 - 2\omega_5 \omega_2^2 c_s^2 + \omega_2^2 \omega_3^2 c_s^2 + 2\omega_5 v_2^2 \omega_3^2 - 3\omega_5 \omega_2 v_2^2 \omega_3^2 + \\
&4\omega_5 \omega_2^2 v_2^2 - \omega_5 \omega_2^2 \omega_3^2 c_s^2 + 2\omega_5 \omega_2 v_2^2 \omega_3 - 2\omega_5 \omega_2 \omega_3 c_s^2 \\
C_5 &= -12\omega_5 c_s^2 + 3\omega_3^2 c_s^2 - 3\omega_5 \omega_3^2 c_s^2 - 3\omega_5 \omega_3 - \omega_5 v_2^2 \omega_3^2 + 6\omega_3 + 3v_2^2 \omega_3^2 - 3\omega_3^2 - 6v_2^2 \omega_3 + 15\omega_5 \omega_3 c_s^2 + 3\omega_5 v_2^2 \omega_3 - 6\omega_3 c_s^2 + \omega_5 \omega_3^2 \\
C_6 &= -12\omega_5 c_s^2 + 6\omega_3^2 c_s^2 - 3\omega_5 \omega_3^2 c_s^2 - 6\omega_5 \omega_3 - 5\omega_5 v_2^2 \omega_3^2 + 12\omega_3 + 6v_2^2 \omega_3^2 - 6\omega_3^2 - 12v_2^2 \omega_3 - 12\omega_5 v_2^2 + 18\omega_5 \omega_3 c_s^2 + 18\omega_5 v_2^2 \omega_3 - 12\omega_3 c_s^2 + 2\omega_5 \omega_3^2 \\
C_7 &= -6\omega_3^2 c_s^2 + 25\omega_2^2 c_s^2 \omega_4^2 - 9\omega_3^2 \omega_4 - 6\omega_3^2 v_1^2 - 60\omega_2^2 v_1^2 \omega_4 - 11\omega_2^2 \omega_4 + 27\omega_2^2 v_1^2 \omega_4^2 + 6\omega_3^2 + 36\omega_2^2 \omega_4 + 12\omega_2^2 c_s^2 - 36\omega_2^2 c_s^2 \omega_4 + \omega_3^2 \omega_4^2 - 12\omega_2^2 + \\
&12\omega_2^2 v_1^2 + 15\omega_2^2 v_1^2 \omega_4 + 24c_s^2 \omega_4^2 - 42\omega_2 v_1^2 \omega_4^2 - 2\omega_2^2 c_s^2 \omega_4^2 - 24\omega_2 \omega_4 + 24\omega_2 c_s^2 \omega_4 + 9\omega_2^2 c_s^2 \omega_4 + 12\omega_2 \omega_4^2 + 12v_1^2 \omega_4^2 - 48\omega_2 c_s^2 \omega_4^2 - 3\omega_3^2 v_1^2 \omega_4^2 + 48\omega_2 v_1^2 \omega_4 \\
C_8 &= -3\omega_3^2 c_s^4 \omega_4^2 - 12\omega_2^2 c_s^2 v_1^2 \omega_4 + 24\omega_2 c_s^4 \omega_4 - 8\omega_2^2 c_s^2 \omega_4^2 - 72\omega_2^2 c_s^2 v_1^2 \omega_4^2 - 18\omega_2^2 v_1^4 \omega_4 - 12\omega_3^2 v_1^2 + 24c_s^4 \omega_4^2 + 24\omega_2 v_1^4 \omega_4^2 - 96c_s^2 v_1^2 \omega_4^2 - \\
&72\omega_2^2 v_1^2 \omega_4 + 24\omega_2^2 v_1^2 \omega_4^2 + 3\omega_3^2 v_1^4 \omega_4^2 - 24\omega_2^2 c_s^2 v_1^2 - 48\omega_2 v_1^4 \omega_4 + 24\omega_2^2 c_s^2 \omega_4 + 48\omega_2^2 c_s^2 v_1^2 \omega_4 + 24\omega_2^2 v_1^2 + 6\omega_3^2 c_s^2 v_1^2 \omega_4^2 + 6\omega_3^2 c_s^4 \omega_4 - 48\omega_2 c_s^4 \omega_4^2 + \\
&156\omega_2 c_s^2 v_1^2 \omega_4^2 + 18\omega_3^2 v_1^2 \omega_4 - 24\omega_2 v_1^2 \omega_4^2 + 72\omega_2^2 v_1^4 \omega_4 - 24\omega_2^2 v_1^4 + \omega_2^2 c_s^2 \omega_4^2 - 24\omega_2 c_s^2 \omega_4 + 24\omega_2^2 c_s^4 \omega_4^2 - 24\omega_2^2 c_s^4 \omega_4 - 6\omega_3^2 c_s^2 \omega_4 + 12\omega_2 c_s^2 \omega_4^2 - \\
&24\omega_2^2 v_1^4 \omega_4^2 + 12\omega_2^2 c_s^2 v_1^2 - 24\omega_2 c_s^2 v_1^2 \omega_4 - 3\omega_3^2 v_1^2 \omega_4^2 + 12\omega_3^2 v_1^4 + 48\omega_2 v_1^2 \omega_4 \\
C_9 &= 6\omega_2^2 c_s^2 - 20\omega_2^2 c_s^2 \omega_4^2 + 6\omega_3^2 \omega_4 + 6\omega_2^2 v_1^2 + 24\omega_2^2 v_1^2 \omega_4 + 8\omega_2^2 \omega_4^2 - 16\omega_2^2 v_1^2 \omega_4^2 - 6\omega_3^2 - 24\omega_2^2 \omega_4 - 12\omega_2^2 c_s^2 + 24\omega_2^2 c_s^2 \omega_4 - \omega_3^2 \omega_4^2 + 12\omega_2^2 - \\
&12\omega_2^2 v_1^2 - 6\omega_2^2 v_1^2 \omega_4 - 24c_s^2 \omega_4^2 + 24\omega_2 v_1^2 \omega_4^2 + \omega_2^2 c_s^2 \omega_4^2 + 12\omega_2 \omega_4 - 12\omega_2 c_s^2 \omega_4 - 6\omega_2^2 c_s^2 \omega_4 - 6\omega_2 \omega_4^2 - 12v_1^2 \omega_4^2 + 42\omega_2 c_s^2 \omega_4^2 + 2\omega_2^2 v_1^2 \omega_4^2 - 12\omega_2 v_1^2 \omega_4 \\
C_{10} &= 24\omega_2 \omega_3^2 \omega_4 - 6\omega_3^2 \omega_4 - 7\omega_2^2 \omega_3^2 \omega_4 - 12\omega_3^2 \omega_4 - 12\omega_2 \omega_3^2 \omega_4 - 6\omega_2^2 \omega_3 \omega_4 + \omega_2^2 \omega_3^2 \omega_4 - 6\omega_2^2 \omega_3^2 + 12\omega_2^2 \omega_3^2 \omega_4 + 12\omega_2^2 \omega_3^2 + 12\omega_2^2 \omega_3 \omega_4 - \\
&10\omega_2^2 \omega_3^2 \omega_4 + 3\omega_2^2 \omega_3^2 - 6\omega_3^2 \omega_3^2 \\
C_{11} &= -24\omega_3 v_1^2 \omega_4^2 - 2\omega_2^2 \omega_3 c_s^2 \omega_4^2 - 30\omega_2^2 \omega_3 c_s^2 \omega_4 - 18\omega_2^2 c_s^2 \omega_4^2 + 9\omega_2^2 \omega_3 v_1^2 \omega_4 - 10\omega_2^2 \omega_3 v_1^2 \omega_4^2 + 12\omega_2^2 v_1^2 \omega_4 + 6\omega_2^2 v_1^2 \omega_4^2 - 30\omega_2^2 \omega_3 v_1^2 \omega_4 + \\
&12\omega_2^2 \omega_3 v_1^2 + 12\omega_3 c_s^2 \omega_4^2 + \omega_2^2 \omega_3 v_1^2 \omega_4^2 + 12\omega_2^2 \omega_3 c_s^2 + 12\omega_2^2 c_s^2 \omega_4 + 22\omega_2^2 \omega_3 c_s^2 \omega_4^2 + 9\omega_2^2 \omega_3 c_s^2 \omega_4 - 6\omega_3^2 v_1^2 \omega_4 + 12\omega_2 \omega_3 c_s^2 \omega_4 - 12\omega_2 v_1^2 \omega_4^2 + \\
&3\omega_2^2 c_s^2 \omega_4^2 + 36\omega_2 \omega_3 v_1^2 \omega_4^2 - 6\omega_2^2 c_s^2 \omega_4 - 6\omega_2^2 \omega_3 c_s^2 + 12\omega_2 \omega_3 v_1^2 \omega_4 + 12\omega_2 c_s^2 \omega_4^2 - \omega_2^2 v_1^2 \omega_4^2 - 30\omega_2 \omega_3 c_s^2 \omega_4^2 - 6\omega_3^2 \omega_3 v_1^2 \\
C_{12} &= 12\omega_2 \omega_3^2 \omega_4 - 12\omega_2^2 \omega_3^2 c_s^2 \omega_4 - \omega_2^2 \omega_3^2 \omega_4^2 + 6\omega_2^2 \omega_3 c_s^2 \omega_4 - 48\omega_2^2 \omega_3^2 c_s^2 \omega_4^2 - 3\omega_2^2 \omega_3^2 \omega_4 - 36\omega_2^2 c_s^2 \omega_4^2 + 6\omega_2^2 \omega_3 v_1^2 \omega_4^2 - 24\omega_2 \omega_3^2 c_s^2 \omega_4^2 + \\
&42\omega_2^2 \omega_3^2 v_1^2 \omega_4 + 6\omega_3^2 \omega_3^2 v_1^2 + 6\omega_2^2 \omega_3^2 v_1^2 \omega_4^2 - 12\omega_2^2 \omega_3^2 v_1^2 \omega_4 + \omega_2^2 \omega_3^2 \omega_4^2 - 12\omega_2^2 \omega_3 v_1^2 \omega_4^2 - 12\omega_2^2 \omega_3 c_s^2 \omega_4^2 + 12\omega_2 \omega_3^2 v_1^2 \omega_4^2 + 6\omega_3^2 \omega_3^2 c_s^2 + 42\omega_2^2 \omega_3^2 c_s^2 \omega_4 + \\
&6\omega_2^2 \omega_3^2 c_s^2 \omega_4^2 - 6\omega_2 \omega_3^2 \omega_4^2 + 24\omega_2^2 v_1^2 \omega_4^2 + 6\omega_2^2 \omega_3^2 \omega_4 + 78\omega_2 \omega_3^2 c_s^2 \omega_4^2 + 6\omega_2^2 \omega_3^2 - 12\omega_2^2 \omega_3^2 v_1^2 \omega_4 + 6\omega_2^2 \omega_3^2 v_1^2 \omega_4^2 + 6\omega_2^2 \omega_3^2 \omega_4 - 24\omega_2 \omega_3^2 v_1^2 \omega_4 + \\
&42\omega_2^2 \omega_3^2 c_s^2 \omega_4^2 + 6\omega_2^2 \omega_3^2 c_s^2 \omega_4 + 7\omega_2^2 \omega_3^2 \omega_4^2 - 12\omega_2^2 \omega_3^2 c_s^2 \omega_4^2 - 21\omega_2^2 \omega_3^2 \omega_4 - 30\omega_2 \omega_3^2 v_1^2 \omega_4^2 - 3\omega_2^2 \omega_3^2 - 12\omega_2^2 \omega_3^2 c_s^2 \omega_4 - 12\omega_2^2 \omega_3^2 c_s^2 - 3\omega_2^2 \omega_3^2 \omega_4^2 + \\
&6\omega_2^2 \omega_3^2 v_1^2 \omega_4 - 24\omega_2 \omega_3^2 c_s^2 \omega_4 - 12\omega_2^2 \omega_3^2 v_1^2 + 6\omega_2^2 v_1^2 \omega_4^2 - 12\omega_2^2 \omega_3^2 v_1^2 \omega_4 \\
C_{13} &= -12\omega_2^2 \omega_3^2 c_s^2 \omega_4 - \omega_2^2 \omega_3^2 \omega_4^2 + 6\omega_2^2 \omega_3 c_s^2 \omega_4^2 - 32\omega_2^2 \omega_3^2 c_s^2 \omega_4^2 - 6\omega_2^2 \omega_3^2 \omega_4 + 3\omega_2^2 \omega_3^2 v_1^2 \omega_4^2 - 12\omega_3^2 c_s^2 \omega_2^2 - 24\omega_2 \omega_3^2 c_s^2 \omega_4^2 + 36\omega_2^2 \omega_3^2 v_1^2 \omega_4 + \\
&6\omega_2^2 \omega_3^2 v_1^2 - 12\omega_2^2 \omega_3^2 v_1^2 \omega_4 + 2\omega_2^2 \omega_3^2 \omega_4^2 - 18\omega_2^2 \omega_3 v_1^2 \omega_4^2 - 12\omega_2^2 \omega_3 c_s^2 \omega_4^2 + 6\omega_2^2 \omega_3^2 c_s^2 + 36\omega_2^2 \omega_3^2 c_s^2 \omega_4 + 4\omega_2^2 \omega_3^2 c_s^2 \omega_4^2 + 24\omega_3^2 v_1^2 \omega_4^2 + 3\omega_2^2 \omega_3^2 \omega_4 + \\
&36\omega_2 \omega_3^2 c_s^2 \omega_4^2 - 24\omega_2^2 \omega_3^2 v_1^2 \omega_4 + 12\omega_2^2 \omega_3^2 \omega_4 - 12\omega_2 \omega_3^2 v_1^2 \omega_4 + 48\omega_2^2 \omega_3^2 c_s^2 \omega_4^2 + 12\omega_2^2 \omega_3^2 c_s^2 \omega_4 + 3\omega_2^2 \omega_3^2 \omega_4^2 - 12\omega_2^2 \omega_3^2 c_s^2 \omega_4^2 - 6\omega_2^2 \omega_3^2 \omega_4 - \\
&30\omega_2 \omega_3^2 v_1^2 \omega_4 - 24\omega_2^2 \omega_3^2 c_s^2 \omega_4 - 12\omega_2^2 \omega_3^2 c_s^2 - 6\omega_2^2 \omega_3^2 \omega_4^2 + 12\omega_2^2 \omega_3^2 v_1^2 \omega_4 - 12\omega_2 \omega_3^2 c_s^2 \omega_4 - 12\omega_2^2 \omega_3^2 v_1^2 + 12\omega_3^2 v_1^2 \omega_4^2 + 12\omega_2^2 \omega_3^2 v_1^2 \omega_4 \\
C_{14} &= 6\omega_2^2 c_s^2 - 44\omega_2^2 c_s^2 \omega_4^2 + 9\omega_2^2 \omega_4 + 6\omega_3^2 v_1^2 + 48\omega_2^2 v_1^2 \omega_4 + 11\omega_2^2 \omega_4^2 - 8\omega_2^2 v_1^2 \omega_4^2 - 6\omega_3^2 - 36\omega_2^2 \omega_4 - 12\omega_2^2 c_s^2 + 48\omega_2^2 c_s^2 \omega_4 - \omega_3^2 \omega_4^2 + 12\omega_2^2 - \\
&12\omega_2^2 v_1^2 - 12\omega_2^2 v_1^2 \omega_4 - 48c_s^2 \omega_4^2 + 4\omega_2^2 c_s^2 \omega_4^2 + 24\omega_2 \omega_4 - 36\omega_2 c_s^2 \omega_4 - 12\omega_2^2 c_s^2 \omega_4 - 12\omega_2 \omega_4^2 + 12v_1^2 \omega_4^2 + 90\omega_2 c_s^2 \omega_4^2 + \omega_2^2 v_1^2 \omega_4^2 - 36\omega_2 v_1^2 \omega_4 \\
C_{15} &= -30\omega_5 \omega_2 \omega_3^2 c_s^2 - 6\omega_2 \omega_3^2 c_s^2 + 12\omega_5^2 \omega_3 c_s^2 + \omega_2^2 \omega_2 v_2^2 \omega_3^2 - 12\omega_2^2 v_2^2 \omega_3 - 10\omega_5^2 \omega_2 v_2^2 \omega_3^2 + 12\omega_5 \omega_2^2 c_s^2 + 36\omega_5^2 \omega_2 v_2^2 \omega_3 + 12\omega_2 \omega_3^2 c_s^2 + 6\omega_5^2 v_2^2 \omega_3^2 + \\
&12\omega_2^2 \omega_2 c_s^2 + 9\omega_5 \omega_2 \omega_3^2 c_s^2 - 6\omega_5 \omega_3^2 c_s^2 - 30\omega_5^2 \omega_2 \omega_3 c_s^2 - \omega_5^2 v_2^2 \omega_3^2 + 9\omega_5 \omega_2 v_2^2 \omega_3^2 - 24\omega_2^2 \omega_2 v_2^2 + 22\omega_5^2 \omega_2 \omega_3^2 c_s^2 + 12\omega_5 v_2^2 \omega_3^2 + 3\omega_5^2 \omega_3^2 c_s^2 - \\
&30\omega_5 \omega_2 v_2^2 \omega_3^2 - 6\omega_5 v_2^2 \omega_3^2 - 6\omega_2 v_2^2 \omega_3^2 - 2\omega_5^2 \omega_2 \omega_3^2 c_s^2 + 12\omega_5 \omega_2 v_2^2 \omega_3 + 12\omega_5 \omega_2 \omega_3 c_s^2 + 12\omega_2 v_2^2 \omega_3^2 - 18\omega_5^2 \omega_3^2 c_s^2 \\
C_{16} &= 12\omega_5 \omega_2 \omega_3^2 - 6\omega_5 \omega_2 \omega_3^2 - 12\omega_5 \omega_2^2 + 12\omega_5 \omega_2^2 \omega_3^2 - 6\omega_2^2 \omega_3^2 + 12\omega_2^2 \omega_3^2 + 24\omega_5 \omega_2^2 \omega_3 - 7\omega_5 \omega_2^2 \omega_3^2 - 6\omega_5 \omega_3^2 + 3\omega_2^2 \omega_3^2 - 10\omega_5 \omega_2^2 \omega_3^2 - \\
&12\omega_5 \omega_2^2 \omega_3 + \omega_5 \omega_2^2 \omega_3^2 - 6\omega_3^2 \omega_3^2 \\
C_{17} &= 4\omega_5^2 \omega_2^2 \omega_3^2 c_s^4 \omega_4^2 + 2\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 c_s^2 - 8\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 - 2\omega_5^2 \omega_2^2 \omega_3^2 c_s^2 v_1^2 \omega_4 + 10\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_4 - 4\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 + 12\omega_5^2 \omega_2^2 \omega_3^2 c_s^2 v_1^2 \omega_4^2 - \\
&2\omega_5^2 \omega_2^2 \omega_3^2 c_s^4 \omega_4^2 - 2\omega_5 \omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 + 2\omega_5 \omega_2^2 \omega_3^2 c_s^2 v_1^2 \omega_4^2 - 38\omega_5^2 \omega_2 v_2^2 \omega_3^2 v_1^2 \omega_4^2 + \omega_5^2 \omega_2^2 \omega_3^2 c_s^4 \omega_4 - 8\omega_5^2 \omega_2 \omega_3^2 c_s^2 v_1^2 \omega_4^2 + 10\omega_5 \omega_3^2 v_2^2 \omega_3^2 v_1^2 \omega_4^2 + \\
&2\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 v_1^2 + 4\omega_5^2 \omega_2 \omega_3^2 c_s^4 \omega_4^2 + 20\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 v_1^2 \omega_4^2 + 2\omega_5 \omega_3^2 v_2^2 \omega_3^2 v_1^2 \omega_4 - 2\omega_5 \omega_2^2 \omega_3^2 c_s^4 \omega_4^2 - 4\omega_3^2 v_2^2 \omega_3^2 v_1^2 \omega_4^2 - 4\omega_5 \omega_3^2 \omega_2 c_s^2 v_1^2 \omega_4^2 - 4\omega_5^2 \omega_2 v_2^2 \omega_3^2 c_s^2 \omega_4 + \\
&20\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 v_1^2 \omega_4^2 - 4\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 v_1^2 + 2\omega_5 \omega_2^2 v_2^2 \omega_3^2 v_1^2 \omega_4^2 + 4\omega_5^2 \omega_2 \omega_3^2 c_s^2 v_1^2 \omega_4^2 + 10\omega_5^2 \omega_2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 + 10\omega_5^2 \omega_2^2 \omega_3^2 \omega_2 c_s^2 v_1^2 \omega_4^2 + 20\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 v_1^2 \omega_4^2 + \\
&\omega_5 \omega_3^2 \omega_3^2 c_s^4 \omega_4^2 + 10\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 v_1^2 \omega_4 + 4\omega_5^2 \omega_2^2 c_s^2 v_1^2 \omega_4^2 + 2\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_4 - 4\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 c_s^2 - \omega_5^2 \omega_3^2 \omega_3^2 c_s^4 \omega_4^2 - 4\omega_5^2 \omega_2 v_2^2 \omega_3^2 v_1^2 \omega_4 - 2\omega_5^2 \omega_2^2 \omega_3^2 c_s^4 \omega_4 - \\
&2\omega_5 \omega_3^2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 - 4\omega_5 \omega_2^2 \omega_3^2 c_s^2 v_1^2 \omega_4^2 + 4\omega_5^2 \omega_2^2 \omega_3^2 c_s^4 \omega_4^2 - 8\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 - 4\omega_5^2 \omega_2^2 \omega_3^2 c_s^2 v_1^2 \omega_4^2 + 2\omega_3^2 v_2^2 \omega_3^2 v_1^2 \omega_4^2 - 3\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 v_1^2 \omega_4 + \\
&4\omega_5 \omega_2^2 \omega_3^2 c_s^4 \omega_4^2 - 4\omega_5^2 \omega_2^2 \omega_3^2 c_s^2 v_1^2 \omega_4^2 + 20\omega_5^2 v_2^2 \omega_3^2 v_1^2 \omega_4^2 - 38\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 v_1^2 \omega_4^2 - 3\omega_5 \omega_3^2 v_2^2 \omega_3^2 v_1^2 \omega_4^2 - 36\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 v_1^2 \omega_4^2 - 2\omega_5^2 \omega_2 \omega_3^2 c_s^4 \omega_4^2 + \\
&20\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 c_s^2 v_1^2 \omega_4^2 + \omega_5^2 \omega_2^2 \omega_3^2 c_s^2 v_1^2 \omega_4^2 + \omega_5^2 \omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 + 20\omega_5^2 \omega_2 v_2^2 \omega_3^2 v_1^2 \omega_4^2 - 2\omega_5^2 \omega_2^2 \omega_3^2 c_s^4 \omega_4 + 10\omega_5 \omega_3^2 \omega_3^2 c_s^2 v_1^2 \omega_4^2 + 4\omega_5 \omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 - \\
&12\omega_5^2 \omega_2^2 \omega_3^2 c_s^4 \omega_4^2 - 4\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_4 - 4\omega_3^2 \omega_3^2 c_s^2 v_1^2 \omega_4^2 - 2\omega_5^2 \omega_2^2 \omega_3^2 c_s^2 v_1^2 \omega_4 - 3\omega_5 \omega_2^2 \omega_3^2 c_s^2 v_1^2 \omega_4^2 + \omega_5 \omega_3^2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 + 4\omega_5^2 \omega_2^2 \omega_3^2 c_s^4 \omega_4 - \\
&8\omega_5^2 \omega_2^2 \omega_3^2 c_s^2 v_1^2 \omega_4^2 + 12\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 - 4\omega_5^2 \omega_3^2 c_s^2 v_1^2 \omega_4^2 + 2\omega_3^2 \omega_3^2 c_s^2 v_1^2 \omega_4^2 - 3\omega_5^2 \omega_3^2 v_2^2 \omega_3^2 c_s^2 \omega_4 + \omega_5^2 \omega_2^2 \omega_3^2 c_s^2 v_1^2 \omega_4 + 4\omega_5^2 \omega_2^2 \omega_3^2 c_s^4 \omega_4^2 - 4\omega_5^2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 + \\
&4\omega_5^2 \omega_3^2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 - 2\omega_5 \omega_2^2 \omega_3^2 c_s^4 \omega_4^2 - 4\omega_5 \omega_2^2 v_2^2 \omega_3^2 v_1^2 \omega_4^2 - 4\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 v_1^2 \omega_4 - 3\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 v_1^2 \omega_4^2 - 4\omega_5^2 \omega_2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 - 4\omega_5 \omega_2^2 v_2^2 \omega_3^2 v_1^2 \omega_4^2
\end{aligned}$$

$$\begin{aligned}
C_{18} &= -6\omega_5^2\omega_2^3\omega_3^2c_s^2 + 24\omega_5^2\omega_2^3\omega_3c_s^2 - 30\omega_5^2\omega_2v_2^2\omega_3^3 - 6\omega_5\omega_2^3\omega_3^3c_s^2 - 12\omega_3^3\omega_2^3c_s^2 + 24\omega_5^2\omega_2v_2^2\omega_3^2 + 48\omega_5^2\omega_2^3v_2^2 - 78\omega_5^2\omega_2^3v_2^2\omega_3 + 34\omega_5^2\omega_2^3v_2^2\omega_3^2 + \\
&12\omega_5^2\omega_2^3\omega_3^2c_s^2 - 4\omega_5^2\omega_2^3v_2^2\omega_3^3 + 6\omega_3^3\omega_2^3c_s^2 + 24\omega_5\omega_2^3\omega_3^2c_s^2 + 12\omega_5^2v_2^2\omega_3^3 + 24\omega_5^2\omega_2^3v_2^2\omega_3 - 12\omega_5\omega_2^3\omega_3c_s^2 - 12\omega_3^3v_2^2\omega_3^2 - 12\omega_5^2\omega_2\omega_3^2c_s^2 - \\
&12\omega_5\omega_2^3v_2^2\omega_3 + \omega_5^2\omega_2^3\omega_3^2c_s^2 + 6\omega_3^3v_2^2\omega_3^2 + 6\omega_5^2\omega_2\omega_3^2c_s^2 + 22\omega_5^2\omega_2^3v_2^2\omega_3^2 + 24\omega_5\omega_2^3v_2^2\omega_3 - 12\omega_5^2\omega_2^3c_s^2 - 14\omega_5^2\omega_2^3\omega_3^2c_s^2 - 48\omega_5^2\omega_2^3v_2^2\omega_3^2 - 6\omega_5\omega_2^3v_2^2\omega_3^3 \\
C_{19} &= -6\omega_5^2\omega_3^3c_s^2\omega_4 + 6\omega_5^2\omega_3c_s^2\omega_4^2 - 14\omega_5^2\omega_3^3c_s^2\omega_4^2 - 4\omega_5^2\omega_3^3v_1^2\omega_4^2 - 12\omega_3^3c_s^2\omega_4^2 + 24\omega_5^2\omega_3v_1^2\omega_4^2 + 24\omega_5^2\omega_3^3v_1^2\omega_4 + 6\omega_3^3\omega_3^3v_1^2 + 34\omega_5^2\omega_3^3v_1^2\omega_4^2 - \\
&6\omega_5^2\omega_3^3v_1^2\omega_4 - 30\omega_5^2\omega_3v_1^2\omega_4^2 - 12\omega_5^2\omega_3^3c_s^2\omega_4^2 + 24\omega_5\omega_3^3v_1^2\omega_4^2 + 6\omega_5^2\omega_3^3c_s^2\omega_4^2 + 24\omega_5^2\omega_3^3c_s^2\omega_4 + \omega_3^3\omega_3^3c_s^2\omega_4^2 + 48\omega_3^3v_1^2\omega_4^2 + 24\omega_5\omega_3^3c_s^2\omega_4^2 + \\
&22\omega_5^2\omega_3^3v_1^2\omega_4^2 - 12\omega_5^2\omega_3^3v_1^2\omega_4 + 12\omega_5^2\omega_3^2c_s^2\omega_4^2 - 6\omega_5^2\omega_3^2c_s^2\omega_4^2 - 78\omega_5^2\omega_3^3v_1^2\omega_4^2 - 12\omega_5^2\omega_3^3c_s^2 - 12\omega_5\omega_3^3c_s^2\omega_4 - 12\omega_5^2\omega_3^3v_1^2 + 12\omega_5^2v_1^2\omega_4^2 - 48\omega_5^2\omega_2^3v_1^2\omega_4^2 \\
C_{20} &= -48\omega_5^2\omega_3c_s^2 - 11\omega_5^2\omega_3^2 + 12\omega_3^2c_s^2 - 42\omega_5^2v_2^2\omega_3 + \omega_5^2\omega_3^3 + 12\omega_5^2v_2^2 - 36\omega_5\omega_3^2c_s^2 - 6\omega_3^3c_s^2 + 27\omega_5^2v_2^2\omega_3^2 + 9\omega_5\omega_3^3c_s^2 + 12\omega_5^2\omega_3 - 3\omega_5^2v_2^2\omega_3^3 - \\
&24\omega_5\omega_3 - 6v_2^2\omega_3^3 - 60\omega_5v_2^2\omega_3^2 - 2\omega_5^2\omega_3^2c_s^2 + 15\omega_5v_2^2\omega_3^2 + 12v_2^2\omega_3^2 - 12\omega_3^2 - 9\omega_5\omega_3^2 + 24\omega_5\omega_3c_s^2 + 24\omega_5^2c_s^2 + 6\omega_3^3 + 48\omega_5v_2^2\omega_3 + 25\omega_5^2\omega_3^2c_s^2 + 36\omega_5\omega_3^2 \\
C_{21} &= -12\omega_5^2\omega_2^3\omega_3^2c_s^2 + 78\omega_5^2\omega_2^3\omega_3c_s^2 + 7\omega_5^2\omega_2^3\omega_3^2 - 12\omega_5^2\omega_2v_2^2\omega_3^3 - 12\omega_5\omega_2^3\omega_3^3c_s^2 - 12\omega_3^3\omega_2^3c_s^2 + 6\omega_5^2\omega_2v_2^2\omega_3^2 + 24\omega_5^2\omega_2^3v_2^2 - \omega_5^2\omega_2^3\omega_3^3 - \\
&30\omega_5^2\omega_2^3v_2^2\omega_3 + 6\omega_5\omega_2^3v_2^2\omega_3^2 - 3\omega_5^2\omega_2^3\omega_3^2 + 6\omega_5^2\omega_2^3v_2^2\omega_3^2 + 42\omega_5^2\omega_2^3\omega_3^2c_s^2 - 12\omega_5\omega_2^3v_2^2\omega_3^2 + \omega_5^2\omega_2^3\omega_3^3 - 6\omega_5^2\omega_2^3\omega_3 + 6\omega_3^3\omega_3^2c_s^2 + 42\omega_5\omega_2^3\omega_3^2c_s^2 + \\
&6\omega_5^2v_2^2\omega_3^3 + 6\omega_5\omega_2^3\omega_3^2 + 12\omega_5^2\omega_2^3v_2^2\omega_3 - 24\omega_5\omega_2^3\omega_3^2c_s^2 - 12\omega_3^3v_2^2\omega_3^2 - 12\omega_5^2\omega_2\omega_3^2c_s^2 + 6\omega_5\omega_2^3\omega_3^2c_s^2 - 24\omega_5\omega_2^3v_2^2\omega_3 + 12\omega_5\omega_2^3\omega_3 - 3\omega_5\omega_2^3\omega_3^3 + \\
&6\omega_5^2\omega_2^3\omega_3^2c_s^2 - 24\omega_5^2\omega_2\omega_3^2c_s^2 + 6\omega_5^2v_2^2\omega_3^3 - 12\omega_5\omega_2^3\omega_3^2c_s^2 + 6\omega_5^2\omega_2\omega_3^2c_s^2 + 6\omega_5^2\omega_2^3v_2^2\omega_3^2 + 42\omega_5\omega_2^3v_2^2\omega_3^2 - 3\omega_3^3\omega_3^3 - 21\omega_5\omega_2^3\omega_3^2 - 36\omega_5^2\omega_2^3c_s^2 - \\
&48\omega_5^2\omega_2^3\omega_3^2c_s^2 + 6\omega_5\omega_2^3\omega_3^2 - 12\omega_5^2\omega_2^3v_2^2\omega_3^2 + 6\omega_5^2\omega_3^2 - 12\omega_5\omega_2^3v_2^2\omega_3^2 \\
C_{22} &= 90\omega_5^2\omega_3c_s^2 + 11\omega_5^2\omega_3^2 - 12\omega_3^2c_s^2 - \omega_5^2\omega_3^3 + 12\omega_5^2v_2^2 + 48\omega_5\omega_3^2c_s^2 + 6\omega_3^3c_s^2 - 8\omega_5^2v_2^2\omega_3^2 - 12\omega_5\omega_3^2c_s^2 - 12\omega_5^2\omega_3 + \omega_5^2v_2^2\omega_3^3 + 24\omega_5\omega_3 + \\
&6v_2^2\omega_3^3 + 48\omega_5v_2^2\omega_3^2 + 4\omega_5^2\omega_3^2c_s^2 - 12\omega_5v_2^2\omega_3^2 - 12v_2^2\omega_3^2 + 12\omega_3^2 + 9\omega_5\omega_3^2 - 36\omega_5\omega_3c_s^2 - 48\omega_5^2c_s^2 - 6\omega_3^3 - 36\omega_5v_2^2\omega_3 - 44\omega_5^2\omega_3^2c_s^2 - 36\omega_5\omega_3^2 \\
C_{23} &= -12\omega_5^2\omega_2^3\omega_3^2c_s^2 + 36\omega_5^2\omega_2^3\omega_3c_s^2 + 3\omega_5^2\omega_2^3\omega_3^2 - 18\omega_5^2\omega_2v_2^2\omega_3^3 - 12\omega_5\omega_2^3\omega_3^3c_s^2 - 12\omega_3^3\omega_2^3c_s^2 + 24\omega_5^2\omega_2^3v_2^2 - \omega_5^2\omega_2^3\omega_3^3 - 30\omega_5^2\omega_2^3v_2^2\omega_3 + \\
&12\omega_5\omega_2^3v_2^2\omega_3^2 - 6\omega_5^2\omega_2^3\omega_3^2 + 48\omega_5^2\omega_2^3\omega_3^2c_s^2 - 24\omega_5\omega_2^3v_2^2\omega_3^2 + 3\omega_5^2\omega_2^3v_2^2\omega_3^2 + 2\omega_5^2\omega_2^3\omega_3^2 + 6\omega_3^3\omega_3^2c_s^2 + 36\omega_5\omega_2^3\omega_3^2c_s^2 + 12\omega_5^2v_2^2\omega_3^3 + 12\omega_5\omega_2^3\omega_3^2 - \\
&12\omega_5\omega_2^3\omega_3c_s^2 - 12\omega_3^3v_2^2\omega_3^2 - 12\omega_5^2\omega_2\omega_3^2c_s^2 + 12\omega_5\omega_2^3\omega_3^2c_s^2 - 12\omega_5\omega_2^3v_2^2\omega_3 - 6\omega_5\omega_2^3\omega_3^2 + 4\omega_5^2\omega_2^3\omega_3^2c_s^2 - 24\omega_5^2\omega_2^3\omega_3c_s^2 + 6\omega_3^3v_2^2\omega_3^2 - 24\omega_5\omega_2^3\omega_3^2c_s^2 + \\
&6\omega_5^2\omega_2\omega_3^2c_s^2 + 36\omega_5\omega_2^3v_2^2\omega_3^2 - 6\omega_5\omega_2^3\omega_3^2 - 12\omega_5^2\omega_2^3c_s^2 - 32\omega_5^2\omega_2^3\omega_3^2c_s^2 + 3\omega_5\omega_2^3\omega_3^2 + 12\omega_5^2\omega_2^3v_2^2\omega_3^2 - 12\omega_5\omega_2^3v_2^2\omega_3^2 \\
C_{24} &= 24\omega_5\omega_3c_s^4 - 18\omega_5v_2^4\omega_3^3 + 12\omega_5^2\omega_3c_s^2 - 24\omega_5v_2^3\omega_3c_s^2 - 24v_2^4\omega_3^4 - 24\omega_5^2v_2^2\omega_3 + 24\omega_5^2\omega_3^2c_s^4 + 12v_2^4\omega_3^3 + 24\omega_5^2c_s^4 + 6\omega_5^2v_2^2\omega_3^3c_s^2 + 24\omega_5\omega_3^2c_s^2 + \\
&72\omega_5v_2^4\omega_3^2 - 48\omega_5v_2^4\omega_3 + 24\omega_5^2v_2^2\omega_3^2 - 6\omega_5\omega_3^3c_s^2 - 72\omega_5^2v_2^2\omega_3^2c_s^2 - 3\omega_5^2\omega_3^3c_s^4 - 3\omega_5^2v_2^2\omega_3^3 + 156\omega_5^2v_2^2\omega_3c_s^2 + 24\omega_5^2v_2^4\omega_3 - 12v_2^2\omega_3^3 - 72\omega_5v_2^2\omega_3^2 - \\
&12\omega_5v_2^2\omega_3^2c_s^2 - 96\omega_5^2v_2^2c_s^2 + \omega_5^2\omega_3^3c_s^2 + 18\omega_5v_2^3\omega_3^3 + 6\omega_5\omega_3^3c_s^4 - 24v_2^2\omega_3^2c_s^2 + 24v_2^2\omega_3^2 - 48\omega_5\omega_3c_s^4 - 24\omega_5\omega_3c_s^2 + 3\omega_5^2v_2^4\omega_3^3 + 12v_2^2\omega_3^3c_s^2 - \\
&24\omega_5\omega_3^2c_s^4 + 48\omega_5v_2^2\omega_3 - 8\omega_5^2\omega_3^2c_s^2 - 24\omega_5^2v_2^4\omega_3^2 + 48\omega_5v_2^2\omega_3^2c_s^2 \\
C_{25} &= 42\omega_5^2\omega_3c_s^2 + 8\omega_5^2\omega_3^2 - 12\omega_3^2c_s^2 + 24\omega_5^2v_2^2\omega_3 - \omega_5^2\omega_3^3 - 12\omega_5^2v_2^2 + 24\omega_5\omega_3^2c_s^2 + 6\omega_3^3c_s^2 - 16\omega_5^2v_2^2\omega_3^2 - 6\omega_5\omega_3^3c_s^2 - 6\omega_5^2\omega_3 + 2\omega_5^2v_2^2\omega_3^3 + \\
&12\omega_5\omega_3 + 6v_2^2\omega_3^3 + 24\omega_5v_2^2\omega_3^2 + \omega_5^2\omega_3^2c_s^2 - 6\omega_5v_2^2\omega_3^2 - 12v_2^2\omega_3^2 + 12\omega_3^2 + 6\omega_5\omega_3^2 - 12\omega_5\omega_3c_s^2 - 24\omega_5^2c_s^2 - 6\omega_3^3 - 12\omega_5v_2^2\omega_3 - 20\omega_5^2\omega_3^2c_s^2 - 24\omega_5\omega_3^2
\end{aligned}$$

2.3 MRT2

2.3.1 Definitions

Collision operator C :

$$C(f) = \mathbf{M}_2^{-1} \mathbf{S} \left(\mu_2^{(eq)} - \mathbf{M}_2 f \right),$$

where

$$\mathbf{S} = \text{diag}(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5),$$

$\omega_1, \omega_2, \dots, \omega_5 \in (0, 2)$.

Matrix \mathbf{M}_2 corresponds to the transformation matrix to the raw moment basis defined by

$$\mu_2 = \begin{pmatrix} m_{(0,0)} \\ m_{(1,0)} \\ m_{(0,1)} \\ m_{(2,0)} + m_{(0,2)} \\ m_{(2,0)} - m_{(0,2)} \end{pmatrix},$$

and is given by

$$\mathbf{M}_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & 1 & -1 \end{pmatrix}.$$

The equilibrium moments $\mu_2^{(eq)}$ are defined by

$$\mu_2^{(eq)} = \mathbf{M}_2 \mathbf{M}^{-1} \mu^{(eq)},$$

i.e.,

$$\mu_2^{(eq)} = \begin{pmatrix} \rho \\ \rho v_1 \\ \rho v_2 \\ \rho(v_1^2 + v_2^2 + 2c_s^2) \\ \rho(v_1^2 - v_2^2) \end{pmatrix}.$$

2.3.2 Conservation of mass equation



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$$\begin{aligned} & \frac{\partial \rho}{\partial t} + \frac{\delta_1 v_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\delta_1 \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_1 v_2}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\delta_1 \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_2) \frac{\delta_1}{2\omega_2} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial t} + (-2 + \omega_2) \frac{\delta_1^2 v_1}{2\omega_2 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + \\ & (-2 + \omega_2) \frac{\delta_1^2 \rho}{2\omega_2 \delta_t} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + (2 - \omega_3) \frac{\delta_1^2 v_2}{2\omega_3 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (\omega_2 - \omega_2 \omega_3 + \omega_3) \frac{\delta_1^2 v_1}{\omega_2 \omega_3 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_2} + \\ & (\omega_2 - \omega_2 \omega_3 + \omega_3) \frac{\delta_1^2 \rho}{\omega_2 \omega_3 \delta_t} \frac{\partial v_1}{\partial x_1} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_3) \frac{\delta_1}{2\omega_3} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial t} + (\omega_2 - \omega_2 \omega_3 + \omega_3) \frac{\delta_1^2 v_2}{\omega_2 \omega_3 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_1} + (2 - \omega_2) \frac{\delta_1^2 v_1}{2\omega_2 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} \\ & + (-2 + \omega_3) \frac{\delta_1^2 v_2}{2\omega_3 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_3) \frac{\delta_1^2 \rho}{2\omega_3 \delta_t} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + (-2 + \omega_2) \frac{\delta_1 \rho}{2\omega_2} \frac{\partial^2 v_1}{\partial t \partial x_1} + (-2 + \omega_2) \frac{\delta_1^2 c_s^2}{2\omega_2 \delta_t} \frac{\partial^2 \rho}{\partial x_1^2} + \\ & (-2 + \omega_2) \frac{\delta_1^2 v_1 \rho}{2\omega_2 \delta_t} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega_3) \frac{\delta_1 \rho}{2\omega_3} \frac{\partial^2 v_2}{\partial t \partial x_2} + (\omega_2 - \omega_2 \omega_3 + \omega_3) \frac{\delta_1^2 v_2 v_1}{\omega_2 \omega_3 \delta_t} \frac{\partial^2 \rho}{\partial x_1 \partial x_2} + (2 - \omega_3) \frac{\delta_1^2 v_2 \rho}{2\omega_3 \delta_t} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + \\ & (2 - \omega_2) \frac{\delta_1^2 v_1 \rho}{2\omega_2 \delta_t} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_3) \frac{\delta_1^2 c_s^2}{2\omega_3 \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + (-2 + \omega_3) \frac{\delta_1^2 v_2 \rho}{2\omega_3 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + (12 - 12\omega_2 + \omega_2^2) \frac{\delta_1 \rho \delta_t}{12\omega_2^2} \frac{\partial^3 v_1}{\partial t^2 \partial x_1} + \\ & (12 - 6\omega_2 + \omega_2 \omega_4 - 6\omega_4) \frac{\delta_1^2 v_1 \rho}{6\omega_2 \omega_4} \frac{\partial^3 v_1}{\partial t \partial x_1^2} + C_1 \frac{\delta_1^3 v_1}{6\omega_2^2 \delta_t \omega_4} \frac{\partial^3 \rho}{\partial x_1^3} + C_2 \frac{\delta_1^3 \rho}{12\omega_2^2 \delta_t \omega_4} \frac{\partial^3 v_1}{\partial x_1^3} + (12 + \omega_3^2 - 12\omega_3) \frac{\delta_1 \rho \delta_t}{12\omega_3^2} \frac{\partial^3 v_2}{\partial t^2 \partial x_2} + \\ & (-2\omega_2 \omega_3^2 - 6\omega_2 + 3\omega_3^2 + 9\omega_2 \omega_3 - 6\omega_3) \frac{\delta_1^2 v_2 \rho}{6\omega_2 \omega_3^2} \frac{\partial^3 v_1}{\partial t \partial x_1 \partial x_2} + (-6\omega_2 + 9\omega_2 \omega_3 - 6\omega_3 - 2\omega_2^2 \omega_3 + 3\omega_2^2) \frac{\delta_1^2 v_1 \rho}{6\omega_2^2 \omega_3} \frac{\partial^3 v_2}{\partial t \partial x_1 \partial x_2} + \\ & C_3 \frac{\delta_1^3 v_2}{2\omega_2^2 \omega_3^2 \delta_t \omega_4} \frac{\partial^3 \rho}{\partial x_1^3 \partial x_2} + (-6\omega_2 \omega_3^2 + 6\omega_3^2 - 6\omega_2^2 \omega_3 + \omega_2^2 \omega_3^2 + 6\omega_2^2) \frac{\delta_1^3 v_2 v_1 \rho}{6\omega_2^2 \omega_3^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3 \partial x_2} + \\ & (6\omega_2^2 c_s^2 - 12c_s^2 \omega_4 - 6\omega_2 v_1^2 \omega_4 - 12\omega_2 v_1^2 + 12v_1^2 \omega_4 + 18\omega_2 c_s^2 \omega_4 - 3\omega_2^2 c_s^2 \omega_4 + 6\omega_2^2 v_1^2 \omega_4 + \omega_2^2 v_1^2 \omega_4 - 12\omega_2 c_s^2) \frac{\delta_1^3 \rho}{12\omega_2^2 \delta_t \omega_4} \frac{\partial^3 v_2}{\partial x_1^3 \partial x_2} \\ & + (12 - 6\omega_5 - 6\omega_3 + \omega_5 \omega_3) \frac{\delta_1^2 v_2 \rho}{6\omega_5 \omega_3} \frac{\partial^3 v_2}{\partial t \partial x_2^2} + C_4 \frac{\delta_1^3 v_1}{2\omega_5 \omega_2^2 \omega_3^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + \\ & (-12\omega_5 c_s^2 - 12v_2^2 \omega_3 + 18\omega_5 \omega_3 c_s^2 + 6v_2^2 \omega_3^2 + 6\omega_3^2 c_s^2 + 12\omega_5 v_2^2 - 3\omega_5 \omega_3^2 c_s^2 - 12\omega_3 c_s^2 + \omega_5 v_2^2 \omega_3^2 - 6\omega_5 v_2^2 \omega_3) \frac{\delta_1^3 \rho}{12\omega_5 \omega_3^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} \\ & + (-6\omega_2 \omega_3^2 + 6\omega_3^2 - 6\omega_2^2 \omega_3 + \omega_2^2 \omega_3^2 + 6\omega_2^2) \frac{\delta_1^3 v_2 v_1 \rho}{6\omega_2^2 \omega_3^2 \delta_t} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_5 \frac{\delta_1^3 v_2}{6\omega_5 \omega_3^2 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + C_6 \frac{\delta_1^3 \rho}{12\omega_5 \omega_3^2 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & (-2 + 3\omega_2 - \omega_2^2) \frac{\delta_1 \rho \delta_t^2}{2\omega_2^2} \frac{\partial^4 v_1}{\partial t^3 \partial x_1} + (2\omega_4^2 - 4\omega_2 \omega_4 - \omega_2 \omega_4^2 - 2\omega_2^2 \omega_4 - \omega_2^2 \omega_4^2 + 2\omega_2^2 + 8\omega_2^2 \omega_4 - 4\omega_2^2) \frac{\delta_1^2 v_1 \rho \delta_t}{2\omega_2^2 \omega_4^2} \frac{\partial^4 v_1}{\partial t^2 \partial x_1^2} + \\ & C_7 \frac{\delta_1^3 \rho}{12\omega_2^2 \omega_4^2} \frac{\partial^4 v_1}{\partial t \partial x_1^3} + C_8 \frac{\delta_1^4}{24\omega_2^2 \delta_t \omega_4^2} \frac{\partial^4 \rho}{\partial x_1^4} + C_9 \frac{\delta_1^4 v_1 \rho}{12\omega_2^2 \delta_t \omega_4^2} \frac{\partial^4 v_1}{\partial x_1^4} + (-2 - \omega_3^2 + 3\omega_3) \frac{\delta_1 \rho \delta_t^2}{2\omega_3^2} \frac{\partial^4 v_2}{\partial t^3 \partial x_2} + \\ & (7\omega_2 \omega_3^3 - 24\omega_2 \omega_3^2 + 12\omega_3^2 + 12\omega_2 \omega_3 - 6\omega_3^3 - 24\omega_2^2 \omega_3 + 13\omega_2^2 \omega_3^2 + 12\omega_2^2 - \omega_2^2 \omega_3^3) \frac{\delta_1^2 v_2 \rho \delta_t}{12\omega_2^2 \omega_3^2} \frac{\partial^4 v_1}{\partial t^2 \partial x_1 \partial x_2} + \\ & (-24\omega_2 \omega_3^2 + 12\omega_3^2 + 12\omega_2 \omega_3 - \omega_2^2 \omega_3^2 - 24\omega_2^2 \omega_3 + 13\omega_2^2 \omega_3^2 - 6\omega_2^3 + 12\omega_2^2 + 7\omega_2^2 \omega_3) \frac{\delta_1^2 v_1 \rho \delta_t}{12\omega_2^2 \omega_3^2} \frac{\partial^4 v_2}{\partial t^2 \partial x_1 \partial x_2} + \\ & C_{10} \frac{\delta_1^3 v_2 v_1 \rho}{6\omega_2^2 \omega_3^2 \omega_4} \frac{\partial^4 v_1}{\partial t \partial x_1^2 \partial x_2} + C_{11} \frac{\delta_1^3 \rho}{12\omega_2^2 \omega_3^2 \omega_4} \frac{\partial^4 v_2}{\partial t \partial x_1^2 \partial x_2} + C_{12} \frac{\delta_1^4 v_2 v_1}{6\omega_2^2 \omega_3^2 \delta_t \omega_4} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{13} \frac{\delta_1^4 v_2 \rho}{12\omega_2^2 \omega_3^2 \delta_t \omega_4} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + \\ & C_{14} \frac{\delta_1^4 v_1 \rho}{12\omega_2^2 \delta_t \omega_4^2} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + (-\omega_5^2 \omega_3^2 - 4\omega_2^2 + 2\omega_3^2 - \omega_5^2 \omega_3 + 2\omega_5^2 - 4\omega_5 \omega_3 - 2\omega_5 \omega_3^2 + 8\omega_5 \omega_3^2) \frac{\delta_1^2 v_2 \rho \delta_t}{2\omega_5^2 \omega_3^2} \frac{\partial^4 v_2}{\partial t^2 \partial x_2^2} + \\ & C_{15} \frac{\delta_1^3 \rho}{12\omega_5^2 \omega_2 \omega_3^2} \frac{\partial^4 v_1}{\partial t \partial x_1 \partial x_2^2} + C_{16} \frac{\delta_1^3 v_2 v_1 \rho}{6\omega_5 \omega_2 \omega_3^2} \frac{\partial^4 v_2}{\partial t \partial x_1 \partial x_2^2} + C_{17} \frac{\delta_1^4}{4\omega_5^2 \omega_2^2 \omega_3^2 \delta_t \omega_4^2} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_{18} \frac{\delta_1^4 v_1 \rho}{12\omega_5^2 \omega_2^2 \omega_3^2 \delta_t} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + \\ & C_{19} \frac{\delta_1^4 v_2 \rho}{12\omega_5^2 \omega_2^2 \omega_3^2 \delta_t} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_{20} \frac{\delta_1^3 \rho}{12\omega_5^2 \omega_3^2} \frac{\partial^4 v_2}{\partial t \partial x_2^3} + C_{21} \frac{\delta_1^4 v_2 v_1}{6\omega_5^2 \omega_2^2 \omega_3^2 \delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^2} + C_{22} \frac{\delta_1^4 v_2 \rho}{12\omega_5^2 \omega_3^2 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^2} + C_{23} \frac{\delta_1^4 v_1 \rho}{12\omega_5^2 \omega_2^2 \omega_3^2 \delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^2} \\ & + C_{24} \frac{\delta_1^4}{24\omega_5^2 \omega_3^2 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{25} \frac{\delta_1^4 v_2 \rho}{12\omega_5^2 \omega_3^2 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:

$$\begin{aligned}
C_1 &= 3\omega_2^2 c_s^2 - 12c_s^2 \omega_4 + 3\omega_2 v_1^2 \omega_4 + 6\omega_2 - 3\omega_2 \omega_4 - 6\omega_2 v_1^2 + 15\omega_2 c_s^2 \omega_4 - 3\omega_2^2 c_s^2 \omega_4 + 3\omega_2^2 v_1^2 + \omega_2^2 \omega_4 - \omega_2^2 v_1^2 \omega_4 - 3\omega_2^2 - 6\omega_2 c_s^2 \\
C_2 &= 6\omega_2^2 c_s^2 - 12c_s^2 \omega_4 + 18\omega_2 v_1^2 \omega_4 + 12\omega_2 - 6\omega_2 \omega_4 - 12\omega_2 v_1^2 - 12v_1^2 \omega_4 + 18\omega_2 c_s^2 \omega_4 - 3\omega_2^2 c_s^2 \omega_4 + 6\omega_2^2 v_1^2 + 2\omega_2^2 \omega_4 - 5\omega_2^2 v_1^2 \omega_4 - 6\omega_2^2 - 12\omega_2 c_s^2 \\
C_3 &= -2\omega_2 v_1^2 \omega_3^2 - 2\omega_2 \omega_3 c_s^2 \omega_4 + \omega_2^2 v_1^2 \omega_3^2 \omega_4 - 2\omega_3^2 c_s^2 \omega_4 - 2\omega_2 \omega_3^2 c_s^2 - \omega_2^2 \omega_3^2 c_s^2 \omega_4 + 4v_1^2 \omega_3^2 \omega_4 + 2\omega_2 v_1^2 \omega_3 \omega_4 + \omega_2^2 \omega_3 c_s^2 \omega_4 - 4\omega_2 v_1^2 \omega_3^2 \omega_4 + \\
&\omega_2^2 v_1^2 \omega_3^2 - 3\omega_2^2 v_1^2 \omega_3 \omega_4 + 4\omega_2 \omega_3^2 c_s^2 \omega_4 + 2\omega_2^2 v_1^2 \omega_4 + \omega_2^2 \omega_3^2 c_s^2 \\
C_4 &= -2\omega_5 \omega_2 \omega_3 c_s^2 - 2\omega_2^2 \omega_3 c_s^2 - \omega_5 \omega_2^2 \omega_3^2 c_s^2 + \omega_5 \omega_2^2 v_2^2 \omega_3^2 + 4\omega_5 \omega_2^2 v_2^2 - 4\omega_5 \omega_2^2 v_2^2 \omega_3 - 2\omega_2^2 v_2^2 \omega_3 + 2\omega_5 \omega_2 v_2^2 \omega_3 + 2\omega_5 v_2^2 \omega_3^2 + 4\omega_5 \omega_2^2 \omega_3 c_s^2 - \\
&2\omega_5 \omega_2^2 c_s^2 - 3\omega_5 \omega_2 v_2^2 \omega_3^2 + \omega_5 \omega_2 \omega_3^2 c_s^2 + \omega_2^2 \omega_3^2 c_s^2 + \omega_2^2 v_2^2 \omega_3^2 \\
C_5 &= -12\omega_5 c_s^2 - 6v_2^2 \omega_3 + 15\omega_5 \omega_3 c_s^2 + 3v_2^2 \omega_3^2 - 3\omega_3^2 + 3\omega_3^2 c_s^2 + 6\omega_3 - 3\omega_5 \omega_3 - 3\omega_5 \omega_3^2 c_s^2 - 6\omega_3 c_s^2 - \omega_5 v_2^2 \omega_3^2 + 3\omega_5 v_2^2 \omega_3 + \omega_5 \omega_3^2 \\
C_6 &= -12\omega_5 c_s^2 - 12v_2^2 \omega_3 + 18\omega_5 \omega_3 c_s^2 + 6v_2^2 \omega_3^2 - 6\omega_3^2 + 6\omega_3^2 c_s^2 + 12\omega_3 - 12\omega_5 v_2^2 - 6\omega_5 \omega_3 - 3\omega_5 \omega_3^2 c_s^2 - 12\omega_3 c_s^2 - 5\omega_5 v_2^2 \omega_3^2 + 18\omega_5 v_2^2 \omega_3 + 2\omega_5 \omega_3^2 \\
C_7 &= 9\omega_2^3 c_s^2 \omega_4 + 12\omega_2^2 c_s^2 - 48\omega_2 c_s^2 \omega_4^2 - 3\omega_2^2 v_1^2 \omega_4^2 + 48\omega_2 v_1^2 \omega_4 + 12v_1^2 \omega_4^2 - 24\omega_2 \omega_4 + 12\omega_2 \omega_4^2 - 6\omega_2^3 c_s^2 + 15\omega_2^3 v_1^2 \omega_4 - 42\omega_2 v_1^2 \omega_4^2 - 2\omega_2^3 c_s^2 \omega_4^2 + \\
&24c_s^2 \omega_4^2 + 24\omega_2 c_s^2 \omega_4 - 9\omega_2^3 \omega_4 - 11\omega_2^2 \omega_4^2 + 27\omega_2^2 v_1^2 \omega_4^2 - 36\omega_2^2 c_s^2 \omega_4 + 12\omega_2^2 v_1^2 + 25\omega_2^2 c_s^2 \omega_4^2 + 6\omega_2^3 + 36\omega_2^2 \omega_4 - 60\omega_2^2 v_1^2 \omega_4 - 6\omega_2^3 v_1^2 - 12\omega_2^2 + \omega_2^3 \omega_4^2 \\
C_8 &= -6\omega_2^3 c_s^2 \omega_4 - 24\omega_2^2 v_1^4 \omega_4^2 + 12\omega_2^2 c_s^2 \omega_4^2 - 24\omega_2^2 v_1^2 c_s^2 - 24\omega_2^2 c_s^4 \omega_4 - 3\omega_2^3 v_1^2 \omega_4^2 + 48\omega_2 v_1^2 \omega_4 - 24\omega_2 v_1^2 c_s^2 \omega_4 + 12\omega_2^3 v_1^4 + 156\omega_2 v_1^2 c_s^2 \omega_4^2 + \\
&18\omega_2^3 v_1^2 \omega_4 + 24\omega_2^2 c_s^4 \omega_4^2 - 24\omega_2^2 v_1^4 - 24\omega_2 v_1^2 \omega_4^2 + 72\omega_2^2 v_1^4 \omega_4 + \omega_2^3 c_s^2 \omega_4^2 - 24\omega_2 c_s^2 \omega_4 + 6\omega_2^3 v_1^2 c_s^2 \omega_4^2 + 6\omega_2^3 c_s^4 \omega_4 + 24\omega_2^2 v_1^2 \omega_4^2 - 48\omega_2 c_s^4 \omega_4^2 + \\
&24\omega_2^2 c_s^2 \omega_4 + 3\omega_2^3 v_1^4 \omega_4^2 + 12\omega_2^3 v_1^2 c_s^2 + 24\omega_2^2 v_1^2 - 48\omega_2 v_1^4 \omega_4 + 48\omega_2^2 v_1^2 c_s^2 \omega_4 - 96v_1^2 c_s^2 \omega_4^2 - 18\omega_2^3 v_1^4 \omega_4 - 8\omega_2^2 c_s^2 \omega_4^2 + 24\omega_2 v_1^4 \omega_4^2 - 72\omega_2^2 v_1^2 c_s^2 \omega_4^2 - \\
&72\omega_2^2 v_1^2 \omega_4 - 3\omega_2^3 c_s^4 \omega_4^2 - 12\omega_2^3 v_1^2 c_s^2 \omega_4 + 24c_s^4 \omega_4^2 + 24\omega_2 c_s^4 \omega_4 - 12\omega_2^3 v_1^2 \\
C_9 &= -6\omega_2^3 c_s^2 \omega_4 - 12\omega_2^2 c_s^2 + 42\omega_2 c_s^2 \omega_4^2 + 2\omega_2^3 v_1^2 \omega_4^2 - 12\omega_2 v_1^2 \omega_4 - 12v_1^2 \omega_4^2 + 12\omega_2 \omega_4 - 6\omega_2 \omega_4^2 + 6\omega_2^3 c_s^2 - 6\omega_2^3 v_1^2 \omega_4 + 24\omega_2 v_1^2 \omega_4^2 + \omega_2^3 c_s^2 \omega_4^2 - \\
&24c_s^2 \omega_4^2 - 12\omega_2 c_s^2 \omega_4 + 6\omega_2^3 \omega_4 + 8\omega_2^2 \omega_4^2 - 16\omega_2^2 v_1^2 \omega_4^2 + 24\omega_2^2 c_s^2 \omega_4 - 12\omega_2^2 v_1^2 - 20\omega_2^2 c_s^2 \omega_4^2 - 6\omega_2^3 - 24\omega_2^2 \omega_4 + 24\omega_2^2 v_1^2 \omega_4 + 6\omega_2^3 v_1^2 + 12\omega_2^2 - \omega_2^3 \omega_4^2 \\
C_{10} &= 12\omega_2^2 \omega_3^2 \omega_4 - 10\omega_2^2 \omega_3^3 \omega_4 + 12\omega_2^2 \omega_3 \omega_4 - 6\omega_2^3 \omega_4 - 7\omega_2^3 \omega_3^2 \omega_4 - 6\omega_2^3 \omega_3^2 + 3\omega_2^3 \omega_3^3 + 24\omega_2 \omega_3^3 \omega_4 + \omega_2^3 \omega_3^3 \omega_4 + 12\omega_2^2 \omega_3^2 - 6\omega_2^2 \omega_3 \omega_4 - 12\omega_2^3 \omega_4 - \\
&12\omega_2 \omega_3^2 \omega_4 - 6\omega_2^2 \omega_3^2 \\
C_{11} &= -6\omega_2^3 c_s^2 \omega_4 - 6\omega_2^3 v_2^2 \omega_3 + 12\omega_2 c_s^2 \omega_4^2 + 12\omega_2^2 \omega_3 c_s^2 - \omega_2^3 v_1^2 \omega_4^2 + 12\omega_2 \omega_3 c_s^2 \omega_4 + 36\omega_2 v_1^2 \omega_3 \omega_4^2 + 12\omega_2 v_1^2 \omega_3 \omega_4 - 6\omega_2^3 v_1^2 \omega_4 - 12\omega_2 v_1^2 \omega_4^2 - \\
&30\omega_2 \omega_3 c_s^2 \omega_4 + 3\omega_2^3 c_s^2 \omega_4^2 + 12\omega_2 c_s^2 \omega_4 - 30\omega_2^2 \omega_3 c_s^2 \omega_4 - 2\omega_2^3 \omega_3 c_s^2 \omega_4^2 - 6\omega_2^3 \omega_3 c_s^2 + 6\omega_2^2 v_1^2 \omega_4^2 + 9\omega_2^3 v_1^2 \omega_3 \omega_4 - 10\omega_2^2 v_1^2 \omega_3 \omega_4^2 + 12\omega_2^2 v_1^2 \omega_3 + \\
&12\omega_2^2 c_s^2 \omega_4 - 18\omega_2^2 c_s^2 \omega_4^2 - 30\omega_2^2 v_1^2 \omega_3 \omega_4 - 24v_1^2 \omega_3 \omega_4^2 + 12\omega_2^2 v_1^2 \omega_4 + \omega_2^3 v_1^2 \omega_3 \omega_4^2 + 9\omega_2^3 \omega_3 c_s^2 \omega_4 + 22\omega_2^2 \omega_3 c_s^2 \omega_4^2 \\
C_{12} &= 7\omega_2^2 \omega_3^2 \omega_4^2 + 6\omega_2^2 v_1^2 \omega_3^2 \omega_4^2 + 6\omega_2^3 v_1^2 \omega_4^2 - 12\omega_2^3 v_1^2 \omega_3^2 \omega_4 + 78\omega_2 \omega_3^2 c_s^2 \omega_4^2 + 6\omega_2^3 \omega_3^2 c_s^2 - 12\omega_2^2 v_1^2 \omega_3^2 \omega_4 + 6\omega_2^3 \omega_3^2 c_s^2 \omega_4 + 6\omega_2^2 \omega_3^2 \omega_4 + \\
&42\omega_2^2 \omega_3^2 c_s^2 \omega_4^2 + 6\omega_2^3 v_1^2 \omega_3^2 \omega_4^2 + 6\omega_2^3 v_1^2 \omega_3^2 \omega_4 - 12\omega_2^2 \omega_3^2 c_s^2 \omega_4^2 - 12\omega_2^2 \omega_3^2 c_s^2 \omega_4 - 12\omega_2^2 v_1^2 \omega_3^2 \omega_4^2 + 6\omega_2^3 v_1^2 \omega_3^2 - 3\omega_2^2 \omega_3^2 \omega_4^2 - 24\omega_2 \omega_3^2 c_s^2 \omega_4 - 21\omega_2^2 \omega_3^2 \omega_4 + \\
&24v_1^2 \omega_3^2 \omega_4^2 + 42\omega_2^2 v_1^2 \omega_3^2 \omega_4 - 48\omega_2^2 \omega_3^2 c_s^2 \omega_4^2 - 12\omega_2^3 \omega_3^2 c_s^2 \omega_4 - 30\omega_2 v_1^2 \omega_3^2 \omega_4^2 + 6\omega_2^3 \omega_3^2 c_s^2 \omega_4^2 - 3\omega_2^2 \omega_3^2 \omega_4 - 24\omega_2 \omega_3^2 c_s^2 \omega_4^2 - 12\omega_2^2 \omega_3^2 c_s^2 + \\
&6\omega_2^2 v_1^2 \omega_3^2 \omega_4^2 - \omega_2^3 \omega_3^2 \omega_4^2 - 3\omega_2^3 \omega_3^2 + 12\omega_2 \omega_3^2 \omega_4 + 6\omega_2^2 \omega_3^2 \omega_4 - 6\omega_2 \omega_3^2 \omega_4^2 + 12\omega_2 v_1^2 \omega_3^2 \omega_4^2 - 12\omega_2^2 v_1^2 \omega_3 \omega_4^2 + 6\omega_2^3 \omega_3^2 c_s^2 \omega_4^2 - 24\omega_2 v_1^2 \omega_3^2 \omega_4 + 6\omega_2^2 \omega_3^2 + \\
&\omega_2^3 \omega_3^2 \omega_4^2 - 12\omega_2^2 v_1^2 \omega_3^2 - 12\omega_2^2 \omega_3 c_s^2 \omega_4^2 - 36\omega_2^3 c_s^2 \omega_4^2 + 42\omega_2^2 \omega_3^2 c_s^2 \omega_4 \\
C_{13} &= 3\omega_2^2 \omega_3^2 \omega_4^2 + 12\omega_2^3 v_1^2 \omega_4^2 - 12\omega_2^2 v_1^2 \omega_3^2 \omega_4 + 36\omega_2 \omega_3^2 c_s^2 \omega_4^2 + 6\omega_2^3 \omega_3^2 c_s^2 - 24\omega_2^2 v_1^2 \omega_3^2 \omega_4 + 12\omega_2^3 \omega_3^2 c_s^2 \omega_4 + 12\omega_2^2 \omega_3^2 \omega_4 + 48\omega_2^2 \omega_3^2 c_s^2 \omega_4^2 + \\
&12\omega_2^3 v_1^2 \omega_3^2 \omega_4 - 24\omega_2^2 \omega_3^2 c_s^2 \omega_4 - 12\omega_2^2 \omega_3^2 c_s^2 \omega_4^2 + 12\omega_2^2 v_1^2 \omega_3^2 \omega_4^2 + 6\omega_2^3 v_1^2 \omega_3^2 - 6\omega_2^2 \omega_3^2 \omega_4^2 + 3\omega_2^3 v_1^2 \omega_3^2 \omega_4^2 - 12\omega_2 \omega_3^2 c_s^2 \omega_4 - 6\omega_2^2 \omega_3^2 \omega_4 + 24v_1^2 \omega_3^2 \omega_4^2 + \\
&36\omega_2^2 v_1^2 \omega_3 \omega_4 - 32\omega_2^2 \omega_3^2 c_s^2 \omega_4 - 12\omega_2^2 \omega_3^2 c_s^2 \omega_4 - 30\omega_2 v_1^2 \omega_3^2 \omega_4^2 + 6\omega_2^3 \omega_3^2 c_s^2 \omega_4^2 - 6\omega_2^2 \omega_3^2 \omega_4 - 24\omega_2 \omega_3^2 c_s^2 \omega_4^2 - 12\omega_2^2 \omega_3^2 c_s^2 - \omega_2^3 \omega_3^2 \omega_4^2 + 3\omega_2^3 \omega_3^2 \omega_4 - \\
&18\omega_2^2 v_1^2 \omega_3 \omega_4^2 + 4\omega_2^3 \omega_3^2 c_s^2 \omega_4^2 - 12\omega_2 v_1^2 \omega_3^2 \omega_4 + 2\omega_2^2 \omega_3^2 \omega_4 - 12\omega_2^2 v_1^2 \omega_3^2 - 12\omega_2^2 \omega_3 c_s^2 \omega_4^2 - 12\omega_2^3 c_s^2 \omega_4^2 + 36\omega_2^2 \omega_3^2 c_s^2 \omega_4 \\
C_{14} &= -12\omega_2^3 c_s^2 \omega_4 - 12\omega_2^2 c_s^2 + 90\omega_2 c_s^2 \omega_4^2 + \omega_2^3 v_1^2 \omega_4^2 - 36\omega_2 v_1^2 \omega_4 + 12v_1^2 \omega_4^2 + 24\omega_2 \omega_4 - 12\omega_2 \omega_4^2 + 6\omega_2^3 c_s^2 - 12\omega_2^3 v_1^2 \omega_4 + 4\omega_2^3 c_s^2 \omega_4^2 - 48c_s^2 \omega_4^2 - \\
&36\omega_2 c_s^2 \omega_4 + 9\omega_2^2 \omega_4 + 11\omega_2^2 \omega_4^2 - 8\omega_2^2 v_1^2 \omega_4^2 + 48\omega_2^2 c_s^2 \omega_4 - 12\omega_2^2 v_1^2 - 44\omega_2^2 c_s^2 \omega_4^2 - 6\omega_2^3 - 36\omega_2^2 \omega_4 + 48\omega_2^2 v_1^2 \omega_4 + 6\omega_2^3 v_1^2 + 12\omega_2^2 - \omega_2^3 \omega_4^2 \\
C_{15} &= 12\omega_5 \omega_2 \omega_3 c_s^2 + 36\omega_5^2 \omega_2 v_2^2 \omega_3 - 12\omega_5^2 v_2^2 \omega_3 - 6\omega_2 v_2^2 \omega_3^2 - 2\omega_5^2 \omega_2 \omega_3^2 c_s^2 + 3\omega_5^2 \omega_3^2 c_s^2 + 12\omega_2 v_2^2 \omega_3^2 + 12\omega_2 \omega_3^2 c_s^2 - \omega_5^2 v_2^2 \omega_3^2 + \omega_5^2 \omega_2 v_2^2 \omega_3^2 - \\
&6\omega_2 \omega_3^2 c_s^2 - 24\omega_5^2 \omega_2 v_2^2 + 6\omega_5^2 v_2^2 \omega_3^2 - 18\omega_5^2 \omega_3^2 c_s^2 + 22\omega_5^2 \omega_2 \omega_3^2 c_s^2 - 10\omega_5^2 \omega_2 v_2^2 \omega_3^2 - 6\omega_5 v_2^2 \omega_3^2 + 12\omega_5^2 \omega_3 c_s^2 - 30\omega_5^2 \omega_2 \omega_3 c_s^2 + 12\omega_5 \omega_2 v_2^2 \omega_3 + \\
&12\omega_5 \omega_3^2 c_s^2 + 12\omega_5 v_2^2 \omega_3^2 + 9\omega_5 \omega_2 \omega_3^2 c_s^2 + 12\omega_5^2 \omega_2 c_s^2 + 9\omega_5 \omega_2 v_2^2 \omega_3^2 - 30\omega_5 \omega_2 v_2^2 \omega_3^2 - 30\omega_5 \omega_2 \omega_3^2 c_s^2 - 6\omega_5 \omega_3^2 c_s^2 \\
C_{16} &= -7\omega_5 \omega_2^2 \omega_3^2 + 24\omega_5 \omega_2^2 \omega_3 + 12\omega_5 \omega_2^2 \omega_3^2 + \omega_5 \omega_2^2 \omega_3^2 - 12\omega_5 \omega_2^2 \omega_3 - 12\omega_5 \omega_2^2 - 10\omega_5 \omega_2^2 \omega_3^2 - 6\omega_2^2 \omega_3^2 + 3\omega_2^2 \omega_3^2 - 6\omega_5 \omega_2^2 \omega_3^2 + 12\omega_2^2 \omega_3^2 - \\
&6\omega_5 \omega_2 \omega_3^2 + 12\omega_5 \omega_2 \omega_3^2 - 6\omega_2^2 \omega_3^2 \\
C_{17} &= -3\omega_5 \omega_2^3 v_1^2 \omega_3^2 c_s^2 \omega_4^2 - 4\omega_5^2 \omega_2^3 v_2^2 v_1^2 \omega_3^2 - 4\omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 + 12\omega_5^2 \omega_2^3 v_1^2 \omega_3^2 c_s^2 \omega_4^2 - 3\omega_5^2 \omega_2^3 v_2^2 v_1^2 \omega_3^2 \omega_4^2 - 12\omega_2^2 \omega_2^2 \omega_3^2 c_s^4 \omega_4^2 - 4\omega_5^2 \omega_2 v_2^2 \omega_3^2 c_s^2 \omega_4 - \\
&2\omega_5^2 \omega_2^2 \omega_3^2 c_s^4 \omega_4 + \omega_5^2 \omega_2^3 v_1^2 \omega_3^2 c_s^2 \omega_4 + 10\omega_5^2 \omega_2^3 v_1^2 \omega_3^2 c_s^2 \omega_4^2 + 10\omega_5^2 \omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_4 + 2\omega_5^2 \omega_2^3 v_2^2 v_1^2 \omega_3^2 \omega_4 + 20\omega_5^2 \omega_2^3 v_2^2 v_1^2 \omega_4^2 - 2\omega_5^2 \omega_2 \omega_3^2 c_s^4 \omega_4^2 - \\
&8\omega_5^2 \omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_4^2 + 2\omega_2^3 v_2^2 v_1^2 \omega_3^2 \omega_4^2 + 2\omega_5^2 \omega_2^3 v_1^2 c_s^2 \omega_4^2 - 4\omega_5^2 \omega_2^3 v_1^2 c_s^2 \omega_4^2 + 10\omega_5 \omega_2^3 v_2^2 v_1^2 \omega_3^2 \omega_4^2 - 2\omega_5 \omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_4^2 + 4\omega_5 \omega_2^2 \omega_3^2 c_s^4 \omega_4^2 - 8\omega_5^2 \omega_2 v_1^2 \omega_3^2 c_s^2 \omega_4^2 - \\
&4\omega_5^2 \omega_2^3 v_2^2 c_s^2 \omega_4^2 - 4\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 c_s^2 + 2\omega_2^3 v_1^2 \omega_3^2 c_s^2 \omega_4^2 + 2\omega_5^2 \omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_4 + 20\omega_5^2 \omega_2^3 v_2^2 v_1^2 \omega_3^2 \omega_4^2 - 8\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 - 2\omega_5 \omega_2^3 \omega_3^2 c_s^4 \omega_4^2 - 2\omega_5 \omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_4^2 + \\
&10\omega_2^2 \omega_2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 + \omega_5^2 \omega_2^3 v_1^2 \omega_3^2 c_s^2 \omega_4^2 + 4\omega_5^2 \omega_2^3 \omega_3^2 c_s^4 \omega_4^2 + 20\omega_5^2 \omega_2^2 v_2^2 v_1^2 \omega_3 \omega_4^2 - 4\omega_5 \omega_2^2 v_1^2 \omega_3^2 c_s^2 \omega_4^2 - 3\omega_5 \omega_2^3 v_2^2 v_1^2 \omega_3^2 \omega_4^2 + 2\omega_2^2 \omega_2^3 v_2^2 v_1^2 \omega_3^2 - \\
&4\omega_2^3 v_2^2 v_1^2 \omega_3^2 \omega_4^2 - 3\omega_5^2 \omega_2^3 v_2^2 v_1^2 \omega_3^2 \omega_4 + 4\omega_5^2 \omega_2^2 \omega_3^2 c_s^4 \omega_4 - 4\omega_5 \omega_2^3 v_1^2 \omega_3^2 c_s^2 \omega_4^2 + 4\omega_5^2 \omega_2^2 v_1^2 \omega_3^2 c_s^2 \omega_4 + 4\omega_5 \omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 + 20\omega_5^2 v_2^2 v_1^2 \omega_3^2 \omega_4^2 + \omega_5^2 \omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_4^2 - \\
&36\omega_5^2 \omega_2^2 v_2^2 v_1^2 \omega_3^2 \omega_4^2 + 4\omega_5^2 \omega_2 v_1^2 \omega_3^2 c_s^2 \omega_4^2 - 2\omega_5^2 \omega_2^2 \omega_3^2 c_s^4 \omega_4^2 - 4\omega_5^2 \omega_2 v_2^2 v_1^2 \omega_3^2 \omega_4 + 4\omega_5^2 \omega_2 \omega_3^2 c_s^4 \omega_4^2 - 4\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_4 - 2\omega_5^2 \omega_2^2 v_1^2 \omega_3^2 c_s^2 \omega_4 + \\
&10\omega_5^2 \omega_2^2 v_2^2 v_1^2 \omega_3^2 \omega_4 + \omega_5^2 \omega_2^3 v_1^2 c_s^4 \omega_4 - 2\omega_5^2 \omega_2^3 \omega_3^2 c_s^4 \omega_4^2 + 2\omega_5 \omega_2^2 v_2^2 v_1^2 \omega_3^2 \omega_4^2 + 10\omega_5 \omega_2^2 v_1^2 \omega_3^2 c_s^2 \omega_4^2 - 38\omega_5^2 \omega_2^2 v_2^2 v_1^2 \omega_3 \omega_4^2 + 20\omega_5^2 \omega_2 v_2^2 v_1^2 \omega_3^2 \omega_4^2 + \\
&4\omega_5^2 \omega_2^2 \omega_3^2 c_s^4 \omega_4^2 - 4\omega_2^2 \omega_2^2 v_1^2 \omega_3^2 c_s^2 \omega_4^2 - 4\omega_5^2 \omega_2^2 v_1^2 \omega_3^2 c_s^2 \omega_4^2 + 4\omega_5^2 \omega_2 \omega_3^2 c_s^4 \omega_4^2 - 2\omega_5^2 \omega_2^2 v_1^2 \omega_3^2 c_s^2 \omega_4 - 2\omega_5^2 \omega_2^2 \omega_3^2 c_s^4 \omega_4 + 20\omega_5^2 \omega_2^2 v_2^2 v_1^2 \omega_3^2 \omega_4^2 - \\
&\omega_5^2 \omega_2^3 \omega_3^2 c_s^4 \omega_4^2 - 4\omega_5 \omega_2^2 v_2^2 v_1^2 \omega_3 \omega_4^2 - 8\omega_5^2 \omega_2^2 v_1^2 \omega_3^2 c_s^2 \omega_4^2 - 4\omega_5^2 \omega_2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 + 2\omega_5 \omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 + \omega_5 \omega_2^3 \omega_3^2 c_s^4 \omega_4^2 - 38\omega_5^2 \omega_2 v_2^2 v_1^2 \omega_3^2 \omega_4^2 + \\
&12\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 + \omega_5 \omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_4^2 - 4\omega_5 \omega_2^2 v_2^2 v_1^2 \omega_3^2 \omega_4^2 + 4\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_4^2 - 4\omega_2^3 v_1^2 \omega_3^2 c_s^2 \omega_4^2 - 4\omega_5^2 \omega_2^2 v_2^2 v_1^2 \omega_3^2 \omega_4 - 3\omega_5^2 \omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_4^2
\end{aligned}$$

$$\begin{aligned}
C_{18} &= 34\omega_5^2\omega_3^2v_2^2\omega_3^2 - 14\omega_5^2\omega_3^2\omega_3^2c_s^2 + 6\omega_5^2\omega_3^2c_s^2 - 4\omega_5^2\omega_3^2v_2^2\omega_3^3 + 6\omega_5^2\omega_2\omega_3^2c_s^2 - 12\omega_5^2\omega_3^2c_s^2 + 12\omega_5^2v_2^2\omega_3^3 - 12\omega_5^2v_2^2\omega_3^2 + \omega_5^2\omega_2^2\omega_3^3c_s^2 - 30\omega_5^2\omega_2v_2^2\omega_3^3 + \\
&6\omega_5^2v_2^2\omega_3^3 - 12\omega_5^2\omega_2\omega_3^2c_s^2 - 12\omega_5^2\omega_3^2c_s^2 - 12\omega_5\omega_2^2\omega_3c_s^2 - 78\omega_5^2\omega_2^2v_2^2\omega_3 + 24\omega_5^2\omega_2v_2^2\omega_3^2 + 24\omega_5\omega_3^2\omega_3^2c_s^2 + 24\omega_5\omega_2^2v_2^2\omega_3^2 + 22\omega_5^2\omega_2^2v_2^2\omega_3^3 - \\
&6\omega_5\omega_2^2v_2^2\omega_3^3 - 48\omega_5^2\omega_2^2v_2^2\omega_3^2 + 12\omega_5^2\omega_2^2\omega_3^2c_s^2 + 24\omega_5^2\omega_2^2v_2^2\omega_3 - 6\omega_5\omega_2^2\omega_3^2c_s^2 + 48\omega_5^2\omega_2^2v_2^2 - 6\omega_5^2\omega_2^2\omega_3^2c_s^2 + 24\omega_5^2\omega_2^2\omega_3c_s^2 - 12\omega_5\omega_3^2v_2^2\omega_3 \\
C_{19} &= 34\omega_5^2v_1^2\omega_3^3\omega_4^2 + 12\omega_5^2v_1^2\omega_4^2 - 6\omega_5^2v_1^2\omega_3^3\omega_4 + 24\omega_2\omega_3^2c_s^2\omega_4^2 + 6\omega_5^2\omega_3^2c_s^2 + 12\omega_5^2\omega_3^2c_s^2\omega_4^2 + 22\omega_5^2v_1^2\omega_3^2\omega_4^2 - 6\omega_5^2\omega_3^2c_s^2\omega_4^2 - 48\omega_5^2v_1^2\omega_3^2\omega_4^2 + \\
&6\omega_5^2v_1^2\omega_3^3 - 4\omega_5^2v_1^2\omega_3^3\omega_4^2 - 12\omega_2\omega_3^2c_s^2\omega_4 + 48v_1^2\omega_3^3\omega_4^2 + 24\omega_5^2v_1^2\omega_3^3\omega_4 - 14\omega_5^2\omega_3^2c_s^2\omega_4^2 - 6\omega_5^2\omega_3^2c_s^2\omega_4 - 78\omega_2v_1^2\omega_3^3\omega_4^2 + 6\omega_5^2\omega_3^2c_s^2\omega_4^2 - 12\omega_5^2\omega_3^2c_s^2 + \\
&24\omega_5^2v_1^2\omega_3\omega_4^2 + 24\omega_2v_1^2\omega_3^2\omega_4^2 - 30\omega_5^2v_1^2\omega_3\omega_4^2 + \omega_5^2\omega_3^2c_s^2\omega_4^2 - 12\omega_2v_1^2\omega_3^3\omega_4 - 12\omega_5^2v_1^2\omega_3^3 - 12\omega_5^2\omega_3^2c_s^2\omega_4^2 - 12\omega_5^2\omega_3^2c_s^2\omega_4 + 24\omega_5^2\omega_3^2c_s^2\omega_4 \\
C_{20} &= -11\omega_5^2\omega_3^2 - 42\omega_5^2v_2^2\omega_3 - 6\omega_5^2c_s^2 + \omega_5^2\omega_3^3 - 2\omega_5^2\omega_3^2c_s^2 + 24\omega_5\omega_3c_s^2 + 12v_2^2\omega_3^2 - 12\omega_3^2 - 3\omega_5^2v_2^2\omega_3^3 + 12\omega_5^2c_s^2 + 27\omega_5^2v_2^2\omega_3^2 + 12\omega_5^2v_2^2 + 6\omega_3^3 + \\
&25\omega_5^2\omega_3^2c_s^2 - 6v_2^2\omega_3^3 + 12\omega_5^2\omega_3 + 15\omega_5v_2^2\omega_3^3 - 48\omega_5^2\omega_3c_s^2 - 24\omega_5\omega_3 - 36\omega_5\omega_3^2c_s^2 - 60\omega_5v_2^2\omega_3^2 - 9\omega_5\omega_3^3 + 24\omega_5^2c_s^2 + 48\omega_5v_2^2\omega_3 + 36\omega_5\omega_3^3 + 9\omega_5\omega_3^2c_s^2 \\
C_{21} &= -3\omega_5\omega_2^2\omega_3^3 + 6\omega_5^2\omega_3^2v_2^2\omega_3^2 + 12\omega_5\omega_3^2\omega_3 + 6\omega_5\omega_2^2v_2^2\omega_3^3 - 48\omega_5^2\omega_3^2\omega_3^2c_s^2 + 6\omega_5^2\omega_3^2c_s^2 - 12\omega_5\omega_2^2\omega_3^2c_s^2 + 6\omega_5^2\omega_2\omega_3^2c_s^2 - 12\omega_5\omega_2^2\omega_3^2 - 12\omega_5^2\omega_3^2c_s^2 + \\
&6\omega_5^2v_2^2\omega_3^3 - 12\omega_5^2v_2^2\omega_3^2 + 6\omega_5^2\omega_3^2\omega_3^2c_s^2 - 12\omega_5^2\omega_2v_2^2\omega_3^3 - 24\omega_5^2\omega_2\omega_3^2c_s^2 + 6\omega_5\omega_2^2\omega_3^3 + 6\omega_5^2v_2^2\omega_3^3 - 12\omega_5^2\omega_2\omega_3^2c_s^2 - 36\omega_5^2\omega_3^2c_s^2 - 21\omega_5\omega_3^2\omega_3^2 - 24\omega_5\omega_2^2\omega_3^2c_s^2 - \\
&30\omega_5^2\omega_3^2v_2^2\omega_3 + 6\omega_5\omega_2^2\omega_3^2c_s^2 + 6\omega_5^2\omega_2v_2^2\omega_3^3 + 42\omega_5\omega_2^2\omega_3^2c_s^2 - \omega_5^2\omega_2^2\omega_3^3 + 42\omega_5\omega_2^2v_2^2\omega_3^2 + 6\omega_5^2\omega_2^2\omega_3^2 + 6\omega_5^2\omega_2^2v_2^2\omega_3^3 - 12\omega_5\omega_2^2v_2^2\omega_3^3 - 12\omega_5^2\omega_2^2v_2^2\omega_3^2 - 3\omega_5^2\omega_3^3 + \\
&7\omega_5^2\omega_2^2\omega_3^2 + 42\omega_5^2\omega_2^2\omega_3^2c_s^2 + 12\omega_5^2\omega_2^2v_2^2\omega_3 - 6\omega_5^2\omega_2^2\omega_3 - 12\omega_5\omega_2^2\omega_3^2c_s^2 + \omega_5^2\omega_2^2\omega_3^3 + 24\omega_5^2\omega_2^2v_2^2 - 3\omega_5^2\omega_2^2\omega_3^2 - 12\omega_5^2\omega_2^2\omega_3^2c_s^2 + 78\omega_5^2\omega_2^2\omega_3^2c_s^2 - 24\omega_5\omega_2^2v_2^2\omega_3 \\
C_{22} &= 11\omega_5^2\omega_3^2 + 6\omega_5^2c_s^2 - \omega_5^2\omega_3^3 + 4\omega_5^2\omega_3^2c_s^2 - 36\omega_5\omega_3c_s^2 - 12v_2^2\omega_3^2 + 12\omega_3^2 + \omega_5^2v_2^2\omega_3^3 - 12\omega_3^2c_s^2 - 8\omega_5^2v_2^2\omega_3^2 + 12\omega_5^2v_2^2 - 6\omega_3^3 - 44\omega_5^2\omega_3^2c_s^2 + \\
&6v_2^2\omega_3^3 - 12\omega_5\omega_3 - 12\omega_5v_2^2\omega_3^3 + 90\omega_5^2\omega_3c_s^2 + 24\omega_5\omega_3 + 48\omega_5\omega_3^2c_s^2 + 48\omega_5v_2^2\omega_3^2 + 9\omega_5\omega_3^3 - 48\omega_5^2c_s^2 - 36\omega_5v_2^2\omega_3 - 36\omega_5\omega_3^2 - 12\omega_5\omega_3^2c_s^2 \\
C_{23} &= -6\omega_5\omega_2^2\omega_3^3 + 12\omega_5\omega_2^2v_2^2\omega_3^2 - 32\omega_5^2\omega_2^2\omega_3^2c_s^2 + 6\omega_5^2\omega_3^2c_s^2 - 24\omega_5\omega_2^2\omega_3^2c_s^2 + 3\omega_5^2\omega_2^2v_2^2\omega_3^3 + 6\omega_5^2\omega_2\omega_3^2c_s^2 - 24\omega_5\omega_2^2v_2^2\omega_3^2 + 12\omega_5\omega_2^2\omega_3^2 - \\
&12\omega_5^2\omega_3^2c_s^2 + 12\omega_5^2v_2^2\omega_3^3 - 12\omega_5^2v_2^2\omega_3^2 + 4\omega_5^2\omega_2^2\omega_3^2c_s^2 - 18\omega_5^2\omega_2v_2^2\omega_3^3 - 24\omega_5^2\omega_2\omega_3^2c_s^2 + 3\omega_5\omega_2^2\omega_3^3 + 6\omega_5^2v_2^2\omega_3^3 - 12\omega_5^2\omega_2\omega_3^2c_s^2 - 12\omega_5^2\omega_3^2c_s^2 - \\
&6\omega_5\omega_2^2\omega_3^2 - 12\omega_5\omega_2^2\omega_3^2c_s^2 - 30\omega_5^2\omega_2^2v_2^2\omega_3 + 12\omega_5\omega_2^2\omega_3^2c_s^2 + 36\omega_5\omega_2^2\omega_3^2c_s^2 - \omega_5^2\omega_2^2\omega_3^3 + 36\omega_5\omega_2^2v_2^2\omega_3^2 - 12\omega_5\omega_2^2v_2^2\omega_3^3 + 12\omega_5^2v_2^2\omega_3^2 + 3\omega_5^2\omega_3^2\omega_3^2 + \\
&48\omega_5^2\omega_2^2c_s^2 - 12\omega_5\omega_2^2\omega_3^2c_s^2 + 2\omega_5^2\omega_2^2\omega_3^2 + 24\omega_5^2\omega_2^2v_2^2 - 6\omega_5^2\omega_2^2\omega_3^2 - 12\omega_5^2\omega_2^2\omega_3^2c_s^2 + 36\omega_5^2\omega_2^2\omega_3^2c_s^2 - 12\omega_5\omega_2^2v_2^2\omega_3 \\
C_{24} &= 72\omega_5v_2^4\omega_3^2 - 24\omega_5^2v_2^2\omega_3 + 24\omega_5^2c_s^4 - 18\omega_5v_2^4\omega_3^3 + 48\omega_5v_2^2\omega_3^2c_s^2 + 6\omega_5\omega_3^3c_s^4 + \omega_5^2\omega_3^3c_s^2 - 24\omega_5\omega_3c_s^2 + 24v_2^2\omega_3^2 - 3\omega_5^2v_2^2\omega_3^3 + 156\omega_5^2v_2^2\omega_3c_s^2 - \\
&48\omega_5^2\omega_3c_s^4 + 24\omega_5^2v_2^2\omega_3^2 - 8\omega_5^2\omega_3^2c_s^2 - 12v_2^2\omega_3^3 - 12\omega_5v_2^2\omega_3^3c_s^2 - 24\omega_5\omega_3^2c_s^4 - 48\omega_5v_2^4\omega_3 + 18\omega_5v_2^2\omega_3^3 + 12\omega_5^2\omega_3c_s^2 + 24\omega_5\omega_3c_s^4 + 12v_2^2\omega_3^3c_s^2 + \\
&24\omega_5\omega_3^2c_s^2 - 96\omega_5^2v_2^2c_s^2 - 72\omega_5v_2^2\omega_3^2 - 72\omega_5^2v_2^2\omega_3^2c_s^2 + 24\omega_5^2v_2^4\omega_3 + 24\omega_5^2\omega_3^2c_s^4 - 24\omega_5^2v_2^4\omega_3^2 + 12v_2^4\omega_3^3 - 24\omega_5v_2^2\omega_3c_s^2 + 48\omega_5v_2^2\omega_3 - 3\omega_5^2\omega_3^3c_s^4 - \\
&24v_2^4\omega_3^2 + 3\omega_5^2v_2^4\omega_3^2 + 6\omega_5^2v_2^2\omega_3^2c_s^2 - 24v_2^2\omega_3^2c_s^2 - 6\omega_5\omega_3^2c_s^2 \\
C_{25} &= 8\omega_5^2\omega_3^2 + 24\omega_5^2v_2^2\omega_3 + 6\omega_5^2c_s^2 - \omega_5^2\omega_3^3 + \omega_5^2\omega_3^2c_s^2 - 12\omega_5\omega_3c_s^2 - 12v_2^2\omega_3^2 + 12\omega_3^2 + 2\omega_5^2v_2^2\omega_3^3 - 12\omega_3^2c_s^2 - 16\omega_5^2v_2^2\omega_3^2 - 12\omega_5^2v_2^2 - 6\omega_3^3 - \\
&20\omega_5^2\omega_3^2c_s^2 + 6v_2^2\omega_3^3 - 6\omega_5^2\omega_3 - 6\omega_5v_2^2\omega_3^3 + 42\omega_5^2\omega_3c_s^2 + 12\omega_5\omega_3 + 24\omega_5\omega_3^2c_s^2 + 24\omega_5v_2^2\omega_3^2 + 6\omega_5\omega_3^3 - 24\omega_5^2c_s^2 - 12\omega_5v_2^2\omega_3 - 24\omega_5\omega_3^2 - 6\omega_5\omega_3^2c_s^2
\end{aligned}$$

2.4 CLBM1

2.4.1 Definitions

Collision operator C :

$$C(f) = \mathbf{K}^{-1} \mathbf{S} \left(\boldsymbol{\kappa}^{(eq)} - \mathbf{K} f \right),$$

where

$$\mathbf{S} = \text{diag}(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5),$$

$\omega_1, \omega_2, \dots, \omega_5 \in (0, 2)$.

Matrix \mathbf{K} corresponds to the transformation matrix to the central moment basis defined by

$$\boldsymbol{\kappa} = \left(k_{(0,0)}, k_{(1,0)}, k_{(0,1)}, k_{(2,0)}, k_{(0,2)} \right)^T,$$

and is given by

$$\mathbf{K} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -v_1 & 1 - v_1 & -v_1 & -v_1 - 1 & -v_1 \\ -v_2 & -v_2 & 1 - v_2 & -v_2 & -v_2 - 1 \\ v_1^2 & (1 - v_1)^2 & v_1^2 & (v_1 + 1)^2 & v_1^2 \\ v_2^2 & v_2^2 & (1 - v_2)^2 & v_2^2 & (v_2 + 1)^2 \end{pmatrix}.$$

The equilibrium central moments are defined by

$$\boldsymbol{\kappa}^{(eq)} = \mathbf{K} \mathbf{M}^{-1} \boldsymbol{\mu}^{(eq)},$$

i.e.,

$$\kappa^{(eq)} = \left(\rho, 0, 0, \rho c_s^2, \rho c_s^2 \right)^T.$$

2.4.2 Conservation of mass equation



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$$\begin{aligned} & \frac{\partial \rho}{\partial t} + \frac{v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_2) \frac{\delta_l}{2\omega_2} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial t} + (-2 + \omega_2) \frac{v_1 \delta_l^2}{2\delta_t \omega_2} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + \\ & (-2 + \omega_2) \frac{\rho \delta_l^2}{2\delta_t \omega_2} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + (2 - \omega_3) \frac{v_2 \delta_l^2}{2\omega_3 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (\omega_3 - \omega_3 \omega_2 + \omega_2) \frac{v_1 \delta_l^2}{\omega_3 \delta_t \omega_2} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_2} + \\ & (\omega_3 - \omega_3 \omega_2 + \omega_2) \frac{\rho \delta_l^2}{\omega_3 \delta_t \omega_2} \frac{\partial v_1}{\partial x_1} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_3) \frac{\delta_l}{2\omega_3} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial t} + (\omega_3 - \omega_3 \omega_2 + \omega_2) \frac{v_2 \delta_l^2}{\omega_3 \delta_t \omega_2} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_1} + (2 - \omega_2) \frac{v_1 \delta_l^2}{2\delta_t \omega_2} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} \\ & + (-2 + \omega_3) \frac{v_2 \delta_l^2}{2\omega_3 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_3) \frac{\rho \delta_l^2}{2\omega_3 \delta_t} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + (-2 + \omega_2) \frac{\rho \delta_l}{2\omega_2} \frac{\partial^2 v_1}{\partial t \partial x_1} + (-2 + \omega_2) \frac{c_s^2 \delta_l^2}{2\delta_t \omega_2} \frac{\partial^2 \rho}{\partial x_1^2} + \\ & (-2 + \omega_2) \frac{\rho v_1 \delta_l^2}{2\delta_t \omega_2} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + (-2 + \omega_3) \frac{\rho \delta_l}{2\omega_3} \frac{\partial^2 v_2}{\partial t \partial x_2} + (\omega_3 - \omega_3 \omega_2 + \omega_2) \frac{v_1 v_2 \delta_l^2}{\omega_3 \delta_t \omega_2} \frac{\partial^2 \rho}{\partial x_1 \partial x_2} + (2 - \omega_3) \frac{\rho v_2 \delta_l^2}{2\omega_3 \delta_t} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + \\ & (2 - \omega_2) \frac{\rho v_1 \delta_l^2}{2\delta_t \omega_2} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_3) \frac{c_s^2 \delta_l^2}{2\omega_3 \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + (-2 + \omega_3) \frac{\rho v_2 \delta_l^2}{2\omega_3 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + (12 + \omega_2^2 - 12\omega_2) \frac{\delta_t \rho \delta_l}{12\omega_2^2} \frac{\partial^3 v_1}{\partial t^2 \partial x_1} + \\ & (12 + \omega_2^2 - 12\omega_2) \frac{\rho v_1 \delta_l^2}{6\omega_2^2} \frac{\partial^3 v_1}{\partial t \partial x_1^2} + C_1 \frac{v_1 \delta_l^3}{6\delta_t \omega_4 \omega_2} \frac{\partial^3 \rho}{\partial x_1^3} + C_2 \frac{\rho \delta_l^3}{12\delta_t \omega_4 \omega_2^2} \frac{\partial^3 v_1}{\partial x_1^3} + (12 - 12\omega_3 + \omega_3^2) \frac{\delta_t \rho \delta_l}{12\omega_3^2} \frac{\partial^3 v_2}{\partial t^2 \partial x_2} + \\ & (-6\omega_3 - 2\omega_3^2 \omega_2 + 3\omega_3^2 + 9\omega_3 \omega_2 - 6\omega_2) \frac{\rho v_2 \delta_l^2}{6\omega_3^2 \omega_2} \frac{\partial^3 v_1}{\partial t \partial x_1 \partial x_2} + (-6\omega_3 + 9\omega_3 \omega_2 + 3\omega_2^2 - 6\omega_2 - 2\omega_3 \omega_2^2) \frac{\rho v_1 \delta_l^2}{6\omega_3 \omega_2^2} \frac{\partial^3 v_2}{\partial t \partial x_1 \partial x_2} + \\ & C_3 \frac{v_2 \delta_l^3}{2\omega_3^2 \delta_t \omega_4 \omega_2^2} \frac{\partial^3 \rho}{\partial x_1^3 \partial x_2} + (\omega_3^2 \omega_2^2 - 6\omega_3^2 \omega_2 + 6\omega_3^2 + 6\omega_2^2 - 6\omega_3 \omega_2^2) \frac{\rho v_1 v_2 \delta_l^3}{6\omega_3^2 \delta_t \omega_2^2} \frac{\partial^3 v_1}{\partial x_1^3 \partial x_2} + \\ & (6v_1^2 \omega_4 \omega_2 - 3c_s^2 \omega_4 \omega_2^2 - 12c_s^2 \omega_2 + 6c_s^2 \omega_2^2 + v_1^2 \omega_4 \omega_2^2 - 12c_s^2 \omega_4 + 18c_s^2 \omega_4 \omega_2 + 12v_1^2 \omega_2 - 12v_1^2 \omega_4 - 6v_1^2 \omega_2^2) \frac{\rho \delta_l^3}{12\delta_t \omega_4 \omega_2^2} \frac{\partial^3 v_2}{\partial x_1^3 \partial x_2} \\ & + (12 - 12\omega_3 + \omega_3^2) \frac{\rho v_2 \delta_l^2}{6\omega_3^2} \frac{\partial^3 v_2}{\partial t \partial x_2^2} + C_4 \frac{v_1 \delta_l^3}{2\omega_3^2 \delta_t \omega_5 \omega_2^2} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + \\ & (-12\omega_3 c_s^2 + \omega_3^2 v_2^2 \omega_5 - 12c_s^2 \omega_5 - 6\omega_3^2 v_2^2 - 3\omega_3^2 c_s^2 \omega_5 - 12v_2^2 \omega_5 + 18\omega_3 c_s^2 \omega_5 + 12\omega_3 v_2^2 + 6\omega_3 v_2^2 \omega_5 + 6\omega_3^2 c_s^2) \frac{\rho \delta_l^3}{12\omega_3^2 \delta_t \omega_5} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} \\ & + (\omega_3^2 \omega_2^2 - 6\omega_3^2 \omega_2 + 6\omega_3^2 + 6\omega_2^2 - 6\omega_3 \omega_2^2) \frac{\rho v_1 v_2 \delta_l^3}{6\omega_3^2 \delta_t \omega_2^2} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_5 \frac{v_2 \delta_l^3}{6\omega_3 \delta_t \omega_5} \frac{\partial^3 \rho}{\partial x_2^3} + C_6 \frac{\rho \delta_l^3}{12\omega_3^2 \delta_t \omega_5} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & (-2 - \omega_2^2 + 3\omega_2) \frac{\delta_t \rho \delta_l}{2\omega_2^2} \frac{\partial^4 v_1}{\partial t^3 \partial x_1} + (-2 - \omega_2^2 + 3\omega_2) \frac{3\delta_t \rho v_1 \delta_l^2}{2\omega_2^2} \frac{\partial^4 v_1}{\partial t^2 \partial x_1^2} + C_7 \frac{\rho \delta_l^3}{12\omega_4^2 \omega_2^2} \frac{\partial^4 v_1}{\partial t \partial x_1^3} + C_8 \frac{\delta_l^4}{24\delta_t \omega_4^2 \omega_2^2} \frac{\partial^4 \rho}{\partial x_1^4} + \\ & C_9 \frac{\rho v_1 \delta_l^4}{12\delta_t \omega_4^2 \omega_2^2} \frac{\partial^4 v_1}{\partial x_1^4} + (-2 + 3\omega_3 - \omega_3^2) \frac{\delta_t^2 \rho \delta_l}{2\omega_3^2} \frac{\partial^4 v_2}{\partial t^3 \partial x_2} + \\ & (7\omega_3^2 \omega_2 + 13\omega_3^2 \omega_2^2 - 24\omega_3^2 \omega_2 + 12\omega_3^2 - 6\omega_3^3 - \omega_3^3 \omega_2^2 + 12\omega_3 \omega_2 + 12\omega_2^2 - 24\omega_3 \omega_2^2) \frac{\delta_t \rho v_2 \delta_l^2}{12\omega_3^2 \omega_2^2} \frac{\partial^4 v_1}{\partial t^2 \partial x_1 \partial x_2} + \\ & (-\omega_3^2 \omega_2^2 + 13\omega_3^2 \omega_2^2 - 24\omega_3^2 \omega_2 + 12\omega_3^2 - 6\omega_3^3 + 12\omega_3 \omega_2 + 12\omega_2^2 - 24\omega_3 \omega_2^2 + 7\omega_3 \omega_2^3) \frac{\delta_t \rho v_1 \delta_l^2}{12\omega_3^2 \omega_2^2} \frac{\partial^4 v_2}{\partial t^2 \partial x_1 \partial x_2} + \\ & (-7\omega_3^2 \omega_2^2 + 18\omega_3^2 \omega_2 + 6\omega_3^2 \omega_2^2 + \omega_3^3 \omega_2^2 - 12\omega_3^3 - 7\omega_3^3 \omega_2^2 - 6\omega_2^3 - 6\omega_3 \omega_2^2 + 12\omega_3 \omega_2^3) \frac{\rho v_1 v_2 \delta_l^3}{6\omega_3^2 \omega_2^2} \frac{\partial^4 v_1}{\partial t \partial x_1^2 \partial x_2} + \\ & C_{10} \frac{\rho \delta_l^3}{12\omega_3 \omega_4^2 \omega_2^2} \frac{\partial^4 v_2}{\partial t \partial x_1^2 \partial x_2} + C_{11} \frac{v_1 v_2 \delta_l^4}{6\omega_3^2 \delta_t \omega_4^2 \omega_2^2} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2} + C_{12} \frac{\rho v_2 \delta_l^4}{12\omega_3^2 \delta_t \omega_4^2 \omega_2^2} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2} + C_{13} \frac{\rho v_1 \delta_l^4}{12\delta_t \omega_4^2 \omega_2^2} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2} + \\ & (-2 + 3\omega_3 - \omega_3^2) \frac{3\delta_t \rho v_2 \delta_l^2}{2\omega_3^2} \frac{\partial^4 v_2}{\partial t^3 \partial x_2^2} + C_{14} \frac{\rho \delta_l^3}{12\omega_3^2 \omega_5^2 \omega_2} \frac{\partial^4 v_1}{\partial t \partial x_1 \partial x_2^2} + \\ & (-7\omega_3^2 \omega_2^2 + 12\omega_3^2 \omega_2 + 6\omega_3^2 \omega_2^2 + \omega_3^3 \omega_2^2 - 6\omega_3^3 \omega_2 - 6\omega_3^3 - 7\omega_3^3 \omega_2^2 - 12\omega_2^3 + 18\omega_3 \omega_2^3) \frac{\rho v_1 v_2 \delta_l^3}{6\omega_3^2 \omega_2^2} \frac{\partial^4 v_2}{\partial t \partial x_1 \partial x_2^2} + \\ & C_{15} \frac{\delta_l^4}{4\omega_3^2 \delta_t \omega_4^2 \omega_5^2 \omega_2^2} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_{16} \frac{\rho v_1 \delta_l^4}{12\omega_3^2 \delta_t \omega_5^2 \omega_2^2} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{17} \frac{\rho v_2 \delta_l^4}{12\omega_3^2 \delta_t \omega_5^2 \omega_2^2} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_{18} \frac{\rho \delta_l^3}{12\omega_3^2 \omega_5^2} \frac{\partial^4 v_2}{\partial t \partial x_2^3} + \\ & C_{19} \frac{v_1 v_2 \delta_l^4}{6\omega_3^2 \delta_t \omega_5^2 \omega_2^2} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{20} \frac{\rho v_2 \delta_l^4}{12\omega_3^2 \delta_t \omega_5^2} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{21} \frac{\rho v_1 \delta_l^4}{12\omega_3^2 \delta_t \omega_5^2 \omega_2^2} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + C_{22} \frac{\delta_l^4}{24\omega_3^2 \delta_t \omega_5^2} \frac{\partial^4 \rho}{\partial x_2^4} + C_{23} \frac{\rho v_2 \delta_l^4}{12\omega_3^2 \delta_t \omega_5^2} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:

$$\begin{aligned} C_1 &= 6 - v_1^2 \omega_4 \omega_2 + 9c_s^2 \omega_2 - 3\omega_4 - 18c_s^2 + 9c_s^2 \omega_4 - 3c_s^2 \omega_4 \omega_2 + 3v_1^2 \omega_2 - 6v_1^2 + 3v_1^2 \omega_4 - 3\omega_2 + \omega_4 \omega_2 \\ C_2 &= 6v_1^2 \omega_4 \omega_2 - 3c_s^2 \omega_4 \omega_2^2 - 12c_s^2 \omega_2 + 6c_s^2 \omega_2^2 - 5v_1^2 \omega_4 \omega_2^2 - 12c_s^2 \omega_4 + 18c_s^2 \omega_4 \omega_2 - 36v_1^2 \omega_2 + 2\omega_4 \omega_2^2 - 6\omega_2^2 + 12v_1^2 \omega_4 + 12\omega_2 - 6\omega_4 \omega_2 + 18v_1^2 \omega_2^2 \end{aligned}$$

$$C_3 = -2\omega_3^2 c_s^2 \omega_4 - 3\omega_3 v_1^2 \omega_4 \omega_2^2 - 2\omega_3 c_s^2 \omega_4 \omega_2 + \omega_3^2 c_s^2 \omega_2^2 - 2\omega_3^2 c_s^2 \omega_2 + 2\omega_3 v_1^2 \omega_4 \omega_2 + \omega_3 c_s^2 \omega_4 \omega_2^2 + 2v_1^2 \omega_4 \omega_2^2 + \omega_3^2 v_1^2 \omega_4 \omega_2^2 + 4\omega_3^2 c_s^2 \omega_4 \omega_2 - \omega_3^2 v_1^2 \omega_2^2 - 2\omega_3^2 v_1^2 \omega_4 \omega_2 - \omega_3^2 c_s^2 \omega_4 \omega_2^2 + 2\omega_3^2 v_1^2 \omega_2$$

$$C_4 = 2\omega_3^2 v_2^2 \omega_5 + \omega_3^2 c_s^2 \omega_2^2 - \omega_3^2 c_s^2 \omega_5 \omega_2 + 2\omega_3 v_2^2 \omega_5 \omega_2 + \omega_3^2 c_s^2 \omega_5 \omega_2 - \omega_3^2 v_2^2 \omega_2^2 - 2\omega_3 v_2^2 \omega_5 \omega_2^2 + 2\omega_3 v_2^2 \omega_2^2 - 3\omega_3^2 v_2^2 \omega_5 \omega_2 + 4\omega_3 c_s^2 \omega_5 \omega_2^2 - 2c_s^2 \omega_5 \omega_2^2 + \omega_3^2 v_2^2 \omega_5 \omega_2^2 - 2\omega_3 c_s^2 \omega_5 \omega_2 - 2\omega_3 c_s^2 \omega_2^2$$

$$C_5 = 6 + 9\omega_3 c_s^2 + 9c_s^2 \omega_5 - 3\omega_3 + 3v_2^2 \omega_5 - 18c_s^2 + \omega_3 \omega_5 - 3\omega_3 c_s^2 \omega_5 + 3\omega_3 v_2^2 - 3\omega_5 - 6v_2^2 - \omega_3 v_2^2 \omega_5$$

$$C_6 = -12\omega_3 c_s^2 - 5\omega_3^2 v_2^2 \omega_5 - 12c_s^2 \omega_5 + 12\omega_3 - 6\omega_3^2 + 18\omega_3^2 v_2^2 - 3\omega_3^2 c_s^2 \omega_5 + 12v_2^2 \omega_5 + 2\omega_3^2 \omega_5 - 6\omega_3 \omega_5 + 18\omega_3 c_s^2 \omega_5 - 36\omega_3 v_2^2 + 6\omega_3 v_2^2 \omega_5 + 6\omega_3^2 c_s^2$$

$$C_7 = 12\omega_4^2 \omega_2 + 24c_s^2 \omega_4^2 + 72v_1^2 \omega_4 \omega_2 - 36c_s^2 \omega_4 \omega_2^2 + 9c_s^2 \omega_4 \omega_2^3 + 12c_s^2 \omega_2^2 + 27v_1^2 \omega_4 \omega_2^3 + \omega_4^2 \omega_2^3 - 6c_s^2 \omega_2^3 - 11\omega_4^2 \omega_2^2 - 108v_1^2 \omega_4 \omega_2^2 + 24c_s^2 \omega_4 \omega_2 + 6\omega_3^2 + 36\omega_4 \omega_2^2 - 48c_s^2 \omega_4 \omega_2 + 15v_1^2 \omega_4^2 \omega_2^2 - 12\omega_2^2 - 3v_1^2 \omega_4^2 \omega_2^2 - 36v_1^2 \omega_4^2 - 9\omega_4 \omega_2^3 - 2c_s^2 \omega_4^2 \omega_2^2 - 18v_1^2 \omega_2^3 - 24\omega_4 \omega_2 + 25c_s^2 \omega_4^2 \omega_2^2 + 18v_1^2 \omega_4^2 \omega_2 + 36v_1^2 \omega_2^2$$

$$C_8 = 24c_s^2 \omega_4 \omega_2^2 - 30v_1^4 \omega_4 \omega_2^3 - 6c_s^2 \omega_4 \omega_2^3 + 72c_s^2 v_1^2 \omega_4 \omega_2 + 24c_s^4 \omega_4 \omega_2 + 72v_1^4 \omega_4 \omega_2^2 + 108c_s^2 v_1^2 \omega_2^3 - 24c_s^4 \omega_4 \omega_2^2 + 144c_s^2 v_1^2 \omega_4 \omega_2^2 - 72v_1^4 \omega_2^2 + 30v_1^2 \omega_4 \omega_2^3 + 6c_s^4 \omega_4 \omega_2^3 - 72c_s^2 v_1^2 \omega_4 \omega_2^3 - 216c_s^2 v_1^2 \omega_2^3 + 36v_1^4 \omega_2^3 - 72v_1^2 \omega_4 \omega_2^2 - 24c_s^2 \omega_4 \omega_2 + 6c_s^2 v_1^2 \omega_4^2 \omega_2^3 - 3c_s^4 \omega_4^2 \omega_2^3 + 12c_s^2 \omega_4^2 \omega_2 + 12v_1^2 \omega_4^2 \omega_2^2 + 24c_s^4 \omega_4^2 - 12c_s^2 v_1^2 \omega_4^2 \omega_2^2 + 24c_s^4 \omega_4^2 \omega_2^2 - 3v_1^4 \omega_4^2 \omega_2^3 + c_s^2 \omega_4^2 \omega_2^3 - 36v_1^2 \omega_2^3 - 12v_1^4 \omega_4^2 \omega_2^2 - 48c_s^4 \omega_4^2 \omega_2 - 36c_s^2 v_1^2 \omega_4^2 \omega_2 - 8c_s^2 \omega_2^2 \omega_2^2 + 3v_1^4 \omega_4^2 \omega_2^3 + 72v_1^2 \omega_2^2$$

$$C_9 = 6\omega_4^2 \omega_2 + 24c_s^2 \omega_4^2 + 60v_1^2 \omega_4 \omega_2 + 72c_s^2 \omega_4 \omega_2^2 - 24c_s^2 \omega_4 \omega_2^3 - 60c_s^2 \omega_2^2 - 24v_1^2 \omega_4 \omega_2^3 - \omega_4^2 \omega_2^3 + 30c_s^2 \omega_2^3 + 2\omega_4^2 \omega_2^2 + 24v_1^2 \omega_4 \omega_2^2 - 12c_s^2 \omega_4 \omega_2 - 18\omega_2^3 - 24\omega_4 \omega_2^2 - 30c_s^2 \omega_4^2 \omega_2 + 2v_1^2 \omega_4^2 \omega_2^2 + 36\omega_2^2 + 2v_1^2 \omega_4^2 \omega_2^2 - 12v_1^4 \omega_4^2 + 12\omega_4 \omega_2^3 + c_s^2 \omega_4^2 \omega_2^3 + 42v_1^2 \omega_2^3 - 12\omega_4 \omega_2 - 2c_s^2 \omega_4^2 \omega_2 - 12v_1^2 \omega_4^2 \omega_2 - 84v_1^2 \omega_2^2$$

$$C_{10} = 24\omega_3 v_1^2 \omega_4^2 + 12c_s^2 \omega_4 \omega_2^2 + 30\omega_3 v_1^2 \omega_4 \omega_2^2 + 12\omega_3 c_s^2 \omega_4 \omega_2 - 6c_s^2 \omega_4 \omega_2^3 - 9\omega_3 v_1^2 \omega_4 \omega_2^3 + 9\omega_3 c_s^2 \omega_4 \omega_2^3 - 12\omega_3 v_1^2 \omega_2^2 + 6v_1^2 \omega_4 \omega_2^3 - 12\omega_3 v_1^2 \omega_4 \omega_2 + 6\omega_3 v_1^2 \omega_2^3 - 30\omega_3 c_s^2 \omega_4 \omega_2^2 - 12v_1^2 \omega_4 \omega_2^2 + 22\omega_3 c_s^2 \omega_4^2 \omega_2^2 - 36\omega_3 v_1^2 \omega_4^2 \omega_2 + 12c_s^2 \omega_4^2 \omega_2 - 6v_1^2 \omega_4^2 \omega_2^2 - 2\omega_3 c_s^2 \omega_4^2 \omega_2^3 - v_1^2 \omega_4^2 \omega_2^3 + 12\omega_3 c_s^2 \omega_4^2 + 3c_s^2 \omega_4^2 \omega_2^2 - 6\omega_3 c_s^2 \omega_2^3 + \omega_3 v_1^2 \omega_4^2 \omega_2^3 - 18c_s^2 \omega_4^2 \omega_2^2 + 12v_1^2 \omega_4^2 \omega_2 - 30\omega_3 c_s^2 \omega_4^2 \omega_2 + 12\omega_3 c_s^2 \omega_2^2 + 8\omega_3 v_1^2 \omega_4^2 \omega_2$$

$$C_{11} = -6\omega_3^2 v_1^2 \omega_4^2 \omega_2^2 + 36\omega_3^2 c_s^2 \omega_2^2 \omega_2 - 3\omega_3^2 \omega_4^2 \omega_2^2 + 36\omega_3^2 c_s^2 \omega_4^2 \omega_2^2 - 6\omega_3^2 v_1^2 \omega_2^3 + \omega_3^2 \omega_4^2 \omega_2^3 + 12\omega_3^2 \omega_4 \omega_2 + 12\omega_3^2 v_1^2 \omega_2^2 - 12\omega_3^2 c_s^2 \omega_4^2 \omega_2^3 + 6\omega_3^2 v_1^2 \omega_4^2 \omega_2^3 - 3\omega_3^2 \omega_2^3 + 6\omega_3^2 c_s^2 \omega_4^2 \omega_2^3 - 21\omega_3^2 \omega_4 \omega_2^2 - 12\omega_3^2 c_s^2 \omega_4^2 \omega_2 + 6\omega_3^2 \omega_2^2 - 6\omega_3^2 v_1^2 \omega_4^2 \omega_2^2 + 6\omega_3^2 \omega_4 \omega_2^3 + 12\omega_3^2 v_1^2 \omega_4^2 \omega_2 - 36\omega_3^2 c_s^2 \omega_2^2 \omega_2^2 - 36\omega_3^2 c_s^2 \omega_2^2 - 12\omega_3 c_s^2 \omega_2^2 \omega_2^2 - \omega_3^2 \omega_4^2 \omega_2^3 + 72\omega_3^2 c_s^2 \omega_4 \omega_2^2 - 24\omega_3^2 v_1^2 \omega_4 \omega_2 + 6\omega_3 c_s^2 \omega_2^2 \omega_2^2 + 18\omega_3^2 c_s^2 \omega_2^3 + 6v_1^2 \omega_4^2 \omega_2^3 - 24\omega_3^2 c_s^2 \omega_4 \omega_2^3 + 7\omega_3^2 \omega_4^2 \omega_2^2 - 12\omega_3^2 c_s^2 \omega_4^2 - 3\omega_3^2 \omega_4 \omega_2^2 - 6\omega_3^2 \omega_2^2 \omega_2 - 12\omega_3 v_1^2 \omega_4^2 \omega_2^2 + 12\omega_3^2 c_s^2 \omega_4 \omega_2^2 - 24\omega_3^2 c_s^2 \omega_4 \omega_2 + 6\omega_3^2 \omega_4 \omega_2^2 + 12\omega_3^2 v_1^2 \omega_4 \omega_2^2 - 24\omega_3^2 c_s^2 \omega_4 \omega_2^2 + 6\omega_3 v_1^2 \omega_4^2 \omega_2^2$$

$$C_{12} = -12\omega_3^2 v_1^2 \omega_2^2 \omega_2^2 + 36\omega_3^2 c_s^2 \omega_4^2 \omega_2 - 6\omega_3^2 \omega_4^2 \omega_2^2 + 48\omega_3^2 c_s^2 \omega_4^2 \omega_2^2 - 6\omega_3^2 v_1^2 \omega_2^3 + 2\omega_3^2 \omega_4^2 \omega_2^3 + 3\omega_3^2 v_1^2 \omega_4^2 \omega_2^3 + 12\omega_3^2 v_1^2 \omega_2^2 - 12\omega_3^2 c_s^2 \omega_4^2 \omega_2^3 + 4\omega_3^2 c_s^2 \omega_2^2 \omega_2^2 - 6\omega_3^2 \omega_4 \omega_2^2 - 24\omega_3^2 c_s^2 \omega_4^2 \omega_2 + 12\omega_3^2 v_1^2 \omega_4^2 \omega_2 + 3\omega_3^2 \omega_4 \omega_2^3 - 24\omega_3^2 v_1^2 \omega_4^2 + 30\omega_3^2 v_1^2 \omega_2^2 \omega_2 - 32\omega_3^2 c_s^2 \omega_4^2 \omega_2^2 - 24\omega_3^2 v_1^2 \omega_4 \omega_2^2 - 12\omega_3^2 c_s^2 \omega_2^2 - 12\omega_3 c_s^2 \omega_4^2 \omega_2^2 + 12\omega_3^2 v_1^2 \omega_2^2 + 12\omega_3^2 v_1^2 \omega_4^2 \omega_2^2 + 12v_1^2 \omega_4^2 \omega_2^2 - 12\omega_3^2 c_s^2 \omega_4 \omega_2^2 + 3\omega_3^2 \omega_4^2 \omega_2^2 - 12\omega_3^2 c_s^2 \omega_4^2 - 6\omega_3^2 \omega_4 \omega_2^2 - 18\omega_3 v_1^2 \omega_4^2 \omega_2^2 + 12\omega_3^2 c_s^2 \omega_4 \omega_2^2 - 12\omega_3^2 c_s^2 \omega_4 \omega_2 + 12\omega_3^2 \omega_4 \omega_2^2 - 12\omega_3^2 v_1^2 \omega_4 \omega_2^2 - 24\omega_3^2 c_s^2 \omega_4 \omega_2^2$$

$$C_{13} = -12\omega_2^2 \omega_2 - 60v_1^2 \omega_4 \omega_2 + 96c_s^2 \omega_4 \omega_2^2 - 30c_s^2 \omega_4 \omega_2^3 - 60c_s^2 \omega_2^2 - 6v_1^2 \omega_4 \omega_2^3 - \omega_2^2 \omega_2^3 + 30c_s^2 \omega_2^3 + 11\omega_4^2 \omega_2^2 + 48v_1^2 \omega_4 \omega_2^2 - 36c_s^2 \omega_4 \omega_2 - 6\omega_2^3 - 36\omega_4 \omega_2^2 + 18c_s^2 \omega_4^2 \omega_2 - 14v_1^2 \omega_4^2 \omega_2^2 + 12\omega_2^2 + v_1^2 \omega_4^2 \omega_2^2 + 12v_1^2 \omega_4^2 + 9\omega_4 \omega_2^3 + 4c_s^2 \omega_4^2 \omega_2^2 - 6v_1^2 \omega_2^2 + 24\omega_4 \omega_2 - 26c_s^2 \omega_4^2 \omega_2^2 + 12v_1^2 \omega_4^2 \omega_2 + 12v_1^2 \omega_2^2$$

$$C_{14} = -12\omega_3^2 v_2^2 \omega_5 - 18\omega_3^2 c_s^2 \omega_5^2 + \omega_3^2 v_2^2 \omega_5^2 \omega_2 + 9\omega_3^2 c_s^2 \omega_5 \omega_2 - 12\omega_3^2 v_2^2 \omega_2 - 12\omega_3 v_2^2 \omega_5 \omega_2 - 30\omega_3 c_s^2 \omega_5^2 \omega_2 - 30\omega_3^2 c_s^2 \omega_5 \omega_2 + 12\omega_3^2 c_s^2 \omega_2 + 8\omega_3^2 v_2^2 \omega_5^2 \omega_2 + 12c_s^2 \omega_5^2 \omega_2 + 12\omega_3^2 c_s^2 \omega_5 - 6\omega_3^2 v_2^2 \omega_5 + 6\omega_3^2 v_2^2 \omega_2 + 3\omega_3^2 c_s^2 \omega_5 + 30\omega_3^2 v_2^2 \omega_5 \omega_2 + 6\omega_3^2 v_2^2 \omega_5 + 22\omega_3^2 c_s^2 \omega_5^2 \omega_2 + 12\omega_3 v_2^2 \omega_5^2 + 24v_2^2 \omega_5^2 \omega_2 - 2\omega_3^2 c_s^2 \omega_5^2 \omega_2 - 9\omega_3^2 v_2^2 \omega_5 \omega_2 - \omega_3^2 v_2^2 \omega_5^2 - 6\omega_3^2 c_s^2 \omega_5 + 12\omega_3 c_s^2 \omega_5 \omega_2 + 12\omega_3 c_s^2 \omega_5^2 - 36\omega_3 v_2^2 \omega_5^2 \omega_2 - 6\omega_3^2 c_s^2 \omega_2$$

$$C_{15} = -2\omega_3 c_s^4 \omega_1^2 \omega_5^2 \omega_2^3 - 2\omega_3^2 c_s^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^3 + 8\omega_3^2 c_s^2 v_1^2 \omega_1^2 \omega_5^2 \omega_2^2 - 2\omega_3^2 c_s^4 \omega_1 \omega_5^2 \omega_2^3 + 4\omega_3^2 v_1^2 v_2^2 \omega_1^2 \omega_2^3 + \omega_3^2 c_s^4 \omega_1^2 \omega_5^2 \omega_2^3 - 2\omega_3^2 v_1^2 v_2^2 \omega_1^2 \omega_2^3 - 2\omega_3^2 v_1^2 v_2^2 \omega_4 \omega_5^2 \omega_2^3 + 4\omega_3^2 v_1^2 v_2^2 \omega_4 \omega_5^2 \omega_2^2 - 12\omega_3^2 c_s^4 \omega_1^2 \omega_5^2 \omega_2^2 - 2\omega_3^2 c_s^4 \omega_1^2 \omega_5^2 \omega_2 - 4\omega_3^2 c_s^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2 + \omega_3^2 c_s^4 \omega_1 \omega_5^2 \omega_2^3 + 8\omega_3^2 c_s^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 - 8\omega_3^2 c_s^2 v_1^2 \omega_1^2 \omega_5^2 \omega_2^3 + 4\omega_3 c_s^4 \omega_1^2 \omega_5^2 \omega_2^2 - 10\omega_3^2 v_1^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^3 - 4\omega_3^2 c_s^2 v_1^2 \omega_1^2 \omega_5^2 \omega_2^2 + 4\omega_3^2 c_s^4 \omega_1^2 \omega_5^2 \omega_2^2 + 4\omega_3^2 v_1^2 v_2^2 \omega_1 \omega_5^2 \omega_2^2 + 4\omega_3^2 v_1^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 - 14\omega_3^2 v_1^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2 + 10\omega_3 c_s^2 v_1^2 \omega_1^2 \omega_5^2 \omega_2^3 + 14\omega_3^2 v_1^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 + 2\omega_3^2 c_s^2 v_1^2 \omega_1^2 \omega_5^2 \omega_2^2 - \omega_3^2 c_s^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^3 + 4\omega_3 v_1^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^3 + 10\omega_3^2 c_s^2 v_2^2 \omega_1 \omega_5^2 \omega_2^2 - \omega_3^2 c_s^2 v_1^2 \omega_1^2 \omega_5^2 \omega_2^2 - 4\omega_3^2 c_s^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2 - 3\omega_3^2 v_1^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 - 4\omega_3 c_s^2 v_1^2 \omega_1^2 \omega_5^2 \omega_2^2 + 4\omega_3^2 c_s^4 \omega_1^2 \omega_5^2 \omega_2 - 4\omega_3^2 c_s^2 v_1^2 \omega_1^2 \omega_5^2 \omega_2^3 - 3\omega_3^2 c_s^2 v_2^2 \omega_1 \omega_5^2 \omega_2^2 + 2\omega_3^2 c_s^2 v_1^2 \omega_1^2 \omega_5^2 \omega_2^2 - 3\omega_3^2 c_s^2 v_1^2 \omega_1^2 \omega_5^2 \omega_2^2 + 2\omega_3^2 v_1^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 + 2\omega_3^2 c_s^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 - 10\omega_3^2 v_1^2 v_2^2 \omega_1 \omega_5^2 \omega_2^2 + 3\omega_3^2 v_1^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 + 12\omega_3^2 v_1^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2 - 8\omega_3^2 c_s^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 + \omega_3^2 c_s^2 v_1^2 \omega_1^2 \omega_5^2 \omega_2^3 + 12\omega_3 v_1^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 - 2\omega_3^2 v_1^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2 + 2\omega_3^2 c_s^2 v_1^2 \omega_1^2 \omega_2^2 + 3\omega_3^2 v_1^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 - 4\omega_3 c_s^2 v_1^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 - 14\omega_3 v_1^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 + \omega_3^2 c_s^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 - 2\omega_3^2 c_s^2 v_1^2 \omega_1^2 \omega_5^2 \omega_2^2 - 2\omega_3^2 c_s^4 \omega_1^2 \omega_5^2 \omega_2 - 4c_s^2 v_1^2 \omega_1^2 \omega_5^2 \omega_2^3 + 4\omega_3^2 c_s^4 \omega_1^2 \omega_5^2 \omega_2^2 + 10\omega_3^2 c_s^2 v_1^2 \omega_1^2 \omega_5^2 \omega_2^2 - 4\omega_3^2 c_s^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 + 2\omega_3^2 c_s^2 v_2^2 \omega_1 \omega_5^2 \omega_2^2 - 4\omega_3^2 c_s^2 v_1^2 \omega_1^2 \omega_5^2 \omega_2^2 - 2\omega_3^2 v_1^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 + 4\omega_3^2 c_s^4 \omega_1^2 \omega_5^2 \omega_2^2 - 4\omega_3^2 c_s^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 - 2\omega_3^2 c_s^4 \omega_1^2 \omega_5^2 \omega_2^2 + 14\omega_3^2 v_1^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 + 4\omega_3^2 v_1^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 - 4\omega_3^2 c_s^2 v_2^2 \omega_1 \omega_5^2 \omega_2^2 + 2\omega_3^2 c_s^2 v_1^2 \omega_1^2 \omega_5^2 \omega_2^2 - 4\omega_3^2 c_s^2 v_1^2 \omega_1^2 \omega_5^2 \omega_2^2 + 2\omega_3^2 c_s^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 + 10\omega_3^2 c_s^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 - \omega_3^2 c_s^4 \omega_1^2 \omega_5^2 \omega_2^3 + 4\omega_3^2 v_1^2 v_2^2 \omega_1 \omega_5^2 \omega_2^2 + 4\omega_3^2 c_s^4 \omega_1^2 \omega_5^2 \omega_2^2 - 28\omega_3^2 v_1^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2 - 2\omega_3^2 c_s^4 \omega_1 \omega_5^2 \omega_2^2 + 2\omega_3^2 c_s^2 v_2^2 \omega_1^2 \omega_5^2 \omega_2^2$$

$$C_{16} = -12c_s^2 \omega_5^2 \omega_2^3 + 22\omega_3^2 v_2^2 \omega_5^2 \omega_2^3 - 12\omega_3^2 c_s^2 \omega_2^3 + 24\omega_3^2 c_s^2 \omega_5 \omega_2^3 - 30\omega_3^2 v_2^2 \omega_5^2 \omega_2 - 48\omega_3^2 v_2^2 \omega_5^2 \omega_2^2 + 24\omega_3^2 v_2^2 \omega_5^2 \omega_2 + 22\omega_3^2 v_2^2 \omega_5^2 \omega_2^2 + 24\omega_3 c_s^2 \omega_5^2 \omega_2^3 + 12\omega_3^2 v_2^2 \omega_5^2 \omega_2^3 + 12\omega_3^2 v_2^2 \omega_2^3 - 6\omega_3^2 c_s^2 \omega_5^2 \omega_2^3 - 4\omega_3^2 v_2^2 \omega_5^2 \omega_2^3 - 18\omega_3^2 v_2^2 \omega_5^2 \omega_2^3 - 12\omega_3 c_s^2 \omega_5^2 \omega_2^3 + 6\omega_3^2 v_2^2 \omega_5^2 \omega_2^3 + \omega_3^2 c_s^2 \omega_5^2 \omega_2^3 - 12\omega_3^2 c_s^2 \omega_5^2 \omega_2 + 6\omega_3^2 c_s^2 \omega_2^3 + 24\omega_3 v_2^2 \omega_5^2 \omega_2^2 - 6\omega_3^2 c_s^2 \omega_2^2 \omega_2^2 + 6\omega_3^2 c_s^2 \omega_2^2 \omega_2 + 12\omega_3^2 v_2^2 \omega_5^2 + 12\omega_3^2 c_s^2 \omega_5^2 \omega_2^2 - 6\omega_3^2 v_2^2 \omega_2^2 - 14\omega_3^2 c_s^2 \omega_5^2 \omega_2^2 - 24\omega_3^2 v_2^2 \omega_5^2 \omega_2^2$$

$$C_{17} = 22\omega_3^2 v_1^2 \omega_4^2 \omega_2^2 + 24\omega_3^2 c_s^2 \omega_4 \omega_2^2 + 12\omega_3^2 c_s^2 \omega_4^2 \omega_2^2 - 6\omega_3^2 v_1^2 \omega_2^2 + 24\omega_3^2 v_1^2 \omega_4^2 \omega_2 - 4\omega_3^2 v_1^2 \omega_4^2 \omega_2^3 + 12\omega_3^2 v_1^2 \omega_2^2 - 6\omega_3^2 c_s^2 \omega_4^2 \omega_2^3 + 22\omega_3^2 v_1^2 \omega_2^2 \omega_2^3 + \omega_3^2 c_s^2 \omega_4^2 \omega_2^3 - 48\omega_3^2 v_1^2 \omega_4^2 \omega_2^3 - 18\omega_3^2 v_1^2 \omega_4^2 \omega_2 - 14\omega_3^2 c_s^2 \omega_4^2 \omega_2^2 - 12\omega_3^2 c_s^2 \omega_2^2 - 12\omega_3 c_s^2 \omega_4^2 \omega_2^2 + 24\omega_3^2 c_s^2 \omega_4 \omega_2^2 + 12\omega_3^2 v_1^2 \omega_4 \omega_2 + 6\omega_3 c_s^2 \omega_4^2 \omega_2^2 + 6\omega_3^2 c_s^2 \omega_2^2 + 12v_1^2 \omega_4^2 \omega_2^2 - 6\omega_3^2 c_s^2 \omega_4 \omega_2^2 - 12\omega_3^2 c_s^2 \omega_4^2 + 6\omega_3^2 v_1^2 \omega_4 \omega_2^2 - 30\omega_3 v_1^2 \omega_4^2 \omega_2^2 - 12\omega_3^2 c_s^2 \omega_4 \omega_2 - 24\omega_3^2 v_1^2 \omega_4 \omega_2^2 + 24\omega_3 v_1^2 \omega_4^2 \omega_2^2$$

$$C_{18} = -18\omega_3^2 v_2^2 - 108\omega_3^2 v_2^2 \omega_5 + 25\omega_3^2 c_s^2 \omega_5^2 - 11\omega_3^2 \omega_5^2 - 36v_2^2 \omega_5^2 - 9\omega_3^2 \omega_5 + \omega_3^2 \omega_5^2 - 12\omega_3^2 + 36\omega_3^2 v_2^2 + 6\omega_3^3 - 36\omega_3^2 c_s^2 \omega_5 + 15\omega_3^2 v_2^2 \omega_5^2 + 36\omega_3^2 \omega_5 + 24c_s^2 \omega_5^2 - 24\omega_3 \omega_5 - 2\omega_3^2 c_s^2 \omega_5^2 + 27\omega_3^2 v_2^2 \omega_5 + 24\omega_3 c_s^2 \omega_5 + 18\omega_3 v_2^2 \omega_5^2 - 6\omega_3^2 c_s^2 - 3\omega_3^2 v_2^2 \omega_5^2 + 9\omega_3^2 c_s^2 \omega_5 + 72\omega_3 v_2^2 \omega_5 - 48\omega_3 c_s^2 \omega_5^2 + 12\omega_3^2 c_s^2 + 12\omega_3 \omega_5^2$$

$$C_{19} = -21\omega_3^2 \omega_5 \omega_2^3 - 12c_s^2 \omega_5^2 \omega_2^3 + 6\omega_3^2 \omega_2^3 - 6\omega_3^2 v_2^2 \omega_5^2 \omega_2^3 - 36\omega_3^2 c_s^2 \omega_2^3 + 72\omega_3^2 c_s^2 \omega_5 \omega_2^3 - 12\omega_3^2 v_2^2 \omega_5^2 \omega_2 - 6\omega_3 \omega_5^2 \omega_2^3 + 6\omega_3^2 \omega_5 \omega_2^2 - 24\omega_3^2 c_s^2 \omega_5 \omega_2^2 - 6\omega_3^2 v_2^2 \omega_5^2 \omega_2^2 + 6\omega_3^2 v_2^2 \omega_5^2 \omega_2 - 3\omega_3^2 \omega_2^3 - 12\omega_3 c_s^2 \omega_5^2 \omega_2^2 - \omega_3^2 \omega_5^2 \omega_2^3 + 6\omega_3^2 v_2^2 \omega_5^2 \omega_2^2 + 12\omega_3^2 c_s^2 \omega_5 \omega_2^2 + 36\omega_3 c_s^2 \omega_5^2 \omega_2^3 - 24\omega_3 v_2^2 \omega_5 \omega_2^3 + 12\omega_3^2 v_2^2 \omega_2^3 -$$

$$\begin{aligned}
& 24\omega_3^3 c_s^2 \omega_5 \omega_2^3 + \omega_3^3 \omega_5^2 \omega_2^2 + 12\omega_3 v_2^2 \omega_5^2 \omega_2^3 - 24\omega_3 c_s^2 \omega_5 \omega_2^3 + 6\omega_3^3 c_s^2 \omega_5^2 \omega_2^3 - 3\omega_3^3 \omega_5 \omega_2^2 - 12\omega_3^2 c_s^2 \omega_5^2 \omega_2 + 18\omega_3^3 c_s^2 \omega_2^3 + 6\omega_3^3 \omega_5 \omega_2^3 - 12\omega_3^3 c_s^2 \omega_5^2 \omega_2^2 + \\
& 6\omega_3^3 c_s^2 \omega_2^2 \omega_2 - 3\omega_3^3 \omega_5^2 \omega_2^2 + 12\omega_3 \omega_5 \omega_2^3 + 6\omega_3^3 v_2^2 \omega_5^2 + 36\omega_3^2 c_s^2 \omega_5^2 \omega_2^2 - 6\omega_3^3 v_2^2 \omega_2^3 + 7\omega_3^2 \omega_5^2 \omega_2^2 - 36\omega_3^2 c_s^2 \omega_5^2 \omega_2^2 + 12\omega_3^2 v_2^2 \omega_5 \omega_2^3 \\
C_{20} = & -6\omega_3^3 v_2^2 + 48\omega_3^2 v_2^2 \omega_5 - 26\omega_3^2 c_s^2 \omega_5^2 + 11\omega_3^2 \omega_5^2 + 12v_2^2 \omega_5^2 + 9\omega_3^3 \omega_5 - \omega_3^3 \omega_5^2 + 12\omega_3^2 + 12\omega_3^2 v_2^2 - 6\omega_3^3 + 96\omega_3^2 c_s^2 \omega_5 - 14\omega_3^2 v_2^2 \omega_5^2 - 36\omega_3^2 \omega_5 + \\
& 24\omega_3 \omega_5 + 4\omega_3^3 c_s^2 \omega_5^2 - 6\omega_3^3 v_2^2 \omega_5 - 36\omega_3 c_s^2 \omega_5 + 12\omega_3 v_2^2 \omega_5^2 + 30\omega_3^3 c_s^2 + \omega_3^3 v_2^2 \omega_5^2 - 30\omega_3^3 c_s^2 \omega_5 - 60\omega_3 v_2^2 \omega_5 + 18\omega_3 c_s^2 \omega_5^2 - 60\omega_3^2 c_s^2 - 12\omega_3 \omega_5^2 \\
C_{21} = & -6\omega_3^2 \omega_5 \omega_2^3 - 12c_s^2 \omega_5^2 \omega_2^3 - 12\omega_3^2 v_2^2 \omega_5^2 \omega_2^3 - 12\omega_3^2 c_s^2 \omega_2^3 + 36\omega_3^2 c_s^2 \omega_5 \omega_2^3 - 18\omega_3^2 v_2^2 \omega_5^2 \omega_2 + 12\omega_3^2 \omega_5 \omega_2^2 - 24\omega_3^2 c_s^2 \omega_5 \omega_2^2 + 12\omega_3^2 v_2^2 \omega_5^2 \omega_2^2 - \\
& 24\omega_3 c_s^2 \omega_5^2 \omega_2^2 - \omega_3^3 \omega_5^2 \omega_2^2 + 12\omega_3^3 c_s^2 \omega_5 \omega_2^2 + 36\omega_3 c_s^2 \omega_5^2 \omega_2^2 + 12\omega_3 v_2^2 \omega_5 \omega_2^2 + 12\omega_3^2 v_2^2 \omega_2^3 - 12\omega_3^2 c_s^2 \omega_5 \omega_2^2 + 3\omega_3^3 v_2^2 \omega_5^2 \omega_2^2 + 2\omega_3^3 \omega_5^2 \omega_2^2 + 30\omega_3 v_2^2 \omega_5^2 \omega_2^2 - \\
& 12\omega_3 c_s^2 \omega_5 \omega_2^2 + 4\omega_3^3 c_s^2 \omega_5^2 \omega_2^2 - 6\omega_3^3 \omega_5 \omega_2^2 - 12\omega_3^2 c_s^2 \omega_5^2 \omega_2 + 6\omega_3^3 c_s^2 \omega_2^3 + 3\omega_3^3 \omega_5 \omega_2^2 - 12\omega_3^2 c_s^2 \omega_5^2 \omega_2^2 + 12\omega_3^2 v_2^2 \omega_5 \omega_2^2 + 6\omega_3^3 c_s^2 \omega_5^2 \omega_2 - 6\omega_3^3 \omega_5^2 \omega_2^2 - \\
& 24\omega_3^2 v_2^2 \omega_5 \omega_2^2 + 12\omega_3^2 v_2^2 \omega_5^2 + 48\omega_3^2 c_s^2 \omega_5^2 \omega_2^2 - 6\omega_3^3 v_2^2 \omega_2^3 - 24v_2^2 \omega_5^2 \omega_2^2 + 3\omega_3^2 \omega_5^2 \omega_2^2 - 32\omega_3^2 c_s^2 \omega_5^2 \omega_2^2 - 12\omega_3^2 v_2^2 \omega_5 \omega_2^2 \\
C_{22} = & -30\omega_3^3 v_2^4 \omega_5 - 3\omega_3^3 c_s^4 \omega_5^2 - 36\omega_3^3 v_2^2 + 144\omega_3^2 c_s^2 v_2^2 \omega_5 + 24\omega_3 c_s^4 \omega_5 - 72\omega_3^2 v_2^2 \omega_5 + 6\omega_3^3 c_s^2 v_2^2 \omega_5^2 - 8\omega_3^2 c_s^2 \omega_5^2 - 216\omega_3^2 c_s^2 v_2^2 + 72\omega_3^2 v_2^2 + \\
& 24\omega_3^2 c_s^2 \omega_5 - 72\omega_3^3 c_s^2 v_2^2 \omega_5 + 12\omega_3^2 v_2^2 \omega_5^2 + 6\omega_3^3 c_s^4 \omega_5 + 3\omega_3^3 v_2^4 \omega_5^2 - 12\omega_3^2 c_s^2 v_2^2 \omega_5^2 - 48\omega_3 c_s^4 \omega_5^2 - 72\omega_3^2 v_2^4 + 72\omega_3 c_s^2 v_2^2 \omega_5 + \omega_3^3 c_s^2 \omega_5^2 + 30\omega_3^3 v_2^2 \omega_5 - \\
& 24\omega_3 c_s^2 \omega_5 + 24\omega_3^2 c_s^4 \omega_5^2 + 72\omega_3^2 v_2^4 \omega_5 - 12\omega_3^2 v_2^4 \omega_5^2 - 24\omega_3^2 c_s^4 \omega_5 + 24c_s^4 \omega_5^2 - 3\omega_3^3 v_2^2 \omega_5^2 - 6\omega_3^3 c_s^2 \omega_5 + 36\omega_3^3 v_2^2 + 12\omega_3 c_s^2 \omega_5^2 - 36\omega_3 c_s^2 v_2^2 \omega_5^2 + 108\omega_3^3 c_s^2 v_2^2 \\
C_{23} = & 42\omega_3^3 v_2^2 + 24\omega_3^2 v_2^2 \omega_5 - 2\omega_3^2 c_s^2 \omega_5^2 + 2\omega_3^2 \omega_5^2 - 12v_2^2 \omega_5^2 + 12\omega_3^3 \omega_5 - \omega_3^3 \omega_5^2 + 36\omega_3^2 - 84\omega_3^2 v_2^2 - 18\omega_3^3 + 72\omega_3^2 c_s^2 \omega_5 + 2\omega_3^2 v_2^2 \omega_5^2 - 24\omega_3^2 \omega_5 + \\
& 24c_s^2 \omega_5^2 - 12\omega_3 \omega_5 + \omega_3^3 c_s^2 \omega_5^2 - 24\omega_3^3 v_2^2 \omega_5 - 12\omega_3 c_s^2 \omega_5 - 12\omega_3 v_2^2 \omega_5^2 + 30\omega_3^3 c_s^2 + 2\omega_3^3 v_2^2 \omega_5^2 - 24\omega_3^3 c_s^2 \omega_5 + 60\omega_3 v_2^2 \omega_5 - 30\omega_3 c_s^2 \omega_5^2 - 60\omega_3^2 c_s^2 + 6\omega_3 \omega_5^2
\end{aligned}$$

2.5 CLBM2

2.5.1 Definitions

Collision operator C :

$$C(f) = \mathbf{K}^{-1} \mathbf{S} \left(\boldsymbol{\kappa}^{(eq)} - \mathbf{K} f \right),$$

where

$$\mathbf{S} = \text{diag}(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5),$$

$\omega_1, \omega_2, \dots, \omega_5 \in (0, 2)$.

Matrix \mathbf{K} corresponds to the transformation matrix to the central moment basis defined by

$$\boldsymbol{\kappa} = \begin{pmatrix} k_{(0,0)} \\ k_{(1,0)} \\ k_{(0,1)} \\ k_{(2,0)} + k_{(0,2)} \\ k_{(2,0)} - k_{(0,2)} \end{pmatrix}.$$

The transformation matrix \mathbf{K} satisfies

$$\mathbf{K} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -v_1 & 1 - v_1 & -v_1 & -v_1 - 1 & -v_1 \\ -v_2 & -v_2 & 1 - v_2 & -v_2 & -v_2 - 1 \\ v_1^2 + v_2^2 & v_1^2 + v_2^2 - 2v_1 + 1 & v_1^2 + v_2^2 - 2v_2 + 1 & v_1^2 + v_2^2 + 2v_1 + 1 & v_1^2 + v_2^2 + 2v_2 + 1 \\ v_1^2 - v_2^2 & v_1^2 - v_2^2 - 2v_1 + 1 & v_1^2 - v_2^2 + 2v_2 - 1 & v_1^2 - v_2^2 + 2v_1 + 1 & v_1^2 - v_2^2 - 2v_2 - 1 \end{pmatrix}.$$

The equilibrium central moments are defined by

$$\boldsymbol{\kappa}^{(eq)} = \mathbf{K} \mathbf{M}^{-1} \boldsymbol{\mu}^{(eq)},$$

i.e.,

$$\boldsymbol{\kappa}^{(eq)} = \left(\rho, 0, 0, 2\rho c_s^2, 0 \right)^T.$$

2.5.2 Conservation of mass equation



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$$\begin{aligned}
& \frac{\partial \rho}{\partial t} + \frac{\delta_1 v_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\delta_1 \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_1 v_2}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\delta_1 \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_2) \frac{\delta_1}{2\omega_2} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial t} + (-2 + \omega_2) \frac{\delta_1^2 v_1}{2\omega_2 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + \\
& (-2 + \omega_2) \frac{\delta_1^2 \rho}{2\omega_2 \delta_t} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + (2 - \omega_3) \frac{\delta_1^2 v_2}{2\omega_3 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (\omega_2 + \omega_3 - \omega_2 \omega_3) \frac{\delta_1^2 v_1}{\omega_2 \omega_3 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_2} + \\
& (\omega_2 + \omega_3 - \omega_2 \omega_3) \frac{\delta_1^2 \rho}{\omega_2 \omega_3 \delta_t} \frac{\partial v_1}{\partial x_1} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_3) \frac{\delta_1}{2\omega_3} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial t} + (\omega_2 + \omega_3 - \omega_2 \omega_3) \frac{\delta_1^2 v_2}{\omega_2 \omega_3 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_1} + (2 - \omega_2) \frac{\delta_1^2 v_1}{2\omega_2 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} \\
& + (-2 + \omega_3) \frac{\delta_1^2 v_2}{2\omega_3 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_3) \frac{\delta_1^2 \rho}{2\omega_3 \delta_t} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + (-2 + \omega_2) \frac{\delta_1 \rho}{2\omega_2} \frac{\partial^2 v_1}{\partial t \partial x_1} + (-2 + \omega_2) \frac{\delta_1^2 c_s^2}{2\omega_2 \delta_t} \frac{\partial^2 \rho}{\partial x_1^2} + \\
& (-2 + \omega_2) \frac{\delta_1^2 v_1 \rho}{2\omega_2 \delta_t} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega_3) \frac{\delta_1 \rho}{2\omega_3} \frac{\partial^2 v_2}{\partial t \partial x_2} + (\omega_2 + \omega_3 - \omega_2 \omega_3) \frac{\delta_1^2 v_2 \rho}{\omega_2 \omega_3 \delta_t} \frac{\partial^2 \rho}{\partial x_1 \partial x_2} + (2 - \omega_3) \frac{\delta_1^2 v_2 \rho}{2\omega_3 \delta_t} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + \\
& (2 - \omega_2) \frac{\delta_1^2 v_1 \rho}{2\omega_2 \delta_t} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_3) \frac{\delta_1^2 c_s^2}{2\omega_3 \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + (-2 + \omega_3) \frac{\delta_1^2 v_2 \rho}{2\omega_3 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + (12 - 12\omega_2 + \omega_2^2) \frac{\delta_1 \delta_t \rho}{12\omega_2^2} \frac{\partial^3 v_1}{\partial t^2 \partial x_1} + \\
& (12 - 12\omega_2 + \omega_2^2) \frac{\delta_1^2 v_1 \rho}{6\omega_2^2} \frac{\partial^3 v_1}{\partial t \partial x_1^2} + C_1 \frac{\delta_1^3 v_1}{6\omega_2 \delta_t \omega_4} \frac{\partial^3 \rho}{\partial x_1^3} + C_2 \frac{\delta_1^3 \rho}{12\omega_2^2 \delta_t \omega_4} \frac{\partial^3 v_1}{\partial x_1^3} + (12 - 12\omega_3 + \omega_3^2) \frac{\delta_1 \delta_t \rho}{12\omega_3^2} \frac{\partial^3 v_2}{\partial t^2 \partial x_2} + \\
& (-6\omega_2 - 6\omega_3 - 2\omega_2 \omega_3 + 9\omega_2 \omega_3 + 3\omega_3^2) \frac{\delta_1^2 v_2 \rho}{6\omega_2 \omega_3} \frac{\partial^3 v_1}{\partial t \partial x_1 \partial x_2} + (-6\omega_2 - 2\omega_2^2 \omega_3 + 3\omega_2^2 - 6\omega_3 + 9\omega_2 \omega_3) \frac{\delta_1^2 v_1 \rho}{6\omega_2^2 \omega_3} \frac{\partial^3 v_2}{\partial t \partial x_1 \partial x_2} + \\
& C_3 \frac{\delta_1^3 v_2}{2\omega_2^2 \omega_3^2 \delta_t \omega_4} \frac{\partial^3 \rho}{\partial x_1^3 \partial x_2} + (-6\omega_2^2 \omega_3 + 6\omega_2^2 + \omega_2^2 \omega_3^2 - 6\omega_2 \omega_3^2 + 6\omega_3^2) \frac{\delta_1^3 v_2 v_1 \rho}{6\omega_2^2 \omega_3^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3 \partial x_2} + \\
& (-6\omega_2^2 v_1^2 + 6\omega_2^2 c_s^2 - 12v_1^2 \omega_4 - 12c_s^2 \omega_4 - 3\omega_2^2 c_s^2 \omega_4 + \omega_2^2 v_1^2 \omega_4 + 6\omega_2 v_1^2 \omega_4 + 18\omega_2 c_s^2 \omega_4 - 12\omega_2 c_s^2 + 12\omega_2 v_1^2) \frac{\delta_1^3 \rho}{12\omega_2^2 \delta_t \omega_4} \frac{\partial^3 v_2}{\partial x_1^3 \partial x_2} \\
& + (12 - 12\omega_3 + \omega_3^2) \frac{\delta_1^2 v_2 \rho}{6\omega_3^2} \frac{\partial^3 v_2}{\partial t \partial x_2^2} + C_4 \frac{\delta_1^3 v_1}{2\omega_2^2 \omega_3^2 \delta_t \omega_5} \frac{\partial^3 \rho}{\partial x_1^3 \partial x_2^2} + \\
& (\omega_3^2 v_2^2 \omega_5 + 6\omega_3^2 c_s^2 - 6\omega_3^2 v_2^2 - 3\omega_3^2 c_s^2 \omega_5 - 12c_s^2 \omega_5 + 12\omega_3 v_2^2 + 18\omega_3 c_s^2 \omega_5 - 12\omega_3 c_s^2 - 12v_2^2 \omega_5 + 6\omega_3 v_2^2 \omega_5) \frac{\delta_1^3 \rho}{12\omega_3^2 \delta_t \omega_5} \frac{\partial^3 v_1}{\partial x_1^3 \partial x_2^2} \\
& + (-6\omega_2^2 \omega_3 + 6\omega_2^2 + \omega_2^2 \omega_3^2 - 6\omega_2 \omega_3^2 + 6\omega_3^2) \frac{\delta_1^3 v_2 v_1 \rho}{6\omega_2^2 \omega_3^2 \delta_t} \frac{\partial^3 v_2}{\partial x_1^3 \partial x_2^2} + C_5 \frac{\delta_1^3 v_2}{6\omega_3 \delta_t \omega_5} \frac{\partial^3 \rho}{\partial x_2^3} + C_6 \frac{\delta_1^3 \rho}{12\omega_3^2 \delta_t \omega_5} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& (-2 + 3\omega_2 - \omega_2^2) \frac{\delta_1 \delta_t^2 \rho}{2\omega_2^2} \frac{\partial^4 v_1}{\partial t^3 \partial x_1} + (-2 + 3\omega_2 - \omega_2^2) \frac{3\delta_1^2 \delta_t v_1 \rho}{2\omega_2^2} \frac{\partial^4 v_1}{\partial t^2 \partial x_1^2} + C_7 \frac{\delta_1^3 \rho}{12\omega_2^3 \omega_4^2} \frac{\partial^4 v_1}{\partial t \partial x_1^3} + C_8 \frac{\delta_1^4}{24\omega_2^3 \delta_t \omega_4^2} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& C_9 \frac{\delta_1^4 v_1 \rho}{12\omega_2^3 \delta_t \omega_4^2} \frac{\partial^4 v_1}{\partial x_1^4} + (-2 + 3\omega_3 - \omega_3^2) \frac{\delta_1 \delta_t^2 \rho}{2\omega_3^2} \frac{\partial^4 v_2}{\partial t^3 \partial x_2} + \\
& (-24\omega_2^2 \omega_3 + 12\omega_2^2 + 13\omega_2^2 \omega_3^2 - \omega_2^2 \omega_3^3 + 7\omega_2 \omega_3^3 - 24\omega_2 \omega_3^2 + 12\omega_2 \omega_3 - 6\omega_3^3 + 12\omega_3^2) \frac{\delta_1^2 v_2 \delta_t \rho}{12\omega_2^2 \omega_3^2} \frac{\partial^4 v_1}{\partial t^2 \partial x_1 \partial x_2} + \\
& (-\omega_2^3 \omega_3^2 - 24\omega_2^2 \omega_3 + 12\omega_2^2 + 13\omega_2^2 \omega_3^2 + 7\omega_2^3 \omega_3 - 6\omega_2^3 - 24\omega_2 \omega_3^2 + 12\omega_2 \omega_3 + 12\omega_3^2) \frac{\delta_1^2 \delta_t v_1 \rho}{12\omega_2^2 \omega_3^2} \frac{\partial^4 v_2}{\partial t^2 \partial x_1 \partial x_2} + \\
& (-7\omega_2^3 \omega_3^2 + \omega_2^3 \omega_3^3 - 6\omega_2^2 \omega_3 + 6\omega_2^2 \omega_3^2 - 7\omega_2^2 \omega_3^3 + 12\omega_2^3 \omega_3 - 6\omega_2^3 + 18\omega_2 \omega_3^2 - 12\omega_3^3) \frac{\delta_1^3 v_2 v_1 \rho}{6\omega_2^3 \omega_3^3} \frac{\partial^4 v_1}{\partial t \partial x_1^2 \partial x_2} + \\
& C_{10} \frac{\delta_1^3 \rho}{12\omega_2^3 \omega_3 \omega_4^2} \frac{\partial^4 v_2}{\partial t \partial x_1^2 \partial x_2} + C_{11} \frac{\delta_1^4 v_2 v_1}{6\omega_2^3 \omega_3^3 \delta_t \omega_4^2} \frac{\partial^4 \rho}{\partial x_1^4 \partial x_2} + C_{12} \frac{\delta_1^4 v_2 \rho}{12\omega_2^3 \omega_3^3 \delta_t \omega_4^2} \frac{\partial^4 v_1}{\partial x_1^4 \partial x_2} + C_{13} \frac{\delta_1^4 v_1 \rho}{12\omega_2^3 \delta_t \omega_4^2} \frac{\partial^4 v_2}{\partial x_1^4 \partial x_2} + \\
& (-2 + 3\omega_3 - \omega_3^2) \frac{3\delta_1^2 v_2 \delta_t \rho}{2\omega_3^3} \frac{\partial^4 v_2}{\partial t^2 \partial x_2^2} + C_{14} \frac{\delta_1^3 \rho}{12\omega_2 \omega_3^3 \omega_5^2} \frac{\partial^4 v_1}{\partial t \partial x_1 \partial x_2^2} + \\
& (-7\omega_2^3 \omega_3^2 + \omega_2^3 \omega_3^3 + 6\omega_2^2 \omega_3^2 - 7\omega_2^2 \omega_3^3 + 18\omega_2^3 \omega_3 - 12\omega_2^3 + 12\omega_2 \omega_3^2 - 6\omega_2 \omega_3^2 - 6\omega_3^3) \frac{\delta_1^3 v_2 v_1 \rho}{6\omega_2^3 \omega_3^3} \frac{\partial^4 v_2}{\partial t \partial x_1 \partial x_2^2} + \\
& C_{15} \frac{\delta_1^4}{4\omega_2^3 \omega_3^3 \delta_t \omega_4^2 \omega_5^2} \frac{\partial^4 \rho}{\partial x_1^4 \partial x_2^2} + C_{16} \frac{\delta_1^4 v_1 \rho}{12\omega_2^3 \omega_3^3 \delta_t \omega_5^2} \frac{\partial^4 v_1}{\partial x_1^4 \partial x_2^2} + C_{17} \frac{\delta_1^4 v_2 \rho}{12\omega_2^3 \omega_3^3 \delta_t \omega_4^2} \frac{\partial^4 v_2}{\partial x_1^4 \partial x_2^2} + C_{18} \frac{\delta_1^3 \rho}{12\omega_3^3 \omega_5^2} \frac{\partial^4 v_2}{\partial t \partial x_2^3} + \\
& C_{19} \frac{\delta_1^4 v_2 v_1}{6\omega_2^3 \omega_3^3 \delta_t \omega_5^2} \frac{\partial^4 \rho}{\partial x_1^4 \partial x_2^3} + C_{20} \frac{\delta_1^4 v_2 \rho}{12\omega_3^3 \delta_t \omega_5^2} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{21} \frac{\delta_1^4 v_1 \rho}{12\omega_2^3 \omega_3^3 \delta_t \omega_5^2} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + C_{22} \frac{\delta_1^4}{24\omega_3^3 \delta_t \omega_5^2} \frac{\partial^4 \rho}{\partial x_2^4} + C_{23} \frac{\delta_1^4 v_2 \rho}{12\omega_3^3 \delta_t \omega_5^2} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= 6 - 3\omega_2 + \omega_2 \omega_4 + 3v_1^2 \omega_4 + 9c_s^2 \omega_4 - 6v_1^2 - 18c_s^2 - \omega_2 v_1^2 \omega_4 - 3\omega_4 - 3\omega_2 c_s^2 \omega_4 + 9\omega_2 c_s^2 + 3\omega_2 v_1^2 \\
C_2 &= 18\omega_2^2 v_1^2 + 12\omega_2 + 6\omega_2^2 c_s^2 - 6\omega_2 \omega_4 + 12v_1^2 \omega_4 - 12c_s^2 \omega_4 - 6\omega_2^2 - 3\omega_2^2 c_s^2 \omega_4 - 5\omega_2^2 v_1^2 \omega_4 + 6\omega_2 v_1^2 \omega_4 + 18\omega_2 c_s^2 \omega_4 + 2\omega_2^2 \omega_4 - 12\omega_2 c_s^2 - 36\omega_2 v_1^2 \\
C_3 &= 2\omega_2 \omega_3 v_1^2 \omega_4 - 2\omega_2 \omega_3 c_s^2 \omega_4 + \omega_2^2 \omega_3^2 c_s^2 - \omega_2^2 \omega_3^2 v_1^2 + \omega_2^2 \omega_3^2 v_1^2 \omega_4 - \omega_2^2 \omega_3^2 c_s^2 \omega_4 + 2\omega_2^2 v_1^2 \omega_4 + \omega_2^2 \omega_3 c_s^2 \omega_4 - 3\omega_2^2 \omega_3 v_1^2 \omega_4 + 2\omega_2 \omega_3^2 v_1^2 + \\
& 4\omega_2 \omega_3^2 c_s^2 \omega_4 - 2\omega_2 \omega_3^2 c_s^2 - 2\omega_2 \omega_3^2 v_1^2 \omega_4 - 2\omega_3^2 c_s^2 \omega_4 \\
C_4 &= \omega_2^2 \omega_3^2 c_s^2 + 4\omega_2^2 \omega_3 c_s^2 \omega_5 - 3\omega_2 \omega_3^2 v_2^2 \omega_5 - \omega_2^2 \omega_3^2 v_2^2 + 2\omega_3^2 v_2^2 \omega_5 - 2\omega_2 \omega_3 v_2^2 \omega_5 + \omega_2 \omega_3^2 c_s^2 \omega_5 + \omega_2^2 \omega_3^2 v_2^2 \omega_5 - 2\omega_2 \omega_3 c_s^2 \omega_5 - 2\omega_2^2 c_s^2 \omega_5 + \\
& 2\omega_2^2 \omega_3 v_2^2 - \omega_2^2 \omega_3^2 c_s^2 \omega_5 + 2\omega_2 \omega_3 v_2^2 \omega_5 - 2\omega_2^2 \omega_3 c_s^2 \\
C_5 &= 6 - 3\omega_3 - 6v_2^2 + 9c_s^2 \omega_5 + 3\omega_3 v_2^2 - 18c_s^2 - 3\omega_3 c_s^2 \omega_5 + 9\omega_3 c_s^2 + 3v_2^2 \omega_5 + \omega_3 \omega_5 - \omega_3 v_2^2 \omega_5 - 3\omega_5 \\
C_6 &= 2\omega_3^2 \omega_5 - 5\omega_3^2 v_2^2 \omega_5 + 6\omega_3^2 c_s^2 + 12\omega_3 + 18\omega_3^2 v_2^2 - 3\omega_3^2 c_s^2 \omega_5 - 12c_s^2 \omega_5 - 36\omega_3 v_2^2 + 18\omega_3 c_s^2 \omega_5 - 12\omega_3 c_s^2 + 12v_2^2 \omega_5 - 6\omega_3 \omega_5 + 6\omega_3 v_2^2 \omega_5 - 6\omega_3^2 \\
C_7 &= 36\omega_2^2 v_1^2 + 12\omega_2^2 c_s^2 + 25\omega_2^2 c_s^2 \omega_4 + 15\omega_2^2 v_1^2 \omega_4 - 24\omega_2 \omega_4 - 12\omega_2^2 - 36v_1^2 \omega_4 + 12\omega_2 \omega_4^2 - 18\omega_3^2 v_1^2 - 6\omega_3^2 c_s^2 + 24c_s^2 \omega_4 - 36\omega_2^2 c_s^2 \omega_4 - 108\omega_2^2 v_1^2 \omega_4 + \\
& 6\omega_2^2 - 9\omega_3^2 \omega_4 - 2\omega_2^2 c_s^2 \omega_4 + 72\omega_2 v_1^2 \omega_4 + 24\omega_2 c_s^2 \omega_4 - 3\omega_2^2 v_1^2 \omega_4 - 11\omega_2^2 \omega_4^2 + 36\omega_2^2 \omega_4 + 9\omega_2^2 c_s^2 \omega_4 + 18\omega_2 v_1^2 \omega_4 - 48\omega_2 c_s^2 \omega_4 + 27\omega_2^2 v_1^2 \omega_4 + \omega_2^2 \omega_4^2
\end{aligned}$$

$$72\omega_3^2 v_2^4 + 24\omega_3^2 c_s^4 \omega_5^2 - 24\omega_3 c_s^2 \omega_5^2 + \omega_3^3 c_s^2 \omega_5^2 + 12\omega_3 c_s^2 \omega_5^2 - 6\omega_3^3 c_s^2 \omega_5 - 24\omega_3^2 c_s^4 \omega_5 + 72\omega_3 v_2^2 c_s^2 \omega_5 + 108\omega_3^3 v_2^2 c_s^2 - 12\omega_3^2 v_2^4 \omega_5^2 + 36\omega_3^3 v_2^4 - 3\omega_3^3 v_2^2 \omega_5^2$$

$$C_{23} = 30\omega_3^3 c_s^2 - 2\omega_3^2 c_s^2 \omega_5^2 - \omega_3^3 \omega_5^2 + 42\omega_3^3 v_2^2 - 24\omega_3^2 \omega_5 + 24\omega_3^2 v_2^2 \omega_5 + 2\omega_3^2 v_2^2 \omega_5^2 + 2\omega_3^2 \omega_5^2 - 60\omega_3^2 c_s^2 + 12\omega_3^3 \omega_5 - 84\omega_3^3 v_2^2 + 72\omega_3^3 c_s^2 \omega_5 - 12\omega_3^2 v_2^2 \omega_5^2 - 24\omega_3^3 v_2^2 \omega_5 + 6\omega_3 \omega_5^2 - 12\omega_3 c_s^2 \omega_5 - 12v_2^2 \omega_5^2 + \omega_3^3 c_s^2 \omega_5^2 - 30\omega_3 c_s^2 \omega_5^2 - 24\omega_3^3 c_s^2 \omega_5 - 18\omega_3^3 - 12\omega_3 \omega_5 + 60\omega_3 v_2^2 \omega_5 + 2\omega_3^3 v_2^2 \omega_5^2 + 24c_s^2 \omega_5^2 + 36\omega_3^2$$

3 Comparison of SRT, MRT, and CLBM

3.1 Conservation of mass equation

$$\begin{aligned} & \frac{\partial \rho}{\partial t} + v_1 \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \rho \frac{\delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + v_2 \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \rho \frac{\delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + C_{D_{x\rho}, D_t v_1}^{(0)} \delta_l \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial t} + C_{D_{x\rho}, D_x v_1}^{(0)} \frac{\delta_l^2}{\delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + \\ & C_{D_{x v_1}, D_x v_1}^{(0)} \frac{\delta_l^2}{\delta_t} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + C_{D_{x\rho}, D_y v_1}^{(0)} \frac{\delta_l^2}{\delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + C_{D_{x\rho}, D_y v_2}^{(0)} \frac{\delta_l^2}{\delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_2} + C_{D_{x v_1}, D_y v_2}^{(0)} \frac{\delta_l^2}{\delta_t} \frac{\partial v_1}{\partial x_1} \frac{\partial v_2}{\partial x_2} + \\ & C_{D_y \rho, D_t v_2}^{(0)} \delta_l \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial t} + C_{D_y \rho, D_x v_1}^{(0)} \frac{\delta_l^2}{\delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_1} + C_{D_y \rho, D_x v_2}^{(0)} \frac{\delta_l^2}{\delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + C_{D_y \rho, D_y v_2}^{(0)} \frac{\delta_l^2}{\delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + \\ & C_{D_y v_2, D_y v_2}^{(0)} \frac{\delta_l^2}{\delta_t} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + C_{D_t D_x v_1}^{(0)} \delta_l \frac{\partial^2 v_1}{\partial t \partial x_1} + C_{D_x^2 \rho}^{(0)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 \rho}{\partial x_1^2} + C_{D_x^2 v_1}^{(0)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 v_1}{\partial x_1^2} + C_{D_t D_y v_2}^{(0)} \delta_l \frac{\partial^2 v_2}{\partial t \partial x_2} + C_{D_x D_y \rho}^{(0)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 \rho}{\partial x_1 \partial x_2} \\ & + C_{D_x D_y v_1}^{(0)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + C_{D_x D_y v_2}^{(0)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + C_{D_y^2 \rho}^{(0)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + C_{D_y^2 v_2}^{(0)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + C_{D_t^2 D_x v_1}^{(0)} \delta_l \delta_t \frac{\partial^3 v_1}{\partial t^2 \partial x_1} + \\ & C_{D_t^2 D_x v_1}^{(0)} \delta_l^2 \frac{\partial^3 v_1}{\partial t \partial x_1^2} + C_{D_x^3 \rho}^{(0)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + C_{D_x^3 v_1}^{(0)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + C_{D_t^2 D_y v_2}^{(0)} \delta_l \delta_t \frac{\partial^3 v_2}{\partial t^2 \partial x_2} + C_{D_t D_x D_y v_1}^{(0)} \delta_l^2 \frac{\partial^3 v_1}{\partial t \partial x_1 \partial x_2} + \\ & C_{D_t D_x D_y v_2}^{(0)} \delta_l^2 \frac{\partial^3 v_2}{\partial t \partial x_1 \partial x_2} + C_{D_x^2 D_y \rho}^{(0)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + C_{D_x^2 D_y v_1}^{(0)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} + C_{D_x^2 D_y v_2}^{(0)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + C_{D_t D_y^2 v_2}^{(0)} \delta_l^2 \frac{\partial^3 v_2}{\partial t \partial x_2^2} + \\ & C_{D_x D_y^2 \rho}^{(0)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + C_{D_x D_y^2 v_1}^{(0)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_{D_x D_y^2 v_2}^{(0)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_{D_y^3 \rho}^{(0)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + C_{D_y^3 v_2}^{(0)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & C_{D_t^3 D_x v_1}^{(0)} \delta_l \delta_t^2 \frac{\partial^4 v_1}{\partial t^3 \partial x_1} + C_{D_t^2 D_x^2 v_1}^{(0)} \delta_l^2 \delta_t \frac{\partial^4 v_1}{\partial t^2 \partial x_1^2} + C_{D_t D_x^3 v_1}^{(0)} \delta_l^3 \frac{\partial^4 v_1}{\partial t \partial x_1^3} + C_{D_x^4 \rho}^{(0)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + C_{D_x^4 v_1}^{(0)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + \\ & C_{D_t^3 D_y v_2}^{(0)} \delta_l \delta_t^2 \frac{\partial^4 v_2}{\partial t^3 \partial x_2} + C_{D_t^2 D_x D_y v_1}^{(0)} \delta_l^2 \delta_t \frac{\partial^4 v_1}{\partial t^2 \partial x_1 \partial x_2} + C_{D_t^2 D_x D_y v_2}^{(0)} \delta_l^2 \delta_t \frac{\partial^4 v_2}{\partial t^2 \partial x_1 \partial x_2} + C_{D_t D_x^2 D_y v_1}^{(0)} \delta_l^3 \frac{\partial^4 v_1}{\partial t \partial x_1^2 \partial x_2} + \\ & C_{D_t D_x^2 D_y v_2}^{(0)} \delta_l^3 \frac{\partial^4 v_2}{\partial t \partial x_1^2 \partial x_2} + C_{D_x^3 D_y \rho}^{(0)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{D_x^3 D_y v_1}^{(0)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + C_{D_x^3 D_y v_2}^{(0)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_{D_t^2 D_y^2 v_2}^{(0)} \delta_l^2 \delta_t \frac{\partial^4 v_2}{\partial t^2 \partial x_2^2} + \\ & C_{D_t D_x D_y^2 v_1}^{(0)} \delta_l^3 \frac{\partial^4 v_1}{\partial t \partial x_1 \partial x_2^2} + C_{D_t D_x D_y^2 v_2}^{(0)} \delta_l^3 \frac{\partial^4 v_2}{\partial t \partial x_1 \partial x_2^2} + C_{D_x^2 D_y^2 \rho}^{(0)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_{D_x^2 D_y^2 v_1}^{(0)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{D_x^2 D_y^2 v_2}^{(0)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} \\ & + C_{D_t D_x^3 v_2}^{(0)} \delta_l^3 \frac{\partial^4 v_2}{\partial t \partial x_2^3} + C_{D_x D_y^3 \rho}^{(0)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{D_x D_y^3 v_1}^{(0)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{D_x D_y^3 v_2}^{(0)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + C_{D_y^4 \rho}^{(0)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\ & C_{D_y^4 v_2}^{(0)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:

coefficient $C_{D_{x\rho}, D_t v_1}^{(0)}$ **at** $\frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial t}$:

$$C_{D_{x\rho}, D_t v_1}^{(0), \text{SRT}} = (-2 + \omega) \frac{1}{2\omega}$$

$$C_{D_{x\rho}, D_t v_1}^{(0), \text{MRT1}} = (-2 + \omega_2) \frac{1}{2\omega_2}$$

$$C_{D_{x\rho}, D_t v_1}^{(0), \text{MRT2}} = C_{D_{x\rho}, D_t v_1}^{(0), \text{MRT1}}$$

$$C_{D_{x\rho}, D_t v_1}^{(0), \text{CLBM1}} = C_{D_{x\rho}, D_t v_1}^{(0), \text{MRT1}}$$

$$C_{D_{x\rho}, D_t v_1}^{(0), \text{CLBM2}} = C_{D_{x\rho}, D_t v_1}^{(0), \text{MRT1}}$$

coefficient $C_{D_{x\rho}, D_x v_1}^{(0)}$ **at** $\frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1}$:

$$C_{D_{x\rho}, D_x v_1}^{(0), \text{SRT}} = (-2 + \omega) \frac{v_1}{2\omega}$$

$$C_{D_{x\rho}, D_x v_1}^{(0), \text{MRT1}} = (-2 + \omega_2) \frac{v_1}{2\omega_2}$$

$$C_{D_{x\rho}, D_x v_1}^{(0), \text{MRT2}} = C_{D_{x\rho}, D_x v_1}^{(0), \text{MRT1}}$$

$$C_{D_x \rho, D_x v_1}^{(0), \text{CLBM1}} = C_{D_x \rho, D_x v_1}^{(0), \text{MRT1}}$$

$$C_{D_x \rho, D_x v_1}^{(0), \text{CLBM2}} = C_{D_x \rho, D_x v_1}^{(0), \text{MRT1}}$$

coefficient $C_{D_x v_1, D_x v_1}^{(0)}$ **at** $\left(\frac{\partial v_1}{\partial x_1}\right)^2$:

$$C_{D_x v_1, D_x v_1}^{(0), \text{SRT}} = (-2 + \omega) \frac{\rho}{2\omega}$$

$$C_{D_x v_1, D_x v_1}^{(0), \text{MRT1}} = (-2 + \omega_2) \frac{\rho}{2\omega_2}$$

$$C_{D_x v_1, D_x v_1}^{(0), \text{MRT2}} = C_{D_x v_1, D_x v_1}^{(0), \text{MRT1}}$$

$$C_{D_x v_1, D_x v_1}^{(0), \text{CLBM1}} = C_{D_x v_1, D_x v_1}^{(0), \text{MRT1}}$$

$$C_{D_x v_1, D_x v_1}^{(0), \text{CLBM2}} = C_{D_x v_1, D_x v_1}^{(0), \text{MRT1}}$$

coefficient $C_{D_x \rho, D_y v_1}^{(0)}$ **at** $\frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2}$:

$$C_{D_x \rho, D_y v_1}^{(0), \text{SRT}} = (2 - \omega) \frac{v_2}{2\omega}$$

$$C_{D_x \rho, D_y v_1}^{(0), \text{MRT1}} = (2 - \omega_3) \frac{v_2}{2\omega_3}$$

$$C_{D_x \rho, D_y v_1}^{(0), \text{MRT2}} = C_{D_x \rho, D_y v_1}^{(0), \text{MRT1}}$$

$$C_{D_x \rho, D_y v_1}^{(0), \text{CLBM1}} = C_{D_x \rho, D_y v_1}^{(0), \text{MRT1}}$$

$$C_{D_x \rho, D_y v_1}^{(0), \text{CLBM2}} = C_{D_x \rho, D_y v_1}^{(0), \text{MRT1}}$$

coefficient $C_{D_x \rho, D_y v_2}^{(0)}$ **at** $\frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_2}$:

$$C_{D_x \rho, D_y v_2}^{(0), \text{SRT}} = (2 - \omega) \frac{v_1}{\omega}$$

$$C_{D_x \rho, D_y v_2}^{(0), \text{MRT1}} = (\omega_2 + \omega_3 - \omega_2 \omega_3) \frac{v_1}{\omega_2 \omega_3}$$

$$C_{D_x \rho, D_y v_2}^{(0), \text{MRT2}} = C_{D_x \rho, D_y v_2}^{(0), \text{MRT1}}$$

$$C_{D_x \rho, D_y v_2}^{(0), \text{CLBM1}} = C_{D_x \rho, D_y v_2}^{(0), \text{MRT1}}$$

$$C_{D_x \rho, D_y v_2}^{(0), \text{CLBM2}} = C_{D_x \rho, D_y v_2}^{(0), \text{MRT1}}$$

coefficient $C_{D_x v_1, D_y v_2}^{(0)}$ **at** $\frac{\partial v_1}{\partial x_1} \frac{\partial v_2}{\partial x_2}$:

$$C_{D_x v_1, D_y v_2}^{(0), \text{SRT}} = (2 - \omega) \frac{\rho}{\omega}$$

$$C_{D_x v_1, D_y v_2}^{(0), \text{MRT1}} = (\omega_2 + \omega_3 - \omega_2 \omega_3) \frac{\rho}{\omega_2 \omega_3}$$

$$C_{D_x v_1, D_y v_2}^{(0), \text{MRT2}} = C_{D_x v_1, D_y v_2}^{(0), \text{MRT1}}$$

$$C_{D_x v_1, D_y v_2}^{(0), \text{CLBM1}} = C_{D_x v_1, D_y v_2}^{(0), \text{MRT1}}$$

$$C_{D_x v_1, D_y v_2}^{(0), \text{CLBM2}} = C_{D_x v_1, D_y v_2}^{(0), \text{MRT1}}$$

coefficient $C_{D_y \rho, D_t v_2}^{(0)}$ **at** $\frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial t}$:

$$C_{D_y \rho, D_t v_2}^{(0), \text{SRT}} = (-2 + \omega) \frac{1}{2\omega}$$

$$C_{D_y \rho, D_t v_2}^{(0), \text{MRT1}} = (-2 + \omega_3) \frac{1}{2\omega_3}$$

$$C_{D_y \rho, D_t v_2}^{(0), \text{MRT2}} = C_{D_y \rho, D_t v_2}^{(0), \text{MRT1}}$$

$$C_{D_y \rho, D_t v_2}^{(0), \text{CLBM1}} = C_{D_y \rho, D_t v_2}^{(0), \text{MRT1}}$$

$$C_{D_y \rho, D_t v_2}^{(0), \text{CLBM2}} = C_{D_y \rho, D_t v_2}^{(0), \text{MRT1}}$$

coefficient $C_{D_y \rho, D_x v_1}^{(0)}$ **at** $\frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_1}$:

$$C_{D_y \rho, D_x v_1}^{(0), \text{SRT}} = (2 - \omega) \frac{v_2}{\omega}$$

$$C_{D_y \rho, D_x v_1}^{(0), \text{MRT1}} = (\omega_2 + \omega_3 - \omega_2 \omega_3) \frac{v_2}{\omega_2 \omega_3}$$

$$C_{D_y \rho, D_x v_1}^{(0), \text{MRT2}} = C_{D_y \rho, D_x v_1}^{(0), \text{MRT1}}$$

$$C_{D_y \rho, D_x v_1}^{(0), \text{CLBM1}} = C_{D_y \rho, D_x v_1}^{(0), \text{MRT1}}$$

$$C_{D_y \rho, D_x v_1}^{(0), \text{CLBM2}} = C_{D_y \rho, D_x v_1}^{(0), \text{MRT1}}$$

coefficient $C_{D_y \rho, D_x v_2}^{(0)}$ **at** $\frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1}$:

$$C_{D_y \rho, D_x v_2}^{(0), \text{SRT}} = (2 - \omega) \frac{v_1}{2\omega}$$

$$C_{D_y \rho, D_x v_2}^{(0), \text{MRT1}} = (2 - \omega_2) \frac{v_1}{2\omega_2}$$

$$C_{D_y \rho, D_x v_2}^{(0), \text{MRT2}} = C_{D_y \rho, D_x v_2}^{(0), \text{MRT1}}$$

$$C_{D_y \rho, D_x v_2}^{(0), \text{CLBM1}} = C_{D_y \rho, D_x v_2}^{(0), \text{MRT1}}$$

$$C_{D_y \rho, D_x v_2}^{(0), \text{CLBM2}} = C_{D_y \rho, D_x v_2}^{(0), \text{MRT1}}$$

coefficient $C_{D_y \rho, D_y v_2}^{(0)}$ **at** $\frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2}$:

$$C_{D_y \rho, D_y v_2}^{(0), \text{SRT}} = (-2 + \omega) \frac{v_2}{2\omega}$$

$$C_{D_y \rho, D_y v_2}^{(0), \text{MRT1}} = (-2 + \omega_3) \frac{v_2}{2\omega_3}$$

$$C_{D_y \rho, D_y v_2}^{(0), \text{MRT2}} = C_{D_y \rho, D_y v_2}^{(0), \text{MRT1}}$$

$$C_{D_y \rho, D_y v_2}^{(0), \text{CLBM1}} = C_{D_y \rho, D_y v_2}^{(0), \text{MRT1}}$$

$$C_{D_y \rho, D_y v_2}^{(0), \text{CLBM2}} = C_{D_y \rho, D_y v_2}^{(0), \text{MRT1}}$$

coefficient $C_{D_y v_2, D_y v_2}^{(0)}$ **at** $\left(\frac{\partial v_2}{\partial x_2} \right)^2$:

$$C_{D_y v_2, D_y v_2}^{(0), \text{SRT}} = (-2 + \omega) \frac{\rho}{2\omega}$$

$$C_{D_y v_2, D_y v_2}^{(0), \text{MRT1}} = (-2 + \omega_3) \frac{\rho}{2\omega_3}$$

$$C_{D_y v_2, D_y v_2}^{(0), \text{MRT2}} = C_{D_y v_2, D_y v_2}^{(0), \text{MRT1}}$$

$$C_{D_y v_2, D_y v_2}^{(0), \text{CLBM1}} = C_{D_y v_2, D_y v_2}^{(0), \text{MRT1}}$$

$$C_{D_y v_2, D_y v_2}^{(0), \text{CLBM2}} = C_{D_y v_2, D_y v_2}^{(0), \text{MRT1}}$$

coefficient $C_{D_t D_x v_1}^{(0)}$ **at** $\frac{\partial^2 v_1}{\partial t \partial x_1}$:

$$C_{D_t D_x v_1}^{(0), \text{SRT}} = (-2 + \omega) \frac{\rho}{2\omega}$$

$$C_{D_t D_x v_1}^{(0), \text{MRT1}} = (-2 + \omega_2) \frac{\rho}{2\omega_2}$$

$$C_{D_t D_x v_1}^{(0), \text{MRT2}} = C_{D_t D_x v_1}^{(0), \text{MRT1}}$$

$$C_{D_t D_x v_1}^{(0), \text{CLBM1}} = C_{D_t D_x v_1}^{(0), \text{MRT1}}$$

$$C_{D_t D_x v_1}^{(0), \text{CLBM2}} = C_{D_t D_x v_1}^{(0), \text{MRT1}}$$

$$\text{coefficient } C_{D_x^2 \rho}^{(0)} \text{ at } \frac{\partial^2 \rho}{\partial x_1^2} :$$

$$C_{D_x^2 \rho}^{(0), \text{SRT}} = (-2 + \omega) \frac{c_s^2}{2\omega}$$

$$C_{D_x^2 \rho}^{(0), \text{MRT1}} = (-2 + \omega_2) \frac{c_s^2}{2\omega_2}$$

$$C_{D_x^2 \rho}^{(0), \text{MRT2}} = C_{D_x^2 \rho}^{(0), \text{MRT1}}$$

$$C_{D_x^2 \rho}^{(0), \text{CLBM1}} = C_{D_x^2 \rho}^{(0), \text{MRT1}}$$

$$C_{D_x^2 \rho}^{(0), \text{CLBM2}} = C_{D_x^2 \rho}^{(0), \text{MRT1}}$$

$$\text{coefficient } C_{D_x^2 v_1}^{(0)} \text{ at } \frac{\partial^2 v_1}{\partial x_1^2} :$$

$$C_{D_x^2 v_1}^{(0), \text{SRT}} = (-2 + \omega) \frac{\rho v_1}{2\omega}$$

$$C_{D_x^2 v_1}^{(0), \text{MRT1}} = (-2 + \omega_2) \frac{\rho v_1}{2\omega_2}$$

$$C_{D_x^2 v_1}^{(0), \text{MRT2}} = C_{D_x^2 v_1}^{(0), \text{MRT1}}$$

$$C_{D_x^2 v_1}^{(0), \text{CLBM1}} = C_{D_x^2 v_1}^{(0), \text{MRT1}}$$

$$C_{D_x^2 v_1}^{(0), \text{CLBM2}} = C_{D_x^2 v_1}^{(0), \text{MRT1}}$$

$$\text{coefficient } C_{D_t D_y v_2}^{(0)} \text{ at } \frac{\partial^2 v_2}{\partial t \partial x_2} :$$

$$C_{D_t D_y v_2}^{(0), \text{SRT}} = (-2 + \omega) \frac{\rho}{2\omega}$$

$$C_{D_t D_y v_2}^{(0), \text{MRT1}} = (-2 + \omega_3) \frac{\rho}{2\omega_3}$$

$$C_{D_t D_y v_2}^{(0), \text{MRT2}} = C_{D_t D_y v_2}^{(0), \text{MRT1}}$$

$$C_{D_t D_y v_2}^{(0), \text{CLBM1}} = C_{D_t D_y v_2}^{(0), \text{MRT1}}$$

$$C_{D_t D_y v_2}^{(0), \text{CLBM2}} = C_{D_t D_y v_2}^{(0), \text{MRT1}}$$

$$\text{coefficient } C_{D_x D_y \rho}^{(0)} \text{ at } \frac{\partial^2 \rho}{\partial x_1 \partial x_2} :$$

$$C_{D_x D_y \rho}^{(0), \text{SRT}} = (2 - \omega) \frac{v_2 v_1}{\omega}$$

$$C_{D_x D_y \rho}^{(0), \text{MRT1}} = (\omega_2 + \omega_3 - \omega_2 \omega_3) \frac{v_2 v_1}{\omega_2 \omega_3}$$

$$C_{D_x D_y \rho}^{(0), \text{MRT2}} = C_{D_x D_y \rho}^{(0), \text{MRT1}}$$

$$C_{D_x D_y \rho}^{(0), \text{CLBM1}} = C_{D_x D_y \rho}^{(0), \text{MRT1}}$$

$$C_{D_x D_y \rho}^{(0), \text{CLBM2}} = C_{D_x D_y \rho}^{(0), \text{MRT1}}$$

coefficient $C_{D_x D_y v_1}^{(0)}$ **at** $\frac{\partial^2 v_1}{\partial x_1 \partial x_2}$:

$$C_{D_x D_y v_1}^{(0), \text{SRT}} = (2 - \omega) \frac{v_2 \rho}{2\omega}$$

$$C_{D_x D_y v_1}^{(0), \text{MRT1}} = (2 - \omega_3) \frac{v_2 \rho}{2\omega_3}$$

$$C_{D_x D_y v_1}^{(0), \text{MRT2}} = C_{D_x D_y v_1}^{(0), \text{MRT1}}$$

$$C_{D_x D_y v_1}^{(0), \text{CLBM1}} = C_{D_x D_y v_1}^{(0), \text{MRT1}}$$

$$C_{D_x D_y v_1}^{(0), \text{CLBM2}} = C_{D_x D_y v_1}^{(0), \text{MRT1}}$$

coefficient $C_{D_x D_y v_2}^{(0)}$ **at** $\frac{\partial^2 v_2}{\partial x_1 \partial x_2}$:

$$C_{D_x D_y v_2}^{(0), \text{SRT}} = (2 - \omega) \frac{\rho v_1}{2\omega}$$

$$C_{D_x D_y v_2}^{(0), \text{MRT1}} = (2 - \omega_2) \frac{\rho v_1}{2\omega_2}$$

$$C_{D_x D_y v_2}^{(0), \text{MRT2}} = C_{D_x D_y v_2}^{(0), \text{MRT1}}$$

$$C_{D_x D_y v_2}^{(0), \text{CLBM1}} = C_{D_x D_y v_2}^{(0), \text{MRT1}}$$

$$C_{D_x D_y v_2}^{(0), \text{CLBM2}} = C_{D_x D_y v_2}^{(0), \text{MRT1}}$$

coefficient $C_{D_y^2 \rho}^{(0)}$ **at** $\frac{\partial^2 \rho}{\partial x_2^2}$:

$$C_{D_y^2 \rho}^{(0), \text{SRT}} = (-2 + \omega) \frac{c_s^2}{2\omega}$$

$$C_{D_y^2 \rho}^{(0), \text{MRT1}} = (-2 + \omega_3) \frac{c_s^2}{2\omega_3}$$

$$C_{D_y^2 \rho}^{(0), \text{MRT2}} = C_{D_y^2 \rho}^{(0), \text{MRT1}}$$

$$C_{D_y^2 \rho}^{(0), \text{CLBM1}} = C_{D_y^2 \rho}^{(0), \text{MRT1}}$$

$$C_{D_y^2 \rho}^{(0), \text{CLBM2}} = C_{D_y^2 \rho}^{(0), \text{MRT1}}$$

coefficient $C_{D_y^2 v_2}^{(0)}$ **at** $\frac{\partial^2 v_2}{\partial x_2^2}$:

$$C_{D_y^2 v_2}^{(0), \text{SRT}} = (-2 + \omega) \frac{v_2 \rho}{2\omega}$$

$$C_{D_y^2 v_2}^{(0), \text{MRT1}} = (-2 + \omega_3) \frac{v_2 \rho}{2\omega_3}$$

$$C_{D_y^2 v_2}^{(0), \text{MRT2}} = C_{D_y^2 v_2}^{(0), \text{MRT1}}$$

$$C_{D_y^2 v_2}^{(0), \text{CLBM1}} = C_{D_y^2 v_2}^{(0), \text{MRT1}}$$

$$C_{D_y^2 v_2}^{(0), \text{CLBM2}} = C_{D_y^2 v_2}^{(0), \text{MRT1}}$$

coefficient $C_{D_t^2 D_x v_1}^{(0)}$ **at** $\frac{\partial^3 v_1}{\partial t^2 \partial x_1}$:

$$C_{D_t^2 D_x v_1}^{(0), \text{SRT}} = (12 - 12\omega + \omega^2) \frac{\rho}{12\omega^2}$$

$$C_{D_t^2 D_x v_1}^{(0), \text{MRT1}} = (12 - 12\omega_2 + \omega_2^2) \frac{\rho}{12\omega_2^2}$$

$$C_{D_t^2 D_x v_1}^{(0), \text{MRT2}} = C_{D_t^2 D_x v_1}^{(0), \text{MRT1}}$$

$$C_{D_t^2 D_x v_1}^{(0), \text{CLBM1}} = C_{D_t^2 D_x v_1}^{(0), \text{MRT1}}$$

$$C_{D_t^2 D_x v_1}^{(0), \text{CLBM2}} = C_{D_t^2 D_x v_1}^{(0), \text{MRT1}}$$

coefficient $C_{D_t D_x^2 v_1}^{(0)}$ **at** $\frac{\partial^3 v_1}{\partial t \partial x_1^2}$:

$$C_{D_t D_x^2 v_1}^{(0), \text{SRT}} = (12 - 12\omega + \omega^2) \frac{\rho v_1}{6\omega^2}$$

$$C_{D_t D_x^2 v_1}^{(0), \text{MRT1}} = (12 - 6\omega_4 + \omega_4\omega_2 - 6\omega_2) \frac{\rho v_1}{6\omega_4\omega_2}$$

$$C_{D_t D_x^2 v_1}^{(0), \text{MRT2}} = C_{D_t D_x^2 v_1}^{(0), \text{MRT1}}$$

$$C_{D_t D_x^2 v_1}^{(0), \text{CLBM1}} = (12 - 12\omega_2 + \omega_2^2) \frac{\rho v_1}{6\omega_2^2}$$

$$C_{D_t D_x^2 v_1}^{(0), \text{CLBM2}} = C_{D_t D_x^2 v_1}^{(0), \text{CLBM1}}$$

coefficient $C_{D_x^3 \rho}^{(0)}$ **at** $\frac{\partial^3 \rho}{\partial x_1^3}$:

$$C_{D_x^3 \rho}^{(0), \text{SRT}} = (6 - \omega^2 v_1^2 - 6\omega + 18\omega c_s^2 + \omega^2 - 6v_1^2 - 18c_s^2 - 3\omega^2 c_s^2 + 6\omega v_1^2) \frac{v_1}{6\omega^2}$$

$$C_{D_x^3 \rho}^{(0), \text{MRT1}} = (-12\omega_4 c_s^2 - 3\omega_4 \omega_2 - 3\omega_4 \omega_2^2 c_s^2 - 6\omega_2 v_1^2 + \omega_4 \omega_2^2 + 3\omega_4 \omega_2 v_1^2 + 3\omega_2^2 c_s^2 + 6\omega_2 - \omega_4 \omega_2^2 v_1^2 - 6\omega_2 c_s^2 + 15\omega_4 \omega_2 c_s^2 + 3\omega_2^2 v_1^2 - 3\omega_2^2) \frac{v_1}{6\omega_4 \omega_2^2}$$

$$C_{D_x^3 \rho}^{(0), \text{MRT2}} = C_{D_x^3 \rho}^{(0), \text{MRT1}}$$

$$C_{D_x^3 \rho}^{(0), \text{CLBM1}} = (6 + 9\omega_4 c_s^2 - 3\omega_4 + \omega_4 \omega_2 + 3\omega_2 v_1^2 - \omega_4 \omega_2 v_1^2 - 3\omega_2 - 6v_1^2 + 9\omega_2 c_s^2 - 3\omega_4 \omega_2 c_s^2 - 18c_s^2 + 3\omega_4 v_1^2) \frac{v_1}{6\omega_4 \omega_2}$$

$$C_{D_x^3 \rho}^{(0), \text{CLBM2}} = C_{D_x^3 \rho}^{(0), \text{CLBM1}}$$

coefficient $C_{D_x^3 v_1}^{(0)}$ **at** $\frac{\partial^3 v_1}{\partial x_1^3}$:

$$C_{D_x^3 v_1}^{(0), \text{SRT}} = (12 - 5\omega^2 v_1^2 - 12\omega + 24\omega c_s^2 + 2\omega^2 - 24v_1^2 - 24c_s^2 - 3\omega^2 c_s^2 + 24\omega v_1^2) \frac{\rho}{12\omega^2}$$

$$C_{D_x^3 v_1}^{(0), \text{MRT1}} =$$

$$(-12\omega_4 c_s^2 - 6\omega_4 \omega_2 - 3\omega_4 \omega_2^2 c_s^2 - 12\omega_2 v_1^2 + 2\omega_4 \omega_2^2 + 18\omega_4 \omega_2 v_1^2 + 6\omega_2^2 c_s^2 + 12\omega_2 - 5\omega_4 \omega_2^2 v_1^2 - 12\omega_2 c_s^2 + 18\omega_4 \omega_2 c_s^2 + 6\omega_2^2 v_1^2 - 12\omega_4 v_1^2 - 6\omega_2^2) \frac{\rho}{12\omega_4 \omega_2^2}$$

$$C_{D_x^3 v_1}^{(0), \text{MRT2}} = C_{D_x^3 v_1}^{(0), \text{MRT1}}$$

$$C_{D_x^3 v_1}^{(0), \text{CLBM1}} =$$

$$(-12\omega_4 c_s^2 - 6\omega_4 \omega_2 - 3\omega_4 \omega_2^2 c_s^2 - 36\omega_2 v_1^2 + 2\omega_4 \omega_2^2 + 6\omega_4 \omega_2 v_1^2 + 6\omega_2^2 c_s^2 + 12\omega_2 - 5\omega_4 \omega_2^2 v_1^2 - 12\omega_2 c_s^2 + 18\omega_4 \omega_2 c_s^2 + 18\omega_2^2 v_1^2 + 12\omega_4 v_1^2 - 6\omega_2^2) \frac{\rho}{12\omega_4 \omega_2^2}$$

$$C_{D_x^3 v_1}^{(0), \text{CLBM2}} = C_{D_x^3 v_1}^{(0), \text{CLBM1}}$$

coefficient $C_{D_t^2 D_y v_2}^{(0)}$ **at** $\frac{\partial^3 v_2}{\partial t^2 \partial x_2}$:

$$C_{D_t^2 D_y v_2}^{(0), \text{SRT}} = (12 - 12\omega + \omega^2) \frac{\rho}{12\omega^2}$$

$$C_{D_t^2 D_y v_2}^{(0), \text{MRT1}} = (12 + \omega_3^2 - 12\omega_3) \frac{\rho}{12\omega_3^2}$$

$$C_{D_t^2 D_y v_2}^{(0), \text{MRT2}} = C_{D_t^2 D_y v_2}^{(0), \text{MRT1}}$$

$$C_{D_t^2 D_y v_2}^{(0), \text{CLBM1}} = C_{D_t^2 D_y v_2}^{(0), \text{MRT1}}$$

$$C_{D_t^2 D_y v_2}^{(0), \text{CLBM2}} = C_{D_t^2 D_y v_2}^{(0), \text{MRT1}}$$

coefficient $C_{D_t D_x D_y v_1}^{(0)}$ at $\frac{\partial^3 v_1}{\partial t \partial x_1 \partial x_2}$:

$$C_{D_t D_x D_y v_1}^{(0), \text{SRT}} = (-6 + 6\omega - \omega^2) \frac{v_2 \rho}{3\omega^2}$$

$$C_{D_t D_x D_y v_1}^{(0), \text{MRT1}} = (-6\omega_2 + 3\omega_3^2 - 2\omega_2\omega_3^2 - 6\omega_3 + 9\omega_2\omega_3) \frac{v_2 \rho}{6\omega_2\omega_3^2}$$

$$C_{D_t D_x D_y v_1}^{(0), \text{MRT2}} = C_{D_t D_x D_y v_1}^{(0), \text{MRT1}}$$

$$C_{D_t D_x D_y v_1}^{(0), \text{CLBM1}} = C_{D_t D_x D_y v_1}^{(0), \text{MRT1}}$$

$$C_{D_t D_x D_y v_1}^{(0), \text{CLBM2}} = C_{D_t D_x D_y v_1}^{(0), \text{MRT1}}$$

coefficient $C_{D_t D_x D_y v_2}^{(0)}$ at $\frac{\partial^3 v_2}{\partial t \partial x_1 \partial x_2}$:

$$C_{D_t D_x D_y v_2}^{(0), \text{SRT}} = (-6 + 6\omega - \omega^2) \frac{\rho v_1}{3\omega^2}$$

$$C_{D_t D_x D_y v_2}^{(0), \text{MRT1}} = (-2\omega_2^2\omega_3 - 6\omega_2 - 6\omega_3 + 9\omega_2\omega_3 + 3\omega_2^2) \frac{\rho v_1}{6\omega_2^2\omega_3}$$

$$C_{D_t D_x D_y v_2}^{(0), \text{MRT2}} = C_{D_t D_x D_y v_2}^{(0), \text{MRT1}}$$

$$C_{D_t D_x D_y v_2}^{(0), \text{CLBM1}} = C_{D_t D_x D_y v_2}^{(0), \text{MRT1}}$$

$$C_{D_t D_x D_y v_2}^{(0), \text{CLBM2}} = C_{D_t D_x D_y v_2}^{(0), \text{MRT1}}$$

coefficient $C_{D_x^2 D_y \rho}^{(0)}$ at $\frac{\partial^3 \rho}{\partial x_1^2 \partial x_2}$:

$$C_{D_x^2 D_y \rho}^{(0), \text{SRT}} = (\omega^2 v_1^2 + 6\omega c_s^2 + 6v_1^2 - 6c_s^2 - \omega^2 c_s^2 - 6\omega v_1^2) \frac{v_2}{2\omega^2}$$

$$C_{D_x^2 D_y \rho}^{(0), \text{MRT1}} = (-4\omega_4\omega_2 v_1^2 \omega_3^2 - \omega_4\omega_2^2 \omega_3^2 c_s^2 - 2\omega_4\omega_2\omega_3 c_s^2 - 2\omega_2 v_1^2 \omega_3^2 + \omega_2^2 \omega_3^2 c_s^2 + 4\omega_4 v_1^2 \omega_3^2 + 2\omega_4\omega_2 v_1^2 \omega_3 + 2\omega_4\omega_2^2 v_1^2 - 2\omega_4\omega_3^2 c_s^2 - 2\omega_2\omega_3^2 c_s^2 + \omega_2^2 v_1^2 \omega_3^2 - 3\omega_4\omega_2^2 v_1^2 \omega_3 + 4\omega_4\omega_2\omega_3^2 c_s^2 + \omega_4\omega_2^2 v_1^2 \omega_3^2 + \omega_4\omega_2^2 \omega_3 c_s^2) \frac{v_2}{2\omega_4\omega_2^2\omega_3^2}$$

$$C_{D_x^2 D_y \rho}^{(0), \text{MRT2}} = C_{D_x^2 D_y \rho}^{(0), \text{MRT1}}$$

$$C_{D_x^2 D_y \rho}^{(0), \text{CLBM1}} = (-2\omega_4\omega_2 v_1^2 \omega_3^2 - \omega_4\omega_2^2 \omega_3^2 c_s^2 - 2\omega_4\omega_2\omega_3 c_s^2 + 2\omega_2 v_1^2 \omega_3^2 + \omega_2^2 \omega_3^2 c_s^2 + 2\omega_4\omega_2 v_1^2 \omega_3 + 2\omega_4\omega_2^2 v_1^2 - 2\omega_4\omega_3^2 c_s^2 - 2\omega_2\omega_3^2 c_s^2 - \omega_2^2 v_1^2 \omega_3^2 - 3\omega_4\omega_2^2 v_1^2 \omega_3 + 4\omega_4\omega_2\omega_3^2 c_s^2 + \omega_4\omega_2^2 v_1^2 \omega_3^2 + \omega_4\omega_2^2 \omega_3 c_s^2) \frac{v_2}{2\omega_4\omega_2^2\omega_3^2}$$

$$C_{D_x^2 D_y \rho}^{(0), \text{CLBM2}} = C_{D_x^2 D_y \rho}^{(0), \text{CLBM1}}$$

coefficient $C_{D_x^2 D_y v_1}^{(0)}$ at $\frac{\partial^3 v_1}{\partial x_1^2 \partial x_2}$:

$$C_{D_x^2 D_y v_1}^{(0), \text{SRT}} = (12 - 12\omega + \omega^2) \frac{v_2 \rho v_1}{6\omega^2}$$

$$C_{D_x^2 D_y v_1}^{(0), \text{MRT1}} = (-6\omega_2^2\omega_3 + \omega_2^2\omega_3^2 + 6\omega_3^2 - 6\omega_2\omega_3^2 + 6\omega_2^2) \frac{v_2 \rho v_1}{6\omega_2^2\omega_3^2}$$

$$C_{D_x^2 D_y v_1}^{(0), \text{MRT2}} = C_{D_x^2 D_y v_1}^{(0), \text{MRT1}}$$

$$C_{D_x^2 D_y v_1}^{(0), \text{CLBM1}} = C_{D_x^2 D_y v_1}^{(0), \text{MRT1}}$$

$$C_{D_x^2 D_y v_1}^{(0), \text{CLBM2}} = C_{D_x^2 D_y v_1}^{(0), \text{MRT1}}$$

coefficient $C_{D_x^2 D_y v_2}^{(0)}$ **at** $\frac{\partial^3 v_2}{\partial x_1^2 \partial x_2}$:

$$C_{D_x^2 D_y v_2}^{(0), \text{SRT}} = (\omega^2 v_1^2 + 24\omega c_s^2 - 24c_s^2 - 3\omega^2 c_s^2) \frac{\rho}{12\omega^2}$$

$$C_{D_x^2 D_y v_2}^{(0), \text{MRT1}} = (-12\omega_4 c_s^2 - 3\omega_4 \omega_2^2 c_s^2 - 12\omega_2 v_1^2 - 6\omega_4 \omega_2 v_1^2 + 6\omega_2^2 c_s^2 + \omega_4 \omega_2^2 v_1^2 - 12\omega_2 c_s^2 + 18\omega_4 \omega_2 c_s^2 + 6\omega_2^2 v_1^2 + 12\omega_4 v_1^2) \frac{\rho}{12\omega_4 \omega_2^2}$$

$$C_{D_x^2 D_y v_2}^{(0), \text{MRT2}} = C_{D_x^2 D_y v_2}^{(0), \text{MRT1}}$$

$$C_{D_x^2 D_y v_2}^{(0), \text{CLBM1}} = (-12\omega_4 c_s^2 - 3\omega_4 \omega_2^2 c_s^2 + 12\omega_2 v_1^2 + 6\omega_4 \omega_2 v_1^2 + 6\omega_2^2 c_s^2 + \omega_4 \omega_2^2 v_1^2 - 12\omega_2 c_s^2 + 18\omega_4 \omega_2 c_s^2 - 6\omega_2^2 v_1^2 - 12\omega_4 v_1^2) \frac{\rho}{12\omega_4 \omega_2^2}$$

$$C_{D_x^2 D_y v_2}^{(0), \text{CLBM2}} = C_{D_x^2 D_y v_2}^{(0), \text{CLBM1}}$$

coefficient $C_{D_t D_y^2 v_2}^{(0)}$ **at** $\frac{\partial^3 v_2}{\partial t \partial x_2^2}$:

$$C_{D_t D_y^2 v_2}^{(0), \text{SRT}} = (12 - 12\omega + \omega^2) \frac{v_2 \rho}{6\omega^2}$$

$$C_{D_t D_y^2 v_2}^{(0), \text{MRT1}} = (12 - 6\omega_5 + \omega_5 \omega_3 - 6\omega_3) \frac{v_2 \rho}{6\omega_5 \omega_3}$$

$$C_{D_t D_y^2 v_2}^{(0), \text{MRT2}} = C_{D_t D_y^2 v_2}^{(0), \text{MRT1}}$$

$$C_{D_t D_y^2 v_2}^{(0), \text{CLBM1}} = (12 + \omega_3^2 - 12\omega_3) \frac{v_2 \rho}{6\omega_3^2}$$

$$C_{D_t D_y^2 v_2}^{(0), \text{CLBM2}} = C_{D_t D_y^2 v_2}^{(0), \text{CLBM1}}$$

coefficient $C_{D_x D_y^2 \rho}^{(0)}$ **at** $\frac{\partial^3 \rho}{\partial x_1 \partial x_2^2}$:

$$C_{D_x D_y^2 \rho}^{(0), \text{SRT}} = (6v_2^2 + 6\omega c_s^2 + \omega^2 v_2^2 - 6\omega v_2^2 - 6c_s^2 - \omega^2 c_s^2) \frac{v_1}{2\omega^2}$$

$$C_{D_x D_y^2 \rho}^{(0), \text{MRT1}} = (-2\omega_5 \omega_2 \omega_3 c_s^2 + 2\omega_5 v_2^2 \omega_3^2 - \omega_5 \omega_2^2 \omega_3^2 c_s^2 + \omega_5 \omega_2^2 v_2^2 \omega_3^2 - 2\omega_2^2 v_2^2 \omega_3 + \omega_2^2 \omega_3^2 c_s^2 + \omega_2^2 v_2^2 \omega_3^2 - 4\omega_5 \omega_2^2 v_2^2 \omega_3 + 4\omega_5 \omega_2^2 v_2^2 + 2\omega_5 \omega_2 v_2^2 \omega_3 - 2\omega_2^2 \omega_3 c_s^2 + 4\omega_5 \omega_2^2 \omega_3 c_s^2 - 2\omega_5 \omega_2^2 c_s^2 - 3\omega_5 \omega_2 v_2^2 \omega_3^2 + \omega_5 \omega_2 \omega_3^2 c_s^2) \frac{v_1}{2\omega_5 \omega_2^2 \omega_3^2}$$

$$C_{D_x D_y^2 \rho}^{(0), \text{MRT2}} = C_{D_x D_y^2 \rho}^{(0), \text{MRT1}}$$

$$C_{D_x D_y^2 \rho}^{(0), \text{CLBM1}} = (-2\omega_5 \omega_2 \omega_3 c_s^2 + 2\omega_5 v_2^2 \omega_3^2 - \omega_5 \omega_2^2 \omega_3^2 c_s^2 + \omega_5 \omega_2^2 v_2^2 \omega_3^2 + 2\omega_2^2 v_2^2 \omega_3 + \omega_2^2 \omega_3^2 c_s^2 - \omega_2^2 v_2^2 \omega_3^2 - 2\omega_5 \omega_2^2 v_2^2 \omega_3 + 2\omega_5 \omega_2 v_2^2 \omega_3 - 2\omega_2^2 \omega_3 c_s^2 + 4\omega_5 \omega_2^2 \omega_3 c_s^2 - 2\omega_5 \omega_2^2 c_s^2 - 3\omega_5 \omega_2 v_2^2 \omega_3^2 + \omega_5 \omega_2 \omega_3^2 c_s^2) \frac{v_1}{2\omega_5 \omega_2^2 \omega_3^2}$$

$$C_{D_x D_y^2 \rho}^{(0), \text{CLBM2}} = C_{D_x D_y^2 \rho}^{(0), \text{CLBM1}}$$

coefficient $C_{D_x D_y^2 v_1}^{(0)}$ **at** $\frac{\partial^3 v_1}{\partial x_1 \partial x_2^2}$:

$$C_{D_x D_y^2 v_1}^{(0), \text{SRT}} = (24\omega c_s^2 + \omega^2 v_2^2 - 24c_s^2 - 3\omega^2 c_s^2) \frac{\rho}{12\omega^2}$$

$$C_{D_x D_y^2 v_1}^{(0), \text{MRT1}} = (\omega_5 v_2^2 \omega_3^2 - 3\omega_5 \omega_3^2 c_s^2 - 12\omega_3 c_s^2 + 12\omega_5 v_2^2 - 6\omega_5 v_2^2 \omega_3 + 6v_2^2 \omega_3^2 + 6\omega_3^2 c_s^2 - 12\omega_5 c_s^2 - 12v_2^2 \omega_3 + 18\omega_5 \omega_3 c_s^2) \frac{\rho}{12\omega_5 \omega_3^2}$$

$$C_{D_x D_y^2 v_1}^{(0), \text{MRT2}} = C_{D_x D_y^2 v_1}^{(0), \text{MRT1}}$$

$$C_{D_x D_y^2 v_1}^{(0), \text{CLBM1}} = (\omega_5 v_2^2 \omega_3^2 - 3\omega_5 \omega_3^2 c_s^2 - 12\omega_3 c_s^2 - 12\omega_5 v_2^2 + 6\omega_5 v_2^2 \omega_3 - 6v_2^2 \omega_3^2 + 6\omega_3^2 c_s^2 - 12\omega_5 c_s^2 + 12v_2^2 \omega_3 + 18\omega_5 \omega_3 c_s^2) \frac{\rho}{12\omega_5 \omega_3^2}$$

$$C_{D_x D_y^2 v_1}^{(0), \text{CLBM2}} = C_{D_x D_y^2 v_1}^{(0), \text{CLBM1}}$$

coefficient $C_{D_x D_y^2 v_2}^{(0)}$ **at** $\frac{\partial^3 v_2}{\partial x_1 \partial x_2^2}$:

$$C_{D_x D_y^2 v_2}^{(0), \text{SRT}} = (12 - 12\omega + \omega^2) \frac{v_2 \rho v_1}{6\omega^2}$$

$$C_{D_x D_y^2 v_2}^{(0), \text{MRT1}} = (-6\omega_2^2 \omega_3 + \omega_2^2 \omega_3^2 + 6\omega_3^2 - 6\omega_2 \omega_3^2 + 6\omega_2^2) \frac{v_2 \rho v_1}{6\omega_2^2 \omega_3^2}$$

$$C_{D_x D_y^2 v_2}^{(0), \text{MRT2}} = C_{D_x D_y^2 v_2}^{(0), \text{MRT1}}$$

$$C_{D_x D_y^2 v_2}^{(0), \text{CLBM1}} = C_{D_x D_y^2 v_2}^{(0), \text{MRT1}}$$

$$C_{D_x D_y^2 v_2}^{(0), \text{CLBM2}} = C_{D_x D_y^2 v_2}^{(0), \text{MRT1}}$$

coefficient $C_{D_y^3 \rho}^{(0)}$ **at** $\frac{\partial^3 \rho}{\partial x_2^3}$:

$$C_{D_y^3 \rho}^{(0), \text{SRT}} = (6 - 6\omega - 6v_2^2 + 18\omega c_s^2 + \omega^2 - \omega^2 v_2^2 + 6\omega v_2^2 - 18c_s^2 - 3\omega^2 c_s^2) \frac{v_2}{6\omega^2}$$

$$C_{D_y^3 \rho}^{(0), \text{MRT1}} = (-\omega_5 v_2^2 \omega_3^2 - 3\omega_5 \omega_3^2 c_s^2 - 6\omega_3 c_s^2 + 3\omega_5 v_2^2 \omega_3 - 3\omega_3^2 - 3\omega_5 \omega_3 + 3v_2^2 \omega_3^2 + 3\omega_3^2 c_s^2 + 6\omega_3 - 12\omega_5 c_s^2 - 6v_2^2 \omega_3 + 15\omega_5 \omega_3 c_s^2 + \omega_5 \omega_3^2) \frac{v_2}{6\omega_5 \omega_3^2}$$

$$C_{D_y^3 \rho}^{(0), \text{MRT2}} = C_{D_y^3 \rho}^{(0), \text{MRT1}}$$

$$C_{D_y^3 \rho}^{(0), \text{CLBM1}} = (6 - 6v_2^2 + 9\omega_5 c_s^2 - 3\omega_5 + 3\omega_5 v_2^2 - \omega_5 v_2^2 \omega_3 + \omega_5 \omega_3 - 18c_s^2 - 3\omega_3 + 9\omega_5 c_s^2 + 3v_2^2 \omega_3 - 3\omega_5 \omega_3 c_s^2) \frac{v_2}{6\omega_5 \omega_3}$$

$$C_{D_y^3 \rho}^{(0), \text{CLBM2}} = C_{D_y^3 \rho}^{(0), \text{CLBM1}}$$

coefficient $C_{D_y^3 v_2}^{(0)}$ **at** $\frac{\partial^3 v_2}{\partial x_2^3}$:

$$C_{D_y^3 v_2}^{(0), \text{SRT}} = (12 - 12\omega - 24v_2^2 + 24\omega c_s^2 + 2\omega^2 - 5\omega^2 v_2^2 + 24\omega v_2^2 - 24c_s^2 - 3\omega^2 c_s^2) \frac{\rho}{12\omega^2}$$

$$C_{D_y^3 v_2}^{(0), \text{MRT1}} = (-5\omega_5 v_2^2 \omega_3^2 - 3\omega_5 \omega_3^2 c_s^2 - 12\omega_3 c_s^2 - 12\omega_5 v_2^2 + 18\omega_5 v_2^2 \omega_3 - 6\omega_3^2 - 6\omega_5 \omega_3 + 6v_2^2 \omega_3^2 + 6\omega_3^2 c_s^2 + 12\omega_3 - 12\omega_5 c_s^2 - 12v_2^2 \omega_3 + 18\omega_5 \omega_3 c_s^2 + 2\omega_5 \omega_3^2) \frac{\rho}{12\omega_5 \omega_3^2}$$

$$C_{D_y^3 v_2}^{(0), \text{MRT2}} = C_{D_y^3 v_2}^{(0), \text{MRT1}}$$

$$C_{D_y^3 v_2}^{(0), \text{CLBM1}} = (-5\omega_5 v_2^2 \omega_3^2 - 3\omega_5 \omega_3^2 c_s^2 - 12\omega_3 c_s^2 + 12\omega_5 v_2^2 + 6\omega_5 v_2^2 \omega_3 - 6\omega_3^2 - 6\omega_5 \omega_3 + 18v_2^2 \omega_3^2 + 6\omega_3^2 c_s^2 + 12\omega_3 - 12\omega_5 c_s^2 - 36v_2^2 \omega_3 + 18\omega_5 \omega_3 c_s^2 + 2\omega_5 \omega_3^2) \frac{\rho}{12\omega_5 \omega_3^2}$$

$$C_{D_y^3 v_2}^{(0), \text{CLBM2}} = C_{D_y^3 v_2}^{(0), \text{CLBM1}}$$

coefficient $C_{D_t^3 D_x v_1}^{(0)}$ **at** $\frac{\partial^4 v_1}{\partial t^3 \partial x_1}$:

$$C_{D_t^3 D_x v_1}^{(0), \text{SRT}} = (-2 + 3\omega - \omega^2) \frac{\rho}{2\omega^3}$$

$$C_{D_t^3 D_x v_1}^{(0), \text{MRT1}} = (-2 + 3\omega_2 - \omega_2^2) \frac{\rho}{2\omega_2^3}$$

$$C_{D_t^3 D_x v_1}^{(0), \text{MRT2}} = C_{D_t^3 D_x v_1}^{(0), \text{MRT1}}$$

$$C_{D_t^3 D_x v_1}^{(0), \text{CLBM1}} = C_{D_t^3 D_x v_1}^{(0), \text{MRT1}}$$

$$C_{\mathbf{D}_t^3 \mathbf{D}_x v_1}^{(0), \text{CLBM2}} = C_{\mathbf{D}_t^3 \mathbf{D}_x v_1}^{(0), \text{MRT1}}$$

coefficient $C_{\mathbf{D}_t^2 \mathbf{D}_x^2 v_1}^{(0)}$ **at** $\frac{\partial^4 v_1}{\partial t^2 \partial x_1^2}$:

$$C_{\mathbf{D}_t^2 \mathbf{D}_x^2 v_1}^{(0), \text{SRT}} = (-2 + 3\omega - \omega^2) \frac{3\rho v_1}{2\omega^3}$$

$$C_{\mathbf{D}_t^2 \mathbf{D}_x^2 v_1}^{(0), \text{MRT1}} = (2\omega_4^2 - 4\omega_4\omega_2 + 8\omega_4\omega_2^2 - 2\omega_4\omega_2^3 - \omega_4^2\omega_2^2 - \omega_4^2\omega_2 + 2\omega_2^3 - 4\omega_2^2) \frac{\rho v_1}{2\omega_4^2\omega_2^3}$$

$$C_{\mathbf{D}_t^2 \mathbf{D}_x^2 v_1}^{(0), \text{MRT2}} = C_{\mathbf{D}_t^2 \mathbf{D}_x^2 v_1}^{(0), \text{MRT1}}$$

$$C_{\mathbf{D}_t^2 \mathbf{D}_x^2 v_1}^{(0), \text{CLBM1}} = (-2 + 3\omega_2 - \omega_2^2) \frac{3\rho v_1}{2\omega_2^3}$$

$$C_{\mathbf{D}_t^2 \mathbf{D}_x^2 v_1}^{(0), \text{CLBM2}} = C_{\mathbf{D}_t^2 \mathbf{D}_x^2 v_1}^{(0), \text{CLBM1}}$$

coefficient $C_{\mathbf{D}_t \mathbf{D}_x^3 v_1}^{(0)}$ **at** $\frac{\partial^4 v_1}{\partial t \partial x_1^3}$:

$$C_{\mathbf{D}_t \mathbf{D}_x^3 v_1}^{(0), \text{SRT}} = (-36 + 42\omega^2 v_1^2 + 54\omega + \omega^3 - 90\omega c_s^2 - 20\omega^2 - 3\omega^3 v_1^2 + 72v_1^2 - 2\omega^3 c_s^2 + 60c_s^2 + 34\omega^2 c_s^2 - 108\omega v_1^2) \frac{\rho}{12\omega^3}$$

$$C_{\mathbf{D}_t \mathbf{D}_x^3 v_1}^{(0), \text{MRT1}} = (-3\omega_4^2\omega_2^3 v_1^2 + 9\omega_4\omega_2^3 c_s^2 - 24\omega_4\omega_2 + 12\omega_4^2 v_1^2 - 6\omega_2^3 c_s^2 - 36\omega_4\omega_2^2 c_s^2 + 27\omega_4^2\omega_2^2 v_1^2 + 36\omega_4\omega_2^2 + 48\omega_4\omega_2 v_1^2 - 48\omega_4^2\omega_2 c_s^2 + 12\omega_2^2 c_s^2 - 9\omega_4\omega_2^3 - 60\omega_4\omega_2^2 v_1^2 + 25\omega_4^2\omega_2^2 c_s^2 + \omega_4^2\omega_2^2 + 24\omega_4\omega_2 c_s^2 - 42\omega_4^2\omega_2 v_1^2 + 12\omega_2^2 v_1^2 - 11\omega_4^2\omega_2^2 + 12\omega_4^2\omega_2 + 6\omega_2^3 - 2\omega_4^2\omega_2^3 c_s^2 + 15\omega_4\omega_2^3 v_1^2 - 12\omega_2^2 + 24\omega_4^2 c_s^2 - 6\omega_2^3 v_1^2) \frac{\rho}{12\omega_4^2\omega_2^3}$$

$$C_{\mathbf{D}_t \mathbf{D}_x^3 v_1}^{(0), \text{MRT2}} = C_{\mathbf{D}_t \mathbf{D}_x^3 v_1}^{(0), \text{MRT1}}$$

$$C_{\mathbf{D}_t \mathbf{D}_x^3 v_1}^{(0), \text{CLBM1}} = (-3\omega_4^2\omega_2^3 v_1^2 + 9\omega_4\omega_2^3 c_s^2 - 24\omega_4\omega_2 + 36\omega_4^2 v_1^2 - 6\omega_2^3 c_s^2 - 36\omega_4\omega_2^2 c_s^2 + 15\omega_4^2\omega_2^2 v_1^2 + 36\omega_4\omega_2^2 + 72\omega_4\omega_2 v_1^2 - 48\omega_4^2\omega_2 c_s^2 + 12\omega_2^2 c_s^2 - 9\omega_4\omega_2^3 - 108\omega_4\omega_2^2 v_1^2 + 25\omega_4^2\omega_2^2 c_s^2 + \omega_4^2\omega_2^2 + 24\omega_4\omega_2 c_s^2 + 18\omega_4^2\omega_2 v_1^2 + 36\omega_2^2 v_1^2 - 11\omega_4^2\omega_2^2 + 12\omega_4^2\omega_2 + 6\omega_2^3 - 2\omega_4^2\omega_2^3 c_s^2 + 27\omega_4\omega_2^3 v_1^2 - 12\omega_2^2 + 24\omega_4^2 c_s^2 - 18\omega_2^3 v_1^2) \frac{\rho}{12\omega_4^2\omega_2^3}$$

$$C_{\mathbf{D}_t \mathbf{D}_x^3 v_1}^{(0), \text{CLBM2}} = C_{\mathbf{D}_t \mathbf{D}_x^3 v_1}^{(0), \text{CLBM1}}$$

coefficient $C_{\mathbf{D}_x^4 \rho}^{(0)}$ **at** $\frac{\partial^4 \rho}{\partial x_1^4}$:

$$C_{\mathbf{D}_x^4 \rho}^{(0), \text{SRT}} = (42\omega^2 v_1^2 + 30\omega^2 c_s^4 + 36\omega c_s^2 + 216\omega v_1^2 c_s^2 + 108\omega v_1^4 - 3\omega^3 c_s^4 + 6\omega^3 v_1^2 c_s^2 - 3\omega^3 v_1^2 - 144v_1^2 c_s^2 + 48c_s^4 + 72v_1^2 + 3\omega^3 v_1^4 + \omega^3 c_s^2 - 72v_1^4 - 24c_s^2 - 14\omega^2 c_s^2 - 42\omega^2 v_1^4 - 108\omega v_1^2 - 72\omega c_s^4 - 84\omega^2 v_1^2 c_s^2) \frac{1}{24\omega^3}$$

$$C_{\mathbf{D}_x^4 \rho}^{(0), \text{MRT1}} = (12\omega_2^3 v_1^2 c_s^2 - 3\omega_4^2\omega_2^3 c_s^4 - 18\omega_4\omega_2^3 v_1^4 - 3\omega_4^2\omega_2^3 v_1^2 - 6\omega_4\omega_2^3 c_s^2 + 12\omega_2^3 v_1^4 - 24\omega_4\omega_2 v_1^2 c_s^2 + 24\omega_4^2 c_s^4 + 24\omega_4\omega_2^2 c_s^2 + 24\omega_4^2\omega_2^2 v_1^2 + 72\omega_4\omega_2^2 v_1^4 - 12\omega_4\omega_2^3 v_1^2 c_s^2 + 24\omega_4^2\omega_2^2 c_s^4 + 48\omega_4\omega_2 v_1^2 + 12\omega_4^2\omega_2 c_s^2 + 24\omega_4\omega_2 c_s^4 + 24\omega_4^2\omega_2 v_1^4 - 24\omega_2^3 v_1^4 - 72\omega_4^2\omega_2^2 v_1^2 c_s^2 - 72\omega_4\omega_2^2 v_1^2 - 8\omega_4^2\omega_2^2 c_s^2 - 24\omega_4\omega_2^2 c_s^4 - 24\omega_4^2\omega_2^2 v_1^4 - 24\omega_4\omega_2 c_s^2 - 24\omega_4^2\omega_2 v_1^2 + 24\omega_2^3 v_1^2 + 156\omega_4^2\omega_2 v_1^2 c_s^2 - 48\omega_4\omega_2 v_1^4 - 24\omega_2^2 v_1^2 c_s^2 - 48\omega_4^2\omega_2 c_s^4 - 96\omega_4^2 v_1^2 c_s^2 + 3\omega_4^2\omega_2^3 v_1^4 + 6\omega_4\omega_2^3 c_s^4 + 6\omega_4^2\omega_2^3 v_1^2 c_s^2 + \omega_4^2\omega_2^3 c_s^2 + 18\omega_4\omega_2^3 v_1^2 + 48\omega_4\omega_2^2 v_1^2 c_s^2 - 12\omega_2^3 v_1^2) \frac{1}{24\omega_4^2\omega_2^3}$$

$$C_{\mathbf{D}_x^4 \rho}^{(0), \text{MRT2}} = C_{\mathbf{D}_x^4 \rho}^{(0), \text{MRT1}}$$

$$C_{\mathbf{D}_x^4 \rho}^{(0), \text{CLBM1}} = (108\omega_2^3 v_1^2 c_s^2 - 3\omega_4^2\omega_2^3 c_s^4 - 30\omega_4\omega_2^3 v_1^4 - 3\omega_4^2\omega_2^3 v_1^2 - 6\omega_4\omega_2^3 c_s^2 + 36\omega_2^3 v_1^4 + 72\omega_4\omega_2 v_1^2 c_s^2 + 24\omega_4^2 c_s^4 + 24\omega_4\omega_2^2 c_s^2 + 12\omega_4^2\omega_2^2 v_1^2 + 72\omega_4\omega_2^2 v_1^4 - 72\omega_4\omega_2^3 v_1^2 c_s^2 + 24\omega_4^2\omega_2^2 c_s^4 + 12\omega_4^2\omega_2 c_s^2 + 24\omega_4\omega_2 c_s^4 - 72\omega_2^2 v_1^4 - 12\omega_4^2\omega_2^2 v_1^2 c_s^2 - 72\omega_4\omega_2^2 v_1^2 - 8\omega_4^2\omega_2^2 c_s^2 - 24\omega_4\omega_2^2 c_s^4 - 12\omega_4^2\omega_2^2 v_1^4 - 24\omega_4\omega_2 c_s^2 + 72\omega_2^2 v_1^2 - 36\omega_4^2\omega_2 v_1^2 c_s^2 - 216\omega_2^2 v_1^2 c_s^2 - 48\omega_4^2\omega_2 c_s^4 + 3\omega_4^2\omega_2^2 v_1^4 + 6\omega_4\omega_2^2 c_s^4 + 6\omega_4^2\omega_2^2 v_1^2 c_s^2 + \omega_4^2\omega_2^2 c_s^2 + 30\omega_4\omega_2^3 v_1^2 + 144\omega_4\omega_2^2 v_1^2 c_s^2 - 36\omega_2^3 v_1^2) \frac{1}{24\omega_4^2\omega_2^3}$$

$$C_{\mathbf{D}_x^4 \rho}^{(0), \text{CLBM2}} = C_{\mathbf{D}_x^4 \rho}^{(0), \text{CLBM1}}$$

coefficient $C_{\mathbf{D}_x^4 v_1}^{(0)}$ **at** $\frac{\partial^4 v_1}{\partial x_1^4}$:

$$C_{\mathbf{D}_x^4 v_1}^{(0), \text{SRT}} = (24 - 22\omega^2 v_1^2 - 36\omega - \omega^3 + 72\omega c_s^2 + 14\omega^2 + 2\omega^3 v_1^2 - 36v_1^2 + \omega^3 c_s^2 - 48c_s^2 - 26\omega^2 c_s^2 + 54\omega v_1^2) \frac{\rho v_1}{12\omega^3}$$

$$C_{D_x^4 v_1}^{(0), \text{MRT}1} = (2\omega_4^2 \omega_2^3 v_1^2 - 6\omega_4 \omega_2^3 c_s^2 + 12\omega_4 \omega_2 - 12\omega_4^2 v_1^2 + 6\omega_2^3 c_s^2 + 24\omega_4 \omega_2^2 c_s^2 - 16\omega_4^2 \omega_2^2 v_1^2 - 24\omega_4 \omega_2^2 - 12\omega_4 \omega_2 v_1^2 + 42\omega_4^2 \omega_2 c_s^2 - 12\omega_2^2 c_s^2 + 6\omega_4 \omega_2^3 + 24\omega_4 \omega_2^2 v_1^2 - 20\omega_4^2 \omega_2^2 c_s^2 - \omega_4^2 \omega_2^3 - 12\omega_4 \omega_2 c_s^2 + 24\omega_4^2 \omega_2 v_1^2 - 12\omega_2^2 v_1^2 + 8\omega_4^2 \omega_2^2 - 6\omega_4^2 \omega_2 - 6\omega_2^3 + \omega_4^2 \omega_2^3 c_s^2 - 6\omega_4 \omega_2^3 v_1^2 + 12\omega_2^2 - 24\omega_4^2 c_s^2 + 6\omega_2^3 v_1^2) \frac{\rho v_1}{12\omega_4^2 \omega_2^3}$$

$$C_{D_x^4 v_1}^{(0), \text{MRT}2} = C_{D_x^4 v_1}^{(0), \text{MRT}1}$$

$$C_{D_x^4 v_1}^{(0), \text{CLBM}1} = (2\omega_4^2 \omega_2^3 v_1^2 - 24\omega_4 \omega_2^3 c_s^2 - 12\omega_4 \omega_2 - 12\omega_4^2 v_1^2 + 30\omega_2^3 c_s^2 + 72\omega_4 \omega_2^2 c_s^2 + 2\omega_4^2 \omega_2^2 v_1^2 - 24\omega_4 \omega_2^2 + 60\omega_4 \omega_2 v_1^2 - 30\omega_4^2 \omega_2 c_s^2 - 60\omega_2^2 c_s^2 + 12\omega_4 \omega_2^3 + 24\omega_4 \omega_2^2 v_1^2 - 2\omega_4^2 \omega_2^2 c_s^2 - \omega_4^2 \omega_2^3 - 12\omega_4 \omega_2 c_s^2 - 12\omega_4^2 \omega_2 v_1^2 - 84\omega_2^2 v_1^2 + 2\omega_4^2 \omega_2^2 + 6\omega_4^2 \omega_2 - 18\omega_2^3 + \omega_4^2 \omega_2^3 c_s^2 - 24\omega_4 \omega_2^3 v_1^2 + 36\omega_2^2 + 24\omega_4^2 c_s^2 + 42\omega_2^3 v_1^2) \frac{\rho v_1}{12\omega_4^2 \omega_2^3}$$

$$C_{D_x^4 v_1}^{(0), \text{CLBM}2} = C_{D_x^4 v_1}^{(0), \text{CLBM}1}$$

$$\text{coefficient } C_{D_t^3 D_y v_2}^{(0)} \text{ at } \frac{\partial^4 v_2}{\partial t^3 \partial x_2} :$$

$$C_{D_t^3 D_y v_2}^{(0), \text{SRT}} = (-2 + 3\omega - \omega^2) \frac{\rho}{2\omega^3}$$

$$C_{D_t^3 D_y v_2}^{(0), \text{MRT}1} = (-2 - \omega_3^2 + 3\omega_3) \frac{\rho}{2\omega_3^3}$$

$$C_{D_t^3 D_y v_2}^{(0), \text{MRT}2} = C_{D_t^3 D_y v_2}^{(0), \text{MRT}1}$$

$$C_{D_t^3 D_y v_2}^{(0), \text{CLBM}1} = C_{D_t^3 D_y v_2}^{(0), \text{MRT}1}$$

$$C_{D_t^3 D_y v_2}^{(0), \text{CLBM}2} = C_{D_t^3 D_y v_2}^{(0), \text{MRT}1}$$

$$\text{coefficient } C_{D_t^2 D_x D_y v_1}^{(0)} \text{ at } \frac{\partial^4 v_1}{\partial t^2 \partial x_1 \partial x_2} :$$

$$C_{D_t^2 D_x D_y v_1}^{(0), \text{SRT}} = (36 - 54\omega - \omega^3 + 20\omega^2) \frac{v_2 \rho}{12\omega^3}$$

$$C_{D_t^2 D_x D_y v_1}^{(0), \text{MRT}1} = (-24\omega_2^2 \omega_3 + 13\omega_2^2 \omega_3^2 - \omega_2^2 \omega_3^3 + 12\omega_3^2 + 7\omega_2 \omega_3^3 - 6\omega_3^3 - 24\omega_2 \omega_3^2 + 12\omega_2 \omega_3 + 12\omega_2^2) \frac{v_2 \rho}{12\omega_2^2 \omega_3^3}$$

$$C_{D_t^2 D_x D_y v_1}^{(0), \text{MRT}2} = C_{D_t^2 D_x D_y v_1}^{(0), \text{MRT}1}$$

$$C_{D_t^2 D_x D_y v_1}^{(0), \text{CLBM}1} = C_{D_t^2 D_x D_y v_1}^{(0), \text{MRT}1}$$

$$C_{D_t^2 D_x D_y v_1}^{(0), \text{CLBM}2} = C_{D_t^2 D_x D_y v_1}^{(0), \text{MRT}1}$$

$$\text{coefficient } C_{D_t^2 D_x D_y v_2}^{(0)} \text{ at } \frac{\partial^4 v_2}{\partial t^2 \partial x_1 \partial x_2} :$$

$$C_{D_t^2 D_x D_y v_2}^{(0), \text{SRT}} = (36 - 54\omega - \omega^3 + 20\omega^2) \frac{\rho v_1}{12\omega^3}$$

$$C_{D_t^2 D_x D_y v_2}^{(0), \text{MRT}1} = (-\omega_2^3 \omega_3^2 - 24\omega_2^2 \omega_3 + 13\omega_2^2 \omega_3^2 + 7\omega_2^3 \omega_3 + 12\omega_3^2 - 24\omega_2 \omega_3^2 + 12\omega_2 \omega_3 - 6\omega_2^3 + 12\omega_2^2) \frac{\rho v_1}{12\omega_2^3 \omega_3^2}$$

$$C_{D_t^2 D_x D_y v_2}^{(0), \text{MRT}2} = C_{D_t^2 D_x D_y v_2}^{(0), \text{MRT}1}$$

$$C_{D_t^2 D_x D_y v_2}^{(0), \text{CLBM}1} = C_{D_t^2 D_x D_y v_2}^{(0), \text{MRT}1}$$

$$C_{D_t^2 D_x D_y v_2}^{(0), \text{CLBM}2} = C_{D_t^2 D_x D_y v_2}^{(0), \text{MRT}1}$$

$$\text{coefficient } C_{D_t D_x^2 D_y v_1}^{(0)} \text{ at } \frac{\partial^4 v_1}{\partial t \partial x_1^2 \partial x_2} :$$

$$C_{D_t D_x^2 D_y v_1}^{(0), \text{SRT}} = (-24 + 36\omega + \omega^3 - 14\omega^2) \frac{v_2 \rho v_1}{6\omega^3}$$

$$C_{D_t D_x^2 D_y v_1}^{(0), \text{MRT}1} = (-6\omega_2^3 \omega_3^2 - 6\omega_4 \omega_2^2 \omega_3 + \omega_4 \omega_2^3 \omega_3^3 - 12\omega_4 \omega_3^3 - 7\omega_4 \omega_2^3 \omega_3^2 + 3\omega_2^3 \omega_3^3 + 12\omega_4 \omega_2^3 \omega_3 + 12\omega_2^2 \omega_3^2 - 10\omega_4 \omega_2^2 \omega_3^3 + 12\omega_4 \omega_2^2 \omega_3^2 - 6\omega_4 \omega_2^3 - 6\omega_2^2 \omega_3^3 - 12\omega_4 \omega_2 \omega_3^2 + 24\omega_4 \omega_2 \omega_3^3) \frac{v_2 \rho v_1}{6\omega_4 \omega_2^3 \omega_3^3}$$

$$C_{D_t D_x^2 D_y v_1}^{(0), \text{MRT}2} = C_{D_t D_x^2 D_y v_1}^{(0), \text{MRT}1}$$

$$C_{D_t D_x^2 D_y v_1}^{(0), \text{CLBM}1} = (-7\omega_2^3 \omega_3^2 - 6\omega_2^2 \omega_3 + \omega_2^3 \omega_3^3 + 6\omega_2^2 \omega_3^2 + 12\omega_2^3 \omega_3 - 7\omega_2^2 \omega_3^3 + 18\omega_2 \omega_3^3 - 12\omega_3^3 - 6\omega_2^3) \frac{v_2 \rho v_1}{6\omega_2^3 \omega_3^3}$$

$$C_{D_t D_x^2 D_y v_1}^{(0), \text{CLBM}2} = C_{D_t D_x^2 D_y v_1}^{(0), \text{CLBM}1}$$

coefficient $C_{D_t D_x^2 D_y v_2}^{(0)}$ **at** $\frac{\partial^4 v_2}{\partial t \partial x_1^2 \partial x_2}$:

$$C_{D_t D_x^2 D_y v_2}^{(0), \text{SRT}} = (-2\omega^2 v_1^2 - 90\omega c_s^2 + \omega^3 v_1^2 - 2\omega^3 c_s^2 + 60c_s^2 + 34\omega^2 c_s^2) \frac{\rho}{12\omega^3}$$

$$C_{D_t D_x^2 D_y v_2}^{(0), \text{MRT}1} = (-\omega_4^2 \omega_3^3 v_1^2 - 6\omega_2^3 \omega_3 c_s^2 - 6\omega_4 \omega_2^3 c_s^2 + 9\omega_4 \omega_2^3 v_1^2 \omega_3 - 10\omega_4^2 \omega_2^2 v_1^2 \omega_3 + 12\omega_4 \omega_2 \omega_3 c_s^2 + 12\omega_4 \omega_2^2 c_s^2 + 9\omega_4 \omega_2^3 \omega_3 c_s^2 - 6\omega_2^3 v_1^2 \omega_3 + 6\omega_4^2 \omega_2^2 v_1^2 + 12\omega_4 \omega_2 v_1^2 \omega_3 + 22\omega_4^2 \omega_2^2 \omega_3 c_s^2 + 12\omega_4^2 \omega_2 c_s^2 + \omega_4^2 \omega_2^3 v_1^2 \omega_3 + 12\omega_4 \omega_2^2 v_1^2 - 18\omega_4^2 \omega_2^2 c_s^2 + 12\omega_2^2 \omega_3 c_s^2 - 30\omega_4^2 \omega_2 \omega_3 c_s^2 - 12\omega_4^2 \omega_2 v_1^2 + 12\omega_4^2 \omega_3 c_s^2 - 30\omega_4 \omega_2^2 v_1^2 \omega_3 + 3\omega_4^2 \omega_2^3 c_s^2 - 6\omega_4 \omega_2^2 v_1^2 - 2\omega_4^2 \omega_2^3 c_s^2 - 30\omega_4 \omega_2^2 \omega_3 c_s^2 - 24\omega_4^2 v_1^2 \omega_3 + 36\omega_4^2 \omega_2 v_1^2 \omega_3 + 12\omega_2^2 v_1^2 \omega_3) \frac{\rho}{12\omega_4^2 \omega_2^3 \omega_3}$$

$$C_{D_t D_x^2 D_y v_2}^{(0), \text{MRT}2} = C_{D_t D_x^2 D_y v_2}^{(0), \text{MRT}1}$$

$$C_{D_t D_x^2 D_y v_2}^{(0), \text{CLBM}1} = (-\omega_4^2 \omega_3^3 v_1^2 - 6\omega_2^3 \omega_3 c_s^2 - 6\omega_4 \omega_2^3 c_s^2 - 9\omega_4 \omega_2^3 v_1^2 \omega_3 + 8\omega_4^2 \omega_2^2 v_1^2 \omega_3 + 12\omega_4 \omega_2 \omega_3 c_s^2 + 12\omega_4 \omega_2^2 c_s^2 + 9\omega_4 \omega_2^3 \omega_3 c_s^2 + 6\omega_2^3 v_1^2 \omega_3 - 6\omega_4^2 \omega_2^2 v_1^2 - 12\omega_4 \omega_2 v_1^2 \omega_3 + 22\omega_4^2 \omega_2^2 \omega_3 c_s^2 + 12\omega_4^2 \omega_2 c_s^2 + \omega_4^2 \omega_2^3 v_1^2 \omega_3 - 12\omega_4 \omega_2^2 v_1^2 - 18\omega_4^2 \omega_2^2 c_s^2 + 12\omega_2^2 \omega_3 c_s^2 - 30\omega_4^2 \omega_2 \omega_3 c_s^2 + 12\omega_4^2 \omega_2 v_1^2 + 12\omega_4^2 \omega_3 c_s^2 + 30\omega_4 \omega_2^2 v_1^2 \omega_3 + 3\omega_4^2 \omega_2^3 c_s^2 + 6\omega_4 \omega_2^2 v_1^2 - 2\omega_4^2 \omega_2^3 c_s^2 - 30\omega_4 \omega_2^2 \omega_3 c_s^2 + 24\omega_4^2 v_1^2 \omega_3 - 36\omega_4^2 \omega_2 v_1^2 \omega_3 - 12\omega_2^2 v_1^2 \omega_3) \frac{\rho}{12\omega_4^2 \omega_2^3 \omega_3}$$

$$C_{D_t D_x^2 D_y v_2}^{(0), \text{CLBM}2} = C_{D_t D_x^2 D_y v_2}^{(0), \text{CLBM}1}$$

coefficient $C_{D_x^3 D_y \rho}^{(0)}$ **at** $\frac{\partial^4 \rho}{\partial x_1^3 \partial x_2}$:

$$C_{D_x^3 D_y \rho}^{(0), \text{SRT}} = (24 - 36\omega - \omega^3 + 180\omega c_s^2 + 14\omega^2 + 6\omega^3 c_s^2 - 120c_s^2 - 72\omega^2 c_s^2) \frac{v_2 v_1}{6\omega^3}$$

$$C_{D_x^3 D_y \rho}^{(0), \text{MRT}1} = (6\omega_4 \omega_2^3 \omega_3^3 - 36\omega_2^4 \omega_3^3 c_s^2 + 6\omega_4^2 \omega_2^3 v_1^2 + 6\omega_2^3 v_1^2 \omega_3^3 - 12\omega_4 \omega_2^2 \omega_3^2 c_s^2 + 78\omega_4^2 \omega_2 \omega_3^3 c_s^2 - 12\omega_2^2 \omega_3^3 c_s^2 - 3\omega_4 \omega_2^3 \omega_3^2 - 3\omega_2^3 \omega_3^3 + 6\omega_4^2 \omega_2^3 v_1^2 \omega_3 - 12\omega_2^2 \omega_3^3 \omega_2^2 c_s^2 - 24\omega_4 \omega_2 v_1^2 \omega_3^3 - 24\omega_4^2 \omega_2 \omega_3^2 c_s^2 - 12\omega_4^2 \omega_2^2 v_1^2 \omega_3^3 - 12\omega_4 \omega_2^3 v_1^2 \omega_3^3 + 42\omega_4 \omega_2^2 \omega_3^3 c_s^2 - 21\omega_4 \omega_2^2 \omega_3^3 + 6\omega_4^2 \omega_2^3 \omega_3^3 c_s^2 - 12\omega_2^2 \omega_2^3 \omega_3 c_s^2 + 6\omega_4^2 \omega_2^3 v_1^2 \omega_3^3 + 6\omega_4 \omega_2^3 v_1^2 \omega_3^3 + 6\omega_4 \omega_2^2 \omega_3^3 - 6\omega_4^2 \omega_2 \omega_3^3 + 6\omega_2^2 \omega_3^3 - 12\omega_4^2 \omega_2^3 v_1^2 \omega_3 - 24\omega_4 \omega_2 \omega_3^3 c_s^2 + 12\omega_4^2 \omega_2 v_1^2 \omega_3^3 + 42\omega_4^2 \omega_2^2 \omega_3^2 c_s^2 + 7\omega_4^2 \omega_2^2 \omega_3^3 + 6\omega_4 \omega_2^3 \omega_3^3 c_s^2 - 12\omega_2^3 v_1^2 \omega_3^3 - 30\omega_4^2 \omega_2 v_1^2 \omega_3^3 + 6\omega_2^3 \omega_3^3 c_s^2 + 24\omega_4^2 v_1^2 \omega_3^3 + 12\omega_4 \omega_2 \omega_3^3 - 3\omega_4^2 \omega_2^2 \omega_3^3 - \omega_4^2 \omega_2^3 \omega_3^3 - 48\omega_4^2 \omega_2^3 c_s^2 - 12\omega_4 \omega_2^2 v_1^2 \omega_3^3 + 6\omega_4^2 \omega_2^3 \omega_3 c_s^2 + \omega_4^2 \omega_2^3 \omega_3^2 + 42\omega_4 \omega_2^2 v_1^2 \omega_3^3 + 6\omega_4^2 \omega_2^3 v_1^2 \omega_3^3 - 12\omega_4 \omega_2^3 \omega_3^3 c_s^2) \frac{v_2 v_1}{6\omega_4^2 \omega_2^3 \omega_3^3}$$

$$C_{D_x^3 D_y \rho}^{(0), \text{MRT}2} = C_{D_x^3 D_y \rho}^{(0), \text{MRT}1}$$

$$C_{D_x^3 D_y \rho}^{(0), \text{CLBM}1} = (6\omega_4 \omega_2^3 \omega_3^3 - 12\omega_4^2 \omega_3^3 c_s^2 + 6\omega_4^2 \omega_2^3 v_1^2 - 6\omega_2^3 v_1^2 \omega_3^3 - 24\omega_4 \omega_2^2 \omega_3^2 c_s^2 + 36\omega_4^2 \omega_2 \omega_3^3 c_s^2 - 36\omega_2^2 \omega_3^3 c_s^2 - 3\omega_4 \omega_2^3 \omega_3^2 - 3\omega_2^3 \omega_3^3 + 6\omega_4^2 \omega_2^3 v_1^2 \omega_3 - 12\omega_4^2 \omega_2^3 \omega_3^2 c_s^2 - 24\omega_4 \omega_2 v_1^2 \omega_3^3 - 12\omega_4^2 \omega_2 \omega_3^2 c_s^2 - 6\omega_4^2 \omega_2^2 v_1^2 \omega_3^3 + 72\omega_4 \omega_2^2 \omega_3^3 c_s^2 - 21\omega_4 \omega_2^2 \omega_3^3 + 6\omega_4^2 \omega_2^3 \omega_3^3 c_s^2 - 12\omega_4^2 \omega_2^2 \omega_3 c_s^2 - 6\omega_4^2 \omega_2^3 v_1^2 \omega_3^3 + 6\omega_4 \omega_2^3 \omega_3^3 + 6\omega_2^2 \omega_3^3 - 12\omega_4^2 \omega_2^3 v_1^2 \omega_3 - 24\omega_4 \omega_2 \omega_3^3 c_s^2 + 12\omega_4^2 \omega_2 v_1^2 \omega_3^3 + 42\omega_4^2 \omega_2^2 \omega_3^2 c_s^2 + 7\omega_4^2 \omega_2^2 \omega_3^3 + 18\omega_2^3 \omega_3^3 c_s^2 + 12\omega_4 \omega_2 \omega_3^3 - 3\omega_4^2 \omega_2^2 \omega_3^3 - \omega_4^2 \omega_2^3 \omega_3^3 - 36\omega_4^2 \omega_2^2 \omega_3^2 c_s^2 + 6\omega_4^2 \omega_2^3 \omega_3 c_s^2 + \omega_4^2 \omega_2^3 \omega_3^2 + 12\omega_4 \omega_2^2 v_1^2 \omega_3^3 + 6\omega_4^2 \omega_2 v_1^2 \omega_3^3 - 24\omega_4 \omega_2^2 \omega_3^3 c_s^2) \frac{v_2 v_1}{6\omega_4^2 \omega_2^3 \omega_3^3}$$

$$C_{D_x^3 D_y \rho}^{(0), \text{CLBM}2} = C_{D_x^3 D_y \rho}^{(0), \text{CLBM}1}$$

coefficient $C_{D_x^3 D_y v_1}^{(0)}$ **at** $\frac{\partial^4 v_1}{\partial x_1^3 \partial x_2}$:

$$C_{D_x^3 D_y v_1}^{(0), \text{SRT}} = (12 - 12\omega^2 v_1^2 - 18\omega - \omega^3 + 144\omega c_s^2 + 8\omega^2 + 3\omega^3 v_1^2 - 12v_1^2 + 4\omega^3 c_s^2 - 96c_s^2 - 56\omega^2 c_s^2 + 18\omega v_1^2) \frac{v_2 \rho}{12\omega^3}$$

$$C_{D_x^3 D_y v_1}^{(0), \text{MRT}1} = (3\omega_4 \omega_2^3 \omega_3^3 - 12\omega_4^2 \omega_3^3 c_s^2 + 12\omega_4^2 \omega_2^3 v_1^2 + 6\omega_2^3 v_1^2 \omega_3^3 - 24\omega_4 \omega_2^2 \omega_3^2 c_s^2 + 36\omega_4^2 \omega_2 \omega_3^3 c_s^2 - 12\omega_2^2 \omega_3^3 c_s^2 - 6\omega_4 \omega_2^3 \omega_3^2 - 12\omega_4^2 \omega_2^3 \omega_3^2 c_s^2 - 24\omega_4 \omega_2 v_1^2 \omega_3^3 + 12\omega_4^2 \omega_2^2 v_1^2 \omega_3^3 + 36\omega_4 \omega_2^2 \omega_3^3 c_s^2 - 6\omega_4 \omega_2^2 \omega_3^3 + 4\omega_4^2 \omega_2^3 \omega_3^3 c_s^2 - 12\omega_4^2 \omega_2^2 \omega_3 c_s^2 + 12\omega_4 \omega_2^3 v_1^2 \omega_3^3 + 12\omega_4 \omega_2^2 \omega_3^3 - 18\omega_4^2 \omega_2^3 v_1^2 \omega_3 - 12\omega_4 \omega_2 \omega_3^3 c_s^2 + 48\omega_4^2 \omega_2^2 \omega_3^2 c_s^2 + 3\omega_4^2 \omega_2^3 \omega_3^3 + 12\omega_4 \omega_2^3 \omega_3^3 c_s^2 - 12\omega_2^2 v_1^2 \omega_3^3 - 30\omega_4^2 \omega_2 v_1^2 \omega_3^3 + 6\omega_2^3 \omega_3^3 c_s^2 + 24\omega_4^2 v_1^2 \omega_3^3 - 6\omega_4^2 \omega_2^2 \omega_3^3 - \omega_4^2 \omega_2^3 \omega_3^3 - 32\omega_4^2 \omega_2^2 \omega_3^2 c_s^2 - 24\omega_4 \omega_2^2 v_1^2 \omega_3^3 + 6\omega_4^2 \omega_2^3 \omega_3 c_s^2 + 3\omega_4^2 \omega_2^3 v_1^2 \omega_3^3 + 2\omega_4^2 \omega_2^3 \omega_3^2 + 36\omega_4 \omega_2^2 v_1^2 \omega_3^3 - 12\omega_4 \omega_2^2 \omega_3^3 c_s^2) \frac{v_2 \rho}{12\omega_4^2 \omega_2^3 \omega_3^3}$$

$$C_{\text{D}_x^3 \text{D}_y v_1}^{(0), \text{MRT}^2} = C_{\text{D}_x^3 \text{D}_y v_1}^{(0), \text{MRT}^1}$$

$$C_{\text{D}_x^3 \text{D}_y v_1}^{(0), \text{CLBM}^1} = (3\omega_4\omega_2^3\omega_3^3 - 12\omega_4^2\omega_3^3c_s^2 + 12\omega_4^2\omega_2^3v_1^2 - 6\omega_2^3v_1^2\omega_3^3 - 24\omega_4\omega_2^2\omega_3^2c_s^2 + 36\omega_4^2\omega_2\omega_3^3c_s^2 - 12\omega_2^2\omega_3^3c_s^2 - 6\omega_4\omega_2^3\omega_3^2 - 12\omega_4^2\omega_2^3\omega_3^2c_s^2 + 12\omega_4\omega_2v_1^2\omega_3^3 - 24\omega_4^2\omega_2\omega_3^2c_s^2 + 12\omega_4^2\omega_2^2v_1^2\omega_3^2 + 36\omega_4\omega_2^2\omega_3^3c_s^2 - 6\omega_4\omega_2^2\omega_3^3 + 4\omega_4^2\omega_2^3\omega_3^3c_s^2 - 12\omega_4^2\omega_2^3\omega_3c_s^2 - 12\omega_4^2\omega_2^2v_1^2\omega_3^3 + 12\omega_4\omega_2^3v_1^2\omega_3^2 + 12\omega_4\omega_2^2\omega_3^3 - 18\omega_4^2\omega_2^3v_1^2\omega_3 - 12\omega_4\omega_2\omega_3^3c_s^2 + 48\omega_4^2\omega_2^2\omega_3^3c_s^2 + 3\omega_4^2\omega_2^2\omega_3^3 + 12\omega_4\omega_2^3\omega_3^3c_s^2 + 12\omega_2^3v_1^2\omega_3^3 + 30\omega_4^2\omega_2v_1^2\omega_3^3 + 6\omega_2^3\omega_3^3c_s^2 - 24\omega_4^2v_1^2\omega_3^3 - 6\omega_4^2\omega_2^2\omega_3^3 - \omega_4^2\omega_2^3\omega_3^3 - 32\omega_4^2\omega_2^2\omega_3^3c_s^2 - 24\omega_4\omega_2^2v_1^2\omega_3^2 + 6\omega_4^2\omega_2^3\omega_3c_s^2 + 3\omega_4^2\omega_2^3v_1^2\omega_3^3 + 2\omega_4^2\omega_2^3\omega_3^3 - 12\omega_4\omega_2^2v_1^2\omega_3^3 - 12\omega_4\omega_2^3\omega_3^3c_s^2) \frac{v_2\rho}{12\omega_4^2\omega_2^3\omega_3^3}$$

$$C_{\text{D}_x^3 \text{D}_y v_1}^{(0), \text{CLBM}^2} = C_{\text{D}_x^3 \text{D}_y v_1}^{(0), \text{CLBM}^1}$$

$$\text{coefficient } C_{\text{D}_x^3 \text{D}_y v_2}^{(0)} \text{ at } \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} :$$

$$C_{\text{D}_x^3 \text{D}_y v_2}^{(0), \text{SRT}} = (36 - 20\omega^2v_1^2 - 54\omega - \omega^3 + 144\omega c_s^2 + 20\omega^2 + \omega^3v_1^2 - 36v_1^2 + 4\omega^3c_s^2 - 96c_s^2 - 56\omega^2c_s^2 + 54\omega v_1^2) \frac{\rho v_1}{12\omega^3}$$

$$C_{\text{D}_x^3 \text{D}_y v_2}^{(0), \text{MRT}^1} = (\omega_2^2\omega_3^3v_1^2 - 12\omega_4\omega_2^3c_s^2 + 24\omega_4\omega_2 + 12\omega_4^2v_1^2 + 6\omega_2^3c_s^2 + 48\omega_4\omega_2^2c_s^2 - 8\omega_4^2\omega_2^2v_1^2 - 36\omega_4\omega_2^2 - 36\omega_4\omega_2v_1^2 + 90\omega_4^2\omega_2c_s^2 - 12\omega_2^2c_s^2 + 9\omega_4\omega_2^3 + 48\omega_4\omega_2^2v_1^2 - 44\omega_4^2\omega_2^2c_s^2 - \omega_4^2\omega_2^3 - 36\omega_4\omega_2c_s^2 - 12\omega_2^2v_1^2 + 11\omega_4^2\omega_2^2 - 12\omega_4^2\omega_2 - 6\omega_2^3 + 4\omega_4^2\omega_2^3c_s^2 - 12\omega_4\omega_2^3v_1^2 + 12\omega_2^2 - 48\omega_4^2c_s^2 + 6\omega_2^3v_1^2) \frac{\rho v_1}{12\omega_4^2\omega_2^3}$$

$$C_{\text{D}_x^3 \text{D}_y v_2}^{(0), \text{MRT}^2} = C_{\text{D}_x^3 \text{D}_y v_2}^{(0), \text{MRT}^1}$$

$$C_{\text{D}_x^3 \text{D}_y v_2}^{(0), \text{CLBM}^1} = (\omega_4^2\omega_2^3v_1^2 - 30\omega_4\omega_2^3c_s^2 + 24\omega_4\omega_2 + 12\omega_4^2v_1^2 + 30\omega_2^3c_s^2 + 96\omega_4\omega_2^2c_s^2 - 14\omega_4^2\omega_2^2v_1^2 - 36\omega_4\omega_2^2 - 60\omega_4\omega_2v_1^2 + 18\omega_4^2\omega_2c_s^2 - 60\omega_2^2c_s^2 + 9\omega_4\omega_2^3 + 48\omega_4\omega_2^2v_1^2 - 26\omega_4^2\omega_2^2c_s^2 - \omega_4^2\omega_2^3 - 36\omega_4\omega_2c_s^2 + 12\omega_4^2\omega_2v_1^2 + 12\omega_2^2v_1^2 + 11\omega_4^2\omega_2^2 - 12\omega_4^2\omega_2 - 6\omega_2^3 + 4\omega_4^2\omega_2^3c_s^2 - 6\omega_4\omega_2^3v_1^2 + 12\omega_2^2 - 6\omega_2^3v_1^2) \frac{\rho v_1}{12\omega_4^2\omega_2^3}$$

$$C_{\text{D}_x^3 \text{D}_y v_2}^{(0), \text{CLBM}^2} = C_{\text{D}_x^3 \text{D}_y v_2}^{(0), \text{CLBM}^1}$$

$$\text{coefficient } C_{\text{D}_t^2 \text{D}_y^2 v_2}^{(0)} \text{ at } \frac{\partial^4 v_2}{\partial t^2 \partial x_2^2} :$$

$$C_{\text{D}_t^2 \text{D}_y^2 v_2}^{(0), \text{SRT}} = (-2 + 3\omega - \omega^2) \frac{3v_2\rho}{2\omega^3}$$

$$C_{\text{D}_t^2 \text{D}_y^2 v_2}^{(0), \text{MRT}^1} = (-\omega_5^2\omega_3^2 - \omega_5^2\omega_3 - 4\omega_3^2 - 4\omega_5\omega_3 + 2\omega_3^3 + 2\omega_5^2 - 2\omega_5\omega_3^3 + 8\omega_5\omega_3^2) \frac{v_2\rho}{2\omega_5^2\omega_3^3}$$

$$C_{\text{D}_t^2 \text{D}_y^2 v_2}^{(0), \text{MRT}^2} = C_{\text{D}_t^2 \text{D}_y^2 v_2}^{(0), \text{MRT}^1}$$

$$C_{\text{D}_t^2 \text{D}_y^2 v_2}^{(0), \text{CLBM}^1} = (-2 - \omega_3^2 + 3\omega_3) \frac{3v_2\rho}{2\omega_3^3}$$

$$C_{\text{D}_t^2 \text{D}_y^2 v_2}^{(0), \text{CLBM}^2} = C_{\text{D}_t^2 \text{D}_y^2 v_2}^{(0), \text{CLBM}^1}$$

$$\text{coefficient } C_{\text{D}_t \text{D}_x \text{D}_y^2 v_1}^{(0)} \text{ at } \frac{\partial^4 v_1}{\partial t \partial x_1 \partial x_2^2} :$$

$$C_{\text{D}_t \text{D}_x \text{D}_y^2 v_1}^{(0), \text{SRT}} = (-90\omega c_s^2 + \omega^3v_2^2 - 2\omega^2v_2^2 - 2\omega^3c_s^2 + 60c_s^2 + 34\omega^2c_s^2) \frac{\rho}{12\omega^3}$$

$$C_{\text{D}_t \text{D}_x \text{D}_y^2 v_1}^{(0), \text{MRT}^1} = (36\omega_5^2\omega_2v_2^2\omega_3 + 12\omega_5\omega_2\omega_3c_s^2 + 12\omega_5v_2^2\omega_3^2 + 12\omega_5\omega_3^2c_s^2 - 2\omega_5^2\omega_2\omega_3^2c_s^2 - 6\omega_5v_2^2\omega_3^3 + 12\omega_5^2\omega_3c_s^2 - 6\omega_5\omega_3^3c_s^2 - 24\omega_5^2\omega_2v_2^2 + \omega_5^2\omega_2v_2^2\omega_3^3 - 10\omega_5^2\omega_2v_2^2\omega_3^2 + 22\omega_5^2\omega_2\omega_3^2c_s^2 + 3\omega_5^2\omega_3^3c_s^2 + 12\omega_2v_2^2\omega_3^2 + 12\omega_2\omega_3^2c_s^2 - 30\omega_5^2\omega_2\omega_3c_s^2 + 12\omega_5\omega_2v_2^2\omega_3 - 12\omega_5^2v_2^2\omega_3 - 6\omega_2v_2^2\omega_3^3 + 9\omega_5\omega_2\omega_3^3c_s^2 + 9\omega_5\omega_2v_2^2\omega_3^3 - 18\omega_5^2\omega_3^2c_s^2 - 6\omega_2\omega_3^3c_s^2 + 6\omega_5^2v_2^2\omega_3 - 30\omega_5\omega_2v_2^2\omega_3 - 30\omega_5\omega_2\omega_3^2c_s^2 + 12\omega_5^2\omega_2c_s^2 - \omega_5^2v_2^2\omega_3^3) \frac{\rho}{12\omega_5^2\omega_2\omega_3^3}$$

$$C_{\text{D}_t \text{D}_x \text{D}_y^2 v_1}^{(0), \text{MRT}^2} = C_{\text{D}_t \text{D}_x \text{D}_y^2 v_1}^{(0), \text{MRT}^1}$$

$$C_{\text{D}_t \text{D}_x \text{D}_y^2 v_1}^{(0), \text{CLBM}^1} = (-36\omega_5^2\omega_2v_2^2\omega_3 + 12\omega_5\omega_2\omega_3c_s^2 - 12\omega_5v_2^2\omega_3^2 + 12\omega_5\omega_3^2c_s^2 - 2\omega_5^2\omega_2\omega_3^2c_s^2 + 6\omega_5v_2^2\omega_3^3 + 12\omega_5^2\omega_3c_s^2 - 6\omega_5\omega_3^3c_s^2 + 24\omega_5^2\omega_2v_2^2 + \omega_5^2\omega_2v_2^2\omega_3^3 + 8\omega_5^2\omega_2v_2^2\omega_3^2 + 22\omega_5^2\omega_2\omega_3^2c_s^2 + 3\omega_5^2\omega_3^3c_s^2 - 12\omega_2v_2^2\omega_3^2 + 12\omega_2\omega_3^2c_s^2 - 30\omega_5^2\omega_2\omega_3c_s^2 - 12\omega_5\omega_2v_2^2\omega_3 + 12\omega_5^2v_2^2\omega_3 + 6\omega_2v_2^2\omega_3^3 + 9\omega_5\omega_2\omega_3^3c_s^2 - 9\omega_5\omega_2v_2^2\omega_3^3 - 18\omega_5^2\omega_3^2c_s^2 - 6\omega_2\omega_3^3c_s^2 - 6\omega_5^2v_2^2\omega_3 + 30\omega_5\omega_2v_2^2\omega_3 - 30\omega_5\omega_2\omega_3^2c_s^2 + 12\omega_5^2\omega_2c_s^2 - \omega_5^2v_2^2\omega_3^3) \frac{\rho}{12\omega_5^2\omega_2\omega_3^3}$$

$$C_{\text{D}_t \text{D}_x \text{D}_y^2 v_1}^{(0), \text{CLBM}^2} = C_{\text{D}_t \text{D}_x \text{D}_y^2 v_1}^{(0), \text{CLBM}^1}$$

$$C_{\text{D}_x^2 \text{D}_y^2 v_1}^{(0), \text{CLBM1}} = (22\omega_5^2 \omega_2^3 v_2^2 \omega_3^2 - 14\omega_5^2 \omega_2^3 \omega_3^2 c_s^2 - 4\omega_5^2 \omega_2^3 v_2^2 \omega_3^3 + 6\omega_5^2 \omega_2 \omega_3^3 c_s^2 - 12\omega_5^2 \omega_3^3 c_s^2 + \omega_5^2 \omega_2^3 \omega_3^3 c_s^2 - 30\omega_5^2 \omega_2 v_2^2 \omega_3^3 + 24\omega_5^2 \omega_2 v_2^2 \omega_3^2 - 12\omega_5^2 \omega_2^2 \omega_3^3 c_s^2 - 12\omega_5 \omega_2^3 \omega_3^3 c_s^2 - 18\omega_5^2 \omega_2^3 v_2^2 \omega_3 + 22\omega_5^2 \omega_2^3 v_2^2 \omega_3^3 - 24\omega_5 \omega_2^3 v_2^2 \omega_3^2 + 24\omega_5 \omega_2^3 \omega_3^3 c_s^2 + 6\omega_2^3 \omega_3^3 c_s^2 + 12\omega_5^2 \omega_2^2 \omega_3^3 c_s^2 - 48\omega_5^2 \omega_2^2 v_2^2 \omega_3^3 + 6\omega_5 \omega_2^3 v_2^2 \omega_3^3 - 6\omega_5 \omega_2^3 \omega_3^3 c_s^2 + 24\omega_5^2 \omega_2^2 v_2^2 \omega_3 - 6\omega_2^3 v_2^2 \omega_3^3 + 12\omega_5 \omega_2^3 v_2^2 \omega_3 - 6\omega_5^2 \omega_2^3 \omega_3^3 c_s^2 + 24\omega_5^2 \omega_2^3 \omega_3^3 c_s^2 + 12\omega_2^3 v_2^2 \omega_3^2 - 12\omega_2^3 \omega_3^3 c_s^2 + 12\omega_5^2 v_2^2 \omega_3^3) \frac{\rho v_1}{12\omega_5^2 \omega_2^3 \omega_3^3}$$

$$C_{\text{D}_x^2 \text{D}_y^2 v_1}^{(0), \text{CLBM2}} = C_{\text{D}_x^2 \text{D}_y^2 v_1}^{(0), \text{CLBM1}}$$

$$\text{coefficient } C_{\text{D}_x^2 \text{D}_y^2 v_2}^{(0)} \text{ at } \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} :$$

$$C_{\text{D}_x^2 \text{D}_y^2 v_2}^{(0), \text{SRT}} = (50\omega^2 v_1^2 + 72\omega c_s^2 - 4\omega^3 v_1^2 + 84v_1^2 + \omega^3 c_s^2 - 48c_s^2 - 26\omega^2 c_s^2 - 126\omega v_1^2) \frac{v_2 \rho}{12\omega^3}$$

$$C_{\text{D}_x^2 \text{D}_y^2 v_2}^{(0), \text{MRT1}} = (-12\omega_4^2 \omega_3^3 c_s^2 + 12\omega_4^2 \omega_3^3 v_1^2 + 6\omega_2^3 v_1^2 \omega_3^3 + 24\omega_4^2 \omega_2 \omega_3^3 c_s^2 - 12\omega_2^3 \omega_3^3 c_s^2 + 24\omega_4^2 \omega_2^2 v_1^2 \omega_3 - 6\omega_4^2 \omega_2^3 \omega_3^2 c_s^2 - 12\omega_4 \omega_2 v_1^2 \omega_3^3 - 48\omega_4^2 \omega_2^2 v_1^2 \omega_3^2 - 6\omega_4 \omega_2^3 v_1^2 \omega_3^3 + 24\omega_4 \omega_2^3 \omega_3^3 c_s^2 + \omega_4^2 \omega_2^3 \omega_3^3 c_s^2 - 12\omega_4^2 \omega_2^2 \omega_3^3 c_s^2 + 34\omega_4^2 \omega_2^2 v_1^2 \omega_3^3 - 30\omega_4^2 \omega_2^3 v_1^2 \omega_3 - 12\omega_4 \omega_2 \omega_3^3 c_s^2 + 24\omega_4^2 \omega_2 v_1^2 \omega_3^2 + 12\omega_4^2 \omega_2^2 \omega_3^3 c_s^2 - 12\omega_2^3 v_1^2 \omega_3^3 - 78\omega_4^2 \omega_2 v_1^2 \omega_3^3 + 6\omega_2^3 \omega_3^3 c_s^2 + 48\omega_4^2 v_1^2 \omega_3^3 - 14\omega_4^2 \omega_2^2 \omega_3^3 c_s^2 + 6\omega_4^2 \omega_2^3 \omega_3^3 c_s^2 - 4\omega_4^2 \omega_2^3 v_1^2 \omega_3^3 + 24\omega_4 \omega_2^2 v_1^2 \omega_3^3 + 22\omega_4^2 \omega_2^3 v_1^2 \omega_3^2 - 6\omega_4 \omega_2^3 \omega_3^3 c_s^2) \frac{v_2 \rho}{12\omega_4^2 \omega_2^3 \omega_3^3}$$

$$C_{\text{D}_x^2 \text{D}_y^2 v_2}^{(0), \text{MRT2}} = C_{\text{D}_x^2 \text{D}_y^2 v_2}^{(0), \text{MRT1}}$$

$$C_{\text{D}_x^2 \text{D}_y^2 v_2}^{(0), \text{CLBM1}} = (-12\omega_4^2 \omega_3^3 c_s^2 + 12\omega_4^2 \omega_3^3 v_1^2 - 6\omega_2^3 v_1^2 \omega_3^3 + 24\omega_4^2 \omega_2 \omega_3^3 c_s^2 - 12\omega_2^3 \omega_3^3 c_s^2 + 24\omega_4^2 \omega_2^2 v_1^2 \omega_3 - 6\omega_4^2 \omega_2^3 \omega_3^2 c_s^2 + 12\omega_4 \omega_2 v_1^2 \omega_3^3 - 48\omega_4^2 \omega_2^2 v_1^2 \omega_3^2 + 6\omega_4 \omega_2^3 v_1^2 \omega_3^3 + 24\omega_4 \omega_2^3 \omega_3^3 c_s^2 + \omega_4^2 \omega_2^3 \omega_3^3 c_s^2 - 12\omega_4^2 \omega_2^2 \omega_3^3 c_s^2 + 22\omega_4^2 \omega_2^2 v_1^2 \omega_3^3 - 30\omega_4^2 \omega_2^3 v_1^2 \omega_3 - 12\omega_4 \omega_2 \omega_3^3 c_s^2 + 24\omega_4^2 \omega_2 v_1^2 \omega_3^2 + 12\omega_4^2 \omega_2^2 \omega_3^3 c_s^2 + 12\omega_2^3 v_1^2 \omega_3^3 - 18\omega_4^2 \omega_2 v_1^2 \omega_3^3 + 6\omega_2^3 \omega_3^3 c_s^2 - 14\omega_4^2 \omega_2^2 \omega_3^3 c_s^2 + 6\omega_4^2 \omega_2^3 \omega_3^3 c_s^2 - 4\omega_4^2 \omega_2^3 v_1^2 \omega_3^3 - 24\omega_4 \omega_2^2 v_1^2 \omega_3^3 + 22\omega_4^2 \omega_2^3 v_1^2 \omega_3^2 - 6\omega_4 \omega_2^3 \omega_3^3 c_s^2) \frac{v_2 \rho}{12\omega_4^2 \omega_2^3 \omega_3^3}$$

$$C_{\text{D}_x^2 \text{D}_y^2 v_2}^{(0), \text{CLBM2}} = C_{\text{D}_x^2 \text{D}_y^2 v_2}^{(0), \text{CLBM1}}$$

$$\text{coefficient } C_{\text{D}_t \text{D}_y^3 v_2}^{(0)} \text{ at } \frac{\partial^4 v_2}{\partial t \partial x_2^2} :$$

$$C_{\text{D}_t \text{D}_y^3 v_2}^{(0), \text{SRT}} = (-36 + 54\omega + \omega^3 + 72v_2^2 - 90\omega c_s^2 - 3\omega^3 v_2^2 - 20\omega^2 + 42\omega^2 v_2^2 - 108\omega v_2^2 - 2\omega^3 c_s^2 + 60c_s^2 + 34\omega^2 c_s^2) \frac{\rho}{12\omega^3}$$

$$C_{\text{D}_t \text{D}_y^3 v_2}^{(0), \text{MRT1}} = (-11\omega_5^2 \omega_3^3 - 60\omega_5 v_2^2 \omega_3^3 - 36\omega_5 \omega_2^3 c_s^2 + 24\omega_5^2 c_s^2 + 15\omega_5 v_2^2 \omega_3^3 - 48\omega_5^2 \omega_3^3 c_s^2 + \omega_5^2 \omega_3^3 + 9\omega_5 \omega_3^3 c_s^2 + 12\omega_5^2 \omega_3 + 48\omega_5 v_2^2 \omega_3 - 12\omega_3^3 - 6v_2^2 \omega_3^3 + 12\omega_5^2 v_2^2 - 2\omega_5^2 \omega_3^3 c_s^2 - 24\omega_5 \omega_3 - 42\omega_5^2 v_2^2 \omega_3 + 6\omega_3^3 + 12v_2^2 \omega_3^3 + 12\omega_3^3 c_s^2 + 25\omega_5^2 \omega_3^3 c_s^2 + 27\omega_5^2 v_2^2 \omega_3^3 - 9\omega_5 \omega_3^3 - 6\omega_3^3 c_s^2 + 24\omega_5 \omega_3 c_s^2 + 36\omega_5 \omega_3^3 - 3\omega_5^2 v_2^2 \omega_3^3) \frac{\rho}{12\omega_5^2 \omega_3^3}$$

$$C_{\text{D}_t \text{D}_y^3 v_2}^{(0), \text{MRT2}} = C_{\text{D}_t \text{D}_y^3 v_2}^{(0), \text{MRT1}}$$

$$C_{\text{D}_t \text{D}_y^3 v_2}^{(0), \text{CLBM1}} = (-11\omega_5^2 \omega_3^3 - 108\omega_5 v_2^2 \omega_3^3 - 36\omega_5 \omega_2^3 c_s^2 + 24\omega_5^2 c_s^2 + 27\omega_5 v_2^2 \omega_3^3 - 48\omega_5^2 \omega_3^3 c_s^2 + \omega_5^2 \omega_3^3 + 9\omega_5 \omega_3^3 c_s^2 + 12\omega_5^2 \omega_3 + 72\omega_5 v_2^2 \omega_3 - 12\omega_3^3 - 18v_2^2 \omega_3^3 - 36\omega_5^2 v_2^2 - 2\omega_5^2 \omega_3^3 c_s^2 - 24\omega_5 \omega_3 + 18\omega_5^2 v_2^2 \omega_3 + 6\omega_3^3 + 36v_2^2 \omega_3^3 + 12\omega_3^3 c_s^2 + 25\omega_5^2 \omega_3^3 c_s^2 + 15\omega_5^2 v_2^2 \omega_3^3 - 9\omega_5 \omega_3^3 - 6\omega_3^3 c_s^2 + 24\omega_5 \omega_3 c_s^2 + 36\omega_5 \omega_3^3 - 3\omega_5^2 v_2^2 \omega_3^3) \frac{\rho}{12\omega_5^2 \omega_3^3}$$

$$C_{\text{D}_t \text{D}_y^3 v_2}^{(0), \text{CLBM2}} = C_{\text{D}_t \text{D}_y^3 v_2}^{(0), \text{CLBM1}}$$

$$\text{coefficient } C_{\text{D}_x \text{D}_y^3 \rho}^{(0)} \text{ at } \frac{\partial^4 \rho}{\partial x_1 \partial x_2^2} :$$

$$C_{\text{D}_x \text{D}_y^3 \rho}^{(0), \text{SRT}} = (24 - 36\omega - \omega^3 + 180\omega c_s^2 + 14\omega^2 + 6\omega^3 c_s^2 - 120c_s^2 - 72\omega^2 c_s^2) \frac{v_2 v_1}{6\omega^3}$$

$$C_{\text{D}_x \text{D}_y^3 \rho}^{(0), \text{MRT1}} = (\omega_5^2 \omega_2^2 \omega_3^3 - 6\omega_5^2 \omega_2^3 \omega_3 + 6\omega_2^3 \omega_3^2 + 6\omega_5 \omega_2^2 v_2^2 \omega_3^3 + 6\omega_5^2 \omega_2^3 v_2^2 \omega_3^2 - 48\omega_5^2 \omega_2^3 \omega_3^3 c_s^2 - 12\omega_5 \omega_2^2 \omega_3^3 c_s^2 - 3\omega_2^3 \omega_3^3 - 12\omega_5 \omega_2^2 v_2^2 \omega_3^2 + 6\omega_5^2 \omega_2^3 c_s^2 - 3\omega_5^2 \omega_2^2 \omega_3^3 - 36\omega_5^2 \omega_2^3 c_s^2 + 6\omega_5^2 \omega_2^3 \omega_3^3 c_s^2 - \omega_5^2 \omega_2^3 \omega_3^3 - 24\omega_5^2 \omega_2^2 \omega_3^3 c_s^2 - 12\omega_5^2 \omega_2 v_2^2 \omega_3^3 + 6\omega_5^2 \omega_2 v_2^2 \omega_3^2 - 12\omega_5^2 \omega_2 \omega_3^3 c_s^2 + 7\omega_5^2 \omega_2^3 \omega_3^3 - 24\omega_5 \omega_2^3 \omega_3^3 c_s^2 - 30\omega_5^2 \omega_2^3 v_2^2 \omega_3 + 6\omega_5 \omega_2^2 \omega_3^3 c_s^2 + 6\omega_5^2 \omega_2^2 v_2^2 \omega_3^3 + 42\omega_5 \omega_2^3 v_2^2 \omega_3 + 42\omega_5 \omega_2^3 \omega_3^3 c_s^2 + 6\omega_5 \omega_2^3 \omega_3^3 + 6\omega_2^3 \omega_3^3 c_s^2 - 21\omega_5 \omega_2^3 \omega_3^2 + 42\omega_5^2 \omega_2^3 \omega_3^3 c_s^2 - 12\omega_5^2 \omega_2^2 v_2^2 \omega_3^3 - 12\omega_5 \omega_2^3 v_2^2 \omega_3^3 - 12\omega_5 \omega_2^3 \omega_3^3 c_s^2 + 12\omega_5 \omega_2^3 \omega_3 - 3\omega_5 \omega_2^2 \omega_3^3 + 12\omega_5^2 \omega_2^2 v_2^2 \omega_3 + 6\omega_2^3 v_2^2 \omega_3^3 - 24\omega_5 \omega_2^3 v_2^2 \omega_3 + 6\omega_5 \omega_2^2 \omega_3^3 - 12\omega_5^2 \omega_2^3 \omega_3^3 c_s^2 + 24\omega_5^2 \omega_2^2 v_2^2 + 78\omega_5^2 \omega_2^3 \omega_3 c_s^2 - 12\omega_2^3 v_2^2 \omega_3^2 - 12\omega_2^3 \omega_3^3 c_s^2 + 6\omega_5^2 v_2^2 \omega_3^3) \frac{v_2 v_1}{6\omega_5^2 \omega_2^3 \omega_3^3}$$

$$C_{\text{D}_x \text{D}_y^3 \rho}^{(0), \text{MRT2}} = C_{\text{D}_x \text{D}_y^3 \rho}^{(0), \text{MRT1}}$$

$$C_{\text{D}_x \text{D}_y^3 \rho}^{(0), \text{CLBM1}} = (\omega_5^2 \omega_2^2 \omega_3^3 - 6\omega_5^2 \omega_2^3 \omega_3 + 6\omega_2^3 \omega_3^2 - 6\omega_5^2 \omega_2^3 v_2^2 \omega_3^3 - 36\omega_5^2 \omega_2^3 \omega_3^3 c_s^2 - 24\omega_5 \omega_2^2 \omega_3^3 c_s^2 - 3\omega_2^3 \omega_3^3 + 6\omega_5^2 \omega_2 \omega_3^3 c_s^2 - 3\omega_5^2 \omega_2^2 \omega_3^3 - 12\omega_5^2 \omega_2^3 c_s^2 +$$

$$6\omega_5^2\omega_3^3\omega_s^2c_s^2 - \omega_5^2\omega_3^3\omega_s^3 - 12\omega_5^2\omega_2\omega_3c_s^2 - 12\omega_5^2\omega_2v_2^3\omega_3^3 + 6\omega_5^2\omega_2v_2^2\omega_3^2 - 12\omega_5^2\omega_2\omega_3^2c_s^2 + 7\omega_5^2\omega_3^3\omega_s^2 - 24\omega_5\omega_3^3\omega_s^2c_s^2 + 12\omega_5^2\omega_3^3v_2^2\omega_3 + 12\omega_5\omega_2^2\omega_3^3c_s^2 + 6\omega_5^2\omega_3^3v_2^2\omega_3^3 + 12\omega_5\omega_3^3v_2^2\omega_3^2 + 72\omega_5\omega_3^3\omega_s^2c_s^2 + 6\omega_5\omega_3^3\omega_s^3 + 18\omega_3^3\omega_s^3c_s^2 - 21\omega_5\omega_3^3\omega_s^2 + 36\omega_5^2\omega_2^2\omega_3^2c_s^2 - 6\omega_5^2\omega_2^2v_2^2\omega_3^2 - 24\omega_5\omega_2^2\omega_3^3c_s^2 + 12\omega_5\omega_2^2\omega_3 - 3\omega_5\omega_2^2\omega_3^3 - 6\omega_3^3v_2^2\omega_3^3 - 24\omega_5\omega_3^3v_2^2\omega_3 + 6\omega_5\omega_2^2\omega_3^2 - 12\omega_5^2\omega_3^3c_s^2 + 36\omega_5^2\omega_2^2\omega_3c_s^2 + 12\omega_3^3v_2^2\omega_3^2 - 36\omega_3^3\omega_s^2c_s^2 + 6\omega_5^2v_2^2\omega_3^3) \frac{v_2v_1}{6\omega_5^2\omega_3^3\omega_s^3}$$

$$C_{D_x D_y^3 \rho}^{(0), \text{CLBM2}} = C_{D_x D_y^3 \rho}^{(0), \text{CLBM1}}$$

$$\text{coefficient } C_{D_x D_y^3 v_1}^{(0)} \text{ at } \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} :$$

$$C_{D_x D_y^3 v_1}^{(0), \text{SRT}} = (36 - 54\omega - \omega^3 - 36v_2^2 + 144\omega c_s^2 + \omega^3 v_2^2 + 20\omega^2 - 20\omega^2 v_2^2 + 54\omega v_2^2 + 4\omega^3 c_s^2 - 96c_s^2 - 56\omega^2 c_s^2) \frac{v_2 \rho}{12\omega^3}$$

$$C_{D_x D_y^3 v_1}^{(0), \text{MRT1}} = (11\omega_5^2\omega_3^2 + 48\omega_5v_2^2\omega_3^2 + 48\omega_5\omega_3^2c_s^2 - 48\omega_5^2c_s^2 - 12\omega_5v_2^2\omega_3^3 + 90\omega_5^2\omega_3c_s^2 - \omega_5^2\omega_3^3 - 12\omega_5\omega_3^3c_s^2 - 12\omega_5^2\omega_3 - 36\omega_5v_2^2\omega_3 + 12\omega_3^2 + 6v_2^2\omega_3^3 + 12\omega_5^2v_2^2 + 4\omega_5^2\omega_3^3c_s^2 + 24\omega_5\omega_3 - 6\omega_3^3 - 12v_2^2\omega_3^2 - 12\omega_3^2c_s^2 - 44\omega_5^2\omega_3^2c_s^2 - 8\omega_5^2v_2^2\omega_3^2 + 9\omega_5\omega_3^3 + 6\omega_3^3c_s^2 - 36\omega_5\omega_3c_s^2 - 36\omega_5\omega_3^2 + \omega_5^2v_2^2\omega_3^3) \frac{v_2 \rho}{12\omega_5^2\omega_3^3}$$

$$C_{D_x D_y^3 v_1}^{(0), \text{MRT2}} = C_{D_x D_y^3 v_1}^{(0), \text{MRT1}}$$

$$C_{D_x D_y^3 v_1}^{(0), \text{CLBM1}} = (11\omega_5^2\omega_3^2 + 48\omega_5v_2^2\omega_3^2 + 96\omega_5\omega_3^2c_s^2 - 6\omega_5v_2^2\omega_3^3 + 18\omega_5^2\omega_3c_s^2 - \omega_5^2\omega_3^3 - 30\omega_5\omega_3^3c_s^2 - 12\omega_5^2\omega_3 - 60\omega_5v_2^2\omega_3 + 12\omega_3^2 - 6v_2^2\omega_3^3 + 12\omega_5^2v_2^2 + 4\omega_5^2\omega_3^3c_s^2 + 24\omega_5\omega_3 + 12\omega_5^2v_2^2\omega_3 - 6\omega_3^3 + 12v_2^2\omega_3^2 - 60\omega_3^2c_s^2 - 26\omega_5^2\omega_3^2c_s^2 - 14\omega_5^2v_2^2\omega_3^2 + 9\omega_5\omega_3^3 + 30\omega_3^3c_s^2 - 36\omega_5\omega_3c_s^2 - 36\omega_5\omega_3^2 + \omega_5^2v_2^2\omega_3^3) \frac{v_2 \rho}{12\omega_5^2\omega_3^3}$$

$$C_{D_x D_y^3 v_1}^{(0), \text{CLBM2}} = C_{D_x D_y^3 v_1}^{(0), \text{CLBM1}}$$

$$\text{coefficient } C_{D_x D_y^3 v_2}^{(0)} \text{ at } \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} :$$

$$C_{D_x D_y^3 v_2}^{(0), \text{SRT}} = (12 - 18\omega - \omega^3 - 12v_2^2 + 144\omega c_s^2 + 3\omega^3 v_2^2 + 8\omega^2 - 12\omega^2 v_2^2 + 18\omega v_2^2 + 4\omega^3 c_s^2 - 96c_s^2 - 56\omega^2 c_s^2) \frac{\rho v_1}{12\omega^3}$$

$$C_{D_x D_y^3 v_2}^{(0), \text{MRT1}} = (2\omega_5^2\omega_2^2\omega_3^3 + 12\omega_5\omega_2^2v_2^2\omega_3^3 - 32\omega_5^2\omega_2^2\omega_3^2c_s^2 - 24\omega_5\omega_2^2\omega_3^2c_s^2 - 24\omega_5\omega_2^2v_2^2\omega_3^2 + 3\omega_5^2\omega_2^2v_2^2\omega_3^3 + 6\omega_5^2\omega_2\omega_3^3c_s^2 - 6\omega_5^2\omega_2^2\omega_3^2 - 12\omega_5^2\omega_2^3c_s^2 + 4\omega_5^2\omega_3^3\omega_s^2c_s^2 - \omega_5^2\omega_3^3\omega_s^3 - 24\omega_5^2\omega_2^2\omega_3c_s^2 - 18\omega_5^2\omega_2v_2^2\omega_3^3 - 12\omega_5^2\omega_2\omega_3^2c_s^2 + 3\omega_5^2\omega_3^3\omega_s^2 - 12\omega_5\omega_3^3\omega_s^2c_s^2 - 30\omega_5^2\omega_3^3v_2^2\omega_3 + 12\omega_5\omega_2^2\omega_3^3c_s^2 + 36\omega_5\omega_2^2v_2^2\omega_3^2 + 36\omega_5\omega_2^2\omega_3^2c_s^2 + 3\omega_5\omega_2^3\omega_3^3 + 6\omega_3^3\omega_s^2c_s^2 - 6\omega_5\omega_3^3\omega_s^2 + 48\omega_5^2\omega_2^2\omega_3^2c_s^2 + 12\omega_5^2\omega_2^2v_2^2\omega_3^2 - 12\omega_5\omega_2^2v_2^2\omega_3^3 - 12\omega_5\omega_2^2\omega_3^3c_s^2 - 6\omega_5\omega_2^2\omega_3^3 + 6\omega_2^2v_2^2\omega_3^3 - 12\omega_5\omega_2^2v_2^2\omega_3 + 12\omega_5\omega_2^2\omega_3^2 - 12\omega_5^2\omega_2^2\omega_3^2c_s^2 + 24\omega_5^2\omega_2^2v_2^2 + 36\omega_5^2\omega_2^2\omega_3c_s^2 - 12\omega_2^2v_2^2\omega_3^2 - 12\omega_2^2\omega_3^2c_s^2 + 12\omega_5^2v_2^2\omega_3^3) \frac{\rho v_1}{12\omega_5^2\omega_3^3\omega_s^3}$$

$$C_{D_x D_y^3 v_2}^{(0), \text{MRT2}} = C_{D_x D_y^3 v_2}^{(0), \text{MRT1}}$$

$$C_{D_x D_y^3 v_2}^{(0), \text{CLBM1}} = (2\omega_5^2\omega_2^2\omega_3^3 + 12\omega_5\omega_2^2v_2^2\omega_3^3 - 12\omega_5^2\omega_2^2v_2^2\omega_3^2 - 32\omega_5^2\omega_2^2\omega_3^2c_s^2 - 24\omega_5\omega_2^2\omega_3^2c_s^2 - 24\omega_5\omega_2^2v_2^2\omega_3^2 + 3\omega_5^2\omega_2^2v_2^2\omega_3^3 + 6\omega_5^2\omega_2\omega_3^3c_s^2 - 6\omega_5^2\omega_2^2\omega_3^2 - 12\omega_5^2\omega_2^3c_s^2 + 4\omega_5^2\omega_3^3\omega_s^2c_s^2 - \omega_5^2\omega_3^3\omega_s^3 - 24\omega_5^2\omega_2^2\omega_3c_s^2 - 18\omega_5^2\omega_2v_2^2\omega_3^3 - 12\omega_5^2\omega_2\omega_3^2c_s^2 + 3\omega_5^2\omega_3^3\omega_s^2 - 12\omega_5\omega_3^3\omega_s^2c_s^2 + 30\omega_5^2\omega_3^3v_2^2\omega_3 + 12\omega_5\omega_2^2\omega_3^3c_s^2 - 12\omega_5\omega_2^2v_2^2\omega_3^2 + 36\omega_5\omega_2^2\omega_3^2c_s^2 + 3\omega_5\omega_2^3\omega_3^3 + 6\omega_3^3\omega_s^2c_s^2 - 6\omega_5\omega_3^3\omega_s^2 + 48\omega_5^2\omega_2^2\omega_3^2c_s^2 + 12\omega_5^2\omega_2^2v_2^2\omega_3^2 - 12\omega_5\omega_2^2v_2^2\omega_3^3 - 12\omega_5\omega_2^2\omega_3^3c_s^2 - 6\omega_5\omega_2^2\omega_3^3 + 6\omega_2^2v_2^2\omega_3^3 + 12\omega_5\omega_2^2v_2^2\omega_3 + 12\omega_5\omega_2^2\omega_3^2 - 12\omega_5^2\omega_2^2\omega_3^2c_s^2 - 24\omega_5^2\omega_2^2v_2^2 + 36\omega_5^2\omega_2^2\omega_3c_s^2 + 12\omega_2^2v_2^2\omega_3^2 - 12\omega_2^2\omega_3^2c_s^2 + 12\omega_5^2v_2^2\omega_3^3) \frac{\rho v_1}{12\omega_5^2\omega_3^3\omega_s^3}$$

$$C_{D_x D_y^3 v_2}^{(0), \text{CLBM2}} = C_{D_x D_y^3 v_2}^{(0), \text{CLBM1}}$$

$$\text{coefficient } C_{D_y^4 \rho}^{(0)} \text{ at } \frac{\partial^4 \rho}{\partial x_2^4} :$$

$$C_{D_y^4 \rho}^{(0), \text{SRT}} = (72v_2^2 + 30\omega^2c_s^4 + 36\omega c_s^2 - 3\omega^3v_2^2 - 84\omega^2v_2^2c_s^2 - 3\omega^3c_s^4 + 108\omega v_2^4 + 42\omega^2v_2^2 + 48c_s^4 + 6\omega^3v_2^2c_s^2 - 108\omega v_2^2 + \omega^3c_s^2 - 144v_2^2c_s^2 - 42\omega^2v_2^4 - 24c_s^2 - 72v_2^4 - 14\omega^2c_s^2 + 3\omega^3v_2^4 - 72\omega c_s^4 + 216\omega v_2^2c_s^2) \frac{1}{24\omega^3}$$

$$C_{D_y^4 \rho}^{(0), \text{MRT1}} = (24\omega_5^2v_2^4\omega_3 + 24\omega_5^2\omega_3^4c_s^4 - 24v_2^4\omega_3^2 + 6\omega_5^2v_2^2\omega_3^3c_s^2 - 72\omega_5v_2^2\omega_3^2 + 24\omega_5\omega_3^2c_s^2 + 12v_2^4\omega_3^3 + 24\omega_5\omega_3c_s^4 + 18\omega_5v_2^2\omega_3^3 + 12\omega_5^2\omega_3c_s^2 - 24\omega_5v_2^2\omega_3^3c_s^2 - 6\omega_5\omega_3^3c_s^2 - 72\omega_5^2v_2^2\omega_3^2c_s^2 - 3\omega_5^2\omega_3^3c_s^4 + 3\omega_5^2v_2^4\omega_3^3 + 48\omega_5v_2^2\omega_3 - 24\omega_5^2v_2^4\omega_3^2 - 12v_2^2\omega_3^3 - 12\omega_5v_2^2\omega_3^3c_s^2 + \omega_5^2\omega_3^3c_s^2 - 18\omega_5v_2^4\omega_3^3 + 6\omega_5\omega_3^3c_s^4 + 156\omega_5^2v_2^2\omega_3c_s^2 - 24\omega_5^2v_2^2\omega_3 + 24v_2^2\omega_3^2 + 72\omega_5v_2^4\omega_3 - 24v_2^2\omega_3^2c_s^2 - 48\omega_5v_2^4\omega_3 - 24\omega_5\omega_3^4c_s^4 - 8\omega_5^2\omega_3^2c_s^2 + 24\omega_5^2v_2^2\omega_3^2 + 48\omega_5v_2^2\omega_3^2c_s^2 + 12v_2^2\omega_3^3c_s^2 - 48\omega_5^2\omega_3c_s^4 - 96\omega_5^2v_2^2c_s^2 + 24\omega_5^2c_s^4 - 24\omega_5\omega_3c_s^2 - 3\omega_5^2v_2^2\omega_3^3) \frac{1}{24\omega_5^2\omega_3^3}$$

$$C_{D_y^4 \rho}^{(0), \text{MRT2}} = C_{D_y^4 \rho}^{(0), \text{MRT1}}$$

$$C_{D_y^4 \rho}^{(0), \text{CLBM1}} = (24\omega_5^2\omega_3^2c_s^4 - 72v_2^4\omega_3^2 + 6\omega_5^2v_2^2\omega_3^3c_s^2 - 72\omega_5v_2^2\omega_3^2 + 24\omega_5\omega_3^2c_s^2 + 36v_2^4\omega_3^3 + 24\omega_5\omega_3c_s^4 + 30\omega_5v_2^2\omega_3^3 + 12\omega_5^2\omega_3c_s^2 + 72\omega_5v_2^2\omega_3c_s^2 -$$

$$6\omega_5\omega_3^3c_s^2 - 12\omega_5^2v_2^2\omega_3^2c_s^2 - 3\omega_5^2\omega_3^3c_s^4 + 3\omega_5^2v_2^4\omega_3^3 - 12\omega_5^2v_2^4\omega_3^2 - 36v_2^2\omega_3^3 - 72\omega_5v_2^2\omega_3^3c_s^2 + \omega_5^2\omega_3^3c_s^2 - 30\omega_5v_2^4\omega_3^3 + 6\omega_5\omega_3^3c_s^4 - 36\omega_5^2v_2^2\omega_3c_s^2 + 72v_2^2\omega_3^2 + 72\omega_5v_2^4\omega_3^2 - 216v_2^2\omega_3^2c_s^2 - 24\omega_5\omega_3^2c_s^4 - 8\omega_5^2\omega_3^2c_s^2 + 12\omega_5^2v_2^2\omega_3^2 + 144\omega_5v_2^2\omega_3^2c_s^2 + 108v_2^2\omega_3^3c_s^2 - 48\omega_5^2\omega_3c_s^4 + 24\omega_5^2c_s^4 - 24\omega_5\omega_3c_s^2 - 3\omega_5^2v_2^2\omega_3^3) \frac{1}{24\omega_5^2\omega_3^3}$$

$$C_{D_y^4 \rho}^{(0), \text{CLBM2}} = C_{D_y^4 \rho}^{(0), \text{CLBM1}}$$

coefficient $C_{D_y^4 v_2}^{(0)}$ **at** $\frac{\partial^4 v_2}{\partial x_2^4}$:

$$C_{D_y^4 v_2}^{(0), \text{SRT}} = (24 - 36\omega - \omega^3 - 36v_2^2 + 72\omega c_s^2 + 2\omega^3 v_2^2 + 14\omega^2 - 22\omega^2 v_2^2 + 54\omega v_2^2 + \omega^3 c_s^2 - 48c_s^2 - 26\omega^2 c_s^2) \frac{v_2 \rho}{12\omega^3}$$

$$C_{D_y^4 v_2}^{(0), \text{MRT1}} = (8\omega_5^2\omega_3^2 + 24\omega_5v_2^2\omega_3^2 + 24\omega_5\omega_3^3c_s^2 - 24\omega_5^2c_s^2 - 6\omega_5v_2^2\omega_3^3 + 42\omega_5^2\omega_3c_s^2 - \omega_5^2\omega_3^3 - 6\omega_5\omega_3^3c_s^2 - 6\omega_5^2\omega_3 - 12\omega_5v_2^2\omega_3 + 12\omega_3^2 + 6v_2^2\omega_3^3 - 12\omega_5^2v_2^2 + \omega_5^2\omega_3^3c_s^2 + 12\omega_5\omega_3 + 24\omega_5^2v_2^2\omega_3 - 6\omega_3^3 - 12v_2^2\omega_3^2 - 12\omega_3^2c_s^2 - 20\omega_5^2\omega_3^2c_s^2 - 16\omega_5^2v_2^2\omega_3^2 + 6\omega_5\omega_3^3 + 6\omega_3^3c_s^2 - 12\omega_5\omega_3c_s^2 - 24\omega_5\omega_3^2 + 2\omega_5^2v_2^2\omega_3^3) \frac{v_2 \rho}{12\omega_5^2\omega_3^3}$$

$$C_{D_y^4 v_2}^{(0), \text{MRT2}} = C_{D_y^4 v_2}^{(0), \text{MRT1}}$$

$$C_{D_y^4 v_2}^{(0), \text{CLBM1}} = (2\omega_5^2\omega_3^2 + 24\omega_5v_2^2\omega_3^2 + 72\omega_5\omega_3^3c_s^2 + 24\omega_5^2c_s^2 - 24\omega_5v_2^2\omega_3^3 - 30\omega_5^2\omega_3c_s^2 - \omega_5^2\omega_3^3 - 24\omega_5\omega_3^3c_s^2 + 6\omega_5^2\omega_3 + 60\omega_5v_2^2\omega_3 + 36\omega_3^2 + 42v_2^2\omega_3^3 - 12\omega_5^2v_2^2 + \omega_5^2\omega_3^3c_s^2 - 12\omega_5\omega_3 - 12\omega_5^2v_2^2\omega_3 - 18\omega_3^3 - 84v_2^2\omega_3^2 - 60\omega_3^2c_s^2 - 2\omega_5^2\omega_3^2c_s^2 + 2\omega_5^2v_2^2\omega_3^2 + 12\omega_5\omega_3^3 + 30\omega_3^3c_s^2 - 12\omega_5\omega_3c_s^2 - 24\omega_5\omega_3^2 + 2\omega_5^2v_2^2\omega_3^3) \frac{v_2 \rho}{12\omega_5^2\omega_3^3}$$

$$C_{D_y^4 v_2}^{(0), \text{CLBM2}} = C_{D_y^4 v_2}^{(0), \text{CLBM1}}$$

References

- [1] T. Krüger, H. Kusumaatmaja, A. Kuzmin, O. Shardt, G. Silva, E. M. Viggien, The lattice Boltzmann method, Springer International Publishing 10 (978-3) (2017) 4–15.