

D2Q9 NSE,

a supplementary material for

Lattice Boltzmann Method Analysis Tool (LBMAT)

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1 Global definitions

In \mathbb{R}^2 , the position and velocity vectors are given by $\mathbf{x} = (x_1, x_2)^T$ and $\mathbf{v} = (v_1, v_2)^T$, respectively.

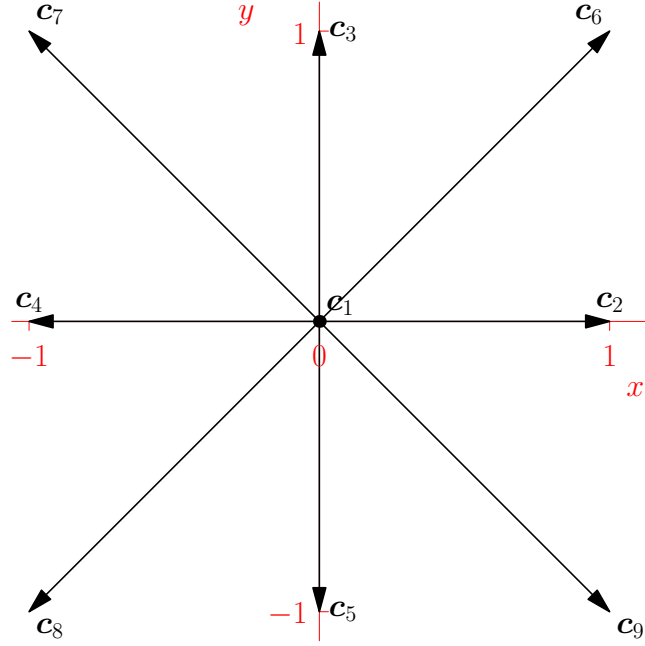
1.1 Discrete velocity vectors

Discrete velocity vectors and the lattice speed of sound are defined by

$$\{\mathbf{c}_i\}_{i=1}^9 = \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right),$$

$$c_s = \frac{1}{\sqrt{3}},$$

respectively [1].



1.2 Raw and central moments

The raw and central moments are defined by

$$m_{\alpha} := \sum_{i=1}^9 f_i c_i^{\alpha},$$

and

$$k_{\alpha} := \sum_{i=1}^9 f_i (c_i - v)^{\alpha},$$

respectively, where $\alpha = (\alpha_1, \alpha_2) \in \mathbb{Z}^2$ denotes a multi-index (as a row vector) and $c_i^{\alpha} := \prod_{j=1}^2 [c_i]_j^{\alpha_j}$.

1.3 Transformation matrix M

Matrix \mathbf{M} , that defines macroscopic quantities (moments) $\boldsymbol{\mu}$ by

$$\boldsymbol{\mu} = \mathbf{M} \mathbf{f},$$

with $\mathbf{f} = (f_1, f_2, \dots, f_9)^T$, is selected such that

$$\boldsymbol{\mu} = \left(m_{(0,0)}, m_{(1,0)}, m_{(0,1)}, m_{(1,1)}, m_{(2,0)}, m_{(0,2)}, m_{(2,1)}, m_{(1,2)}, m_{(2,2)} \right)^T,$$

i.e., \mathbf{M} is given by

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

1.4 Equilibrium

The corresponding equilibrium raw moments are defined using the continuous Maxwell–Boltzmann distribution function [1]

$$f^{(eq)}(\boldsymbol{\xi}) = \frac{\rho}{2\pi c_s^2} \exp\left(-\frac{\|\boldsymbol{\xi} - \mathbf{v}\|^2}{2c_s^2}\right)$$

as

$$m_{\boldsymbol{\alpha}}^{(eq)} = \int_{\mathbb{R}^2} \boldsymbol{\xi}^{\boldsymbol{\alpha}} f^{(eq)}(\boldsymbol{\xi}) d\boldsymbol{\xi},$$

where $\alpha_i \in \{0, 1, 2\}$, $i = 1, 2$. Hence, the equilibrium moments $\boldsymbol{\mu}^{(eq)}$ satisfy

$$\boldsymbol{\mu}^{(eq)} = \begin{pmatrix} \rho \\ \rho v_1 \\ \rho v_2 \\ \rho v_1 v_2 \\ \rho(v_1^2 + c_s^2) \\ \rho(v_2^2 + c_s^2) \\ \rho v_2(c_s^2 + v_1^2) \\ \rho v_1(c_s^2 + v_2^2) \\ \rho(c_s^4 + c_s^2(v_1^2 + v_2^2) + v_1^2 v_2^2) \end{pmatrix}.$$

2 Spatial EPDEs

2.1 SRT

2.1.1 Definitions

Collision operator \mathcal{C} :

$$\mathcal{C}(\mathbf{f}) = \omega \left(\mathbf{M}^{-1} \boldsymbol{\mu}^{(eq)} - \mathbf{f} \right),$$

$\omega \in (0, 2)$.

2.1.2 Conservation of mass: ρ



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$$\begin{aligned}
& \frac{\partial \rho}{\partial t} + \frac{\delta_1 v_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\delta_1 \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_1 v_2}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\delta_1 \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + 3c_s^2 + v_1^2) \frac{\delta_1^3 v_1}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3v_1^2) \frac{\delta_1^3 \rho}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\
& \frac{\delta_1^3 c_s^2 \rho}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{\delta_1^3 c_s^2 \rho}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{\delta_1^3 v_2}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + c_s^2 + 3v_2^2) \frac{\delta_1^3 \rho}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& (6v_1^4 + \omega c_s^2 + 2c_s^4 + 3\omega v_1^2 - 2c_s^2 - 3\omega v_1^4 + 24c_s^2 v_1^2 - 6v_1^2 - \omega c_s^4 - 12\omega c_s^2 v_1^2) \frac{\delta_1^4}{24\omega\delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& (-4 + 2\omega - 3\omega c_s^2 - 5\omega v_1^2 + 6c_s^2 + 10v_1^2) \frac{\delta_1^4 \rho v_1}{12\omega\delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + (2 - \omega + 3\omega c_s^2 + \omega v_1^2 - 6c_s^2 - 2v_1^2) \frac{\delta_1^4 \rho v_1}{12\omega\delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& (-2 + \omega) \frac{\delta_1^4 c_s^4}{6\omega\delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + (2 - \omega + 3\omega c_s^2 + \omega v_2^2 - 6c_s^2 - 2v_2^2) \frac{\delta_1^4 v_2 \rho}{12\omega\delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& (\omega c_s^2 + 6v_2^4 + 3\omega v_2^2 + 2c_s^4 - 2c_s^2 + 24c_s^2 v_2^2 - 3\omega v_2^4 - 12\omega c_s^2 v_2^2 - 6v_2^2 - \omega c_s^4) \frac{\delta_1^4}{24\omega\delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\
& (-4 + 2\omega - 3\omega c_s^2 - 5\omega v_2^2 + 6c_s^2 + 10v_2^2) \frac{\delta_1^4 v_2 \rho}{12\omega\delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0.
\end{aligned}$$

2.1.3 Conservation of momentum: ρv_1



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$$\begin{aligned}
& v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (c_s^2 + v_1^2) \frac{\delta_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2\delta_1 \rho v_1}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_1 v_2 v_1}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\delta_1 v_2 \rho}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{\delta_1 \rho v_1}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\
& (-2 + \omega - 2\omega c_s^2 - 3\omega v_1^2 + 4c_s^2 + 6v_1^2) \frac{\delta_1^2}{\omega\delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + (2 - \omega) \frac{3\delta_1^2 \rho v_1}{\omega\delta_t} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + (-2 + \omega) \frac{\delta_1^2 c_s^2}{2\omega\delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\
& (-2 + \omega) \frac{\delta_1^2 c_s^2}{2\omega\delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (-2 + \omega - 3\omega c_s^2 - \omega v_1^2 + 6c_s^2 + 2v_1^2) \frac{\delta_1^2 v_1}{2\omega\delta_t} \frac{\partial^2 \rho}{\partial x_1^2} + \\
& (-2 + \omega - \omega c_s^2 - 3\omega v_1^2 + 2c_s^2 + 6v_1^2) \frac{\delta_1^2 \rho}{2\omega\delta_t} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega) \frac{\delta_1^2 c_s^2 \rho}{2\omega\delta_t} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega) \frac{\delta_1^2 c_s^2 \rho}{2\omega\delta_t} \frac{\partial^2 v_1}{\partial x_2^2} + C_1 \frac{\delta_1^3}{12\omega^2\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + \\
& (-24 + 24\omega - 4\omega^2 - 36\omega c_s^2 - 60\omega v_1^2 + 36c_s^2 + 11\omega^2 v_1^2 + 60v_1^2 + 5\omega^2 c_s^2) \frac{\delta_1^3 \rho v_1}{6\omega^2\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + \\
& (12 - 12\omega + 3\omega^2 + 36\omega c_s^2 + 12\omega v_1^2 - 36c_s^2 - 3\omega^2 v_1^2 - 12v_1^2 - 11\omega^2 c_s^2) \frac{\delta_1^3 \rho v_1}{12\omega^2\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + \\
& (-12 + 12\omega - \omega^2) \frac{\delta_1^3 c_s^4}{6\omega^2\delta_t} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} - \frac{\delta_1^3 c_s^2 \rho v_1}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{\delta_1^3 v_2 v_1}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\
& (6 - 6\omega + \omega^2 + 18\omega c_s^2 + 6\omega v_2^2 - 18c_s^2 - \omega^2 v_2^2 - 3\omega^2 c_s^2 - 6v_2^2) \frac{\delta_1^3 v_2 \rho}{6\omega^2\delta_t} \frac{\partial^3 v_1}{\partial x_2^3} + (-1 + c_s^2 + 3v_2^2) \frac{\delta_1^3 \rho v_1}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& C_2 \frac{\delta_1^4 v_1}{12\omega^3\delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + C_3 \frac{\delta_1^4 \rho}{12\omega^3\delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_4 \frac{\delta_1^4 \rho}{12\omega^3\delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_5 \frac{\delta_1^4 c_s^2 v_1}{12\omega^3\delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\
& (-24 + 36\omega - 12\omega^2 - 18\omega c_s^2 - 108\omega v_1^2 + 12c_s^2 + 36\omega^2 v_1^2 - \omega^3 c_s^2 + 72v_1^2 + 8\omega^2 c_s^2) \frac{\delta_1^4 c_s^2 \rho}{12\omega^3\delta_t} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_6 \frac{\delta_1^4 c_s^2 v_2}{12\omega^3\delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + \\
& + (2 - \omega + 3\omega c_s^2 + \omega v_2^2 - 6c_s^2 - 2v_2^2) \frac{\delta_1^4 v_2 \rho v_1}{12\omega\delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& (-12 + 18\omega - 6\omega^2 - 54\omega v_2^2 - \omega^3 c_s^2 + 18\omega^2 v_2^2 + 2\omega^2 c_s^2 + 36v_2^2) \frac{\delta_1^4 c_s^2 \rho}{12\omega^3\delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\
& (\omega c_s^2 + 6v_2^4 + 3\omega v_2^2 + 2c_s^4 - 2c_s^2 + 24c_s^2 v_2^2 - 3\omega v_2^4 - 12\omega c_s^2 v_2^2 - 6v_2^2 - \omega c_s^4) \frac{\delta_1^4 v_1}{24\omega\delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_7 \frac{\delta_1^4 \rho}{24\omega^3\delta_t} \frac{\partial^4 v_1}{\partial x_2^4} + \\
& (-4 + 2\omega - 3\omega c_s^2 - 5\omega v_2^2 + 6c_s^2 + 10v_2^2) \frac{\delta_1^4 v_2 \rho v_1}{12\omega\delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= 36v_1^4 + \omega^2 c_s^4 + 12\omega c_s^2 + 24\omega^2 c_s^2 v_1^2 + 12c_s^4 + 7\omega^2 v_1^4 + 36\omega v_1^2 - 12c_s^2 - 7\omega^2 v_1^2 - 36\omega v_1^4 + 144c_s^2 v_1^2 - 36v_1^2 - \omega^2 c_s^2 - 12\omega c_s^4 - 144\omega c_s^2 v_1^2 \\
C_2 &= 12 - 9\omega^3 v_1^4 - 18\omega + 144v_1^4 + 8\omega^2 + 82\omega^2 c_s^4 + 198\omega c_s^2 - \omega^3 + 404\omega^2 c_s^2 v_1^2 - 5\omega^3 c_s^4 + 144c_s^4 + 90\omega^2 v_1^4 + 234\omega v_1^2 - 34\omega^3 c_s^2 v_1^2 - 132c_s^2 - \\
& 98\omega^2 v_1^2 + 6\omega^3 c_s^2 - 216\omega v_1^4 + 672c_s^2 v_1^2 - 156v_1^2 - 78\omega^2 c_s^2 + 10\omega^3 v_1^2 - 216\omega c_s^4 - 1008\omega c_s^2 v_1^2 \\
C_3 &= 12 - 29\omega^3 v_1^4 - 18\omega + 504v_1^4 + 8\omega^2 + 14\omega^2 c_s^4 + 54\omega c_s^2 - \omega^3 + 252\omega^2 c_s^2 v_1^2 - \omega^3 c_s^4 + 24c_s^4 + 310\omega^2 v_1^4 + 378\omega v_1^2 - 18\omega^3 c_s^2 v_1^2 - 36c_s^2 - \\
& 154\omega^2 v_1^2 + 2\omega^3 c_s^2 - 756\omega v_1^4 + 432c_s^2 v_1^2 - 252v_1^2 - 22\omega^2 c_s^2 + 14\omega^3 v_1^2 - 36\omega c_s^4 - 648\omega c_s^2 v_1^2 \\
C_4 &= 4\omega^3 v_1^4 - 36v_1^4 + 20\omega^2 c_s^4 + 36\omega c_s^2 - 42\omega^2 c_s^2 v_1^2 - \omega^3 c_s^4 + 36c_s^4 - 26\omega^2 v_1^4 - 54\omega v_1^2 + 12\omega^3 c_s^2 v_1^2 - 24c_s^2 + 26\omega^2 v_1^2 + 54\omega v_1^4 - 36c_s^2 v_1^2 + \\
& 36v_1^2 - 12\omega^2 c_s^2 - 4\omega^3 v_1^2 - 54\omega c_s^4 + 54\omega c_s^2 v_1^2 \\
C_5 &= 24 - 36\omega + 14\omega^2 + 108\omega c_s^2 - \omega^3 + 36\omega v_1^2 - 72c_s^2 - 14\omega^2 v_1^2 + 5\omega^3 c_s^2 - 24v_1^2 - 46\omega^2 c_s^2 + \omega^3 v_1^2 \\
C_6 &= 24 - 36\omega + 14\omega^2 + 108\omega c_s^2 - \omega^3 + 36\omega v_2^2 - 72c_s^2 + 3\omega^3 c_s^2 - 14\omega^2 v_2^2 - 42\omega^2 c_s^2 - 24v_2^2 + \omega^3 v_2^2
\end{aligned}$$

$$C_7 = -84\omega^2 c_s^2 v_2^2 + 30\omega^2 c_s^4 + 3\omega^3 v_2^4 + 36\omega c_s^2 - 72v_2^4 - 3\omega^3 c_s^4 - 108\omega v_2^2 + 48c_s^4 - 42\omega^2 v_2^4 - 24c_s^2 + \omega^3 c_s^2 - 144c_s^2 v_2^2 + 108\omega v_2^4 + 6\omega^3 c_s^2 v_2^2 + 42\omega^2 v_2^2 - 14\omega^2 c_s^2 + 216\omega c_s^2 v_2^2 + 72v_2^2 - 3\omega^3 v_2^2 - 72\omega c_s^4$$

2.1.4 Conservation of momentum: ρv_2



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$$\begin{aligned} & v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{\delta_1 v_2 v_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\delta_1 v_2 \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_1 \rho v_1}{\delta_t} \frac{\partial v_2}{\partial x_1} + (c_s^2 + v_2^2) \frac{\delta_1}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{2\delta_1 v_2 \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega) \frac{\delta_1^2 c_s^2}{2\omega \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\ & (-2 + \omega) \frac{\delta_1^2 c_s^2}{2\omega \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (-2 + \omega - 2\omega c_s^2 - 3\omega v_2^2 + 4c_s^2 + 6v_2^2) \frac{\delta_1^2}{\omega \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (2 - \omega) \frac{3\delta_1^2 v_2 \rho}{\omega \delta_t} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + \\ & (-2 + \omega) \frac{\delta_1^2 c_s^2 \rho}{2\omega \delta_t} \frac{\partial^2 v_2}{\partial x_1^2} + (-2 + \omega) \frac{\delta_1^2 c_s^2 \rho}{2\omega \delta_t} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + (-2 + \omega - 3\omega c_s^2 - \omega v_2^2 + 6c_s^2 + 2v_2^2) \frac{\delta_1^2 v_2}{2\omega \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + \\ & (-2 + \omega - \omega c_s^2 - 3\omega v_2^2 + 2c_s^2 + 6v_2^2) \frac{\delta_1^2 \rho}{2\omega \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + (-1 + 3c_s^2 + v_1^2) \frac{\delta_1^3 v_2 v_1}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3v_1^2) \frac{\delta_1^3 v_2 \rho}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + \\ & (6 - 6\omega + \omega^2 + 18\omega c_s^2 + 6\omega v_1^2 - 18c_s^2 - \omega^2 v_1^2 - 6v_1^2 - 3\omega^2 c_s^2) \frac{\delta_1^3 \rho v_1}{6\omega^2 \delta_t} \frac{\partial^3 v_2}{\partial x_1^3} + (-12 + 12\omega - \omega^2) \frac{\delta_1^3 c_s^4}{6\omega^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3 \partial x_2} - \\ & \frac{\delta_1^3 c_s^2 v_2 \rho}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^3 \partial x_2} + (12 - 12\omega + 3\omega^2 + 36\omega c_s^2 + 12\omega v_2^2 - 36c_s^2 - 3\omega^2 v_2^2 - 11\omega^2 c_s^2 - 12v_2^2) \frac{\delta_1^3 v_2 \rho}{12\omega^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + \\ & C_1 \frac{\delta_1^3}{12\omega^2 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-24 + 24\omega - 4\omega^2 - 36\omega c_s^2 - 60\omega v_2^2 + 36c_s^2 + 11\omega^2 v_2^2 + 5\omega^2 c_s^2 + 60v_2^2) \frac{\delta_1^3 v_2 \rho}{6\omega^2 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & (6v_1^4 + \omega c_s^2 + 2c_s^4 + 3\omega v_1^2 - 2c_s^2 - 3\omega v_1^4 + 24c_s^2 v_1^2 - 6v_1^2 - \omega c_s^4 - 12\omega c_s^2 v_1^2) \frac{\delta_1^4 v_2}{24\omega \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\ & (-4 + 2\omega - 3\omega c_s^2 - 5\omega v_1^2 + 6c_s^2 + 10v_1^2) \frac{\delta_1^4 v_2 \rho v_1}{12\omega \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_2 \frac{\delta_1^4 \rho}{24\omega^3 \delta_t} \frac{\partial^4 v_2}{\partial x_1^4} + C_3 \frac{\delta_1^4 c_s^2 v_1}{12\omega^3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4 \partial x_2} + \\ & (-12 + 18\omega - 6\omega^2 - 54\omega v_1^2 + 18\omega^2 v_1^2 - \omega^3 c_s^2 + 36v_1^2 + 2\omega^2 c_s^2) \frac{\delta_1^4 c_s^2 \rho}{12\omega^3 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4 \partial x_2} + \\ & (2 - \omega + 3\omega c_s^2 + \omega v_1^2 - 6c_s^2 - 2v_1^2) \frac{\delta_1^4 v_2 \rho v_1}{12\omega \delta_t} \frac{\partial^4 v_2}{\partial x_1^4 \partial x_2} + C_4 \frac{\delta_1^4 c_s^2 v_2}{12\omega^3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4 \partial x_2^2} + \\ & (-24 + 36\omega - 12\omega^2 - 18\omega c_s^2 - 108\omega v_2^2 + 12c_s^2 - \omega^3 c_s^2 + 36\omega^2 v_2^2 + 8\omega^2 c_s^2 + 72v_2^2) \frac{\delta_1^4 c_s^2 \rho}{12\omega^3 \delta_t} \frac{\partial^4 v_2}{\partial x_1^4 \partial x_2^2} + C_5 \frac{\delta_1^4 \rho}{12\omega^3 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} \\ & + C_6 \frac{\delta_1^4 v_2}{12\omega^3 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_7 \frac{\delta_1^4 \rho}{12\omega^3 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:

$$\begin{aligned} C_1 &= 24\omega^2 c_s^2 v_2^2 + \omega^2 c_s^4 + 12\omega c_s^2 + 36v_2^4 + 36\omega v_2^2 + 12c_s^4 + 7\omega^2 v_2^4 - 12c_s^2 + 144c_s^2 v_2^2 - 36\omega v_2^4 - 7\omega^2 v_2^2 - \omega^2 c_s^2 - 144\omega c_s^2 v_2^2 - 36v_2^2 - 12\omega c_s^4 \\ C_2 &= 3\omega^3 v_1^4 - 72v_1^4 + 30\omega^2 c_s^4 + 36\omega c_s^2 - 84\omega^2 c_s^2 v_1^2 - 3\omega^3 c_s^4 + 48c_s^4 - 42\omega^2 v_1^4 - 108\omega v_1^2 + 6\omega^3 c_s^2 v_1^2 - 24c_s^2 + 42\omega^2 v_1^2 + \omega^3 c_s^2 + 108\omega v_1^4 - \\ & 144c_s^2 v_1^2 + 72v_1^2 - 14\omega^2 c_s^2 - 3\omega^3 v_1^2 - 72\omega c_s^4 + 216\omega c_s^2 v_1^2 \\ C_3 &= 24 - 36\omega + 14\omega^2 + 108\omega c_s^2 - \omega^3 + 36\omega v_1^2 - 72c_s^2 - 14\omega^2 v_1^2 + 3\omega^3 c_s^2 - 24v_1^2 - 42\omega^2 c_s^2 + \omega^3 v_1^2 \\ C_4 &= 24 - 36\omega + 14\omega^2 + 108\omega c_s^2 - \omega^3 + 36\omega v_2^2 - 72c_s^2 + 5\omega^3 c_s^2 - 14\omega^2 v_2^2 - 46\omega^2 c_s^2 - 24v_2^2 + \omega^3 v_2^2 \\ C_5 &= -42\omega^2 c_s^2 v_2^2 + 20\omega^2 c_s^4 + 4\omega^3 v_2^4 + 36\omega c_s^2 - 36v_2^4 - \omega^3 c_s^4 - 54\omega v_2^2 + 36c_s^4 - 26\omega^2 v_2^4 - 24c_s^2 - 36c_s^2 v_2^2 + 54\omega v_2^4 + 12\omega^3 c_s^2 v_2^2 + 26\omega^2 v_2^2 - \\ & 12\omega^2 c_s^2 + 54\omega c_s^2 v_2^2 + 36v_2^2 - 4\omega^3 v_2^2 - 54\omega c_s^4 \\ C_6 &= 12 + 404\omega^2 c_s^2 v_2^2 - 18\omega + 8\omega^2 + 82\omega^2 c_s^4 - 9\omega^3 v_2^4 + 198\omega c_s^2 + 144v_2^4 - \omega^3 - 5\omega^3 c_s^4 + 234\omega v_2^2 + 144c_s^4 + 90\omega^2 v_2^4 - 132c_s^2 + 6\omega^3 c_s^2 + \\ & 672c_s^2 v_2^2 - 216\omega v_2^4 - 34\omega^3 c_s^2 v_2^2 - 98\omega^2 v_2^2 - 78\omega^2 c_s^2 - 1008\omega c_s^2 v_2^2 - 156v_2^2 + 10\omega^3 v_2^2 - 216\omega c_s^4 \\ C_7 &= 12 + 252\omega^2 c_s^2 v_2^2 - 18\omega + 8\omega^2 + 14\omega^2 c_s^4 - 29\omega^3 v_2^4 + 54\omega c_s^2 + 504v_2^4 - \omega^3 - \omega^3 c_s^4 + 378\omega v_2^2 + 24c_s^4 + 310\omega^2 v_2^4 - 36c_s^2 + 2\omega^3 c_s^2 + \\ & 432c_s^2 v_2^2 - 756\omega v_2^4 - 18\omega^3 c_s^2 v_2^2 - 154\omega^2 v_2^2 - 22\omega^2 c_s^2 - 648\omega c_s^2 v_2^2 - 252v_2^2 + 14\omega^3 v_2^2 - 36\omega c_s^4 \end{aligned}$$

2.2 MRT

2.2.1 Definitions

Collision operator \mathcal{C} :

$$\mathcal{C}(f) = \mathbf{M}^{-1} \mathbf{S} \left(\mu^{(eq)} - \mathbf{M} f \right),$$

where

$$\mathbf{S} = \text{diag}(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9),$$

$$\omega_1, \omega_2, \dots, \omega_9 \in (0, 2).$$

2.2.2 Conservation of mass: ρ



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$$\begin{aligned} & \frac{\partial \rho}{\partial t} + \frac{\delta_l v_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\delta_l \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_l v_2}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\delta_l \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + v_1^2 + 3c_s^2) \frac{\delta_l^3 v_1}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + 3v_1^2 + c_s^2) \frac{\delta_l^3 \rho}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\ & \frac{\delta_l^3 c_s^2 \rho}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{\delta_l^3 c_s^2 \rho}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + v_2^2 + 3c_s^2) \frac{\delta_l^3 v_2}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + 3v_2^2 + c_s^2) \frac{\delta_l^3 \rho}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & (2c_s^4 + 24c_s^2 v_1^2 + c_s^2 \omega_5 + 6v_1^4 - 3v_1^4 \omega_5 - c_s^4 \omega_5 - 12c_s^2 v_1^2 \omega_5 - 6v_1^2 + 3v_1^2 \omega_5 - 2c_s^2) \frac{\delta_l^4 \rho}{24\delta_t \omega_5} \frac{\partial^4 \rho}{\partial x_1^4} + \\ & (-4 - 3c_s^2 \omega_5 + 10v_1^2 - 5v_1^2 \omega_5 + 6c_s^2 + 2\omega_5) \frac{\delta_l^4 v_1 \rho}{12\delta_t \omega_5} \frac{\partial^4 v_1}{\partial x_1^4} + (-3c_s^2 \omega_5 + \omega_7 v_1^2 - \omega_7 - v_1^2 \omega_5 + \omega_5 + 3c_s^2 \omega_7) \frac{\delta_l^4 v_2 v_1}{4\delta_t \omega_7 \omega_5} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} \\ & + (-c_s^2 \omega_5 + 3\omega_7 v_1^2 - \omega_7 - 3v_1^2 \omega_5 + \omega_5 + c_s^2 \omega_7) \frac{\delta_l^4 v_2 \rho}{4\delta_t \omega_7 \omega_5} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + (3\omega_4 \omega_5 - 6c_s^2 \omega_7 \omega_5 + 3c_s^2 \omega_7 \omega_4 \omega_5 - 3c_s^2 \omega_4 \omega_5 + \\ & \omega_7 \omega_4 v_1^2 \omega_5 - \omega_7 \omega_4 + \omega_7 \omega_4 v_1^2 - \omega_7 \omega_4 \omega_5 - 3\omega_4 v_1^2 \omega_5 + 3c_s^2 \omega_7 \omega_4) \frac{\delta_l^4 v_1 \rho}{12\delta_t \omega_7 \omega_4 \omega_5} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + (-2 + \omega_4) \frac{\delta_l^4 c_s^4}{6\delta_t \omega_4} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\ & (-\omega_7 + \omega_4) \frac{\delta_l^4 c_s^2 v_1 \rho}{2\delta_t \omega_7 \omega_4} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + (\omega_4 - \omega_8) \frac{\delta_l^4 c_s^2 v_2 \rho}{2\delta_t \omega_4 \omega_8} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + (\omega_6 + 3c_s^2 \omega_8 - \omega_6 v_2^2 - \omega_8 + v_2^2 \omega_8 - 3\omega_6 c_s^2) \frac{\delta_l^4 v_2 v_1}{4\omega_6 \delta_t \omega_8} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} \\ & + (3c_s^2 \omega_4 \omega_8 - 3\omega_6 \omega_4 v_2^2 - \omega_6 \omega_4 \omega_8 + 3\omega_6 c_s^2 \omega_4 \omega_8 - 6\omega_6 c_s^2 \omega_8 + 3\omega_6 \omega_4 - \omega_4 \omega_8 + \omega_6 \omega_4 v_2^2 \omega_8 - 3\omega_6 c_s^2 \omega_4 + \\ & \omega_4 v_2^2 \omega_8) \frac{\delta_l^4 v_2 \rho}{12\omega_6 \delta_t \omega_4 \omega_8} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + (\omega_6 + c_s^2 \omega_8 - 3\omega_6 v_2^2 - \omega_8 + 3v_2^2 \omega_8 - \omega_6 c_s^2) \frac{\delta_l^4 v_1 \rho}{4\omega_6 \delta_t \omega_8} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\ & (24c_s^2 v_2^2 + 2c_s^4 - 12\omega_6 c_s^2 v_2^2 - \omega_6 c_s^4 + 6v_2^4 - 3\omega_6 v_2^4 - 6v_2^2 + 3\omega_6 v_2^2 - 2c_s^2 + \omega_6 c_s^2) \frac{\delta_l^4 \rho}{24\omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\ & (-4 + 2\omega_6 + 10v_2^2 - 5\omega_6 v_2^2 + 6c_s^2 - 3\omega_6 c_s^2) \frac{\delta_l^4 v_2 \rho}{12\omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0. \end{aligned}$$

2.2.3 Conservation of momentum: ρv_1



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$$\begin{aligned} & v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (v_1^2 + c_s^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2\delta_l v_1 \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_l v_2 v_1}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\delta_l v_2 \rho}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{\delta_l v_1 \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\ & (-2 - 2c_s^2 \omega_5 + 6v_1^2 - 3v_1^2 \omega_5 + 4c_s^2 + \omega_5) \frac{\delta_l^2}{\delta_t \omega_5} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + (2 - \omega_5) \frac{3\delta_l^2 v_1 \rho}{\delta_t \omega_5} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + (-2 + \omega_4) \frac{\delta_l^2 c_s^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\ & (-2 + \omega_4) \frac{\delta_l^2 c_s^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (-2 - 3c_s^2 \omega_5 + 2v_1^2 - v_1^2 \omega_5 + 6c_s^2 + \omega_5) \frac{\delta_l^2 v_1}{2\delta_t \omega_5} \frac{\partial^2 \rho}{\partial x_1^2} + \\ & (-2 - c_s^2 \omega_5 + 6v_1^2 - 3v_1^2 \omega_5 + 2c_s^2 + \omega_5) \frac{\delta_l^2 \rho}{2\delta_t \omega_5} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega_4) \frac{\delta_l^2 c_s^2 \rho}{2\delta_t \omega_4} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_4) \frac{\delta_l^2 c_s^2 \rho}{2\delta_t \omega_4} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + C_1 \frac{\delta_l^3}{12\delta_t \omega_5^2} \frac{\partial^3 \rho}{\partial x_1^3} \\ & + (-24 - 36c_s^2 \omega_5 + 5c_s^2 \omega_5^2 + 60v_1^2 + 11v_1^2 \omega_5^2 - 4\omega_5^2 - 60v_1^2 \omega_5 + 36c_s^2 + 24\omega_5) \frac{\delta_l^3 v_1 \rho}{6\delta_t \omega_5^2} \frac{\partial^3 v_1}{\partial x_1^3} + C_2 \frac{\delta_l^3 v_2 v_1}{\delta_t \omega_7 \omega_4 \omega_5^2} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + \\ & C_3 \frac{\delta_l^3 v_2 \rho}{\delta_t \omega_7 \omega_4 \omega_5^2} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} + C_4 \frac{\delta_l^3 v_1 \rho}{12\delta_t \omega_7 \omega_4^2 \omega_5^2} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + (-12 - \omega_4^2 + 12\omega_4) \frac{\delta_l^3 c_s^4}{6\delta_t \omega_4^2} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + \\ & (12\omega_4 \omega_5 + 12\omega_4^2 - 12\omega_7 \omega_5 - \omega_7 \omega_4^2 \omega_5 - 12\omega_7 \omega_4 + 12\omega_7 \omega_4 \omega_5 - 12\omega_4^2 \omega_5) \frac{\delta_l^3 c_s^2 v_1 \rho}{6\delta_t \omega_7 \omega_4^2 \omega_5} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + \\ & (-\omega_4^2 + \omega_4 \omega_8 + 2\omega_4 - 2\omega_8) \frac{\delta_l^3 c_s^2 v_2 \rho}{\delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_5 \frac{\delta_l^3 v_2 v_1}{12\omega_6 \delta_t \omega_4 \omega_8} \frac{\partial^3 \rho}{\partial x_2^3} + C_6 \frac{\delta_l^3 v_2 \rho}{6\delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_1}{\partial x_2^3} + C_7 \frac{\delta_l^3 v_1 \rho}{12\omega_6 \delta_t \omega_4 \omega_8} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & C_8 \frac{\delta_l^4 v_1}{12\delta_t \omega_5^3} \frac{\partial^4 \rho}{\partial x_1^4} + C_9 \frac{\delta_l^4 \rho}{12\delta_t \omega_5^3} \frac{\partial^4 v_1}{\partial x_1^4} + C_{10} \frac{\delta_l^4 v_2}{4\delta_t \omega_7^2 \omega_4^2 \omega_5^3} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{11} \frac{\delta_l^4 v_2 v_1 \rho}{4\delta_t \omega_7^2 \omega_4^2 \omega_5^3} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + C_{12} \frac{\delta_l^4 \rho}{12\delta_t \omega_7^2 \omega_4^2 \omega_5^3} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\ & C_{13} \frac{\delta_l^4 v_1}{12\omega_9 \delta_t \omega_7^2 \omega_4^3 \omega_8 \omega_5^3} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_{14} \frac{\delta_l^4 \rho}{12\omega_9 \delta_t \omega_7^2 \omega_4^3 \omega_8 \omega_5^3} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{15} \frac{\delta_l^4 v_2 v_1 \rho}{2\omega_9 \omega_6 \delta_t \omega_7^2 \omega_4^3 \omega_8^2 \omega_5^3} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + \\ & C_{16} \frac{\delta_l^4 v_2}{12\omega_9 \omega_6^2 \delta_t \omega_7 \omega_4^3 \omega_8^2 \omega_5^3} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{17} \frac{\delta_l^4 v_2 v_1 \rho}{12\omega_9 \omega_6^2 \delta_t \omega_7 \omega_4^3 \omega_8^2 \omega_5^3} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{18} \frac{\delta_l^4 \rho}{12\omega_9 \omega_6^2 \delta_t \omega_7 \omega_4^3 \omega_8^2 \omega_5^3} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + C_{19} \frac{\delta_l^4 v_1}{24\omega_9^2 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 \rho}{\partial x_1^4} \\ & + C_{20} \frac{\delta_l^4 \rho}{24\delta_t \omega_4^3 \omega_8^2} \frac{\partial^4 v_1}{\partial x_2^4} + C_{21} \frac{\delta_l^4 v_2 v_1 \rho}{12\omega_9^2 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:

$$C_1 = 12c_s^4 + 7v_1^4\omega_5^2 + 144c_s^2v_1^2 + 12c_s^2\omega_5 - c_s^2\omega_5^2 + 36v_1^4 - 36v_1^4\omega_5 - 12c_s^4\omega_5 - 144c_s^2v_1^2\omega_5 - 36v_1^2 - 7v_1^2\omega_5^2 + 24c_s^2v_1^2\omega_5^2 + 36v_1^2\omega_5 - 12c_s^2 + c_s^4\omega_5^2$$

$$C_2 = 3c_s^2\omega_4\omega_5^2 + \omega_4\omega_5 + 3c_s^2\omega_7\omega_5 - \omega_7\omega_5 - 3c_s^2\omega_7\omega_4\omega_5 - 3c_s^2\omega_5^2 - 3c_s^2\omega_4\omega_5 - \omega_4\omega_5^2 + \omega_7v_1^2\omega_5 - \omega_7\omega_4v_1^2\omega_5 + \omega_4v_1^2\omega_5^2 - \omega_7\omega_4 + \omega_7\omega_4v_1^2 + \omega_7\omega_4\omega_5 - v_1^2\omega_5^2 + \omega_5^2 - \omega_4v_1^2\omega_5 + 3c_s^2\omega_7\omega_4$$

$$C_3 = c_s^2\omega_4\omega_5^2 + \omega_4\omega_5 + c_s^2\omega_7\omega_5 - \omega_7\omega_5 - c_s^2\omega_7\omega_4\omega_5 - c_s^2\omega_5^2 - c_s^2\omega_4\omega_5 - \omega_4\omega_5^2 + 3\omega_7v_1^2\omega_5 - 3\omega_7\omega_4v_1^2\omega_5 + 3\omega_4v_1^2\omega_5^2 - \omega_7\omega_4 + 3\omega_7\omega_4v_1^2 + \omega_7\omega_4\omega_5 - 3v_1^2\omega_5^2 + \omega_5^2 - 3\omega_4v_1^2\omega_5 + c_s^2\omega_7\omega_4$$

$$C_4 = -12\omega_1^2v_1^2\omega_5 - 12c_s^2\omega_4\omega_5^2 + 42c_s^2\omega_7\omega_4\omega_5^2 - 3\omega_7\omega_4^2v_1^2\omega_5^2 + 3\omega_7\omega_4^2\omega_5^2 - 24c_s^2\omega_7\omega_5^2 - 24c_s^2\omega_7\omega_4\omega_5 - 6\omega_7\omega_4^2v_1^2\omega_5 + 6\omega_7\omega_4^2\omega_5 + 12\omega_7\omega_4^2v_1^2 + 12\omega_4^2v_1^2\omega_5^2 + 12\omega_4\omega_5^2 - 12\omega_4^2\omega_5^2 - 12c_s^2\omega_4^2\omega_5 - 12\omega_4v_1^2\omega_5^2 + 36c_s^2\omega_7\omega_4^2 - 18c_s^2\omega_7\omega_4^2\omega_5 - 11c_s^2\omega_7\omega_4^2\omega_5^2 - 6\omega_7\omega_4\omega_5^2 - 12\omega_7\omega_4^2 + 6\omega_7\omega_4v_1^2\omega_5^2 + 12\omega_4^2\omega_5 + 12c_s^2\omega_4^2\omega_5^2$$

$$C_5 = -18c_s^2\omega_4\omega_8 + 6\omega_6\omega_4v_2^2 + 12\omega_6 + 36c_s^2\omega_8 - \omega_6\omega_4\omega_8 + 3\omega_6c_s^2\omega_4\omega_8 - 6\omega_6\omega_4 + 6\omega_4\omega_8 - 12\omega_6v_2^2 + \omega_6\omega_4v_2^2\omega_8 - 12\omega_8 + 12v_2^2\omega_8 + 18\omega_6c_s^2\omega_4 - 6\omega_4v_2^2\omega_8 - 36\omega_6c_s^2$$

$$C_6 = 15c_s^2\omega_4\omega_8 - 3\omega_4^2 - \omega_4^2v_2^2\omega_8 - 6\omega_4v_2^2 - 12c_s^2\omega_8 - 3\omega_4\omega_8 + 6\omega_4 + \omega_4^2\omega_8 + 3c_s^2\omega_4^2 - 6c_s^2\omega_4 + 3\omega_4^2v_2^2 - 3c_s^2\omega_4^2\omega_8 + 3\omega_4v_2^2\omega_8$$

$$C_7 = -6c_s^2\omega_4\omega_8 + 18\omega_6\omega_4v_2^2 + 12\omega_6 + 12c_s^2\omega_8 - \omega_6\omega_4\omega_8 + \omega_6c_s^2\omega_4\omega_8 - 6\omega_6\omega_4 + 6\omega_4\omega_8 - 36\omega_6v_2^2 + 3\omega_6\omega_4v_2^2\omega_8 - 12\omega_8 + 36v_2^2\omega_8 + 6\omega_6c_s^2\omega_4 - 18\omega_4v_2^2\omega_8 - 12\omega_6c_s^2$$

$$C_8 = 12 + 144c_s^4 + 90v_1^4\omega_5^2 + 672c_s^2v_1^2 + 198c_s^2\omega_5 - 9v_1^4\omega_5^3 - 78c_s^2\omega_5^2 + 144v_1^4 - 216v_1^4\omega_5 + 6c_s^2\omega_5^3 - 216c_s^4\omega_5 + 10v_1^2\omega_5^3 - 1008c_s^2v_1^2\omega_5 - 156v_1^2 - 98v_1^2\omega_5^2 + 8\omega_5^2 + 404c_s^2v_1^2\omega_5^2 + 234v_1^2\omega_5 - 132c_s^2 - 5c_s^2\omega_5^2 - 18\omega_5 - \omega_5^3 - 34c_s^2v_1^2\omega_5^3 + 82c_s^4\omega_5^2$$

$$C_9 = 12 + 24c_s^4 + 310v_1^4\omega_5^2 + 432c_s^2v_1^2 + 54c_s^2\omega_5 - 29v_1^4\omega_5^3 - 22c_s^2\omega_5^2 + 504v_1^4 - 756v_1^4\omega_5 + 2c_s^2\omega_5^3 - 36c_s^4\omega_5 + 14v_1^2\omega_5^3 - 648c_s^2v_1^2\omega_5 - 252v_1^2 - 154v_1^2\omega_5^2 + 8\omega_5^2 + 252c_s^2v_1^2\omega_5^2 + 378v_1^2\omega_5 - 36c_s^2 - c_s^4\omega_5^3 - 18\omega_5 - \omega_5^3 - 18c_s^2v_1^2\omega_5^3 + 14c_s^4\omega_5^2$$

$$C_{10} = -20\omega_1^2\omega_4v_1^2\omega_5 + 13\omega_7\omega_4^2v_1^2\omega_5^3 - 8c_s^2\omega_7\omega_4\omega_5^3 - 4c_s^2\omega_7^2\omega_4^2\omega_5^2 + 8c_s^4\omega_7^2\omega_4^2 + 84c_s^2\omega_7\omega_4v_1^2\omega_5^3 + 4c_s^4\omega_7^2\omega_4\omega_5 - 144c_s^2\omega_7^2\omega_4^2v_1^2\omega_5 + 13\omega_7^2\omega_4^2v_1^4\omega_5^2 - 32\omega_7\omega_4^2v_1^2\omega_5^2 - 4\omega_4v_1^4\omega_5^3 - 4c_s^4\omega_7\omega_5^3 + 8\omega_7v_1^2\omega_5^3 + 4c_s^2\omega_4\omega_5^3 - 4c_s^4\omega_7\omega_4^2\omega_5 + 36c_s^2\omega_7^2v_1^2\omega_5^2 - 48c_s^2\omega_7\omega_4v_1^2\omega_5^2 + 24\omega_7^2\omega_4^2v_1^4 - 4\omega_4^2v_1^2\omega_5^3 - 16\omega_7\omega_4v_1^4\omega_5^2 - 8c_s^2\omega_7^2\omega_4^2 + 8c_s^4\omega_7\omega_4^2\omega_5^2 + 20\omega_7\omega_4^2v_1^2\omega_5 - 8c_s^4\omega_7^2\omega_4\omega_5^2 + 20\omega_7\omega_4v_1^4\omega_5^3 + 4\omega_4^2v_1^2\omega_5^2 - 4c_s^4\omega_7\omega_4^2\omega_5^3 + 51c_s^2\omega_7^2\omega_4^2v_1^2\omega_5^2 - 36\omega_7^2\omega_4^2v_1^4\omega_5 - 24c_s^2\omega_4v_1^2\omega_5^3 + 4c_s^2\omega_7\omega_5^3 + 8\omega_7^2v_1^4\omega_5^2 - 4c_s^4\omega_4\omega_5^3 + 20\omega_7^2\omega_4v_1^2\omega_5^2 + 96c_s^2\omega_7^2\omega_4^2v_1^2 + 12c_s^2\omega_7^2\omega_4^2\omega_5 + 8c_s^4\omega_7\omega_4\omega_5^3 - 72c_s^2\omega_7\omega_4^2v_1^2\omega_5 + 32\omega_7\omega_4^2v_1^4\omega_5^2 + 4\omega_4v_1^2\omega_5^3 + 4c_s^4\omega_7^2\omega_4^2\omega_5^2 - 24c_s^2\omega_7^2v_1^2\omega_5^2 - 8\omega_7v_1^4\omega_5^3 - 24\omega_7^2\omega_4^2v_1^2 + 4c_s^4\omega_4^2\omega_5^3 - 4c_s^2\omega_7^2\omega_4\omega_5 + 20\omega_7^2\omega_4v_1^4\omega_5 + 24c_s^2\omega_4^2v_1^2\omega_5^3 - 4c_s^2\omega_7^2\omega_5^2 - 84c_s^2\omega_7^2\omega_4v_1^2\omega_5^2 - 13\omega_7\omega_4^2v_1^2\omega_5^3 + 4c_s^2\omega_7\omega_4^2\omega_5 - 4c_s^4\omega_4^2\omega_5^2 - 13\omega_7^2\omega_4^2v_1^2\omega_5^2 - 8c_s^2\omega_7\omega_4^2\omega_5^2 - 20\omega_7\omega_4v_1^2\omega_5^3 - 4\omega_4^2v_1^4\omega_5^2 + 36\omega_7^2\omega_4^2v_1^2\omega_5 - 20\omega_7^2\omega_4v_1^4\omega_5^2 - 4c_s^2\omega_4^2\omega_5^3 - 8\omega_7^2v_1^2\omega_5^2 + 72c_s^2\omega_7^2\omega_4v_1^2\omega_5 - 36c_s^2\omega_7v_1^2\omega_5^3 - 51c_s^2\omega_7\omega_4^2v_1^2\omega_5^3 + 4c_s^4\omega_7^2\omega_5^2 + 4\omega_4^2v_1^4\omega_5^3 + 16\omega_7\omega_4v_1^2\omega_5^2 + 8c_s^2\omega_7^2\omega_4\omega_5^2 + 4c_s^2\omega_7\omega_4^2\omega_5^3 + 120c_s^2\omega_7\omega_4^2v_1^2\omega_5^2 - 12c_s^2\omega_7^2\omega_4^2\omega_5 - 20\omega_7\omega_4^2v_1^4\omega_5 + 4c_s^2\omega_4^2\omega_5^2$$

$$C_{11} = 64\omega_7^2\omega_4v_1^2\omega_5 + 28\omega_7^2\omega_4\omega_5^2 + 17\omega_7\omega_4^2\omega_5^3 - 43\omega_7\omega_4^2v_1^2\omega_5^3 + 44c_s^2\omega_7\omega_4\omega_5^3 + 25c_s^2\omega_7^2\omega_4^2\omega_5^2 + 12\omega_7\omega_5^3 - 16c_s^2\omega_7\omega_4\omega_5^2 + 104\omega_7\omega_4^2v_1^2\omega_5^2 - 40\omega_7\omega_4^2\omega_5^2 - 28\omega_7v_1^2\omega_5^3 - 16c_s^2\omega_4\omega_5^3 - 32\omega_7^2\omega_4^2 + 8\omega_4\omega_5^3 + 16\omega_7^2v_1^2\omega_5^3 + 48c_s^2\omega_7^2\omega_4^2 - 64\omega_7\omega_4^2v_1^2\omega_5 + 24\omega_7\omega_4^2\omega_5 - 16\omega_4^2v_1^2\omega_5^2 - 20c_s^2\omega_7\omega_5^3 - 24\omega_7^2\omega_4\omega_5 - 68\omega_7^2\omega_4v_1^2\omega_5^2 - 72c_s^2\omega_7^2\omega_4^2\omega_5 - 16\omega_4v_1^2\omega_5^3 + 8\omega_7^2\omega_5^2 + 80\omega_7^2\omega_4^2v_1^2 + 48\omega_7^2\omega_4^2\omega_5 + 32c_s^2\omega_7^2\omega_4\omega_5 + 20c_s^2\omega_7^2\omega_5^2 - 8\omega_4^2\omega_5^3 - 32c_s^2\omega_7\omega_4^2\omega_5 + 43\omega_7^2\omega_4^2v_1^2\omega_5^2 + 56c_s^2\omega_7\omega_7^2\omega_5^2 + 68\omega_7\omega_4v_1^2\omega_5^3 - 120\omega_7^2\omega_4^2v_1^2\omega_5 + 16\omega_7\omega_4\omega_5^2 - 12\omega_7^2\omega_5^2 + 16c_s^2\omega_4^2\omega_5^3 + 28\omega_7^2v_1^2\omega_5^2 - 28\omega_7\omega_4\omega_5^3 - 17\omega_7^2\omega_4^2\omega_5^2 - 48\omega_7\omega_4v_1^2\omega_5^2 - 44c_s^2\omega_7^2\omega_4\omega_5^2 - 25c_s^2\omega_7\omega_4^2\omega_5^3 - 16c_s^2\omega_4^2\omega_5^2$$

$$C_{12} = -19\omega_7^2\omega_4^3v_1^2\omega_5^2 - 48\omega_7\omega_4^2v_1^2\omega_5^3 - 12c_s^2\omega_7\omega_4\omega_5^3 - 6c_s^2\omega_7^2\omega_4^2\omega_5^2 - 12c_s^2\omega_7\omega_4v_1^2\omega_5^3 - 6c_s^4\omega_7\omega_4^3\omega_5^2 - 48c_s^2\omega_7^2v_1^2\omega_5^3 + 12c_s^2\omega_4^3v_1^2\omega_5^3 + 72\omega_7^2\omega_4^3v_1^4 - 108c_s^2\omega_7^2v_1^2\omega_5 + 12\omega_7^2\omega_4^3v_1^4\omega_5^3 - 27\omega_7\omega_4^3v_1^4\omega_5^3 - 5c_s^2\omega_7^2\omega_4^3\omega_5^3 + 12c_s^4\omega_7^2\omega_4^3 + 252c_s^2\omega_7^2\omega_4^3v_1^2 - 4\omega_7^2\omega_4^3v_1^2\omega_5^3 + 12c_s^2\omega_7^2\omega_4^3\omega_5 + 24\omega_7\omega_4^3v_1^2\omega_5^2 - 18\omega_7^2\omega_4^3v_1^4\omega_5^3 + 60\omega_7\omega_4^3v_1^4\omega_5^2 - 36c_s^2\omega_7\omega_4^3v_1^2\omega_5 - 12c_s^2\omega_7^3v_1^2\omega_5^2 + 6c_s^4\omega_7\omega_4^3\omega_5^3 - 36\omega_7\omega_4^3v_1^4\omega_5 - 81c_s^2\omega_7^2\omega_4^3v_1^2\omega_5^3 + 54c_s^2\omega_7\omega_4^3v_1^2\omega_5^2 + 12\omega_4^3v_1^2\omega_5^3 + 12c_s^4\omega_7\omega_4^2\omega_5^2 - 24c_s^4\omega_7^2\omega_4\omega_5^3 - c_s^2\omega_7^2\omega_4^3\omega_5^2 + 12\omega_4^3v_1^2\omega_5^3 - 12\omega_7^2\omega_4v_1^2\omega_5^3 - 24\omega_7\omega_4v_1^4\omega_5^3 - 18c_s^4\omega_7\omega_4^2\omega_5^3 - 12c_s^2\omega_7^2\omega_4^3 + 162c_s^2\omega_7^2\omega_4^3v_1^2\omega_5^2 - 21c_s^2\omega_7\omega_4^3v_1^2\omega_5^3 - 12\omega_4^3v_1^4\omega_5^2 + 90\omega_7^2\omega_4^3v_1^2\omega_5 + 12c_s^2\omega_7^2\omega_4^3\omega_5 + 12c_s^4\omega_7\omega_4\omega_5^3 - 24\omega_7\omega_4^3v_1^4\omega_5^2 + 4\omega_7^2\omega_4^3v_1^4\omega_5^3 + 6c_s^4\omega_7^2\omega_4^3\omega_5^2 + 102c_s^2\omega_7^2\omega_4v_1^2\omega_5^3 - 60\omega_7\omega_4^3v_1^2\omega_5^2 + 18\omega_7^2\omega_4^3v_1^2\omega_5^3 + 6c_s^2\omega_7\omega_4^3\omega_5^2 - 12c_s^2\omega_7^2v_1^2\omega_5^3 - 48c_s^2\omega_7^2\omega_4v_1^2\omega_5^2 + 48\omega_7\omega_4^3v_1^4\omega_5^3 + 13c_s^4\omega_7^2\omega_4^3\omega_5^3 + 19\omega_7^2\omega_4^3v_1^4\omega_5^2 - 306c_s^2\omega_7^2\omega_4^3v_1^2\omega_5 - 12c_s^4\omega_7^2\omega_4^3\omega_5 - 6c_s^2\omega_7\omega_4^3\omega_5^3 + 27\omega_7\omega_4^3v_1^2\omega_5^3 - 12\omega_7^2\omega_4^3v_1^2\omega_5^2 - 12c_s^2\omega_7\omega_4^2\omega_5^2 + 6c_s^2\omega_7^2\omega_4\omega_5^3 + 24\omega_7\omega_4v_1^2\omega_5^3 - 72\omega_7^2\omega_4^3v_1^2 + 12c_s^4\omega_7^2\omega_5^3 + 12\omega_4^3v_1^2\omega_5^2 - 90\omega_7^2\omega_4^3v_1^4\omega_5 + 30c_s^2\omega_7\omega_4^3v_1^2\omega_5^3 + c_s^4\omega_7^2\omega_4^3\omega_5^2 + 60c_s^2\omega_7^2\omega_4^3v_1^2\omega_5^2 + 36\omega_7\omega_4^3v_1^2\omega_5 - 12\omega_4^3v_1^4\omega_5^3 + 18c_s^2\omega_7\omega_4^3\omega_5^3 - 12c_s^2\omega_7\omega_4^2v_1^2\omega_5^3 - c_s^2\omega_7^2\omega_4^3\omega_5^3 + 12c_s^2\omega_7^2\omega_4^3v_1^2\omega_5^3 - 12c_s^4\omega_7^2\omega_4^3\omega_5 + 12\omega_7^2\omega_4v_1^2\omega_5^3 - 12\omega_4^3v_1^2\omega_5^3$$

$$C_{13} = -12\omega_7\omega_4^3v_2^2\omega_8\omega_5^2 + 36\omega_9c_s^4\omega_7^2\omega_4^3\omega_8 - 12\omega_9\omega_7^2\omega_4^3v_2^2v_1^2\omega_5^2 - 12c_s^2\omega_7^2\omega_4^3\omega_8v_1^2\omega_5^3 + 18\omega_9c_s^2\omega_7^2\omega_4^3v_2^2\omega_5^2 - 42\omega_9c_s^4\omega_7\omega_3^3\omega_8\omega_5^2 + 12\omega_9\omega_4^3v_2^2\omega_8\omega_5^3 - 88\omega_9c_s^4\omega_7^2\omega_4^3\omega_8\omega_5^3 - 18c_s^4\omega_7^2\omega_3^3\omega_8\omega_5^2 + 36c_s^2\omega_7^2\omega_4^3v_2^2\omega_5^3 + 18\omega_9c_s^2\omega_7\omega_4^3\omega_8v_1^2\omega_5^3 - 12\omega_9c_s^2\omega_7^2\omega_4^3\omega_8v_1^2\omega_5 + 36c_s^4\omega_7\omega_4^3\omega_8\omega_5^3 + 18\omega_9c_s^2\omega_7^2\omega_4^3\omega_8v_1^2\omega_5^2 + 18\omega_9c_s^4\omega_7^2\omega_4^3\omega_8\omega_5^3 - 12\omega_9\omega_4^3v_2^2\omega_8v_1^2\omega_5^3 - 6\omega_7^2\omega_3^3v_2^2\omega_8v_1^2\omega_5^2 - 12\omega_9c_s^2\omega_4^3\omega_8\omega_5^3 + 36\omega_9c_s^2\omega_7\omega_4^3\omega_8\omega_5^2 + 6\omega_9\omega_7^2\omega_4^3v_2^2\omega_8v_1^2\omega_5^2 + 12\omega_9\omega_4^3v_2^2\omega_8\omega_5^3 + 30\omega_9c_s^4\omega_7\omega_4^3\omega_8\omega_5^3 + 150\omega_9c_s^4\omega_7^2\omega_4^3\omega_8\omega_5^2 + 6\omega_9\omega_7^2\omega_4^3v_2^2\omega_8v_1^2\omega_5^2 + 12\omega_7\omega_4^3v_2^2\omega_8\omega_5^3 - 18\omega_9c_s^2\omega_7^2\omega_4^3\omega_8\omega_5^2 + 6\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 + 6\omega_9c_s^2\omega_7\omega_4^3\omega_8\omega_5^3 + 12\omega_9c_s^2\omega_4^3\omega_8\omega_5^2 + 12c_s^2\omega_7\omega_4^3\omega_8v_1^2\omega_5^3 - 6c_s^2\omega_7^2\omega_4^3\omega_8\omega_5^3 + 12\omega_9c_s^4\omega_4^3\omega_8v_1^2\omega_5^3 + 12\omega_9\omega_7^2\omega_4^3v_2^2\omega_5^2 - 12\omega_9c_s^2\omega_7^2\omega_4^3\omega_8v_1^2\omega_5^3 - 6\omega_7^2\omega_4^3v_2^2\omega_8v_1^2\omega_5^2 - 12\omega_9c_s^2\omega_7\omega_4^3\omega_8v_1^2\omega_5 - 12\omega_7\omega_4^3v_2^2\omega_8\omega_5^3 + 6\omega_9c_s^2\omega_7^2\omega_4^3v_1^2\omega_5^2 + 12c_s^2\omega_7^2\omega_4^3v_1^2\omega_5^3 - 6c_s^2\omega_7^2\omega_4^3\omega_8v_1^2\omega_5^2 + 24\omega_9\omega_7\omega_4v_2^2\omega_8\omega_5^3 + 12\omega_9c_s^2\omega_7\omega_4\omega_8\omega_5^3 - 36\omega_9c_s^4\omega_7^2\omega_4^3\omega_8\omega_5^2 + 12\omega_9\omega_4^3v_2^2\omega_8\omega_5^2 + 24\omega_9\omega_7\omega_4^3v_2^2\omega_8v_1^2\omega_5^2 + 18\omega_9c_s^2\omega_7\omega_4^3\omega_8v_1^2\omega_5^2 - 72\omega_9c_s^2\omega_7\omega_4v_2^2\omega_8\omega_5^3 + 72\omega_9c_s^2\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 + \omega_9c_s^2\omega_7^2\omega_4^3\omega_8v_1^2\omega_5^3 - 48\omega_9c_s^4\omega_7^2\omega_4^3\omega_8\omega_5 - 12\omega_9c_s^2\omega_7^2\omega_4^3\omega_8v_1^2\omega_5^3 - 24\omega_9\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^2 - 12c_s^2\omega_7\omega_4^3\omega_8v_1^2\omega_5^3 - 18\omega_9\omega_7^2\omega_4^3v_2^2\omega_8v_1^2\omega_5^3 - 12\omega_7\omega_4^3v_2^2\omega_8v_1^2\omega_5^3 + 12\omega_9c_s^4\omega_7\omega_4\omega_8\omega_5^3 - 12\omega_9\omega_7\omega_4^3v_2^2\omega_8v_1^2\omega_5^3 - 6\omega_9c_s^2\omega_7\omega_4^3\omega_8v_1^2\omega_5^3 - 12\omega_9\omega_3^3v_2^2\omega_8\omega_5^3 + 6\omega_9\omega_7^2\omega_4^3v_2^2v_1^2\omega_5^2 + 6c_s^2\omega_7^2\omega_4^3\omega_8v_1^2\omega_5^3 + 12\omega_9c_s^2\omega_7\omega_4^3\omega_8\omega_5 + 6\omega_7^2\omega_4^3v_2^2\omega_5^3 + 12\omega_7^2\omega_4^3v_2^2\omega_8\omega_5^3 + 12c_s^2\omega_7\omega_4^3\omega_8v_1^2\omega_5^2 + 12\omega_7\omega_4^3v_2^2\omega_8v_1^2\omega_5^3 + 6c_s^2\omega_7^2\omega_4^3\omega_5^3 + 12\omega_9c_s^4\omega_7\omega_3^3\omega_8\omega_5 - 2\omega_9c_s^2\omega_7^2\omega_4^3\omega_8v_1^2\omega_5^2 - 36c_s^2\omega_7\omega_3^3v_2^2\omega_8\omega_5^3 + 12c_s^2\omega_7^2\omega_4^3\omega_8\omega_5^3 + 12\omega_7\omega_4^3v_2^2\omega_8v_1^2\omega_5^3 + 12\omega_9\omega_7^2\omega_4^3v_2^2\omega_8v_1^2\omega_5^3 - 12c_s^2\omega_7\omega_4^3\omega_8\omega_5^2 - 18\omega_9c_s^2\omega_7\omega_4^3\omega_8\omega_5^3 + 2\omega_9c_s^2\omega_7^2\omega_4^3\omega_8\omega_5^2 - 12\omega_9c_s^2\omega_7^2\omega_4^3\omega_8v_1^2\omega_5^2 - 24\omega_9\omega_7\omega_4v_2^2\omega_8v_1^2\omega_5^3 - 6\omega_9\omega_7^2\omega_4^3v_2^2\omega_5^2 - 36\omega_9c_s^2\omega_4^3v_2^2\omega_8\omega_5^3 - 12\omega_9c_s^2\omega_7^2\omega_3^3\omega_8 - 6\omega_9c_s^4\omega_7^2\omega_4^3\omega_8\omega_5^2 - 42\omega_9c_s^4\omega_7\omega_4^3\omega_8\omega_5^3 - 96\omega_9c_s^4\omega_7^2\omega_8\omega_5^3 + 36c_s^4\omega_7\omega_4^3\omega_8\omega_5^2 - 12\omega_9c_s^4\omega_4^3\omega_8\omega_5^3 -$$

$$C_{21} = 36\omega_6^2\omega_8 - 84\omega_6^2v_2\omega_8 - 48\omega_4\omega_8^2 + 2\omega_6^2\omega_4\omega_8^2 + 60\omega_6c_s^2\omega_8^2 + 61\omega_6\omega_1^2v_2\omega_8^2 - 33\omega_6^2c_s^2\omega_4\omega_8 + 72\omega_6\omega_4\omega_8^2 - 48\omega_6^2c_s^2\omega_4 - 120\omega_6c_s^2\omega_4\omega_8^2 + 168\omega_6^2\omega_4v_2\omega_8 + 24\omega_6^2\omega_1^2v_2^2 - 60\omega_1^4v_2^2\omega_8^2 - 3\omega_6^2c_s^2\omega_4^2\omega_8^2 + 24\omega_6\omega_4\omega_8 + 24\omega_6^2c_s^2\omega_4^2 - 24\omega_6c_s^2\omega_4\omega_8 + 36\omega_6\omega_4v_2^2\omega_8 + 21\omega_6^2\omega_4^2\omega_8 + 72c_s^2\omega_4\omega_8^2 - 5\omega_6^2\omega_4^2v_2^2\omega_8^2 + 120\omega_4v_2^2\omega_8^2 + 24\omega_6^2\omega_4 - 36\omega_6\omega_8 - 36c_s^2\omega_4^2\omega_8^2 - 72\omega_6^2\omega_4\omega_8 - 72\omega_6\omega_4v_2^2\omega_8 - 60\omega_6^2c_s^2\omega_8 - 12\omega_6\omega_4^2\omega_8 + 12\omega_6c_s^2\omega_4^2\omega_8 + 84\omega_6v_2^2\omega_8^2 + 120\omega_6^2c_s^2\omega_4\omega_8 - 25\omega_6\omega_4^2\omega_8^2 + 39\omega_6c_s^2\omega_4^2\omega_8^2 - 168\omega_6\omega_4v_2^2\omega_8^2 - 48\omega_6^2\omega_4v_2^2 + 24\omega_4^2\omega_8^2 - 51\omega_6^2\omega_4^2v_2^2\omega_8 - 12\omega_6^2\omega_4^2$$

2.2.4 Conservation of momentum: ρv_2



attached text file: output_d2q9_nse_mrt1_symbolic_pde_02.txt

$$\begin{aligned} & v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{\delta_l v_2 v_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\delta_l v_2 \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_l v_1 \rho}{\delta_t} \frac{\partial v_2}{\partial x_1} + (v_2^2 + c_s^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{2\delta_l v_2 \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_4) \frac{\delta_l^2 c_s^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\ & (-2 + \omega_4) \frac{\delta_l^2 c_s^2}{2\delta_t \omega_4} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (-2 + \omega_6 + 6v_2^2 - 3\omega_6 v_2^2 + 4c_s^2 - 2\omega_6 c_s^2) \frac{\delta_l^2}{\omega_6 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (2 - \omega_6) \frac{3\delta_l^2 v_2 \rho}{\omega_6 \delta_t} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + \\ & (-2 + \omega_4) \frac{\delta_l^2 c_s^2 \rho}{2\delta_t \omega_4} \frac{\partial^2 v_2}{\partial x_1^2} + (-2 + \omega_4) \frac{\delta_l^2 c_s^2 \rho}{2\delta_t \omega_4} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + (-2 + \omega_6 + 2v_2^2 - \omega_6 v_2^2 + 6c_s^2 - 3\omega_6 c_s^2) \frac{\delta_l^2 v_2}{2\omega_6 \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + \\ & (-2 + \omega_6 + 6v_2^2 - 3\omega_6 v_2^2 + 2c_s^2 - \omega_6 c_s^2) \frac{\delta_l^2 \rho}{2\omega_6 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + C_1 \frac{\delta_l^3 v_2 v_1}{12\delta_t \omega_7 \omega_4 \omega_5} \frac{\partial^3 \rho}{\partial x_1^3} + C_2 \frac{\delta_l^3 v_2 \rho}{12\delta_t \omega_7 \omega_4 \omega_5} \frac{\partial^3 v_1}{\partial x_1^3} + C_3 \frac{\delta_l^3 v_1 \rho}{6\delta_t \omega_7 \omega_4} \frac{\partial^3 v_2}{\partial x_1^3} + \\ & (-12 - \omega_4^2 + 12\omega_4) \frac{\delta_l^3 c_s^4}{6\delta_t \omega_4^2} \frac{\partial^3 \rho}{\partial x_1^3 \partial x_2} + (-\omega_4^2 - 2\omega_7 + 2\omega_4 + \omega_7 \omega_4) \frac{\delta_l^3 c_s^2 v_1 \rho}{\delta_t \omega_7 \omega_4^2} \frac{\partial^3 v_1}{\partial x_1^3 \partial x_2} + \\ & (-12\omega_6 \omega_4^2 + 12\omega_4^2 + 12\omega_6 \omega_4 \omega_8 + 12\omega_6 \omega_4 - 12\omega_4 \omega_8 - \omega_6 \omega_4^2 \omega_8 - 12\omega_6 \omega_8) \frac{\delta_l^3 c_s^2 v_2 \rho}{6\omega_6 \delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_2}{\partial x_1^3 \partial x_2} + C_4 \frac{\delta_l^3 v_2 v_1}{\omega_6^3 \delta_t \omega_4 \omega_8} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + \\ & C_5 \frac{\delta_l^3 v_2 \rho}{12\omega_6^3 \delta_t \omega_4 \omega_8} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_6 \frac{\delta_l^3 v_1 \rho}{\omega_6^3 \delta_t \omega_4 \omega_8} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_7 \frac{\delta_l^3 \rho}{12\omega_6^3 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\ & (-24 + 5\omega_6^2 c_s^2 + 24\omega_6 + 11\omega_6^2 v_2^2 - 4\omega_6^2 + 60v_2^2 - 60\omega_6 v_2^2 + 36c_s^2 - 36\omega_6 c_s^2) \frac{\delta_l^3 v_2 \rho}{6\omega_6^3 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_8 \frac{\delta_l^4 v_2}{24\delta_t \omega_7^2 \omega_4^2 \omega_5^2} \frac{\partial^4 \rho}{\partial x_1^4} + \\ & C_9 \frac{\delta_l^4 v_2 v_1 \rho}{12\delta_t \omega_7^2 \omega_4^2 \omega_5^2} \frac{\partial^4 v_1}{\partial x_1^4} + C_{10} \frac{\delta_l^4 \rho}{24\delta_t \omega_7^2 \omega_4^2} \frac{\partial^4 v_2}{\partial x_1^4} + C_{11} \frac{\delta_l^4 v_1}{12\omega_9 \omega_6 \delta_t \omega_7^2 \omega_4^2 \omega_8^2} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{12} \frac{\delta_l^4 \rho}{12\omega_9 \omega_6 \delta_t \omega_7^2 \omega_4^2 \omega_8^2} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + \\ & C_{13} \frac{\delta_l^4 v_2 v_1 \rho}{12\omega_9 \omega_6^3 \delta_t \omega_7^2 \omega_4^2 \omega_8^2} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_{14} \frac{\delta_l^4 v_2}{12\omega_9 \omega_6^3 \delta_t \omega_7^2 \omega_4^2 \omega_8^2} \frac{\partial^4 \rho}{\partial x_2^2 \partial x_2^2} + C_{15} \frac{\delta_l^4 v_2 v_1 \rho}{2\omega_9 \omega_6^3 \delta_t \omega_7^2 \omega_4^2 \omega_8^2} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + \\ & C_{16} \frac{\delta_l^4 \rho}{12\omega_9 \omega_6^3 \delta_t \omega_7^2 \omega_4^2 \omega_8^2} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_{17} \frac{\delta_l^4 v_1}{4\omega_6^3 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{18} \frac{\delta_l^4 \rho}{12\omega_6^3 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{19} \frac{\delta_l^4 v_2 v_1 \rho}{4\omega_6^3 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\ & C_{20} \frac{\delta_l^4 v_2}{12\omega_6^3 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{21} \frac{\delta_l^4 \rho}{12\omega_6^3 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:

$$\begin{aligned} C_1 &= -6\omega_4\omega_5 - 36c_s^2\omega_5 + 12\omega_7v_1^2 + 3c_s^2\omega_7\omega_4\omega_5 + 18c_s^2\omega_4\omega_5 - 12\omega_7 + \omega_7\omega_4v_1^2\omega_5 + 6\omega_7\omega_4 - 6\omega_7\omega_4v_1^2 - \omega_7\omega_4\omega_5 - 12v_1^2\omega_5 + 6\omega_4v_1^2\omega_5 - 18c_s^2\omega_7\omega_4 + 12\omega_5 + 36c_s^2\omega_7 \\ C_2 &= -6\omega_4\omega_5 - 12c_s^2\omega_5 + 36\omega_7v_1^2 + c_s^2\omega_7\omega_4\omega_5 + 6c_s^2\omega_4\omega_5 - 12\omega_7 + 3\omega_7\omega_4v_1^2\omega_5 + 6\omega_7\omega_4 - 18\omega_7\omega_4v_1^2 - \omega_7\omega_4\omega_5 - 36v_1^2\omega_5 + 18\omega_4v_1^2\omega_5 - 6c_s^2\omega_7\omega_4 + 12\omega_5 + 12c_s^2\omega_7 \\ C_3 &= -6\omega_4v_1^2 - 3\omega_4^2 - \omega_7\omega_4^2v_1^2 + 6\omega_4 - 3\omega_7\omega_4 - 3c_s^2\omega_7\omega_4^2 + 3c_s^2\omega_4^2 + 3\omega_7\omega_4v_1^2 - 6c_s^2\omega_4 + \omega_7\omega_4^2 + 15c_s^2\omega_7\omega_4 - 12c_s^2\omega_7 + 3\omega_4^2v_1^2 \\ C_4 &= 3c_s^2\omega_4\omega_8 - 3\omega_6^2c_s^2 - \omega_6\omega_4v_2^2 + 3\omega_6^2c_s^2\omega_4 - \omega_6^2v_2^2 + \omega_6\omega_4\omega_8 - 3\omega_6c_s^2\omega_4\omega_8 + 3\omega_6c_s^2\omega_8 + \omega_6^2 + \omega_6\omega_4 - \omega_4\omega_8 - \omega_6^2\omega_4 - \omega_6\omega_4v_2^2\omega_8 + \omega_6v_2^2\omega_8 + \omega_6^2\omega_4v_2^2 - \omega_6\omega_8 - 3\omega_6c_s^2\omega_4 + \omega_4v_2^2\omega_8 \\ C_5 &= 12\omega_6\omega_4^2 - 11\omega_6^2c_s^2\omega_4^2\omega_8 - 12\omega_6^2c_s^2\omega_4 + 12\omega_4^2v_2^2\omega_8 + 6\omega_6^2\omega_4v_2^2\omega_8 + 12\omega_6^2\omega_4^2v_2^2 + 12\omega_6^2c_s^2\omega_4^2 - 24\omega_6c_s^2\omega_4\omega_8 - 6\omega_6\omega_4^2v_2^2\omega_8 + 3\omega_6^2\omega_4^2\omega_8 + 12\omega_6^2\omega_4 - 12\omega_6c_s^2\omega_4^2 - 12\omega_4^2\omega_8 - 6\omega_6^2\omega_4\omega_8 - 24\omega_6^2c_s^2\omega_8 + 6\omega_6\omega_4^2\omega_8 - 18\omega_6c_s^2\omega_4^2\omega_8 + 42\omega_6^2c_s^2\omega_4\omega_8 - 12\omega_6^2\omega_4v_2^2 + 36c_s^2\omega_4^2\omega_8 - 3\omega_6^2\omega_4^2v_2^2\omega_8 - 12\omega_6\omega_4^2v_2^2 - 12\omega_6^2\omega_4^2 \\ C_6 &= c_s^2\omega_4\omega_8 - \omega_6^2c_s^2 - 3\omega_6\omega_4v_2^2 + \omega_6^2c_s^2\omega_4 - 3\omega_6^2v_2^2 + \omega_6\omega_4\omega_8 - \omega_6c_s^2\omega_4\omega_8 + \omega_6c_s^2\omega_8 + \omega_6^2 + \omega_6\omega_4 - \omega_4\omega_8 - \omega_6^2\omega_4 - 3\omega_6\omega_4v_2^2\omega_8 + 3\omega_6v_2^2\omega_8 + 3\omega_6^2\omega_4v_2^2 - \omega_6\omega_8 - \omega_6c_s^2\omega_4 + 3\omega_4v_2^2\omega_8 \\ C_7 &= 144c_s^2v_2^2 - \omega_6^2c_s^2 + 12c_s^4 - 144\omega_6c_s^2v_2^2 - 12\omega_6c_s^4 - 7\omega_6^2v_2^2 + 36v_2^4 - 36\omega_6v_2^4 + 7\omega_6^2v_2^4 - 36v_2^2 + 36\omega_6v_2^2 + \omega_6^2c_s^4 - 12c_s^2 + 24\omega_6^2c_s^2v_2^2 + 12\omega_6c_s^2 \\ C_8 &= 96\omega_7^2\omega_4v_1^2\omega_5 + 48\omega_7^2v_1^4\omega_5 - 24c_s^4\omega_7\omega_5^2 - 24\omega_4v_1^4\omega_5^2 + c_s^2\omega_7^2\omega_4^2\omega_5^2 - 12c_s^4\omega_7^2\omega_4^2 - 48c_s^4\omega_7^2\omega_4\omega_5 + 24c_s^2\omega_4\omega_5^2 + 150c_s^2\omega_7^2\omega_4^2v_1^2\omega_5 - 3\omega_7^2\omega_4^2v_1^4\omega_5^2 + 48\omega_7v_1^2\omega_5^2 - 48c_s^2\omega_7\omega_4\omega_5^2 + 30\omega_7\omega_4^2v_1^2\omega_5^2 - 24c_s^2\omega_7^2\omega_4 - 48\omega_7\omega_4v_1^4\omega_5 + 432c_s^2\omega_7\omega_4v_1^2\omega_5^2 + 24c_s^2\omega_7\omega_5^2 - 36\omega_7^2\omega_1^4v_1^2 - 144c_s^2\omega_4v_1^2\omega_5^2 + 96\omega_7\omega_4v_1^4\omega_5^2 + 12c_s^2\omega_7^2\omega_4^2 + 216c_s^2\omega_7^2v_1^2\omega_5 - 12c_s^4\omega_7\omega_4^2\omega_5^2 - 144c_s^2\omega_7\omega_4v_1^2\omega_5 - 24\omega_7\omega_4^2v_1^2\omega_5 - 24c_s^4\omega_4\omega_5^2 - 12\omega_4^2v_1^2\omega_5^2 - 12c_s^2\omega_7^2\omega_4^2v_1^2\omega_5 + 36\omega_7^2\omega_4^2v_1^4\omega_5 - 72\omega_7^2\omega_4v_1^2\omega_5 - 144c_s^2\omega_7^2\omega_4^2v_1^2 + 24c_s^4\omega_7^2\omega_4 - 14c_s^2\omega_7^2\omega_4^2\omega_5 + 72c_s^2\omega_7\omega_4^2v_1^2\omega_5 - 30\omega_7\omega_4^2v_1^4\omega_5 - c_s^4\omega_7^2\omega_4^2\omega_5^2 + 72c_s^2\omega_4^2v_1^2\omega_5^2 + 24c_s^4\omega_7^2\omega_5 + 36\omega_7^2\omega_4^2v_1^2 + 48\omega_7\omega_4v_1^2\omega_5 + 48c_s^2\omega_7^2\omega_4\omega_5 + 288c_s^2\omega_7^2\omega_4v_1^2 - 48\omega_7^2v_1^2\omega_5 - 96\omega_7^2\omega_4v_1^4\omega_5 + 72\omega_7^2\omega_4v_1^2\omega_5^2 + 48c_s^4\omega_7\omega_4\omega_5^2 + 12c_s^2\omega_7^2\omega_5 - 48\omega_7v_1^2\omega_5^2 + 3\omega_7^2\omega_4^2v_1^2\omega_5 + 12c_s^2\omega_7\omega_4^2\omega_5^2 + 12\omega_4^2v_1^2\omega_5^2 - 36\omega_7^2\omega_4^2v_1^2\omega_5 - 24c_s^2\omega_7^2\omega_5 - 432c_s^2\omega_7^2\omega_4v_1^2\omega_5 - 96\omega_7\omega_4v_1^2\omega_5^2 - 126c_s^2\omega_7\omega_4^2v_1^2\omega_5^2 + 14c_s^4\omega_7^2\omega_4^2\omega_5 - 216c_s^2\omega_7v_1^2\omega_5^2 + 24\omega_7\omega_4^2v_1^4\omega_5 - 12c_s^2\omega_4^2\omega_5^2 \end{aligned}$$

$$\begin{aligned}
& 12\omega_3^3 c_s^4 \omega_4^2 \omega_8^2 + 72\omega_9 \omega_6^2 \omega_7 \omega_4 v_2^2 \omega_8^2 v_1^2 + 24\omega_9 \omega_6^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 + 36\omega_9 \omega_6 \omega_7 \omega_4^2 v_2^2 \omega_8^2 v_1^2 + 12\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 \omega_8 + 6\omega_9 \omega_6^2 c_s^4 \omega_4^3 \omega_8^2 - 6\omega_9 \omega_6^2 \omega_3^3 \omega_8^2 v_1^2 + \\
& 6\omega_6^3 c_s^2 \omega_7 \omega_4^3 \omega_8^2 v_1^2 + 18\omega_9 \omega_6^2 \omega_7 \omega_4^3 v_2^2 \omega_8^2 v_1^2 - 24\omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4^2 \omega_8 - 12\omega_9 \omega_6 c_s^2 \omega_7 \omega_4^3 \omega_8 v_1^2 + 24\omega_9 \omega_6^3 \omega_7 \omega_4 \omega_8 v_1^2 - 6\omega_6^3 \omega_7 \omega_4^3 \omega_8^2 v_1^2 + 12\omega_9 c_s^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 + \\
& 36\omega_6^3 \omega_7 \omega_4^2 v_2^2 \omega_8 v_1^2 - 36\omega_6^3 c_s^2 \omega_7 \omega_4^3 v_2^2 \omega_8 + 12\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 v_1^2 - 12\omega_6^3 \omega_4^2 \omega_8^2 v_1^2 + 12\omega_6^3 c_s^4 \omega_7 \omega_4^3 \omega_8 + 12\omega_9 \omega_6^2 \omega_7 \omega_4^2 \omega_8 v_1^2 + 12\omega_9 \omega_6 c_s^2 \omega_7 \omega_4^2 \omega_8^2 - \\
& 12\omega_6^3 c_s^2 \omega_7 \omega_4^2 \omega_8 + 12\omega_9 \omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - \omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4^3 \omega_8^2 - 108\omega_9 \omega_6 c_s^2 \omega_7 \omega_4^2 v_2^2 \omega_8^2 - 36\omega_9 \omega_6^2 \omega_7 \omega_4^3 v_2^2 v_1^2 - 24\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 \omega_8^2 - 36\omega_9 \omega_6^2 c_s^2 \omega_4^2 v_2^2 \omega_8^2 - \\
& 12\omega_6^3 \omega_7 \omega_4^3 \omega_8 v_1^2 - 24\omega_9 \omega_6^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 - 18\omega_6^3 \omega_7 \omega_4^3 v_2^2 \omega_8^2 v_1^2 - 6\omega_6^3 c_s^2 \omega_7 \omega_4^3 \omega_8^2 + 108\omega_9 \omega_6^3 \omega_7 \omega_4^2 v_2^2 \omega_8 v_1^2 + 78\omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^3 v_2^2 \omega_8 + 60\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 v_2^2 \omega_8 + \\
& 84\omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4 v_2^2 \omega_8^2 - 24\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 - 6\omega_9 \omega_6 c_s^2 \omega_7 \omega_4^3 \omega_8^2 - 12\omega_9 \omega_6^2 c_s^4 \omega_7 \omega_4 \omega_8^2 + 180\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 v_2^2 \omega_8^2 + 24\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 v_2^2 + 12\omega_6^3 c_s^2 \omega_7 \omega_4^3 \omega_8 - \\
& 6\omega_9 \omega_6^2 c_s^2 \omega_4^3 \omega_8^2 - 6\omega_9 \omega_6^2 \omega_7 \omega_4^3 \omega_8^2 v_1^2 + 60\omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4 v_2^2 \omega_8 - 36\omega_9 \omega_6^3 \omega_7 \omega_4^2 v_2^2 v_1^2 + 12\omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4^3 \omega_8 + 12\omega_6^3 c_s^2 \omega_7 \omega_4^2 \omega_8 v_1^2 - 12\omega_9 \omega_6^2 c_s^2 \omega_4^2 \omega_8^2 v_1^2 + \\
& 12\omega_6^3 c_s^2 \omega_7 \omega_4^3 \omega_8 v_1^2 - 12\omega_9 \omega_6^2 c_s^4 \omega_4^2 \omega_8^2 - 36\omega_9 \omega_6^3 \omega_7 \omega_4^3 v_2^2 \omega_8 v_1^2 - 12\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 \omega_8 + 12\omega_9 \omega_6 c_s^2 \omega_7 \omega_4^2 \omega_8^2 v_1^2 - 12\omega_6^3 \omega_7 \omega_4^2 \omega_8 v_1^2 - 36\omega_6^3 \omega_7 \omega_4^3 v_2^2 \omega_8 v_1^2 - \\
& 12\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 v_1^2 - 6\omega_6^3 c_s^4 \omega_7 \omega_4^3 \omega_8^2 - 72\omega_9 \omega_6^2 \omega_7 \omega_4^2 v_2^2 \omega_8^2 v_1^2 - 12\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4 \omega_8 + 36\omega_6^3 c_s^2 \omega_4^2 v_2^2 \omega_8^2 - 54\omega_9 \omega_6 \omega_7 \omega_4^2 v_2^2 \omega_8^2 v_1^2 + 36\omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^2 \omega_8 v_1^2 + \\
& 12\omega_6^3 c_s^2 \omega_7 \omega_4^2 \omega_8^2 + 24\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 \omega_8 v_1^2 + 6\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 \omega_8^2 + 12\omega_9 \omega_6^3 \omega_7 \omega_4^3 \omega_8 v_1^2 - 4\omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4^2 \omega_8^2 + 18\omega_9 \omega_6 \omega_7 \omega_4^3 \omega_8^2 v_1^2 + 18\omega_6^3 c_s^2 \omega_7 \omega_4^3 v_2^2 \omega_8^2
\end{aligned}$$

$$\begin{aligned}
C_{17} = & -144\omega_6 c_s^2 \omega_4^2 v_2^2 \omega_8^2 - 4\omega_6^3 c_s^4 \omega_4^2 \omega_8 + 24\omega_6^3 c_s^2 \omega_4^2 v_2^2 - 8\omega_6^3 v_2^4 \omega_8 + 4\omega_6^3 c_s^4 \omega_4^2 \omega_8^2 + 4\omega_6^3 c_s^2 \omega_8 + 36\omega_6 \omega_4^2 v_2^2 \omega_8^2 - 8\omega_6^2 c_s^2 \omega_4^2 \omega_8 + 13\omega_6^2 \omega_4^2 v_2^4 \omega_8^2 + \\
& 36\omega_6^2 c_s^2 v_2^2 \omega_8^2 - 4\omega_6 c_s^2 \omega_4^2 \omega_8^2 - 4\omega_6^3 c_s^4 \omega_4 + 51\omega_6^2 c_s^2 \omega_4^2 v_2^2 \omega_8^2 - 36\omega_6^3 c_s^2 v_2^2 \omega_8 + 16\omega_6^2 \omega_4^2 v_2^2 \omega_8 - 13\omega_6^3 \omega_4^2 v_2^4 \omega_8 + 4\omega_6^2 \omega_4^2 v_2^2 + 12\omega_6^2 c_s^2 \omega_4^2 v_2^2 \omega_8 + \\
& 4\omega_6^3 c_s^4 \omega_4^2 - 24\omega_4^2 v_2^2 \omega_8^2 + 4\omega_6^3 c_s^2 \omega_4^2 \omega_8 + 20\omega_6^2 \omega_4^2 v_2^2 \omega_8^2 - 20\omega_6^3 \omega_4^2 v_2^2 \omega_8 - 4\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 32\omega_6^2 \omega_4^2 v_2^4 \omega_8 + 84\omega_6^3 c_s^2 \omega_4 v_2^2 \omega_8 + 4\omega_6^2 c_s^2 \omega_4^2 + 4\omega_6^3 \omega_4^2 v_2^4 + \\
& 8\omega_6^2 c_s^2 \omega_4 \omega_8 - 24\omega_6^2 c_s^2 \omega_4^2 v_2^2 + 4\omega_6 c_s^4 \omega_4 \omega_8^2 + 20\omega_6 \omega_4^2 v_2^2 \omega_8 + 4\omega_6^3 \omega_4^2 v_2^2 - 72\omega_6 c_s^2 \omega_4^2 v_2^2 \omega_8^2 + 20\omega_6 \omega_4 v_2^4 \omega_8^2 - 8\omega_6^2 v_2^2 \omega_8^2 + 4\omega_6^3 c_s^4 \omega_8^2 - 13\omega_6^2 \omega_4^2 v_2^2 \omega_8^2 - \\
& 4\omega_6^3 \omega_4^2 v_2^2 - 48\omega_6^2 c_s^2 \omega_4 v_2^2 \omega_8 - 12\omega_6 c_s^4 \omega_4^2 \omega_8^2 + 4\omega_6^3 c_s^2 \omega_4 - 51\omega_6^2 c_s^2 \omega_4^2 v_2^2 \omega_8 - 8c_s^2 \omega_4^2 \omega_8^2 + 13\omega_6^3 \omega_4^2 v_2^2 \omega_8 - 16\omega_6^2 \omega_4^2 v_2^2 \omega_8 + 96c_s^2 \omega_4^2 v_2^2 \omega_8^2 - \\
& 8\omega_6^2 c_s^2 \omega_4 \omega_8 + 8\omega_6^3 v_2^2 \omega_8 + 8\omega_6^2 c_s^2 \omega_4 \omega_8^2 + 4\omega_6 c_s^2 \omega_4^2 \omega_8 - 36\omega_6 \omega_4^2 v_2^2 \omega_8^2 - 4\omega_6^3 c_s^4 \omega_8 - 4\omega_6^3 \omega_4 v_2^2 + 12\omega_6 c_s^2 \omega_4^2 v_2^2 \omega_8^2 - 20\omega_6 \omega_4^2 v_2^4 \omega_8 + 72\omega_6 c_s^2 \omega_4 v_2^2 \omega_8^2 - \\
& 20\omega_6 \omega_4 v_2^2 \omega_8^2 + 8\omega_6^2 v_2^4 \omega_8^2 - 4\omega_6^2 c_s^2 \omega_8^2 + 8c_s^4 \omega_4^2 \omega_8^2 - 4\omega_6^3 c_s^2 \omega_4^2 + 24\omega_4^2 v_2^2 \omega_8^2 - 20\omega_6^2 \omega_4^2 v_2^2 \omega_8^2 - 24\omega_6^2 c_s^2 \omega_4 v_2^2 - 4\omega_6^2 \omega_4^2 v_2^4 + 8\omega_6^3 c_s^4 \omega_4 \omega_8 - \\
& 32\omega_6 \omega_4^2 v_2^2 \omega_8 - 8\omega_6^2 c_s^4 \omega_4 \omega_8^2 + 20\omega_6^2 \omega_4 v_2^2 \omega_8 - 4\omega_6^2 c_s^4 \omega_4^2 - 84\omega_6^2 c_s^2 \omega_4 v_2^2 \omega_8^2 - 4\omega_6 c_s^4 \omega_4^2 \omega_8
\end{aligned}$$

$$\begin{aligned}
C_{18} = & -108\omega_6 c_s^2 \omega_4^2 v_2^2 \omega_8^2 + 12\omega_6^3 \omega_3^3 v_2^4 - \omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 18\omega_6^3 c_s^4 \omega_4^2 \omega_8 - 12\omega_6^3 c_s^2 \omega_4^2 v_2^2 - 90\omega_6 \omega_3^3 v_2^4 \omega_8^2 + 252c_s^2 \omega_3^3 v_2^2 \omega_8^2 + 6\omega_6^2 c_s^4 \omega_4^2 \omega_8^2 - 6\omega_6^3 c_s^2 \omega_4^2 \omega_8 - \omega_6^3 c_s^4 \omega_4^2 \omega_8^2 - \\
& 12\omega_6^2 c_s^2 \omega_4^2 \omega_8 + 12\omega_6^2 \omega_4^2 v_2^4 \omega_8^2 - 12\omega_6^3 \omega_4^2 v_2^2 \omega_8^2 + 102\omega_6^3 c_s^2 \omega_4 v_2^2 \omega_8^2 + 27\omega_6^3 \omega_4^2 v_2^2 \omega_8 - 21\omega_6^3 c_s^2 \omega_3^3 v_2^2 \omega_8 + 162\omega_6^2 c_s^2 \omega_4^2 v_2^2 \omega_8^2 - 19\omega_6^3 \omega_4^2 v_2^2 \omega_8^2 - 12\omega_6^2 c_s^2 \omega_4^2 v_2^2 + \\
& 48\omega_6^3 \omega_4^2 v_2^2 \omega_8 - 5\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 6\omega_6^2 c_s^4 \omega_4^2 \omega_8 - 48\omega_6^3 c_s^2 v_2^2 \omega_8^2 - 12\omega_6^2 c_s^2 \omega_4^2 v_2^2 \omega_8 - 60\omega_6^2 \omega_4^2 v_2^2 \omega_8 - 18\omega_6^3 \omega_4^2 v_2^2 \omega_8^2 + \omega_6^2 c_s^4 \omega_4^2 \omega_8^2 + 18\omega_6^3 c_s^2 \omega_4^2 \omega_8 + 12\omega_6^2 c_s^2 \omega_4^2 v_2^2 + \\
& 24\omega_6^3 \omega_4^2 v_2^2 \omega_8 - 6\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + 6\omega_6^3 c_s^4 \omega_4^2 \omega_8 - 24\omega_6^2 \omega_4^2 v_2^4 \omega_8 - 12\omega_6^3 c_s^2 \omega_4^2 v_2^2 \omega_8 - 4\omega_6^3 \omega_4^2 v_2^2 \omega_8^2 - 12\omega_6^3 \omega_4^2 v_2^4 + 12\omega_6^3 c_s^2 \omega_3^3 v_2^2 \omega_8^2 + 72\omega_3^3 v_2^4 \omega_8^2 + 12\omega_6^2 \omega_4^2 v_2^2 + \\
& 12\omega_6^2 c_s^4 \omega_4^2 \omega_8 + 13\omega_6^3 c_s^4 \omega_4^2 \omega_8^2 + 6\omega_6^2 c_s^2 \omega_4^2 \omega_8 - 36\omega_6 \omega_4^2 v_2^2 \omega_8 + 12\omega_6^2 \omega_4^2 v_2^2 \omega_8^2 - 12\omega_6^2 \omega_4^2 v_2^2 \omega_8^2 + 12c_s^4 \omega_4^2 \omega_8^2 + 60\omega_6^2 c_s^2 \omega_4^2 v_2^2 \omega_8^2 + 12\omega_6^3 \omega_4^2 v_2^2 - 27\omega_6^3 \omega_4^2 v_2^2 \omega_8 - \\
& 12\omega_6 c_s^4 \omega_4^2 \omega_8^2 + 12\omega_6 c_s^2 \omega_4^2 \omega_8^2 + 19\omega_6^2 \omega_3^3 v_2^4 \omega_8^2 + 30\omega_6^3 c_s^2 \omega_4^2 v_2^2 \omega_8 - 24\omega_6^3 c_s^4 \omega_4 \omega_8^2 - 48\omega_6^3 \omega_4^2 v_2^2 \omega_8 - 12\omega_6^3 c_s^2 \omega_4 \omega_8 + 90\omega_6 \omega_4^2 v_2^2 \omega_8^2 - 12\omega_6^2 \omega_4^2 v_2^4 - \\
& 306\omega_6 c_s^2 \omega_4^2 v_2^2 \omega_8^2 - 12c_s^2 \omega_4^2 \omega_8^2 - 36\omega_6 c_s^2 \omega_4^2 v_2^2 \omega_8 + 12\omega_6^3 c_s^4 \omega_4^2 + 12\omega_6 c_s^2 \omega_4^2 \omega_8^2 - 12\omega_6^3 \omega_3^3 v_2^2 - 12\omega_6 c_s^4 \omega_4^2 \omega_8^2 + 6\omega_6^3 c_s^2 \omega_4 \omega_8^2 + 36\omega_6 \omega_4^2 v_2^2 \omega_8 + 60\omega_6^2 \omega_3^3 v_2^4 \omega_8 - \\
& 81\omega_6^2 c_s^2 \omega_4^2 v_2^2 \omega_8^2 + 18\omega_6^3 \omega_4^2 v_2^2 \omega_8^2 + 12\omega_6^3 c_s^4 \omega_4 \omega_8 + 24\omega_6^2 \omega_4^2 v_2^2 \omega_8 - 24\omega_6^3 \omega_4^2 v_2^2 \omega_8 + 54\omega_6^2 c_s^2 \omega_4^2 v_2^2 \omega_8 + 4\omega_6^3 \omega_4^2 v_2^4 \omega_8^2 - 48\omega_6^2 c_s^2 \omega_4 v_2^2 \omega_8^2 - 72\omega_4^2 v_2^2 \omega_8^2
\end{aligned}$$

$$\begin{aligned}
C_{19} = & -17\omega_6^2 \omega_4^2 \omega_8^2 - 20\omega_6^3 c_s^2 \omega_8 - 120\omega_6 \omega_4^2 v_2^2 \omega_8^2 + 56\omega_6^2 c_s^2 \omega_4^2 \omega_8 - 24\omega_6 \omega_4 \omega_8^2 - 28\omega_6^3 \omega_4 \omega_8 + 32\omega_6 c_s^2 \omega_4 \omega_8^2 - 48\omega_6^2 \omega_4 v_2^2 \omega_8 - 16\omega_6^2 \omega_4^2 v_2^2 + \\
& 80\omega_4^2 v_2^2 \omega_8^2 - 25\omega_6^3 c_s^2 \omega_4^2 \omega_8 - 68\omega_6^2 \omega_4 v_2^2 \omega_8^2 + 68\omega_6^3 \omega_4 v_2^2 \omega_8 + 25\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 16\omega_6^2 c_s^2 \omega_4^2 + 12\omega_6^3 \omega_8 - 64\omega_6 \omega_4^2 v_2^2 \omega_8 - 16\omega_6^3 \omega_4 v_2^2 - 12\omega_6^2 \omega_8^2 + \\
& 28\omega_6^2 v_2^2 \omega_8^2 - 40\omega_6^2 \omega_4^2 \omega_8 + 43\omega_6^2 \omega_4^2 v_2^2 \omega_8^2 + 16\omega_6^3 \omega_4^2 v_2^2 - 16\omega_6^3 c_s^2 \omega_4 + 48c_s^2 \omega_4^2 \omega_8^2 - 43\omega_6^3 \omega_4^2 v_2^2 \omega_8 + 16\omega_6^2 \omega_4 \omega_8 - 8\omega_6^3 \omega_4^2 + 44\omega_6^3 c_s^2 \omega_4 \omega_8 - \\
& 28\omega_4^2 v_2^2 \omega_8^2 - 44\omega_6^2 c_s^2 \omega_4 \omega_8^2 + 24\omega_6 \omega_4^2 \omega_8 - 32\omega_6 c_s^2 \omega_4^2 \omega_8 - 16\omega_6^2 c_s^2 \omega_4 \omega_8 + 48\omega_6 \omega_4^2 \omega_8^2 - 72\omega_6 c_s^2 \omega_4^2 \omega_8^2 + 17\omega_6^3 \omega_4^2 \omega_8 + 8\omega_6^3 \omega_4 + 64\omega_6 \omega_4 v_2^2 \omega_8^2 + \\
& 20\omega_6^2 c_s^2 \omega_8^2 + 16\omega_6^3 c_s^2 \omega_4^2 + 28\omega_6^2 \omega_4 \omega_8^2 - 32\omega_4^2 \omega_8^2 + 104\omega_6^2 \omega_4^2 v_2^2 \omega_8 + 8\omega_4^2 \omega_8^2
\end{aligned}$$

$$\begin{aligned}
C_{20} = & 12 + 672c_s^2 v_2^2 - 78\omega_6^2 c_s^2 + 144c_s^4 + 10\omega_6^3 v_2^2 - 1008\omega_6 c_s^2 v_2^2 - 216\omega_6 c_s^4 - 18\omega_6 - 98\omega_6^2 v_2^2 - \omega_6^3 + 144v_2^4 + 6\omega_6^3 c_s^2 - 34\omega_6^3 c_s^2 v_2^2 + 8\omega_6^2 - \\
& 216\omega_6 v_2^4 + 90\omega_6^2 v_2^4 - 156v_2^2 - 5\omega_6^3 c_s^4 + 234\omega_6 v_2^2 + 82\omega_6^2 c_s^4 - 132c_s^2 - 9\omega_6^3 v_2^4 + 404\omega_6^2 c_s^2 v_2^2 + 198\omega_6 c_s^2
\end{aligned}$$

$$\begin{aligned}
C_{21} = & 12 + 432c_s^2 v_2^2 - 22\omega_6^2 c_s^2 + 24c_s^4 + 14\omega_6^3 v_2^2 - 648\omega_6 c_s^2 v_2^2 - 36\omega_6 c_s^4 - 18\omega_6 - 154\omega_6^2 v_2^2 - \omega_6^3 + 504v_2^4 + 2\omega_6^3 c_s^2 - 18\omega_6^3 c_s^2 v_2^2 + 8\omega_6^2 - \\
& 756\omega_6 v_2^4 + 310\omega_6^2 v_2^4 - 252v_2^2 - \omega_6^3 c_s^4 + 378\omega_6 v_2^2 + 14\omega_6^2 c_s^4 - 36c_s^2 - 29\omega_6^3 v_2^4 + 252\omega_6^2 c_s^2 v_2^2 + 54\omega_6 c_s^2
\end{aligned}$$

2.3 MRT2

2.3.1 Definitions

Collision operator C :

$$C(f) = M_2^{-1} S \left(\mu_2^{(eq)} - M_2 f \right),$$

where

$$S = \text{diag}(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9),$$

$$\omega_1, \omega_2, \dots, \omega_9 \in (0, 2).$$

Matrix \mathbf{M}_2 corresponds to the transformation matrix to the raw moment basis defined by

$$\boldsymbol{\mu}_2 = \begin{pmatrix} m_{(0,0)} \\ m_{(1,0)} \\ m_{(0,1)} \\ m_{(1,1)} \\ m_{(2,0)} + m_{(0,2)} \\ m_{(2,0)} - m_{(0,2)} \\ m_{(2,1)} \\ m_{(1,2)} \\ m_{(2,2)} \end{pmatrix},$$

and is given by

$$\mathbf{M}_2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}.$$

The equilibrium moments $\boldsymbol{\mu}_2^{(eq)}$ are defined by

$$\boldsymbol{\mu}_2^{(eq)} = \mathbf{M}_2 \mathbf{M}^{-1} \boldsymbol{\mu}^{(eq)},$$

i.e.,

$$\boldsymbol{\mu}_2^{(eq)} = \begin{pmatrix} \rho \\ \rho v_1 \\ \rho v_2 \\ \rho v_1 v_2 \\ \rho(v_1^2 + v_2^2 + 2c_s^2) \\ \rho(v_1^2 - v_2^2) \\ \rho v_2(v_1^2 + c_s^2) \\ \rho v_1(v_2^2 + c_s^2) \\ \rho(v_1^2 v_2^2 + c_s^2 v_2^2 + c_s^2 v_1^2 + c_s^4) \end{pmatrix}.$$

2.3.2 Conservation of mass: ρ



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$$\begin{aligned} & \frac{\partial \rho}{\partial t} + \frac{v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + 3c_s^2 + v_1^2) \frac{v_1 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3v_1^2) \frac{\rho \delta_l^3}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\ & \frac{\rho c_s^2 \delta_l^3}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{\rho c_s^2 \delta_l^3}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{v_2 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + c_s^2 + 3v_2^2) \frac{\rho \delta_l^3}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & (-c_s^4 \omega_5 + 24v_1^2 c_s^2 - 2c_s^2 + 3v_1^2 \omega_5 + 6v_1^4 + 2c_s^4 + c_s^2 \omega_5 - 6v_1^2 - 12v_1^2 c_s^2 \omega_5 - 3v_1^4 \omega_5) \frac{\delta_l^4}{24\delta_t \omega_5} \frac{\partial^4 \rho}{\partial x_1^4} + \\ & (-4 + 6c_s^2 - 5v_1^2 \omega_5 - 3c_s^2 \omega_5 + 10v_1^2 + 2\omega_5) \frac{\rho v_1 \delta_l^4}{12\delta_t \omega_5} \frac{\partial^4 v_1}{\partial x_1^4} + (v_1^2 \omega_7 - \omega_7 - v_1^2 \omega_5 - 3c_s^2 \omega_5 + \omega_5 + 3c_s^2 \omega_7) \frac{v_2 v_1 \delta_l^4}{4\delta_t \omega_7 \omega_5} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + \\ & (3v_1^2 \omega_7 - \omega_7 - 3v_1^2 \omega_5 - c_s^2 \omega_5 + \omega_5 + c_s^2 \omega_7) \frac{\rho v_2 \delta_l^4}{4\delta_t \omega_7 \omega_5} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + (3c_s^2 \omega_7 \omega_4 - 3c_s^2 \omega_4 \omega_5 - 6c_s^2 \omega_7 \omega_5 - \omega_7 \omega_4 - 3v_1^2 \omega_4 \omega_5 + \end{aligned}$$

$$\begin{aligned}
& v_1^2 \omega_7 \omega_4 + 3 \omega_4 \omega_5 + v_1^2 \omega_7 \omega_4 \omega_5 - \omega_7 \omega_4 \omega_5 + 3 c_s^2 \omega_7 \omega_4 \omega_5) \frac{\rho v_1 \delta_l^4}{12 \delta_t \omega_7 \omega_4 \omega_5} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + (-2 + \omega_4) \frac{c_s^4 \delta_l^4}{6 \delta_t \omega_4} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2^2} + \\
& (-\omega_7 + \omega_4) \frac{\rho v_1 c_s^2 \delta_l^4}{2 \delta_t \omega_7 \omega_4} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + (\omega_4 - \omega_8) \frac{\rho v_2 c_s^2 \delta_l^4}{2 \delta_t \omega_4 \omega_8} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + (\omega_6 + 3 c_s^2 \omega_8 - 3 \omega_6 c_s^2 - \omega_6 v_2^2 - \omega_8 + v_2^2 \omega_8) \frac{v_2 v_1 \delta_l^4}{4 \omega_6 \delta_t \omega_8} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} \\
& + (-3 \omega_6 c_s^2 \omega_4 - \omega_4 \omega_8 + 3 \omega_6 c_s^2 \omega_4 \omega_8 + \omega_6 v_2^2 \omega_4 \omega_8 + 3 \omega_6 \omega_4 - 6 \omega_6 c_s^2 \omega_8 - 3 \omega_6 v_2^2 \omega_4 - \omega_6 \omega_4 \omega_8 + v_2^2 \omega_4 \omega_8 + \\
& 3 c_s^2 \omega_4 \omega_8) \frac{\rho v_2 \delta_l^4}{12 \omega_6 \delta_t \omega_4 \omega_8} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + (\omega_6 + c_s^2 \omega_8 - \omega_6 c_s^2 - 3 \omega_6 v_2^2 - \omega_8 + 3 v_2^2 \omega_8) \frac{\rho v_1 \delta_l^4}{4 \omega_6 \delta_t \omega_8} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\
& (-2 c_s^2 + 24 v_2^2 c_s^2 + 6 v_2^4 + \omega_6 c_s^2 - 3 \omega_6 v_2^4 - 6 v_2^2 + 2 c_s^4 - 12 \omega_6 v_2^2 c_s^2 + 3 \omega_6 v_2^2 - \omega_6 c_s^4) \frac{\delta_l^4}{24 \omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\
& (-4 + 2 \omega_6 + 6 c_s^2 - 3 \omega_6 c_s^2 + 10 v_2^2 - 5 \omega_6 v_2^2) \frac{\rho v_2 \delta_l^4}{12 \omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0.
\end{aligned}$$

2.3.3 Conservation of momentum: ρv_1



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$$\begin{aligned}
& v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (c_s^2 + v_1^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2 \rho v_1 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho v_2 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{\rho v_1 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\
& (-2 + 4 c_s^2 - 3 v_1^2 \omega_5 - 2 c_s^2 \omega_5 + 6 v_1^2 + \omega_5) \frac{\delta_l^2}{\delta_t \omega_5} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + (2 - \omega_5) \frac{3 \rho v_1 \delta_l^2}{\delta_t \omega_5} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + (-2 + \omega_4) \frac{c_s^2 \delta_l^2}{2 \delta_t \omega_4} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\
& (-2 + \omega_4) \frac{c_s^2 \delta_l^2}{2 \delta_t \omega_4} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (-2 + 6 c_s^2 - v_1^2 \omega_5 - 3 c_s^2 \omega_5 + 2 v_1^2 + \omega_5) \frac{v_1 \delta_l^2}{2 \delta_t \omega_5} \frac{\partial^2 \rho}{\partial x_1^2} + \\
& (-2 + 2 c_s^2 - 3 v_1^2 \omega_5 - c_s^2 \omega_5 + 6 v_1^2 + \omega_5) \frac{\rho \delta_l^2}{2 \delta_t \omega_5} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega_4) \frac{\rho c_s^2 \delta_l^2}{2 \delta_t \omega_4} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_4) \frac{\rho c_s^2 \delta_l^2}{2 \delta_t \omega_4} \frac{\partial^2 v_1}{\partial x_2^2} + C_1 \frac{\delta_l^3}{12 \delta_t \omega_5^2} \frac{\partial^3 \rho}{\partial x_1^3} \\
& + (-24 + 11 v_1^2 \omega_5^2 + 36 c_s^2 - 4 \omega_5^2 - 60 v_1^2 \omega_5 - 36 c_s^2 \omega_5 + 60 v_1^2 + 24 \omega_5 + 5 c_s^2 \omega_5^2) \frac{\rho v_1 \delta_l^3}{6 \delta_t \omega_5^2} \frac{\partial^3 v_1}{\partial x_1^3} + C_2 \frac{v_2 v_1 \delta_l^3}{\delta_t \omega_7 \omega_4 \omega_5^2} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + \\
& C_3 \frac{\rho v_2 \delta_l^3}{\delta_t \omega_7 \omega_4 \omega_5^2} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} + C_4 \frac{\rho v_1 \delta_l^3}{12 \delta_t \omega_7 \omega_4^2 \omega_5^2} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + (-12 - \omega_4^2 + 12 \omega_4) \frac{c_s^4 \delta_l^3}{6 \delta_t \omega_4^2} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + \\
& (-\omega_7 \omega_4^2 \omega_5 + 12 \omega_4^2 - 12 \omega_4^2 \omega_5 - 12 \omega_7 \omega_4 + 12 \omega_4 \omega_5 - 12 \omega_7 \omega_5 + 12 \omega_7 \omega_4 \omega_5) \frac{\rho v_1 c_s^2 \delta_l^3}{6 \delta_t \omega_7 \omega_4^2 \omega_5} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + \\
& (-\omega_4^2 + \omega_4 \omega_8 + 2 \omega_4 - 2 \omega_8) \frac{\rho v_2 c_s^2 \delta_l^3}{\delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_5 \frac{v_2 v_1 \delta_l^3}{12 \omega_6 \delta_t \omega_4 \omega_8} \frac{\partial^3 \rho}{\partial x_2^2} + C_6 \frac{\rho v_2 \delta_l^3}{6 \delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_1}{\partial x_2^2} + C_7 \frac{\rho v_1 \delta_l^3}{12 \omega_6 \delta_t \omega_4 \omega_8} \frac{\partial^3 v_2}{\partial x_2^2} + \\
& C_8 \frac{v_1 \delta_l^4}{12 \delta_t \omega_5^3} \frac{\partial^4 \rho}{\partial x_1^4} + C_9 \frac{\rho \delta_l^4}{12 \delta_t \omega_5^3} \frac{\partial^4 v_1}{\partial x_1^4} + C_{10} \frac{v_2 \delta_l^4}{4 \delta_t \omega_7^2 \omega_4^2 \omega_5^3} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{11} \frac{\rho v_2 v_1 \delta_l^4}{4 \delta_t \omega_7^2 \omega_4^2 \omega_5^3} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + C_{12} \frac{\rho \delta_l^4}{12 \delta_t \omega_7^2 \omega_4^2 \omega_5^3} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& C_{13} \frac{v_1 \delta_l^4}{12 \omega_9 \delta_t \omega_7^2 \omega_4^2 \omega_5^3} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_{14} \frac{\rho \delta_l^4}{12 \omega_9 \delta_t \omega_7^2 \omega_4^2 \omega_5^3} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{15} \frac{\rho v_2 v_1 \delta_l^4}{2 \omega_9 \omega_6 \delta_t \omega_7^2 \omega_4^2 \omega_5^3} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + \\
& C_{16} \frac{v_2 \delta_l^4}{12 \omega_9 \omega_6^2 \delta_t \omega_7 \omega_4^2 \omega_5^3} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{17} \frac{\rho v_2 v_1 \delta_l^4}{12 \omega_9 \omega_6^2 \delta_t \omega_7 \omega_4^2 \omega_5^3} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{18} \frac{\rho \delta_l^4}{12 \omega_9 \omega_6^2 \delta_t \omega_7 \omega_4^2 \omega_5^3} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + C_{19} \frac{v_1 \delta_l^4}{24 \omega_6^2 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 \rho}{\partial x_2^4} \\
& + C_{20} \frac{\rho \delta_l^4}{24 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 v_1}{\partial x_2^4} + C_{21} \frac{\rho v_2 v_1 \delta_l^4}{12 \omega_6^2 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= -12 c_s^4 \omega_5 + 144 v_1^2 c_s^2 - 7 v_1^2 \omega_5^2 - 12 c_s^2 + 36 v_1^2 \omega_5 + c_s^4 \omega_5^2 + 36 v_1^4 + 12 c_s^4 + 12 c_s^2 \omega_5 - 36 v_1^2 - 144 v_1^2 c_s^2 \omega_5 + 7 v_1^4 \omega_5^2 - 36 v_1^4 \omega_5 + 24 v_1^2 c_s^2 \omega_5^2 - c_s^2 \omega_5^2 \\
C_2 &= -v_1^2 \omega_5^2 + 3 c_s^2 \omega_7 \omega_4 - 3 c_s^2 \omega_4 \omega_5 + 3 c_s^2 \omega_7 \omega_5 + 3 c_s^2 \omega_4 \omega_5^2 + \omega_5^2 - \omega_7 \omega_4 - v_1^2 \omega_4 \omega_5 + v_1^2 \omega_7 \omega_4 + \omega_4 \omega_5 - \omega_4 \omega_5^2 - \omega_7 \omega_5 - 3 c_s^2 \omega_5^2 + v_1^2 \omega_7 \omega_5 - \\
& v_1^2 \omega_7 \omega_4 \omega_5 + v_1^2 \omega_4 \omega_5^2 + \omega_7 \omega_4 \omega_5 - 3 c_s^2 \omega_7 \omega_4 \omega_5 \\
C_3 &= -3 v_1^2 \omega_5^2 + c_s^2 \omega_7 \omega_4 - c_s^2 \omega_4 \omega_5 + c_s^2 \omega_7 \omega_5 + c_s^2 \omega_4 \omega_5^2 + \omega_5^2 - \omega_7 \omega_4 - 3 v_1^2 \omega_4 \omega_5 + 3 v_1^2 \omega_7 \omega_4 + \omega_4 \omega_5 - \omega_4 \omega_5^2 - \omega_7 \omega_5 - c_s^2 \omega_5^2 + 3 v_1^2 \omega_7 \omega_5 - \\
& 3 v_1^2 \omega_7 \omega_4 \omega_5 + 3 v_1^2 \omega_4 \omega_5^2 + \omega_7 \omega_4 \omega_5 - c_s^2 \omega_7 \omega_4 \omega_5 \\
C_4 &= 6 \omega_7 \omega_4^2 \omega_5 - 18 c_s^2 \omega_7 \omega_4^2 \omega_5 - 12 \omega_4^2 \omega_5^2 - 24 c_s^2 \omega_7 \omega_5^2 - 6 v_1^2 \omega_7 \omega_4^2 \omega_5 + 12 v_1^2 \omega_4^2 \omega_5^2 - 3 v_1^2 \omega_7 \omega_4^2 \omega_5^2 - 12 v_1^2 \omega_4^2 \omega_5 - 12 c_s^2 \omega_4 \omega_5^2 + 36 c_s^2 \omega_7 \omega_4^2 + \\
& 12 \omega_4^2 \omega_5 - 11 c_s^2 \omega_7 \omega_4^2 \omega_5^2 + 3 \omega_7 \omega_4^2 \omega_5^2 + 42 c_s^2 \omega_7 \omega_4 \omega_5^2 - 6 \omega_7 \omega_4 \omega_5^2 + 12 c_s^2 \omega_4^2 \omega_5^2 + 6 v_1^2 \omega_7 \omega_4 \omega_5^2 + 12 v_1^2 \omega_7 \omega_4^2 + 12 \omega_4 \omega_5^2 - 12 c_s^2 \omega_4^2 \omega_5 - 12 \omega_7 \omega_4^2 - \\
& 12 v_1^2 \omega_4 \omega_5^2 - 24 c_s^2 \omega_7 \omega_4 \omega_5 \\
C_5 &= 18 \omega_6 c_s^2 \omega_4 + 12 \omega_6 + 6 \omega_4 \omega_8 + 36 c_s^2 \omega_8 - 36 \omega_6 c_s^2 + 3 \omega_6 c_s^2 \omega_4 \omega_8 + \omega_6 v_2^2 \omega_4 \omega_8 - 6 \omega_6 \omega_4 + 6 \omega_6 v_2^2 \omega_4 - \omega_6 \omega_4 \omega_8 - 12 \omega_6 v_2^2 - 12 \omega_8 - 6 v_2^2 \omega_4 \omega_8 - \\
& 18 c_s^2 \omega_4 \omega_8 + 12 v_2^2 \omega_8 \\
C_6 &= -3 c_s^2 \omega_4^2 \omega_8 - v_2^2 \omega_4^2 \omega_8 + 3 v_2^2 \omega_4^2 - 3 \omega_4^2 - 6 v_2^2 \omega_4 - 3 \omega_4 \omega_8 - 12 c_s^2 \omega_8 + 6 \omega_4 + \omega_4^2 \omega_8 + 3 c_s^2 \omega_4^2 - 6 c_s^2 \omega_4 + 3 v_2^2 \omega_4 \omega_8 + 15 c_s^2 \omega_4 \omega_8 \\
C_7 &= 6 \omega_6 c_s^2 \omega_4 + 12 \omega_6 + 6 \omega_4 \omega_8 + 12 c_s^2 \omega_8 - 12 \omega_6 c_s^2 + \omega_6 c_s^2 \omega_4 \omega_8 + 3 \omega_6 v_2^2 \omega_4 \omega_8 - 6 \omega_6 \omega_4 + 18 \omega_6 v_2^2 \omega_4 - \omega_6 \omega_4 \omega_8 - 36 \omega_6 v_2^2 - 12 \omega_8 - \\
& 18 v_2^2 \omega_4 \omega_8 - 6 c_s^2 \omega_4 \omega_8 + 36 v_2^2 \omega_8
\end{aligned}$$

$$\begin{aligned}
& (-12 - \omega_4^2 + 12\omega_4) \frac{c_s^4 \delta_t^3}{6\delta_t \omega_4^2} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + (-\omega_4^2 - 2\omega_7 + 2\omega_4 + \omega_7 \omega_4) \frac{\rho v_1 c_s^2 \delta_t^3}{\delta_t \omega_7 \omega_4} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} + \\
& (-12\omega_6 \omega_4^2 + 12\omega_4^2 - 12\omega_4 \omega_8 + 12\omega_6 \omega_4 - \omega_6 \omega_4^2 \omega_8 + 12\omega_6 \omega_4 \omega_8 - 12\omega_6 \omega_8) \frac{\rho v_2 c_s^2 \delta_t^3}{6\omega_6 \delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + C_4 \frac{v_2 v_1 \delta_t^3}{\omega_6^2 \delta_t \omega_4 \omega_8} \frac{\partial^3 \rho}{\partial x_1 \partial x_2} + \\
& C_5 \frac{\rho v_2 \delta_t^3}{12\omega_6^2 \delta_t \omega_4^2 \omega_8} \frac{\partial^3 v_1}{\partial x_1 \partial x_2} + C_6 \frac{\rho v_1 \delta_t^3}{\omega_6^2 \delta_t \omega_4 \omega_8} \frac{\partial^3 v_2}{\partial x_1 \partial x_2} + C_7 \frac{\delta_t^3}{12\omega_6^2 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\
& (-24 + 24\omega_6 + 36c_s^2 + 11\omega_6^2 v_2^2 - 36\omega_6 c_s^2 + 60v_2^2 + 5\omega_6^2 c_s^2 - 60\omega_6 v_2^2 - 4\omega_6^2) \frac{\rho v_2 \delta_t^3}{6\omega_6^2 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_8 \frac{v_2 \delta_t^4}{24\delta_t \omega_7^2 \omega_4^2 \omega_5^2} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& C_9 \frac{\rho v_2 v_1 \delta_t^4}{12\delta_t \omega_7^2 \omega_4^2 \omega_5^2} \frac{\partial^4 v_1}{\partial x_1^4} + C_{10} \frac{\rho \delta_t^4}{24\delta_t \omega_7^2 \omega_4^2} \frac{\partial^4 v_2}{\partial x_1^4} + C_{11} \frac{v_1 \delta_t^4}{12\omega_9 \omega_6 \delta_t \omega_7^2 \omega_4^2 \omega_8^2} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2} + C_{12} \frac{\rho \delta_t^4}{12\omega_9 \omega_6 \delta_t \omega_7^2 \omega_4^2 \omega_8^2} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2} + \\
& C_{13} \frac{\rho v_2 v_1 \delta_t^4}{12\omega_9 \omega_6^2 \delta_t \omega_7^2 \omega_4^2 \omega_8^2} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2} + C_{14} \frac{v_2 \delta_t^4}{12\omega_9 \omega_6^2 \delta_t \omega_7^2 \omega_4^2 \omega_8^2} \frac{\partial^4 \rho}{\partial x_2^2 \partial x_2} + C_{15} \frac{\rho v_2 v_1 \delta_t^4}{2\omega_9 \omega_6^2 \delta_t \omega_7^2 \omega_4^2 \omega_8^2} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2} + \\
& C_{16} \frac{\rho \delta_t^4}{12\omega_9 \omega_6^2 \delta_t \omega_7^2 \omega_4^2 \omega_8^2} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2} + C_{17} \frac{v_1 \delta_t^4}{4\omega_3^2 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^2} + C_{18} \frac{\rho \delta_t^4}{12\omega_6^2 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^2} + C_{19} \frac{\rho v_2 v_1 \delta_t^4}{4\omega_6^2 \delta_t \omega_4^2 \omega_8^2} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^2} + \\
& C_{20} \frac{v_2 \delta_t^4}{12\omega_6^2 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{21} \frac{\rho \delta_t^4}{12\omega_6^2 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$C_1 = -18c_s^2 \omega_7 \omega_4 + 12v_1^2 \omega_7 + 18c_s^2 \omega_4 \omega_5 - 12\omega_7 - 12v_1^2 \omega_5 - 36c_s^2 \omega_5 + 6\omega_7 \omega_4 + 6v_1^2 \omega_4 \omega_5 - 6v_1^2 \omega_7 \omega_4 - 6\omega_4 \omega_5 + 12\omega_5 + v_1^2 \omega_7 \omega_4 \omega_5 - \omega_7 \omega_4 \omega_5 + 3c_s^2 \omega_7 \omega_4 \omega_5 + 36c_s^2 \omega_7$$

$$C_2 = -6c_s^2 \omega_7 \omega_4 + 36v_1^2 \omega_7 + 6c_s^2 \omega_4 \omega_5 - 12\omega_7 - 36v_1^2 \omega_5 - 12c_s^2 \omega_5 + 6\omega_7 \omega_4 + 18v_1^2 \omega_4 \omega_5 - 18v_1^2 \omega_7 \omega_4 - 6\omega_4 \omega_5 + 12\omega_5 + 3v_1^2 \omega_7 \omega_4 \omega_5 - \omega_7 \omega_4 \omega_5 + c_s^2 \omega_7 \omega_4 \omega_5 + 12c_s^2 \omega_7$$

$$C_3 = -6v_1^2 \omega_4 + 15c_s^2 \omega_7 \omega_4 - 3\omega_4^2 - 3c_s^2 \omega_7 \omega_4^2 + 6\omega_4 + 3v_2^2 \omega_4^2 - 3\omega_7 \omega_4 + 3v_1^2 \omega_7 \omega_4 + 3c_s^2 \omega_4^2 - 6c_s^2 \omega_4 - v_1^2 \omega_7 \omega_4^2 + \omega_7 \omega_4^2 - 12c_s^2 \omega_7$$

$$C_4 = \omega_6 v_2^2 \omega_8 - 3\omega_6 c_s^2 \omega_4 - \omega_6^2 v_2^2 - \omega_4 \omega_8 + \omega_6^2 v_2^2 \omega_4 - 3\omega_6 c_s^2 \omega_4 \omega_8 - \omega_6 v_2^2 \omega_4 \omega_8 + \omega_6 \omega_4 - 3\omega_6^2 c_s^2 - \omega_6^2 \omega_4 + 3\omega_6 c_s^2 \omega_8 - \omega_6 v_2^2 \omega_4 + \omega_6 \omega_4 \omega_8 + v_2^2 \omega_4 \omega_8 + 3c_s^2 \omega_4 \omega_8 + 3\omega_6^2 c_s^2 \omega_4 + \omega_6^2 - \omega_6 \omega_8$$

$$C_5 = -3\omega_6^2 v_2^2 \omega_4^2 \omega_8 - 11\omega_6^2 c_s^2 \omega_4^2 \omega_8 + 12\omega_6 \omega_4^2 + 36c_s^2 \omega_4^2 \omega_8 + 12v_2^2 \omega_4^2 \omega_8 + 12\omega_6^2 v_2^2 \omega_4^2 - 24\omega_6^2 c_s^2 \omega_8 - 6\omega_6^2 \omega_4 \omega_8 - 12\omega_6^2 v_2^2 \omega_4 - 12\omega_6 c_s^2 \omega_4^2 - 24\omega_6 c_s^2 \omega_4 \omega_8 + 6\omega_6 \omega_4^2 \omega_8 + 12\omega_6^2 \omega_4 - 6\omega_6 v_2^2 \omega_4^2 \omega_8 - 18\omega_6 c_s^2 \omega_4^2 \omega_8 + 12\omega_6^2 c_s^2 \omega_4^2 - 12\omega_4^2 \omega_8 + 3\omega_6^2 \omega_4^2 \omega_8 - 12\omega_6^2 c_s^2 \omega_4 - 12\omega_6 v_2^2 \omega_4^2 + 42\omega_6^2 c_s^2 \omega_4 \omega_8 + 6\omega_6^2 v_2^2 \omega_4 \omega_8 - 12\omega_6^2 \omega_4^2$$

$$C_6 = 3\omega_6 v_2^2 \omega_8 - \omega_6 c_s^2 \omega_4 - 3\omega_6^2 v_2^2 - \omega_4 \omega_8 + 3\omega_6^2 v_2^2 \omega_4 - \omega_6 c_s^2 \omega_4 \omega_8 - 3\omega_6 v_2^2 \omega_4 \omega_8 + \omega_6 \omega_4 - \omega_6^2 c_s^2 - \omega_6^2 \omega_4 + \omega_6 c_s^2 \omega_8 - 3\omega_6 v_2^2 \omega_4 + \omega_6 \omega_4 \omega_8 + 3v_2^2 \omega_4 \omega_8 + c_s^2 \omega_4 \omega_8 + \omega_6^2 c_s^2 \omega_4 + \omega_6^2 - \omega_6 \omega_8$$

$$C_7 = -12c_s^2 - 7\omega_6^2 v_2^2 + 144v_2^2 c_s^2 + 36v_2^4 + 24\omega_6^2 v_2^2 c_s^2 + \omega_6^2 c_s^4 + 12\omega_6 c_s^2 - 36\omega_6 v_2^4 - 36v_2^2 - \omega_6^2 c_s^2 + 12c_s^4 + 7\omega_6^2 v_2^4 - 144\omega_6 v_2^2 c_s^2 + 36\omega_6 v_2^2 - 12\omega_6 c_s^4$$

$$\begin{aligned}
C_8 = & -126v_1^2 c_s^2 \omega_7 \omega_4^2 \omega_5^2 - 216v_1^2 c_s^2 \omega_7 \omega_5^2 + 48c_s^4 \omega_7 \omega_4 \omega_5^2 - 24c_s^4 \omega_4 \omega_5^2 - 3v_1^4 \omega_7^2 \omega_4^2 \omega_5^2 + 96v_1^2 \omega_7^2 \omega_4 \omega_5 + 72v_1^4 \omega_7^2 \omega_4 + 96v_1^4 \omega_7 \omega_4 \omega_5^2 + 36v_1^2 \omega_7^2 \omega_4^2 - \\
& c_s^4 \omega_7^2 \omega_4^2 \omega_5^2 + 24c_s^2 \omega_7 \omega_5^2 + 48c_s^2 \omega_7^2 \omega_4 \omega_5 - 48v_1^2 \omega_7^2 \omega_5 - 24v_1^2 \omega_7 \omega_4^2 \omega_5 - 12v_1^2 \omega_7^2 \omega_5^2 + 30v_1^2 \omega_7 \omega_4^2 \omega_5^2 + 24c_s^2 \omega_4 \omega_5^2 - 72v_1^2 \omega_7^2 \omega_4 - 48v_1^4 \omega_7 \omega_4 \omega_5 + \\
& 14c_s^4 \omega_7^2 \omega_4^2 \omega_5 - 432v_1^2 c_s^2 \omega_7^2 \omega_4 \omega_5 - 36v_1^4 \omega_7^2 \omega_4^2 + 12c_s^2 \omega_7 \omega_4^2 \omega_5^2 + 48v_1^4 \omega_7^2 \omega_5 - 24c_s^4 \omega_7 \omega_5^2 + 72v_1^2 c_s^2 \omega_7 \omega_4^2 \omega_5 - 144v_1^2 c_s^2 \omega_4 \omega_5^2 + 36v_1^4 \omega_7^2 \omega_4^2 \omega_5 + \\
& 12v_1^4 \omega_4^2 \omega_5^2 + 24c_s^4 \omega_7^2 \omega_4 - 144v_1^2 c_s^2 \omega_7^2 \omega_4^2 + 3v_1^2 \omega_7^2 \omega_4^2 \omega_5^2 - 48c_s^2 \omega_7 \omega_4^2 \omega_5^2 - 144v_1^2 c_s^2 \omega_7 \omega_4 \omega_5 + 216v_1^2 c_s^2 \omega_7^2 \omega_5 - 24v_1^4 \omega_4 \omega_5^2 - 96v_1^4 \omega_7^2 \omega_4 \omega_5 + \\
& 72v_1^2 c_s^2 \omega_4^2 \omega_5^2 + 48v_1^2 \omega_7 \omega_5^2 - 24c_s^2 \omega_7^2 \omega_5 + c_s^2 \omega_7^2 \omega_4^2 \omega_5^2 - 12c_s^2 \omega_7^2 \omega_5^2 - 96v_1^2 \omega_7 \omega_4^2 \omega_5 + 24v_1^4 \omega_7 \omega_4^2 \omega_5 + 12c_s^2 \omega_7^2 \omega_4^2 - 48c_s^4 \omega_7^2 \omega_4 \omega_5 + \\
& 150v_1^2 c_s^2 \omega_7^2 \omega_5 - 30v_1^4 \omega_7 \omega_4^2 \omega_5^2 - 24c_s^2 \omega_7^2 \omega_4 - 12v_1^2 c_s^2 \omega_7^2 \omega_4^2 \omega_5^2 - 14c_s^2 \omega_7^2 \omega_4^2 \omega_5 + 48v_1^2 \omega_7 \omega_4 \omega_5 + 24v_1^4 \omega_4 \omega_5^2 + 432v_1^2 c_s^2 \omega_7 \omega_4 \omega_5^2 + 24c_s^4 \omega_7^2 \omega_5 - \\
& 12c_s^4 \omega_7 \omega_4^2 \omega_5^2 - 48v_1^4 \omega_7 \omega_5^2 + 12c_s^4 \omega_4^2 \omega_5^2 - 12c_s^4 \omega_7^2 \omega_4^2 + 288v_1^2 c_s^2 \omega_7^2 \omega_4 - 36v_1^2 \omega_7^2 \omega_5^2
\end{aligned}$$

$$\begin{aligned}
C_9 = & -12\omega_7 \omega_4^2 \omega_5 - 168v_1^2 \omega_7^2 \omega_4 \omega_5 + 12c_s^2 \omega_7 \omega_4^2 \omega_5 - 12\omega_4^2 \omega_5^2 - 60v_1^2 \omega_7^2 \omega_4^2 + 72\omega_7^2 \omega_4 \omega_5 - 36\omega_7^2 \omega_5 - 60c_s^2 \omega_7 \omega_5^2 - 120c_s^2 \omega_7^2 \omega_4 \omega_5 + 84v_1^2 \omega_7^2 \omega_5 + \\
& 36v_1^2 \omega_7 \omega_4^2 \omega_5 + 24v_1^2 \omega_4^2 \omega_5^2 + 24\omega_7^2 \omega_4^2 - 51v_1^2 \omega_7 \omega_4^2 \omega_5^2 - 48\omega_7^2 \omega_4 - 48c_s^2 \omega_4 \omega_5^2 + 120v_1^2 \omega_7 \omega_4 - 33c_s^2 \omega_7 \omega_4^2 \omega_5^2 + 21\omega_7 \omega_4^2 \omega_5^2 - 5v_1^2 \omega_7^2 \omega_4^2 \omega_5^2 + \\
& 120c_s^2 \omega_7 \omega_4 \omega_5^2 - 72\omega_7 \omega_4^2 \omega_5^2 - 84v_1^2 \omega_7 \omega_5^2 + 60c_s^2 \omega_7^2 \omega_5 - 3c_s^2 \omega_7^2 \omega_4^2 \omega_5^2 + 24c_s^2 \omega_7^2 \omega_5^2 + 168v_1^2 \omega_7 \omega_4 \omega_5^2 + 2\omega_7^2 \omega_4^2 \omega_5^2 - 36c_s^2 \omega_7^2 \omega_4^2 + 36\omega_7 \omega_5^2 - \\
& 25\omega_7^2 \omega_4^2 \omega_5 + 24\omega_4 \omega_5^2 + 72c_s^2 \omega_7^2 \omega_4 + 39c_s^2 \omega_7^2 \omega_4^2 \omega_5 - 72v_1^2 \omega_7 \omega_4 \omega_5 - 48v_1^2 \omega_4 \omega_5^2 + 24\omega_7 \omega_4 \omega_5 + 61v_1^2 \omega_7^2 \omega_4^2 \omega_5 - 24c_s^2 \omega_7 \omega_4 \omega_5
\end{aligned}$$

$$\begin{aligned}
C_{10} = & -3v_1^2 \omega_7^2 \omega_4^3 + 24v_1^4 \omega_7^2 \omega_4 - 24c_s^4 \omega_7 \omega_4^2 - 24v_1^2 c_s^2 \omega_7 \omega_4 - 24c_s^2 \omega_7 \omega_4 + 24v_1^2 \omega_7^2 \omega_4^2 + 24c_s^4 \omega_7^2 + 6c_s^4 \omega_7 \omega_4^3 - 12v_1^2 c_s^2 \omega_7 \omega_4^3 + 3v_1^4 \omega_7^2 \omega_4^3 + \\
& 24c_s^2 \omega_7 \omega_4^2 - 24v_1^2 \omega_7^2 \omega_4 - 12v_1^2 \omega_4^3 - 24v_1^4 \omega_7^2 \omega_4^2 + 24c_s^4 \omega_7 \omega_4 + 48v_1^2 c_s^2 \omega_7 \omega_4^2 + 24v_1^2 \omega_4^2 - 6c_s^2 \omega_7 \omega_4^3 - 48c_s^4 \omega_7^2 \omega_4 - 72v_1^2 c_s^2 \omega_7^2 \omega_4^2 + 12v_1^2 c_s^2 \omega_4^3 + \\
& 72v_1^4 \omega_7 \omega_4^2 - 96v_1^2 c_s^2 \omega_7^2 + c_s^2 \omega_7^2 \omega_4^3 - 24v_1^2 c_s^2 \omega_4^2 - 18v_1^4 \omega_7 \omega_4^3 + 6v_1^2 c_s^2 \omega_7^2 \omega_4^3 + 48v_1^2 \omega_7 \omega_4 - 8c_s^2 \omega_7^2 \omega_4^2 - 72v_1^2 \omega_7 \omega_4^2 + 12c_s^2 \omega_7^2 \omega_4 - 24v_1^4 \omega_4^2 - \\
& 3c_s^4 \omega_7^2 \omega_4^3 + 18v_1^4 \omega_7 \omega_4^3 + 24c_s^4 \omega_7^2 \omega_4^2 + 156v_1^2 c_s^2 \omega_7^2 \omega_4 + 12v_1^4 \omega_4^3 - 48v_1^4 \omega_7 \omega_4
\end{aligned}$$

$$\begin{aligned}
C_{11} = & 6\omega_6 c_s^2 \omega_7 \omega_4^3 \omega_8 \omega_5^2 - 24\omega_9 \omega_6 v_2^2 \omega_7^2 \omega_4 \omega_8 \omega_5 + 72\omega_9 v_2^2 c_s^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 + 6\omega_9 v_1^2 c_s^2 \omega_7^2 \omega_4^3 \omega_8 \omega_5 + 18\omega_9 c_s^4 \omega_7^2 \omega_4^3 \omega_8 \omega_5 + 3\omega_9 \omega_6 c_s^4 \omega_7^2 \omega_4^3 \omega_8 \omega_5^2 - \\
& 6v_2^2 v_1^2 \omega_7^2 \omega_4^3 \omega_8 \omega_5^2 + 12\omega_6 v_2^2 v_1^2 \omega_7^2 \omega_4^2 \omega_5^2 + 144\omega_9 \omega_6 v_2^2 c_s^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5^2 - 3\omega_9 \omega_6 v_1^2 c_s^2 \omega_7 \omega_4^3 \omega_8 \omega_5^2 - 36\omega_9 \omega_6 c_s^4 \omega_7^2 \omega_4 \omega_8 \omega_5 + 12\omega_9 \omega_6 v_2^2 v_1^2 \omega_7^2 \omega_4^2 \omega_8 + \\
& 6\omega_9 v_2^2 \omega_7^2 \omega_4^3 \omega_8 \omega_5 - \omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4^3 \omega_8 \omega_5^2 - 6\omega_9 c_s^2 \omega_7^2 \omega_4^3 \omega_8 \omega_5 - 18\omega_6 c_s^4 \omega_7 \omega_4^3 \omega_8 \omega_5^2 + 9\omega_9 \omega_6 v_2^2 v_1^2 \omega_7^2 \omega_4^3 \omega_8 \omega_5 + 12\omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4 \omega_8 \omega_5 - \\
& 6\omega_6 v_1^2 c_s^2 \omega_7 \omega_4^3 \omega_8 \omega_5^2 + 12\omega_9 \omega_6 c_s^4 \omega_4^2 \omega_8 \omega_5^2 + 36\omega_6 v_2^2 c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_5^2 + 6\omega_6 v_2^2 \omega_7 \omega_4^3 \omega_8 \omega_5^2 + 36\omega_9 \omega_6 c_s^4 \omega_7^2 \omega_4^3 \omega_5 + 6\omega_9 \omega_6 v_2^2 v_1^2 \omega_4^3 \omega_8 \omega_5^2 + \\
& 12\omega_9 v_2^2 v_1^2 \omega_7 \omega_4^3 \omega_8 \omega_5^2 - 12\omega_9 \omega_6 v_1^2 c_s^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5^2 - 48\omega_9 \omega_6 v_2^2 \omega_7 \omega_4^2 \omega_8 \omega_5^2 - 6\omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4^3 \omega_8 \omega_5^2 + 6c_s^2 \omega_7^2 \omega_4^3 \omega_8 \omega_5^2 - 36\omega_9 c_s^4 \omega_7^2 \omega_4^3 \omega_5 - \\
& 42\omega_9 \omega_6 c_s^4 \omega_7 \omega_4^2 \omega_8 \omega_5^2 - 36\omega_6 c_s^2 \omega_7^2 \omega_4^3 \omega_5^2 + 72\omega_9 \omega_6 v_2^2 c_s^2 \omega_7^2 \omega_4 \omega_8 \omega_5 - 6\omega_9 \omega_6 v_2^2 \omega_7^2 \omega_4^3 \omega_8 \omega_5^2 + 12\omega_6 c_s^2 \omega_7^2 \omega_4^3 \omega_8 \omega_5^2 + 36\omega_9 \omega_6 v_2^2 c_s^2 \omega_7^2 \omega_4^2 \omega_8 - \\
& 12\omega_6 v_1^2 c_s^2 \omega_7^2 \omega_4^3 \omega_5^2 + 36\omega_6 v_2^2 c_s^2 \omega_7^2 \omega_4^2 \omega_5^2 + 12\omega_6 v_2^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5^2 - 18c_s^4 \omega_7^2 \omega_4^3 \omega_8 \omega_5^2 + 18\omega_6 v_2^2 c_s^2 \omega_7^2 \omega_4^3 \omega_8 \omega_5^2 - 12\omega_6 v_1^2 c_s^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5^2 - \\
& 12\omega_9 \omega_6 v_2^2 v_1^2 \omega_7 \omega_4^2 \omega_8 \omega_5 + 6v_2^2 \omega_7^2 \omega_4^3 \omega_8 \omega_5^2 - 12\omega_9 \omega_6 v_2^2 \omega_7^2 \omega_4^2 \omega_8 - 36\omega_6 c_s^4 \omega_7^2 \omega_4^3 \omega_8 \omega_5^2 + 18\omega_9 \omega_6 v_2^2 c_s^2 \omega_7^2 \omega_4^3 \omega_8 \omega_5^2 - 18\omega_9 \omega_6 c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_5^2 + \\
& 24\omega_9 v_2^2 v_1^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 - 36\omega_9 \omega_6 c_s^4 \omega_7^2 \omega_4^2 \omega_5 - 18v_2^2 c_s^2 \omega_7^2 \omega_4^3 \omega_8 \omega_5^2 + 48\omega_9 \omega_6 v_2^2 v_1^2 \omega_7 \omega_4^2 \omega_8 \omega_5^2 + 27\omega_9 \omega_6 v_2^2 c_s^2 \omega_7^2 \omega_4^3 \omega_8 \omega_5 + 18\omega_9 \omega_6 v_1^2 c_s^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 + \\
& 12\omega_9 \omega_6 v_2^2 \omega_7 \omega_4^2 \omega_8 \omega_5 + 24\omega_9 v_2^2 \omega_7 \omega_4^2 \omega_8 \omega_5^2 - 6\omega_9 \omega_6 v_2^2 v_1^2 \omega_7^2 \omega_4^3 \omega_8 - 12\omega_6 v_2^2 v_1^2 \omega_7^2 \omega_4^3 \omega_5^2 + 36\omega_9 v_2^2 c_s^2 \omega_7 \omega_4^3 \omega_8 \omega_5^2 + 12\omega_6 v_2^2 v_1^2 \omega_7 \omega_4^2 \omega_8 \omega_5^2 +
\end{aligned}$$

$$\begin{aligned}
C_{18} = & 12\omega_6^2 v_2^3 \omega_4^3 - 12c_s^2 \omega_4^3 \omega_8^2 - 5\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 6\omega_6^2 c_s^4 \omega_4^3 \omega_8 - 72v_2^2 \omega_4^3 \omega_8^2 + 18\omega_6^3 v_2^2 \omega_4^2 \omega_8^2 + 60\omega_6^2 v_2^4 \omega_4^3 \omega_8 + 54\omega_6^2 v_2^2 c_s^2 \omega_4^3 \omega_8 + 252v_2^2 c_s^2 \omega_4^3 \omega_8^2 + \\
& 4\omega_6^3 v_2^4 \omega_4^3 \omega_8^2 + 24\omega_6^2 v_2^2 \omega_4^2 \omega_8 + 12\omega_6^3 v_2^4 \omega_4^3 - 108\omega_6 v_2^2 c_s^2 \omega_4^2 \omega_8^2 - \omega_6^3 c_s^4 \omega_4^3 \omega_8^2 - 12\omega_6^2 c_s^2 \omega_4^2 \omega_8 + 102\omega_6^3 v_2^2 c_s^2 \omega_4 \omega_8^2 + 6\omega_6^2 c_s^4 \omega_4^2 \omega_8^2 - 6\omega_6^3 c_s^2 \omega_4^3 \omega_8 + \\
& 12\omega_6^3 v_2^4 \omega_4^3 \omega_8^2 + 27\omega_6^3 v_2^2 \omega_4^3 \omega_8 + 12\omega_6^3 c_s^4 \omega_8^2 - 12\omega_6^2 v_2^2 c_s^2 \omega_4^3 + 162\omega_6^2 v_2^2 c_s^2 \omega_4^2 \omega_8^2 - 19\omega_6^2 v_2^2 \omega_4^3 \omega_8^2 + 48\omega_6^3 v_2^4 \omega_4^3 \omega_8 - 36\omega_6 v_2^2 c_s^2 \omega_4^3 \omega_8 - \omega_6^2 c_s^2 \omega_4^3 \omega_8^2 - \\
& 18\omega_6^3 c_s^4 \omega_4^2 \omega_8 - 12\omega_6^3 v_2^4 \omega_4^2 + 12\omega_6^3 v_2^2 c_s^2 \omega_4^3 + 72v_2^4 \omega_4^3 \omega_8^2 - 18\omega_6^3 v_2^4 \omega_4^2 \omega_8^2 - 60\omega_6^2 v_2^2 \omega_4^3 \omega_8 + 12c_s^4 \omega_4^3 \omega_8^2 + 13\omega_6^3 c_s^4 \omega_4^2 \omega_8^2 + 6\omega_6^2 c_s^2 \omega_4^3 \omega_8 - \\
& 306\omega_6 v_2^2 c_s^2 \omega_4^3 \omega_8^2 + 12\omega_6^2 c_s^4 \omega_4^2 \omega_8 - 12\omega_6^2 v_2^2 c_s^2 \omega_4^2 \omega_8 - 4\omega_6^3 v_2^2 \omega_4^3 \omega_8^2 - 24\omega_6^2 v_2^2 \omega_4^2 \omega_8 - 12\omega_6^2 v_2^2 \omega_4^3 \omega_8^2 - 27\omega_6^3 v_2^4 \omega_4^3 \omega_8 - 12\omega_6^3 v_2^2 c_s^2 \omega_4^2 \omega_8^2 - 6\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 + \\
& 6\omega_6^3 c_s^4 \omega_4^3 \omega_8 - 12\omega_6^3 v_2^2 c_s^2 \omega_4 \omega_8 + \omega_6^2 c_s^4 \omega_4^3 \omega_8^2 + 18\omega_6^3 c_s^2 \omega_4^2 \omega_8 + 60\omega_6^2 v_2^2 c_s^2 \omega_4^3 \omega_8^2 + 19\omega_6^2 v_2^4 \omega_4^3 \omega_8^2 - 48\omega_6^3 v_2^2 \omega_4^2 \omega_8 + 30\omega_6^3 v_2^2 c_s^2 \omega_4^2 \omega_8 - 36\omega_6 v_2^2 \omega_4^3 \omega_8 + \\
& 24\omega_6^3 v_2^2 \omega_4 \omega_8 + 12\omega_6^3 v_2^2 \omega_4^2 - 12\omega_6^3 c_s^2 \omega_4^2 \omega_8 + 90\omega_6 v_2^2 \omega_4^3 \omega_8^2 + 12\omega_6^2 c_s^2 \omega_4^3 \omega_8^2 - 12\omega_6^2 v_2^2 \omega_4^3 + 12\omega_6^3 v_2^2 c_s^2 \omega_4^2 \omega_8^2 - 24\omega_6^3 c_s^4 \omega_4^2 \omega_8^2 + 12\omega_6^3 v_2^4 \omega_4^2 \omega_8^2 - \\
& 12\omega_6^3 v_2^2 \omega_4^3 - 12\omega_6^2 c_s^4 \omega_4^2 \omega_8^2 - 48\omega_6^2 v_2^2 c_s^2 \omega_4^2 \omega_8^2 + 36\omega_6 v_2^2 \omega_4^3 \omega_8 - 21\omega_6^3 v_2^2 c_s^2 \omega_4^3 \omega_8 + 12\omega_6^3 c_s^4 \omega_4 \omega_8 - 24\omega_6^3 v_2^4 \omega_4 \omega_8 - 12\omega_6^2 c_s^4 \omega_4^3 \omega_8^2 - 90\omega_6 v_2^4 \omega_4^3 \omega_8^2 - \\
& 12\omega_6^3 v_2^2 \omega_4 \omega_8^2 + 6\omega_6^3 c_s^2 \omega_4^2 \omega_8^2 - 81\omega_6^3 v_2^2 c_s^2 \omega_4^2 \omega_8^2 - 48\omega_6^3 v_2^2 c_s^2 \omega_8^2 + 12\omega_6^2 c_s^2 \omega_4^2 \omega_8^2
\end{aligned}$$

$$\begin{aligned}
C_{19} = & 17\omega_6^3 \omega_4^2 \omega_8 + 48\omega_6 \omega_4^2 \omega_8^2 + 104\omega_6^2 v_2^2 \omega_4^2 \omega_8 + 56\omega_6^2 c_s^2 \omega_4^2 \omega_8 + 32\omega_6 c_s^2 \omega_4^2 \omega_8^2 + 64\omega_6 v_2^2 \omega_4 \omega_8^2 + 16\omega_6^3 c_s^2 \omega_4^2 + 28\omega_6^2 \omega_4 \omega_8^2 - 16\omega_6^2 v_2^2 \omega_4^2 + \\
& 20\omega_6^2 c_s^2 \omega_8^2 + 12\omega_6^3 \omega_8 + 48c_s^2 \omega_4^2 \omega_8^2 + 16\omega_6^2 \omega_4 \omega_8 + 80v_2^2 \omega_4^2 \omega_8^2 + 43\omega_6^2 v_2^2 \omega_4^2 \omega_8^2 - 12\omega_6^2 \omega_8^2 + 25\omega_6^2 c_s^2 \omega_4^2 \omega_8^2 - 16\omega_6^3 c_s^2 \omega_4 - 28\omega_6^3 v_2^2 \omega_8 - \\
& 25\omega_6^3 c_s^2 \omega_4 \omega_8 + 24\omega_6 \omega_4^2 \omega_8 - 43\omega_6^3 v_2^2 \omega_4^2 \omega_8 - 44\omega_6^2 c_s^2 \omega_4 \omega_8^2 - 68\omega_6^2 v_2^2 \omega_4 \omega_8^2 - 64\omega_6 v_2^2 \omega_4^2 \omega_8 - 32\omega_6^2 c_s^2 \omega_4^2 \omega_8 + 68\omega_6^3 v_2^2 \omega_4 \omega_8 + 16\omega_6^3 v_2^2 \omega_4^2 + \\
& 44\omega_6^3 c_s^2 \omega_4 \omega_8 - 8\omega_6^3 \omega_4^2 - 16\omega_6^2 c_s^2 \omega_4^2 - 40\omega_6^2 \omega_4^2 \omega_8 + 28\omega_6^2 v_2^2 \omega_8^2 - 17\omega_6^2 \omega_4^2 \omega_8^2 + 8\omega_6^3 \omega_4 - 32\omega_4^2 \omega_8^2 - 28\omega_6^3 \omega_4 \omega_8 - 24\omega_6 \omega_4 \omega_8^2 - 16\omega_6^3 v_2^2 \omega_4 - \\
& 16\omega_6^2 c_s^2 \omega_4 \omega_8 - 48\omega_6^2 v_2^2 \omega_4 \omega_8 - 20\omega_6^3 c_s^2 \omega_8 - 120\omega_6 v_2^2 \omega_4^2 \omega_8 + 8\omega_6^2 \omega_4^2 - 72\omega_6 c_s^2 \omega_4^2 \omega_8^2
\end{aligned}$$

$$\begin{aligned}
C_{20} = & 12 - 5\omega_6^3 c_s^4 - 18\omega_6 + 10\omega_6^3 v_2^2 - 132c_s^2 - 98\omega_6^2 v_2^2 + 672v_2^2 c_s^2 + 144v_2^4 + 404\omega_6^2 v_2^2 c_s^2 + 82\omega_6^2 c_s^4 + 198\omega_6 c_s^2 - 216\omega_6 v_2^4 - 156v_2^2 - 78\omega_6^2 c_s^2 + \\
& 144c_s^4 + 90\omega_6^2 v_2^4 - 1008\omega_6 v_2^2 c_s^2 + 234\omega_6 v_2^2 - 216\omega_6 c_s^4 - \omega_6^3 - 9\omega_6^3 v_2^4 + 6\omega_6^3 c_s^2 + 8\omega_6^2 - 34\omega_6^3 v_2^2 c_s^2
\end{aligned}$$

$$\begin{aligned}
C_{21} = & 12 - \omega_6^3 c_s^4 - 18\omega_6 + 14\omega_6^3 v_2^2 - 36c_s^2 - 154\omega_6^2 v_2^2 + 432v_2^2 c_s^2 + 504v_2^4 + 252\omega_6^2 v_2^2 c_s^2 + 14\omega_6^2 c_s^4 + 54\omega_6 c_s^2 - 756\omega_6 v_2^4 - 252v_2^2 - 22\omega_6^2 c_s^2 + \\
& 24c_s^4 + 310\omega_6^2 v_2^4 - 648\omega_6 v_2^2 c_s^2 + 378\omega_6 v_2^2 - 36\omega_6 c_s^4 - \omega_6^3 - 29\omega_6^3 v_2^4 + 2\omega_6^3 c_s^2 + 8\omega_6^2 - 18\omega_6^3 v_2^2 c_s^2
\end{aligned}$$

2.4 CLBM1

2.4.1 Definitions

Collision operator C :

$$C(f) = \mathbf{K}^{-1} \mathbf{S} \left(\boldsymbol{\kappa}^{(eq)} - \mathbf{K} f \right),$$

where

$$\mathbf{S} = \text{diag}(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9),$$

$\omega_1, \omega_2, \dots, \omega_9 \in (0, 2)$.

Matrix \mathbf{K} corresponds to the transformation matrix to the central moment basis defined by

$$\boldsymbol{\kappa} = \left(k_{(0,0)}, k_{(1,0)}, k_{(0,1)}, k_{(1,1)}, k_{(2,0)}, k_{(0,2)}, k_{(2,1)}, k_{(1,2)}, k_{(2,2)} \right)^T,$$

and is given by

$$\begin{aligned}
\mathbf{K}_{1,i} &= (\mathbf{c}_i - \mathbf{v})^{(0,0)} \\
\mathbf{K}_{2,i} &= (\mathbf{c}_i - \mathbf{v})^{(1,0)} \\
\mathbf{K}_{3,i} &= (\mathbf{c}_i - \mathbf{v})^{(0,1)} \\
\mathbf{K}_{4,i} &= (\mathbf{c}_i - \mathbf{v})^{(1,1)} \\
\mathbf{K}_{5,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,0)} \\
\mathbf{K}_{6,i} &= (\mathbf{c}_i - \mathbf{v})^{(0,2)} \\
\mathbf{K}_{7,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,1)} \\
\mathbf{K}_{8,i} &= (\mathbf{c}_i - \mathbf{v})^{(1,2)} \\
\mathbf{K}_{9,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,2)},
\end{aligned}$$

$\forall i \in \{1, 2, \dots, 9\}$.

The equilibrium central moments are defined by

$$\kappa^{(eq)} = \mathbf{KM}^{-1} \mu^{(eq)},$$

i.e.,

$$\kappa^{(eq)} = \left(\rho, 0, 0, 0, \rho c_s^2, \rho c_s^2, 0, 0, \rho c_s^4 \right)^T.$$

2.4.2 Conservation of mass: ρ



attached text file: output_d2q9_nse_clbm1_symbolic_pde_00.txt

$$\begin{aligned} & \frac{\partial \rho}{\partial t} + \frac{v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + 3c_s^2 + v_1^2) \frac{v_1 \delta_l^3}{12 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3v_1^2) \frac{\rho \delta_l^3}{12 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\ & \frac{\rho c_s^2 \delta_l^3}{6 \delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{\rho c_s^2 \delta_l^3}{6 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{v_2 \delta_l^3}{12 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + c_s^2 + 3v_2^2) \frac{\rho \delta_l^3}{12 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & (-c_s^4 \omega_5 - 2c_s^2 - 6v_1^2 - 3v_1^4 \omega_5 + c_s^2 \omega_5 - 12c_s^2 v_1^2 \omega_5 + 2c_s^4 + 6v_1^4 + 24c_s^2 v_1^2 + 3v_1^2 \omega_5) \frac{\delta_l^4}{24 \omega_5 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\ & (-4 + 6c_s^2 + 10v_1^2 + 2\omega_5 - 3c_s^2 \omega_5 - 5v_1^2 \omega_5) \frac{\rho v_1 \delta_l^4}{12 \omega_5 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + \\ & (v_1^2 \omega_7 \omega_5 - \omega_7 + 3\omega_5 - 9c_s^2 \omega_5 + v_1^2 \omega_7 + 3c_s^2 \omega_7 \omega_5 + 3c_s^2 \omega_7 - 3v_1^2 \omega_5 - \omega_7 \omega_5) \frac{\rho v_1 \delta_l^4}{12 \omega_7 \omega_5 \delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + (-2 + \omega_4) \frac{c_s^4 \delta_l^4}{6 \omega_4 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\ & (v_2^2 \omega_8 \omega_6 - \omega_8 \omega_6 - 9c_s^2 \omega_6 + 3c_s^2 \omega_8 \omega_6 + 3c_s^2 \omega_8 - \omega_8 - 3v_2^2 \omega_6 + v_2^2 \omega_8 + 3\omega_6) \frac{\rho v_2 \delta_l^4}{12 \omega_8 \omega_6 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\ & (-3v_2^4 \omega_6 + c_s^2 \omega_6 + 24c_s^2 v_2^2 + 6v_2^4 - 2c_s^2 + 3v_2^2 \omega_6 - c_s^4 \omega_6 - 6v_2^2 + 2c_s^4 - 12c_s^2 v_2^2 \omega_6) \frac{\delta_l^4}{24 \omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\ & (-4 - 3c_s^2 \omega_6 + 6c_s^2 - 5v_2^2 \omega_6 + 10v_2^2 + 2\omega_6) \frac{\rho v_2 \delta_l^4}{12 \omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0. \end{aligned}$$

2.4.3 Conservation of momentum: ρv_1



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$$\begin{aligned} & v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (c_s^2 + v_1^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2\rho v_1 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_1 v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho v_2 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{\rho v_1 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\ & (-2 + 4c_s^2 + 6v_1^2 + \omega_5 - 2c_s^2 \omega_5 - 3v_1^2 \omega_5) \frac{\delta_l^2}{\omega_5 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + (2 - \omega_5) \frac{3\rho v_1 \delta_l^2}{\omega_5 \delta_t} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + (-2 + \omega_4) \frac{c_s^2 \delta_l^2}{2\omega_4 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\ & (-2 + \omega_4) \frac{c_s^2 \delta_l^2}{2\omega_4 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (-2 + 6c_s^2 + 2v_1^2 + \omega_5 - 3c_s^2 \omega_5 - v_1^2 \omega_5) \frac{v_1 \delta_l^2}{2\omega_5 \delta_t} \frac{\partial^2 \rho}{\partial x_1^2} + \\ & (-2 + 2c_s^2 + 6v_1^2 + \omega_5 - c_s^2 \omega_5 - 3v_1^2 \omega_5) \frac{\rho \delta_l^2}{2\omega_5 \delta_t} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega_4) \frac{\rho c_s^2 \delta_l^2}{2\omega_4 \delta_t} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_4) \frac{\rho c_s^2 \delta_l^2}{2\omega_4 \delta_t} \frac{\partial^2 v_1}{\partial x_2^2} + C_1 \frac{\delta_l^3}{12 \omega_5^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + \\ & (-24 - 4\omega_5^2 + 36c_s^2 + 60v_1^2 + 24\omega_5 - 36c_s^2 \omega_5 + 11v_1^2 \omega_5^2 - 60v_1^2 \omega_5 + 5c_s^2 \omega_5^2) \frac{\rho v_1 \delta_l^3}{6 \omega_5^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + C_2 \frac{\rho v_1 \delta_l^3}{12 \omega_7 \omega_4 \omega_5^2 \delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + \\ & (-12 - \omega_4^2 + 12\omega_4) \frac{c_s^4 \delta_l^3}{6 \omega_4^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} - \frac{\rho c_s^2 v_1 \delta_l^3}{6 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{v_1 v_2 \delta_l^3}{12 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + C_3 \frac{\rho v_2 \delta_l^3}{6 \omega_4 \omega_8 \delta_t} \frac{\partial^3 v_1}{\partial x_2^3} + \\ & (-1 + c_s^2 + 3v_2^2) \frac{\rho v_1 \delta_l^3}{12 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_4 \frac{v_1 \delta_l^4}{12 \omega_5^3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + C_5 \frac{\rho \delta_l^4}{12 \omega_5^3 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_6 \frac{\rho \delta_l^4}{12 \omega_7^2 \omega_4^3 \omega_5^3 \delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_7 \frac{c_s^2 v_1 \delta_l^4}{12 \omega_7^2 \omega_4^3 \omega_5^3 \omega_9 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\ & + C_8 \frac{\rho c_s^2 \delta_l^4}{12 \omega_7 \omega_4^3 \omega_8 \omega_5^2 \omega_9 \delta_t} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_9 \frac{c_s^2 v_2 \delta_l^4}{12 \omega_7 \omega_4^3 \omega_8^2 \omega_5 \omega_9 \omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + \\ & (v_2^2 \omega_8 \omega_6 - \omega_8 \omega_6 - 9c_s^2 \omega_6 + 3c_s^2 \omega_8 \omega_6 + 3c_s^2 \omega_8 - \omega_8 - 3v_2^2 \omega_6 + v_2^2 \omega_8 + 3\omega_6) \frac{\rho v_1 v_2 \delta_l^4}{12 \omega_8 \omega_6 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\ & C_{10} \frac{\rho c_s^2 \delta_l^4}{12 \omega_7 \omega_4^3 \omega_8 \omega_5 \omega_9 \omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\ & (-3v_2^4 \omega_6 + c_s^2 \omega_6 + 24c_s^2 v_2^2 + 6v_2^4 - 2c_s^2 + 3v_2^2 \omega_6 - c_s^4 \omega_6 - 6v_2^2 + 2c_s^4 - 12c_s^2 v_2^2 \omega_6) \frac{v_1 \delta_l^4}{24 \omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{11} \frac{\rho \delta_l^4}{24 \omega_4^3 \omega_8^3 \delta_t} \frac{\partial^4 v_1}{\partial x_2^4} + \\ & (-4 - 3c_s^2 \omega_6 + 6c_s^2 - 5v_2^2 \omega_6 + 10v_2^2 + 2\omega_6) \frac{\rho v_1 v_2 \delta_l^4}{12 \omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:

2.4.4 Conservation of momentum: ρv_2



attached text file: output_d2q9_nse_clbm1_symbolic_pde_02.txt

$$\begin{aligned}
& v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{v_1 v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho v_2 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\rho v_1 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_1} + (c_s^2 + v_2^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{2 \rho v_2 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_4) \frac{c_s^2 \delta_l^2}{2 \omega_4 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\
& (-2 + \omega_4) \frac{c_s^2 \delta_l^2}{2 \omega_4 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (-2 - 2c_s^2 \omega_6 + 4c_s^2 - 3v_2^2 \omega_6 + 6v_2^2 + \omega_6) \frac{\delta_l^2}{\omega_6 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (2 - \omega_6) \frac{3 \rho v_2 \delta_l^2}{\omega_6 \delta_t} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + \\
& (-2 + \omega_4) \frac{\rho c_s^2 \delta_l^2}{2 \omega_4 \delta_t} \frac{\partial^2 v_2}{\partial x_1^2} + (-2 + \omega_4) \frac{\rho c_s^2 \delta_l^2}{2 \omega_4 \delta_t} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + (-2 - 3c_s^2 \omega_6 + 6c_s^2 - v_2^2 \omega_6 + 2v_2^2 + \omega_6) \frac{v_2 \delta_l^2}{2 \omega_6 \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + \\
& (-2 - c_s^2 \omega_6 + 2c_s^2 - 3v_2^2 \omega_6 + 6v_2^2 + \omega_6) \frac{\rho \delta_l^2}{2 \omega_6 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + (-1 + 3c_s^2 + v_1^2) \frac{v_1 v_2 \delta_l^3}{12 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3v_1^2) \frac{\rho v_2 \delta_l^3}{12 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + \\
& C_1 \frac{\rho v_1 \delta_l^3}{6 \omega_7 \omega_4 \delta_t} \frac{\partial^3 v_2}{\partial x_1^3} + (-12 - \omega_4 + 12 \omega_4) \frac{c_s^4 \delta_l^3}{6 \omega_4^3 \delta_t} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} - \frac{\rho c_s^2 v_2 \delta_l^3}{6 \delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + C_2 \frac{\rho v_2 \delta_l^3}{12 \omega_4 \omega_8 \omega_6^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_3 \frac{\delta_l^3}{12 \omega_6^2 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\
& (-24 - 36c_s^2 \omega_6 + 36c_s^2 + 5c_s^2 \omega_6^2 - 60v_2^2 \omega_6 + 60v_2^2 + 24\omega_6 - 4\omega_6^2 + 11v_2^2 \omega_6^2) \frac{\rho v_2 \delta_l^3}{6 \omega_6^3 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& (-c_s^4 \omega_5 - 2c_s^2 - 6v_1^2 - 3v_1^4 \omega_5 + c_s^2 \omega_5 - 12c_s^2 v_1^2 \omega_5 + 2c_s^4 + 6v_1^4 + 24c_s^2 v_1^2 + 3v_1^2 \omega_5) \frac{v_2 \delta_l^4}{24 \omega_5 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& (-4 + 6c_s^2 + 10v_1^2 + 2\omega_5 - 3c_s^2 \omega_5 - 5v_1^2 \omega_5) \frac{\rho v_1 v_2 \delta_l^4}{12 \omega_5 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_4 \frac{\rho \delta_l^4}{24 \omega_7^2 \omega_4^3 \delta_t} \frac{\partial^4 v_2}{\partial x_1^4} + C_5 \frac{c_s^2 v_2 \delta_l^4}{12 \omega_7^2 \omega_4^3 \omega_8 \omega_5 \omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + \\
& C_6 \frac{\rho c_s^2 \delta_l^4}{12 \omega_7 \omega_4^3 \omega_8 \omega_5 \omega_6 \delta_t} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + \\
& (v_1^2 \omega_7 \omega_5 - \omega_7 + 3\omega_5 - 9c_s^2 \omega_5 + v_1^2 \omega_7 + 3c_s^2 \omega_7 \omega_5 + 3c_s^2 \omega_7 - 3v_1^2 \omega_5 - \omega_7 \omega_5) \frac{\rho v_1 v_2 \delta_l^4}{12 \omega_7 \omega_5 \delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& C_7 \frac{c_s^2 v_2 \delta_l^4}{12 \omega_7 \omega_4^3 \omega_8 \omega_5 \omega_6^3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_8 \frac{\rho c_s^2 \delta_l^4}{12 \omega_7 \omega_4^3 \omega_8 \omega_5 \omega_6^2 \delta_t} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_9 \frac{\rho \delta_l^4}{12 \omega_4^3 \omega_6^3 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{10} \frac{v_2 \delta_l^4}{12 \omega_6^3 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{11} \frac{\rho \delta_l^4}{12 \omega_6^3 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} \\
& = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= 6 - 3\omega_7 - 3\omega_4 - 18c_s^2 - 6v_1^2 - v_1^2 \omega_7 \omega_4 + \omega_7 \omega_4 + 3v_1^2 \omega_7 + 3v_1^2 \omega_4 + 9c_s^2 \omega_7 - 3c_s^2 \omega_7 \omega_4 + 9c_s^2 \omega_4 \\
C_2 &= 12\omega_4 \omega_6 - 6\omega_4 v_2^2 \omega_8 \omega_6 + 6\omega_4 \omega_8 \omega_6 + 36c_s^2 \omega_4 \omega_8 + 36c_s^2 \omega_4 \omega_6^2 + 18c_s^2 \omega_8 \omega_6^2 - 12\omega_4 \omega_8 - 36c_s^2 \omega_4 \omega_6 - 36c_s^2 \omega_6^2 - 3\omega_4 v_2^2 \omega_8 \omega_6^2 - 12\omega_4 \omega_6^2 + \\
& 3\omega_4 \omega_8 \omega_6^2 + 6v_2^2 \omega_8 \omega_6^2 - 6\omega_8 \omega_6^2 - 12\omega_4 v_2^2 \omega_6 - 18c_s^2 \omega_4 \omega_8 \omega_6 + 12\omega_4 v_2^2 \omega_8 - 11c_s^2 \omega_4 \omega_8 \omega_6^2 + 12\omega_6^2 - 12v_2^2 \omega_6^2 + 12\omega_4 v_2^2 \omega_6^2 \\
C_3 &= -36v_2^4 \omega_6 + 12c_s^2 \omega_6 + 144c_s^2 v_2^2 + 36v_2^4 - 12c_s^2 + 7v_2^4 \omega_6^2 - c_s^2 \omega_6^2 + 36v_2^2 \omega_6 - 12c_s^4 \omega_6 - 36v_2^2 + 12c_s^4 + 24c_s^2 v_2^2 \omega_6^2 - 144c_s^2 v_2^2 \omega_6 - 7v_2^2 \omega_6^2 + c_s^4 \omega_6^2 \\
C_4 &= -72v_1^2 \omega_7 \omega_4^2 + 24c_s^4 \omega_7^2 \omega_4^2 + 24c_s^4 \omega_7^2 - 3c_s^4 \omega_7^2 \omega_4^3 + 30v_1^2 \omega_7 \omega_4^3 - 36c_s^2 v_1^2 \omega_7^2 \omega_4 + 12c_s^2 \omega_7^2 \omega_4 + 36v_1^4 \omega_4^3 - 12c_s^2 v_1^2 \omega_7^2 \omega_4^2 - 8c_s^2 \omega_7^2 \omega_4^2 + 72v_1^4 \omega_7 \omega_4^2 - \\
& 30v_1^4 \omega_7 \omega_4^3 + c_s^2 \omega_7^2 \omega_4^3 + 6c_s^2 v_1^2 \omega_7^2 \omega_4^3 - 72v_1^4 \omega_4^2 - 48c_s^4 \omega_7^2 \omega_4 - 6c_s^2 \omega_7 \omega_4^3 + 3v_1^4 \omega_7^2 \omega_4^3 - 72c_s^2 v_1^2 \omega_7 \omega_4^3 + 24c_s^4 \omega_7 \omega_4 - 216c_s^2 v_1^2 \omega_4^2 + 144c_s^2 v_1^2 \omega_7 \omega_4^2 - \\
& 12v_1^4 \omega_7^2 \omega_4^2 + 24c_s^2 \omega_7 \omega_4^2 + 108c_s^2 v_1^2 \omega_4^3 - 3v_1^2 \omega_7^2 \omega_4^3 + 6c_s^4 \omega_7 \omega_4^3 + 72v_1^2 \omega_4^2 + 72c_s^2 v_1^2 \omega_7 \omega_4 - 24c_s^2 \omega_7 \omega_4 - 36v_1^2 \omega_4^3 - 24c_s^4 \omega_7 \omega_4^2 + 12v_1^2 \omega_7^2 \omega_4^2 \\
C_5 &= -36c_s^2 \omega_7^2 \omega_4 \omega_8 \omega_5 \omega_6 + 18c_s^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_6 + 6\omega_7 \omega_4^2 \omega_8 \omega_5 \omega_6 + 54c_s^2 \omega_7^2 \omega_4 \omega_8 \omega_5 \omega_6 - 12\omega_7^2 \omega_8 \omega_5 \omega_6 - 12\omega_7^2 \omega_4^2 \omega_8 \omega_5 + \\
& 12v_1^2 \omega_7 \omega_4 \omega_8 \omega_5 \omega_6 - 12\omega_7 \omega_8 \omega_5 \omega_6 - 36c_s^2 \omega_7^2 \omega_8 \omega_5 \omega_6 + 5\omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_6 - 12v_1^2 \omega_7^2 \omega_4 \omega_8 \omega_5 \omega_6 + 36c_s^2 \omega_7^2 \omega_4 \omega_8 \omega_5 \omega_6 + 36c_s^2 \omega_7 \omega_4 \omega_8 \omega_5 \omega_6 - \\
& 36c_s^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_6 + 18v_1^2 \omega_7^2 \omega_4 \omega_8 \omega_5 \omega_6 - 36c_s^2 \omega_7^2 \omega_4 \omega_8 \omega_5 \omega_6 - 6\omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_6 - 6\omega_7^2 \omega_4 \omega_8 \omega_5 \omega_6 + 12v_1^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_6 + 12\omega_7^2 \omega_4 \omega_8 \omega_5 \omega_6 + \\
& 12\omega_7^2 \omega_8 \omega_5 \omega_6 - 12v_1^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_6 + 12v_1^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_6 + 12\omega_4 \omega_8 \omega_5 \omega_6 - 12\omega_7^2 \omega_4 \omega_8 \omega_5 \omega_6 - 9c_s^2 \omega_7^2 \omega_4 \omega_8 \omega_5 \omega_6 - 36c_s^2 \omega_7 \omega_8 \omega_5 \omega_6 - \\
& 6v_1^2 \omega_7 \omega_4 \omega_8 \omega_5 \omega_6 - 15c_s^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_6 - 12v_1^2 \omega_7^2 \omega_4 \omega_8 \omega_5 \omega_6 - 18c_s^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 + 12\omega_7^2 \omega_4^2 \omega_8 \omega_5 + 12\omega_7^2 \omega_4 \omega_8 \omega_5 \omega_6 - 12\omega_7 \omega_4 \omega_8 \omega_5 \omega_6 + \\
& 6v_1^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 + 18c_s^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_6 + 12v_1^2 \omega_7^2 \omega_8 \omega_5 \omega_6 - 12\omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_6 - \omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_6 + 12\omega_7^2 \omega_4 \omega_8 \omega_5 \omega_6 - 5v_1^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_6 - \\
& 18c_s^2 \omega_7 \omega_4 \omega_8 \omega_5 \omega_6 - 12v_1^2 \omega_7^2 \omega_4 \omega_8 \omega_5 \omega_6 - 12v_1^2 \omega_7 \omega_8 \omega_5 \omega_6 + 54c_s^2 \omega_7 \omega_4 \omega_8 \omega_5 \omega_6 + 36c_s^2 \omega_7^2 \omega_8 \omega_5 \omega_6 - 18\omega_7^2 \omega_4 \omega_8 \omega_5 \omega_6 + 36c_s^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 + \\
& 12\omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_6 - 12v_1^2 \omega_7^2 \omega_8 \omega_5 \omega_6 + 6v_1^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_6 + 12v_1^2 \omega_7^2 \omega_4 \omega_8 \omega_5 \omega_6 + v_1^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_6 + 3\omega_7 \omega_4 \omega_8 \omega_5 \omega_6 + 6v_1^2 \omega_7^2 \omega_8 \omega_5 \omega_6 - \\
& 36c_s^2 \omega_4 \omega_8 \omega_5 \omega_6 - 36c_s^2 \omega_7^2 \omega_4 \omega_8 \omega_5 \omega_6 - 3v_1^2 \omega_7 \omega_4^2 \omega_8 \omega_5 \omega_6 + 6\omega_7^2 \omega_4^2 \omega_8 \omega_5 - 36c_s^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 + 18c_s^2 \omega_4 \omega_8 \omega_5 \omega_6 + 3c_s^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_6 + \\
& 18v_1^2 \omega_7 \omega_4 \omega_8 \omega_5 \omega_6 + 36c_s^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_6 - 18\omega_7 \omega_4 \omega_8 \omega_5 \omega_6 + 12v_1^2 \omega_7^2 \omega_4^2 \omega_8 \omega_5 - 12v_1^2 \omega_4 \omega_8 \omega_5 \omega_6 \\
C_6 &= 12\omega_7 \omega_4^3 \omega_9 + 12\omega_7 \omega_4^2 \omega_8 \omega_5 \omega_6 - 36v_1^2 \omega_7 \omega_4 \omega_8 \omega_5 \omega_6 + 18v_1^2 \omega_4^3 \omega_8 \omega_5 \omega_6 + 12c_s^2 \omega_4^2 \omega_8 \omega_5 \omega_6 - 18\omega_7 \omega_4^2 \omega_8 \omega_5 \omega_6 - 12c_s^2 \omega_7 \omega_4 \omega_8 \omega_5 \omega_6 + \\
& 6\omega_4^3 \omega_8 \omega_5 \omega_6 + 12c_s^2 \omega_7 \omega_4^3 \omega_8 \omega_5 - 6c_s^2 \omega_7 \omega_4^3 \omega_8 \omega_9 - 12\omega_7 \omega_4^3 \omega_8 \omega_5 - 18v_1^2 \omega_7 \omega_4^3 \omega_8 \omega_5 + 12\omega_7 \omega_4^2 \omega_8 \omega_5 \omega_6 + 6c_s^2 \omega_4^3 \omega_8 \omega_5 \omega_6 + 18\omega_4^2 \omega_8 \omega_5 \omega_6 - \\
& 12\omega_4^2 \omega_8 \omega_5 \omega_6 + 36v_1^2 \omega_7 \omega_4^3 \omega_8 \omega_9 - c_s^2 \omega_7 \omega_4^3 \omega_8 \omega_5 \omega_6 - 36v_1^2 \omega_7 \omega_4^3 \omega_8 \omega_5 \omega_6 - 6\omega_7 \omega_4^3 \omega_8 \omega_5 \omega_6 - 12c_s^2 \omega_7 \omega_4^3 \omega_8 \omega_9 + 36v_1^2 \omega_7 \omega_4^3 \omega_8 \omega_5 - 6c_s^2 \omega_4^3 \omega_8 \omega_5 \omega_6 - \\
& 12\omega_4 \omega_8 \omega_5 \omega_6 + 5\omega_7 \omega_4^3 \omega_8 \omega_9 \omega_6 + 12c_s^2 \omega_7 \omega_4^3 \omega_8 \omega_5 \omega_6 - 6\omega_7 \omega_4^3 \omega_8 \omega_9 - 5c_s^2 \omega_7 \omega_4^3 \omega_8 \omega_5 \omega_6 + 12\omega_7 \omega_4 \omega_8 \omega_9 \omega_6 - 12c_s^2 \omega_7 \omega_8 \omega_5 \omega_6 - \\
& 36v_1^2 \omega_7 \omega_4^3 \omega_8 \omega_5 \omega_6 - 12\omega_7 \omega_4^3 \omega_8 \omega_5 \omega_6 + 36v_1^2 \omega_4^3 \omega_8 \omega_5 \omega_6 + 54v_1^2 \omega_7 \omega_4^3 \omega_8 \omega_9 \omega_6 + 12\omega_7 \omega_4^3 \omega_8 \omega_5 \omega_6 - 6\omega_4^3 \omega_8 \omega_5 \omega_6 - 6c_s^2 \omega_7 \omega_4^3 \omega_8 \omega_5 - \\
& 12c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_5 \omega_6 - 12c_s^2 \omega_7 \omega_4^3 \omega_9 + 18c_s^2 \omega_7 \omega_4 \omega_8 \omega_5 \omega_9 \omega_6 + 18c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6 + 18v_1^2 \omega_7 \omega_4^3 \omega_8 \omega_9 + 6c_s^2 \omega_7 \omega_4^3 \omega_8 \omega_5 \omega_6 + \omega_7 \omega_4^3 \omega_8 \omega_5 \omega_9 \omega_6 - \\
& 54v_1^2 \omega_4^3 \omega_8 \omega_5 \omega_9 \omega_6 - 12c_s^2 \omega_7 \omega_4^3 \omega_8 \omega_5 \omega_6 + 12c_s^2 \omega_4 \omega_8 \omega_5 \omega_9 \omega_6 - 5c_s^2 \omega_7 \omega_4^3 \omega_8 \omega_9 \omega_6 - 3v_1^2 \omega_7 \omega_4^3 \omega_8 \omega_5 \omega_9 \omega_6 + 12c_s^2 \omega_7 \omega_4^3 \omega_8 \omega_9 \omega_6 - 18v_1^2 \omega_4^3 \omega_8 \omega_5 \omega_6 - \\
& 36v_1^2 \omega_7 \omega_4^3 \omega_9 - 18c_s^2 \omega_4^3 \omega_8 \omega_5 \omega_9 \omega_6 + 18v_1^2 \omega_7 \omega_4^3 \omega_8 \omega_5 \omega_6 + 36v_1^2 \omega_7 \omega_4^3 \omega_8 \omega_5 \omega_6 + 6\omega_7 \omega_4^3 \omega_8 \omega_5 - 15v_1^2 \omega_7 \omega_4^3 \omega_8 \omega_9 \omega_6 + 36v_1^2 \omega_4 \omega_8 \omega_5 \omega_9 \omega_6 - 36v_1^2 \omega_7 \omega_4^3 \omega_9 \omega_6 \\
C_7 &= -36c_s^2 \omega_7 \omega_8^2 \omega_9 \omega_6^2 - 6c_s^2 \omega_7 \omega_4^2 \omega_8^2 \omega_9 \omega_6^2 + 18\omega_7 \omega_4 v_2^2 \omega_8 \omega_9 \omega_6^3 + 18c_s^2 \omega_7 \omega_4^2 \omega_8^2 \omega_6^3 + 12\omega_7 \omega_4 \omega_8^2 \omega_9 \omega_6^3 + 18\omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 \omega_6^2 + 12\omega_7 v_2^2 \omega_8^2 \omega_9 \omega_6^3 + \\
& 12\omega_7 \omega_4^2 v_2^2 \omega_8 \omega_9 + 36c_s^2 \omega_4 \omega_8^2 \omega_6^3 + 12\omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6 - 36c_s^2 \omega_7 \omega_4^2 \omega_8 \omega_9 \omega_6^2 + 12\omega_7 \omega_4^2 v_2^2 \omega_8 \omega_6^3 - 6\omega_4^2 v_2^2 \omega_8 \omega_6^3 - 18c_s^2 \omega_7 \omega_4^2 \omega_8^2 \omega_6^2 + 36c_s^2 \omega_7 \omega_4 \omega_8 \omega_6^3 +
\end{aligned}$$

$$\begin{aligned}
& 5c_s^2\omega_7\omega_1^2\omega_8\omega_9\omega_6^3 + 36c_s^2\omega_7\omega_8\omega_9\omega_6^3 - 12\omega_7\omega_4^2\omega_8^2\omega_9 - 36c_s^2\omega_4\omega_8^2\omega_9\omega_6^2 + 36c_s^2\omega_7\omega_4^2\omega_9\omega_6^3 - 12\omega_7v_2^2\omega_8^2\omega_9\omega_6^2 - 18\omega_7\omega_4\omega_8^2\omega_9\omega_6^2 - 6\omega_7\omega_1^2v_2^2\omega_8\omega_9\omega_6^3 + \\
& 6\omega_7\omega_1^2\omega_8\omega_9\omega_6^3 + 12\omega_7\omega_4^2\omega_9\omega_6^2 - 6\omega_7\omega_4^2v_2^2\omega_8^2\omega_6^2 + 12\omega_7\omega_8^2\omega_9\omega_6^2 + 12\omega_7\omega_4^2v_2^2\omega_9\omega_6^3 + 6\omega_4^2v_2^2\omega_8^2\omega_9\omega_6^2 - 6\omega_7\omega_4^2\omega_8^2\omega_6^3 - 12\omega_7\omega_1^2v_2^2\omega_8\omega_9\omega_6 + \\
& 6\omega_7\omega_4^2v_2^2\omega_8^2\omega_6^3 - 12\omega_7\omega_4^2\omega_9\omega_6^3 - 18\omega_7\omega_4^2\omega_8\omega_9\omega_6^2 - 12\omega_4v_2^2\omega_8^2\omega_9\omega_6^2 + 6\omega_7\omega_4^2\omega_8^2\omega_6^2 - 12\omega_7\omega_4\omega_8\omega_6^3 - 36c_s^2\omega_7\omega_4^2\omega_8^2\omega_9\omega_6 - 12\omega_7\omega_4^2v_2^2\omega_9\omega_6^2 + \\
& 36c_s^2\omega_7\omega_4^2\omega_8^2\omega_9 - 12\omega_7\omega_8^2\omega_9\omega_6^3 + 12\omega_4\omega_8^2\omega_9\omega_6^2 + 6\omega_4^2\omega_8^2\omega_6^3 + 54c_s^2\omega_7\omega_4\omega_8\omega_9\omega_6^3 - 12\omega_7\omega_4^2v_2^2\omega_8\omega_6^3 - \omega_7\omega_4^2\omega_8^2\omega_9\omega_6^3 + 12\omega_7\omega_4\omega_8^2\omega_6^3 - \\
& 12\omega_7\omega_4^2\omega_8\omega_6^2 + 54c_s^2\omega_7\omega_4\omega_8^2\omega_9\omega_6^2 + 2\omega_7\omega_4^2\omega_8^2\omega_9\omega_6^2 + 12\omega_7\omega_1^2v_2^2\omega_8\omega_6^2 + 12\omega_7\omega_4\omega_9\omega_6^3 - 12\omega_7\omega_4^2v_2^2\omega_8^2\omega_9\omega_6 - 12\omega_4\omega_8^2\omega_6^3 - 40c_s^2\omega_7\omega_4\omega_8^2\omega_9\omega_6^3 + \\
& 18c_s^2\omega_4^2\omega_8^2\omega_9\omega_6^2 + 12\omega_7\omega_8\omega_9\omega_6^3 - 36c_s^2\omega_7\omega_4^2\omega_8\omega_9\omega_6 + 12\omega_7\omega_4^2\omega_8\omega_6^3 + 12\omega_4v_2^2\omega_8^2\omega_6^3 - 12\omega_7\omega_4v_2^2\omega_8^2\omega_9\omega_6^3 - 36c_s^2\omega_7\omega_4\omega_8^2\omega_6^3 - 12\omega_7\omega_4v_2^2\omega_9\omega_6^3 + \\
& 36c_s^2\omega_7\omega_4\omega_8\omega_6^2 + 54c_s^2\omega_7\omega_1^2\omega_8\omega_9\omega_6^2 + 12\omega_7\omega_4^2\omega_8^2\omega_9\omega_6 - 18c_s^2\omega_4^2\omega_8^2\omega_6^3 - 12\omega_7v_2^2\omega_8\omega_9\omega_6^3 - 18\omega_7\omega_4\omega_8\omega_9\omega_6^3 - 2\omega_7\omega_4^2v_2^2\omega_8^2\omega_9\omega_6^2 - 6\omega_4^2\omega_8^2\omega_9\omega_6^2 - \\
& 36c_s^2\omega_7\omega_8\omega_9\omega_6^3 - 18c_s^2\omega_7\omega_1^2\omega_8\omega_9\omega_6^3 - 36c_s^2\omega_7\omega_4^2\omega_8\omega_6^3 + 18\omega_7\omega_4v_2^2\omega_8^2\omega_9\omega_6^2 + \omega_7\omega_4^2v_2^2\omega_8^2\omega_9\omega_6^3 - 12\omega_7\omega_4v_2^2\omega_8^2\omega_6^3 - 36c_s^2\omega_7\omega_4\omega_9\omega_6^3
\end{aligned}$$

$$\begin{aligned}
C_8 = & -6c_s^2\omega_7\omega_4^3\omega_8\omega_6 + 6\omega_4^3\omega_8\omega_6^2 + 36\omega_7\omega_4^2v_2^2\omega_9\omega_6 - 12\omega_7\omega_4^2\omega_6^2 - 12c_s^2\omega_1^2\omega_8\omega_9\omega_6 + 36\omega_7\omega_4^2v_2^2\omega_6^2 + 36\omega_7\omega_4^3v_2^2\omega_9\omega_6^2 - 24\omega_7\omega_4^2\omega_8\omega_9\omega_6 - \\
& 24c_s^2\omega_7\omega_4^2\omega_9\omega_6^2 + 12c_s^2\omega_1^2\omega_8\omega_6^2 - 12c_s^2\omega_7\omega_4\omega_8\omega_9\omega_6 - 6\omega_7\omega_4^3\omega_8\omega_6^2 + 6c_s^2\omega_7\omega_1^3\omega_8\omega_9 - 12\omega_7\omega_4^3\omega_6 - 6\omega_4^3\omega_8\omega_9\omega_6 + 18\omega_4^3v_2^2\omega_8\omega_9\omega_6 - 12\omega_7\omega_4^2\omega_9\omega_6 + \\
& 24\omega_7\omega_4^2\omega_9\omega_6^2 + 6\omega_7\omega_4^3\omega_8\omega_6 - 36\omega_7\omega_4v_2^2\omega_8\omega_9\omega_6 + 18c_s^2\omega_7\omega_4\omega_8\omega_9\omega_6^2 + 12\omega_4^2\omega_8\omega_9\omega_6 + 12\omega_7\omega_4^3\omega_6^2 + 72\omega_7\omega_4^2v_2^2\omega_8\omega_9\omega_6 + 6c_s^2\omega_1^3\omega_8\omega_9\omega_6 + \\
& 12c_s^2\omega_7\omega_4^2\omega_9\omega_6 - 36\omega_7\omega_4^3v_2^2\omega_9\omega_6 + 6c_s^2\omega_7\omega_4^3\omega_8\omega_6^2 + 6\omega_7\omega_1^3\omega_8\omega_9\omega_6 - 72\omega_7\omega_4^2v_2^2\omega_9\omega_6^2 - 6\omega_7\omega_4^3\omega_8\omega_9 + 12\omega_7\omega_4\omega_8\omega_9\omega_6 - 18\omega_4^3v_2^2\omega_8\omega_6^2 + \\
& 12c_s^2\omega_7\omega_4^3\omega_6 - 12\omega_7\omega_4\omega_9\omega_6^2 + 12\omega_7\omega_1^3\omega_9\omega_6 + 12\omega_7\omega_4^2\omega_8\omega_6^2 - 12c_s^2\omega_7\omega_4^2\omega_8\omega_9 + 18\omega_7\omega_4^3v_2^2\omega_8\omega_9 - c_s^2\omega_7\omega_1^3\omega_8\omega_9\omega_6^2 - 36\omega_7\omega_4^2v_2^2\omega_8\omega_6^2 - \\
& 6c_s^2\omega_7\omega_4^3\omega_6 + 12c_s^2\omega_7\omega_1^2\omega_6^2 - 36\omega_7\omega_4^3v_2^2\omega_6^2 + 12c_s^2\omega_7\omega_1^3\omega_9\omega_6^2 - 12\omega_4^2\omega_8\omega_6^2 - 18\omega_7\omega_4^3v_2^2\omega_8\omega_6 + 24c_s^2\omega_7\omega_4^2\omega_8\omega_9\omega_6 - 18\omega_7\omega_4^3v_2^2\omega_8\omega_9\omega_6 - \\
& 12c_s^2\omega_7\omega_4^2\omega_8\omega_6^2 + 18\omega_7\omega_4^3v_2^2\omega_8\omega_6^2 - 4c_s^2\omega_7\omega_4^2\omega_8\omega_9\omega_6^2 - 12c_s^2\omega_7\omega_8\omega_9\omega_6^2 + 12\omega_7\omega_4^2\omega_8\omega_9 - 6c_s^2\omega_7\omega_1^3\omega_8\omega_9\omega_6 + 12c_s^2\omega_7\omega_4\omega_9\omega_6^2 - \\
& 12c_s^2\omega_7\omega_4^3\omega_9\omega_6 + 36\omega_7\omega_4^2v_2^2\omega_6 - 36\omega_4^2v_2^2\omega_8\omega_9\omega_6 + 36\omega_4^2v_2^2\omega_8\omega_6^2 + 36\omega_7\omega_4v_2^2\omega_9\omega_6^2 - 36\omega_7\omega_4^2v_2^2\omega_8\omega_9 - 12c_s^2\omega_7\omega_1^3\omega_6^2 - 12\omega_7\omega_4^3\omega_9\omega_6^2
\end{aligned}$$

$$\begin{aligned}
C_9 = & -12c_s^4\omega_4^2\omega_6^2\omega_8 + 36\omega_4^3v_2^2\omega_6^3 + 108c_s^2\omega_4^2v_2^2\omega_6^3 + 4\omega_4^3v_2^2\omega_8^2\omega_6^2 + 36c_s^2\omega_4v_2^2\omega_8\omega_6^3 + 6c_s^4\omega_1^3\omega_8\omega_6^3 + 90\omega_4^3v_2^2\omega_8^2\omega_6 + 36\omega_4^2v_2^2\omega_6^3 + 6c_s^2\omega_4\omega_8^2\omega_6^3 - \\
& 12c_s^2\omega_4^2\omega_8\omega_6^2 + 12c_s^2\omega_1^3v_2^2\omega_8^2\omega_6^3 - 6c_s^4\omega_4^3\omega_8\omega_6^2 + 6\omega_4^2v_2^2\omega_8^2\omega_6^3 - 108c_s^2\omega_4^3v_2^2\omega_6^2 + 19\omega_4^3v_2^2\omega_8^2\omega_6^2 - 36c_s^2\omega_1^2v_2^2\omega_8^2\omega_6 - 36\omega_4^3v_2^2\omega_6^2 + 18c_s^2\omega_4^2\omega_8\omega_6^3 + \\
& 60c_s^2\omega_4^3v_2^2\omega_6^2 + 12c_s^2\omega_4^3\omega_8^2\omega_6 - 306c_s^2\omega_1^3v_2^2\omega_8^2\omega_6 - c_s^2\omega_4^3\omega_8^2\omega_6^2 - 4\omega_4^3v_2^2\omega_8^2\omega_6^3 - 90\omega_4^3v_2^2\omega_8^2\omega_6 + 13c_s^4\omega_4^2\omega_8^2\omega_6^3 + 18c_s^2\omega_4^2v_2^2\omega_8^2\omega_6^2 + \\
& 252c_s^2\omega_4^3v_2^2\omega_8 - 19\omega_4^3v_2^2\omega_8^2\omega_6^2 - 6\omega_4^3v_2^2\omega_8^2\omega_6^3 + 72\omega_4^3v_2^2\omega_8^2\omega_6^3 + 6c_s^4\omega_4^2\omega_8^2\omega_6^2 - 3c_s^2\omega_4^2v_2^2\omega_8^2\omega_6^3 + 12c_s^4\omega_4\omega_8\omega_6^3 + 36\omega_4^2v_2^2\omega_8\omega_6^3 - 72\omega_4^3v_2^2\omega_8\omega_6^2 - \\
& 6c_s^2\omega_4^3\omega_8\omega_6^3 + 12c_s^2\omega_4^2\omega_8^2\omega_6 + 36\omega_4^3v_2^2\omega_6^2 - 24c_s^4\omega_4\omega_8^2\omega_6^3 + 54c_s^2\omega_4^2v_2^2\omega_8\omega_6^3 + 12c_s^4\omega_4^2\omega_8\omega_6^2 + 12c_s^4\omega_4^3\omega_8\omega_6^2 - 36\omega_4^3v_2^2\omega_6^3 - 108c_s^2\omega_1^3v_2^2\omega_8\omega_6 + \\
& 12c_s^4\omega_8\omega_6^3 + 39\omega_4^3v_2^2\omega_8\omega_6^3 + 6c_s^2\omega_4^3\omega_8\omega_6^2 - 36\omega_4^3v_2^2\omega_8\omega_6 - 36\omega_4^2v_2^2\omega_6^3 - 12c_s^4\omega_4^2\omega_8^2\omega_6 + 36c_s^2\omega_4^2v_2^2\omega_8\omega_6^2 - 18c_s^4\omega_4^2\omega_8\omega_6^3 - 108c_s^2\omega_4^2v_2^2\omega_6^3 + \\
& 72\omega_4^3v_2^2\omega_8\omega_6^2 - 36\omega_4^2v_2^2\omega_8\omega_6^3 + c_s^4\omega_1^3\omega_8^2\omega_6^2 - 72\omega_4^3v_2^2\omega_8^2\omega_6^2 + 198c_s^2\omega_4^3v_2^2\omega_8\omega_6^2 - 5c_s^2\omega_4^2\omega_8^2\omega_6^3 - 12c_s^2\omega_4^2\omega_8^2\omega_6^2 - 18c_s^2\omega_4v_2^2\omega_8^2\omega_6^3 - c_s^4\omega_4^3\omega_8^2\omega_6^3 - \\
& 39\omega_4^3v_2^2\omega_8\omega_6^3 + 36c_s^4\omega_4^3v_2^2\omega_8\omega_6 - 99c_s^2\omega_1^3v_2^2\omega_8\omega_6^3 - 6c_s^2\omega_4^2\omega_8^2\omega_6^2 - 12c_s^2\omega_4\omega_8\omega_6^3
\end{aligned}$$

$$\begin{aligned}
C_{10} = & 12 - 216v_2^4\omega_6 + 198c_s^2\omega_6 + 672c_s^2v_2^2 + 144v_2^4 - 132c_s^2 + 90v_2^4\omega_6^2 - 78c_s^2\omega_6^2 + 6c_s^2\omega_6^3 - 9v_2^4\omega_6^3 + 234v_2^2\omega_6 - 34c_s^2v_2^2\omega_6^3 - 216c_s^4\omega_6 - \\
& 156v_2^2 + 144c_s^4 + 404c_s^2v_2^2\omega_6 - \omega_6^3 - 1008c_s^2v_2^2\omega_6 - 5c_s^4\omega_6^3 - 18\omega_6 + 10v_2^2\omega_6^3 + 8\omega_6^2 - 98v_2^2\omega_6^2 + 82c_s^4\omega_6^2
\end{aligned}$$

$$\begin{aligned}
C_{11} = & 12 - 756v_2^4\omega_6 + 54c_s^2\omega_6 + 432c_s^2v_2^2 + 504v_2^4 - 36c_s^2 + 310v_2^4\omega_6^2 - 22c_s^2\omega_6^2 + 2c_s^2\omega_6^3 - 29v_2^4\omega_6^3 + 378v_2^2\omega_6 - 18c_s^2v_2^2\omega_6^3 - 36c_s^4\omega_6 - 252v_2^2 + \\
& 24c_s^4 + 252c_s^2v_2^2\omega_6^2 - \omega_6^3 - 648c_s^2v_2^2\omega_6 - c_s^4\omega_6^3 - 18\omega_6 + 14v_2^2\omega_6^3 + 8\omega_6^2 - 154v_2^2\omega_6^2 + 14c_s^4\omega_6^2
\end{aligned}$$

2.5 CLBM2

2.5.1 Definitions

Collision operator C :

$$C(f) = \mathbf{K}^{-1} \mathbf{S} \left(\kappa^{(eq)} - \mathbf{K}f \right),$$

where

$$\mathbf{S} = \text{diag}(\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8, \omega_9),$$

$$\omega_1, \omega_2, \dots, \omega_9 \in (0, 2).$$

Matrix \mathbf{K} corresponds to the transformation matrix to the central moment basis defined by

$$\kappa = \begin{pmatrix} k_{(0,0)} \\ k_{(1,0)} \\ k_{(0,1)} \\ k_{(1,1)} \\ k_{(2,0)} + k_{(0,2)} \\ k_{(2,0)} - k_{(0,2)} \\ k_{(2,1)} \\ k_{(1,2)} \\ k_{(2,2)} \end{pmatrix},$$

thus, the transformation matrix \mathbf{K} satisfies

$$\begin{aligned}
\mathbf{K}_{1,i} &= (\mathbf{c}_i - \mathbf{v})^{(0,0)} \\
\mathbf{K}_{2,i} &= (\mathbf{c}_i - \mathbf{v})^{(1,0)} \\
\mathbf{K}_{3,i} &= (\mathbf{c}_i - \mathbf{v})^{(0,1)} \\
\mathbf{K}_{4,i} &= (\mathbf{c}_i - \mathbf{v})^{(1,1)} \\
\mathbf{K}_{5,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,0)} + (\mathbf{c}_i - \mathbf{v})^{(0,2)} \\
\mathbf{K}_{6,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,0)} - (\mathbf{c}_i - \mathbf{v})^{(0,2)} \\
\mathbf{K}_{7,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,1)} \\
\mathbf{K}_{8,i} &= (\mathbf{c}_i - \mathbf{v})^{(1,2)} \\
\mathbf{K}_{9,i} &= (\mathbf{c}_i - \mathbf{v})^{(2,2)},
\end{aligned}$$

$\forall i \in \{1, 2, \dots, 9\}$.

The equilibrium central moments are defined by

$$\boldsymbol{\kappa}^{(eq)} = \mathbf{K} \mathbf{M}^{-1} \boldsymbol{\mu}^{(eq)},$$

i.e.,

$$\boldsymbol{\kappa}^{(eq)} = \left(\rho, 0, 0, 0, 2\rho c_s^2, 0, 0, 0, \rho c_s^4 \right)^T.$$

2.5.2 Conservation of mass: ρ



attached text file: output_d2q9_nse_clbm2_symbolic_pde_00.txt

$$\begin{aligned}
& \frac{\partial \rho}{\partial t} + \frac{v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + 3c_s^2 + v_1^2) \frac{v_1 \delta_l^3}{12 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3v_1^2) \frac{\rho \delta_l^3}{12 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\
& \frac{\rho \delta_l^3 c_s^2}{6 \delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{\rho \delta_l^3 c_s^2}{6 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{v_2 \delta_l^3}{12 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + c_s^2 + 3v_2^2) \frac{\rho \delta_l^3}{12 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& (\omega_5 c_s^2 + 3\omega_5 v_1^2 + 6v_1^4 + 2c_s^4 - 2c_s^2 - 6v_1^2 - 12\omega_5 v_1^2 c_s^2 + 24v_1^2 c_s^2 - 3\omega_5 v_1^4 - \omega_5 c_s^4) \frac{\delta_l^4}{24 \omega_5 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& (-4 - 3\omega_5 c_s^2 - 5\omega_5 v_1^2 + 2\omega_5 + 6c_s^2 + 10v_1^2) \frac{\rho v_1 \delta_l^4}{12 \omega_5 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + \\
& (3\omega_7 \omega_5 c_s^2 + \omega_7 \omega_5 v_1^2 - \omega_7 - 9\omega_5 c_s^2 - 3\omega_5 v_1^2 + 3\omega_5 + 3\omega_7 c_s^2 + \omega_7 v_1^2 - \omega_7 \omega_5) \frac{\rho v_1 \delta_l^4}{12 \omega_7 \omega_5 \delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + (-2 + \omega_4) \frac{\delta_l^4 c_s^4}{6 \omega_4 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} \\
& + (v_2^2 \omega_8 \omega_6 - \omega_8 \omega_6 - \omega_8 + 3\omega_8 \omega_6 c_s^2 + 3\omega_8 c_s^2 - 3v_2^2 \omega_6 + 3\omega_6 + v_2^2 \omega_8 - 9\omega_6 c_s^2) \frac{\rho v_2 \delta_l^4}{12 \omega_8 \omega_6 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& (-3v_2^4 \omega_6 - \omega_6 c_s^4 + 2c_s^4 + 6v_2^4 + 3v_2^2 \omega_6 - 2c_s^2 + 24v_2^2 c_s^2 - 6v_2^2 - 12v_2^2 \omega_6 c_s^2 + \omega_6 c_s^2) \frac{\delta_l^4}{24 \omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\
& (-4 - 5v_2^2 \omega_6 + 6c_s^2 + 10v_2^2 + 2\omega_6 - 3\omega_6 c_s^2) \frac{\rho v_2 \delta_l^4}{12 \omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0.
\end{aligned}$$

2.5.3 Conservation of momentum: ρv_1



attached text file: output_d2q9_nse_clbm2_symbolic_pde_01.txt

$$\begin{aligned}
& v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (c_s^2 + v_1^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2\rho v_1 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho v_2 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{\rho v_1 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\
& (-2 - 2\omega_5 c_s^2 - 3\omega_5 v_1^2 + \omega_5 + 4c_s^2 + 6v_1^2) \frac{\delta_l^2}{\omega_5 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + (2 - \omega_5) \frac{3\rho v_1 \delta_l^2}{\omega_5 \delta_t} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + (-2 + \omega_4) \frac{\delta_l^2 c_s^2}{2\omega_4 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} +
\end{aligned}$$

$$\begin{aligned}
& (-2 + \omega_4) \frac{\delta_l^2 c_s^2}{2\omega_4 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (-2 - 3\omega_5 c_s^2 - \omega_5 v_1^2 + \omega_5 + 6c_s^2 + 2v_1^2) \frac{v_1 \delta_l^2}{2\omega_5 \delta_t} \frac{\partial^2 \rho}{\partial x_1^2} + \\
& (-2 - \omega_5 c_s^2 - 3\omega_5 v_1^2 + \omega_5 + 2c_s^2 + 6v_1^2) \frac{\rho \delta_l^2}{2\omega_5 \delta_t} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega_4) \frac{\rho \delta_l^2 c_s^2}{2\omega_4 \delta_t} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_4) \frac{\rho \delta_l^2 c_s^2}{2\omega_4 \delta_t} \frac{\partial^2 v_1}{\partial x_2^2} + C_1 \frac{\delta_l^3}{12\omega_5^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3} \\
& + (-24 - 36\omega_5 c_s^2 - 60\omega_5 v_1^2 + 24\omega_5 - 4\omega_5^2 + 36c_s^2 + 60v_1^2 + 5\omega_5^2 c_s^2 + 11\omega_5^2 v_1^2) \frac{\rho v_1 \delta_l^3}{6\omega_5^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + C_2 \frac{\rho v_1 \delta_l^3}{12\omega_7 \omega_4 \omega_5^2 \delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + \\
& (-12 - \omega_4^2 + 12\omega_4) \frac{\delta_l^3 c_s^4}{6\omega_4^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} - \frac{\rho v_1 \delta_l^3 c_s^2}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{v_2 v_1 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + C_3 \frac{\rho v_2 \delta_l^3}{6\omega_4 \omega_8 \delta_t} \frac{\partial^3 v_1}{\partial x_2^3} + \\
& (-1 + c_s^2 + 3v_2^2) \frac{\rho v_1 \delta_l^3}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_4 \frac{v_1 \delta_l^4}{12\omega_5^2 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + C_5 \frac{\rho \delta_l^4}{12\omega_5^2 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_6 \frac{\rho \delta_l^4}{12\omega_7^2 \omega_4^3 \omega_5^3 \delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_7 \frac{v_1 \delta_l^4 c_s^2}{12\omega_7^2 \omega_4^3 \omega_5^3 \omega_9 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} \\
& + C_8 \frac{\rho \delta_l^4 c_s^2}{12\omega_7 \omega_4^3 \omega_8 \omega_5^2 \omega_9 \delta_t} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_9 \frac{v_2 \delta_l^4 c_s^2}{12\omega_7 \omega_4^3 \omega_8^2 \omega_5 \omega_9 \omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + \\
& (v_2^2 \omega_8 \omega_6 - \omega_8 \omega_6 - \omega_8 + 3\omega_8 \omega_6 c_s^2 + 3\omega_8 c_s^2 - 3v_2^2 \omega_6 + 3\omega_6 + v_2^2 \omega_8 - 9\omega_6 c_s^2) \frac{\rho v_2 v_1 \delta_l^4}{12\omega_8 \omega_6 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& C_{10} \frac{\rho \delta_l^4 c_s^2}{12\omega_7 \omega_4^3 \omega_8 \omega_5 \omega_9 \omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\
& (-3v_2^2 \omega_6 - \omega_6 c_s^4 + 2c_s^4 + 6v_2^2 + 3v_2^2 \omega_6 - 2c_s^2 + 24v_2^2 c_s^2 - 6v_2^2 - 12v_2^2 \omega_6 c_s^2 + \omega_6 c_s^2) \frac{v_1 \delta_l^4}{24\omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{11} \frac{\rho \delta_l^4}{24\omega_4^3 \omega_8^2 \delta_t} \frac{\partial^4 v_1}{\partial x_2^4} + \\
& (-4 - 5v_2^2 \omega_6 + 6c_s^2 + 10v_2^2 + 2\omega_6 - 3\omega_6 c_s^2) \frac{\rho v_2 v_1 \delta_l^4}{12\omega_6 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= 7\omega_5^2 v_1^4 + 24\omega_5^2 v_1^2 c_s^2 + \omega_5^2 c_s^4 + 12\omega_5 c_s^2 + 36\omega_5 v_1^2 + 36v_1^4 + 12c_s^4 - 12c_s^2 - 36v_1^2 - \omega_5^2 c_s^2 - 7\omega_5^2 v_1^2 - 144\omega_5 v_1^2 c_s^2 + 144v_1^2 c_s^2 - 36\omega_5 v_1^4 - 12\omega_5 c_s^4 \\
C_2 &= 12\omega_4 \omega_5 - 3\omega_7 \omega_4 \omega_5^2 v_1^2 - 11\omega_7 \omega_4 \omega_5^2 c_s^2 + 6\omega_7 \omega_4 \omega_5 + 3\omega_7 \omega_4 \omega_5^2 + 36\omega_7 \omega_4 c_s^2 + 12\omega_7 \omega_4 v_1^2 + 12\omega_4 \omega_5^2 v_1^2 - 12\omega_4 \omega_5^2 - 12\omega_7 \omega_4 + 36\omega_4 \omega_5^2 c_s^2 + \\
& 12\omega_5^2 - 12\omega_4 \omega_5 v_1^2 - 36\omega_4 \omega_5 c_s^2 - 6\omega_7 \omega_5^2 - 36\omega_5^2 c_s^2 - 6\omega_7 \omega_4 \omega_5 v_1^2 - 12\omega_5^2 v_1^2 - 18\omega_7 \omega_4 \omega_5 c_s^2 + 18\omega_7 \omega_5^2 c_s^2 + 6\omega_7 \omega_5^2 v_1^2 \\
C_3 &= 6 - 3\omega_4 + \omega_4 \omega_8 + 3\omega_4 v_2^2 - 3\omega_8 + 9\omega_4 c_s^2 - \omega_4 v_2^2 \omega_8 + 9\omega_8 c_s^2 - 18c_s^2 - 6v_2^2 + 3v_2^2 \omega_8 - 3\omega_4 \omega_8 c_s^2 \\
C_4 &= 12 + 90\omega_5^2 v_1^4 + 404\omega_5^2 v_1^2 c_s^2 + 82\omega_5^2 c_s^4 + 198\omega_5 c_s^2 + 234\omega_5 v_1^2 + 144v_1^4 - 5\omega_5^3 c_s^4 - 9\omega_5^3 v_1^4 + 144c_s^4 - 18\omega_5 + 8\omega_5^2 + 10\omega_5^3 v_1^2 - 132c_s^2 - \\
& 156v_1^2 + 6\omega_5^3 c_s^2 - \omega_5^3 - 34\omega_5^2 v_1^2 c_s^2 - 78\omega_5^2 c_s^2 - 98\omega_5^2 v_1^2 - 1008\omega_5 v_1^2 c_s^2 + 672v_2^2 c_s^2 - 216\omega_5 v_1^4 - 216\omega_5 c_s^4 \\
C_5 &= 12 + 310\omega_5^2 v_1^4 + 252\omega_5^2 v_1^2 c_s^2 + 14\omega_5^2 c_s^4 + 54\omega_5 c_s^2 + 378\omega_5 v_1^2 + 504v_1^4 - \omega_5^3 c_s^4 - 29\omega_5^3 v_1^4 + 24c_s^4 - 18\omega_5 + 8\omega_5^2 + 14\omega_5^3 v_1^2 - 36c_s^2 - 252v_1^2 + \\
& 2\omega_5^3 c_s^2 - \omega_5^3 - 18\omega_5^2 v_1^2 c_s^2 - 22\omega_5^2 c_s^2 - 154\omega_5^2 v_1^2 - 648\omega_5 v_1^2 c_s^2 + 432v_2^2 c_s^2 - 756\omega_5 v_1^4 - 36\omega_5 c_s^4 \\
C_6 &= \omega_7^2 \omega_4^3 \omega_5^2 c_s^4 + 39\omega_7 \omega_4^3 \omega_5^3 v_1^2 - 6\omega_7 \omega_4^3 \omega_5^3 c_s^2 + 36\omega_7^3 \omega_5^2 v_1^2 + 19\omega_7^2 \omega_5^3 \omega_5^2 v_1^4 + 198\omega_7 \omega_4^3 \omega_5^2 v_1^2 c_s^2 - 72\omega_7^2 \omega_4^3 v_1^2 + 6\omega_7^2 \omega_4^3 \omega_5^3 v_1^2 - 24\omega_7^2 \omega_4 \omega_5^3 c_s^4 - \\
& 36\omega_7^2 \omega_5^3 v_1^4 + 12\omega_7 \omega_4^2 \omega_5^2 c_s^4 + 60\omega_7^2 \omega_4^2 \omega_5^2 v_1^2 c_s^2 + 90\omega_7^2 \omega_4^2 \omega_5 v_1^2 + 12\omega_7^2 \omega_4^2 \omega_5 c_s^2 + 108\omega_4^3 \omega_5^3 v_1^2 c_s^2 - 5\omega_7^2 \omega_4^2 \omega_5^2 c_s^2 - 12\omega_7^2 \omega_4^2 c_s^2 - 36\omega_4^3 \omega_5^3 v_1^2 + \\
& 4\omega_7^2 \omega_4^3 \omega_5^3 v_1^4 + 18\omega_7^2 \omega_4^3 \omega_5^2 v_1^2 c_s^2 + 6\omega_7 \omega_4^3 \omega_5^2 c_s^2 - 72\omega_7 \omega_4^3 \omega_5^2 v_1^2 - 108\omega_4^3 \omega_5^3 v_1^2 c_s^2 - 12\omega_7^2 \omega_4^3 \omega_5^2 c_s^4 + 36\omega_7 \omega_4^3 \omega_5^2 v_1^2 c_s^2 - \omega_7^2 \omega_4^3 \omega_5^3 c_s^4 - 6\omega_7^2 \omega_4^3 \omega_5^2 c_s^2 - \\
& 36\omega_7 \omega_4^3 \omega_5^2 v_1^4 - 18\omega_7^2 \omega_4^3 \omega_5^2 v_1^2 c_s^2 + 36\omega_7 \omega_4^3 \omega_5^2 v_1^4 - 12\omega_7 \omega_4^3 \omega_5^3 c_s^2 - 18\omega_7 \omega_4^3 \omega_5^3 c_s^4 + 36\omega_7 \omega_4^3 \omega_5^2 v_1^2 c_s^2 + 72\omega_7 \omega_4^3 \omega_5^2 v_1^4 + 12\omega_7^2 \omega_4^3 \omega_5^2 c_s^2 - \\
& 36\omega_7^2 \omega_4^3 \omega_5 v_1^2 c_s^2 - 108\omega_4^3 \omega_5^2 v_1^2 c_s^2 + 36\omega_4^3 \omega_5^2 v_1^4 - 4\omega_7^2 \omega_4^3 \omega_5^2 v_1^4 + 12\omega_7^2 \omega_4^3 \omega_5^2 v_1^2 c_s^2 - 6\omega_7 \omega_4^3 \omega_5^2 c_s^4 + 18\omega_7 \omega_4^3 \omega_5^2 v_1^2 c_s^2 + 12\omega_7^2 \omega_4^3 c_s^4 + \\
& 6\omega_7^2 \omega_4^3 \omega_5^2 c_s^4 + 36\omega_7 \omega_4^3 \omega_5 v_1^2 - 36\omega_7 \omega_4^3 \omega_5^2 v_1^2 + 12\omega_7 \omega_4^3 \omega_5^2 c_s^4 + 6\omega_7 \omega_4^3 \omega_5^2 c_s^4 + 54\omega_7 \omega_4^3 \omega_5^2 v_1^2 c_s^2 - 36\omega_4^3 \omega_5^2 v_1^4 - 19\omega_7^2 \omega_4^3 \omega_5^2 v_1^2 - 108\omega_7 \omega_4^3 \omega_5 v_1^2 c_s^2 - \\
& \omega_7^2 \omega_4^3 \omega_5^2 c_s^2 - 39\omega_7 \omega_4^3 \omega_5^3 v_1^4 - 306\omega_7^2 \omega_4^3 \omega_5 v_1^2 c_s^2 - 12\omega_7^2 \omega_4^3 \omega_5^2 c_s^4 + 13\omega_7^2 \omega_4^3 \omega_5^2 c_s^4 + 12\omega_7^2 \omega_4^3 c_s^4 + 72\omega_7^2 \omega_4^3 v_1^4 - 6\omega_7^2 \omega_4^3 \omega_5^2 v_1^4 + 36\omega_4^3 \omega_5^3 v_1^4 + \\
& 6\omega_7^2 \omega_4^3 \omega_5^2 c_s^2 - 3\omega_7^2 \omega_4^3 \omega_5^2 v_1^2 c_s^2 + 252\omega_7^2 \omega_4^3 v_1^2 c_s^2 - 12\omega_7 \omega_4^3 \omega_5^2 c_s^2 - 90\omega_7^2 \omega_4^3 \omega_5 v_1^4 \\
C_7 &= -12\omega_4^2 \omega_8 \omega_5^3 \omega_9 - 36\omega_7^2 \omega_4 \omega_5^2 \omega_9 c_s^2 + 18\omega_7 \omega_4^2 \omega_8 \omega_5^2 v_1^2 \omega_9 - 18\omega_7 \omega_4 \omega_8 \omega_5^3 \omega_9 - 18\omega_7^2 \omega_4^2 \omega_8 \omega_5^2 c_s^2 - 36\omega_4^2 \omega_8 \omega_5^2 \omega_9 c_s^2 - 6\omega_7^2 \omega_4^2 \omega_8 \omega_5^2 v_1^2 + \\
& 12\omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_9 - 12\omega_7^2 \omega_4^2 \omega_8 \omega_9 - 12\omega_4^2 \omega_8 \omega_5^2 v_1^2 \omega_9 + 12\omega_7 \omega_4^2 \omega_8 \omega_5^2 v_1^2 - 6\omega_7^2 \omega_4^2 \omega_5^3 v_1^2 - 12\omega_7^2 \omega_4 \omega_5^2 v_1^2 \omega_9 - 12\omega_7^2 \omega_8 \omega_5^3 \omega_9 + 12\omega_7 \omega_4^2 \omega_8 \omega_5 \omega_9 + \\
& 54\omega_7 \omega_4^2 \omega_8 \omega_5^2 \omega_9 c_s^2 + 12\omega_7^2 \omega_4 \omega_8 \omega_5^3 - 18\omega_7^2 \omega_4^2 \omega_5^3 c_s^2 + 36\omega_7 \omega_4^2 \omega_8 \omega_5^2 c_s^2 + 12\omega_7^2 \omega_4 \omega_8 \omega_5^3 \omega_9 - 12\omega_7 \omega_4^2 \omega_8 \omega_5^2 + 6\omega_7^2 \omega_4^2 \omega_5^3 - 6\omega_7^2 \omega_4^2 \omega_8 \omega_5^2 \omega_9 c_s^2 + \\
& 6\omega_7^2 \omega_4^2 \omega_8 \omega_5^3 v_1^2 + 18\omega_7^2 \omega_4 \omega_8 \omega_5^2 v_1^2 \omega_9 + 18\omega_7^2 \omega_4^2 \omega_8 \omega_5^2 c_s^2 + 12\omega_4^2 \omega_8 \omega_5^2 \omega_9 - 36\omega_7^2 \omega_8 \omega_5^2 \omega_9 c_s^2 + 12\omega_7^2 \omega_4 \omega_5^2 \omega_9 - 18\omega_7^2 \omega_4 \omega_8 \omega_5^2 \omega_9 - 2\omega_7^2 \omega_4^2 \omega_8 \omega_5^2 v_1^2 \omega_9 - \\
& 36\omega_7 \omega_4^2 \omega_8 \omega_5^3 c_s^2 + 12\omega_7 \omega_4^2 \omega_8 \omega_5^3 + 12\omega_7^2 \omega_8 \omega_5^2 \omega_9 - 12\omega_7^2 \omega_8 \omega_5^2 v_1^2 \omega_9 + 54\omega_7^2 \omega_4 \omega_8 \omega_5^2 \omega_9 c_s^2 - 12\omega_7 \omega_4^2 \omega_8 \omega_5^3 v_1^2 + 18\omega_7 \omega_4 \omega_8 \omega_5^3 v_1^2 \omega_9 - \\
& 36\omega_7^2 \omega_4^2 \omega_8 \omega_5 \omega_9 c_s^2 - 36\omega_4 \omega_8 \omega_5^3 \omega_9 c_s^2 + 12\omega_4^2 \omega_8 \omega_5^3 v_1^2 \omega_9 + 6\omega_7 \omega_4^2 \omega_8 \omega_5^3 \omega_9 - 6\omega_7^2 \omega_4^2 \omega_5^2 \omega_9 - 18\omega_7 \omega_4^2 \omega_8 \omega_5^3 \omega_9 c_s^2 + 12\omega_4 \omega_8 \omega_5^3 \omega_9 + 18\omega_7^2 \omega_4^2 \omega_5^2 \omega_9 c_s^2 - \\
& 12\omega_7 \omega_4 \omega_8 \omega_5^3 - \omega_7^2 \omega_4^2 \omega_8 \omega_5^3 \omega_9 - 12\omega_4 \omega_8 \omega_5^3 v_1^2 \omega_9 - 12\omega_7^2 \omega_4 \omega_5^3 + 54\omega_7 \omega_4 \omega_8 \omega_5^3 \omega_9 c_s^2 - 12\omega_7^2 \omega_4^2 \omega_8 \omega_5 v_1^2 \omega_9 + 6\omega_7^2 \omega_4^2 \omega_5^2 v_1^2 \omega_9 - 6\omega_7 \omega_4^2 \omega_8 \omega_5^3 v_1^2 \omega_9 + \\
& 12\omega_7 \omega_8 \omega_5^3 \omega_9 + 36\omega_4^2 \omega_8 \omega_5^2 \omega_9 c_s^2 + 12\omega_7^2 \omega_8 \omega_5^3 v_1^2 \omega_9 - 12\omega_7 \omega_4^2 \omega_8 \omega_5 v_1^2 \omega_9 - 18\omega_7 \omega_4^2 \omega_8 \omega_5^2 \omega_9 - 36\omega_7^2 \omega_4 \omega_8 \omega_5^3 c_s^2 - 40\omega_7^2 \omega_4 \omega_8 \omega_5^3 \omega_9 c_s^2 + 12\omega_7^2 \omega_4 \omega_8 v_1^2 \omega_9 + \\
& \omega_7^2 \omega_4^2 \omega_8 \omega_5^3 v_1^2 \omega_9 - 12\omega_7^2 \omega_4 \omega_8 \omega_5^3 v_1^2 + 6\omega_7^2 \omega_4^2 \omega_8 \omega_5^2 - 36\omega_7 \omega_8 \omega_5^2 \omega_9 c_s^2 - 36\omega_7 \omega_4^2 \omega_8 \omega_5 \omega_9 c_s^2 - 12\omega_7^2 \omega_4 \omega_8 \omega_5^3 v_1^2 \omega_9 + 12\omega_7 \omega_4 \omega_8 \omega_5^3 v_1^2 + 12\omega_7^2 \omega_4 \omega_5^3 v_1^2 + \\
& 36\omega_7^2 \omega_8 \omega_5^3 \omega_9 c_s^2 - 6\omega_7^2 \omega_4^2 \omega_8 \omega_5^3 - 12\omega_7 \omega_8 \omega_5^3 v_1^2 \omega_9 + 36\omega_7^2 \omega_4 \omega_8 \omega_5^3 c_s^2 + 36\omega_7 \omega_4 \omega_8 \omega_5^3 c_s^2 + 2\omega_7^2 \omega_4^2 \omega_8 \omega_5^2 \omega_9 + 36\omega_7^2 \omega_4^2 \omega_8 \omega_9 c_s^2 + 5\omega_7^2 \omega_4^2 \omega_8 \omega_5^3 \omega_9 c_s^2 \\
C_8 &= 6\omega_7 \omega_4^3 \omega_5 \omega_9 c_s^2 - 12\omega_4^3 \omega_8 \omega_5^2 - 24\omega_4^2 \omega_8 \omega_5^2 \omega_9 c_s^2 - 6\omega_7 \omega_4^3 \omega_8 \omega_5 - 18\omega_7 \omega_4^3 \omega_8 \omega_5 v_1^2 \omega_9 + 36\omega_4 \omega_8 \omega_5^2 v_1^2 \omega_9 - 12\omega_4^3 \omega_8 \omega_5 \omega_9 c_s^2 + 18\omega_7 \omega_4 \omega_8 \omega_5^2 \omega_9 c_s^2 + \\
& 12\omega_3^3 \omega_8 \omega_5 c_s^2 + 18\omega_7 \omega_4^3 \omega_5 v_1^2 \omega_9 - 72\omega_4^2 \omega_8 \omega_5^2 v_1^2 \omega_9 - 36\omega_7 \omega_4^2 \omega_8 \omega_5^2 v_1^2 - 24\omega_7 \omega_4^2 \omega_8 \omega_5 \omega_9 - 4\omega_7 \omega_4^2 \omega_8 \omega_5^2 \omega_9 c_s^2 - 36\omega_4^3 \omega_8 \omega_5 v_1^2 \omega_9 - 12\omega_4^3 \omega_8 \omega_5^2 \omega_9 - \\
& 6\omega_7 \omega_4^3 \omega_8 \omega_5 \omega_9 c_s^2 - 12\omega_7 \omega_4^2 \omega_8 \omega_5^2 c_s^2 + 36\omega_4^3 \omega_8 \omega_5 v_1^2 + 12\omega_4 \omega_8 \omega_5^2 \omega_9 c_s^2 + 6\omega_7 \omega_4^3 \omega_8 \omega_5 \omega_9 + 12\omega_7 \omega_4^2 \omega_8 \omega_5^2 + 18\omega_7 \omega_4^3 \omega_8 \omega_5^2 v_1^2 - 6\omega_7 \omega_4^3 \omega_5^2 c_s^2 - \\
& 36\omega_7 \omega_4^3 \omega_5 v_1^2 \omega_9 - 18\omega_7 \omega_4^3 \omega_5^2 v_1^2 + 6\omega_7 \omega_4^3 \omega_8 \omega_5^2 c_s^2 - 6\omega_7 \omega_4^3 \omega_5 \omega_9 + 24\omega_4^3 \omega_8 \omega_5^2 \omega_9 - 12\omega_7 \omega_4^3 \omega_5 \omega_9 c_s^2 - 12\omega_7 \omega_8 \omega_5^2 \omega_9 c_s^2 + 6\omega_7 \omega_4^3 \omega_8 \omega_5 + \\
& 18\omega_7 \omega_4^3 \omega_8 v_1^2 \omega_9 - 12\omega_7 \omega_4^2 \omega_5^2 + 6\omega_7 \omega_4^3 \omega_8 \omega_9 c_s^2 + 12\omega_4^3 \omega_8 \omega_5^2 c_s^2 - 18\omega_7 \omega_4^3 \omega_8 \omega_5 v_1^2 - 6\omega_7 \omega_4^3 \omega_8 \omega_5 c_s^2 + 12\omega_7 \omega_4 \omega_8 \omega_5 \omega_9 + 36\omega_4^3 \omega_8 \omega_5^2 v_1^2 + \\
& 12\omega_7 \omega_4^3 \omega_8 \omega_9 - 12\omega_7^2 \omega_8 \omega_5 \omega_9 + 12\omega_4^3 \omega_8 \omega_5^2 + 12\omega_4^3 \omega_8 \omega_5 \omega_9 c_s^2 - 12\omega_4 \omega_8 \omega_5^2 \omega_9 - 12\omega_4^3 \omega_8 \omega_5^2 c_s^2 + 72\omega_7 \omega_4^3 \omega_8 \omega_5 v_1^2 \omega_9 - 12\omega_4^3 \omega_8 \omega_5 - \\
& 12\omega_7 \omega_4^3 \omega_8 \omega_9 c_s^2 - 12\omega_7 \omega_4 \omega_8 \omega_5 \omega_9 c_s^2 + 12\omega_4^3 \omega_8 \omega_5 \omega_9 + 12\omega_4^3 \omega_8 \omega_5^2 \omega_9 c_s^2 - 36\omega_4^3 \omega_8 \omega_5^2 v_1^2 + 24\omega_7 \omega_4^3 \omega_8 \omega_5 \omega_9 c_s^2 - 36\omega_7 \omega_4^3 \omega_8 v_1^2 \omega_9 + 36\omega_7 \omega_4^3 \omega_5^2 v_1^2 + \\
& 36\omega_4^3 \omega_8 \omega_5 v_1^2 \omega_9 + 12\omega_7 \omega_4^3 \omega_5 \omega_9 - \omega_7 \omega_4^3 \omega_8 \omega_5^2 \omega_9 c_s^2 - 6\omega_7 \omega_4^3 \omega_8 \omega_5^2 + 12\omega_7 \omega_4^3 \omega_5^2 c_s^2 - 36\omega_7 \omega_4 \omega_8 \omega_5 v_1^2 \omega_9 + 6\omega_7 \omega_4^3 \omega_5^2 + 36\omega_4^3 \omega_8 \omega_5^2 v_1^2 \omega_9
\end{aligned}$$

$$\begin{aligned}
C_9 = & 18\omega_7\omega_4^2\omega_8^2\omega_9c_s^2 + 12\omega_4v_2^2\omega_8^2\omega_5\omega_6 - 12\omega_4\omega_8^2\omega_5\omega_6 - 6\omega_7\omega_4^2\omega_8^2\omega_5\omega_6 + 36\omega_7\omega_4\omega_8\omega_5\omega_6c_s^2 + 6\omega_7\omega_4^2v_2^2\omega_8^2\omega_5\omega_6 - 36\omega_7\omega_4\omega_8^2\omega_5\omega_9\omega_6c_s^2 + \\
& 12\omega_4^2\omega_8^2\omega_9 - 12\omega_7\omega_8^2\omega_5\omega_9\omega_6 - 12\omega_7\omega_4v_2^2\omega_8^2\omega_5\omega_6 + 54\omega_7\omega_4\omega_8\omega_5\omega_9\omega_6c_s^2 + 12\omega_4^2v_2^2\omega_8^2\omega_6 - 12\omega_7\omega_4v_2^2\omega_8^2\omega_5\omega_6 - \\
& 18\omega_7\omega_4^2\omega_8\omega_5\omega_6c_s^2 - 36\omega_7\omega_8^2\omega_5\omega_9c_s^2 - 9\omega_7\omega_4^2\omega_8\omega_5\omega_9\omega_6c_s^2 - 12\omega_7v_2^2\omega_8\omega_5\omega_9\omega_6 + 54\omega_7\omega_4\omega_8^2\omega_5\omega_9c_s^2 + 36\omega_4^2\omega_8^2\omega_6c_s^2 + 18\omega_7\omega_4^2\omega_8\omega_5\omega_9\omega_6c_s^2 + \\
& 6\omega_7\omega_4^2v_2^2\omega_8^2\omega_9 + \omega_7\omega_4^2v_2^2\omega_8^2\omega_5\omega_9\omega_6 - 12\omega_7\omega_4v_2^2\omega_8^2\omega_5\omega_9\omega_6 + 12\omega_7\omega_4\omega_8^2\omega_5\omega_9\omega_6 + 36\omega_4^2\omega_8^2\omega_5\omega_9c_s^2 - 12\omega_7\omega_4\omega_8\omega_5\omega_6 - 6\omega_7\omega_4^2\omega_8^2\omega_9 + \\
& 6\omega_7\omega_4^2v_2^2\omega_8^2\omega_5\omega_9\omega_6 - \omega_7\omega_4^2\omega_8^2\omega_5\omega_9\omega_6 - 36\omega_4\omega_8^2\omega_5\omega_9c_s^2 - 12\omega_4^2\omega_8^2\omega_5\omega_9 + 3\omega_7\omega_4^2\omega_8^2\omega_5\omega_9\omega_6c_s^2 - 18\omega_7\omega_4\omega_8^2\omega_5\omega_6 - 15\omega_7\omega_4^2\omega_8^2\omega_5\omega_9c_s^2 - \\
& 6\omega_7\omega_4^2\omega_8^2\omega_5\omega_9\omega_6 - 36\omega_7\omega_4\omega_8\omega_5\omega_9\omega_6c_s^2 - 36\omega_4^2\omega_8^2\omega_5\omega_6c_s^2 - 36\omega_4^2\omega_8^2\omega_9c_s^2 - 36\omega_7\omega_4\omega_8^2\omega_5\omega_6c_s^2 + 12\omega_7\omega_4\omega_8^2\omega_5\omega_6 + 12\omega_4^2\omega_8^2\omega_5\omega_6 + \\
& 18\omega_7\omega_4^2\omega_8^2\omega_5\omega_6c_s^2 + 18\omega_7\omega_4v_2^2\omega_8^2\omega_5\omega_9\omega_6 + 6\omega_7\omega_4^2\omega_8^2\omega_6 + 36\omega_4\omega_8^2\omega_5\omega_6c_s^2 + 12\omega_7\omega_8\omega_5\omega_9\omega_6 - 12\omega_7v_2^2\omega_8^2\omega_5\omega_9 - 6\omega_7\omega_4^2v_2^2\omega_8^2\omega_6 - \\
& 12\omega_4^2v_2^2\omega_8^2\omega_9 + 12\omega_7\omega_8^2\omega_5\omega_9 + 6\omega_7\omega_4^2\omega_8\omega_5\omega_6 + 18\omega_7\omega_4v_2^2\omega_8^2\omega_5\omega_9 - 3\omega_7\omega_4^2v_2^2\omega_8\omega_5\omega_9\omega_6 - 18\omega_7\omega_4\omega_8\omega_5\omega_9\omega_6 - 36\omega_7\omega_8\omega_5\omega_9\omega_6c_s^2 - \\
& 6\omega_7\omega_4^2v_2^2\omega_8\omega_5\omega_6 + 12\omega_4^2v_2^2\omega_8^2\omega_5\omega_9 - 18\omega_7\omega_4^2\omega_8^2\omega_6c_s^2 + 12\omega_7v_2^2\omega_8^2\omega_5\omega_9\omega_6 + 5\omega_7\omega_4^2\omega_8^2\omega_5\omega_9 + 12\omega_7\omega_4\omega_8\omega_5\omega_9\omega_6 + 12\omega_4\omega_8^2\omega_5\omega_9 - \\
& 12\omega_4v_2^2\omega_8^2\omega_5\omega_9 + 12\omega_7\omega_4v_2^2\omega_8\omega_5\omega_6 - 12\omega_4^2\omega_8^2\omega_6 - 5\omega_7\omega_4^2v_2^2\omega_8^2\omega_5\omega_9 + 36\omega_7\omega_8^2\omega_5\omega_9\omega_6c_s^2 + 3\omega_7\omega_4^2\omega_8\omega_5\omega_9\omega_6 \\
\\
C_{10} = & -12\omega_7\omega_4^2\omega_8\omega_5\omega_6 + 18\omega_7\omega_4^3v_2^2\omega_8\omega_5\omega_6 - 36\omega_7\omega_4v_2^2\omega_8\omega_5\omega_9 + 36\omega_7\omega_4^2v_2^2\omega_8\omega_5\omega_6 - 6\omega_7\omega_4^3\omega_8\omega_9 - 36\omega_4^2v_2^2\omega_8\omega_5\omega_9 - 6\omega_7\omega_4^3\omega_8\omega_5\omega_9\omega_6 + \\
& 12\omega_4^3\omega_8\omega_5\omega_9c_s^2 + 12\omega_4^2\omega_8\omega_5\omega_6c_s^2 - 36\omega_4^3v_2^2\omega_8\omega_5\omega_6 + 18\omega_7\omega_4\omega_8\omega_5\omega_9\omega_6c_s^2 - 18\omega_7\omega_4^2\omega_8\omega_5\omega_9 - 12\omega_4^3\omega_8\omega_9c_s^2 - 18\omega_7\omega_4^3v_2^2\omega_8\omega_6 + \\
& 54\omega_7\omega_4^2v_2^2\omega_8\omega_5\omega_9 - 5\omega_7\omega_4^3\omega_8\omega_5\omega_6c_s^2 - 12\omega_7\omega_4^2\omega_8\omega_5\omega_6c_s^2 + 12\omega_4^3\omega_8\omega_5\omega_6 + 5\omega_7\omega_4^3\omega_8\omega_5\omega_9 - 5\omega_7\omega_4^2\omega_8\omega_5\omega_9\omega_6c_s^2 + 12\omega_4^3\omega_8\omega_9 - 36\omega_4^3v_2^2\omega_8\omega_9 - \\
& 18\omega_7\omega_4^2\omega_8\omega_5\omega_6c_s^2 - 12\omega_4^2\omega_8\omega_5\omega_6 + 36\omega_7\omega_4v_2^2\omega_8\omega_5\omega_9\omega_6 - 18\omega_7\omega_4^3v_2^2\omega_8\omega_5\omega_6 - 54\omega_7\omega_4^2v_2^2\omega_8\omega_5\omega_9\omega_6 + 18\omega_7\omega_4^2\omega_8\omega_5\omega_9\omega_6 - 6\omega_7\omega_4^3\omega_8\omega_6c_s^2 + \\
& 12\omega_7\omega_4\omega_8\omega_5\omega_9\omega_6c_s^2 + 6\omega_7\omega_4^3\omega_8\omega_5\omega_6c_s^2 + 6\omega_7\omega_4^3\omega_8\omega_9c_s^2 + 36\omega_4^3v_2^2\omega_8\omega_6 - 12\omega_4^3\omega_8\omega_6 - 6\omega_7\omega_4^3\omega_8\omega_5\omega_6 + 12\omega_7\omega_4\omega_8\omega_5\omega_9 + 6\omega_7\omega_4^3\omega_8\omega_6 + \\
& 12\omega_7\omega_4^2\omega_8\omega_5\omega_6c_s^2 + 18\omega_7\omega_4^3v_2^2\omega_8\omega_5\omega_9\omega_6 + 12\omega_4^2\omega_8\omega_5\omega_9 - 12\omega_4^2\omega_8\omega_5\omega_9c_s^2 + 12\omega_7\omega_4^2\omega_8\omega_5\omega_6 - 12\omega_4^3\omega_8\omega_5\omega_6c_s^2 - 3\omega_7\omega_4^3v_2^2\omega_8\omega_5\omega_9\omega_6 + 36\omega_4^3v_2^2\omega_8\omega_5\omega_9 - \\
& 12\omega_7\omega_8\omega_5\omega_9\omega_6c_s^2 - 12\omega_7\omega_4\omega_8\omega_5\omega_9c_s^2 - 12\omega_4^3\omega_8\omega_5\omega_9 - 6\omega_7\omega_4^3\omega_8\omega_5\omega_6c_s^2 - 36\omega_7\omega_4^2v_2^2\omega_8\omega_5\omega_6 + 18\omega_7\omega_4^3v_2^2\omega_8\omega_9 + 18\omega_7\omega_4^2\omega_8\omega_5\omega_9c_s^2 - \\
& 12\omega_7\omega_4\omega_8\omega_5\omega_9\omega_6 + 6\omega_7\omega_4^3\omega_8\omega_6 + 6\omega_7\omega_4^3\omega_8\omega_5\omega_6c_s^2 - 15\omega_7\omega_4^3v_2^2\omega_8\omega_5\omega_9 + 12\omega_4^3\omega_8\omega_6c_s^2 - \omega_7\omega_4^3\omega_8\omega_5\omega_9\omega_6c_s^2 + 36\omega_4^3v_2^2\omega_8\omega_5\omega_6 + \omega_7\omega_4^3\omega_8\omega_5\omega_9\omega_6 \\
\\
C_{11} = & 30\omega_4^3v_2^2\omega_8 + 24\omega_4^2\omega_8c_s^2 + 72\omega_4^2v_2^4\omega_8 + 24\omega_4\omega_8c_s^4 + 36\omega_3^3v_2^4 + 12\omega_4\omega_8^2c_s^2 - 12\omega_4^2v_2^2\omega_8^2c_s^2 + 24\omega_4^2\omega_8^2c_s^4 + 24\omega_8^2c_s^4 + 72\omega_4v_2^2\omega_8c_s^2 + \\
& 108\omega_4^3v_2^2c_s^2 + \omega_4^3\omega_8^2c_s^2 + 6\omega_4^3v_2^2\omega_8^2c_s^2 + 6\omega_4^3\omega_8c_s^4 - 12\omega_4^2v_2^2\omega_8^2 - 3\omega_4^3v_2^2\omega_8^2 - 72\omega_4^2v_2^4 - 3\omega_4^3\omega_8^2c_s^4 - 30\omega_4^3v_2^4\omega_8 - 36\omega_4v_2^2\omega_8^2c_s^2 + 72\omega_4^3v_2^2 - 6\omega_4^3\omega_8c_s^2 - \\
& 72\omega_4^2v_2^2\omega_8 - 72\omega_4^2v_2^2\omega_8^2c_s^2 + 12\omega_4^2v_2^2\omega_8^2 - 24\omega_4\omega_8c_s^2 + 3\omega_4^3v_2^2\omega_8^2 - 24\omega_4^2\omega_8c_s^4 + 144\omega_4^2v_2^2\omega_8^2c_s^2 - 216\omega_4^2v_2^2c_s^2 - 8\omega_4^2\omega_8^2c_s^2 - 36\omega_4^3v_2^2 - 48\omega_4\omega_8^2c_s^4
\end{aligned}$$

2.5.4 Conservation of momentum: ρv_2



attached text file: output_d2q9_nse_clbm2_symbolic_pde_02.txt

$$\begin{aligned}
& v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{v_2 v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho v_2 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\rho v_1 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_1} + (c_s^2 + v_2^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{2\rho v_2 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_4) \frac{\delta_l^2 c_s^2}{2\omega_4 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\
& (-2 + \omega_4) \frac{\delta_l^2 c_s^2}{2\omega_4 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (-2 - 3v_2^2\omega_6 + 4c_s^2 + 6v_2^2 + \omega_6 - 2\omega_6c_s^2) \frac{\delta_l^2}{\omega_6 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (2 - \omega_6) \frac{3\rho v_2 \delta_l^2}{\omega_6 \delta_t} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + \\
& (-2 + \omega_4) \frac{\rho \delta_l^2 c_s^2}{2\omega_4 \delta_t} \frac{\partial^2 v_2}{\partial x_1^2} + (-2 + \omega_4) \frac{\rho \delta_l^2 c_s^2}{2\omega_4 \delta_t} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + (-2 - v_2^2\omega_6 + 6c_s^2 + 2v_2^2 + \omega_6 - 3\omega_6c_s^2) \frac{v_2 \delta_l^2}{2\omega_6 \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + \\
& (-2 - 3v_2^2\omega_6 + 2c_s^2 + 6v_2^2 + \omega_6 - \omega_6c_s^2) \frac{\rho \delta_l^2}{2\omega_6 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + (-1 + 3c_s^2 + v_1^2) \frac{v_2 v_1 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3v_1^2) \frac{\rho v_2 \delta_l^3}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + \\
& C_1 \frac{\rho v_1 \delta_l^3}{6\omega_7 \omega_8 \delta_t} \frac{\partial^3 v_2}{\partial x_1^3} + (-12 - \omega_4^2 + 12\omega_4) \frac{\delta_l^3 c_s^4}{6\omega_4^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} - \frac{\rho v_2 \delta_l^3 c_s^2}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + C_2 \frac{\rho v_2 \delta_l^3 c_s^2}{12\omega_4 \omega_8 \omega_6^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_3 \frac{\delta_l^3}{12\omega_6^2 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\
& (-24 + 5\omega_6^2c_s^2 - 60v_2^2\omega_6 + 36c_s^2 + 60v_2^2 + 24\omega_6 - 4\omega_6^2 - 36\omega_6c_s^2 + 11v_2^2\omega_6^2) \frac{\rho v_2 \delta_l^3}{6\omega_6^2 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& (\omega_5c_s^2 + 3\omega_5v_1^2 + 6v_1^4 + 2c_s^4 - 2c_s^2 - 6v_1^2 - 12\omega_5v_1^2c_s^2 + 24v_1^2c_s^2 - 3\omega_5v_1^4 - \omega_5c_s^4) \frac{v_2 \delta_l^4}{24\omega_5 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& (-4 - 3\omega_5c_s^2 - 5\omega_5v_1^2 + 2\omega_5 + 6c_s^2 + 10v_1^2) \frac{\rho v_2 v_1 \delta_l^4}{12\omega_5 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_4 \frac{\rho \delta_l^4}{24\omega_7^2 \omega_4^3 \delta_t} \frac{\partial^4 v_2}{\partial x_1^4} + C_5 \frac{v_1 \delta_l^4 c_s^2}{12\omega_7^2 \omega_4^3 \omega_8 \omega_5 \omega_9 \omega_6 \delta_t} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + \\
& C_6 \frac{\rho \delta_l^4 c_s^2}{12\omega_7 \omega_4^3 \omega_8 \omega_5 \omega_9 \omega_6 \delta_t} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + \\
& (3\omega_7\omega_5c_s^2 + \omega_7\omega_5v_1^2 - \omega_7 - 9\omega_5c_s^2 - 3\omega_5v_1^2 + 3\omega_5 + 3\omega_7c_s^2 + \omega_7v_1^2 - \omega_7\omega_5) \frac{\rho v_2 v_1 \delta_l^4}{12\omega_7 \omega_5 \delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& C_7 \frac{v_2 \delta_l^4 c_s^2}{12\omega_7 \omega_4^2 \omega_8^2 \omega_9 \omega_6^3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_8 \frac{\rho \delta_l^4 c_s^2}{12\omega_7 \omega_4^3 \omega_8 \omega_9 \omega_6^2 \delta_t} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_9 \frac{\rho \delta_l^4}{12\omega_4^3 \omega_8^2 \omega_6^3 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{10} \frac{v_2 \delta_l^4}{12\omega_6^3 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{11} \frac{\rho \delta_l^4}{12\omega_6^3 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} \\
& = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 = & 6 - 3\omega_7 - 3\omega_4 - 3\omega_7\omega_4c_s^2 - \omega_7\omega_4v_1^2 + 9\omega_4c_s^2 + 3\omega_4v_1^2 + \omega_7\omega_4 - 18c_s^2 - 6v_1^2 + 9\omega_7c_s^2 + 3\omega_7v_1^2 \\
C_2 = & -36\omega_6^2c_s^2 - 12\omega_4\omega_6^2 - 6\omega_8\omega_6^2 - 18\omega_4\omega_8\omega_6c_s^2 - 3\omega_4v_2^2\omega_8\omega_6^2 - 12\omega_4v_2^2\omega_6 - 12\omega_4\omega_8 + 6v_2^2\omega_8\omega_6^2 + 12\omega_4v_2^2\omega_6^2 - 6\omega_4v_2^2\omega_8\omega_6 - 36\omega_4\omega_6c_s^2 + \\
& 12\omega_4\omega_6 + 12\omega_4v_2^2\omega_8 + 18\omega_8\omega_6^2c_s^2 + 3\omega_4\omega_8\omega_6^2 + 36\omega_4\omega_6^2c_s^2 + 36\omega_4\omega_8c_s^2 + 6\omega_4\omega_8\omega_6 + 12\omega_6^2 - 12v_2^2\omega_6^2 - 11\omega_4\omega_8\omega_6^2c_s^2 \\
C_3 = & -\omega_6^2c_s^2 - 36v_2^4\omega_6 - 12\omega_6c_s^4 + 24v_2^2\omega_6^2c_s^2 + 7v_2^4\omega_6^2 + 12c_s^4 + 36v_2^4 + 36v_2^2\omega_6 - 12c_s^2 + 144v_2^2c_s^2 - 36v_2^2 - 144v_2^2\omega_6c_s^2 + \omega_6^2c_s^4 + 12\omega_6c_s^2 - 7v_2^2\omega_6^2
\end{aligned}$$

$$C_4 = 36\omega_4^3v_1^4 + 72\omega_7\omega_4v_2^2c_s^2 + 6\omega_7\omega_4^3c_s^4 - 3\omega_7^2\omega_4^3v_1^2 + 108\omega_4^3v_2^2c_s^2 - 30\omega_7\omega_4^3v_1^4 + \omega_7^2\omega_4^3c_s^2 - 72\omega_4^2v_1^4 - 24\omega_7\omega_4c_s^2 - 12\omega_7^2\omega_4^2v_1^2c_s^2 - 48\omega_7^2\omega_4c_s^4 - 72\omega_7\omega_4^3v_2^2c_s^2 + 72\omega_7\omega_4^3v_1^4 - 8\omega_7^2\omega_4^2c_s^2 - 24\omega_7\omega_4^2c_s^4 + 12\omega_4^2v_1^2 + 12\omega_7^2\omega_4^2v_1^2c_s^2 - 36\omega_7^2\omega_4^2v_1^2c_s^2 + 72\omega_4^2v_1^4 + 24\omega_7\omega_4c_s^4 + 24\omega_7\omega_4^3v_1^2c_s^2 + 24\omega_7\omega_4^3c_s^2 - 12\omega_7^2\omega_4^2v_1^4 - 72\omega_7\omega_4^2v_1^2 + 24\omega_7^2\omega_4^2c_s^4 - 36\omega_4^3v_1^2 + 144\omega_7\omega_4^2v_1^2c_s^2 + 30\omega_7\omega_4^3v_1^2 - 3\omega_7^2\omega_4^3c_s^4 - 6\omega_7\omega_4^3c_s^2 + 3\omega_7^2\omega_4^3v_1^4 + 6\omega_7^2\omega_4^3v_1^2c_s^2$$

$$C_5 = 18\omega_4^2\omega_8\omega_5\omega_9\omega_6c_s^2 + 12\omega_7^2\omega_4\omega_5v_1^2\omega_6 - 5\omega_7^2\omega_4^2\omega_8v_1^2\omega_9\omega_6 + 36\omega_7\omega_4\omega_8\omega_5\omega_6c_s^2 + 36\omega_7^2\omega_4\omega_5\omega_6c_s^2 + 12\omega_7\omega_4\omega_8\omega_5v_1^2\omega_6 - 12\omega_4^2\omega_8\omega_5v_1^2\omega_9\omega_6 - 12\omega_7^2\omega_8v_1^2\omega_9\omega_6 - 12\omega_7^2\omega_4^2\omega_8\omega_5 - 6\omega_7^2\omega_4^2\omega_8\omega_5 - 6\omega_7\omega_4^2\omega_8\omega_5v_1^2\omega_6 + 54\omega_7\omega_4\omega_8\omega_5\omega_9\omega_6c_s^2 - 3\omega_7\omega_4^2\omega_8\omega_5v_1^2\omega_9\omega_6 - 36\omega_7^2\omega_4\omega_9\omega_6c_s^2 + 18\omega_7^2\omega_4\omega_8v_1^2\omega_9\omega_6 - 18\omega_7\omega_4^2\omega_8\omega_5\omega_6c_s^2 - 9\omega_7\omega_4^2\omega_8\omega_5\omega_9\omega_6c_s^2 + 6\omega_7^2\omega_4^2\omega_8\omega_5v_1^2\omega_6 - 12\omega_7^2\omega_4v_1^2\omega_9\omega_6 + 12\omega_7^2\omega_4\omega_9\omega_6 + 54\omega_7^2\omega_4\omega_8\omega_9\omega_6c_s^2 + 18\omega_7\omega_4\omega_8\omega_5v_1^2\omega_9\omega_6 - 12\omega_7^2\omega_4\omega_5\omega_6 + 18\omega_7^2\omega_4^2\omega_8\omega_5\omega_6c_s^2 - 12\omega_7\omega_4\omega_8\omega_5\omega_6 - 36\omega_4\omega_8\omega_5\omega_9\omega_6c_s^2 + 12\omega_7^2\omega_4\omega_8\omega_5\omega_6 + 12\omega_7^2\omega_8\omega_9\omega_6 - 36\omega_7^2\omega_4\omega_8\omega_5\omega_6c_s^2 - 36\omega_7^2\omega_8\omega_9\omega_6c_s^2 - 12\omega_7^2\omega_4\omega_8\omega_5v_1^2\omega_6 + 6\omega_7^2\omega_8\omega_5v_1^2\omega_9\omega_6 - 18\omega_7^2\omega_4\omega_8\omega_9\omega_6 - 15\omega_7^2\omega_4\omega_8\omega_9\omega_6c_s^2 + 36\omega_7^2\omega_4\omega_5c_s^2 - 36\omega_7^2\omega_4^2\omega_5\omega_6c_s^2 + 3\omega_7^2\omega_4^2\omega_8\omega_5\omega_9\omega_6c_s^2 - 12\omega_7^2\omega_8\omega_5\omega_9\omega_6 - 12\omega_7^2\omega_4^2\omega_5v_1^2\omega_6 - 12\omega_7^2\omega_4^2\omega_5v_1^2\omega_6 - 36\omega_7^2\omega_4^2\omega_5v_1^2\omega_6 - \omega_7^2\omega_4^2\omega_8\omega_5\omega_9\omega_6 + 12\omega_7^2\omega_4^2\omega_5\omega_6 + 12\omega_7^2\omega_4^2\omega_5v_1^2 - 12\omega_7^2\omega_4\omega_8\omega_5v_1^2\omega_9\omega_6 + 36\omega_7^2\omega_8\omega_5\omega_9\omega_6c_s^2 + 12\omega_7^2\omega_4\omega_8\omega_5\omega_9\omega_6 + 6\omega_7^2\omega_4^2\omega_8\omega_5 + 36\omega_7^2\omega_4\omega_9\omega_6c_s^2 - 12\omega_7\omega_8\omega_5v_1^2\omega_9\omega_6 + 12\omega_7\omega_8\omega_5\omega_9\omega_6 - 12\omega_7^2\omega_4^2v_1^2\omega_9 + 6\omega_7\omega_4^2\omega_8\omega_5\omega_6 - 18\omega_7\omega_4\omega_8\omega_5\omega_9\omega_6 - 36\omega_7\omega_8\omega_5\omega_9\omega_6c_s^2 + 6\omega_7^2\omega_4^2\omega_8v_1^2\omega_9 + 12\omega_7^2\omega_4^2v_1^2\omega_9\omega_6 + 12\omega_7^2\omega_8\omega_5v_1^2\omega_9\omega_6 - 6\omega_4^2\omega_8\omega_5\omega_9\omega_6 + 5\omega_7^2\omega_4^2\omega_8\omega_9\omega_6 - 36\omega_7^2\omega_4\omega_8\omega_5\omega_9\omega_6c_s^2 + 12\omega_4\omega_8\omega_5\omega_9\omega_6 - 18\omega_7^2\omega_4^2\omega_8\omega_5c_s^2 - 6\omega_7^2\omega_4^2\omega_8\omega_5\omega_6 - 6\omega_7^2\omega_4^2\omega_8\omega_5v_1^2 + 3\omega_7\omega_4^2\omega_8\omega_5\omega_9\omega_6 + 18\omega_7^2\omega_4^2\omega_8\omega_9c_s^2 + \omega_7^2\omega_4^2\omega_8\omega_5v_1^2\omega_9\omega_6$$

$$C_6 = 12\omega_7\omega_4^3v_1\omega_9 - 12\omega_7\omega_4^2\omega_5\omega_6 - 18\omega_7^2\omega_8\omega_5\omega_9\omega_6c_s^2 + 36\omega_4^2\omega_8\omega_5v_1^2\omega_6 - 6\omega_4^3\omega_8\omega_5\omega_9\omega_6 - 6\omega_7\omega_4^3\omega_8\omega_9 + 12\omega_7\omega_4^2\omega_9\omega_6 + 36\omega_4\omega_8\omega_5v_1^2\omega_9\omega_6 + 18\omega_7\omega_4^2\omega_8\omega_9\omega_6c_s^2 + 12\omega_4^2\omega_8\omega_5\omega_6c_s^2 - 36\omega_7\omega_4^2\omega_8\omega_5v_1^2\omega_6 + 18\omega_7\omega_4\omega_8\omega_5\omega_9\omega_6c_s^2 - 12\omega_7\omega_4\omega_8\omega_9\omega_6c_s^2 - 3\omega_7\omega_4^2\omega_8\omega_5v_1^2\omega_9\omega_6 - 12\omega_7\omega_4^2\omega_8\omega_5\omega_6c_s^2 + 6\omega_4^3\omega_8\omega_5\omega_6 - 5\omega_7\omega_4^2\omega_8\omega_5\omega_9\omega_6c_s^2 + 12\omega_7\omega_4\omega_8\omega_5\omega_9\omega_6 - 12\omega_4^2\omega_8\omega_5\omega_6c_s^2 + 12\omega_4\omega_8\omega_5\omega_9\omega_6 + 12\omega_4\omega_8\omega_5\omega_9\omega_6c_s^2 + 12\omega_7\omega_4\omega_8\omega_5\omega_9\omega_6 - 12\omega_7\omega_4^2\omega_8\omega_5v_1^2\omega_9\omega_6 - 12\omega_7\omega_4^2\omega_8\omega_5v_1^2\omega_9\omega_6 - 54\omega_4^2\omega_8\omega_5v_1^2\omega_9\omega_6 + 6\omega_7\omega_4^3\omega_8\omega_5 + 36\omega_7\omega_4^3v_1^2\omega_9\omega_6 - 12\omega_7\omega_4^3\omega_5 + 18\omega_7\omega_4^3\omega_8v_1^2\omega_9 - 36\omega_7\omega_4^3v_1^2\omega_9 - 12\omega_7\omega_4^3\omega_9\omega_6c_s^2 + 36\omega_7\omega_4^3\omega_5v_1^2\omega_6 + 6\omega_7\omega_4^3\omega_8\omega_9c_s^2 - 12\omega_7\omega_4^3\omega_9\omega_6 + 12\omega_7\omega_4^3\omega_5c_s^2 - 18\omega_7\omega_4^3\omega_8\omega_5v_1^2 - 6\omega_7\omega_4^3\omega_8\omega_5\omega_6 - 6\omega_7\omega_4^3\omega_8\omega_5c_s^2 + 6\omega_4^3\omega_8\omega_5\omega_9\omega_6c_s^2 - 15\omega_7\omega_4^3\omega_8v_1^2\omega_9\omega_6 + 36\omega_7\omega_4^3\omega_5v_1^2 + 12\omega_7\omega_4^3\omega_5\omega_6 + 12\omega_7\omega_4^3\omega_5\omega_6c_s^2 - 12\omega_7\omega_4^3\omega_9c_s^2 + 5\omega_7\omega_4^3\omega_8\omega_9\omega_6 + 12\omega_7\omega_4^3\omega_8\omega_5\omega_6 - 5\omega_7\omega_4^3\omega_8\omega_9\omega_6c_s^2 - 6\omega_4^3\omega_8\omega_5\omega_6c_s^2 + 18\omega_4^3\omega_8\omega_5v_1^2\omega_9\omega_6 - 36\omega_7\omega_4^3\omega_5v_1^2\omega_6 - 12\omega_7\omega_8\omega_5\omega_9\omega_6c_s^2 - 18\omega_7\omega_4^2\omega_8\omega_9\omega_6 - 12\omega_7\omega_4^2\omega_5\omega_6c_s^2 - 18\omega_4^3\omega_8\omega_5v_1^2\omega_6 + 18\omega_7^2\omega_8\omega_5\omega_9\omega_6 + 6\omega_7\omega_4^3\omega_8\omega_5\omega_6c_s^2 - 36\omega_7\omega_4^2v_1^2\omega_9\omega_6 - 12\omega_4\omega_8\omega_5\omega_9\omega_6 + 12\omega_7\omega_4^2\omega_9\omega_6c_s^2 - \omega_7\omega_4^3\omega_8\omega_5\omega_9\omega_6c_s^2 + 18\omega_7\omega_4^3\omega_8\omega_5v_1^2\omega_6 + \omega_7\omega_4^3\omega_8\omega_5\omega_9\omega_6$$

$$C_7 = 36\omega_7\omega_4^2\omega_8\omega_9c_s^2 - 12\omega_7\omega_4^2v_2^2\omega_8^2\omega_9\omega_6 - 12\omega_7\omega_4^2v_2^2\omega_8\omega_6^3 + 12\omega_7\omega_4^2\omega_8\omega_6^3 - 40\omega_7\omega_4\omega_8^2\omega_9\omega_6^3c_s^2 - 18\omega_7\omega_4\omega_8\omega_9\omega_6^3 - 36\omega_7\omega_4^2\omega_8\omega_9\omega_6^3c_s^2 + 12\omega_7\omega_4^2\omega_8^2\omega_9\omega_6 + 12\omega_7\omega_4^2\omega_8\omega_6^3 + 12\omega_7\omega_4^2\omega_8\omega_9\omega_6^3 - 18\omega_7\omega_4^2\omega_8^2\omega_9\omega_6^3 - 18\omega_7\omega_4^2\omega_8\omega_6^3c_s^2 + 5\omega_7\omega_4^2\omega_8^2\omega_9\omega_6^3c_s^2 + 12\omega_7\omega_4^2\omega_8\omega_9\omega_6^3 + 18\omega_4\omega_8^2\omega_9\omega_6^3c_s^2 + 18\omega_7\omega_4v_2^2\omega_8^2\omega_9\omega_6^3 + 12\omega_7\omega_4^2\omega_8\omega_9\omega_6^3 + 12\omega_7\omega_4^2v_2^2\omega_8^2\omega_9 + 54\omega_7\omega_4\omega_8^2\omega_9\omega_6^3c_s^2 + 6\omega_4^2v_2^2\omega_8^2\omega_9\omega_6^3 + \omega_7\omega_4^2v_2^2\omega_8^2\omega_9\omega_6^3 - 12\omega_7\omega_4^2\omega_8^2\omega_9 + 18\omega_7\omega_4^2\omega_8^2\omega_6^3c_s^2 + 12\omega_7\omega_4^2\omega_9\omega_6^3 - 12\omega_7\omega_8^2\omega_9\omega_6^3 - 12\omega_4v_2^2\omega_8^2\omega_9\omega_6^3 - 6\omega_7\omega_4^2\omega_8^2\omega_9\omega_6^3c_s^2 + 12\omega_4\omega_8^2\omega_9\omega_6^3 + 36\omega_4\omega_8^2\omega_9\omega_6^3c_s^2 - 12\omega_4\omega_8^2\omega_6^3 - 12\omega_7\omega_4v_2^2\omega_8^2\omega_9\omega_6^3 - 12\omega_7\omega_4^2v_2^2\omega_9\omega_6^3 + 2\omega_7\omega_4^2\omega_8\omega_9\omega_6^3 - 6\omega_7^2v_2^2\omega_8^2\omega_6^3 - 2\omega_7\omega_4^2v_2^2\omega_8^2\omega_9\omega_6^3 - 36\omega_4\omega_8^2\omega_9\omega_6^3c_s^2 - 12\omega_7\omega_4v_2^2\omega_8^2\omega_6^3 - 36\omega_7\omega_4^2\omega_9\omega_6^3c_s^2 + 36\omega_7\omega_8^2\omega_9\omega_6^3c_s^2 - 36\omega_7\omega_4^2\omega_8\omega_9\omega_6^3 + 12\omega_7\omega_4^2\omega_8\omega_9\omega_6^3 + 6\omega_7\omega_4^2v_2^2\omega_8\omega_9\omega_6^3 + 54\omega_7\omega_4\omega_8\omega_9\omega_6^3c_s^2 + 12\omega_7\omega_4v_2^2\omega_8\omega_6^3 - 18\omega_7\omega_4^2\omega_8\omega_9\omega_6^3 - 18\omega_7\omega_4v_2^2\omega_8\omega_9\omega_6^3 - 36\omega_7\omega_4^2\omega_8\omega_9\omega_6^3 + 18\omega_7\omega_4^2v_2^2\omega_8\omega_9\omega_6^3 + 18\omega_7\omega_4^2\omega_8\omega_9\omega_6^3 + 12\omega_7\omega_8\omega_9\omega_6^3 + 18\omega_7\omega_4^2v_2^2\omega_8\omega_9\omega_6^3 - 36\omega_7\omega_4\omega_9\omega_6^3c_s^2 + 6\omega_7\omega_4^2\omega_8\omega_6^3 - 12\omega_7\omega_4\omega_8\omega_6^3 - 12\omega_7\omega_4^2v_2^2\omega_8^2\omega_6^3 + 12\omega_7\omega_4^2\omega_8\omega_9\omega_6 - 12\omega_7v_2^2\omega_8\omega_9\omega_6^3 - 12\omega_7\omega_4v_2^2\omega_9\omega_6^3 + 12\omega_7\omega_4\omega_8^2\omega_9\omega_6^3 + 6\omega_7\omega_4^2v_2^2\omega_8^2\omega_6^3 - 36\omega_7\omega_8\omega_9\omega_6^3c_s^2 + 12\omega_4v_2^2\omega_8^2\omega_6^3 - 6\omega_7\omega_4^2\omega_8^2\omega_6^3 - 18\omega_4^2\omega_8^2\omega_6^3c_s^2 - 18\omega_7\omega_4\omega_8^2\omega_9\omega_6^3 + 36\omega_7\omega_4\omega_8\omega_6^3c_s^2 - 36\omega_7\omega_4\omega_8^2\omega_6^3c_s^2 + 54\omega_7\omega_4^2\omega_8\omega_9\omega_6^3c_s^2 - 6\omega_4^2\omega_8^2\omega_9\omega_6^3$$

$$C_8 = 36\omega_7\omega_4^3v_2^2\omega_9\omega_6^2 + 36\omega_7\omega_4^3v_2^2\omega_6 - 6\omega_4^3\omega_8\omega_6^2c_s^2 - 6\omega_7\omega_4^3\omega_8\omega_9 - 18\omega_7\omega_4^3v_2^2\omega_8\omega_9\omega_6 - 12\omega_7\omega_4^3\omega_9\omega_6 + 12\omega_7\omega_4^3\omega_9\omega_6^2c_s^2 + 24\omega_7\omega_4^3\omega_8\omega_9\omega_6c_s^2 + 36\omega_7\omega_4^3v_2^2\omega_9\omega_6 - \omega_7\omega_4^3\omega_8\omega_9\omega_6^2c_s^2 + 12\omega_7\omega_4^3\omega_8\omega_6^2 - 36\omega_7\omega_4^3v_2^2\omega_8\omega_6^2 - 12\omega_7\omega_4\omega_8\omega_9\omega_6c_s^2 - 6\omega_4^3\omega_8\omega_9\omega_6 + 18\omega_4^3v_2^2\omega_8\omega_9\omega_6 + 12\omega_7\omega_4^3\omega_6^2c_s^2 - 18\omega_7\omega_4^3v_2^2\omega_8\omega_6 - 12\omega_4^3\omega_8\omega_9\omega_6c_s^2 + 12\omega_4^3\omega_8\omega_9\omega_6 + 18\omega_7\omega_4^3v_2^2\omega_9\omega_6 - 36\omega_7\omega_4^3v_2^2\omega_9\omega_6 - 36\omega_7\omega_4^3v_2^2\omega_6^2 - 24\omega_7\omega_4^3\omega_9\omega_6c_s^2 - 6\omega_7\omega_4^3\omega_8\omega_6c_s^2 - 12\omega_7\omega_4^3\omega_8\omega_6^2 - 6\omega_7\omega_4^3\omega_8\omega_6^2c_s^2 + 12\omega_7\omega_4^3\omega_6^2 + 12\omega_7\omega_4^3\omega_9\omega_6c_s^2 + 6\omega_7\omega_4^3\omega_8\omega_9\omega_6c_s^2 + 6\omega_7\omega_4^3\omega_8\omega_9c_s^2 + 12\omega_7\omega_4^3\omega_9\omega_6 - 12\omega_7\omega_4\omega_9\omega_6^2 - 36\omega_7\omega_4^3v_2^2\omega_8\omega_9 - 12\omega_7\omega_8\omega_9\omega_6^2c_s^2 + 12\omega_7\omega_4^2\omega_8\omega_9 + 36\omega_4^3v_2^2\omega_8\omega_6^2 + 6\omega_7\omega_4^3\omega_8\omega_9\omega_6 + 36\omega_7\omega_4^3v_2^2\omega_6^2 - 6\omega_7\omega_4^3\omega_8\omega_9\omega_6c_s^2 - 12\omega_7\omega_4^3\omega_6^2 + 72\omega_7\omega_4^3v_2^2\omega_8\omega_9\omega_6 + 18\omega_7\omega_4\omega_8\omega_9\omega_6^2c_s^2 - 12\omega_7\omega_4^3\omega_8\omega_9c_s^2 - 24\omega_7\omega_4^3\omega_8\omega_9\omega_6 - 18\omega_4^3v_2^2\omega_8\omega_6^2 - 12\omega_7\omega_4^3\omega_9\omega_6^2 + 6\omega_4^3\omega_8\omega_9\omega_6c_s^2 + 12\omega_7\omega_4\omega_9\omega_6^2c_s^2 + 6\omega_7\omega_4^3\omega_8\omega_6 - 36\omega_4^3v_2^2\omega_8\omega_9\omega_6 - 12\omega_7\omega_4^3\omega_6 - 12\omega_7\omega_4^3\omega_9\omega_6c_s^2 + 36\omega_7\omega_4v_2^2\omega_9\omega_6^2 - 12\omega_7\omega_4^3\omega_8\omega_6^2c_s^2 - 36\omega_7\omega_4v_2^2\omega_8\omega_9\omega_6 - 4\omega_7\omega_4^3\omega_8\omega_9\omega_6^2c_s^2$$

$$C_9 = 18\omega_7^2v_2^2\omega_8^2\omega_6^2c_s^2 - 90\omega_4^3v_2^2\omega_8^2\omega_6 - 36\omega_7^2v_2^2\omega_8^2\omega_6^2 + 6\omega_4^3\omega_8\omega_6^2c_s^2 - 36\omega_4^3v_2^2\omega_8^2\omega_6^2 + 198\omega_4^3v_2^2\omega_8\omega_6^2c_s^2 + \omega_4^3\omega_8^2\omega_6^2c_s^4 - 4\omega_4^3v_2^2\omega_8^2\omega_6^3 + 13\omega_4^3\omega_8^2\omega_6^3c_s^4 + 12\omega_4^3\omega_8^2\omega_6c_s^2 - 19\omega_4^3v_2^2\omega_8^2\omega_6^2 - 6\omega_4^3v_2^4\omega_8^2\omega_6^3 + 18\omega_4^3\omega_8\omega_6^3c_s^2 + 36\omega_4^3v_2^2\omega_8^2\omega_6^2 - 306\omega_4^3v_2^2\omega_8^2\omega_6c_s^2 + 90\omega_4^3v_2^2\omega_8^2\omega_6 + 12\omega_4^3\omega_8^2\omega_6c_s^2 - \omega_4^3\omega_8^2\omega_6^3c_s^4 - 99\omega_4^3v_2^2\omega_8\omega_6^3c_s^2 - 6\omega_4^3\omega_8\omega_6^3c_s^2 - 12\omega_4^3\omega_8^2c_s^2 + 4\omega_4^3v_2^2\omega_8^2\omega_6^3 - 3\omega_4^3v_2^2\omega_8^2\omega_6^3c_s^2 + 252\omega_4^3v_2^2\omega_8^2c_s^2 - 12\omega_4^3\omega_8\omega_6^2c_s^2 + 6\omega_4^3\omega_8^2\omega_6^3c_s^2 - 18\omega_4^3v_2^2\omega_8^2\omega_6^3c_s^2 + 6\omega_4^3v_2^2\omega_8^2\omega_6^3 - 72\omega_4^3v_2^2\omega_8^2 + 12\omega_4\omega_8\omega_6^3c_s^4 + 19\omega_4^3v_2^2\omega_8^2\omega_6^2 + 6\omega_4^3\omega_8\omega_6^2c_s^4 + 6\omega_4^3\omega_8\omega_6^3c_s^4 + 12\omega_4^3v_2^2\omega_8^2\omega_6^3c_s^2 + 12\omega_4^3\omega_8^2c_s^4 + 72\omega_4^3v_2^2\omega_8\omega_6^2 + 54\omega_4^3v_2^2\omega_8\omega_6^3c_s^2 - 36\omega_4^3v_2^2\omega_8\omega_6^3 - 36\omega_4^3v_2^2\omega_8^2\omega_6^3 + 12\omega_4^3\omega_8\omega_6^3c_s^4 + 108\omega_4^3v_2^2\omega_6^3c_s^2 - 12\omega_4\omega_8\omega_6^3c_s^2 + 36\omega_4^3v_2^2\omega_8\omega_6^2 + 36\omega_4^3v_2^2\omega_8\omega_6^3c_s^2 - 6\omega_4^3\omega_8^2\omega_6^2c_s^2 + 36\omega_4^3v_2^2\omega_8^2\omega_6^2 + 12\omega_4^3\omega_8\omega_6^2c_s^4 - 24\omega_4\omega_8^2\omega_6^3c_s^4 - 39\omega_4^3v_2^2\omega_8\omega_6^3 - 108\omega_4^3v_2^2\omega_6^2c_s^2 - \omega_4^3\omega_8^2\omega_6^2c_s^2 + 36\omega_4^3v_2^2\omega_8\omega_6^2c_s^2 + 72\omega_4^3v_2^2\omega_8^2 + 36\omega_4^3v_2^2\omega_8\omega_6^3 - 72\omega_4^3v_2^2\omega_8\omega_6^2 + 60\omega_4^3v_2^2\omega_8^2\omega_6^2c_s^2 - 6\omega_4^3\omega_8\omega_6^2c_s^4 + 12\omega_8^2\omega_6^3c_s^4 - 36\omega_4^3v_2^4\omega_8\omega_6 - 36\omega_4^3v_2^2\omega_8^2\omega_6c_s^2 - 18\omega_4^3\omega_8\omega_6^3c_s^4 - 12\omega_4^3\omega_8^2\omega_6c_s^4 - 5\omega_4^3\omega_8^2\omega_6^3c_s^2 - 108\omega_4^3v_2^2\omega_6^3c_s^2 - 108\omega_4^3v_2^2\omega_8\omega_6c_s^2 + 39\omega_4^3v_2^2\omega_8\omega_6^3$$

$$C_{10} = 12 - 78\omega_6^2c_s^2 - 34v_2^2\omega_6^3c_s^2 - 216v_2^4\omega_6 - 216\omega_6c_s^4 + 404v_2^2\omega_6^2c_s^2 + 90v_2^4\omega_6^2 + 6\omega_6^3c_s^2 + 144c_s^4 - 9v_2^4\omega_6^3 + 144v_2^4 + 234v_2^2\omega_6 - 5\omega_6^3c_s^4 - 132c_s^2 + 672v_2^2c_s^2 - 156v_2^2 - 1008v_2^2\omega_6c_s^2 - 18\omega_6 - \omega_6^3 + 82\omega_6^2c_s^4 + 10v_2^2\omega_6^3 + 8\omega_6^2 + 198\omega_6c_s^2 - 98v_2^2\omega_6^2$$

$$C_{11} = 12 - 22\omega_6^2c_s^2 - 18v_2^2\omega_6^3c_s^2 - 756v_2^4\omega_6 - 36\omega_6c_s^4 + 252v_2^2\omega_6^2c_s^2 + 310v_2^4\omega_6^2 + 2\omega_6^3c_s^2 + 24c_s^4 - 29v_2^4\omega_6^3 + 504v_2^4 + 378v_2^2\omega_6 - \omega_6^3c_s^4 - 36c_s^2 + 432v_2^2c_s^2 - 252v_2^2 - 648v_2^2\omega_6c_s^2 - 18\omega_6 - \omega_6^3 + 14\omega_6^2c_s^4 + 14v_2^2\omega_6^3 + 8\omega_6^2 + 54\omega_6c_s^2 - 154v_2^2\omega_6^2$$

2.6 CuLBM1

2.6.1 Definitions

Based on [2], collision operator \mathcal{C} :

$$\mathcal{C}(f) = \mathbf{M}^{-1} \mathbf{G}^{-1} \left(\mathbf{S} \left(\gamma^{(eq)} - \mathbf{G}(\mathbf{M}f) \right) \right),$$

where

$$\mathbf{S} = \text{diag}(0, 0, 0, \omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6),$$

$\omega_1, \omega_2, \dots, \omega_6 \in (0, 2)$.

The nonlinear operator \mathbf{G} (with its inverse \mathbf{G}^{-1}) transforms the raw moment vector $\boldsymbol{\mu}$ defined by matrix \mathbf{M} to the cumulant vector

$$\boldsymbol{\gamma} = \mathbf{G}(\boldsymbol{\mu}) = \left(\gamma_{(0,0)}, \gamma_{(1,0)}, \gamma_{(0,1)}, \gamma_{(2,0)}, \gamma_{(0,2)}, \gamma_{(1,1)}, \gamma_{(2,1)}, \gamma_{(1,2)}, \gamma_{(2,2)} \right)^T$$

as

$$\gamma_{(0,0)} = m_{(0,0)},$$

$$\gamma_{(1,0)} = \frac{m_{(1,0)}}{m_{(0,0)}},$$

$$\gamma_{(0,1)} = \frac{m_{(0,1)}}{m_{(0,0)}},$$

$$\gamma_{(2,0)} = -\frac{m_{(1,0)}^2}{m_{(0,0)}^2} + \frac{m_{(2,0)}}{m_{(0,0)}},$$

$$\gamma_{(0,2)} = -\frac{m_{(0,1)}^2}{m_{(0,0)}^2} + \frac{m_{(0,2)}}{m_{(0,0)}},$$

$$\gamma_{(1,1)} = -\frac{m_{(1,0)}m_{(0,1)}}{m_{(0,0)}^2} + \frac{m_{(1,1)}}{m_{(0,0)}},$$

$$\gamma_{(2,1)} = \frac{m_{(2,1)}}{m_{(0,0)}} - \frac{m_{(0,1)}m_{(2,0)}}{m_{(0,0)}^2} - 2\frac{m_{(1,0)}m_{(1,1)}}{m_{(0,0)}^2} + 2\frac{m_{(1,0)}^2m_{(0,1)}}{m_{(0,0)}^3},$$

$$\gamma_{(1,2)} = \frac{m_{(1,2)}}{m_{(0,0)}} - \frac{m_{(1,0)}m_{(0,2)}}{m_{(0,0)}^2} - 2\frac{m_{(0,1)}m_{(1,1)}}{m_{(0,0)}^2} + 2\frac{m_{(0,1)}^2m_{(1,0)}}{m_{(0,0)}^3},$$

$$\begin{aligned} \gamma_{(2,2)} = & -6\frac{m_{(1,0)}^2m_{(0,1)}^2}{m_{(0,0)}^4} + 2\frac{m_{(0,1)}^2m_{(2,0)} + m_{(1,0)}^2m_{(0,2)}}{m_{(0,0)}^2} + 8\frac{m_{(1,0)}m_{(0,1)}m_{(1,1)}}{m_{(0,0)}^3} - 2\frac{m_{(1,0)}m_{(1,2)} + m_{(0,1)}m_{(2,1)}}{m_{(0,0)}^2} + \frac{m_{(2,2)}}{m_{(0,0)}} - \\ & \frac{m_{(2,0)}m_{(0,2)} + 2m_{(1,1)}^2}{m_{(0,0)}^2}. \end{aligned}$$

The equilibrium cumulant vector $\boldsymbol{\gamma}^{(eq)}$ is defined by

$$\boldsymbol{\gamma}^{(eq)} = \left(\rho, v_1, v_2, c_s^2, c_s^2, 0, 0, 0, 0 \right)^T.$$

2.6.2 Conservation of mass: ρ



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$$\begin{aligned} & \frac{\partial \rho}{\partial t} + \frac{\delta_l v_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\delta_l \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\delta_l \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + 3c_s^2 + v_1^2) \frac{\delta_l^3 v_1}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3v_1^2) \frac{\delta_l^3 \rho}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\ & \frac{\delta_l^3 \rho c_s^2}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{\delta_l^3 \rho c_s^2}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{v_2 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + c_s^2 + 3v_2^2) \frac{\delta_l^3 \rho}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & (-2c_s^2 - \omega_1 c_s^4 - 3\omega_1 v_1^4 - 6v_1^2 + 3\omega_1 v_1^2 + 6v_1^4 - 12\omega_1 v_1^2 c_s^2 + 24v_1^2 c_s^2 + 2c_s^4 + \omega_1 c_s^2) \frac{\delta_l^4}{24\omega_1 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\ & (-4 + 6c_s^2 + 2\omega_1 + 10v_1^2 - 5\omega_1 v_1^2 - 3\omega_1 c_s^2) \frac{\delta_l^4 v_1 \rho}{12\omega_1 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + \\ & (3\omega_1 + 3\omega_1 c_s^2 \omega_4 + 3c_s^2 \omega_4 - 3\omega_1 v_1^2 + \omega_1 v_1^2 \omega_4 + v_1^2 \omega_4 - \omega_1 \omega_4 - 9\omega_1 c_s^2 - \omega_4) \frac{\delta_l^4 v_1 \rho}{12\omega_1 \delta_t \omega_4} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + (-2 + \omega_3) \frac{\delta_l^4 c_s^4}{6\omega_3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} \end{aligned}$$

$$\begin{aligned}
& + (3\omega_2\omega_6c_s^2 + v_2^2\omega_6 + 3\omega_2 - \omega_2\omega_6 - \omega_6 + 3\omega_6c_s^2 - 9\omega_2c_s^2 + \omega_2v_2^2\omega_6 - 3\omega_2v_2^2) \frac{v_2\delta_l^4\rho}{12\omega_2\omega_6\delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& (-12\omega_2v_2^2c_s^2 - 2c_s^2 - 3\omega_2v_2^4 - 6v_2^2 - \omega_2c_s^4 + 6v_2^4 + 24v_2^2c_s^2 + \omega_2c_s^2 + 2c_s^4 + 3\omega_2v_2^2) \frac{\delta_l^4}{24\omega_2\delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + \\
& (-4 + 6c_s^2 + 2\omega_2 + 10v_2^2 - 3\omega_2c_s^2 - 5\omega_2v_2^2) \frac{v_2\delta_l^4\rho}{12\omega_2\delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0.
\end{aligned}$$

2.6.3 Conservation of momentum: ρv_1



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$$\begin{aligned}
& v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (c_s^2 + v_1^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2\delta_l v_1 \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 \delta_l v_1}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{v_2 \delta_l \rho}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{\delta_l v_1 \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\
& (-2 + 4c_s^2 + \omega_1 + 6v_1^2 - 3\omega_1 v_1^2 - 2\omega_1 c_s^2) \frac{\delta_l^2}{\omega_1 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + (2 - \omega_1) \frac{3\delta_l^2 v_1 \rho}{\omega_1 \delta_t} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + (-2 + \omega_3) \frac{\delta_l^2 c_s^2}{2\omega_3 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\
& (-2 + \omega_3) \frac{\delta_l^2 c_s^2}{2\omega_3 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (-2 + 6c_s^2 + \omega_1 + 2v_1^2 - \omega_1 v_1^2 - 3\omega_1 c_s^2) \frac{\delta_l^2 v_1}{2\omega_1 \delta_t} \frac{\partial^2 \rho}{\partial x_1^2} + \\
& (-2 + 2c_s^2 + \omega_1 + 6v_1^2 - 3\omega_1 v_1^2 - \omega_1 c_s^2) \frac{\delta_l^2 \rho}{2\omega_1 \delta_t} \frac{\partial^2 v_1}{\partial x_1^2} + (-2 + \omega_3) \frac{\delta_l^2 \rho c_s^2}{2\omega_3 \delta_t} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_3) \frac{\delta_l^2 \rho c_s^2}{2\omega_3 \delta_t} \frac{\partial^2 v_1}{\partial x_2^2} + C_1 \frac{\delta_l^3}{12\omega_1^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3} \\
& + (-24 + 36c_s^2 + 24\omega_1 + 5\omega_1^2 c_s^2 + 11\omega_1^2 v_1^2 + 60v_1^2 - 60\omega_1 v_1^2 - 4\omega_1^2 - 36\omega_1 c_s^2) \frac{\delta_l^3 v_1 \rho}{6\omega_1^2 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + C_2 \frac{\delta_l^3 v_1 \rho}{12\omega_1^2 \omega_3 \delta_t \omega_4} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + \\
& (-12 + 12\omega_3 - \omega_3^2) \frac{\delta_l^3 c_s^4}{6\omega_3^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} - \frac{\delta_l^3 v_1 \rho c_s^2}{6\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{v_2 \delta_l^3 v_1}{12\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + C_3 \frac{v_2 \delta_l^3 \rho}{6\omega_6 \omega_3 \delta_t} \frac{\partial^3 v_1}{\partial x_2^3} + \\
& (-1 + c_s^2 + 3v_2^2) \frac{\delta_l^3 v_1 \rho}{12\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_4 \frac{\delta_l^4 v_1}{12\omega_1^3 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + C_5 \frac{\delta_l^4 \rho}{12\omega_1^3 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_6 \frac{\delta_l^4 \rho}{12\omega_1^3 \omega_3^2 \delta_t \omega_4^2} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_7 \frac{\delta_l^4 v_1 c_s^2}{12\omega_1^3 \omega_3^2 \delta_t \omega_4^2} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\
& C_8 \frac{\delta_l^4 \rho c_s^2}{12\omega_1^3 \omega_3^2 \delta_t \omega_4} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2} + C_9 \frac{v_2 \delta_l^4 c_s^2}{12\omega_2 \omega_6^2 \omega_3^2 \delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + \\
& (3\omega_2\omega_6c_s^2 + v_2^2\omega_6 + 3\omega_2 - \omega_2\omega_6 - \omega_6 + 3\omega_6c_s^2 - 9\omega_2c_s^2 + \omega_2v_2^2\omega_6 - 3\omega_2v_2^2) \frac{v_2\delta_l^4 v_1 \rho}{12\omega_2\omega_6\delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{10} \frac{\delta_l^4 \rho c_s^2}{12\omega_2\omega_6\omega_3^2 \delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} \\
& + (-12\omega_2v_2^2c_s^2 - 2c_s^2 - 3\omega_2v_2^4 - 6v_2^2 - \omega_2c_s^4 + 6v_2^4 + 24v_2^2c_s^2 + \omega_2c_s^2 + 2c_s^4 + 3\omega_2v_2^2) \frac{\delta_l^4 v_1}{24\omega_2\delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{11} \frac{\delta_l^4 \rho}{24\omega_6^2 \omega_3^2 \delta_t} \frac{\partial^4 v_1}{\partial x_2^4} \\
& + (-4 + 6c_s^2 + 2\omega_2 + 10v_2^2 - 3\omega_2c_s^2 - 5\omega_2v_2^2) \frac{v_2\delta_l^4 v_1 \rho}{12\omega_2\delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= -12c_s^2 + 24\omega_1^2 v_1^2 c_s^2 - \omega_1^2 c_s^2 - 12\omega_1 c_s^4 - 36\omega_1 v_1^4 - 7\omega_1^2 v_1^2 - 36v_1^2 + 36\omega_1 v_1^2 + 36v_1^4 + 7\omega_1^2 v_1^4 - 144\omega_1 v_1^2 c_s^2 + \omega_1^2 c_s^4 + 144v_1^2 c_s^2 + 12c_s^4 + 12\omega_1 c_s^2 \\
C_2 &= 6\omega_1^2 v_1^2 \omega_4 - 6\omega_1 v_1^2 \omega_3 \omega_4 - 6\omega_1^2 \omega_4 + 36\omega_3 c_s^2 \omega_4 - 36\omega_1^2 c_s^2 + 12v_1^2 \omega_3 \omega_4 - 18\omega_1 \omega_3 c_s^2 \omega_4 + 12\omega_1 \omega_3 - 12\omega_1 v_1^2 \omega_3 - 12\omega_3 \omega_4 - 12\omega_1^2 v_1^2 - \\
& 36\omega_1 \omega_3 c_s^2 + 6\omega_1 \omega_3 \omega_4 - 3\omega_1^2 v_1^2 \omega_3 \omega_4 + 12\omega_1^2 + 12\omega_1^2 v_1^2 \omega_3 - 11\omega_1^2 \omega_3 c_s^2 \omega_4 + 3\omega_1^2 \omega_3 \omega_4 - 12\omega_1^2 \omega_3 + 36\omega_1^2 \omega_3 c_s^2 + 18\omega_1^2 c_s^2 \omega_4 \\
C_3 &= 6 + \omega_6 \omega_3 - 18c_s^2 + 3v_2^2 \omega_3 + 3v_2^2 \omega_6 - 6v_2^2 + 9\omega_3 c_s^2 - 3\omega_6 + 9\omega_6 c_s^2 - 3\omega_3 - 3\omega_6 \omega_3 c_s^2 - v_2^2 \omega_6 \omega_3 \\
C_4 &= 12 - 132c_s^2 - 18\omega_1 + 404\omega_1^2 v_1^2 c_s^2 - 78\omega_1^2 c_s^2 - 216\omega_1 c_s^4 + 10\omega_1^3 v_1^2 + 6\omega_1^3 c_s^2 - 216\omega_1 v_1^4 - 98\omega_1^2 v_1^2 - 156v_1^2 + 234\omega_1 v_1^2 - 5\omega_1^3 c_s^4 + 8\omega_1^2 + \\
& 144v_1^4 - \omega_1^3 - 34\omega_1^3 v_1^2 c_s^2 + 90\omega_1^2 v_1^4 - 1008\omega_1 v_1^2 c_s^2 + 82\omega_1^2 c_s^4 + 672v_1^2 c_s^2 + 144c_s^4 - 9\omega_1^3 v_1^4 + 198\omega_1 c_s^2 \\
C_5 &= 12 - 36c_s^2 - 18\omega_1 + 252\omega_1^2 v_1^2 c_s^2 - 22\omega_1^2 c_s^2 - 36\omega_1 c_s^4 + 14\omega_1^3 v_1^2 + 2\omega_1^3 c_s^2 - 756\omega_1 v_1^4 - 154\omega_1^2 v_1^2 - 252v_1^2 + 378\omega_1 v_1^2 - \omega_1^3 c_s^4 + 8\omega_1^2 + \\
& 504v_1^4 - \omega_1^3 - 18\omega_1^3 v_1^2 c_s^2 + 310\omega_1^2 v_1^4 - 648\omega_1 v_1^2 c_s^2 + 14\omega_1^2 c_s^4 + 432v_1^2 c_s^2 + 24c_s^4 - 29\omega_1^3 v_1^4 + 54\omega_1 c_s^2 \\
C_6 &= \omega_1^2 \omega_3^3 c_s^4 \omega_4^2 - 39\omega_1^3 v_1^4 \omega_3^3 \omega_4 + 18\omega_1^3 \omega_3^2 c_s^2 \omega_4 - 12\omega_3^3 c_s^2 \omega_4^2 - 108\omega_1^3 v_1^2 \omega_3^2 c_s^2 - 24\omega_1^3 \omega_3 c_s^4 \omega_4^2 - 6\omega_1^2 \omega_3^2 c_s^2 \omega_4^2 - 36\omega_1^2 v_1^2 \omega_3^2 \omega_4 + 19\omega_1^2 v_1^4 \omega_3^3 \omega_4^2 + \\
& 6\omega_1^3 \omega_3^3 c_s^4 \omega_4 + 36\omega_1^2 v_1^2 \omega_3^2 c_s^2 \omega_4 - 108\omega_1^2 v_1^2 \omega_3^2 c_s^2 - 72\omega_1^2 v_1^2 \omega_3^3 \omega_4 - 12\omega_1 \omega_3^2 c_s^4 \omega_4^2 - 6\omega_1^3 v_1^4 \omega_3^2 \omega_4^2 + 36\omega_1^3 v_1^2 \omega_3 c_s^2 \omega_4 - 4\omega_1^3 v_1^2 \omega_3^3 \omega_4^2 + 12\omega_1 \omega_3^3 c_s^2 \omega_4^2 + \\
& 12\omega_1^3 v_1^2 \omega_3^2 c_s^2 \omega_4^2 + 39\omega_1^3 v_1^2 \omega_3^3 \omega_4 - 18\omega_1^3 v_1^2 \omega_3 c_s^2 \omega_4 - 99\omega_1^3 v_1^2 \omega_3^2 c_s^2 \omega_4 + 108\omega_1^3 v_1^2 \omega_3^2 c_s^2 - 36\omega_1^3 v_1^2 \omega_3^3 + 18\omega_1^2 v_1^2 \omega_3^2 c_s^2 \omega_4^2 - 36\omega_1^2 v_1^4 \omega_3^3 + \\
& 36\omega_1^3 v_1^4 \omega_3^2 \omega_4 - 19\omega_1^2 v_1^2 \omega_3^2 \omega_4^2 + 36\omega_1^3 v_1^2 \omega_3^2 - 36\omega_1 v_1^2 \omega_3^2 c_s^2 \omega_4^2 - 12\omega_1^2 \omega_3^2 c_s^2 \omega_4 + 12\omega_1^3 v_1^2 \omega_3 c_s^4 \omega_4 + 72\omega_1^2 v_1^4 \omega_3^3 \omega_4 + 6\omega_1^3 v_1^2 \omega_3^2 \omega_4^2 - \omega_1^3 \omega_3^3 c_s^4 \omega_4^2 - \\
& 6\omega_1^2 \omega_3^3 c_s^4 \omega_4 + 4\omega_1^3 v_1^4 \omega_3^3 \omega_4^2 - 5\omega_1^3 \omega_3^2 c_s^2 \omega_4^2 + 252v_1^2 \omega_3^2 c_s^2 \omega_4^2 - 90\omega_1 v_1^4 \omega_3^3 \omega_4^2 + 12\omega_1^3 c_s^4 \omega_4^2 - 3\omega_1^3 v_1^2 \omega_3^2 c_s^2 \omega_4^2 + 12\omega_1 \omega_3^2 c_s^2 \omega_4^2 + 198\omega_1^2 v_1^2 \omega_3^3 c_s^2 \omega_4 - \\
& 12\omega_1 \omega_3^3 c_s^4 \omega_4^2 - \omega_1^2 \omega_3^3 c_s^2 \omega_4^2 - 108\omega_1 v_1^2 \omega_3^2 c_s^2 \omega_4 - 18\omega_1^3 \omega_3^2 c_s^4 \omega_4 + 72v_1^4 \omega_3^3 \omega_4^2 + 12\omega_3^3 c_s^4 \omega_4^2 + 36\omega_1 v_1^2 \omega_3^3 \omega_4 + 6\omega_1^2 \omega_3^2 c_s^4 \omega_4^2 + 6\omega_1^3 \omega_3 c_s^2 \omega_4^2 - \\
& 6\omega_1^3 \omega_3^2 c_s^2 \omega_4 - 36\omega_1^3 v_1^4 \omega_3^2 - 12\omega_1^3 \omega_3 c_s^2 \omega_4 + 12\omega_1^2 \omega_3^2 c_s^4 \omega_4 + 90\omega_1 v_1^2 \omega_3^3 \omega_4^2 - 306\omega_1 v_1^2 \omega_3^2 c_s^2 \omega_4^2 + 6\omega_1^2 \omega_3^3 c_s^2 \omega_4 + 13\omega_1^3 \omega_3^2 c_s^4 \omega_4^2 + 60\omega_1^2 v_1^2 \omega_3^3 c_s^2 \omega_4^2 + \\
& 36\omega_1^3 v_1^4 \omega_3^3 - 72v_1^4 \omega_3^3 \omega_4^2 - 36\omega_1 v_1^4 \omega_3^3 \omega_4 + 36\omega_1^2 v_1^4 \omega_3^3 + 54\omega_1^3 v_1^2 \omega_3^2 c_s^2 \omega_4 \\
C_7 &= 4\omega_1^2 v_1^2 \omega_3^2 \omega_4^2 - 18\omega_1^3 \omega_3 \omega_4 - 18\omega_1^3 \omega_3^2 c_s^2 \omega_4 - 36\omega_1^3 \omega_3 c_s^2 - 12\omega_1^3 v_1^2 \omega_3 + 12\omega_1^2 \omega_3^2 c_s^2 \omega_4^2 - 6\omega_1^3 v_1^2 \omega_3^2 \omega_4 + 36\omega_1^3 c_s^2 \omega_4^2 - 4\omega_1^2 \omega_3^2 \omega_4^2 - 12\omega_1^3 \omega_4^2 + \\
& 12\omega_3^3 \omega_4 - 36\omega_1^3 c_s^2 \omega_4 - 6\omega_1^2 \omega_3^2 \omega_4 - 36\omega_1^2 \omega_3^2 c_s^2 + 12\omega_1^3 v_1^2 \omega_3^2 + 12\omega_1^2 \omega_4^2 + 18\omega_1^2 \omega_3^2 c_s^2 \omega_4 + \omega_1^3 v_1^2 \omega_3^2 \omega_4^2 + 12\omega_3^3 \omega_3 \omega_4^2 - 12\omega_1^2 v_1^2 \omega_4^2 + 6\omega_1^2 v_1^2 \omega_3^2 \omega_4 + \\
& 5\omega_1^3 \omega_3^2 c_s^2 \omega_4^2 - 12\omega_1^3 v_1^2 \omega_3 \omega_4^2 - 54\omega_1 \omega_3^2 c_s^2 \omega_4^2 + 12\omega_1^2 v_1^2 \omega_3 \omega_4 - \omega_1^3 \omega_3^2 \omega_4^2 - 36\omega_1^2 c_s^2 \omega_4^2 - 18\omega_1 v_1^2 \omega_3 \omega_4^2 - 12\omega_1^3 \omega_3^2 + 36\omega_3^2 c_s^2 \omega_4^2 + 36\omega_1^2 \omega_3 c_s^2 \omega_4 - \\
& 12\omega_1^3 v_1^2 \omega_4 - 12\omega_1^2 \omega_3 \omega_4 + 12v_1^2 \omega_3^2 \omega_4^2 - 40\omega_1^3 \omega_3 c_s^2 \omega_4^2 + 12\omega_1^3 v_1^2 \omega_4^2 - 6\omega_1^2 \omega_3 \omega_4^2 + 54\omega_1^3 \omega_3 c_s^2 \omega_4 + 12\omega_1^2 \omega_3^2 - 12\omega_1^2 v_1^2 \omega_3^2 + 18\omega_1^2 \omega_3 c_s^2 \omega_4^2 + \\
& 6\omega_1^3 \omega_3^2 \omega_4 + 6\omega_1^2 v_1^2 \omega_3 \omega_4^2 + 36\omega_1^3 \omega_3^2 c_s^2 + 18\omega_1 \omega_3^2 \omega_4^2 + 12\omega_1^3 \omega_3 + 18\omega_1^3 v_1^2 \omega_3 \omega_4 - 12\omega_3^2 \omega_4^2
\end{aligned}$$

$$\begin{aligned}
C_8 &= 12\omega_1^2\omega_3^3c_s^2 - 36\omega_1v_1^2\omega_3\omega_4 + 24\omega_1\omega_3^3 + 72\omega_1v_1^2\omega_3^2 - 24\omega_1\omega_3^2 + 36v_1^2\omega_3^3 - 12\omega_1\omega_3c_s^2\omega_4 - 72\omega_1v_1^2\omega_3^2 - 24\omega_1^2\omega_3^2c_s^2 - 4\omega_1^2\omega_3^2c_s^2\omega_4 + \\
&12\omega_1\omega_3\omega_4 + 12\omega_3^2\omega_4 + 24\omega_1\omega_3^2c_s^2 - 12\omega_1\omega_3^2\omega_4 + 36\omega_1^2v_1^2\omega_3 + 18\omega_1^2\omega_3c_s^2\omega_4 - 12\omega_1^2\omega_3 + 12\omega_1^2\omega_3c_s^2 - 36v_1^2\omega_3^2\omega_4 + 24\omega_1^2\omega_3^2 - 24\omega_1\omega_3^2c_s^2 - \\
&\omega_1^2\omega_3^2c_s^2\omega_4 - 12\omega_3^2c_s^2\omega_4 - 72\omega_1^2v_1^2\omega_3^2 + 12\omega_3^2c_s^2 - 12\omega_1^2c_s^2\omega_4 - 12\omega_1^2\omega_3^2 + 36\omega_1v_1^2\omega_3^2\omega_4 + 36\omega_1^2v_1^2\omega_3^2 + 12\omega_1\omega_3^2c_s^2\omega_4 - 12\omega_3^3 \\
C_9 &= -12\omega_2\omega_6^2 - 12\omega_6\omega_3 - 12\omega_2v_2^2\omega_6^2\omega_3 - \omega_2\omega_6^2\omega_3^2 - 9\omega_2\omega_6\omega_3^2c_s^2 + 12\omega_2\omega_3 + 18\omega_2\omega_3^2c_s^2 + 18\omega_6^2\omega_3c_s^2 - 12v_2^2\omega_6^2 - 36\omega_2\omega_6^2\omega_3c_s^2 - 36\omega_2\omega_6c_s^2 - \\
&18\omega_6\omega_3^2c_s^2 + 6v_2^2\omega_6^2\omega_3 + v_2^2\omega_6^2\omega_3^2 - 36\omega_6^2c_s^2 - 6\omega_2\omega_3^2 + \omega_2v_2^2\omega_6^2\omega_3^2 + 12\omega_2\omega_6 + 6\omega_6\omega_3^2 + 12\omega_2\omega_6^2\omega_3 - 3\omega_2v_2^2\omega_6\omega_3^2 - \omega_6^2\omega_3^2 - 18\omega_2\omega_6\omega_3 - \\
&12\omega_2v_2^2\omega_3 - 6v_2^2\omega_6\omega_3^2 + 12\omega_6^2 + 12\omega_2v_2^2\omega_6^2 + 3\omega_2\omega_6^2\omega_3^2c_s^2 - 12\omega_2v_2^2\omega_6 + 36\omega_2\omega_6^2c_s^2 + 36\omega_6\omega_3c_s^2 + 6\omega_2v_2^2\omega_3^2 + 12v_2^2\omega_6\omega_3 - 6\omega_6^2\omega_3 + \\
&18\omega_2v_2^2\omega_6\omega_3 + 54\omega_2\omega_6\omega_3c_s^2 + 3\omega_2\omega_6\omega_3^2 + 3\omega_6^2\omega_3^2c_s^2 - 36\omega_2\omega_3c_s^2 \\
C_{10} &= 12\omega_6\omega_3 - 5\omega_2\omega_6\omega_3c_s^2 - 12\omega_2\omega_3 - 18\omega_2\omega_3^2c_s^2 - 12\omega_2\omega_6c_s^2 + 6\omega_6\omega_3^2c_s^2 + 6\omega_2\omega_3^2c_s^2 + 36v_2^2\omega_3^2 - 6\omega_2\omega_3^3 - \omega_2\omega_6\omega_3^2c_s^2 - \omega_6\omega_3^3 + 18\omega_2\omega_3^2 - \\
&18v_2^2\omega_3^3 + \omega_6\omega_3^2c_s^2 - 6\omega_6\omega_3^2 + 3v_2^2\omega_6\omega_3^3 + 12\omega_3^2c_s^2 - 3\omega_2v_2^2\omega_6\omega_3^2 + 36\omega_2v_2^2\omega_3 + 18v_2^2\omega_6\omega_3^2 - 12\omega_3^3 - 12\omega_6\omega_3c_s^2 - 54\omega_2v_2^2\omega_3^2 - 6\omega_3^2c_s^2 - \\
&36v_2^2\omega_6\omega_3 + 18\omega_2\omega_6\omega_3c_s^2 + \omega_2\omega_6\omega_3^2 + 18\omega_2v_2^2\omega_3^2 + 12\omega_2\omega_3c_s^2 + 6\omega_3^3 \\
C_{11} &= 24\omega_6\omega_3c_s^4 + 108v_2^2\omega_3^2c_s^2 - 12v_2^4\omega_6^2\omega_3^2 + 12\omega_6^2\omega_3^2c_s^2 + 6v_2^2\omega_6^2\omega_3^2c_s^2 + 3v_2^4\omega_6^2\omega_3^2 + 24\omega_6^2\omega_3^2c_s^4 + 24\omega_6\omega_3^2c_s^2 + 72v_2^2\omega_3^2 - 12v_2^2\omega_6^2\omega_3^2c_s^2 + \\
&12v_2^2\omega_6^2\omega_3^2 - 216v_2^2\omega_3^2c_s^2 - 3v_2^2\omega_6^2\omega_3^3 - 36v_2^2\omega_3^3 + 72v_2^2\omega_6\omega_3c_s^2 - 6\omega_6\omega_3^3c_s^2 - 3\omega_6^2\omega_3^3c_s^4 + 30v_2^2\omega_6\omega_3^3 + 144v_2^2\omega_6\omega_3^2c_s^2 + 24\omega_6^2c_s^4 + \omega_6^2\omega_3^2c_s^2 - \\
&36v_2^2\omega_6^2\omega_3c_s^2 - 72v_2^2\omega_6\omega_3^2 + 6\omega_6\omega_3^2c_s^4 - 30v_2^2\omega_6\omega_3^3 + 36v_2^4\omega_3^3 - 48\omega_6^2\omega_3c_s^4 - 72v_2^2\omega_6\omega_3^2c_s^2 - 24\omega_6\omega_3c_s^2 - 72v_2^4\omega_3^2 - 24\omega_6\omega_3^2c_s^4 + 72v_2^2\omega_6\omega_3^2 - 8\omega_6^2\omega_3^2c_s^2
\end{aligned}$$

2.6.4 Conservation of momentum: ρv_2



attached text file: output_d2q9_nse_culbm1_symbolic_pde_02.txt

$$\begin{aligned}
&v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{v_2 \delta_1 v_1}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{v_2 \delta_1 \rho}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\delta_1 v_1 \rho}{\delta_t} \frac{\partial v_2}{\partial x_1} + (c_s^2 + v_2^2) \frac{\delta_1}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{2v_2 \delta_1 \rho}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_3) \frac{\delta_1^2 c_s^2}{2\omega_3 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\
&(-2 + \omega_3) \frac{\delta_1^2 c_s^2}{2\omega_3 \delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (-2 + 4c_s^2 + \omega_2 + 6v_2^2 - 2\omega_2 c_s^2 - 3\omega_2 v_2^2) \frac{\delta_1^2}{\omega_2 \delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + (2 - \omega_2) \frac{3v_2 \delta_1^2 \rho}{\omega_2 \delta_t} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + \\
&(-2 + \omega_3) \frac{\delta_1^2 \rho c_s^2}{2\omega_3 \delta_t} \frac{\partial^2 v_2}{\partial x_1^2} + (-2 + \omega_3) \frac{\delta_1^2 \rho c_s^2}{2\omega_3 \delta_t} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + (-2 + 6c_s^2 + \omega_2 + 2v_2^2 - 3\omega_2 c_s^2 - \omega_2 v_2^2) \frac{v_2 \delta_1^2}{2\omega_2 \delta_t} \frac{\partial^2 \rho}{\partial x_2^2} + \\
&(-2 + 2c_s^2 + \omega_2 + 6v_2^2 - \omega_2 c_s^2 - 3\omega_2 v_2^2) \frac{\delta_1^2 \rho}{2\omega_2 \delta_t} \frac{\partial^2 v_2}{\partial x_2^2} + (-1 + 3c_s^2 + v_1^2) \frac{v_2 \delta_1^3 v_1}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3v_1^2) \frac{v_2 \delta_1^3 \rho}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + \\
&C_1 \frac{\delta_1^3 v_1 \rho}{6\omega_3 \delta_t \omega_4} \frac{\partial^3 v_2}{\partial x_1^3} + (-12 + 12\omega_3 - \omega_3^2) \frac{\delta_1^3 c_s^4}{6\omega_3^2 \delta_t} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} - \frac{v_2 \delta_1^3 \rho c_s^2}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + C_2 \frac{v_2 \delta_1^3 \rho}{12\omega_2^2 \omega_6 \omega_3 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_3 \frac{\delta_1^3}{12\omega_2^2 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + \\
&(-24 + 36c_s^2 + 11\omega_2^2 v_2^2 - 4\omega_2^2 + 5\omega_2^2 c_s^2 + 24\omega_2 + 60v_2^2 - 36\omega_2 c_s^2 - 60\omega_2 v_2^2) \frac{v_2 \delta_1^3 \rho}{6\omega_2^2 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
&(-2c_s^2 - \omega_1 c_s^4 - 3\omega_1 v_1^4 - 6v_1^2 + 3\omega_1 v_1^2 + 6v_1^4 - 12\omega_1 v_1^2 c_s^2 + 24v_1^2 c_s^2 + 2c_s^4 + \omega_1 c_s^2) \frac{v_2 \delta_1^4}{24\omega_1 \delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + \\
&(-4 + 6c_s^2 + 2\omega_1 + 10v_1^2 - 5\omega_1 v_1^2 - 3\omega_1 c_s^2) \frac{v_2 \delta_1^4 v_1 \rho}{12\omega_1 \delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_4 \frac{\delta_1^4 \rho}{24\omega_3^3 \delta_t \omega_4^2} \frac{\partial^4 v_2}{\partial x_1^4} + C_5 \frac{\delta_1^4 v_1 c_s^2}{12\omega_1 \omega_3^2 \delta_t \omega_4^2} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + \\
&C_6 \frac{\delta_1^4 \rho c_s^2}{12\omega_1 \omega_3^3 \delta_t \omega_4} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + (3\omega_1 + 3\omega_1 c_s^2 \omega_4 + 3c_s^2 \omega_4 - 3\omega_1 v_1^2 + \omega_1 v_1^2 \omega_4 + v_1^2 \omega_4 - \omega_1 \omega_4 - 9\omega_1 c_s^2 - \omega_4) \frac{v_2 \delta_1^4 v_1 \rho}{12\omega_1 \delta_t \omega_4} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
&C_7 \frac{v_2 \delta_1^4 c_s^2}{12\omega_2^2 \omega_6^2 \omega_3^2 \delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_8 \frac{\delta_1^4 \rho c_s^2}{12\omega_2^2 \omega_6 \omega_3^2 \delta_t} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_9 \frac{\delta_1^4 \rho}{12\omega_2^2 \omega_6^2 \omega_3^2 \delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^2} + C_{10} \frac{v_2 \delta_1^4}{12\omega_2^2 \delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{11} \frac{\delta_1^4 \rho}{12\omega_2^2 \delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= 6 - 18c_s^2 - 3\omega_3 c_s^2 \omega_4 - v_1^2 \omega_3 \omega_4 + 9c_s^2 \omega_4 + 9\omega_3 c_s^2 + \omega_3 \omega_4 - 6v_1^2 + 3v_1^2 \omega_3 - 3\omega_3 + 3v_1^2 \omega_4 - 3\omega_4 \\
C_2 &= -12\omega_6 \omega_3 + 6\omega_2^2 v_2^2 \omega_6 - 12\omega_2^2 v_2^2 + 12\omega_2 \omega_3 + 36\omega_2^2 \omega_3 c_s^2 + 12\omega_2^2 + 3\omega_2^2 \omega_6 \omega_3 - 36\omega_2^2 c_s^2 + 12\omega_2^2 v_2^2 \omega_3 - 11\omega_2^2 \omega_6 \omega_3 c_s^2 - 3\omega_2^2 v_2^2 \omega_6 \omega_3 - \\
&6\omega_2^2 \omega_6 + 6\omega_2 \omega_6 \omega_3 - 12\omega_2 v_2^2 \omega_3 + 36\omega_6 \omega_3 c_s^2 + 12v_2^2 \omega_6 \omega_3 - 6\omega_2 v_2^2 \omega_6 \omega_3 - 18\omega_2 \omega_6 \omega_3 c_s^2 - 12\omega_2^2 \omega_3 + 18\omega_2^2 \omega_6 c_s^2 - 36\omega_2 \omega_3 c_s^2 \\
C_3 &= -144\omega_2 v_2^2 c_s^2 - 12c_s^2 - 7\omega_2^2 v_2^2 - 36\omega_2 v_2^4 - \omega_2^2 c_s^2 - 36v_2^2 - 12\omega_2 c_s^4 + \omega_2^2 c_s^4 + 36v_2^4 + 144v_2^2 c_s^2 + 12\omega_2 c_s^2 + 24\omega_2^2 v_2^2 c_s^2 + 12c_s^4 + 7\omega_2^2 v_2^4 + 36\omega_2 v_2^2 \\
C_4 &= 24\omega_3^2 c_s^4 \omega_4^2 + 72v_1^2 \omega_3^2 + \omega_3^2 c_s^2 \omega_4^2 - 24\omega_3 c_s^2 \omega_4 - 36v_1^2 \omega_3^3 - 12v_1^2 \omega_3^2 c_s^2 \omega_4^2 + 144v_1^2 \omega_3^2 c_s^2 \omega_4 - 6\omega_3^3 c_s^2 \omega_4 + 12\omega_3 c_s^2 \omega_4^2 - 24\omega_3^2 c_s^4 \omega_4 + 36v_1^4 \omega_3^3 + \\
&6v_1^2 \omega_3^3 c_s^2 \omega_4^2 + 72v_1^4 \omega_3^2 \omega_4 + 72v_1^2 \omega_3^2 c_s^2 \omega_4 + 108v_1^2 \omega_3^3 c_s^2 + 30v_1^2 \omega_3^3 \omega_4 - 8\omega_3^2 c_s^2 \omega_4^2 + 3v_1^4 \omega_3^3 \omega_4^2 - 72v_1^4 \omega_3^3 - 3\omega_3^3 c_s^4 \omega_4^2 + 12v_1^2 \omega_3^2 \omega_4^2 + 24\omega_3 c_s^4 \omega_4 + \\
&6\omega_3^3 c_s^4 \omega_4 - 72v_1^2 \omega_3^2 \omega_4 - 48\omega_3 c_s^4 \omega_4^2 - 216v_1^2 \omega_3^2 c_s^2 + 24\omega_2^2 c_s^2 \omega_4 - 30v_1^4 \omega_3^3 \omega_4 + 24c_s^4 \omega_4^2 - 3v_1^2 \omega_3^3 \omega_4^2 - 12v_1^4 \omega_3^3 \omega_4^2 - 72v_1^2 \omega_3^3 c_s^2 \omega_4 - 36v_1^2 \omega_3^3 c_s^2 \omega_4^2 \\
C_5 &= 18\omega_1 v_1^2 \omega_3 \omega_4 + 12\omega_1 \omega_3 \omega_4^2 + 36\omega_3 c_s^2 \omega_4 + 6\omega_1 v_1^2 \omega_3^2 - 6\omega_3 \omega_4^2 + 12v_1^2 \omega_3 \omega_4 - 6\omega_1 \omega_3^2 - 36\omega_1 c_s^2 \omega_4 + 54\omega_1 \omega_3 c_s^2 \omega_4 - 36\omega_1 \omega_3 c_s^2 \omega_4^2 - 36c_s^2 \omega_4^2 + \\
&12\omega_1 \omega_3 + 6v_1^2 \omega_3 \omega_4^2 + 36\omega_1 c_s^2 \omega_4^2 - 12\omega_1 v_1^2 \omega_3 - 12\omega_3 \omega_4 + 18\omega_3 c_s^2 \omega_4^2 + 12\omega_4^2 - 36\omega_1 \omega_3 c_s^2 - 12\omega_1 v_1^2 \omega_3 \omega_4^2 - 18\omega_1 \omega_3 \omega_4 - 12\omega_1 \omega_4^2 + 6\omega_3^2 \omega_4 + \\
&3\omega_1 \omega_3^2 c_s^2 \omega_4^2 + 18\omega_1 \omega_3 c_s^2 + 3\omega_1 \omega_3^2 \omega_4 + \omega_1 v_1^2 \omega_3^2 \omega_4^2 + 3\omega_3^2 c_s^2 \omega_4^2 - 12v_1^2 \omega_4^2 + v_1^2 \omega_3^2 \omega_4^2 + 12\omega_1 v_1^2 \omega_4^2 - 6v_1^2 \omega_3 \omega_4 - 12\omega_1 v_1^2 \omega_4 - 18\omega_3^2 c_s^2 \omega_4 - \\
&\omega_1 \omega_3^2 \omega_4^2 - 3\omega_1 v_1^2 \omega_3 \omega_4 + 12\omega_1 \omega_4 - 9\omega_1 \omega_3^2 c_s^2 \omega_4 - \omega_3^2 \omega_4^2 \\
C_6 &= -6\omega_1 \omega_3^3 + 36v_1^2 \omega_3^2 - 12\omega_3 c_s^2 \omega_4 - 54\omega_1 v_1^2 \omega_3^2 - 36v_1^2 \omega_3 \omega_4 + 18\omega_1 \omega_3^2 - 18v_1^2 \omega_3^3 - 12\omega_1 c_s^2 \omega_4 + 18\omega_1 \omega_3 c_s^2 \omega_4 + 18\omega_1 v_1^2 \omega_3^3 - \omega_1 \omega_3^3 c_s^2 \omega_4 - \\
&12\omega_1 \omega_3 + \omega_3^3 c_s^2 \omega_4 + 36\omega_1 v_1^2 \omega_3 + 12\omega_3 \omega_4 + 12\omega_1 \omega_3 c_s^2 + 12\omega_3^2 c_s^2 - 6\omega_3^2 \omega_4 - 18\omega_1 \omega_3^2 c_s^2 + \omega_1 \omega_3^2 \omega_4 + 3v_1^2 \omega_3^3 \omega_4 + 18v_1^2 \omega_3^2 \omega_4 + 6\omega_1 \omega_3^3 c_s^2 - 12\omega_3^3 + \\
&6\omega_3^2 c_s^2 \omega_4 - \omega_3^3 \omega_4 - 6\omega_3^2 c_s^2 - 3\omega_1 v_1^2 \omega_3^2 \omega_4 - 5\omega_1 \omega_3^2 c_s^2 \omega_4 + 6\omega_3^3
\end{aligned}$$

$$C_7 = 12\omega_2^3\omega_6^2\omega_3 + 18\omega_2\omega_6^2\omega_3^2 + 6\omega_2^2v_2^2\omega_6\omega_3^2 + 18\omega_2^3v_2^2\omega_6\omega_3 - 12\omega_2^2v_2^2\omega_3^2 - 36\omega_2^2\omega_6^2c_s^2 + 54\omega_2^3\omega_6\omega_3c_s^2 - 6\omega_2^2\omega_6\omega_3^2 + 5\omega_2^3\omega_6^2\omega_3^2c_s^2 + 12v_2^2\omega_6^2\omega_3^2 - 36\omega_2^3\omega_6c_s^2 - 12\omega_2^2\omega_6\omega_3 + 12\omega_2^2\omega_6^2\omega_3^2c_s^2 + 36\omega_2^3\omega_6^2\omega_3^2 - 6\omega_2^2v_2^2\omega_6\omega_3^2 + 36\omega_2^3\omega_6\omega_3c_s^2 - 12\omega_2^2v_2^2\omega_6^2 - \omega_2^3\omega_6^2\omega_3^2 - 18\omega_2v_2^2\omega_6^2\omega_3^2 + 12\omega_2^2v_2^2\omega_6\omega_3 + 12\omega_2^3\omega_3^2 - 36\omega_2^3\omega_3c_s^2 + \omega_2^3v_2^2\omega_6^2\omega_3^2 - 12\omega_2^2\omega_6^2\omega_3^2 + 6\omega_2^3\omega_6\omega_3^2 + 18\omega_2^2\omega_6\omega_3^2c_s^2 + 6\omega_2^2v_2^2\omega_6^2\omega_3 + 12\omega_2^3v_2^2\omega_3^2 + 36\omega_2^2\omega_6^2c_s^2 + 12\omega_2^3\omega_3 - 12\omega_2^3v_2^2\omega_6 - 6\omega_2^2\omega_6^2\omega_3 + 18\omega_2^2\omega_6\omega_3c_s^2 - 12\omega_2^3\omega_6^2 - 54\omega_2\omega_6^2\omega_3^2c_s^2 - 4\omega_2^2\omega_6^2\omega_3^2 + 12\omega_2^3v_2^2\omega_6^2 + 12\omega_2^2\omega_6 - 40\omega_2^3\omega_6^2\omega_3c_s^2 - 12\omega_2^3v_2^2\omega_3 - 36\omega_2^2\omega_3^2c_s^2 - 12\omega_2^3\omega_3^2 + 12\omega_2^2\omega_6^2 - 18\omega_2^3\omega_6\omega_3 + 4\omega_2^2v_2^2\omega_6^2\omega_3^2 - 12\omega_2^3v_2^2\omega_6^2\omega_3 + 36\omega_2^2\omega_6^2c_s^2 - 18\omega_2^3\omega_6\omega_3^2c_s^2$$

$$C_8 = 12\omega_2\omega_6\omega_3c_s^2 - 72\omega_2^2v_2^2\omega_3^2 + 24\omega_2\omega_3^2c_s^2 + 12\omega_2^2\omega_3c_s^2 - 12\omega_6\omega_3^2c_s^2 + 36\omega_2^2v_2^2\omega_3^3 - 24\omega_2\omega_3^3c_s^2 + 24\omega_2\omega_3^3 - 24\omega_2\omega_3^2 + 36\omega_2^2v_2^2\omega_3 + 36v_2^2\omega_3^3 + 18\omega_2^2\omega_6\omega_3c_s^2 + 12\omega_6\omega_3^2 + 24\omega_2^2\omega_3^2 + 12\omega_2^2\omega_3^3c_s^2 + 36\omega_2v_2^2\omega_6\omega_3^2 - 4\omega_2^2\omega_6\omega_3^2c_s^2 + 12\omega_2\omega_6\omega_3 - 36v_2^2\omega_6\omega_3^2 - 12\omega_2^2\omega_3^3 - \omega_2^2\omega_6\omega_3^3c_s^2 - 24\omega_2^2\omega_3^2c_s^2 + 72\omega_2v_2^2\omega_3^2 + 12\omega_3^3c_s^2 - 36\omega_2v_2^2\omega_6\omega_3 - 12\omega_2\omega_6\omega_3c_s^2 - 12\omega_2\omega_6\omega_3^2 - 12\omega_2^2\omega_3 - 72\omega_2v_2^2\omega_3^3 - 12\omega_2^2\omega_6c_s^2 - 12\omega_3^3$$

$$C_9 = -36\omega_2^2v_2^4\omega_3^2 - \omega_2^2\omega_6^2\omega_3^2c_s^2 - 24\omega_2^3\omega_6^2\omega_3c_s^4 + 18\omega_2^2v_2^2\omega_6^2\omega_3^2c_s^2 - 39\omega_2^2v_2^4\omega_6\omega_3^3 + 54\omega_2^3v_2^2\omega_6\omega_3^2c_s^2 - 12\omega_2\omega_6^2\omega_3^2c_s^4 - 90\omega_2v_2^4\omega_6^2\omega_3^3 + 252v_2^2\omega_6^2\omega_3^2c_s^2 - 12\omega_2^3\omega_6\omega_3^2c_s^2 - 6\omega_2^2\omega_6\omega_3^2c_s^4 - 72\omega_2^2v_2^2\omega_6\omega_3^3 + 72v_2^4\omega_6^2\omega_3^3 + 36\omega_2^3v_2^2\omega_6\omega_3^2 - 18\omega_2^3\omega_6\omega_3^2c_s^4 + 36\omega_2^2v_2^4\omega_3^3 - 5\omega_2^3\omega_6^2\omega_3^2c_s^2 - 36\omega_2v_2^2\omega_6^2\omega_3^2c_s^2 + 36\omega_2^2v_2^2\omega_3^3 - 18\omega_2^3v_2^2\omega_6^2\omega_3^2c_s^2 + 12\omega_2^2\omega_6\omega_3^2c_s^4 - 99\omega_2^2v_2^2\omega_6\omega_3^2c_s^2 - 12\omega_2\omega_6^2\omega_3^2c_s^4 + 39\omega_2^3v_2^2\omega_6\omega_3^3 - 6\omega_2^2\omega_6^2\omega_3^2c_s^2 + 90\omega_2v_2^2\omega_6^2\omega_3^3 + 60\omega_2^2v_2^2\omega_6^2\omega_3^2c_s^2 - 36\omega_2^3v_2^2\omega_6\omega_3^2 + 72\omega_2^2v_2^4\omega_6\omega_3^3 - 72v_2^2\omega_6^2\omega_3^3 - 306\omega_2v_2^2\omega_6^2\omega_3^2c_s^2 + 12\omega_6^2\omega_3^3c_s^4 + 6\omega_2^3\omega_6\omega_3^3c_s^4 + 12\omega_2^3\omega_6^2c_s^4 + 19\omega_2^2v_2^4\omega_6^2\omega_3^3 + 12\omega_2^3v_2^2\omega_6^2\omega_3^2c_s^2 + 6\omega_2^3v_2^2\omega_6^2\omega_3^2 + 6\omega_2^2\omega_6^2\omega_3^2c_s^4 - 36\omega_2^3v_2^2\omega_3^3 + 198\omega_2^2v_2^2\omega_6\omega_3^2c_s^2 - 108\omega_2^3v_2^2\omega_3^2c_s^2 - 12\omega_2^2\omega_6\omega_3^2c_s^2 + 12\omega_2\omega_6^2\omega_3^2c_s^2 - 36\omega_2^2v_2^4\omega_3^3 - 108\omega_2v_2^2\omega_6\omega_3^3c_s^2 - 12\omega_2^2\omega_3^3c_s^2 - 6\omega_2^3\omega_6\omega_3^3c_s^2 - 108\omega_2^2v_2^2\omega_3^2c_s^2 + 36\omega_2^3v_2^2\omega_3^2 - 4\omega_2^2v_2^2\omega_6^2\omega_3^3 + 36\omega_2v_2^2\omega_6\omega_3^3 - \omega_2^3\omega_6^2\omega_3^3c_s^4 + 12\omega_2\omega_6^2\omega_3^2c_s^2 - 6\omega_2^3v_2^2\omega_6^2\omega_3^2 - 19\omega_2^2v_2^2\omega_6^2\omega_3^2 + 36\omega_2^2v_2^2\omega_6\omega_3^2c_s^2 + 6\omega_2^2\omega_6\omega_3^2c_s^2 + 12\omega_2^3\omega_6\omega_3^2c_s^2 + 108\omega_2^2v_2^2\omega_3^2c_s^2 + 6\omega_2^3\omega_6\omega_3^2c_s^2 + \omega_2^2\omega_6^2\omega_3^3c_s^4 - 3\omega_2^3v_2^2\omega_6^2\omega_3^2c_s^2 + 13\omega_2^3\omega_6^2\omega_3^2c_s^4 + 4\omega_2^2v_2^4\omega_6^2\omega_3^3 + 36\omega_2^2v_2^2\omega_6\omega_3^2c_s^2 + 18\omega_2^3\omega_6\omega_3^2c_s^2 - 36\omega_2v_2^4\omega_6\omega_3^3$$

$$C_{10} = 12 + 6\omega_2^3c_s^2 - 1008\omega_2v_2^2c_s^2 - 132c_s^2 - 98\omega_2^2v_2^2 - \omega_2^3 - 216\omega_2v_2^4 + 8\omega_2^2 + 10\omega_2^3v_2^2 - 78\omega_2^2c_s^2 - 18\omega_2 - 156v_2^2 - 216\omega_2c_s^4 - 34\omega_2^3v_2^2c_s^2 - 9\omega_2^2v_2^4 + 82\omega_2^2c_s^4 + 144v_2^4 + 672v_2^2c_s^2 + 198\omega_2c_s^2 + 404\omega_2^2v_2^2c_s^2 - 5\omega_2^3c_s^4 + 144c_s^4 + 90\omega_2^2v_2^4 + 234\omega_2v_2^2$$

$$C_{11} = 12 + 2\omega_2^3c_s^2 - 648\omega_2v_2^2c_s^2 - 36c_s^2 - 154\omega_2^2v_2^2 - \omega_2^3 - 756\omega_2v_2^4 + 8\omega_2^2 + 14\omega_2^3v_2^2 - 22\omega_2^2c_s^2 - 18\omega_2 - 252v_2^2 - 36\omega_2c_s^4 - 18\omega_2^3v_2^2c_s^2 - 29\omega_2^2v_2^4 + 14\omega_2^2c_s^4 + 504v_2^4 + 432v_2^2c_s^2 + 54\omega_2c_s^2 + 252\omega_2^2v_2^2c_s^2 - \omega_2^2c_s^4 + 24c_s^4 + 310\omega_2^2v_2^4 + 378\omega_2v_2^2$$

2.7 CuLBM2

2.7.1 Definitions

Collision operator \mathbf{C} :

$$\mathbf{C}(\mathbf{f}) = \mathbf{M}^{-1}\mathbf{G}^{-1}\left(\mathbf{N}^{-1}\mathbf{S}\mathbf{N}\left(\boldsymbol{\gamma}^{(eq)} - \mathbf{G}(\mathbf{M}\mathbf{f})\right)\right),$$

where

$$\mathbf{S} = \text{diag}(0, 0, 0, \omega_1, \omega_2, \omega_1, \omega_3, \omega_3, \omega_4),$$

$\omega_1, \omega_2, \omega_3, \omega_4 \in (0, 2)$.

Matrix \mathbf{N} defines the combination of cumulants for the collision as

$$\mathbf{N} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

The nonlinear operator \mathbf{G} is the same as in CuLBM1 in Section ?? and, again, the equilibrium cumulant vector $\boldsymbol{\gamma}^{(eq)}$ is defined by

$$\boldsymbol{\gamma}^{(eq)} = \left(\rho, v_1, v_2, 2c_s^2, 0, 0, 0, 0, 0\right)^T.$$

2.7.2 Conservation of mass: ρ



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$$\begin{aligned}
& \frac{\partial \rho}{\partial t} + \frac{v_1 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-1 + 3c_s^2 + v_1^2) \frac{v_1 \delta_l^3}{12 \delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + (-1 + c_s^2 + 3v_1^2) \frac{\rho \delta_l^3}{12 \delta_t} \frac{\partial^3 v_1}{\partial x_1^3} - \\
& \frac{c_s^2 \rho \delta_l^3}{6 \delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} - \frac{c_s^2 \rho \delta_l^3}{6 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{v_2 \delta_l^3}{12 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + (-1 + c_s^2 + 3v_2^2) \frac{\rho \delta_l^3}{12 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + C_1 \frac{\delta_l^4}{24 \delta_t \omega_1 \omega_2} \frac{\partial^4 \rho}{\partial x_1^4} + \\
& (5v_1^2 \omega_2 + 2\omega_1 \omega_2 - 5\omega_1 v_1^2 \omega_2 + 5\omega_1 v_1^2 - 2\omega_1 + 3c_s^2 \omega_1 - 3c_s^2 \omega_1 \omega_2 - 2\omega_2 + 3c_s^2 \omega_2) \frac{\rho v_1 \delta_l^4}{12 \delta_t \omega_1 \omega_2} \frac{\partial^4 v_1}{\partial x_1^4} + \\
& (\omega_1 v_2^2 - \omega_1 - \omega_2 v_2^2 + 3c_s^2 \omega_1 + \omega_2 - 3c_s^2 \omega_2) \frac{v_1 v_2 \delta_l^4}{8 \delta_t \omega_1 \omega_2} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + \\
& (\omega_1 v_2^2 - \omega_1 - \omega_2 v_2^2 + 3c_s^2 \omega_1 + \omega_2 - 3c_s^2 \omega_2) \frac{\rho v_2 \delta_l^4}{24 \delta_t \omega_1 \omega_2} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + C_2 \frac{\rho v_1 \delta_l^4}{24 \omega_3 \delta_t \omega_1 \omega_2} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& (-3v_1^2 \omega_2 + 3\omega_1 v_2^2 + 3\omega_1 v_1^2 - 2\omega_1 - 3\omega_2 v_2^2 + 2c_s^2 \omega_1 + 4c_s^2 \omega_1 \omega_2 + 2\omega_2 - 10c_s^2 \omega_2) \frac{c_s^2 \delta_l^4}{24 \delta_t \omega_1 \omega_2} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + \\
& (-v_1^2 \omega_2 + \omega_1 v_1^2 - \omega_1 + 3c_s^2 \omega_1 + \omega_2 - 3c_s^2 \omega_2) \frac{\rho v_1 \delta_l^4}{24 \delta_t \omega_1 \omega_2} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + \\
& (\omega_1 v_2^2 - \omega_1 - \omega_2 v_2^2 + 3c_s^2 \omega_1 + \omega_2 - 3c_s^2 \omega_2) \frac{\rho v_2 \delta_l^4}{24 \delta_t \omega_1 \omega_2} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + \\
& (-v_1^2 \omega_2 + \omega_1 v_1^2 - \omega_1 + 3c_s^2 \omega_1 + \omega_2 - 3c_s^2 \omega_2) \frac{v_1 v_2 \delta_l^4}{8 \delta_t \omega_1 \omega_2} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_3 \frac{\rho v_2 \delta_l^4}{24 \omega_3 \delta_t \omega_1 \omega_2} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& (-v_1^2 \omega_2 + \omega_1 v_1^2 - \omega_1 + 3c_s^2 \omega_1 + \omega_2 - 3c_s^2 \omega_2) \frac{\rho v_1 \delta_l^4}{24 \delta_t \omega_1 \omega_2} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + C_4 \frac{\delta_l^4}{24 \delta_t \omega_1 \omega_2} \frac{\partial^4 \rho}{\partial x_2^4} + \\
& (2\omega_1 \omega_2 + 5\omega_1 v_2^2 - 2\omega_1 + 5\omega_2 v_2^2 + 3c_s^2 \omega_1 - 3c_s^2 \omega_1 \omega_2 - 2\omega_2 + 3c_s^2 \omega_2 - 5\omega_1 \omega_2 v_2^2) \frac{\rho v_2 \delta_l^4}{12 \delta_t \omega_1 \omega_2} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

$$\begin{aligned}
C_1 &= -3v_1^2 \omega_2 + c_s^4 \omega_1 + 3\omega_1 v_1^2 \omega_2 - 3\omega_1 v_1^2 + c_s^4 \omega_2 - c_s^4 \omega_1 \omega_2 + 12c_s^2 \omega_1 v_1^2 + 3\omega_1 v_1^4 + 3v_1^4 \omega_2 - c_s^2 \omega_1 + 12c_s^2 v_1^2 \omega_2 + c_s^2 \omega_1 \omega_2 - c_s^2 \omega_2 - 3\omega_1 v_1^4 \omega_2 - 12c_s^2 \omega_1 v_1^2 \omega_2 \\
C_2 &= \omega_3 \omega_1 v_1^2 + 6\omega_1 \omega_2 + 9\omega_3 \omega_1 v_2^2 + 6\omega_3 c_s^2 \omega_1 \omega_2 - 6\omega_1 v_1^2 \omega_2 + \omega_3 v_1^2 \omega_2 - 2\omega_3 \omega_1 \omega_2 + 2\omega_3 \omega_2 + 2\omega_3 \omega_1 v_1^2 \omega_2 + 6\omega_3 c_s^2 \omega_1 - 18c_s^2 \omega_1 \omega_2 - 4\omega_3 \omega_1 - 9\omega_3 \omega_2 v_2^2 \\
C_3 &= 9\omega_3 \omega_1 v_1^2 + 6\omega_1 \omega_2 + 2\omega_3 \omega_1 \omega_2 v_2^2 + \omega_3 \omega_1 v_2^2 + 6\omega_3 c_s^2 \omega_1 \omega_2 - 9\omega_3 v_1^2 \omega_2 - 2\omega_3 \omega_1 \omega_2 + 2\omega_3 \omega_2 + 6\omega_3 c_s^2 \omega_1 - 18c_s^2 \omega_1 \omega_2 - 6\omega_1 \omega_2 v_2^2 - 4\omega_3 \omega_1 + \omega_3 \omega_2 v_2^2 \\
C_4 &= -3\omega_1 \omega_2 v_2^4 + c_s^4 \omega_1 + 12c_s^2 \omega_2 v_2^2 + 3\omega_2 v_2^4 - 3\omega_1 v_2^2 + c_s^4 \omega_2 - 12c_s^2 \omega_1 \omega_2 v_2^2 + 12c_s^2 \omega_1 v_2^2 - c_s^4 \omega_1 \omega_2 + 3\omega_1 v_2^4 - 3\omega_2 v_2^2 - c_s^2 \omega_1 + c_s^2 \omega_1 \omega_2 - c_s^2 \omega_2 + 3\omega_1 \omega_2 v_2^2
\end{aligned}$$

2.7.3 Conservation of momentum: ρv_1



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$$\begin{aligned}
& v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (c_s^2 + v_1^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{2\rho v_1 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{v_1 v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{\rho v_2 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_2} + \frac{\rho v_1 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + \\
& (3v_1^2 \omega_2 + \omega_1 \omega_2 - 3\omega_1 v_1^2 \omega_2 + 3\omega_1 v_1^2 - \omega_1 + 2c_s^2 \omega_1 - 2c_s^2 \omega_1 \omega_2 - \omega_2 + 2c_s^2 \omega_2) \frac{\delta_l^2}{\delta_t \omega_1 \omega_2} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + \\
& (-\omega_1 \omega_2 + \omega_1 + \omega_2) \frac{3\rho v_1 \delta_l^2}{\delta_t \omega_1 \omega_2} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + (3\omega_1 v_2^2 - \omega_1 - 3\omega_2 v_2^2 + c_s^2 \omega_1 + \omega_2 - c_s^2 \omega_2) \frac{\delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_2} + \\
& (\omega_1 - \omega_2) \frac{3\rho v_2 \delta_l^2}{\delta_t \omega_1 \omega_2} \frac{\partial v_2}{\partial x_1} \frac{\partial v_2}{\partial x_2} + (3\omega_1 v_2^2 - \omega_1 - 3\omega_2 v_2^2 + 3c_s^2 \omega_1 + c_s^2 \omega_1 \omega_2 + \omega_2 - 5c_s^2 \omega_2) \frac{\delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\
& (-2 + \omega_1) \frac{c_s^2 \delta_l^2}{2\delta_t \omega_1} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + (v_1^2 \omega_2 + \omega_1 \omega_2 - \omega_1 v_1^2 \omega_2 + \omega_1 v_1^2 - \omega_1 + 3c_s^2 \omega_1 - 3c_s^2 \omega_1 \omega_2 - \omega_2 + 3c_s^2 \omega_2) \frac{v_1 \delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial^2 \rho}{\partial x_1^2} + \\
& (3v_1^2 \omega_2 + \omega_1 \omega_2 - 3\omega_1 v_1^2 \omega_2 + 3\omega_1 v_1^2 - \omega_1 + c_s^2 \omega_1 - c_s^2 \omega_1 \omega_2 - \omega_2 + c_s^2 \omega_2) \frac{\rho \delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial^2 v_1}{\partial x_1^2} + \\
& (\omega_1 v_2^2 - \omega_1 - \omega_2 v_2^2 + 3c_s^2 \omega_1 + \omega_2 - 3c_s^2 \omega_2) \frac{v_2 \delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial^2 \rho}{\partial x_1 \partial x_2} + \\
& (3\omega_1 v_2^2 - \omega_1 - 3\omega_2 v_2^2 + c_s^2 \omega_1 + c_s^2 \omega_1 \omega_2 + \omega_2 - 3c_s^2 \omega_2) \frac{\rho \delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + (-2 + \omega_1) \frac{c_s^2 \rho \delta_l^2}{2\delta_t \omega_1} \frac{\partial^2 v_1}{\partial x_2^2} + C_1 \frac{\delta_l^3}{12 \delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 \rho}{\partial x_1^3} + \\
& C_2 \frac{\rho v_1 \delta_l^3}{12 \delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 v_1}{\partial x_1^3} + C_3 \frac{3v_1 v_2 \delta_l^3}{4 \delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + C_4 \frac{\rho v_2 \delta_l^3}{4 \omega_3 \delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} + C_5 \frac{\rho v_1 \delta_l^3}{12 \omega_3 \delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + C_6 \frac{\delta_l^3}{12 \delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} - \\
& \frac{c_s^2 \rho v_1 \delta_l^3}{6 \delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_7 \frac{\rho v_2 \delta_l^3}{4 \delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + (-1 + 3c_s^2 + v_2^2) \frac{v_1 v_2 \delta_l^3}{12 \delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + C_8 \frac{\rho v_2 \delta_l^3}{6 \omega_3 \delta_t \omega_1} \frac{\partial^3 v_1}{\partial x_2^3} + (-1 + c_s^2 + 3v_2^2) \frac{\rho v_1 \delta_l^3}{12 \delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\
& C_9 \frac{v_1 \delta_l^4}{24 \omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 \rho}{\partial x_1^4} + C_{10} \frac{\rho \delta_l^4}{24 \omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_1}{\partial x_1^4} + C_{11} \frac{v_2 \delta_l^4}{24 \omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{12} \frac{\rho v_1 v_2 \delta_l^4}{24 \omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + \\
& C_{13} \frac{\rho \delta_l^4}{24 \omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_{14} \frac{v_1 \delta_l^4}{24 \omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_{15} \frac{\rho \delta_l^4}{24 \omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{16} \frac{\rho v_1 v_2 \delta_l^4}{24 \omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} +
\end{aligned}$$

$$C_{17} \frac{v_2 \delta_1^4}{24 \omega_3^2 \delta_t \omega_1^3 \omega_2^2} \frac{\partial^4 \rho}{\partial x_1 \partial x_3^2} + C_{18} \frac{\rho v_1 v_2 \delta_1^4}{24 \omega_3 \delta_t \omega_1 \omega_2} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^2} + C_{19} \frac{\rho \delta_1^4}{24 \omega_3 \delta_t \omega_1^3 \omega_2^2} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^2} + C_{20} \frac{v_1 \delta_1^4}{24 \delta_t \omega_1 \omega_2} \frac{\partial^4 \rho}{\partial x_2^2} + C_{21} \frac{\rho \delta_1^4}{24 \omega_3^2 \delta_t \omega_1^3} \frac{\partial^4 v_1}{\partial x_2^2} + (2\omega_1 \omega_2 + 5\omega_1 v_2^2 - 2\omega_1 + 5\omega_2 v_2^2 + 3c_s^2 \omega_1 - 3c_s^2 \omega_1 \omega_2 - 2\omega_2 + 3c_s^2 \omega_2 - 5\omega_1 \omega_2 v_2^2) \frac{\rho v_1 v_2 \delta_1^4}{12 \delta_t \omega_1 \omega_2} \frac{\partial^4 v_2}{\partial x_2^2} = 0,$$

where:

$$C_1 = 6c_s^4 \omega_2^2 + c_s^4 \omega_1^2 \omega_2^2 + 6c_s^2 \omega_1^2 \omega_2 + 18\omega_1 v_1^2 \omega_2 - 72c_s^2 \omega_1^2 v_1^2 \omega_2 - 18\omega_1^2 v_1^4 \omega_2 + 9\omega_1^2 v_1^4 + 45c_s^2 \omega_1^2 v_1^2 + 24c_s^2 \omega_1^2 v_1^2 \omega_2^2 + 7\omega_1^2 v_1^4 \omega_2^2 - c_s^2 \omega_1^2 \omega_2^2 - 18\omega_1 v_1^2 \omega_2 - 9v_1^2 \omega_2^2 - 6c_s^4 \omega_1^2 \omega_2 + 6c_s^4 \omega_1^2 - 18\omega_1 v_1^4 \omega_2^2 - 72c_s^2 \omega_1 v_1^2 \omega_2^2 - 9\omega_1^2 v_1^2 + 18\omega_1^2 v_1^2 \omega_2 - 6c_s^2 \omega_2^2 + 45c_s^2 v_1^2 \omega_2^2 + 6c_s^2 \omega_1 \omega_2^2 + 9v_1^4 \omega_2^2 - 6c_s^2 \omega_1^2 - 7\omega_1^2 v_1^2 \omega_2^2 - 6c_s^4 \omega_1 \omega_2^2 + 18\omega_1 v_1^4 \omega_2 + 54c_s^2 \omega_1 v_1^2 \omega_2$$

$$C_2 = -18\omega_1 \omega_2 - 36c_s^2 \omega_1^2 \omega_2 - 60\omega_1 v_1^2 \omega_2^2 + 10c_s^2 \omega_1^2 \omega_2^2 + 54\omega_1 v_1^2 \omega_2 + 33v_1^2 \omega_2^2 + 24\omega_1 \omega_2^2 - 8\omega_1^2 \omega_2^2 - 15\omega_1^2 + 33\omega_1^2 v_1^2 - 60\omega_1^2 v_1^2 \omega_2 + 27c_s^2 \omega_2^2 - 36c_s^2 \omega_1 \omega_2^2 + 18c_s^2 \omega_1 \omega_2 + 27c_s^2 \omega_1^2 + 22\omega_1^2 v_1^2 \omega_2^2 - 15\omega_2^2 + 24\omega_1^2 \omega_2$$

$$C_3 = -\omega_1^2 \omega_2 v_2^2 + 2\omega_1 \omega_2 - 3c_s^2 \omega_1^2 \omega_2 + \omega_1 \omega_2^2 v_2^2 - 2\omega_1 v_1^2 \omega_2 + v_1^2 \omega_2^2 - \omega_1 \omega_2^2 - \omega_2^2 v_2^2 - 2\omega_1^2 + \omega_1^2 v_1^2 + \omega_1^2 v_2^2 + 3c_s^2 \omega_1 \omega_2^2 - 6c_s^2 \omega_1 \omega_2 + 6c_s^2 \omega_1^2 + \omega_1^2 \omega_2$$

$$C_4 = -2\omega_3 \omega_2^2 v_2^2 - 2\omega_1^2 \omega_2 v_2^2 + 9\omega_3 v_1^2 \omega_2^2 - \omega_3 \omega_1 \omega_2^2 - 6c_s^2 \omega_1^2 \omega_2 - 6\omega_3 c_s^2 \omega_1 \omega_2 + 2\omega_1 \omega_2^2 v_2^2 + 3\omega_3 c_s^2 \omega_1 \omega_2^2 - 2\omega_1 \omega_2^2 + 6\omega_3 \omega_1 \omega_2 + \omega_3 \omega_1^2 \omega_2 - 5\omega_3 \omega_1^2 - 18\omega_3 \omega_1 v_1^2 \omega_2 + 9\omega_3 c_s^2 \omega_1^2 + 6c_s^2 \omega_1 \omega_2^2 + \omega_3 \omega_1 \omega_2^2 v_2^2 + 2\omega_3 \omega_1^2 v_2^2 - 3\omega_3 c_s^2 \omega_2^2 - 3\omega_3 c_s^2 \omega_1^2 \omega_2 - \omega_3 \omega_1^2 \omega_2 v_2^2 + 9\omega_3 \omega_1^2 v_1^2 + 2\omega_1^2 \omega_2 - \omega_3 \omega_2^2$$

$$C_5 = -27\omega_3 \omega_2^2 v_2^2 - 3\omega_3 \omega_1^2 v_1^2 \omega_2^2 + 6\omega_3 v_1^2 \omega_2^2 - 12\omega_3 \omega_1 \omega_2^2 - 18c_s^2 \omega_1^2 \omega_2 - 18\omega_1 v_1^2 \omega_2^2 + 18\omega_3 c_s^2 \omega_1 \omega_2^2 + 36c_s^2 \omega_1^2 \omega_2^2 - 3\omega_3 \omega_1^2 v_1^2 \omega_2 + 18\omega_1 \omega_2^2 - 12\omega_2^2 \omega_2^2 + 12\omega_3 \omega_1^2 \omega_2 - 6\omega_1^2 v_1^2 \omega_2 - 15\omega_3 \omega_1^2 - 11\omega_3 c_s^2 \omega_1^2 \omega_2^2 + 27\omega_3 c_s^2 \omega_2^2 - 54c_s^2 \omega_1 \omega_2^2 + 27\omega_3 \omega_1 \omega_2^2 v_2^2 + 27\omega_3 \omega_1^2 v_2^2 + 9\omega_3 c_s^2 \omega_2^2 - 18\omega_3 c_s^2 \omega_1^2 \omega_2 + 3\omega_3 \omega_1 v_1^2 \omega_2^2 - 27\omega_3 \omega_1^2 \omega_2 v_2^2 + 12\omega_1^2 v_1^2 \omega_2^2 + 6\omega_3 \omega_1^2 v_1^2 + 6\omega_1^2 \omega_2 + 3\omega_3 \omega_1^2 \omega_2^2 + 3\omega_3 \omega_2^2$$

$$C_6 = -30c_s^4 \omega_2^2 + 9\omega_1^2 \omega_2 v_2^2 - 2c_s^4 \omega_1^2 \omega_2^2 + 6c_s^2 \omega_1^2 \omega_2 - 9\omega_1 \omega_2^2 v_2^2 + 9\omega_1^2 v_2^4 + 45c_s^2 \omega_1^2 v_2^2 - 6c_s^4 \omega_1^2 \omega_2 + 6c_s^4 \omega_1^2 + 9\omega_2^2 v_2^2 + 45c_s^2 \omega_1 \omega_2^2 v_2^2 - 9\omega_2^2 v_2^4 - 45c_s^2 \omega_2^2 v_2^2 + 6c_s^2 \omega_2^2 - 45c_s^2 \omega_1^2 \omega_2 v_2^2 - 9\omega_1^2 v_2^2 - 6c_s^2 \omega_1 \omega_2^2 - 9\omega_1^2 \omega_2 v_2^4 - 6c_s^2 \omega_1^2 + 9\omega_1 \omega_2^2 v_2^4 + 30c_s^4 \omega_1 \omega_2^2$$

$$C_7 = -11\omega_1^2 \omega_2 v_2^2 - 9c_s^2 \omega_1^2 \omega_2 + 11\omega_1 \omega_2^2 v_2^2 - 5\omega_1 \omega_2^2 - 11\omega_2^2 v_2^2 - 5\omega_1^2 - 9c_s^2 \omega_2^2 + 11\omega_1^2 v_2^2 + 9c_s^2 \omega_1 \omega_2^2 + 9c_s^2 \omega_1^2 + 5\omega_2^2 + 5\omega_1^2 \omega_2$$

$$C_8 = 6 + 9\omega_3 c_s^2 - 3\omega_3 - 18c_s^2 - \omega_3 \omega_1 v_2^2 + 3\omega_1 v_2^2 - 3\omega_1 + 9c_s^2 \omega_1 - 3\omega_3 c_s^2 \omega_1 + 3\omega_3 v_2^2 - 6v_2^2 + \omega_3 \omega_1$$

$$C_9 = 90\omega_3 \omega_1^3 v_1^4 \omega_2^2 + 54\omega_3 c_s^4 \omega_1 \omega_2^2 + 210\omega_3 \omega_1^2 v_1^2 \omega_2^2 - 36c_s^4 \omega_1^2 \omega_2^2 + 404\omega_3 c_s^2 \omega_1^3 v_1^2 \omega_2^2 + 6\omega_3 \omega_1 \omega_2^2 - 171\omega_3 c_s^4 \omega_1 \omega_2^2 - 18\omega_3 \omega_1^3 v_1^4 \omega_2^2 - 68\omega_3 c_s^2 \omega_1^3 v_1^2 \omega_2^2 - 12\omega_3 \omega_1 \omega_2^2 - 51\omega_3 v_1^2 \omega_2^2 + 18c_s^4 \omega_1^3 \omega_2 + 90\omega_3 c_s^4 \omega_1^3 - 98\omega_3 \omega_1^2 v_1^2 \omega_2^2 - 60\omega_3 c_s^2 \omega_1 \omega_2^2 - 12c_s^2 \omega_1^2 v_1^2 \omega_2^2 - 600\omega_3 c_s^2 \omega_1 v_1^2 \omega_2^2 - 117\omega_3 \omega_1 v_1^4 \omega_2^2 + 12c_s^2 \omega_1^2 \omega_2^2 + 90\omega_3 c_s^4 \omega_2^2 + 45\omega_3 \omega_1^3 v_1^4 - 105\omega_3 \omega_1^2 v_1^2 \omega_2 + 411\omega_3 c_s^2 \omega_1 v_1^2 \omega_2^2 + 261\omega_3 c_s^2 v_1^2 \omega_2^2 + 141\omega_3 c_s^2 \omega_1 \omega_2^2 - 600\omega_3 c_s^2 \omega_1^3 v_1^2 \omega_2 - 117\omega_3 \omega_1^3 v_1^4 \omega_2 - 6c_s^2 \omega_1^3 \omega_2 + 99\omega_3 \omega_1 v_1^2 \omega_2^2 + 6c_s^2 \omega_1^3 v_1^2 \omega_2 + 404\omega_3 c_s^2 \omega_1^2 v_1^2 \omega_2^2 - 78\omega_3 c_s^2 \omega_1^2 \omega_2^2 + 20\omega_3 \omega_1^3 v_1^2 \omega_2^2 + 6\omega_3 \omega_1^2 \omega_2 + 141\omega_3 c_s^2 \omega_1^3 \omega_2 - 2\omega_3 \omega_1^3 \omega_2^2 + 54\omega_3 c_s^4 \omega_1^2 \omega_2 + 45\omega_3 v_1^4 \omega_2^2 - 10\omega_3 c_s^4 \omega_1^3 \omega_2^2 - 6c_s^2 \omega_1 \omega_2^2 - 72\omega_3 c_s^2 \omega_1^3 + 90\omega_3 \omega_2^2 v_1^4 \omega_2^2 + 8\omega_3 \omega_1^3 \omega_2^2 + 6c_s^2 \omega_1 v_1^2 \omega_2^2 - 98\omega_3 \omega_1^3 v_1^2 \omega_2^2 + 114\omega_3 c_s^2 \omega_1^2 \omega_2^2 - 816\omega_3 c_s^2 v_1^2 \omega_2^2 - 198\omega_3 \omega_1^2 v_1^4 \omega_2^2 - 51\omega_3 \omega_1^3 v_1^4 + 6\omega_3 \omega_1^3 + 82\omega_3 c_s^4 \omega_1^3 \omega_2^2 + 99\omega_3 \omega_1^2 v_1^4 \omega_2 - 171\omega_3 c_s^4 \omega_1^3 \omega_2 + 82\omega_3 c_s^4 \omega_1^3 \omega_2^2 + 6\omega_3 \omega_2^2 - 12\omega_3 \omega_1^3 \omega_2 + 12\omega_3 c_s^2 \omega_1^3 \omega_2^2 + 8\omega_3 \omega_1^2 \omega_2^2 - 60\omega_3 c_s^2 \omega_1^2 \omega_2 + 129\omega_3 \omega_1^3 v_1^2 \omega_2 + 411\omega_3 c_s^2 \omega_1^2 v_1^2 \omega_2 - 105\omega_3 \omega_1 v_1^2 \omega_2^2 + 18c_s^4 \omega_1 \omega_2^2 - 90\omega_3 c_s^4 \omega_1^2 \omega_2^2 + 261\omega_3 c_s^2 \omega_1^3 v_1^2 + 129\omega_3 \omega_1 v_1^2 \omega_2^2 - 12\omega_3 \omega_1^2 \omega_2^2 - 78\omega_3 c_s^2 \omega_1^2 \omega_2^2 - 72\omega_3 c_s^2 \omega_2^2$$

$$C_{10} = 310\omega_3 \omega_1^3 v_1^4 \omega_2^2 + 6\omega_3 c_s^4 \omega_1 \omega_2^2 + 306\omega_3 \omega_1^2 v_1^2 \omega_2^2 - 12c_s^4 \omega_1^2 \omega_2^2 + 252\omega_3 c_s^2 \omega_1^3 v_1^2 \omega_2^2 + 6\omega_3 \omega_1 \omega_2^2 - 33\omega_3 c_s^4 \omega_1 \omega_2^2 - 58\omega_3 \omega_1^3 v_1^4 \omega_2^2 - 36\omega_3 c_s^2 \omega_1^3 v_1^2 \omega_2^2 - 12\omega_3 \omega_1 \omega_2^2 - 99\omega_3 v_1^2 \omega_2^2 + 6c_s^4 \omega_1^3 \omega_2 + 18\omega_3 c_s^4 \omega_1^3 - 154\omega_3 \omega_1^2 v_1^2 \omega_2^2 - 12\omega_3 c_s^2 \omega_1 \omega_2^2 - 36c_s^2 \omega_1^2 v_1^2 \omega_2^2 - 432\omega_3 c_s^2 \omega_1 v_1^2 \omega_2^2 - 423\omega_3 \omega_1 v_1^4 \omega_2^2 + 12c_s^2 \omega_1^2 \omega_2^2 + 18\omega_3 c_s^4 \omega_2^2 + 171\omega_3 \omega_1^3 v_1^4 - 153\omega_3 \omega_1^2 v_1^2 \omega_2 + 225\omega_3 c_s^2 \omega_1 v_1^2 \omega_2^2 + 207\omega_3 c_s^2 v_1^2 \omega_2^2 + 45\omega_3 c_s^2 \omega_1 \omega_2^2 - 432\omega_3 c_s^2 \omega_1^3 v_1^2 \omega_2 - 423\omega_3 \omega_1^3 v_1^4 \omega_2 - 6c_s^2 \omega_1^3 \omega_2 + 333\omega_3 \omega_1 v_1^4 \omega_2^2 + 18c_s^2 \omega_1^3 \omega_2 + 252\omega_3 c_s^2 \omega_1^2 v_1^2 \omega_2^2 - 22\omega_3 c_s^2 \omega_1^2 \omega_2^2 + 28\omega_3 \omega_1^3 v_1^2 \omega_2^2 + 6\omega_3 \omega_1^2 \omega_2 + 45\omega_3 c_s^2 \omega_1^3 \omega_2 - 2\omega_3 \omega_1^3 \omega_2^2 + 6\omega_3 \omega_1^2 \omega_2 + 171\omega_3 v_1^4 \omega_2^2 - 2\omega_3 \omega_1^3 \omega_2^2 - 6c_s^2 \omega_1^3 \omega_2^2 - 24\omega_3 c_s^2 \omega_1^2 v_1^2 \omega_2^2 + 8\omega_3 \omega_1^3 v_1^2 \omega_2^2 + 18c_s^2 \omega_1 v_1^2 \omega_2^2 - 154\omega_3 \omega_1^2 v_1^2 \omega_2^2 + 18\omega_3 c_s^2 \omega_1^2 \omega_2^2 - 432\omega_3 c_s^2 \omega_1 v_1^2 \omega_2^2 - 666\omega_3 \omega_1 v_1^4 \omega_2^2 - 99\omega_3 \omega_1^3 v_1^4 + 6\omega_3 \omega_1^3 + 14\omega_3 c_s^4 \omega_1^3 \omega_2^2 + 333\omega_3 \omega_1^2 v_1^2 \omega_2 - 33\omega_3 c_s^4 \omega_1^2 \omega_2 + 14\omega_3 c_s^4 \omega_1^2 \omega_2^2 + 6\omega_3 \omega_2^2 - 12\omega_3 \omega_1^3 \omega_2 + 4\omega_3 c_s^2 \omega_1^3 \omega_2^2 + 8\omega_3 \omega_1^2 \omega_2^2 - 12\omega_3 c_s^2 \omega_1^2 \omega_2 + 225\omega_3 \omega_1 v_1^2 \omega_2 + 225\omega_3 c_s^2 \omega_1^2 v_1^2 \omega_2 - 153\omega_3 \omega_1 v_1^2 \omega_2^2 + 6c_s^4 \omega_1 \omega_2^2 - 6\omega_3 c_s^4 \omega_1^2 \omega_2^2 + 207\omega_3 c_s^2 \omega_1^3 v_1^2 + 225\omega_3 \omega_1 v_1^2 \omega_2^2 - 12\omega_3 \omega_1^2 \omega_2^2 - 22\omega_3 c_s^2 \omega_1^3 \omega_2^2 - 24\omega_3 c_s^2 \omega_2^2$$

$$C_{11} = 9\omega_3 \omega_1^2 v_1^2 \omega_2 v_2^2 - 45\omega_3 v_1^2 \omega_2^2 v_2^2 + 12c_s^2 \omega_1^2 \omega_2^2 v_2^2 - 54\omega_3 c_s^4 \omega_1^2 \omega_2 + 12c_s^2 \omega_1^3 \omega_2^2 + 45\omega_3 \omega_1^3 v_1^2 v_2^2 + 18\omega_3 c_s^2 \omega_1^3 \omega_2^2 v_2^2 - 6\omega_3 \omega_1 \omega_2^2 v_2^2 + 5\omega_3 c_s^2 \omega_1^2 \omega_2^2 + 75\omega_3^2 c_s^2 \omega_1^2 \omega_2^2 - \omega_3^2 \omega_2^2 \omega_2^2 - 6\omega_3^2 \omega_1^2 \omega_2^2 + 21\omega_3^2 c_s^2 \omega_1 \omega_2^2 v_2^2 + 6\omega_3^2 c_s^4 \omega_1^3 \omega_2^2 + 72\omega_3^2 \omega_1 v_1^2 \omega_2^2 v_2^2 + 108\omega_3^2 \omega_1^2 v_1^4 \omega_2^2 + 18\omega_3 c_s^4 \omega_1 \omega_2^2 + 6\omega_3 c_s^2 \omega_1 \omega_2^2 v_2^2 - \omega_3^2 \omega_1^3 \omega_2^2 v_2^2 + 72\omega_3 c_s^2 \omega_1^3 v_1^2 \omega_2^2 - 6\omega_3^2 \omega_2^2 + 36c_s^4 \omega_1^2 \omega_2^2 - 72\omega_3^2 c_s^2 \omega_1^2 \omega_2^2 - 24\omega_3^2 \omega_1^3 v_1^2 \omega_2^2 + 2\omega_3^2 c_s^2 \omega_1^3 \omega_2^2 v_2^2 + 6\omega_3^2 c_s^2 \omega_1 \omega_2^2 v_2^2 - 6\omega_3^2 \omega_1^2 \omega_2 + 63\omega_3^2 \omega_1 v_1^2 \omega_2^2 + 126\omega_3^2 \omega_1^3 v_1^2 \omega_2 + 60\omega_3^2 c_s^2 \omega_1^2 \omega_2 - 36c_s^4 \omega_1^3 \omega_2^2 - 72\omega_3^2 c_s^2 \omega_1^3 - 99\omega_3^2 c_s^2 \omega_1^3 v_2^2 + 6\omega_3^2 \omega_1^3 - 6\omega_3^2 c_s^4 \omega_1^3 \omega_2^2 - 297\omega_3^2 c_s^2 \omega_1 v_1^2 \omega_2^2 - 6\omega_3^2 \omega_1 \omega_2^2 v_2^2 - 6\omega_3^2 \omega_1^3 v_2^2 + 54\omega_3^2 v_1^4 \omega_2^2 - 54\omega_3^2 v_1^4 \omega_2 - 486\omega_3^2 c_s^2 \omega_1^3 v_1^2 \omega_2 - 5\omega_3^2 c_s^2 \omega_1^2 \omega_2^2 - 18\omega_3^2 \omega_1 v_1^2 \omega_2^2 - 72\omega_3^2 \omega_1^3 v_1^2 \omega_2 v_2^2 - 6\omega_3^2 c_s^2 \omega_1^2 \omega_2 v_2^2 - 6\omega_3^2 c_s^2 \omega_1 \omega_2^2 + \omega_3^2 \omega_1^3 \omega_2^2 + 405\omega_3^2 c_s^2 \omega_1^3 v_1^2 + 72\omega_3^2 c_s^4 \omega_1^2 \omega_2^2 - 54\omega_3^2 c_s^2 \omega_1 v_1^2 \omega_2^2 - 12c_s^2 \omega_1^2 \omega_2^2 + 6\omega_3^2 \omega_1^2 \omega_2 v_2^2 - 99\omega_3^2 \omega_1^3 v_1^2 - 9\omega_3^2 \omega_1 v_1^2 \omega_2^2 v_2^2 + 18\omega_3^2 c_s^2 \omega_1^2 \omega_2^2 + 24\omega_3^2 \omega_1^3 v_1^2 \omega_2^2 v_2^2 - 108\omega_3^2 \omega_1^3 v_1^2 \omega_2^2 + 6\omega_3^2 c_s^2 \omega_1^3 \omega_2 - 18\omega_3^2 c_s^4 \omega_1^3 \omega_2^2 - 18\omega_3^2 c_s^2 \omega_1^3 v_2^2 + 54\omega_3^2 c_s^2 \omega_1^2 \omega_2^2 v_2^2 + 6\omega_3^2 \omega_1^2 \omega_2^2 - 18\omega_3^2 c_s^2 \omega_1 \omega_2^2 + 24\omega_3^2 \omega_1^3 v_1^2 \omega_2^2 + 54\omega_3^2 \omega_1^3 v_1^4 + 27\omega_3^2 c_s^4 \omega_1 \omega_2^2 - 72\omega_3^2 c_s^2 \omega_1^2 v_1^2 \omega_2^2 + 54\omega_3^2 c_s^4 \omega_1^3 \omega_2^2 - 24\omega_3^2 \omega_1^2 v_1^2 \omega_2^2 v_2^2 + \omega_3^2 \omega_1^2 \omega_2^2 v_2^2 - 54\omega_3^2 \omega_1 v_1^4 \omega_2^2 - 18\omega_3^2 c_s^4 \omega_1^3 \omega_2 + 6\omega_3^2 \omega_1^3 \omega_2 v_2^2 - 54\omega_3 c_s^4 \omega_1^2 \omega_2^2 - 2\omega_3^2 c_s^2 \omega_1^2 \omega_2^2 v_2^2 + 6\omega_3^2 \omega_1 \omega_2^2 - 21\omega_3^2 c_s^2 \omega_1^3 \omega_2 v_2^2 + 12\omega_3^2 c_s^2 \omega_1 \omega_2^2 - 18\omega_3^2 c_s^2 \omega_1^2 \omega_2^2 v_2^2 - 54\omega_3^2 \omega_1 v_1^4 \omega_2^2 - 12c_s^2 \omega_1^3 \omega_2^2 v_2^2 - 6\omega_3^2 \omega_1^3 \omega_2 v_2^2 - 243\omega_3^2 c_s^2 \omega_1^3 v_1^2 \omega_2 - 54\omega_3^2 \omega_1^3 v_1^4 \omega_2 + 90\omega_3^2 c_s^4 \omega_1^3 + 135\omega_3^2 c_s^2 v_1^2 \omega_2^2 - 3\omega_3^2 c_s^2 \omega_1 \omega_2^2 + 45\omega_3^2 \omega_1^2 v_1^2 \omega_2^2 - 9\omega_3^2 v_1^2 \omega_2^2 - 18\omega_3^2 c_s^2 \omega_1^3 \omega_2^2 + 6\omega_3^2 \omega_1 \omega_2^2$$

$$C_{12} = 54c_s^4 \omega_1^3 \omega_2^2 + 396\omega_3 \omega_1^2 v_1^2 \omega_2^2 + 90\omega_3 \omega_1 \omega_2^2 + 45\omega_3 \omega_1 \omega_2^3 + 198\omega_3 v_1^2 \omega_2^3 + 18\omega_1^2 \omega_2^2 v_2^2 - 162\omega_3 c_s^2 \omega_1 \omega_2^2 - 18\omega_1 \omega_2^3 - 36\omega_3 \omega_2^3 v_2^2 - 45\omega_3 \omega_1^3 \omega_2 v_2^2 - 10\omega_3 \omega_1^2 \omega_2^3 v_2^2 + 18\omega_1 \omega_2^3 v_2^2 - 198\omega_3 \omega_1^2 v_1^2 \omega_2 - 27\omega_3 c_s^2 \omega_1 \omega_2^3 - 54c_s^2 \omega_1^3 \omega_2 - 54c_s^2 \omega_1^2 \omega_2^3 + 30\omega_3 c_s^2 \omega_1^2 \omega_2^3 + 90\omega_3 \omega_1^2 \omega_2 - 297\omega_3 c_s^2 \omega_1^3 \omega_2 + 36\omega_3 \omega_1^3 v_2^2 + 45\omega_3 \omega_1 \omega_2^3 v_2^2 + 54c_s^2 \omega_1 \omega_2^3 + 270\omega_3 c_s^2 \omega_1^3 - 10\omega_3 \omega_1^3 \omega_2^2 + 324\omega_3 c_s^2 \omega_1^2 \omega_2^2 - 18\omega_1^3 \omega_2 v_2^2 + 18\omega_1^3 \omega_2 - 18\omega_1^2 \omega_2^3 v_2^2 + 18\omega_1^2 \omega_2^3 + 198\omega_3 \omega_1^3 v_1^2 - 126\omega_3 \omega_1^3 + 10\omega_3 \omega_1^3 \omega_2^2 v_2^2 - 54\omega_3 \omega_1^3 + 135\omega_3 \omega_1^3 \omega_2 + 10\omega_3 \omega_1^2 \omega_2^2 - 162\omega_3 c_s^2 \omega_1^2 \omega_2 - 198\omega_3 \omega_1^3 v_1^2 \omega_2 - 18\omega_1^3 \omega_2^2 - 198\omega_3 \omega_1 v_1^2 \omega_2^2 - 198\omega_3 \omega_1 v_1^2 \omega_2^2 - 180\omega_3 \omega_1^2 \omega_2^2 + 30\omega_3 c_s^2 \omega_1^2 \omega_2^2 + 54\omega_3 c_s^2 \omega_2^2$$

$$C_{13} = 27\omega_3^2 \omega_1^2 v_1^2 \omega_2 v_2^2 + 24\omega_3^2 c_s^2 \omega_1^3 v_1^2 \omega_2^2 - 135\omega_3^2 v_1^2 \omega_2^2 v_2^2 + 7\omega_3^2 \omega_1^4 \omega_2^2 + 72\omega_3 \omega_1^3 v_1^4 \omega_2^2 - 30\omega_3^2 c_s^4 \omega_1^2 \omega_2 + 135\omega_3^2 \omega_1^3 v_1^2 v_2^2 - 36\omega_3 c_s^2 \omega_1^3 \omega_2^2 v_2^2 - 18\omega_3^2 \omega_1 \omega_2^2 v_2^2 - 2\omega_3^2 c_s^4 \omega_1^3 \omega_2^2 - 8\omega_3^2 \omega_1^3 v_1^2 \omega_2^2 - 12\omega_3^2 c_s^2 \omega_1^2 \omega_2^2 + 36\omega_3 \omega_1^2 v_1^2 \omega_2^2 + 21\omega_3^2 c_s^2 \omega_1^2 \omega_2 - \omega_3^2 \omega_1^2 \omega_2^2 + 198\omega_3 c_s^2 \omega_1^3 v_1^2 \omega_2^2 - 6\omega_3^2 \omega_1^3 \omega_2 -$$

$$C_{20} = -3\omega_1\omega_2v_2^4 + c_s^4\omega_1 + 12c_s^2\omega_2v_2^2 + 3\omega_2v_2^4 - 3\omega_1v_2^2 + c_s^4\omega_2 - 12c_s^2\omega_1\omega_2v_2^2 + 12c_s^2\omega_1v_2^2 - c_s^4\omega_1\omega_2 + 3\omega_1v_2^4 - 3\omega_2v_2^2 - c_s^2\omega_1 + c_s^2\omega_1\omega_2 - c_s^2\omega_2 + 3\omega_1\omega_2v_2^2$$

$$C_{21} = 12\omega_3^2\omega_1^2v_2^2 - 12\omega_3^2c_s^2\omega_1^2v_2^2 + 12\omega_3^2c_s^2\omega_1 - 24\omega_3c_s^4\omega_1^2 + 72\omega_3\omega_1^2v_2^4 + 108c_s^2\omega_1^3v_2^2 + 24\omega_3^2c_s^4 + 6\omega_3c_s^4\omega_1^3 + 36\omega_1^3v_2^4 - 30\omega_3\omega_1^3v_2^4 + \omega_3^2c_s^2\omega_1^3 + 6\omega_3^2c_s^2\omega_1^3v_2^2 - 3\omega_3^2\omega_1^3v_2^2 + 24\omega_3c_s^4\omega_1 - 72\omega_1^2v_2^4 - 216c_s^2\omega_1^2v_2^2 + 72\omega_3c_s^2\omega_1v_2^2 - 8\omega_3^2c_s^2\omega_1^2 + 3\omega_3^2\omega_1^2v_2^4 + 144\omega_3c_s^2\omega_1^2v_2^2 + 30\omega_3\omega_1^3v_2^2 - 6\omega_3c_s^2\omega_1^3 - 48\omega_3^2c_s^4\omega_1 + 24\omega_3c_s^2\omega_1^2 + 72\omega_1^2v_2^2 - 24\omega_3c_s^2\omega_1 - 72\omega_3c_s^2\omega_1^2v_2^2 - 72\omega_3\omega_1^2v_2^2 - 12\omega_3\omega_1^2v_2^4 + 24\omega_3^2c_s^4\omega_1^2 - 36\omega_3^2c_s^2\omega_1v_2^2 - 36\omega_1^3v_2^2 - 3\omega_3^2c_s^4\omega_1^3$$

2.7.4 Conservation of momentum: ρv_2



attached text file: output_d2q9_nse_culbm2_symbolic_pde_02.txt

$$\begin{aligned} & v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + \frac{v_1 v_2 \delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + \frac{\rho v_2 \delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + \frac{\rho v_1 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_1} + (c_s^2 + v_2^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \frac{2\rho v_2 \delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + (-2 + \omega_1) \frac{c_s^2 \delta_l^2}{2\delta_t \omega_1} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\ & (-3v_1^2 \omega_2 + 3\omega_1 v_1^2 - \omega_1 + 3c_s^2 \omega_1 + c_s^2 \omega_1 \omega_2 + \omega_2 - 5c_s^2 \omega_2) \frac{\delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + (\omega_1 - \omega_2) \frac{3\rho v_1 \delta_l^2}{\delta_t \omega_1 \omega_2} \frac{\partial v_1}{\partial x_1} \frac{\partial v_1}{\partial x_2} + \\ & (-3v_1^2 \omega_2 + 3\omega_1 v_1^2 - \omega_1 + c_s^2 \omega_1 + \omega_2 - c_s^2 \omega_2) \frac{\delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_1} + \\ & (\omega_1 \omega_2 + 3\omega_1 v_2^2 - \omega_1 + 3\omega_2 v_2^2 + 2c_s^2 \omega_1 - 2c_s^2 \omega_1 \omega_2 - \omega_2 + 2c_s^2 \omega_2 - 3\omega_1 \omega_2 v_2^2) \frac{\delta_l^2}{\delta_t \omega_1 \omega_2} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + \\ & (-\omega_1 \omega_2 + \omega_1 + \omega_2) \frac{3\rho v_2 \delta_l^2}{\delta_t \omega_1 \omega_2} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + (-2 + \omega_1) \frac{c_s^2 \rho \delta_l^2}{2\delta_t \omega_1} \frac{\partial^2 v_2}{\partial x_1^2} + \\ & (-v_1^2 \omega_2 + \omega_1 v_1^2 - \omega_1 + 3c_s^2 \omega_1 + \omega_2 - 3c_s^2 \omega_2) \frac{v_1 \delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial^2 \rho}{\partial x_1 \partial x_2} + \\ & (-3v_1^2 \omega_2 + 3\omega_1 v_1^2 - \omega_1 + c_s^2 \omega_1 + c_s^2 \omega_1 \omega_2 + \omega_2 - 3c_s^2 \omega_2) \frac{\rho \delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + \\ & (\omega_1 \omega_2 + \omega_1 v_2^2 - \omega_1 + \omega_2 v_2^2 + 3c_s^2 \omega_1 - 3c_s^2 \omega_1 \omega_2 - \omega_2 + 3c_s^2 \omega_2 - \omega_1 \omega_2 v_2^2) \frac{v_2 \delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial^2 \rho}{\partial x_2^2} + \\ & (\omega_1 \omega_2 + 3\omega_1 v_2^2 - \omega_1 + 3\omega_2 v_2^2 + c_s^2 \omega_1 - c_s^2 \omega_1 \omega_2 - \omega_2 + c_s^2 \omega_2 - 3\omega_1 \omega_2 v_2^2) \frac{\rho \delta_l^2}{2\delta_t \omega_1 \omega_2} \frac{\partial^2 v_2}{\partial x_2^2} + (-1 + 3c_s^2 + v_1^2) \frac{v_1 v_2 \delta_l^3}{12\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + \\ & (-1 + c_s^2 + 3v_1^2) \frac{\rho v_2 \delta_l^3}{12\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + C_1 \frac{\rho v_1 \delta_l^3}{6\omega_3 \delta_t \omega_1} \frac{\partial^3 v_2}{\partial x_1^3} + C_2 \frac{\delta_l^3}{12\delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + C_3 \frac{\rho v_1 \delta_l^3}{4\delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} - \frac{c_s^2 \rho v_2 \delta_l^3}{6\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + \\ & C_4 \frac{3v_1 v_2 \delta_l^3}{4\delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + C_5 \frac{\rho v_2 \delta_l^3}{12\omega_3 \delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_6 \frac{\rho v_1 \delta_l^3}{4\omega_3 \delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_7 \frac{\delta_l^3}{12\delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 \rho}{\partial x_2^3} + C_8 \frac{\rho v_2 \delta_l^3}{12\delta_t \omega_1^2 \omega_2^2} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & C_9 \frac{v_2 \delta_l^4}{24\delta_t \omega_1 \omega_2} \frac{\partial^4 \rho}{\partial x_1^4} + (5v_1^2 \omega_2 + 2\omega_1 \omega_2 - 5\omega_1 v_1^2 \omega_2 + 5\omega_1 v_1^2 - 2\omega_1 + 3c_s^2 \omega_1 - 3c_s^2 \omega_1 \omega_2 - 2\omega_2 + 3c_s^2 \omega_2) \frac{\rho v_1 v_2 \delta_l^4}{12\delta_t \omega_1 \omega_2} \frac{\partial^4 v_1}{\partial x_1^4} + \\ & C_{10} \frac{\rho \delta_l^4}{24\omega_3^2 \delta_t \omega_1^3} \frac{\partial^4 v_2}{\partial x_1^4} + C_{11} \frac{v_1 \delta_l^4}{24\omega_3^2 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{12} \frac{\rho \delta_l^4}{24\omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + C_{13} \frac{\rho v_1 v_2 \delta_l^4}{24\omega_3 \delta_t \omega_1 \omega_2} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\ & C_{14} \frac{v_2 \delta_l^4}{24\omega_3^2 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_{15} \frac{\rho v_1 v_2 \delta_l^4}{24\omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{16} \frac{\rho \delta_l^4}{24\omega_3^2 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_{17} \frac{v_1 \delta_l^4}{24\omega_3^2 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\ & C_{18} \frac{\rho \delta_l^4}{24\omega_3^2 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{19} \frac{\rho v_1 v_2 \delta_l^4}{24\omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + C_{20} \frac{v_2 \delta_l^4}{24\omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 \rho}{\partial x_2^4} + C_{21} \frac{\rho \delta_l^4}{24\omega_3 \delta_t \omega_1^3 \omega_2^3} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:

$$\begin{aligned} C_1 &= 6 + 9\omega_3 c_s^2 - \omega_3 \omega_1 v_1^2 - 3\omega_3 - 18c_s^2 + 3\omega_1 v_1^2 - 3\omega_1 + 9c_s^2 \omega_1 - 3\omega_3 c_s^2 \omega_1 + \omega_3 \omega_1 + 3\omega_3 v_1^2 - 6v_1^2 \\ C_2 &= -30c_s^4 \omega_2^2 - 2c_s^4 \omega_1^2 \omega_2^2 + 6c_s^2 \omega_1^2 \omega_2 - 9\omega_1 v_1^2 \omega_2^2 - 45c_s^2 \omega_1^2 v_1^2 \omega_2 - 9\omega_1^2 v_1^4 \omega_2 + 9\omega_1^2 v_1^4 + 45c_s^2 \omega_1^2 v_1^2 + 9v_1^2 \omega_2^2 - 6c_s^4 \omega_1^2 \omega_2 + 6c_s^4 \omega_1^2 + 9\omega_1 v_1^4 \omega_2^2 + \\ & 45c_s^2 \omega_1 v_1^2 \omega_2^2 - 9\omega_1^2 v_1^2 + 9\omega_1^2 v_1^2 \omega_2 + 6c_s^2 \omega_2^2 - 45c_s^2 v_1^2 \omega_2^2 - 6c_s^2 \omega_1 \omega_2^2 - 9v_1^4 \omega_2^2 - 6c_s^2 \omega_1^2 + 30c_s^4 \omega_1 \omega_2^2 \\ C_3 &= -9c_s^2 \omega_1^2 \omega_2 + 11\omega_1 v_1^2 \omega_2^2 - 11v_1^2 \omega_2^2 - 5\omega_1 \omega_2^2 - 5\omega_1^2 + 11\omega_1^2 v_1^2 - 11\omega_1^2 v_1^2 \omega_2 - 9c_s^2 \omega_2^2 + 9c_s^2 \omega_1 \omega_2^2 + 9c_s^2 \omega_1^2 + 5\omega_2^2 + 5\omega_1^2 \omega_2 \\ C_4 &= 2\omega_1 \omega_2 - 3c_s^2 \omega_1^2 \omega_2 + \omega_1 v_1^2 \omega_2^2 - v_1^2 \omega_2^2 - \omega_1 \omega_2^2 + \omega_2^2 v_2^2 - 2\omega_1^2 + \omega_1^2 v_1^2 - \omega_1^2 v_1^2 \omega_2 + \omega_1^2 v_2^2 + 3c_s^2 \omega_1 \omega_2^2 - 6c_s^2 \omega_1 \omega_2 + 6c_s^2 \omega_1^2 - 2\omega_1 \omega_2 v_2^2 + \omega_1^2 \omega_2 \\ C_5 &= -3\omega_3 \omega_1^2 \omega_2^2 v_2^2 + 6\omega_3 \omega_2^2 v_2^2 - 6\omega_1^2 \omega_2 v_2^2 - 27\omega_3 v_1^2 \omega_2^2 - 12\omega_3 \omega_1 \omega_2^2 - 18c_s^2 \omega_1^2 \omega_2 - 18\omega_1 \omega_2^2 v_2^2 + 18\omega_3 c_s^2 \omega_1 \omega_2^2 + 36c_s^2 \omega_1^2 \omega_2^2 - 27\omega_3 \omega_1^2 v_1^2 \omega_2 + \\ & 18\omega_1 \omega_2^2 - 12\omega_1^2 \omega_2^2 + 12\omega_3 \omega_1^2 \omega_2 - 15\omega_3 \omega_1^2 - 11\omega_3 c_s^2 \omega_1^2 \omega_2^2 + 27\omega_3 c_s^2 \omega_1^2 - 54c_s^2 \omega_1 \omega_2^2 + 3\omega_3 \omega_1 \omega_2^2 v_2^2 + 6\omega_3 \omega_1^2 v_2^2 + 9\omega_3 c_s^2 \omega_2^2 - 18\omega_3 c_s^2 \omega_1^2 \omega_2 + \\ & 27\omega_3 \omega_1 v_1^2 \omega_2^2 - 3\omega_3 \omega_1^2 \omega_2 v_2^2 + 27\omega_3 \omega_1^2 v_1^2 + 6\omega_1^2 \omega_2 + 12\omega_1^2 \omega_2^2 v_2^2 + 3\omega_3 \omega_1^2 \omega_2^2 + 3\omega_3 \omega_2^2 \\ C_6 &= 9\omega_3 \omega_2^2 v_2^2 - 2\omega_3 v_1^2 \omega_2^2 - \omega_3 \omega_1 \omega_2^2 - 6c_s^2 \omega_1^2 \omega_2 - 18\omega_3 \omega_1 \omega_2 v_2^2 + 2\omega_1 v_1^2 \omega_2^2 - 6\omega_3 c_s^2 \omega_1 \omega_2 + 3\omega_3 c_s^2 \omega_1 \omega_2^2 - \omega_3 \omega_1^2 v_1^2 \omega_2 - 2\omega_1 \omega_2^2 + 6\omega_3 \omega_1 \omega_2 + \\ & \omega_3 \omega_1^2 \omega_2 - 2\omega_1^2 v_1^2 \omega_2 - 5\omega_3 \omega_1^2 + 9\omega_3 c_s^2 \omega_1 + 6c_s^2 \omega_1 \omega_2^2 + 9\omega_3 \omega_1^2 v_2^2 - 3\omega_3 c_s^2 \omega_2^2 - 3\omega_3 c_s^2 \omega_1^2 \omega_2 + \omega_3 \omega_1 v_1^2 \omega_2^2 + 2\omega_3 \omega_1^2 v_1^2 + 2\omega_1^2 \omega_2 - \omega_3 \omega_2^2 \\ C_7 &= 6c_s^4 \omega_2^2 + 18\omega_1 \omega_2 v_2^4 + 18\omega_1^2 \omega_2 v_2^2 + c_s^4 \omega_1^2 \omega_2^2 + 6c_s^2 \omega_1^2 \omega_2 + 7\omega_1^2 \omega_2^2 v_2^4 + 18\omega_1 \omega_2^2 v_2^2 - c_s^2 \omega_1^2 \omega_2^2 + 24c_s^2 \omega_1^2 \omega_2^2 v_2^2 + 9\omega_1^2 v_2^4 + 45c_s^2 \omega_1^2 v_2^2 - \\ & 6c_s^4 \omega_1^2 \omega_2 + 6c_s^4 \omega_1^2 - 9\omega_2^2 v_2^2 + 54c_s^2 \omega_1 \omega_2 v_2^2 - 72c_s^2 \omega_1 \omega_2^2 v_2^2 + 9\omega_2^2 v_2^4 + 45c_s^2 \omega_2^2 v_2^2 - 6c_s^2 \omega_2^2 - 72c_s^2 \omega_1^2 \omega_2 v_2^2 - 9\omega_1^2 v_2^2 + 6c_s^2 \omega_1 \omega_2^2 - 18\omega_1^2 \omega_2 v_2^4 - \\ & 6c_s^2 \omega_1^2 - 18\omega_1 \omega_2 v_2^2 - 18\omega_1 \omega_2^2 v_2^4 - 6c_s^4 \omega_1 \omega_2^2 - 7\omega_1^2 \omega_2^2 v_2^2 \end{aligned}$$

$$C_8 = -60\omega_1^2\omega_2v_2^2 - 18\omega_1\omega_2 - 36c_s^2\omega_1^2\omega_2 - 60\omega_1\omega_2v_2^2 + 10c_s^2\omega_1^2\omega_2^2 + 24\omega_1\omega_2^2 + 33\omega_2^2v_2^2 - 8\omega_1^2\omega_2^2 - 15\omega_1^2 + 27c_s^2\omega_2^2 + 33\omega_1^2v_2^2 - 36c_s^2\omega_1\omega_2^2 + 18c_s^2\omega_1\omega_2 + 27c_s^2\omega_1^2 + 54\omega_1\omega_2v_2^2 - 15\omega_2^2 + 24\omega_1^2\omega_2 + 22\omega_1^2\omega_2^2v_2^2$$

$$C_9 = -3v_1^2\omega_2 + c_s^4\omega_1 + 3\omega_1v_1^2\omega_2 - 3\omega_1v_1^2 + c_s^4\omega_2 - c_s^4\omega_1\omega_2 + 12c_s^2\omega_1v_1^2 + 3\omega_1v_1^4 + 3v_1^4\omega_2 - c_s^2\omega_1 + 12c_s^2v_1^2\omega_2 + c_s^2\omega_1\omega_2 - c_s^2\omega_2 - 3\omega_1v_1^4\omega_2 - 12c_s^2\omega_1v_1^2\omega_2$$

$$C_{10} = 12\omega_3^2c_s^2\omega_1 - 24\omega_3c_s^4\omega_1^2 + 108c_s^2\omega_1^3v_1^2 + 36\omega_3^2v_1^4 + 12\omega_3^2\omega_1^2v_1^2 - 12\omega_3^2c_s^2\omega_1^2v_1^2 + 72\omega_3\omega_1^2v_1^4 + 24\omega_3^2c_s^4 + 6\omega_3c_s^4\omega_1^3 - 72\omega_1^2v_1^4 - 216c_s^2\omega_1^2v_1^2 + 72\omega_3c_s^2\omega_1v_1^2 + \omega_3^2c_s^2\omega_1^3 - 30\omega_3\omega_1^3v_1^4 + 24\omega_3c_s^4\omega_1 + 6\omega_3^2c_s^2\omega_1^2v_1^2 - 8\omega_3^2c_s^2\omega_1^2 - 3\omega_3^2\omega_1^3v_1^2 + 72\omega_1^2v_1^2 - 6\omega_3c_s^2\omega_1^3 - 48\omega_3^2c_s^4\omega_1 + 3\omega_3^2\omega_1^3v_1^4 + 30\omega_3\omega_1^3v_1^2 + 24\omega_3c_s^2\omega_1^2 + 144\omega_3c_s^2\omega_1^2v_1^2 - 24\omega_3c_s^2\omega_1 - 36\omega_3^2c_s^2\omega_1v_1^2 - 36\omega_1^3v_1^2 + 24\omega_3^2c_s^4\omega_1^2 - 72\omega_3\omega_1^2v_1^2 - 72\omega_3c_s^2\omega_1^3v_1^2 - 3\omega_3^2c_s^4\omega_1^3 - 12\omega_3^2\omega_1^2v_1^4$$

$$C_{11} = 2\omega_3^2c_s^2\omega_1^2v_1^2\omega_2^2 - 39\omega_3^2\omega_1^2v_1^4\omega_2^2 + 18\omega_3^2c_s^4\omega_1^2\omega_2 + 6\omega_3^2c_s^4\omega_1^2\omega_2^2 - 3\omega_3^2\omega_1^2\omega_2^2v_2^2 + 81\omega_3^2c_s^2\omega_1^2\omega_2^2 + 123\omega_3^2c_s^2\omega_1^2\omega_2 - 7\omega_3^2\omega_1^2\omega_2^2 - 18\omega_3c_s^2\omega_1^3v_1^2\omega_2^2 - 12\omega_3^2\omega_1^3\omega_2 + 72\omega_3^2c_s^4\omega_1^3\omega_2^2 - 12c_s^2\omega_1^3\omega_2^2 - 90\omega_3c_s^4\omega_1\omega_2\omega_2^2 - 3\omega_3^2\omega_1^3\omega_2^2v_2^2 + 197\omega_3^2c_s^2\omega_1^3v_1^2\omega_2^2 - 6\omega_3c_s^2\omega_1^3v_1^2\omega_2^2 + 72\omega_3^2c_s^2\omega_2^2 - 6\omega_3^2\omega_2^2 - 72c_s^4\omega_1^2\omega_2^2 - 12\omega_3^2c_s^2\omega_1^2\omega_2^2 - 46\omega_3^2\omega_1^2v_1^2\omega_2^2 + 9\omega_3^2c_s^2\omega_1^2\omega_2^2v_2^2 + 6\omega_3^2\omega_1^2\omega_2^2 + 51\omega_3^2\omega_1^2v_1^2\omega_2^2 - 2\omega_3^2c_s^2\omega_1^2\omega_2^2 - 30\omega_3c_s^2\omega_1v_1^2\omega_2^2 + 102\omega_3^2\omega_1^2v_1^2\omega_2 - 24\omega_3^2c_s^2\omega_1^2\omega_2 - 72\omega_3^2c_s^4\omega_1^2\omega_2 + 6\omega_3^2\omega_2^2 - 138\omega_3^2c_s^4\omega_1^2\omega_2^2 - 165\omega_3^2c_s^2\omega_1v_1^2\omega_2^2 - 45\omega_3^2v_1^4\omega_2^2 + 45\omega_3^2\omega_1^2v_1^4\omega_2 - 465\omega_3^2c_s^2\omega_1^2v_1^2\omega_2 + 36c_s^4\omega_1^2\omega_2^2 - 59\omega_3^2c_s^2\omega_1^2\omega_2^2 - 102\omega_3^2\omega_1v_1^2\omega_2^2 - 24c_s^2\omega_1^2v_1^2\omega_2^2 + 30\omega_3c_s^2\omega_1\omega_2^2 + 7\omega_3^2\omega_1^2\omega_2^2 + 6\omega_3c_s^2\omega_1^3v_1^2\omega_2 + 261\omega_3^2c_s^2\omega_1^3v_1^2 + 36\omega_3^2c_s^4\omega_1^2\omega_2^2 + 489\omega_3^2c_s^2\omega_1v_1^2\omega_2^2 + 24c_s^2\omega_1^2\omega_2^2 - 51\omega_3^2\omega_1^3v_1^2 + 42\omega_3c_s^2\omega_1^2v_1^2\omega_2^2 - 42\omega_3^2c_s^2\omega_1^2\omega_2^2 - 6\omega_3c_s^2\omega_1^2\omega_2 - 90\omega_3^2c_s^4\omega_1^2\omega_2^2 - 18\omega_3c_s^4\omega_1^2\omega_2^2 + 12\omega_3^2c_s^2\omega_1^2v_1^2\omega_2^2 - 90\omega_3^2c_s^4\omega_1\omega_2^2 + 39\omega_3^2\omega_1^3v_1^4\omega_2^2 - 24\omega_3c_s^2\omega_1^2\omega_2^2 + 46\omega_3^2\omega_1^2v_1^2\omega_2^2 + 24\omega_3c_s^2\omega_1^2v_1^2\omega_2^2 + 45\omega_3^2\omega_1^2v_1^4 + 225\omega_3^2c_s^2\omega_1\omega_2^2 - 219\omega_3^2c_s^2\omega_1^2v_1^2\omega_2^2 - 54\omega_3c_s^4\omega_1^3\omega_2^2 + 3\omega_3^2\omega_1^3\omega_2^2v_2^2 + 90\omega_3^2\omega_1v_1^4\omega_2^2 + 18\omega_3c_s^4\omega_1^2\omega_2 + 126\omega_3^2c_s^4\omega_1^2\omega_2^2 - 9\omega_3^2c_s^2\omega_1^2\omega_2^2v_2^2 + 6\omega_3c_s^2\omega_1^2\omega_2^2 - 6\omega_3^2\omega_1\omega_2^2 + 12c_s^2\omega_1^2v_1^2\omega_2^2 + 48\omega_3^2c_s^2\omega_1\omega_2^2 + 72\omega_3c_s^4\omega_1^2\omega_2^2 - 45\omega_3^2\omega_1v_1^4\omega_2^2 + 141\omega_3^2c_s^2\omega_1^2v_1^2\omega_2 - 90\omega_3^2\omega_1^2v_1^4\omega_2 + 90\omega_3^2c_s^2\omega_1^2 - 261\omega_3^2c_s^2v_1^2\omega_2^2 - 147\omega_3^2c_s^2\omega_1\omega_2^2 - 51\omega_3^2\omega_1^2v_1^2\omega_2 + 51\omega_3^2v_1^2\omega_2^2 + 3\omega_3^2\omega_1^2\omega_2^2v_2^2 + 18\omega_3c_s^2\omega_1^2\omega_2^2 + 12\omega_3^2\omega_1\omega_2^2$$

$$C_{12} = 138\omega_3\omega_1^3v_1^4\omega_2^2 - 30\omega_3c_s^4\omega_1\omega_2^2 + 18c_s^2\omega_1^3\omega_2^2 + 3\omega_3c_s^2\omega_1^3\omega_2^2v_2^2 + 24c_s^4\omega_1^2\omega_2^2 + 153\omega_3c_s^2\omega_1^3v_1^2\omega_2^2 - 6\omega_3\omega_1\omega_2^2 - 12c_s^2\omega_1^3\omega_2^2 + 69\omega_3c_s^4\omega_1\omega_2^2 + 12\omega_3\omega_1\omega_2^2 + 99\omega_3v_1^2\omega_2^2 - 30c_s^4\omega_1^2\omega_2^2 + 6c_s^4\omega_1^2\omega_2 + 18\omega_3c_s^4\omega_1^3 + 81\omega_3^2v_1^2\omega_2^2 + 36\omega_3c_s^2\omega_1^2\omega_2^2 + 72c_s^2\omega_1^2v_1^2\omega_2^2 + 351\omega_3c_s^2\omega_1v_1^2\omega_2^2 - 18c_s^4\omega_1^3\omega_2^2 + 324\omega_3\omega_1v_1^2\omega_2^2 - 24c_s^2\omega_1^2\omega_2^2 + \omega_3\omega_1^2\omega_2^2v_2^2 - 42\omega_3c_s^2\omega_1^2v_2^2 + 171\omega_3\omega_1^2v_1^4 - 63\omega_3\omega_1^2v_1^2\omega_2 + 12c_s^2\omega_1^2\omega_2^2 - 135\omega_3c_s^2\omega_1v_1^2\omega_2^2 - 207\omega_3c_s^2v_1^2\omega_2^2 - 90c_s^2\omega_1^2v_1^2\omega_2^2 - 45\omega_3c_s^2\omega_1\omega_2^2 - 351\omega_3c_s^2\omega_1^3v_1^2\omega_2 - 324\omega_3\omega_1^3v_1^4\omega_2 - 6c_s^2\omega_1^3\omega_2 + 30c_s^2\omega_1^2\omega_2^2 - 135\omega_3\omega_1v_1^4\omega_2^2 + 18c_s^2\omega_1^3v_1^2\omega_2 - 153\omega_3c_s^2\omega_1^2v_1^2\omega_2^2 - \omega_3\omega_1^3\omega_2^2v_2^2 + 24\omega_3c_s^2\omega_1^2\omega_2^2 + 6\omega_3\omega_1^2\omega_2^2 + 45\omega_3c_s^2\omega_1^2\omega_2^2 + 6\omega_3c_s^4\omega_1^2\omega_2 - 171\omega_3v_1^4\omega_2^2 - 2\omega_3c_s^4\omega_1^2\omega_2^2 - 18c_s^2\omega_1\omega_2^2 - 24\omega_3c_s^2\omega_1^3 - 138\omega_3\omega_1^3v_1^2\omega_2^2 + 7\omega_3\omega_1^3\omega_2^2 + 54c_s^2\omega_1v_1^2\omega_2^2 - 81\omega_3^2v_1^2\omega_2^2 - 12\omega_3c_s^2\omega_1^2\omega_2^2 + 36\omega_3c_s^2\omega_1^2v_1^2\omega_2^2 - 99\omega_3\omega_1^3v_1^2 + 6\omega_3\omega_1^3 - \omega_3\omega_1^3\omega_2^2v_2^2 + 17\omega_3c_s^4\omega_1^3\omega_2^2 + 135\omega_3\omega_1^3v_1^4\omega_2 - 33\omega_3c_s^4\omega_1^3\omega_2 - 25\omega_3c_s^2\omega_1^2\omega_2^2 - 6\omega_3\omega_1^3 - 12\omega_3\omega_1^3\omega_2 - 7\omega_3\omega_1^2\omega_2^2 - 12\omega_3\omega_1^2\omega_2^2 - 180\omega_3\omega_1^3v_1^2\omega_2 + 36c_s^2\omega_1^3v_1^2\omega_2^2 + 63\omega_3c_s^2\omega_1^2v_1^2\omega_2 + 63\omega_3\omega_1v_1^2\omega_2^2 + 18c_s^4\omega_1\omega_2^2 + 12\omega_3c_s^4\omega_1^2\omega_2^2 - 3\omega_3c_s^2\omega_1^2\omega_2^2v_2^2 + 207\omega_3c_s^2\omega_1^3v_1^2 - 180\omega_3\omega_1v_1^2\omega_2^2 - 54c_s^2\omega_1^3v_1^2\omega_2^2 - 24\omega_3c_s^2\omega_1^3\omega_2^2 + 24\omega_3c_s^2\omega_2^2$$

$$C_{13} = \omega_3\omega_1v_1^2 + 6\omega_1\omega_2 + 9\omega_3\omega_1v_2^2 + 6\omega_3c_s^2\omega_1\omega_2 - 6\omega_1v_1^2\omega_2 + \omega_3v_1^2\omega_2 - 2\omega_3\omega_1\omega_2 + 2\omega_3\omega_2 + 2\omega_3\omega_1v_1^2\omega_2 + 6\omega_3c_s^2\omega_1 - 18c_s^2\omega_1\omega_2 - 4\omega_3\omega_1 - 9\omega_3\omega_2v_2^2$$

$$C_{14} = -45\omega_3^2\omega_1^2v_1^2\omega_2v_2^2 + 45\omega_3^2v_1^2\omega_2^2v_2^2 - 27\omega_3^2\omega_1^2v_1^4\omega_2^2 - 36c_s^2\omega_1^2\omega_2^2v_2^2 - 18\omega_3^2c_s^4\omega_1^2\omega_2 + 12c_s^2\omega_1^2\omega_2^2 + 45\omega_3^2\omega_1^2v_1^2v_2^2 + 6\omega_3^2\omega_1\omega_2^2v_2^2 + 10\omega_3^2c_s^4\omega_1^3\omega_2^2 + 41\omega_3^2c_s^2\omega_1^2\omega_2^2 + 93\omega_3^2c_s^2\omega_1^2\omega_2 - 6\omega_3^2\omega_1^2\omega_2 + 3\omega_3^2c_s^2\omega_1\omega_2^2v_2^2 + 35\omega_3^2c_s^4\omega_1^2\omega_2^2 - 45\omega_3^2\omega_1v_1^2\omega_2^2v_2^2 - 24c_s^2\omega_1^3\omega_2^2 - 90\omega_3c_s^4\omega_1\omega_2^2 - 30\omega_3c_s^2\omega_1\omega_2^2v_2^2 + 138\omega_3^2c_s^2\omega_1^3v_1^2\omega_2^2 + 12\omega_3^2\omega_1^2\omega_2^2 + 6\omega_3^2\omega_2^2 - 108c_s^4\omega_1^2\omega_2^2 - 54\omega_3^2c_s^2\omega_1^2\omega_2^2 - 27\omega_3^2\omega_1^2\omega_2^2 + 8\omega_3^2c_s^2\omega_1^2\omega_2^2v_2^2 - 30\omega_3^2c_s^2\omega_1\omega_2^2v_2^2 - 6\omega_3^2\omega_1^2\omega_2 - 9\omega_3^2\omega_1v_1^2\omega_2^2 - 2\omega_3^2c_s^2\omega_1^2\omega_2^2 + 99\omega_3^2\omega_1^3v_1^2\omega_2 + 24\omega_3^2c_s^2\omega_1^2\omega_2 - 36c_s^4\omega_1^2\omega_2^2 - 72\omega_3^2c_s^2\omega_1^3 - 117\omega_3^2c_s^2\omega_1^3\omega_2 + 18\omega_3^2c_s^2\omega_1^2v_2^2 + 6\omega_3^2\omega_1^3 - 91\omega_3^2c_s^4\omega_1^2\omega_2^2 + 27\omega_3^2c_s^2\omega_1v_1^2\omega_2^2 - 12\omega_3c_s^2\omega_1^2\omega_2^2v_2^2 + 6\omega_3^2\omega_1\omega_2^2v_2^2 - 6\omega_3^2\omega_1^3v_2^2 - 54\omega_3^2v_1^4\omega_2^2 - 54\omega_3^2\omega_1^2v_1^4\omega_2 - 459\omega_3^2c_s^2\omega_1^3v_1^2\omega_2 + 72c_s^4\omega_1^3\omega_2^2 + 2\omega_3^2c_s^2\omega_1^3\omega_2^2v_2^2 - 25\omega_3^2c_s^2\omega_1^2\omega_2^2 - 9\omega_3^2\omega_1v_1^2\omega_2^2 - 45\omega_3^2\omega_1^3v_1^2\omega_2v_2^2 - 6\omega_3^2c_s^2\omega_1^2\omega_2v_2^2 + 30\omega_3^2c_s^2\omega_1\omega_2^2 + 405\omega_3^2c_s^2\omega_1^3v_1^2 + 54\omega_3^2c_s^4\omega_1^2\omega_2^2 + 189\omega_3^2c_s^2\omega_1v_1^2\omega_2^2 + 36c_s^2\omega_1^2\omega_2^2 + 6\omega_3^2\omega_1^2\omega_2v_2^2 - 99\omega_3^2\omega_1^3v_1^2 - 45\omega_3^2\omega_1v_1^2\omega_2^2v_2^2 + 18\omega_3^2c_s^2\omega_1^2\omega_2v_2^2 - 48\omega_3c_s^2\omega_1^2\omega_2^2 - 90\omega_3^2\omega_1^2v_1^2\omega_2^2 + 6\omega_3c_s^2\omega_1^2\omega_2 + 18\omega_3^2c_s^4\omega_2^2 + 18\omega_3^2c_s^2\omega_2^2v_2^2 - 36\omega_3c_s^4\omega_1^2\omega_2^2 + 270\omega_3^2c_s^2\omega_1^2v_1^2\omega_2^2 - 12\omega_3^2\omega_1^2\omega_2^2v_2^2 - 90\omega_3^2c_s^4\omega_1^2\omega_2^2 + 27\omega_3^2\omega_1^2v_1^2\omega_2^2 + 54\omega_3^2\omega_1^3v_1^2 + 63\omega_3^2c_s^4\omega_1\omega_2^2 - 138\omega_3^2c_s^2\omega_1^2v_1^2\omega_2^2 + 36\omega_3c_s^2\omega_1^2\omega_2^2v_2^2 + 24c_s^4\omega_1^3\omega_2^2v_2^2 + 54\omega_3^2\omega_1v_1^4\omega_2^2 - 18\omega_3c_s^4\omega_1^2\omega_2 + 6\omega_3^2\omega_1^3\omega_2v_2^2 + 144\omega_3c_s^4\omega_1^2\omega_2^2 - 24\omega_3^2c_s^2\omega_1^2\omega_2^2v_2^2 + 12\omega_3c_s^2\omega_1^3\omega_2^2 - 6\omega_3^2\omega_1\omega_2^2 - 21\omega_3^2c_s^2\omega_1^2\omega_2v_2^2 + 48\omega_3^2c_s^2\omega_1\omega_2^2 + 108\omega_3c_s^4\omega_1^2\omega_2^2 + 48\omega_3c_s^2\omega_1^2\omega_2^2v_2^2 + 54\omega_3^2\omega_1v_1^4\omega_2^2 - 12c_s^2\omega_1^2\omega_2^2v_2^2 + 90\omega_3^2\omega_1^2v_1^2\omega_2^2v_2^2 - 6\omega_3c_s^2\omega_1^2\omega_2v_2^2 - 297\omega_3^2c_s^2\omega_1^2v_1^2\omega_2 - 54\omega_3^2\omega_1^3v_1^4\omega_2 + 90\omega_3^2c_s^4\omega_1^3 - 135\omega_3^2c_s^2\omega_1^2\omega_2^2 - 39\omega_3^2c_s^2\omega_1\omega_2^2 + 99\omega_3^2\omega_1^2v_1^2\omega_2 + 9\omega_3^2v_1^2\omega_2^2 - 6\omega_3^2\omega_1\omega_2^2$$

$$C_{15} = 54\omega_3\omega_1^2\omega_2^2v_2^2 - 18\omega_3\omega_1\omega_2^2 - 81\omega_3\omega_1\omega_2^2 - 198\omega_3v_1^2\omega_2^2 - 100\omega_3\omega_1^2v_1^2\omega_2^2 - 54\omega_3c_s^2\omega_1\omega_2^2 + 18\omega_1\omega_2^2 + 36\omega_3\omega_2^2v_2^2 + 108c_s^2\omega_1^2\omega_2^2 - 27\omega_3\omega_1^3\omega_2v_2^2 - 18\omega_1\omega_2^2v_2^2 - 162\omega_3\omega_1^2v_1^2\omega_2 + 135\omega_3c_s^2\omega_1\omega_2^2 - 54c_s^2\omega_1^2\omega_2 - 36\omega_1^2\omega_2^2 - 84\omega_3c_s^2\omega_1^2\omega_2^2 + 90\omega_3\omega_1^2\omega_2 - 297\omega_3c_s^2\omega_1^2\omega_2 + 36\omega_3\omega_1^3v_2^2 - 27\omega_3\omega_1\omega_2^2v_2^2 - 54c_s^2\omega_1\omega_2^2 + 270\omega_3c_s^2\omega_1^3 - 46\omega_3\omega_1^3\omega_2^2 + 100\omega_3\omega_1^3v_1^2\omega_2^2 + 162\omega_3c_s^2\omega_1^2\omega_2^2 - 18\omega_1^2\omega_2v_2^2 + 18\omega_1^2\omega_2 + 198\omega_3\omega_1^3v_1^2 - 126\omega_3\omega_1^3 - 36\omega_3\omega_1\omega_2^2v_2^2 + 54\omega_3\omega_2^2 + 135\omega_3\omega_1^3\omega_2 + 46\omega_3\omega_1^2\omega_2^2 - 162\omega_3c_s^2\omega_1^2\omega_2 - 216\omega_3\omega_1^2v_1^2\omega_2 + 162\omega_3\omega_1v_1^2\omega_2^2 - 36\omega_3\omega_1\omega_2v_2^2 + 216\omega_3\omega_1v_1^2\omega_2^2 + 36\omega_1^2\omega_2^2v_2^2 - 54\omega_3\omega_1^2\omega_2^2 + 84\omega_3c_s^2\omega_1^2\omega_2^2 - 54\omega_3c_s^2\omega_2^2$$

$$C_{16} = -135\omega_3^2\omega_1^2v_1^2\omega_2v_2^2 + 135\omega_3^2v_1^2\omega_2^2v_2^2 - 9\omega_3^2\omega_1^2v_1^4\omega_2^2 + 54\omega_3\omega_1^3v_1^4\omega_2^2 - 30\omega_3^2c_s^4\omega_1^2\omega_2 + 135\omega_3^2\omega_1^3v_1^2v_2^2 - 90\omega_3c_s^2\omega_1^3\omega_2^2v_2^2 + 18\omega_3^2\omega_1\omega_2^2v_2^2 - 2\omega_3^2c_s^4\omega_1^3\omega_2^2 - \omega_3^2\omega_1^2\omega_2^2v_2^2 + 2\omega_3^2c_s^2\omega_1^2\omega_2^2 + 21\omega_3^2c_s^2\omega_1^2\omega_2 + 144\omega_3c_s^2\omega_1^3\omega_2^2 - 6\omega_3^2\omega_1^3\omega_2 + 9\omega_3^2c_s^2\omega_1\omega_2^2v_2^2 + 2\omega_3^2c_s^4\omega_1^2\omega_2^2 - 135\omega_3^2\omega_1v_1^2\omega_2^2v_2^2 + 30\omega_3c_s^4\omega_1\omega_2^2 + 126\omega_3c_s^2\omega_1\omega_2^2v_2^2 - \omega_3^2\omega_1^2\omega_2^2v_2^2 + 30\omega_3^2c_s^2\omega_1^3v_1^2\omega_2^2 + 12\omega_3^2\omega_1^2\omega_2^2 + 6\omega_3^2\omega_2^2 - 42\omega_3^2c_s^2\omega_1^2\omega_2^2 - 9\omega_3^2\omega_1^3v_1^2\omega_2^2 + 6\omega_3^2c_s^2\omega_1^2\omega_2^2v_2^2 + 54\omega_3\omega_1^3v_1^2\omega_2^2 - 90\omega_3^2c_s^2\omega_1\omega_2^2v_2^2 - 6\omega_3^2\omega_1^2\omega_2 + 45\omega_3^2\omega_1v_1^2\omega_2^2 + 18\omega_3c_s^2\omega_1v_1^2\omega_2^2 + 90\omega_3^2\omega_1^3v_1^2\omega_2 + 36\omega_3^2c_s^2\omega_1^2\omega_2 + 36\omega_1^3v_1^2\omega_2^2 - 24\omega_3^2c_s^2\omega_1^3 + 18\omega_3\omega_1v_1^4\omega_2^2 - 15\omega_3^2c_s^4\omega_1^3\omega_2 + 18\omega_3^2c_s^2\omega_1^3v_1^2 + 6\omega_3^2\omega_1^3 - 10\omega_3^2c_s^4\omega_1^2\omega_2^2 - 45\omega_3^2c_s^2\omega_1v_1^2\omega_2^2 + 72\omega_3c_s^4\omega_1^2\omega_2^2v_2^2 + 18\omega_3\omega_1\omega_2^2v_2^2 - 18\omega_3^2\omega_1^3v_1^2 - 36\omega_3^2v_1^4\omega_2^2 - 216\omega_3^2c_s^2\omega_1^3v_1^2\omega_2 - 2\omega_3^2c_s^2\omega_1^3\omega_2^2 + 36\omega_1^2v_1^4\omega_2^2 - 135\omega_3^2\omega_1^3v_1^2\omega_2v_2^2 + 108c_s^2\omega_1^2v_1^2\omega_2^2 - 54\omega_3^2c_s^2\omega_1^2\omega_2v_2^2 - 30\omega_3^2\omega_1^2\omega_2^2 - 54\omega_3c_s^2\omega_1^2\omega_2^2v_2^2 + 189\omega_3^2c_s^2\omega_1^3v_1^2\omega_2 + 18\omega_3\omega_1^3v_1^4\omega_2 + 30\omega_3^2c_s^4\omega_1^2\omega_2^2 + 108\omega_3^2c_s^2\omega_1v_1^2\omega_2^2 + 18\omega_3^2\omega_1^2\omega_2v_2^2 - 81\omega_3^2\omega_1^3v_1^2 - 135\omega_3^2\omega_1v_1^2\omega_2^2v_2^2 - 144\omega_3c_s^2\omega_1^2v_1^2\omega_2^2 + 72\omega_3^2c_s^2\omega_1^2\omega_2^2v_2^2 + 60\omega_3c_s^2\omega_1^2\omega_2^2 - 90\omega_3^2\omega_1^2v_1^2\omega_2^2 - 6\omega_3c_s^2\omega_1^3\omega_2 - 30\omega_3^2c_s^4\omega_1^3 - 18\omega_3^2c_s^2\omega_2^2v_2^2 + 24\omega_3c_s^4\omega_1^3\omega_2^2 + 108\omega_3^2c_s^2\omega_1^2v_1^2\omega_2^2 - 36\omega_3^2\omega_1^2\omega_2^2v_2^2 - 18\omega_3^2\omega_2^2v_2^2 - 54\omega_3\omega_1^2v_1^4\omega_2^2 - 30\omega_3^2c_s^4\omega_1\omega_2^2 + 9\omega_3^2\omega_1^3v_1^4\omega_2^2 - 54\omega_3\omega_1^3v_1^2\omega_2^2 - 60\omega_3c_s^2\omega_1^2\omega_2^2 + 9\omega_3^2\omega_1^2v_1^2\omega_2^2 + 36\omega_3c_s^2\omega_1^2v_1^2\omega_2^2 + 36\omega_3^2\omega_1^3v_1^4 + 45\omega_3^2c_s^4\omega_1\omega_2^2 - 30\omega_3^2c_s^2\omega_1^2v_1^2\omega_2^2 + 144\omega_3c_s^2\omega_1^2\omega_2^2v_2^2 - 36\omega_3c_s^4\omega_1^3\omega_2^2 + \omega_3\omega_1^2\omega_2^2v_2^2 + 45\omega_3^2\omega_1v_1^4\omega_2^2 + 6\omega_3c_s^4\omega_1^2\omega_2^2 + 18\omega_3\omega_1^2\omega_2v_2^2 - 36\omega_3^2v_1^2\omega_2^2 - 60\omega_3c_s^4\omega_1^2\omega_2^2 - 6\omega_3^2c_s^2\omega_1^2\omega_2^2v_2^2 - 24\omega_3c_s^2\omega_1^3\omega_2^2 - 6\omega_3^2\omega_1^3\omega_2 + 18\omega_3\omega_1^3v_1^2\omega_2 - 9\omega_3^2c_s^2\omega_1^3\omega_2v_2^2 + 36\omega_3^2c_s^2\omega_1\omega_2^2 + 60\omega_3c_s^4\omega_1^2\omega_2^2 - 198\omega_3c_s^2\omega_1^2\omega_2^2v_2^2 + 270\omega_3\omega_1^2v_1^2\omega_2^2v_2^2 + 18\omega_3c_s^2\omega_1^3\omega_2v_2^2 - 81\omega_3^2c_s^2\omega_1^3v_1^2\omega_2 - 45\omega_3^2\omega_1^3v_1^4\omega_2 + 18\omega_3^2c_s^4\omega_1^3 - 36\omega_1^3v_1^4\omega_2^2 - 63\omega_3^2c_s^2v_1^2\omega_2^2 - 18\omega_3\omega_1v_1^2\omega_2^2 - 108c_s^2\omega_1^3v_1^2\omega_2^2 - 3\omega_3^2c_s^2\omega_1\omega_2^2 + 45\omega_3^2\omega_1^2v_1^2\omega_2 - 9\omega_3^2v_1^2\omega_2^2 + \omega_3\omega_1^2\omega_2^2v_2^2 + 36\omega_3c_s^2\omega_1^3\omega_2^2 - 6\omega_3^2\omega_1\omega_2^2$$

$$C_{17} = 9\omega_3^2\omega_1^2v_1^2\omega_2v_2^2 - 45\omega_3^2v_1^2\omega_2^2v_2^2 - 54\omega_3^2c_s^4\omega_1^2\omega_2 + 12c_s^2\omega_1^2\omega_2^2 + 45\omega_3^2\omega_1^2v_1^2v_2^2 - 18\omega_3^2\omega_1\omega_2^2v_2^2 + 5\omega_3^2c_s^2\omega_1^2\omega_2^2 + 75\omega_3^2c_s^2\omega_1^3\omega_2 - \omega_3^2\omega_1^2\omega_2^2 - 54\omega_3^2\omega_1^3\omega_2v_2^2 + 18\omega_3c_s^2\omega_1^3v_1^2\omega_2^2 - 6\omega_3^2\omega_1^3\omega_2 - 54\omega_3^2c_s^2\omega_1\omega_2^2v_2^2 + 6\omega_3^2c_s^4\omega_1^2\omega_2^2 + 72\omega_3^2\omega_1v_1^2\omega_2^2v_2^2 + 18\omega_3c_s^4\omega_1\omega_2^2 - 24\omega_3^2\omega_1^3\omega_2^2v_2^2 + 2\omega_3^2c_s^2\omega_1^3v_1^2\omega_2^2 - 6\omega_3^2\omega_2^2 + 36c_s^4\omega_1^2\omega_2^2 - 72\omega_3^2c_s^2\omega_1^2\omega_2^2 - \omega_3^2\omega_1^3v_1^2\omega_2^2 + 72\omega_3^2c_s^2\omega_1^2\omega_2^2v_2^2 - 297\omega_3^2c_s^2\omega_1\omega_2^2v_2^2 - 6\omega_3^2\omega_1^2\omega_2 - 6\omega_3^2\omega_1v_1^2\omega_2^2 + 108\omega_3^2\omega_1^2\omega_2^2v_2^2 + 6\omega_3c_s^2\omega_1v_1^2\omega_2^2 + 54\omega_3^2\omega_2^2v_2^2 + 6\omega_3^2\omega_1^3v_1^2\omega_2 + 60\omega_3^2c_s^2\omega_1^2\omega_2 - 36c_s^4\omega_1^3\omega_2^2 - 72\omega_3^2c_s^2\omega_1^$$

coefficient $C_{D_x^4 \rho}^{(0)}$ **at** $\frac{\partial^4 \rho}{\partial x_1^4}$:

$$C_{D_x^4 \rho}^{(0), \text{SRT}} = (-3v_1^4 \omega - 6v_1^2 - 2c_s^2 + c_s^2 \omega + 24v_1^2 c_s^2 - 12v_1^2 c_s^2 \omega + 3v_1^2 \omega + 2c_s^4 - c_s^4 \omega + 6v_1^4) \frac{1}{24\omega}$$

$$C_{D_x^4 \rho}^{(0), \text{MRT1}} = (3\omega_5 v_1^2 - 6v_1^2 - 2c_s^2 - 12\omega_5 v_1^2 c_s^2 + \omega_5 c_s^2 + 24v_1^2 c_s^2 + 2c_s^4 - \omega_5 c_s^4 - 3\omega_5 v_1^4 + 6v_1^4) \frac{1}{24\omega_5}$$

$$C_{D_x^4 \rho}^{(0), \text{MRT2}} = C_{D_x^4 \rho}^{(0), \text{MRT1}}$$

$$C_{D_x^4 \rho}^{(0), \text{CLBM1}} = C_{D_x^4 \rho}^{(0), \text{MRT1}}$$

$$C_{D_x^4 \rho}^{(0), \text{CLBM2}} = C_{D_x^4 \rho}^{(0), \text{MRT1}}$$

$$C_{D_x^4 \rho}^{(0), \text{CuLBM1}} = (-6v_1^2 - 3v_1^4 \omega_1 - 2c_s^2 - 12v_1^2 c_s^2 \omega_1 + c_s^2 \omega_1 + 24v_1^2 c_s^2 + 2c_s^4 + 3v_1^2 \omega_1 - c_s^4 \omega_1 + 6v_1^4) \frac{1}{24\omega_1}$$

$$C_{D_x^4 \rho}^{(0), \text{CuLBM2}} = (3\omega_2 v_1^4 - \omega_2 c_s^4 \omega_1 + 3v_1^4 \omega_1 + \omega_2 c_s^4 + 3\omega_2 v_1^2 \omega_1 + 12v_1^2 c_s^2 \omega_1 - c_s^2 \omega_1 + 12\omega_2 v_1^2 c_s^2 - \omega_2 c_s^2 + \omega_2 c_s^2 \omega_1 - 3v_1^2 \omega_1 - 12\omega_2 v_1^2 c_s^2 \omega_1 - 3\omega_2 v_1^4 \omega_1 + c_s^4 \omega_1 - 3\omega_2 v_1^2) \frac{1}{24\omega_2 \omega_1}$$

coefficient $C_{D_x^4 v_1}^{(0)}$ **at** $\frac{\partial^4 v_1}{\partial x_1^4}$:

$$C_{D_x^4 v_1}^{(0), \text{SRT}} = (-4 + 10v_1^2 + 6c_s^2 - 3c_s^2 \omega - 5v_1^2 \omega + 2\omega) \frac{v_1 \rho}{12\omega}$$

$$C_{D_x^4 v_1}^{(0), \text{MRT1}} = (-4 - 5\omega_5 v_1^2 + 2\omega_5 + 10v_1^2 + 6c_s^2 - 3\omega_5 c_s^2) \frac{v_1 \rho}{12\omega_5}$$

$$C_{D_x^4 v_1}^{(0), \text{MRT2}} = C_{D_x^4 v_1}^{(0), \text{MRT1}}$$

$$C_{D_x^4 v_1}^{(0), \text{CLBM1}} = C_{D_x^4 v_1}^{(0), \text{MRT1}}$$

$$C_{D_x^4 v_1}^{(0), \text{CLBM2}} = C_{D_x^4 v_1}^{(0), \text{MRT1}}$$

$$C_{D_x^4 v_1}^{(0), \text{CuLBM1}} = (-4 + 10v_1^2 + 6c_s^2 - 3c_s^2 \omega_1 - 5v_1^2 \omega_1 + 2\omega_1) \frac{v_1 \rho}{12\omega_1}$$

$$C_{D_x^4 v_1}^{(0), \text{CuLBM2}} = (-2\omega_2 + 2\omega_2 \omega_1 - 5\omega_2 v_1^2 \omega_1 + 3c_s^2 \omega_1 + 3\omega_2 c_s^2 - 3\omega_2 c_s^2 \omega_1 + 5v_1^2 \omega_1 - 2\omega_1 + 5\omega_2 v_1^2) \frac{v_1 \rho}{12\omega_2 \omega_1}$$

coefficient $C_{D_x^3 D_y \rho}^{(0)}$ **at** $\frac{\partial^4 \rho}{\partial x_1^3 \partial x_2}$:

$$C_{D_x^3 D_y \rho}^{(0), \text{SRT}} = 0$$

$$C_{D_x^3 D_y \rho}^{(0), \text{MRT1}} = (-\omega_5 v_1^2 + \omega_5 + v_1^2 \omega_7 - 3\omega_5 c_s^2 + 3c_s^2 \omega_7 - \omega_7) \frac{v_1 v_2}{4\omega_5 \omega_7}$$

$$C_{D_x^3 D_y \rho}^{(0), \text{MRT2}} = C_{D_x^3 D_y \rho}^{(0), \text{MRT1}}$$

$$C_{D_x^3 D_y \rho}^{(0), \text{CLBM1}} = 0$$

$$C_{D_x^3 D_y \rho}^{(0), \text{CLBM2}} = 0$$

$$C_{D_x^3 D_y \rho}^{(0), \text{CuLBM1}} = 0$$

$$C_{D_x^3 D_y \rho}^{(0), \text{CuLBM2}} = (-\omega_2 v_2^2 + \omega_2 + v_2^2 \omega_1 + 3c_s^2 \omega_1 - 3\omega_2 c_s^2 - \omega_1) \frac{v_1 v_2}{8\omega_2 \omega_1}$$

coefficient $C_{D_x^3 D_y v_1}^{(0)}$ **at** $\frac{\partial^4 v_1}{\partial x_1^3 \partial x_2}$:

$$C_{D_x^3 D_y v_1}^{(0), \text{SRT}} = 0$$

$$C_{D_x^3 D_y v_1}^{(0), \text{MRT}1} = (-3\omega_5 v_1^2 + \omega_5 + 3v_1^2 \omega_7 - \omega_5 c_s^2 + c_s^2 \omega_7 - \omega_7) \frac{\rho v_2}{4\omega_5 \omega_7}$$

$$C_{D_x^3 D_y v_1}^{(0), \text{MRT}2} = C_{D_x^3 D_y v_1}^{(0), \text{MRT}1}$$

$$C_{D_x^3 D_y v_1}^{(0), \text{CLBM}1} = 0$$

$$C_{D_x^3 D_y v_1}^{(0), \text{CLBM}2} = 0$$

$$C_{D_x^3 D_y v_1}^{(0), \text{CuLBM}1} = 0$$

$$C_{D_x^3 D_y v_1}^{(0), \text{CuLBM}2} = (-\omega_2 v_2^2 + \omega_2 + v_2^2 \omega_1 + 3c_s^2 \omega_1 - 3\omega_2 c_s^2 - \omega_1) \frac{\rho v_2}{24\omega_2 \omega_1}$$

coefficient $C_{D_x^3 D_y v_2}^{(0)}$ **at** $\frac{\partial^4 v_2}{\partial x_1^3 \partial x_2}$:

$$C_{D_x^3 D_y v_2}^{(0), \text{SRT}} = (2 - 2v_1^2 - 6c_s^2 + 3c_s^2 \omega + v_1^2 \omega - \omega) \frac{v_1 \rho}{12\omega}$$

$$C_{D_x^3 D_y v_2}^{(0), \text{MRT}1} = (v_1^2 \omega_7 \omega_4 - \omega_7 \omega_4 - \omega_5 \omega_7 \omega_4 + \omega_5 v_1^2 \omega_7 \omega_4 - 3\omega_5 c_s^2 \omega_4 - 6\omega_5 c_s^2 \omega_7 + 3\omega_5 \omega_4 - 3\omega_5 v_1^2 \omega_4 + 3c_s^2 \omega_7 \omega_4 + 3\omega_5 c_s^2 \omega_7 \omega_4) \frac{v_1 \rho}{12\omega_5 \omega_7 \omega_4}$$

$$C_{D_x^3 D_y v_2}^{(0), \text{MRT}2} = C_{D_x^3 D_y v_2}^{(0), \text{MRT}1}$$

$$C_{D_x^3 D_y v_2}^{(0), \text{CLBM}1} = (-3\omega_5 v_1^2 + 3\omega_5 + \omega_5 v_1^2 \omega_7 - \omega_5 \omega_7 + v_1^2 \omega_7 - 9\omega_5 c_s^2 + 3\omega_5 c_s^2 \omega_7 + 3c_s^2 \omega_7 - \omega_7) \frac{v_1 \rho}{12\omega_5 \omega_7}$$

$$C_{D_x^3 D_y v_2}^{(0), \text{CLBM}2} = C_{D_x^3 D_y v_2}^{(0), \text{CLBM}1}$$

$$C_{D_x^3 D_y v_2}^{(0), \text{CuLBM}1} = (3c_s^2 \omega_4 \omega_1 + 3c_s^2 \omega_4 - 9c_s^2 \omega_1 - 3v_1^2 \omega_1 + v_1^2 \omega_4 - \omega_4 + 3\omega_1 - \omega_4 \omega_1 + v_1^2 \omega_4 \omega_1) \frac{v_1 \rho}{12\omega_4 \omega_1}$$

$$C_{D_x^3 D_y v_2}^{(0), \text{CuLBM}2} = (6\omega_3 c_s^2 \omega_1 + 2\omega_2 \omega_3 - 4\omega_3 \omega_1 + \omega_2 v_1^2 \omega_3 + v_1^2 \omega_3 \omega_1 + 6\omega_2 \omega_1 - 6\omega_2 v_1^2 \omega_1 - 18\omega_2 c_s^2 \omega_1 + 9v_2^2 \omega_3 \omega_1 + 2\omega_2 v_1^2 \omega_3 \omega_1 - 2\omega_2 \omega_3 \omega_1 - 9\omega_2 v_2^2 \omega_3 + 6\omega_2 \omega_3 c_s^2 \omega_1) \frac{v_1 \rho}{24\omega_2 \omega_3 \omega_1}$$

coefficient $C_{D_x^2 D_y^2 \rho}^{(0)}$ **at** $\frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2}$:

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{SRT}} = (-2 + \omega) \frac{c_s^4}{6\omega}$$

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{MRT}1} = (-2 + \omega_4) \frac{c_s^4}{6\omega_4}$$

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{MRT}2} = C_{D_x^2 D_y^2 \rho}^{(0), \text{MRT}1}$$

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{CLBM}1} = C_{D_x^2 D_y^2 \rho}^{(0), \text{MRT}1}$$

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{CLBM}2} = C_{D_x^2 D_y^2 \rho}^{(0), \text{MRT}1}$$

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{CuLBM}1} = (-2 + \omega_3) \frac{c_s^4}{6\omega_3}$$

$$C_{D_x^2 D_y^2 \rho}^{(0), \text{CuLBM}2} = (-3\omega_2 v_2^2 + 2\omega_2 + 3v_2^2 \omega_1 + 2c_s^2 \omega_1 - 10\omega_2 c_s^2 + 4\omega_2 c_s^2 \omega_1 + 3v_1^2 \omega_1 - 2\omega_1 - 3\omega_2 v_1^2) \frac{c_s^2}{24\omega_2 \omega_1}$$

coefficient $C_{D_x^2 D_y^2 v_1}^{(0)}$ **at** $\frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2}$:

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{SRT}} = 0$$

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{MRT}1} = (-\omega_7 + \omega_4) \frac{v_1 \rho c_s^2}{2\omega_7 \omega_4}$$

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{MRT}^2} = C_{D_x^2 D_y^2 v_1}^{(0), \text{MRT}^1}$$

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{CLBM}^1} = 0$$

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{CLBM}^2} = 0$$

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{CuLBM}^1} = 0$$

$$C_{D_x^2 D_y^2 v_1}^{(0), \text{CuLBM}^2} = (\omega_2 + 3c_s^2 \omega_1 - 3\omega_2 c_s^2 + v_1^2 \omega_1 - \omega_1 - \omega_2 v_1^2) \frac{v_1 \rho}{24 \omega_2 \omega_1}$$

coefficient $C_{D_x^2 D_y^2 v_2}^{(0)}$ **at** $\frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2}$:

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{SRT}} = 0$$

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{MRT}^1} = (-\omega_8 + \omega_4) \frac{\rho v_2 c_s^2}{2 \omega_8 \omega_4}$$

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{MRT}^2} = C_{D_x^2 D_y^2 v_2}^{(0), \text{MRT}^1}$$

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{CLBM}^1} = 0$$

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{CLBM}^2} = 0$$

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{CuLBM}^1} = 0$$

$$C_{D_x^2 D_y^2 v_2}^{(0), \text{CuLBM}^2} = (-\omega_2 v_2^2 + \omega_2 + v_2^2 \omega_1 + 3c_s^2 \omega_1 - 3\omega_2 c_s^2 - \omega_1) \frac{\rho v_2}{24 \omega_2 \omega_1}$$

coefficient $C_{D_x D_y^3 \rho}^{(0)}$ **at** $\frac{\partial^4 \rho}{\partial x_1 \partial x_2^3}$:

$$C_{D_x D_y^3 \rho}^{(0), \text{SRT}} = 0$$

$$C_{D_x D_y^3 \rho}^{(0), \text{MRT}^1} = (-\omega_8 - v_2^2 \omega_6 + 3\omega_8 c_s^2 + \omega_6 + \omega_8 v_2^2 - 3\omega_6 c_s^2) \frac{v_1 v_2}{4 \omega_8 \omega_6}$$

$$C_{D_x D_y^3 \rho}^{(0), \text{MRT}^2} = C_{D_x D_y^3 \rho}^{(0), \text{MRT}^1}$$

$$C_{D_x D_y^3 \rho}^{(0), \text{CLBM}^1} = 0$$

$$C_{D_x D_y^3 \rho}^{(0), \text{CLBM}^2} = 0$$

$$C_{D_x D_y^3 \rho}^{(0), \text{CuLBM}^1} = 0$$

$$C_{D_x D_y^3 \rho}^{(0), \text{CuLBM}^2} = (\omega_2 + 3c_s^2 \omega_1 - 3\omega_2 c_s^2 + v_1^2 \omega_1 - \omega_1 - \omega_2 v_1^2) \frac{v_1 v_2}{8 \omega_2 \omega_1}$$

coefficient $C_{D_x D_y^3 v_1}^{(0)}$ **at** $\frac{\partial^4 v_1}{\partial x_1 \partial x_2^3}$:

$$C_{D_x D_y^3 v_1}^{(0), \text{SRT}} = (2 + v_2^2 \omega - 6c_s^2 + 3c_s^2 \omega - 2v_2^2 - \omega) \frac{\rho v_2}{12 \omega}$$

$$C_{D_x D_y^3 v_1}^{(0), \text{MRT}^1} = (-6\omega_8 \omega_6 c_s^2 + 3\omega_6 \omega_4 + 3\omega_8 c_s^2 \omega_4 - 3\omega_6 c_s^2 \omega_4 + \omega_8 v_2^2 \omega_6 \omega_4 - \omega_8 \omega_6 \omega_4 + \omega_8 v_2^2 \omega_4 - 3v_2^2 \omega_6 \omega_4 - \omega_8 \omega_4 + 3\omega_8 \omega_6 c_s^2 \omega_4) \frac{\rho v_2}{12 \omega_8 \omega_6 \omega_4}$$

$$C_{D_x D_y^3 v_1}^{(0), \text{MRT}^2} = C_{D_x D_y^3 v_1}^{(0), \text{MRT}^1}$$

$$C_{D_x D_y^3 v_1}^{(0), \text{CLBM1}} = (-\omega_8 + 3\omega_8\omega_6 c_s^2 - 3v_2^2\omega_6 + 3\omega_8 c_s^2 + 3\omega_6 + \omega_8 v_2^2 - \omega_8\omega_6 + \omega_8 v_2^2\omega_6 - 9\omega_6 c_s^2) \frac{\rho v_2}{12\omega_8\omega_6}$$

$$C_{D_x D_y^3 v_1}^{(0), \text{CLBM2}} = C_{D_x D_y^3 v_1}^{(0), \text{CLBM1}}$$

$$C_{D_x D_y^3 v_1}^{(0), \text{CuLBM1}} = (-3\omega_2 v_2^2 - \omega_2\omega_6 + v_2^2\omega_6 + 3\omega_2 + 3\omega_2\omega_6 c_s^2 - \omega_6 - 9\omega_2 c_s^2 + 3\omega_6 c_s^2 + \omega_2 v_2^2\omega_6) \frac{\rho v_2}{12\omega_2\omega_6}$$

$$C_{D_x D_y^3 v_1}^{(0), \text{CuLBM2}} = (6\omega_3 c_s^2 \omega_1 + 2\omega_2\omega_3 - 4\omega_3\omega_1 - 9\omega_2 v_1^2\omega_3 + 9v_1^2\omega_3\omega_1 + 2\omega_2 v_2^2\omega_3\omega_1 + 6\omega_2\omega_1 - 18\omega_2 c_s^2 \omega_1 - 6\omega_2 v_2^2\omega_1 + v_2^2\omega_3\omega_1 - 2\omega_2\omega_3\omega_1 + \omega_2 v_2^2\omega_3 + 6\omega_2\omega_3 c_s^2 \omega_1) \frac{\rho v_2}{24\omega_2\omega_3\omega_1}$$

coefficient $C_{D_x D_y^3 v_2}^{(0)}$ **at** $\frac{\partial^4 v_2}{\partial x_1 \partial x_2^3}$:

$$C_{D_x D_y^3 v_2}^{(0), \text{SRT}} = 0$$

$$C_{D_x D_y^3 v_2}^{(0), \text{MRT1}} = (-\omega_8 - 3v_2^2\omega_6 + \omega_8 c_s^2 + \omega_6 + 3\omega_8 v_2^2 - \omega_6 c_s^2) \frac{v_1 \rho}{4\omega_8\omega_6}$$

$$C_{D_x D_y^3 v_2}^{(0), \text{MRT2}} = C_{D_x D_y^3 v_2}^{(0), \text{MRT1}}$$

$$C_{D_x D_y^3 v_2}^{(0), \text{CLBM1}} = 0$$

$$C_{D_x D_y^3 v_2}^{(0), \text{CLBM2}} = 0$$

$$C_{D_x D_y^3 v_2}^{(0), \text{CuLBM1}} = 0$$

$$C_{D_x D_y^3 v_2}^{(0), \text{CuLBM2}} = (\omega_2 + 3c_s^2 \omega_1 - 3\omega_2 c_s^2 + v_1^2 \omega_1 - \omega_1 - \omega_2 v_1^2) \frac{v_1 \rho}{24\omega_2 \omega_1}$$

coefficient $C_{D_y^4 \rho}^{(0)}$ **at** $\frac{\partial^4 \rho}{\partial x_2^4}$:

$$C_{D_y^4 \rho}^{(0), \text{SRT}} = (6v_2^4 + 3v_2^2\omega - 2c_s^2 + c_s^2\omega + 2c_s^4 + 24v_2^2 c_s^2 - c_s^4\omega - 6v_2^2 - 12v_2^2 c_s^2\omega - 3v_2^4\omega) \frac{1}{24\omega}$$

$$C_{D_y^4 \rho}^{(0), \text{MRT1}} = (6v_2^4 + 3v_2^2\omega_6 - 12v_2^2\omega_6 c_s^2 - 2c_s^2 - \omega_6 c_s^4 - 3v_2^4\omega_6 + 2c_s^4 + 24v_2^2 c_s^2 + \omega_6 c_s^2 - 6v_2^2) \frac{1}{24\omega_6}$$

$$C_{D_y^4 \rho}^{(0), \text{MRT2}} = C_{D_y^4 \rho}^{(0), \text{MRT1}}$$

$$C_{D_y^4 \rho}^{(0), \text{CLBM1}} = C_{D_y^4 \rho}^{(0), \text{MRT1}}$$

$$C_{D_y^4 \rho}^{(0), \text{CLBM2}} = C_{D_y^4 \rho}^{(0), \text{MRT1}}$$

$$C_{D_y^4 \rho}^{(0), \text{CuLBM1}} = (6v_2^4 + 3\omega_2 v_2^2 - 12\omega_2 v_2^2 c_s^2 - 2c_s^2 - \omega_2 c_s^4 + 2c_s^4 + \omega_2 c_s^2 + 24v_2^2 c_s^2 - 6v_2^2 - 3\omega_2 v_2^4) \frac{1}{24\omega_2}$$

$$C_{D_y^4 \rho}^{(0), \text{CuLBM2}} = (-3\omega_2 v_2^4 \omega_1 - 12\omega_2 v_2^2 c_s^2 \omega_1 - 3\omega_2 v_2^2 - \omega_2 c_s^4 \omega_1 + 12\omega_2 v_2^2 c_s^2 + \omega_2 c_s^4 - 3v_2^2 \omega_1 - c_s^2 \omega_1 - \omega_2 c_s^2 + \omega_2 c_s^2 \omega_1 + 3\omega_2 v_2^2 \omega_1 + 3\omega_2 v_2^4 + c_s^4 \omega_1 + 12v_2^2 c_s^2 \omega_1 + 3v_2^4 \omega_1) \frac{1}{24\omega_2 \omega_1}$$

coefficient $C_{D_y^4 v_2}^{(0)}$ **at** $\frac{\partial^4 v_2}{\partial x_2^4}$:

$$C_{D_y^4 v_2}^{(0), \text{SRT}} = (-4 - 5v_2^2\omega + 6c_s^2 - 3c_s^2\omega + 10v_2^2 + 2\omega) \frac{\rho v_2}{12\omega}$$

$$C_{D_y^4 v_2}^{(0), \text{MRT1}} = (-4 - 5v_2^2\omega_6 + 6c_s^2 + 2\omega_6 - 3\omega_6 c_s^2 + 10v_2^2) \frac{\rho v_2}{12\omega_6}$$

$$C_{D_y^4 v_2}^{(0), \text{MRT2}} = C_{D_y^4 v_2}^{(0), \text{MRT1}}$$

$$C_{D_y^4 v_2}^{(0), \text{CLBM1}} = C_{D_y^4 v_2}^{(0), \text{MRT1}}$$

$$C_{D_y^4 v_2}^{(0), \text{CLBM2}} = C_{D_y^4 v_2}^{(0), \text{MRT1}}$$

$$C_{D_y^4 v_2}^{(0), \text{CuLBM1}} = (-4 - 5\omega_2 v_2^2 + 2\omega_2 + 6c_s^2 - 3\omega_2 c_s^2 + 10v_2^2) \frac{\rho v_2}{12\omega_2}$$

$$C_{D_y^4 v_2}^{(0), \text{CuLBM2}} = (5\omega_2 v_2^2 - 2\omega_2 + 5v_2^2 \omega_1 + 2\omega_2 \omega_1 + 3c_s^2 \omega_1 + 3\omega_2 c_s^2 - 3\omega_2 c_s^2 \omega_1 - 5\omega_2 v_2^2 \omega_1 - 2\omega_1) \frac{\rho v_2}{12\omega_2 \omega_1}$$

3.2 Conservation of momentum: ρv_1

$$\begin{aligned} & v_1 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_1}{\partial t} + (v_1^2 + c_s^2) \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_1} + 2v_1 \rho \frac{\delta_l}{\delta_t} \frac{\partial v_1}{\partial x_1} + v_1 v_2 \frac{\delta_l}{\delta_t} \frac{\partial \rho}{\partial x_2} + \rho v_2 \frac{\delta_l}{\delta_t} \frac{\partial v_1}{\partial x_2} + v_1 \rho \frac{\delta_l}{\delta_t} \frac{\partial v_2}{\partial x_2} + C_{D_x \rho, D_x v_1}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1} + \\ & C_{D_x v_1, D_x v_1}^{(1)} \frac{\delta_l^2}{\delta_t} \left(\frac{\partial v_1}{\partial x_1} \right)^2 + C_{D_x \rho, D_y v_2}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_2} + C_{D_x v_2, D_y v_2}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial v_2}{\partial x_1} \frac{\partial v_2}{\partial x_2} + C_{D_y \rho, D_x v_2}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1} + \\ & C_{D_y \rho, D_y v_1}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2} + C_{D_x^2 \rho}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 \rho}{\partial x_1^2} + C_{D_x^2 v_1}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 v_1}{\partial x_1^2} + C_{D_x D_y \rho}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 \rho}{\partial x_1 \partial x_2} + C_{D_x D_y v_2}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 v_2}{\partial x_1 \partial x_2} + C_{D_y^2 v_1}^{(1)} \frac{\delta_l^2}{\delta_t} \frac{\partial^2 v_1}{\partial x_2^2} \\ & + C_{D_x^3 \rho}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_1^3} + C_{D_x^3 v_1}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_1^3} + C_{D_x^2 D_y \rho}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + C_{D_x^2 D_y v_1}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} + C_{D_x^2 D_y v_2}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} + \\ & C_{D_x D_y^2 \rho}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + C_{D_x D_y^2 v_1}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_{D_x D_y^2 v_2}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_{D_y^3 \rho}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 \rho}{\partial x_2^3} + C_{D_y^3 v_1}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_1}{\partial x_2^3} + C_{D_y^3 v_2}^{(1)} \frac{\delta_l^3}{\delta_t} \frac{\partial^3 v_2}{\partial x_2^3} + \\ & C_{D_x^4 \rho}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^4} + C_{D_x^4 v_1}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^4} + C_{D_x^3 D_y \rho}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{D_x^3 D_y v_1}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + C_{D_x^3 D_y v_2}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + C_{D_x^2 D_y^2 \rho}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} \\ & + C_{D_x^2 D_y^2 v_1}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{D_x^2 D_y^2 v_2}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_{D_x D_y^3 \rho}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{D_x D_y^3 v_1}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + C_{D_x D_y^3 v_2}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + \\ & C_{D_y^4 \rho}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{D_y^4 v_1}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_1}{\partial x_2^4} + C_{D_y^4 v_2}^{(1)} \frac{\delta_l^4}{\delta_t} \frac{\partial^4 v_2}{\partial x_2^4} = 0, \end{aligned}$$

where:

coefficient $C_{D_x \rho, D_x v_1}^{(1)}$ **at** $\frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_1}$:

$$C_{D_x \rho, D_x v_1}^{(1), \text{SRT}} = (-2 + 6v_1^2 + 4c_s^2 - 2c_s^2 \omega - 3v_1^2 \omega + \omega) \frac{1}{\omega}$$

$$C_{D_x \rho, D_x v_1}^{(1), \text{MRT1}} = (-2 - 3\omega_5 v_1^2 + \omega_5 + 6v_1^2 + 4c_s^2 - 2\omega_5 c_s^2) \frac{1}{\omega_5}$$

$$C_{D_x \rho, D_x v_1}^{(1), \text{MRT2}} = C_{D_x \rho, D_x v_1}^{(1), \text{MRT1}}$$

$$C_{D_x \rho, D_x v_1}^{(1), \text{CLBM1}} = C_{D_x \rho, D_x v_1}^{(1), \text{MRT1}}$$

$$C_{D_x \rho, D_x v_1}^{(1), \text{CLBM2}} = C_{D_x \rho, D_x v_1}^{(1), \text{MRT1}}$$

$$C_{D_x \rho, D_x v_1}^{(1), \text{CuLBM1}} = (-2 + 6v_1^2 + 4c_s^2 - 2c_s^2 \omega_1 - 3v_1^2 \omega_1 + \omega_1) \frac{1}{\omega_1}$$

$$C_{D_x \rho, D_x v_1}^{(1), \text{CuLBM2}} = (-\omega_2 + \omega_2 \omega_1 - 3\omega_2 v_1^2 \omega_1 + 2c_s^2 \omega_1 + 2\omega_2 c_s^2 - 2\omega_2 c_s^2 \omega_1 + 3v_1^2 \omega_1 - \omega_1 + 3\omega_2 v_1^2) \frac{1}{\omega_2 \omega_1}$$

coefficient $C_{D_x v_1, D_x v_1}^{(1)}$ **at** $\left(\frac{\partial v_1}{\partial x_1} \right)^2$:

$$C_{D_x v_1, D_x v_1}^{(1), \text{SRT}} = (2 - \omega) \frac{3v_1 \rho}{\omega}$$

$$C_{D_x v_1, D_x v_1}^{(1), \text{MRT1}} = (2 - \omega_5) \frac{3v_1 \rho}{\omega_5}$$

$$C_{D_x v_1, D_x v_1}^{(1), \text{MRT2}} = C_{D_x v_1, D_x v_1}^{(1), \text{MRT1}}$$

$$C_{D_x v_1, D_x v_1}^{(1), \text{CLBM1}} = C_{D_x v_1, D_x v_1}^{(1), \text{MRT1}}$$

$$C_{D_x v_1, D_x v_1}^{(1), \text{CLBM2}} = C_{D_x v_1, D_x v_1}^{(1), \text{MRT1}}$$

$$C_{D_x v_1, D_x v_1}^{(1), \text{CuLBM1}} = (2 - \omega_1) \frac{3v_1 \rho}{\omega_1}$$

$$C_{D_x v_1, D_x v_1}^{(1), \text{CuLBM2}} = (\omega_2 - \omega_2 \omega_1 + \omega_1) \frac{3v_1 \rho}{\omega_2 \omega_1}$$

coefficient $C_{D_x \rho, D_y v_2}^{(1)}$ **at** $\frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_2}$:

$$C_{D_x \rho, D_y v_2}^{(1), \text{SRT}} = 0$$

$$C_{D_x \rho, D_y v_2}^{(1), \text{MRT1}} = 0$$

$$C_{D_x \rho, D_y v_2}^{(1), \text{MRT2}} = 0$$

$$C_{D_x \rho, D_y v_2}^{(1), \text{CLBM1}} = 0$$

$$C_{D_x \rho, D_y v_2}^{(1), \text{CLBM2}} = 0$$

$$C_{D_x \rho, D_y v_2}^{(1), \text{CuLBM1}} = 0$$

$$C_{D_x \rho, D_y v_2}^{(1), \text{CuLBM2}} = (-3\omega_2 v_2^2 + \omega_2 + 3v_2^2 \omega_1 + c_s^2 \omega_1 - \omega_2 c_s^2 - \omega_1) \frac{1}{2\omega_2 \omega_1}$$

coefficient $C_{D_x v_2, D_y v_2}^{(1)}$ **at** $\frac{\partial v_2}{\partial x_1} \frac{\partial v_2}{\partial x_2}$:

$$C_{D_x v_2, D_y v_2}^{(1), \text{SRT}} = 0$$

$$C_{D_x v_2, D_y v_2}^{(1), \text{MRT1}} = 0$$

$$C_{D_x v_2, D_y v_2}^{(1), \text{MRT2}} = 0$$

$$C_{D_x v_2, D_y v_2}^{(1), \text{CLBM1}} = 0$$

$$C_{D_x v_2, D_y v_2}^{(1), \text{CLBM2}} = 0$$

$$C_{D_x v_2, D_y v_2}^{(1), \text{CuLBM1}} = 0$$

$$C_{D_x v_2, D_y v_2}^{(1), \text{CuLBM2}} = (-\omega_2 + \omega_1) \frac{3\rho v_2}{\omega_2 \omega_1}$$

coefficient $C_{D_y \rho, D_x v_2}^{(1)}$ **at** $\frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_1}$:

$$C_{D_y \rho, D_x v_2}^{(1), \text{SRT}} = (-2 + \omega) \frac{c_s^2}{2\omega}$$

$$C_{D_y \rho, D_x v_2}^{(1), \text{MRT1}} = (-2 + \omega_4) \frac{c_s^2}{2\omega_4}$$

$$C_{D_y \rho, D_x v_2}^{(1), \text{MRT2}} = C_{D_y \rho, D_x v_2}^{(1), \text{MRT1}}$$

$$C_{D_y \rho, D_x v_2}^{(1), \text{CLBM1}} = C_{D_y \rho, D_x v_2}^{(1), \text{MRT1}}$$

$$C_{D_y \rho, D_x v_2}^{(1), \text{CLBM2}} = C_{D_y \rho, D_x v_2}^{(1), \text{MRT1}}$$

$$C_{D_y \rho, D_x v_2}^{(1), \text{CuLBM1}} = (-2 + \omega_3) \frac{c_s^2}{2\omega_3}$$

$$C_{D_y \rho, D_x v_2}^{(1), \text{CuLBM2}} = (-3\omega_2 v_2^2 + \omega_2 + 3v_2^2 \omega_1 + 3c_s^2 \omega_1 - 5\omega_2 c_s^2 + \omega_2 c_s^2 \omega_1 - \omega_1) \frac{1}{2\omega_2 \omega_1}$$

coefficient $C_{D_y \rho, D_y v_1}^{(1)}$ **at** $\frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_2}$:

$$C_{D_y \rho, D_y v_1}^{(1), \text{SRT}} = (-2 + \omega) \frac{c_s^2}{2\omega}$$

$$C_{D_y \rho, D_y v_1}^{(1), \text{MRT1}} = (-2 + \omega_4) \frac{c_s^2}{2\omega_4}$$

$$C_{D_y \rho, D_y v_1}^{(1), \text{MRT2}} = C_{D_y \rho, D_y v_1}^{(1), \text{MRT1}}$$

$$C_{D_y \rho, D_y v_1}^{(1), \text{CLBM1}} = C_{D_y \rho, D_y v_1}^{(1), \text{MRT1}}$$

$$C_{D_y \rho, D_y v_1}^{(1), \text{CLBM2}} = C_{D_y \rho, D_y v_1}^{(1), \text{MRT1}}$$

$$C_{D_y \rho, D_y v_1}^{(1), \text{CuLBM1}} = (-2 + \omega_3) \frac{c_s^2}{2\omega_3}$$

$$C_{D_y \rho, D_y v_1}^{(1), \text{CuLBM2}} = (-2 + \omega_1) \frac{c_s^2}{2\omega_1}$$

coefficient $C_{D_x^2 \rho}^{(1)}$ **at** $\frac{\partial^2 \rho}{\partial x_1^2}$:

$$C_{D_x^2 \rho}^{(1), \text{SRT}} = (-2 + 2v_1^2 + 6c_s^2 - 3c_s^2\omega - v_1^2\omega + \omega) \frac{v_1}{2\omega}$$

$$C_{D_x^2 \rho}^{(1), \text{MRT1}} = (-2 - \omega_5 v_1^2 + \omega_5 + 2v_1^2 + 6c_s^2 - 3\omega_5 c_s^2) \frac{v_1}{2\omega_5}$$

$$C_{D_x^2 \rho}^{(1), \text{MRT2}} = C_{D_x^2 \rho}^{(1), \text{MRT1}}$$

$$C_{D_x^2 \rho}^{(1), \text{CLBM1}} = C_{D_x^2 \rho}^{(1), \text{MRT1}}$$

$$C_{D_x^2 \rho}^{(1), \text{CLBM2}} = C_{D_x^2 \rho}^{(1), \text{MRT1}}$$

$$C_{D_x^2 \rho}^{(1), \text{CuLBM1}} = (-2 + 2v_1^2 + 6c_s^2 - 3c_s^2\omega_1 - v_1^2\omega_1 + \omega_1) \frac{v_1}{2\omega_1}$$

$$C_{D_x^2 \rho}^{(1), \text{CuLBM2}} = (-\omega_2 + \omega_2\omega_1 - \omega_2 v_1^2\omega_1 + 3c_s^2\omega_1 + 3\omega_2 c_s^2 - 3\omega_2 c_s^2\omega_1 + v_1^2\omega_1 - \omega_1 + \omega_2 v_1^2) \frac{v_1}{2\omega_2\omega_1}$$

coefficient $C_{D_x^2 v_1}^{(1)}$ **at** $\frac{\partial^2 v_1}{\partial x_1^2}$:

$$C_{D_x^2 v_1}^{(1), \text{SRT}} = (-2 + 6v_1^2 + 2c_s^2 - c_s^2\omega - 3v_1^2\omega + \omega) \frac{\rho}{2\omega}$$

$$C_{D_x^2 v_1}^{(1), \text{MRT1}} = (-2 - 3\omega_5 v_1^2 + \omega_5 + 6v_1^2 + 2c_s^2 - \omega_5 c_s^2) \frac{\rho}{2\omega_5}$$

$$C_{D_x^2 v_1}^{(1), \text{MRT2}} = C_{D_x^2 v_1}^{(1), \text{MRT1}}$$

$$C_{D_x^2 v_1}^{(1), \text{CLBM1}} = C_{D_x^2 v_1}^{(1), \text{MRT1}}$$

$$C_{D_x^2 v_1}^{(1), \text{CLBM2}} = C_{D_x^2 v_1}^{(1), \text{MRT1}}$$

$$C_{D_x^2 v_1}^{(1), \text{CuLBM1}} = (-2 + 6v_1^2 + 2c_s^2 - c_s^2\omega_1 - 3v_1^2\omega_1 + \omega_1) \frac{\rho}{2\omega_1}$$

$$C_{D_x^2 v_1}^{(1), \text{CuLBM2}} = (-\omega_2 + \omega_2\omega_1 - 3\omega_2 v_1^2\omega_1 + c_s^2\omega_1 + \omega_2 c_s^2 - \omega_2 c_s^2\omega_1 + 3v_1^2\omega_1 - \omega_1 + 3\omega_2 v_1^2) \frac{\rho}{2\omega_2\omega_1}$$

coefficient $C_{D_x D_y \rho}^{(1)}$ **at** $\frac{\partial^2 \rho}{\partial x_1 \partial x_2}$:

$$C_{D_x D_y \rho}^{(1), \text{SRT}} = 0$$

$$C_{D_x D_y \rho}^{(1), \text{MRT1}} = 0$$

$$C_{D_x D_y \rho}^{(1), \text{MRT2}} = 0$$

$$C_{D_x D_y \rho}^{(1), \text{CLBM1}} = 0$$

$$C_{D_x D_y \rho}^{(1), \text{CLBM2}} = 0$$

$$C_{D_x D_y \rho}^{(1), \text{CuLBM1}} = 0$$

$$C_{D_x D_y \rho}^{(1), \text{CuLBM2}} = (-\omega_2 v_2^2 + \omega_2 + v_2^2 \omega_1 + 3c_s^2 \omega_1 - 3\omega_2 c_s^2 - \omega_1) \frac{v_2}{2\omega_2 \omega_1}$$

coefficient $C_{D_x D_y v_2}^{(1)}$ **at** $\frac{\partial^2 v_2}{\partial x_1 \partial x_2}$:

$$C_{D_x D_y v_2}^{(1), \text{SRT}} = (-2 + \omega) \frac{\rho c_s^2}{2\omega}$$

$$C_{D_x D_y v_2}^{(1), \text{MRT1}} = (-2 + \omega_4) \frac{\rho c_s^2}{2\omega_4}$$

$$C_{D_x D_y v_2}^{(1), \text{MRT2}} = C_{D_x D_y v_2}^{(1), \text{MRT1}}$$

$$C_{D_x D_y v_2}^{(1), \text{CLBM1}} = C_{D_x D_y v_2}^{(1), \text{MRT1}}$$

$$C_{D_x D_y v_2}^{(1), \text{CLBM2}} = C_{D_x D_y v_2}^{(1), \text{MRT1}}$$

$$C_{D_x D_y v_2}^{(1), \text{CuLBM1}} = (-2 + \omega_3) \frac{\rho c_s^2}{2\omega_3}$$

$$C_{D_x D_y v_2}^{(1), \text{CuLBM2}} = (-3\omega_2 v_2^2 + \omega_2 + 3v_2^2 \omega_1 + c_s^2 \omega_1 - 3\omega_2 c_s^2 + \omega_2 c_s^2 \omega_1 - \omega_1) \frac{\rho}{2\omega_2 \omega_1}$$

coefficient $C_{D_y^2 v_1}^{(1)}$ **at** $\frac{\partial^2 v_1}{\partial x_2^2}$:

$$C_{D_y^2 v_1}^{(1), \text{SRT}} = (-2 + \omega) \frac{\rho c_s^2}{2\omega}$$

$$C_{D_y^2 v_1}^{(1), \text{MRT1}} = (-2 + \omega_4) \frac{\rho c_s^2}{2\omega_4}$$

$$C_{D_y^2 v_1}^{(1), \text{MRT2}} = C_{D_y^2 v_1}^{(1), \text{MRT1}}$$

$$C_{D_y^2 v_1}^{(1), \text{CLBM1}} = C_{D_y^2 v_1}^{(1), \text{MRT1}}$$

$$C_{D_y^2 v_1}^{(1), \text{CLBM2}} = C_{D_y^2 v_1}^{(1), \text{MRT1}}$$

$$C_{D_y^2 v_1}^{(1), \text{CuLBM1}} = (-2 + \omega_3) \frac{\rho c_s^2}{2\omega_3}$$

$$C_{D_y^2 v_1}^{(1), \text{CuLBM2}} = (-2 + \omega_1) \frac{\rho c_s^2}{2\omega_1}$$

coefficient $C_{D_x^3 \rho}^{(1)}$ **at** $\frac{\partial^3 \rho}{\partial x_1^3}$:

$$C_{D_x^3 \rho}^{(1), \text{SRT}} = (24v_1^2 c_s^2 \omega^2 - c_s^2 \omega^2 - 36v_1^4 \omega - 36v_1^2 + 7v_1^4 \omega^2 - 12c_s^2 + 12c_s^2 \omega + 144v_1^2 c_s^2 - 144v_1^2 c_s^2 \omega + 36v_1^2 \omega + 12c_s^4 + c_s^4 \omega^2 - 12c_s^4 \omega - 7v_1^2 \omega^2 + 36v_1^4) \frac{1}{12\omega^2}$$

$$C_{D_x^3 \rho}^{(1), \text{MRT1}} = (36\omega_5 v_1^2 - 36v_1^2 + 7\omega_5^2 v_1^4 + \omega_5^2 c_s^4 - 12c_s^2 - 144\omega_5 v_1^2 c_s^2 + 12\omega_5 c_s^2 + 144v_1^2 c_s^2 + 12c_s^4 - \omega_5^2 c_s^2 - 12\omega_5 c_s^4 + 24\omega_5^2 v_1^2 c_s^2 - 36\omega_5 v_1^4 - 7\omega_5^2 v_1^2 + 36v_1^4) \frac{1}{12\omega_5^2}$$

$$C_{D_x^3 \rho}^{(1), \text{MRT2}} = C_{D_x^3 \rho}^{(1), \text{MRT1}}$$

$$C_{D_x^3 \rho}^{(1), \text{CLBM1}} = C_{D_x^3 \rho}^{(1), \text{MRT1}}$$

$$C_{D_x^3 \rho}^{(1), \text{CLBM2}} = C_{D_x^3 \rho}^{(1), \text{MRT1}}$$

$$C_{D_x^3 \rho}^{(1), \text{CuLBM1}} = (-c_s^2 \omega_1^2 + 24v_1^2 c_s^2 \omega_1^2 - 36v_1^2 - 36v_1^4 \omega_1 + 7v_1^4 \omega_1^2 - 12c_s^2 - 144v_1^2 c_s^2 \omega_1 + 12c_s^2 \omega_1 + 144v_1^2 c_s^2 + 12c_s^4 + 36v_1^2 \omega_1 + c_s^4 \omega_1^2 - 12c_s^4 \omega_1 - 7v_1^2 \omega_1^2 + 36v_1^4) \frac{1}{12\omega_1^2}$$

$$C_{D_x^3 \rho}^{(1), \text{CuLBM2}} = (-6c_s^2 \omega_1^2 - 18\omega_2^2 v_1^4 \omega_1 - 72\omega_2^2 v_1^2 c_s^2 \omega_1 + 18\omega_2 v_1^2 \omega_1^2 + 45v_1^2 c_s^2 \omega_1^2 - 9\omega_2^2 v_1^2 - \omega_2^2 c_s^2 \omega_1^2 + 6\omega_2^2 c_s^2 \omega_1 + 9v_1^4 \omega_1^2 + 45\omega_2^2 v_1^2 c_s^2 -$$

$$6\omega_2 c_s^4 \omega_1^2 - 18\omega_2 v_1^2 \omega_1 - 6\omega_2^2 c_s^2 + 7\omega_2^2 v_1^4 \omega_1^2 + 24\omega_2^2 v_1^2 c_s^2 \omega_1^2 + \omega_2^2 c_s^4 \omega_1^2 + 54\omega_2 v_1^2 c_s^2 \omega_1 + 18\omega_2^2 v_1^2 \omega_1 + 6c_s^4 \omega_1^2 - 18\omega_2 v_1^4 \omega_1^2 + 6\omega_2^2 c_s^4 + 18\omega_2 v_1^4 \omega_1 + 9\omega_2^2 v_1^4 - 7\omega_2^2 v_1^2 \omega_1^2 - 72\omega_2 v_1^2 c_s^2 \omega_1^2 - 9v_1^2 \omega_1^2 - 6\omega_2^2 c_s^4 \omega_1 + 6\omega_2 c_s^2 \omega_1^2) \frac{1}{12\omega_2^2 \omega_1^2}$$

coefficient $C_{D_x^3 v_1}^{(1)}$ **at** $\frac{\partial^3 v_1}{\partial x_1^3}$:

$$C_{D_x^3 v_1}^{(1), \text{SRT}} = (-24 + 5c_s^2 \omega^2 + 60v_1^2 + 36c_s^2 - 36c_s^2 \omega - 60v_1^2 \omega + 24\omega - 4\omega^2 + 11v_1^2 \omega^2) \frac{v_1 \rho}{6\omega^2}$$

$$C_{D_x^3 v_1}^{(1), \text{MRT1}} = (-24 - 60\omega_5 v_1^2 + 24\omega_5 + 60v_1^2 - 4\omega_5^2 + 36c_s^2 - 36\omega_5 c_s^2 + 5\omega_5^2 c_s^2 + 11\omega_5^2 v_1^2) \frac{v_1 \rho}{6\omega_5^2}$$

$$C_{D_x^3 v_1}^{(1), \text{MRT2}} = C_{D_x^3 v_1}^{(1), \text{MRT1}}$$

$$C_{D_x^3 v_1}^{(1), \text{CLBM1}} = C_{D_x^3 v_1}^{(1), \text{MRT1}}$$

$$C_{D_x^3 v_1}^{(1), \text{CLBM2}} = C_{D_x^3 v_1}^{(1), \text{MRT1}}$$

$$C_{D_x^3 v_1}^{(1), \text{CuLBM1}} = (-24 + 5c_s^2 \omega_1^2 + 60v_1^2 + 36c_s^2 - 36c_s^2 \omega_1 - 60v_1^2 \omega_1 - 4\omega_1^2 + 24\omega_1 + 11v_1^2 \omega_1^2) \frac{v_1 \rho}{6\omega_1^2}$$

$$C_{D_x^3 v_1}^{(1), \text{CuLBM2}} = (27c_s^2 \omega_1^2 + 24\omega_2 \omega_1^2 - 60\omega_2 v_1^2 \omega_1^2 + 33\omega_2^2 v_1^2 - 15\omega_2^2 + 10\omega_2^2 c_s^2 \omega_1^2 - 36\omega_2^2 c_s^2 \omega_1 - 18\omega_2 \omega_1 + 54\omega_2 v_1^2 \omega_1 + 27\omega_2^2 c_s^2 + 24\omega_2^2 \omega_1 + 18\omega_2 c_s^2 \omega_1 - 60\omega_2^2 v_1^2 \omega_1 - 15\omega_1^2 + 22\omega_2^2 v_1^2 \omega_1^2 + 33v_1^2 \omega_1^2 - 8\omega_2^2 \omega_1^2 - 36\omega_2 c_s^2 \omega_1^2) \frac{v_1 \rho}{12\omega_2^2 \omega_1^2}$$

coefficient $C_{D_x^2 D_y \rho}^{(1)}$ **at** $\frac{\partial^3 \rho}{\partial x_1^2 \partial x_2}$:

$$C_{D_x^2 D_y \rho}^{(1), \text{SRT}} = 0$$

$$C_{D_x^2 D_y \rho}^{(1), \text{MRT1}} = (v_1^2 \omega_7 \omega_4 - \omega_7 \omega_4 + \omega_5 \omega_7 \omega_4 - \omega_5 v_1^2 \omega_7 \omega_4 + \omega_5 v_1^2 \omega_7 - \omega_5 \omega_7 + \omega_5^2 - 3\omega_5 c_s^2 \omega_4 - \omega_5^2 \omega_4 + \omega_5^2 v_1^2 \omega_4 - 3\omega_5^2 c_s^2 + 3\omega_5^2 c_s^2 \omega_4 + 3\omega_5 c_s^2 \omega_7 + \omega_5 \omega_4 - \omega_5 v_1^2 \omega_4 - \omega_5^2 v_1^2 + 3c_s^2 \omega_7 \omega_4 - 3\omega_5 c_s^2 \omega_7 \omega_4) \frac{v_1 v_2}{\omega_5^2 \omega_7 \omega_4}$$

$$C_{D_x^2 D_y \rho}^{(1), \text{MRT2}} = C_{D_x^2 D_y \rho}^{(1), \text{MRT1}}$$

$$C_{D_x^2 D_y \rho}^{(1), \text{CLBM1}} = 0$$

$$C_{D_x^2 D_y \rho}^{(1), \text{CLBM2}} = 0$$

$$C_{D_x^2 D_y \rho}^{(1), \text{CuLBM1}} = 0$$

$$C_{D_x^2 D_y \rho}^{(1), \text{CuLBM2}} = (6c_s^2 \omega_1^2 + \omega_2 \omega_1^2 + \omega_2^2 v_1^2 + v_2^2 \omega_1^2 + 3\omega_2^2 c_s^2 \omega_1 + 2\omega_2 \omega_1 - 2\omega_2 v_1^2 \omega_1 + \omega_2^2 v_2^2 \omega_1 - \omega_2^2 \omega_1 - 6\omega_2 c_s^2 \omega_1 - 2\omega_1^2 - \omega_2 v_2^2 \omega_1^2 + v_1^2 \omega_1^2 - 3\omega_2 c_s^2 \omega_1^2 - \omega_2^2 v_2^2) \frac{3v_1 v_2}{4\omega_2^2 \omega_1^2}$$

coefficient $C_{D_x^2 D_y v_1}^{(1)}$ **at** $\frac{\partial^3 v_1}{\partial x_1^2 \partial x_2}$:

$$C_{D_x^2 D_y v_1}^{(1), \text{SRT}} = 0$$

$$C_{D_x^2 D_y v_1}^{(1), \text{MRT1}} = (3v_1^2 \omega_7 \omega_4 - \omega_7 \omega_4 + \omega_5 \omega_7 \omega_4 - 3\omega_5 v_1^2 \omega_7 \omega_4 + 3\omega_5 v_1^2 \omega_7 - \omega_5 \omega_7 + \omega_5^2 - \omega_5 c_s^2 \omega_4 - \omega_5^2 \omega_4 + 3\omega_5^2 v_1^2 \omega_4 - \omega_5^2 c_s^2 + \omega_5^2 c_s^2 \omega_4 + \omega_5 c_s^2 \omega_7 + \omega_5 \omega_4 - 3\omega_5 v_1^2 \omega_4 - 3\omega_5^2 v_1^2 + c_s^2 \omega_7 \omega_4 - \omega_5 c_s^2 \omega_7 \omega_4) \frac{\rho v_2}{\omega_5^2 \omega_7 \omega_4}$$

$$C_{D_x^2 D_y v_1}^{(1), \text{MRT2}} = C_{D_x^2 D_y v_1}^{(1), \text{MRT1}}$$

$$C_{D_x^2 D_y v_1}^{(1), \text{CLBM1}} = 0$$

$$C_{D_x^2 D_y v_1}^{(1), \text{CLBM2}} = 0$$

$$C_{D_x^2 D_y v_1}^{(1), \text{CuLBM1}} = 0$$

$$C_{D_x^2 D_y v_1}^{(1), \text{CuLBM2}} = (-\omega_2^2 \omega_3 \omega_1 + 2\omega_2 \omega_1^2 - \omega_2 v_2^2 \omega_3 \omega_1^2 - 2\omega_2^2 v_2^2 \omega_3 + 6\omega_2^2 c_s^2 \omega_1 + 9v_1^2 \omega_3 \omega_1^2 - 5\omega_3 \omega_1^2 + 3\omega_2^2 \omega_3 c_s^2 \omega_1 + 2\omega_2^2 v_2^2 \omega_1 + 9\omega_3 c_s^2 \omega_1^2 - 2\omega_2^2 \omega_1 - 3\omega_2 \omega_3 c_s^2 \omega_1^2 + \omega_2 \omega_3 \omega_1^2 - 18\omega_2 v_1^2 \omega_3 \omega_1 - \omega_2^2 \omega_3 + \omega_2^2 v_2^2 \omega_3 \omega_1 - 2\omega_2 v_2^2 \omega_1^2 + 2v_2^2 \omega_3 \omega_1^2 + 6\omega_2 \omega_3 \omega_1 + 9\omega_2^2 v_1^2 \omega_3 - 6\omega_2 c_s^2 \omega_1^2 - 6\omega_2 \omega_3 c_s^2 \omega_1 - 3\omega_2^2 \omega_3 c_s^2) \frac{\rho v_2}{4\omega_2^2 \omega_3 \omega_1^2}$$

coefficient $C_{D_x^2 D_y v_2}^{(1)}$ **at** $\frac{\partial^3 v_2}{\partial x_1^2 \partial x_2}$:

$$C_{D_x^2 D_y v_2}^{(1), \text{SRT}} = (12 - 11c_s^2 \omega^2 - 12v_1^2 - 36c_s^2 + 36c_s^2 \omega + 12v_1^2 \omega - 12\omega + 3\omega^2 - 3v_1^2 \omega^2) \frac{v_1 \rho}{12\omega^2}$$

$$C_{D_x^2 D_y v_2}^{(1), \text{MRT1}} = (12\omega_5^2 v_1^2 \omega_4^2 - 11\omega_5^2 c_s^2 \omega_7 \omega_4^2 - 12\omega_5^2 c_s^2 \omega_4^2 - 12\omega_5^2 \omega_4^2 - 12\omega_7 \omega_4^2 + 6\omega_5 \omega_7 \omega_4^2 - 6\omega_5 v_1^2 \omega_7 \omega_4^2 + 12v_1^2 \omega_7 \omega_4^2 + 12\omega_5^2 \omega_4 + 42\omega_5^2 c_s^2 \omega_7 \omega_4 - 24\omega_5^2 c_s^2 \omega_7 - 12\omega_5^2 v_1^2 \omega_4 + 3\omega_5^2 \omega_7 \omega_4^2 - 18\omega_5 c_s^2 \omega_7 \omega_4^2 - 12\omega_5^2 c_s^2 \omega_4 + 36c_s^2 \omega_7 \omega_4^2 + 6\omega_5^2 v_1^2 \omega_7 \omega_4 - 12\omega_5 v_1^2 \omega_4^2 + 12\omega_5 \omega_4^2 - 3\omega_5^2 v_1^2 \omega_7 \omega_4^2 + 12\omega_5^2 c_s^2 \omega_4^2 - 6\omega_5^2 \omega_7 \omega_4 - 24\omega_5 c_s^2 \omega_7 \omega_4) \frac{v_1 \rho}{12\omega_5^2 \omega_7 \omega_4^2}$$

$$C_{D_x^2 D_y v_2}^{(1), \text{MRT2}} = C_{D_x^2 D_y v_2}^{(1), \text{MRT1}}$$

$$C_{D_x^2 D_y v_2}^{(1), \text{CLBM1}} = (12v_1^2 \omega_7 \omega_4 - 12\omega_7 \omega_4 + 6\omega_5 \omega_7 \omega_4 - 6\omega_5 v_1^2 \omega_7 \omega_4 + 12\omega_5^2 - 36\omega_5 c_s^2 \omega_4 - 12\omega_5^2 \omega_4 - 11\omega_5^2 c_s^2 \omega_7 \omega_4 + 18\omega_5^2 c_s^2 \omega_7 + 12\omega_5^2 v_1^2 \omega_4 + 6\omega_5^2 v_1^2 \omega_7 - 36\omega_5^2 c_s^2 + 36\omega_5^2 c_s^2 \omega_4 - 6\omega_5^2 \omega_7 - 3\omega_5^2 v_1^2 \omega_7 \omega_4 + 12\omega_5 \omega_4 - 12\omega_5 v_1^2 \omega_4 - 12\omega_5^2 v_1^2 + 36c_s^2 \omega_7 \omega_4 + 3\omega_5^2 \omega_7 \omega_4 - 18\omega_5 c_s^2 \omega_7 \omega_4) \frac{v_1 \rho}{12\omega_5^2 \omega_7 \omega_4}$$

$$C_{D_x^2 D_y v_2}^{(1), \text{CLBM2}} = C_{D_x^2 D_y v_2}^{(1), \text{CLBM1}}$$

$$C_{D_x^2 D_y v_2}^{(1), \text{CuLBM1}} = (-36\omega_3 c_s^2 \omega_1 - 36c_s^2 \omega_1^2 - 12\omega_3 \omega_4 + 12v_1^2 \omega_3 \omega_4 + 12\omega_3 \omega_1 + 36\omega_3 c_s^2 \omega_4 - 12v_1^2 \omega_3 \omega_1 + 18c_s^2 \omega_4 \omega_1^2 + 12v_1^2 \omega_3 \omega_1^2 - 12\omega_3 \omega_1^2 + 36\omega_3 c_s^2 \omega_1^2 + 6v_1^2 \omega_4 \omega_1^2 - 6\omega_4 \omega_1^2 + 6\omega_3 \omega_4 \omega_1 - 11\omega_3 c_s^2 \omega_4 \omega_1^2 - 6v_1^2 \omega_3 \omega_4 \omega_1 + 12\omega_1^2 - 18\omega_3 c_s^2 \omega_4 \omega_1 - 3v_1^2 \omega_3 \omega_4 \omega_1^2 + 3\omega_3 \omega_4 \omega_1^2 - 12v_1^2 \omega_1^2) \frac{v_1 \rho}{12\omega_3 \omega_4 \omega_1^2}$$

$$C_{D_x^2 D_y v_2}^{(1), \text{CuLBM2}} = (-12\omega_2^2 \omega_3 \omega_1 - 11\omega_2^2 \omega_3 c_s^2 \omega_1^2 + 6\omega_2 \omega_1^2 - 6\omega_2 v_1^2 \omega_1^2 - 27\omega_2 v_2^2 \omega_3 \omega_1^2 - 3\omega_2^2 v_1^2 \omega_3 \omega_1^2 - 27\omega_2^2 v_2^2 \omega_3 + 36\omega_2^2 c_s^2 \omega_1^2 - 54\omega_2^2 c_s^2 \omega_1 + 6v_2^2 \omega_3 \omega_1^2 - 15\omega_3 \omega_1^2 + 3\omega_2^2 v_1^2 \omega_3 \omega_1 + 3\omega_2^2 \omega_3 \omega_1^2 + 18\omega_2^2 \omega_3 c_s^2 \omega_1 + 27\omega_3 c_s^2 \omega_1^2 + 18\omega_2^2 \omega_1 - 18\omega_2 \omega_3 c_s^2 \omega_1^2 - 18\omega_2^2 v_1^2 \omega_1 + 12\omega_2 \omega_3 \omega_1^2 - 3\omega_2 v_1^2 \omega_3 \omega_1^2 + 3\omega_2^2 \omega_3 + 27\omega_2^2 v_2^2 \omega_3 \omega_1 + 12\omega_2^2 v_1^2 \omega_1^2 + 27v_2^2 \omega_3 \omega_1^2 + 6\omega_2^2 v_1^2 \omega_3 - 12\omega_2^2 \omega_1^2 - 18\omega_2 c_s^2 \omega_1^2 + 9\omega_2^2 \omega_3 c_s^2) \frac{v_1 \rho}{12\omega_2^2 \omega_3 \omega_1^2}$$

coefficient $C_{D_x D_y^2 \rho}^{(1)}$ **at** $\frac{\partial^3 \rho}{\partial x_1 \partial x_2^2}$:

$$C_{D_x D_y^2 \rho}^{(1), \text{SRT}} = (-12 + 12\omega - \omega^2) \frac{c_s^4}{6\omega^2}$$

$$C_{D_x D_y^2 \rho}^{(1), \text{MRT1}} = (-12 + 12\omega_4 - \omega_4^2) \frac{c_s^4}{6\omega_4^2}$$

$$C_{D_x D_y^2 \rho}^{(1), \text{MRT2}} = C_{D_x D_y^2 \rho}^{(1), \text{MRT1}}$$

$$C_{D_x D_y^2 \rho}^{(1), \text{CLBM1}} = C_{D_x D_y^2 \rho}^{(1), \text{MRT1}}$$

$$C_{D_x D_y^2 \rho}^{(1), \text{CLBM2}} = C_{D_x D_y^2 \rho}^{(1), \text{MRT1}}$$

$$C_{D_x D_y^2 \rho}^{(1), \text{CuLBM1}} = (-12 + 12\omega_3 - \omega_3^2) \frac{c_s^4}{6\omega_3^2}$$

$$C_{D_x D_y^2 \rho}^{(1), \text{CuLBM2}} = (-6c_s^2 \omega_1^2 - 9\omega_2^2 v_2^4 - 9v_2^2 \omega_1^2 - 6\omega_2^2 c_s^2 \omega_1 - 6\omega_2 c_s^4 \omega_1^2 + 6\omega_2^2 c_s^2 - 9\omega_2^2 v_2^2 \omega_1 - 45\omega_2 v_2^2 c_s^2 \omega_1^2 - 9\omega_2 v_2^4 \omega_1^2 - 45\omega_2^2 v_2^2 c_s^2 + 45v_2^2 c_s^2 \omega_1^2 + 9v_2^4 \omega_1^2 - 2\omega_2^2 c_s^4 \omega_1^2 + 45\omega_2^2 v_2^2 c_s^2 \omega_1 + 6c_s^4 \omega_1^2 - 30\omega_2^2 c_s^4 + 9\omega_2^2 v_2^4 \omega_1 + 9\omega_2 v_2^2 \omega_1^2 + 30\omega_2^2 c_s^4 \omega_1 + 6\omega_2 c_s^2 \omega_1^2 + 9\omega_2^2 v_2^2) \frac{1}{12\omega_2^2 \omega_1^2}$$

coefficient $C_{D_x D_y^2 v_1}^{(1)}$ **at** $\frac{\partial^3 v_1}{\partial x_1 \partial x_2^2}$:

$$C_{D_x D_y^2 v_1}^{(1), \text{SRT}} = \frac{-v_1 \rho c_s^2}{6}$$

$$C_{D_x D_y^2 v_1}^{(1), \text{MRT1}} = (-12\omega_7 \omega_4 + 12\omega_5 \omega_7 \omega_4 - 12\omega_5 \omega_7 - \omega_5 \omega_7 \omega_4^2 + 12\omega_5 \omega_4 - 12\omega_5 \omega_4^2 + 12\omega_4^2) \frac{v_1 \rho c_s^2}{6\omega_5 \omega_7 \omega_4^2}$$

$$C_{D_x D_y^2 v_1}^{(1), \text{MRT2}} = C_{D_x D_y^2 v_1}^{(1), \text{MRT1}}$$

$$C_{D_x D_y^2 v_1}^{(1), \text{CLBM1}} = C_{D_x D_y^2 v_1}^{(1), \text{SRT}}$$

$$C_{D_x D_y^2 v_1}^{(1), \text{CLBM2}} = C_{D_x D_y^2 v_1}^{(1), \text{SRT}}$$

$$C_{D_x D_y^2 v_1}^{(1), \text{CuLBM1}} = C_{D_x D_y^2 v_1}^{(1), \text{SRT}}$$

$$C_{D_x D_y^2 v_1}^{(1), \text{CuLBM2}} = C_{D_x D_y^2 v_1}^{(1), \text{SRT}}$$

coefficient $C_{D_x D_y^2 v_2}^{(1)}$ **at** $\frac{\partial^3 v_2}{\partial x_1 \partial x_2^2}$:

$$C_{D_x D_y^2 v_2}^{(1), \text{SRT}} = 0$$

$$C_{D_x D_y^2 v_2}^{(1), \text{MRT1}} = (-2\omega_8 + 2\omega_4 - \omega_4^2 + \omega_8 \omega_4) \frac{\rho v_2 c_s^2}{\omega_8 \omega_4^2}$$

$$C_{D_x D_y^2 v_2}^{(1), \text{MRT2}} = C_{D_x D_y^2 v_2}^{(1), \text{MRT1}}$$

$$C_{D_x D_y^2 v_2}^{(1), \text{CLBM1}} = 0$$

$$C_{D_x D_y^2 v_2}^{(1), \text{CLBM2}} = 0$$

$$C_{D_x D_y^2 v_2}^{(1), \text{CuLBM1}} = 0$$

$$C_{D_x D_y^2 v_2}^{(1), \text{CuLBM2}} = (9c_s^2 \omega_1^2 + 5\omega_2 \omega_1^2 + 5\omega_2^2 + 11v_2^2 \omega_1^2 + 9\omega_2^2 c_s^2 \omega_1 - 9\omega_2^2 c_s^2 + 11\omega_2^2 v_2^2 \omega_1 - 5\omega_2^2 \omega_1 - 5\omega_1^2 - 11\omega_2 v_2^2 \omega_1^2 - 9\omega_2 c_s^2 \omega_1^2 - 11\omega_2^2 v_2^2) \frac{\rho v_2}{4\omega_2^2 \omega_1^2}$$

coefficient $C_{D_y^3 \rho}^{(1)}$ **at** $\frac{\partial^3 \rho}{\partial x_2^3}$:

$$C_{D_y^3 \rho}^{(1), \text{SRT}} = (-1 + 3c_s^2 + v_2^2) \frac{v_1 v_2}{12}$$

$$C_{D_y^3 \rho}^{(1), \text{MRT1}} = (-12\omega_8 - 12v_2^2 \omega_6 + 36\omega_8 c_s^2 + 12\omega_6 - 6\omega_6 \omega_4 + 12\omega_8 v_2^2 - 36\omega_6 c_s^2 - 18\omega_8 c_s^2 \omega_4 + 18\omega_6 c_s^2 \omega_4 + \omega_8 v_2^2 \omega_6 \omega_4 - \omega_8 \omega_6 \omega_4 - 6\omega_8 v_2^2 \omega_4 + 6v_2^2 \omega_6 \omega_4 + 6\omega_8 \omega_4 + 3\omega_8 \omega_6 c_s^2 \omega_4) \frac{v_1 v_2}{12\omega_8 \omega_6 \omega_4}$$

$$C_{D_y^3 \rho}^{(1), \text{MRT2}} = C_{D_y^3 \rho}^{(1), \text{MRT1}}$$

$$C_{D_y^3 \rho}^{(1), \text{CLBM1}} = C_{D_y^3 \rho}^{(1), \text{SRT}}$$

$$C_{D_y^3 \rho}^{(1), \text{CLBM2}} = C_{D_y^3 \rho}^{(1), \text{SRT}}$$

$$C_{D_y^3 \rho}^{(1), \text{CuLBM1}} = C_{D_y^3 \rho}^{(1), \text{SRT}}$$

$$C_{D_y^3 \rho}^{(1), \text{CuLBM2}} = C_{D_y^3 \rho}^{(1), \text{SRT}}$$

coefficient $C_{D_y^3 v_1}^{(1)}$ **at** $\frac{\partial^3 v_1}{\partial x_2^3}$:

$$C_{D_y^3 v_1}^{(1), \text{SRT}} = (6 - 3c_s^2 \omega^2 - v_2^2 \omega^2 + 6v_2^2 \omega - 18c_s^2 + 18c_s^2 \omega - 6v_2^2 - 6\omega + \omega^2) \frac{\rho v_2}{6\omega^2}$$

$$C_{D_y^3 v_1}^{(1), \text{MRT1}} = (3v_2^2 \omega_4^2 + 3c_s^2 \omega_4^2 - 12\omega_8 c_s^2 - 6c_s^2 \omega_4 - 6v_2^2 \omega_4 + \omega_8 \omega_4^2 - \omega_8 v_2^2 \omega_4^2 - 3\omega_8 c_s^2 \omega_4^2 + 15\omega_8 c_s^2 \omega_4 + 6\omega_4 + 3\omega_8 v_2^2 \omega_4 - 3\omega_4^2 - 3\omega_8 \omega_4) \frac{\rho v_2}{6\omega_8 \omega_4^2}$$

$$C_{D_y^3 v_1}^{(1), \text{MRT2}} = C_{D_y^3 v_1}^{(1), \text{MRT1}}$$

$$C_{D_y^3 v_1}^{(1), \text{CLBM1}} = (6 - 3\omega_8 + 9\omega_8 c_s^2 + 9c_s^2 \omega_4 - 18c_s^2 + 3v_2^2 \omega_4 + 3\omega_8 v_2^2 - 6v_2^2 - 3\omega_8 c_s^2 \omega_4 - 3\omega_4 - \omega_8 v_2^2 \omega_4 + \omega_8 \omega_4) \frac{\rho v_2}{6\omega_8 \omega_4}$$

$$C_{D_y^3 v_1}^{(1), \text{CLBM2}} = C_{D_y^3 v_1}^{(1), \text{CLBM1}}$$

$$C_{D_y^3 v_1}^{(1), \text{CuLBM1}} = (6 + 3v_2^2\omega_3 + \omega_6\omega_3 + 3v_2^2\omega_6 - 3\omega_6\omega_3c_s^2 - 18c_s^2 - 3\omega_6 - 3\omega_3 - v_2^2\omega_6\omega_3 + 9\omega_6c_s^2 - 6v_2^2 + 9\omega_3c_s^2) \frac{\rho v_2}{6\omega_6\omega_3}$$

$$C_{D_y^3 v_1}^{(1), \text{CuLBM2}} = (6 - 3\omega_3c_s^2\omega_1 + 3v_2^2\omega_3 + \omega_3\omega_1 - 18c_s^2 + 3v_2^2\omega_1 + 9c_s^2\omega_1 - 3\omega_3 - v_2^2\omega_3\omega_1 - 6v_2^2 + 9\omega_3c_s^2 - 3\omega_1) \frac{\rho v_2}{6\omega_3\omega_1}$$

$$\text{coefficient } C_{D_y^3 v_2}^{(1)} \text{ at } \frac{\partial^3 v_2}{\partial x_2^3} :$$

$$C_{D_y^3 v_2}^{(1), \text{SRT}} = (-1 + c_s^2 + 3v_2^2) \frac{v_1 \rho}{12}$$

$$C_{D_y^3 v_2}^{(1), \text{MRT1}} = (-12\omega_8 - 36v_2^2\omega_6 + 12\omega_8c_s^2 + 12\omega_6 - 6\omega_6\omega_4 + 36\omega_8v_2^2 - 12\omega_6c_s^2 - 6\omega_8c_s^2\omega_4 + 6\omega_6c_s^2\omega_4 + 3\omega_8v_2^2\omega_6\omega_4 - \omega_8\omega_6\omega_4 - 18\omega_8v_2^2\omega_4 + 18v_2^2\omega_6\omega_4 + 6\omega_8\omega_4 + \omega_8\omega_6c_s^2\omega_4) \frac{v_1 \rho}{12\omega_8\omega_6\omega_4}$$

$$C_{D_y^3 v_2}^{(1), \text{MRT2}} = C_{D_y^3 v_2}^{(1), \text{MRT1}}$$

$$C_{D_y^3 v_2}^{(1), \text{CLBM1}} = C_{D_y^3 v_2}^{(1), \text{SRT}}$$

$$C_{D_y^3 v_2}^{(1), \text{CLBM2}} = C_{D_y^3 v_2}^{(1), \text{SRT}}$$

$$C_{D_y^3 v_2}^{(1), \text{CuLBM1}} = C_{D_y^3 v_2}^{(1), \text{SRT}}$$

$$C_{D_y^3 v_2}^{(1), \text{CuLBM2}} = C_{D_y^3 v_2}^{(1), \text{SRT}}$$

$$\text{coefficient } C_{D_x^4 \rho}^{(1)} \text{ at } \frac{\partial^4 \rho}{\partial x_1^4} :$$

$$C_{D_x^4 \rho}^{(1), \text{SRT}} = (12 + 404v_1^2c_s^2\omega^2 - 78c_s^2\omega^2 - 216v_1^4\omega - 34v_1^2c_s^2\omega^3 - 156v_1^2 + 6c_s^2\omega^3 + 90v_1^4\omega^2 - 132c_s^2 + 198c_s^2\omega + 672v_1^2c_s^2 - 1008v_1^2c_s^2\omega - 9v_1^4\omega^3 + 234v_1^2\omega + 144c_s^4 - 5c_s^4\omega^3 + 82c_s^4\omega^2 - 216c_s^4\omega - 18\omega + 10v_1^2\omega^3 + 8\omega^2 - 98v_1^2\omega^2 - \omega^3 + 144v_1^4) \frac{v_1}{12\omega^3}$$

$$C_{D_x^4 \rho}^{(1), \text{MRT1}} = (12 + 234\omega_5v_1^2 - 34\omega_5^3v_1^2c_s^2 - 18\omega_5 - 5\omega_5^3c_s^4 - 156v_1^2 + 90\omega_5^2v_1^4 + 82\omega_5^2c_s^4 + 8\omega_5^2 - 132c_s^2 - 9\omega_5^3v_1^4 - 1008\omega_5v_1^2c_s^2 + 198\omega_5c_s^2 + 672v_1^2c_s^2 - \omega_5^3 + 144c_s^4 - 78\omega_5^2c_s^2 - 216\omega_5c_s^4 + 404\omega_5^2v_1^2c_s^2 + 10\omega_5^3v_1^2 + 6\omega_5^3c_s^2 - 216\omega_5v_1^4 - 98\omega_5^2v_1^2 + 144v_1^4) \frac{v_1}{12\omega_5^3}$$

$$C_{D_x^4 \rho}^{(1), \text{MRT2}} = C_{D_x^4 \rho}^{(1), \text{MRT1}}$$

$$C_{D_x^4 \rho}^{(1), \text{CLBM1}} = C_{D_x^4 \rho}^{(1), \text{MRT1}}$$

$$C_{D_x^4 \rho}^{(1), \text{CLBM2}} = C_{D_x^4 \rho}^{(1), \text{MRT1}}$$

$$C_{D_x^4 \rho}^{(1), \text{CuLBM1}} = (12 - 78c_s^2\omega_1^2 + 404v_1^2c_s^2\omega_1^2 - 156v_1^2 + 6c_s^2\omega_1^3 - 216v_1^4\omega_1 - 34v_1^2c_s^2\omega_1^3 + 90v_1^4\omega_1^2 - 132c_s^2 - 1008v_1^2c_s^2\omega_1 - 9v_1^4\omega_1^3 + 198c_s^2\omega_1 + 672v_1^2c_s^2 + 144c_s^4 - 5c_s^4\omega_1^3 + 234v_1^2\omega_1 + 82c_s^4\omega_1^2 + 10v_1^2\omega_1^3 + 8\omega_1^2 - 216c_s^4\omega_1 - 18\omega_1 - 98v_1^2\omega_1^2 - \omega_1^3 + 144v_1^4) \frac{v_1}{12\omega_1^3}$$

$$C_{D_x^4 \rho}^{(1), \text{CuLBM2}} = (90\omega_2^3v_1^4\omega_3\omega_1^2 - 171\omega_2\omega_3c_s^4\omega_1^3 - 98\omega_2^2v_1^2\omega_3\omega_1^3 + 141\omega_2^3\omega_3c_s^2\omega_1 + 6\omega_2^2\omega_3\omega_1 + 114\omega_2^2\omega_3c_s^2\omega_1^2 + 99\omega_2^2v_1^4\omega_3\omega_1 + 45v_1^4\omega_3\omega_1^3 + 210\omega_2^2v_1^2\omega_3\omega_1^2 - 600\omega_2^3v_1^2\omega_3c_s^2\omega_1 - 18\omega_2^3v_1^4\omega_3\omega_1^3 + 54\omega_2\omega_3c_s^4\omega_1^2 + 12\omega_2^2c_s^2\omega_1^2 + 129\omega_2^3v_1^2\omega_3\omega_1 - 78\omega_2^2\omega_3c_s^2\omega_1^3 + 90\omega_2^2v_1^4\omega_3\omega_1^3 - 98\omega_2^3v_1^2\omega_3\omega_1^2 - 72\omega_2^3\omega_3c_s^2 + 8\omega_2^2\omega_3\omega_1^3 + 18\omega_2^3c_s^4\omega_1 + 12\omega_2^3\omega_3c_s^2\omega_1^3 - 51\omega_2^3v_1^2\omega_3 + 404\omega_2^3v_1^2\omega_3c_s^2\omega_1^2 - 105\omega_2^2v_1^2\omega_3\omega_1 - 72\omega_3c_s^2\omega_1^3 + 261v_1^2\omega_3c_s^2\omega_1^3 - 12\omega_2^2\omega_3\omega_1^2 - 51v_1^2\omega_3\omega_1^3 + 6\omega_2^3v_1^2c_s^2\omega_1 - 60\omega_2^2\omega_3c_s^2\omega_1 + 20\omega_2^3v_1^2\omega_3\omega_1^3 - 198\omega_2^2v_1^4\omega_3\omega_1^2 - 117\omega_2^3v_1^4\omega_3\omega_1 - 68\omega_2^3v_1^2\omega_3c_s^2\omega_1^3 + 18\omega_2c_s^4\omega_1^3 - 78\omega_2^3\omega_3c_s^2\omega_1^2 + 6\omega_3\omega_1^3 - 12\omega_2^2v_1^2c_s^2\omega_1^2 - 816\omega_2^2v_1^2\omega_3c_s^2\omega_1^2 - 12\omega_2\omega_3\omega_1^3 - 60\omega_2\omega_3c_s^2\omega_1^2 - 36\omega_2^2c_s^4\omega_1^2 + 90\omega_2^3\omega_3c_s^4 + 82\omega_2^2\omega_3c_s^4\omega_1^3 - 600\omega_2v_1^2\omega_3c_s^2\omega_1^3 + 129\omega_2v_1^2\omega_3\omega_1^3 + 6\omega_2^3\omega_3 + 141\omega_2\omega_3c_s^2\omega_1^3 - 12\omega_2^3\omega_3\omega_1 - 171\omega_2^3\omega_3c_s^4\omega_1 + 6\omega_2\omega_3\omega_1^2 + 404\omega_2^2v_1^2\omega_3c_s^2\omega_1^3 - 105\omega_2v_1^2\omega_3\omega_1^2 + 411\omega_2v_1^2\omega_3c_s^2\omega_1^2 - 90\omega_2^2\omega_3c_s^4\omega_1^2 + 54\omega_2^2\omega_3c_s^4\omega_1 + 6\omega_2v_1^2c_s^2\omega_1^3 + 8\omega_2^3\omega_3\omega_1^2 - 6\omega_2c_s^2\omega_1^3 - 117\omega_2v_1^4\omega_3\omega_1^3 + 261\omega_2^3v_1^2\omega_3c_s^2 + 82\omega_2^3\omega_3c_s^4\omega_1^2 + 411\omega_2^2v_1^2\omega_3c_s^2\omega_1 - 6\omega_2^2c_s^2\omega_1 + 99\omega_2v_1^4\omega_3\omega_1^2 + 45\omega_2^3v_1^4\omega_3 - 10\omega_2^3\omega_3c_s^4\omega_1^3 - 2\omega_2^3\omega_3\omega_1^3 + 90\omega_3c_s^4\omega_1^3) \frac{v_1}{24\omega_2^3\omega_3\omega_1^3}$$

$$\text{coefficient } C_{D_x^4 v_1}^{(1)} \text{ at } \frac{\partial^4 v_1}{\partial x_1^4} :$$

$$C_{D_x^4 v_1}^{(1), \text{SRT}} = (12 + 252v_1^2c_s^2\omega^2 - 22c_s^2\omega^2 - 756v_1^4\omega - 18v_1^2c_s^2\omega^3 - 252v_1^2 + 2c_s^2\omega^3 + 310v_1^4\omega^2 - 36c_s^2 + 54c_s^2\omega + 432v_1^2c_s^2 - 648v_1^2c_s^2\omega - 29v_1^4\omega^3 +$$

$$378v_1^2\omega + 24c_s^4 - c_s^4\omega^3 + 14c_s^4\omega^2 - 36c_s^4\omega - 18\omega + 14v_1^2\omega^3 + 8\omega^2 - 154v_1^2\omega^2 - \omega^3 + 504v_1^4)\frac{\rho}{12\omega^3}$$

$$C_{D_x^4 v_1}^{(1), \text{MRT1}} = (12 + 378\omega_5 v_1^2 - 18\omega_5^3 v_1^2 c_s^2 - 18\omega_5 - \omega_5^3 c_s^4 - 252v_1^2 + 310\omega_5^2 v_1^4 + 14\omega_5^2 c_s^4 + 8\omega_5^2 - 36c_s^2 - 29\omega_5^3 v_1^4 - 648\omega_5 v_1^2 c_s^2 + 54\omega_5 c_s^2 + 432v_1^2 c_s^2 - \omega_5^3 + 24c_s^4 - 22\omega_5^2 c_s^2 - 36\omega_5 c_s^4 + 252\omega_5^2 v_1^2 c_s^2 + 14\omega_5^3 v_1^2 + 2\omega_5^3 c_s^2 - 756\omega_5 v_1^4 - 154\omega_5^2 v_1^2 + 504v_1^4)\frac{\rho}{12\omega_5^3}$$

$$C_{D_x^4 v_1}^{(1), \text{MRT2}} = C_{D_x^4 v_1}^{(1), \text{MRT1}}$$

$$C_{D_x^4 v_1}^{(1), \text{CLBM1}} = C_{D_x^4 v_1}^{(1), \text{MRT1}}$$

$$C_{D_x^4 v_1}^{(1), \text{CLBM2}} = C_{D_x^4 v_1}^{(1), \text{MRT1}}$$

$$C_{D_x^4 v_1}^{(1), \text{CuLBM1}} = (12 - 22c_s^2\omega_1^2 + 252v_1^2 c_s^2\omega_1^2 - 252v_1^2 + 2c_s^2\omega_1^3 - 756v_1^4\omega_1 - 18v_1^2 c_s^2\omega_1^3 + 310v_1^4\omega_1^2 - 36c_s^2 - 648v_1^2 c_s^2\omega_1 - 29v_1^4\omega_1^3 + 54c_s^2\omega_1 + 432v_1^2 c_s^2 + 24c_s^4 - c_s^4\omega_1^3 + 378v_1^2\omega_1 + 14c_s^4\omega_1^2 + 14v_1^2\omega_1^3 + 8\omega_1^2 - 36c_s^4\omega_1 - 18\omega_1 - 154v_1^2\omega_1^2 - \omega_1^3 + 504v_1^4)\frac{\rho}{12\omega_1^3}$$

$$C_{D_x^4 v_1}^{(1), \text{CuLBM2}} = (310\omega_2^3 v_1^4 \omega_3 \omega_1^2 - 33\omega_2 \omega_3 c_s^4 \omega_1^3 - 154\omega_2^2 v_1^2 \omega_3 \omega_1^3 + 45\omega_2^3 \omega_3 c_s^2 \omega_1 + 6\omega_2^2 \omega_3 \omega_1 + 18\omega_2^2 \omega_3 c_s^2 \omega_1^2 + 333\omega_2^2 v_1^4 \omega_3 \omega_1 + 171v_1^4 \omega_3 \omega_1^3 + 306\omega_2^2 v_1^2 \omega_3 \omega_1^2 - 432\omega_2^2 v_1^2 \omega_3 c_s^2 \omega_1 - 58\omega_2^2 v_1^4 \omega_3 \omega_1^3 + 6\omega_2 \omega_3 c_s^4 \omega_1^2 + 12\omega_2^2 c_s^2 \omega_1^2 + 225\omega_2^3 v_1^2 \omega_3 \omega_1 - 22\omega_2^2 \omega_3 c_s^2 \omega_1^3 + 310\omega_2^2 v_1^4 \omega_3 \omega_1^3 - 154\omega_2^3 v_1^2 \omega_3 \omega_1^2 - 24\omega_2^3 \omega_3 c_s^2 \omega_1^3 + 6\omega_2^3 c_s^4 \omega_1 + 4\omega_2^3 \omega_3 c_s^2 \omega_1^3 - 99\omega_2^3 v_1^2 \omega_3 + 252\omega_2^3 v_1^2 \omega_3 c_s^2 \omega_1^2 - 153\omega_2^2 v_1^2 \omega_3 \omega_1 - 24\omega_3 c_s^2 \omega_1^3 + 207v_1^2 \omega_3 c_s^2 \omega_1^3 - 12\omega_2^2 \omega_3 \omega_1^3 + 18\omega_2^2 v_1^2 c_s^2 \omega_1 - 12\omega_2^2 \omega_3 c_s^2 \omega_1 + 28\omega_2^3 v_1^2 \omega_3 \omega_1^3 - 666\omega_2^2 v_1^4 \omega_3 \omega_1^2 - 423\omega_2^3 v_1^4 \omega_3 \omega_1 - 36\omega_2^3 v_1^2 \omega_3 c_s^2 \omega_1^3 + 6\omega_2 c_s^4 \omega_1^3 - 22\omega_2^3 \omega_3 c_s^2 \omega_1^2 + 6\omega_3 \omega_1^3 - 36\omega_2^2 v_1^2 c_s^2 \omega_1^2 - 432\omega_2^2 v_1^2 \omega_3 c_s^2 \omega_1^3 - 12\omega_2 \omega_3 \omega_1^3 - 12\omega_2 \omega_3 c_s^2 \omega_1^2 - 12\omega_2 c_s^4 \omega_1^2 + 18\omega_2^3 \omega_3 c_s^4 + 14\omega_2^3 \omega_3 c_s^2 \omega_1^3 - 432\omega_2 v_1^2 \omega_3 c_s^2 \omega_1^3 + 225\omega_2 v_1^2 \omega_3 \omega_1^3 + 6\omega_2^3 \omega_3 + 45\omega_2 \omega_3 c_s^2 \omega_1^3 - 12\omega_2^3 \omega_3 \omega_1 - 33\omega_2^3 \omega_3 c_s^4 \omega_1 + 6\omega_2 \omega_3 \omega_1^2 + 252\omega_2^2 v_1^2 \omega_3 c_s^2 \omega_1^3 - 153\omega_2 v_1^2 \omega_3 \omega_1^2 + 225\omega_2 v_1^2 \omega_3 c_s^2 \omega_1^2 - 6\omega_2^2 \omega_3 c_s^4 \omega_1 + 6\omega_2^2 \omega_3 c_s^2 \omega_1^3 + 18\omega_2 v_1^2 c_s^2 \omega_1^3 + 8\omega_2^3 \omega_3 \omega_1^2 - 6\omega_2 c_s^2 \omega_1^3 - 423\omega_2 v_1^4 \omega_3 \omega_1^3 + 207\omega_2^2 v_1^2 \omega_3 c_s^2 + 14\omega_2^3 \omega_3 c_s^4 \omega_1^2 + 225\omega_2^2 v_1^2 \omega_3 c_s^2 \omega_1 - 6\omega_2^2 c_s^2 \omega_1 + 333\omega_2 v_1^4 \omega_3 \omega_1^2 + 171\omega_2^2 v_1^2 \omega_3 - 2\omega_2^3 \omega_3 c_s^4 \omega_1^3 - 2\omega_2^3 \omega_3 \omega_1^3 + 18\omega_3 c_s^4 \omega_1^3)\frac{\rho}{24\omega_2^3 \omega_3 \omega_1^3}$$

$$\text{coefficient } C_{D_x^3 D_y \rho}^{(1)} \text{ at } \frac{\partial^4 \rho}{\partial x_1^4 \partial x_2} :$$

$$C_{D_x^3 D_y \rho}^{(1), \text{SRT}} = 0$$

$$C_{D_x^3 D_y \rho}^{(1), \text{MRT1}} = (4\omega_5^2 v_1^2 \omega_4^2 - 20\omega_5^2 v_1^4 \omega_7 \omega_4 - 20\omega_5 v_1^4 \omega_7 \omega_4^2 - 8\omega_5^3 c_s^2 \omega_7 \omega_4 + 4\omega_5^3 v_1^4 \omega_4^2 - 8\omega_5^2 c_s^2 \omega_7 \omega_4^2 + 72\omega_5 v_1^2 c_s^2 \omega_7 \omega_4 + 84\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4 - 4\omega_5^2 c_s^2 \omega_7^2 \omega_4 - 4\omega_5^2 c_s^2 \omega_7^2 + 4\omega_5^3 c_s^2 \omega_4 - 84\omega_5^2 v_1^2 c_s^2 \omega_7^2 \omega_4 - 12\omega_5 c_s^4 \omega_7^2 \omega_4^2 - 13\omega_5^2 v_1^2 \omega_7^2 \omega_4^2 + 8\omega_5^3 v_1^2 \omega_7 - 4\omega_5^3 c_s^4 \omega_7 \omega_4^2 + 8c_s^4 \omega_7^2 \omega_4^2 + 20\omega_5^2 v_1^2 \omega_7^2 \omega_4 + 96v_1^2 c_s^2 \omega_7^2 \omega_4^2 - 4\omega_5^2 c_s^4 \omega_4^2 + 36\omega_5^2 v_1^2 c_s^2 \omega_7^2 + 20\omega_5 v_1^4 \omega_7 \omega_4^2 + 8\omega_5^3 c_s^4 \omega_7 \omega_4 + 8\omega_5^2 c_s^4 \omega_7 \omega_4^2 - 36\omega_5^3 v_1^2 c_s^2 \omega_7 - 4\omega_5^3 c_s^2 \omega_4^2 + 51\omega_5^2 v_1^2 c_s^2 \omega_7^2 \omega_4^2 + 4\omega_5 c_s^4 \omega_7^2 \omega_4 + 8\omega_5^2 v_1^4 \omega_7^2 - 4\omega_5^3 v_1^4 \omega_4 - 144\omega_5 v_1^2 c_s^2 \omega_7^2 \omega_4^2 + 12\omega_5 c_s^2 \omega_7^2 \omega_4^2 - 51\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^2 + 13\omega_5^2 v_1^4 \omega_7^2 \omega_4^2 + 4\omega_5^3 c_s^2 \omega_7 \omega_4^2 - 4\omega_5^3 c_s^4 \omega_7 - 8c_s^2 \omega_7^2 \omega_4^2 + 8\omega_5^2 c_s^2 \omega_7^2 \omega_4 - 72\omega_5 v_1^2 c_s^2 \omega_7 \omega_4^2 - 4\omega_5^3 c_s^4 \omega_4 + 4\omega_5 c_s^2 \omega_7 \omega_4^2 + 20\omega_5^3 v_1^4 \omega_7 \omega_4 + 32\omega_5^2 v_1^4 \omega_7 \omega_4^2 - 8\omega_5^3 v_1^4 \omega_7 + 20\omega_5 v_1^4 \omega_7^2 \omega_4 - 4\omega_5^2 v_1^4 \omega_4^2 + 16\omega_5^2 v_1^2 \omega_7 \omega_4 - 24\omega_5^3 v_1^2 c_s^2 \omega_4 + 36\omega_5 v_1^2 \omega_7^2 \omega_4^2 + 120\omega_5^2 v_1^2 c_s^2 \omega_7 \omega_4^2 - 4\omega_5^3 v_1^2 \omega_4^2 - 24\omega_5^2 v_1^2 c_s^2 \omega_4^2 + 4\omega_5^2 c_s^4 \omega_7^2 \omega_4^2 + 13\omega_5^3 v_1^2 \omega_7 \omega_4^2 - 24v_1^2 \omega_7^2 \omega_4^2 + 4\omega_5^2 c_s^4 \omega_7^2 + 4\omega_5^3 v_1^2 \omega_4 - 48\omega_5^2 v_1^2 c_s^2 \omega_7 \omega_4 - 8\omega_5^2 c_s^4 \omega_7^2 \omega_4 - 4\omega_5 c_s^4 \omega_7 \omega_4^2 - 20\omega_5^3 v_1^4 \omega_7 \omega_4 - 32\omega_5^2 v_1^4 \omega_7 \omega_4^2 - 20\omega_5 v_1^4 \omega_7^2 \omega_4 + 4\omega_5^3 c_s^2 \omega_7 + 24\omega_5^2 v_1^2 c_s^2 \omega_4^2 - 16\omega_5^2 v_1^4 \omega_7 \omega_4 + 4\omega_5^2 c_s^2 \omega_4^2 - 36\omega_5 v_1^4 \omega_7^2 \omega_4^2 + 4\omega_5^3 c_s^4 \omega_4^2 - 4\omega_5^2 c_s^2 \omega_7^2 \omega_4^2 - 13\omega_5^3 v_1^4 \omega_7 \omega_4^2 - 8\omega_5^2 v_1^2 \omega_7^2 + 24v_1^4 \omega_7^2 \omega_4^2)\frac{v_2}{4\omega_5^3 \omega_7^2 \omega_4^2}$$

$$C_{D_x^3 D_y \rho}^{(1), \text{MRT2}} = C_{D_x^3 D_y \rho}^{(1), \text{MRT1}}$$

$$C_{D_x^3 D_y \rho}^{(1), \text{CLBM1}} = 0$$

$$C_{D_x^3 D_y \rho}^{(1), \text{CLBM2}} = 0$$

$$C_{D_x^3 D_y \rho}^{(1), \text{CuLBM1}} = 0$$

$$C_{D_x^3 D_y \rho}^{(1), \text{CuLBM2}} = (-18\omega_2 \omega_3 c_s^4 \omega_1^3 + 6\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1 - 18\omega_2^2 \omega_3^2 c_s^4 \omega_1 - 6\omega_2^3 \omega_3 c_s^2 \omega_1 + 6\omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_1 - 54\omega_2 v_1^4 \omega_3^2 \omega_1^2 - 6v_2^2 \omega_3^2 \omega_1^3 - 24\omega_2^3 v_1^2 v_2^2 \omega_3^2 \omega_1^2 - 6\omega_2^3 \omega_3^2 c_s^4 \omega_1^2 + 12\omega_2^2 c_s^2 \omega_1^3 + 36\omega_2^3 c_s^4 \omega_1^2 - 9\omega_2^3 v_1^2 \omega_3^2 - 6\omega_2^2 v_2^2 \omega_3^2 \omega_1 + 45v_1^2 v_2^2 \omega_3^2 \omega_1^3 + 90\omega_2^3 c_s^4 \omega_1^3 - \omega_2^3 \omega_3^2 \omega_1^2 + \omega_2^3 v_2^2 \omega_3^2 \omega_1^2 - 18\omega_2^2 \omega_3 c_s^2 \omega_1^3 - 297\omega_2^2 v_1^2 \omega_3^2 c_s^2 \omega_1 - 54\omega_2 v_1^4 \omega_3^2 \omega_1^3 + 135\omega_2^2 v_1^2 \omega_3^2 c_s^2 + 6\omega_2^2 \omega_3^2 \omega_1 + 60\omega_2 \omega_3^2 c_s^2 \omega_1^2 - 6\omega_2^3 v_2^2 \omega_3^2 \omega_1 - 6\omega_2 \omega_3^2 \omega_1^2 + 540\omega_2^2 v_1^2 \omega_3^2 c_s^2 \omega_1^2 + 2\omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_1^3 - 6\omega_2 v_2^2 \omega_3^2 \omega_1^2 - 486\omega_2 v_1^2 \omega_3^2 c_s^2 \omega_1^3 + 45\omega_2 v_2^2 \omega_3^2 \omega_1^2 + 6\omega_2^2 \omega_3^2 c_s^4 \omega_1^3 + 72\omega_2^2 v_1^2 \omega_3^2 c_s^2 \omega_1 + 27\omega_2^3 \omega_3^2 c_s^4 \omega_1 - 6\omega_2 \omega_3^2 \omega_1^3 + 72\omega_2^2 v_1^2 v_2^2 \omega_3^2 \omega_1 + 75\omega_2 \omega_3^2 c_s^2 \omega_1^3 - \omega_2^2 v_2^2 \omega_3^2 \omega_1^3 - 18\omega_2^3 v_2^2 \omega_3^2 \omega_1^2 + 18\omega_2^3 \omega_3 c_s^2 \omega_1^2 + 72\omega_2^2 \omega_3^2 c_s^4 \omega_1^2 + 126\omega_2 v_1^2 \omega_3^2 \omega_1^3 - 21\omega_2 v_2^2 \omega_3^2 c_s^2 \omega_1^3 - 243\omega_2 v_1^2 \omega_3^2 c_s^2 \omega_1^2 - 18\omega_2^3 \omega_3^2 c_s^4 - 45\omega_2^2 v_1^2 v_2^2 \omega_3^2 - 12\omega_2^3 c_s^2 \omega_1^2 + 18v_2^2 \omega_3^2 c_s^2 \omega_1^3 + 18\omega_2^2 v_2^2 \omega_3 c_s^2 \omega_1^3 - 99v_1^2 \omega_3^2 \omega_1^3 + 12\omega_2^2 v_2^2 c_s^2 \omega_1^2 + 54\omega_2^3 v_1^4 \omega_3^2 + 108\omega_2^2 v_1^4 \omega_3^2 \omega_1^2 - 72\omega_2^3 c_s^2 \omega_1^3 - 54\omega_2^2 v_1^4 \omega_3^2 \omega_1 - 72\omega_2^3 v_1^2 \omega_3^2 c_s^2 \omega_1^2 + 6\omega_2^3 \omega_1^3 + 54\omega_2^2 \omega_3 c_s^4 \omega_1^3 + 6\omega_2 \omega_3 c_s^2 \omega_1^3 + 24\omega_2^2 v_1^2 \omega_3^2 \omega_1^2 + 18\omega_2^3 \omega_3 c_s^4 \omega_1 - 18\omega_2^3 v_2^2 \omega_3^2 c_s^2 + \omega_2^2 \omega_3^2 \omega_1^3 + 12\omega_2^2 \omega_3^2 c_s^2 \omega_1 + 405v_1^2 \omega_3^2 c_s^2 \omega_1^3 - 6\omega_2 v_2^2 \omega_3 c_s^2 \omega_1^3 + 5\omega_2^3 \omega_3^2 c_s^2 \omega_1^2 - 2\omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_1^2 - 36\omega_2^2 c_s^4 \omega_1^3 + 63\omega_2^2 v_1^2 \omega_3^2 \omega_1 - 9\omega_2^2 v_1^2 v_2^2 \omega_3^2 \omega_1 + 54v_1^4 \omega_3^2 \omega_1^3 + 6\omega_2 v_2^2 \omega_3^2 \omega_1^2 - 12\omega_2^2 v_2^2 c_s^2 \omega_1^3 - 3\omega_2^3 \omega_3^2 c_s^2 \omega_1 + 21\omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_1 - 99\omega_2 \omega_3^2 c_s^4 \omega_1^3 - 108\omega_2^2 v_1^2 \omega_3^2 \omega_1^2 - 72\omega_2 v_1^2 v_2^2 \omega_3^2 \omega_1^3 - 18\omega_2^2 v_1^2 \omega_3^2 \omega_1 - 72\omega_2^2 \omega_3^2 c_s^2 \omega_1^2 - 54\omega_2^3 \omega_3 c_s^4 \omega_1^2 + 24\omega_2^2 v_1^2 v_2^2 \omega_3^2 \omega_1^3 - 24\omega_2^2 v_1^2 \omega_3^2 \omega_1^3 - 54\omega_2 \omega_3^2 c_s^4 \omega_1^2 - 54\omega_2^2 v_1^2 \omega_3^2 c_s^2 \omega_1 - 6\omega_2^3 \omega_3^2 + 6\omega_2 v_2^2 \omega_3^2 \omega_1^3 + 6\omega_2^2 v_2^2 \omega_3^2 - 5\omega_2^2 \omega_3^2 c_s^2 \omega_1^3 + 6\omega_2^2 \omega_3^2 \omega_1 - 54\omega_2^2 v_1^4 \omega_3^2 \omega_1 + 9\omega_2 v_1^2 v_2^2 \omega_3^2 \omega_1^2)\frac{v_2}{24\omega_2^3 \omega_3^2 \omega_1^3}$$

coefficient $C_{D_x^3 D_y v_1}^{(1)}$ **at** $\frac{\partial^4 v_1}{\partial x_1^3 \partial x_2}$:

$$C_{D_x^3 D_y v_1}^{(1), \text{SRT}} = 0$$

$$C_{D_x^3 D_y v_1}^{(1), \text{MRT1}} = (-16\omega_5^2 v_1^2 \omega_4^2 + 8\omega_5^3 \omega_4 + 44\omega_5^3 c_s^2 \omega_7 \omega_4 + 28\omega_5^2 \omega_7^2 \omega_4 + 56\omega_5^2 c_s^2 \omega_7 \omega_4^2 + 32\omega_5 c_s^2 \omega_7^2 \omega_4 + 20\omega_5^2 c_s^2 \omega_7^2 - 16\omega_5^3 c_s^2 \omega_4 + 8\omega_5^2 \omega_4^2 + 17\omega_5^2 \omega_7 \omega_4^2 + 43\omega_5^2 v_1^2 \omega_7^2 \omega_4^2 - 28\omega_5^3 v_1^2 \omega_7 - 68\omega_5^2 v_1^2 \omega_7^2 \omega_4 - 28\omega_5^3 \omega_7 \omega_4 + 24\omega_5 \omega_7 \omega_4^2 - 64\omega_5 v_1^2 \omega_7 \omega_4^2 + 16\omega_5^3 c_s^2 \omega_4^2 - 17\omega_5^2 \omega_7^2 \omega_4^2 - 16\omega_5^2 c_s^2 \omega_7 \omega_4 - 72\omega_5 c_s^2 \omega_7^2 \omega_4^2 - 8\omega_5^3 \omega_4^2 - 25\omega_5^3 c_s^2 \omega_7 \omega_4^2 + 48c_s^2 \omega_7^2 \omega_4^2 - 44\omega_5^2 c_s^2 \omega_7^2 \omega_4 - 40\omega_5^2 \omega_7 \omega_4^2 - 32\omega_5 c_s^2 \omega_7 \omega_4^2 - 32\omega_7^2 \omega_4^2 - 48\omega_5^2 v_1^2 \omega_7 \omega_4 - 120\omega_5 v_1^2 \omega_7^2 \omega_4^2 + 48\omega_5 \omega_7^2 \omega_4^2 + 16\omega_5^3 v_1^2 \omega_4^2 - 43\omega_5^3 v_1^2 \omega_7 \omega_4^2 + 80v_1^2 \omega_7^2 \omega_4^2 - 16\omega_5^3 v_1^2 \omega_4 + 68\omega_5^3 v_1^2 \omega_7 \omega_4 + 104\omega_5^2 v_1^2 \omega_7 \omega_4^2 - 12\omega_5^2 \omega_7^2 + 64\omega_5 v_1^2 \omega_7^2 \omega_4 - 20\omega_5^3 c_s^2 \omega_7 - 24\omega_5 \omega_7^2 \omega_4 - 16\omega_5^2 c_s^2 \omega_4^2 + 25\omega_5^2 c_s^2 \omega_7^2 \omega_4^2 + 16\omega_5^2 \omega_7 \omega_4 + 28\omega_5^2 v_1^2 \omega_7^2 + 12\omega_5^3 \omega_7) \frac{v_1 \rho v_2}{4\omega_5^3 \omega_7^2 \omega_4^2}$$

$$C_{D_x^3 D_y v_1}^{(1), \text{MRT2}} = C_{D_x^3 D_y v_1}^{(1), \text{MRT1}}$$

$$C_{D_x^3 D_y v_1}^{(1), \text{CLBM1}} = 0$$

$$C_{D_x^3 D_y v_1}^{(1), \text{CLBM2}} = 0$$

$$C_{D_x^3 D_y v_1}^{(1), \text{CuLBM1}} = 0$$

$$C_{D_x^3 D_y v_1}^{(1), \text{CuLBM2}} = (-27\omega_5^2 \omega_3 c_s^2 \omega_1 - 45\omega_2 v_2^2 \omega_3 \omega_1^3 + 90\omega_2^2 \omega_3 \omega_1 + 324\omega_2^2 \omega_3 c_s^2 \omega_1^2 + 54\omega_2^2 c_s^2 \omega_1^3 + 396\omega_2^2 v_1^2 \omega_3 \omega_1^2 + 18\omega_2^2 v_2^2 \omega_1^3 - 198\omega_5^3 v_1^2 \omega_3 \omega_1 + 18\omega_2 \omega_1^3 + 30\omega_2^2 \omega_3 c_s^2 \omega_1^3 + 54\omega_2^2 \omega_3 c_s^2 - 10\omega_2^2 \omega_3 \omega_1^3 + 198\omega_2^2 v_1^2 \omega_3 - 198\omega_2^2 v_1^2 \omega_3 \omega_1 + 270\omega_3 c_s^2 \omega_1^3 - 180\omega_2^2 \omega_3 \omega_1^2 + 198v_1^2 \omega_3 \omega_1^3 - 162\omega_2^2 \omega_3 c_s^2 \omega_1 - 30\omega_5^3 \omega_3 c_s^2 \omega_1^2 - 126\omega_3 \omega_1^3 - 54\omega_2^2 c_s^2 \omega_1^2 + 135\omega_2 \omega_3 \omega_1^3 - 162\omega_2 \omega_3 c_s^2 \omega_1^2 + 10\omega_2^2 v_2^2 \omega_3 \omega_1^3 - 36\omega_5^3 v_2^2 \omega_3 - 198\omega_2 v_1^2 \omega_3 \omega_1^3 - 54\omega_5^2 \omega_3 - 297\omega_2 \omega_3 c_s^2 \omega_1^3 + 45\omega_2^2 v_2^2 \omega_3 \omega_1 + 45\omega_2^2 \omega_3 \omega_1 + 18\omega_2^2 \omega_1^2 - 18\omega_2^2 v_2^2 \omega_1^2 + 90\omega_2 \omega_3 \omega_1^2 - 198\omega_2 v_1^2 \omega_3 \omega_1^2 - 10\omega_2^2 v_2^2 \omega_3 \omega_1^2 + 10\omega_2^2 \omega_3 \omega_1^2 - 54\omega_2 c_s^2 \omega_1^3 + 18\omega_2^2 v_2^2 \omega_1 - 18\omega_2^2 \omega_1^3 - 18\omega_2^2 \omega_1^3 + 54\omega_2^2 c_s^2 \omega_1 + 36v_2^2 \omega_3 \omega_1^3 - 18\omega_2 v_2^2 \omega_1^3) \frac{v_1 \rho v_2}{24\omega_5^3 \omega_3 \omega_1^3}$$

coefficient $C_{D_x^3 D_y v_2}^{(1)}$ **at** $\frac{\partial^4 v_2}{\partial x_1^3 \partial x_2}$:

$$C_{D_x^3 D_y v_2}^{(1), \text{SRT}} = (-42v_1^2 c_s^2 \omega^2 - 12c_s^2 \omega^2 + 54v_1^4 \omega + 12v_1^2 c_s^2 \omega^3 + 36v_1^2 - 26v_1^4 \omega^2 - 24c_s^2 + 36c_s^2 \omega - 36v_1^2 c_s^2 + 54v_1^2 c_s^2 \omega + 4v_1^4 \omega^3 - 54v_1^2 \omega + 36c_s^4 - c_s^4 \omega^3 + 20c_s^4 \omega^2 - 54c_s^4 \omega - 4v_1^2 \omega^3 + 26v_1^2 \omega^2 - 36v_1^4) \frac{\rho}{12\omega^3}$$

$$C_{D_x^3 D_y v_2}^{(1), \text{MRT1}} = (-12\omega_5 c_s^4 \omega_7^2 \omega_4^3 + 12\omega_5^3 c_s^4 \omega_7^2 - 12\omega_5^3 c_s^2 \omega_7 \omega_4 + 6\omega_5^3 c_s^4 \omega_7 \omega_4^3 - 12\omega_5^3 v_1^4 \omega_4^2 + 12c_s^4 \omega_7^2 \omega_4^3 - 12\omega_5^2 c_s^2 \omega_7 \omega_4^2 - 12\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4 - 18\omega_5^3 v_1^4 \omega_7^2 \omega_4^2 - 19\omega_5^2 v_1^2 \omega_7^2 \omega_4^3 - 48\omega_5^3 v_1^2 c_s^2 \omega_7^2 - 36\omega_5 v_1^4 \omega_7 \omega_4^3 - 48\omega_5^2 v_1^2 c_s^2 \omega_7^2 \omega_4 + 12\omega_5^2 v_1^2 \omega_4^3 - 12\omega_5^3 v_1^2 \omega_7^2 \omega_4 - 12\omega_5 c_s^4 \omega_7^2 \omega_4^2 - 12\omega_5^2 v_1^2 \omega_7^2 \omega_4^2 + 4\omega_5^3 v_1^4 \omega_7^2 \omega_4^3 + 6\omega_5^2 c_s^2 \omega_7 \omega_4^3 - 18\omega_5^3 c_s^4 \omega_7 \omega_4^2 + 12\omega_5^3 v_1^4 \omega_4^3 - 21\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^3 + 12\omega_5 c_s^2 \omega_7^2 \omega_4^3 - 306\omega_5 v_1^2 c_s^2 \omega_7^2 \omega_4^3 + 12\omega_5^3 c_s^4 \omega_7 \omega_4 + 12\omega_5^2 c_s^4 \omega_7 \omega_4^2 - 6\omega_5^3 c_s^2 \omega_7 \omega_4 + 162\omega_5^2 v_1^2 c_s^2 \omega_7^2 \omega_4^2 - 12c_s^2 \omega_7^2 \omega_4^3 + 19\omega_5^2 v_1^4 \omega_7^2 \omega_4^3 + 18\omega_5^3 v_1^2 \omega_7^2 \omega_4^2 + 36\omega_5 v_1^4 \omega_7 \omega_4^3 + 12\omega_5^2 v_1^2 c_s^2 \omega_7^2 \omega_4^2 + 30\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^2 - 4\omega_5^3 v_1^2 \omega_7^2 \omega_4^3 + 12\omega_5^2 v_1^4 \omega_7^2 \omega_4^2 + 18\omega_5^3 c_s^2 \omega_7 \omega_4^2 + 60\omega_5^2 v_1^2 c_s^2 \omega_7^2 \omega_4^3 - 6\omega_5^2 c_s^4 \omega_7 \omega_4^3 + 90\omega_5 v_1^2 \omega_7^2 \omega_4^3 - 12\omega_5^2 v_1^4 \omega_4^3 - 81\omega_5^3 v_1^2 c_s^2 \omega_7^2 \omega_4^2 - 24\omega_5^3 v_1^4 \omega_7 \omega_4^2 - 24\omega_5^2 v_1^4 \omega_7 \omega_4^2 + 27\omega_5^3 v_1^2 \omega_7 \omega_4^3 - 72v_1^2 \omega_7^2 \omega_4^3 + \omega_5^2 c_s^4 \omega_7^2 \omega_4^3 - 12\omega_5^2 v_1^2 c_s^2 \omega_4^3 + 54\omega_5^2 v_1^2 c_s^2 \omega_7 \omega_4^3 - 5\omega_5^3 c_s^2 \omega_7^2 \omega_4^2 - 12\omega_5^3 v_1^2 \omega_4^3 + 12\omega_5^2 v_1^2 c_s^2 \omega_7^2 \omega_4^3 - 24\omega_5^3 c_s^4 \omega_7^2 \omega_4 - 36\omega_5 v_1^2 c_s^2 \omega_7 \omega_4^3 - 12\omega_5^2 v_1^2 c_s^2 \omega_7 \omega_4^2 + 12\omega_5^3 v_1^2 \omega_4^2 + 6\omega_5^2 c_s^4 \omega_7^2 \omega_4^2 - 48\omega_5^3 v_1^2 \omega_7 \omega_4^2 + 60\omega_5^2 v_1^4 \omega_7 \omega_4^3 - 90\omega_5^3 v_1^2 \omega_7 \omega_4^2 + 24\omega_5^3 v_1^4 \omega_7 \omega_4^3 + 72v_1^4 \omega_7^2 \omega_4^3 + 24\omega_5^2 v_1^2 \omega_7 \omega_4^2 - 12\omega_5^3 v_1^2 c_s^2 \omega_4^2 + 13\omega_5^3 c_s^4 \omega_7^2 \omega_4^2 - \omega_5^2 c_s^2 \omega_7^2 \omega_4^3 + 6\omega_5^3 c_s^2 \omega_7^2 \omega_4 - 6\omega_5^2 c_s^2 \omega_7^2 \omega_4^2 - \omega_5^3 c_s^4 \omega_7^2 \omega_4^3 + 12\omega_5^3 v_1^2 c_s^2 \omega_4^3 - 60\omega_5^2 v_1^4 \omega_7 \omega_4^3 + 48\omega_5^3 v_1^4 \omega_7 \omega_4^2 + 102\omega_5^3 v_1^2 c_s^2 \omega_7^2 \omega_4) \frac{\rho}{12\omega_5^3 \omega_7^2 \omega_4^3}$$

$$C_{D_x^3 D_y v_2}^{(1), \text{MRT2}} = C_{D_x^3 D_y v_2}^{(1), \text{MRT1}}$$

$$C_{D_x^3 D_y v_2}^{(1), \text{CLBM1}} = (-12\omega_5 c_s^4 \omega_7^2 \omega_4^3 + 12\omega_5^3 c_s^4 \omega_7^2 - 12\omega_5^3 c_s^2 \omega_7 \omega_4 + 6\omega_5^3 c_s^4 \omega_7 \omega_4^3 - 36\omega_5^3 v_1^4 \omega_4^2 + 12c_s^4 \omega_7^2 \omega_4^3 - 12\omega_5^2 c_s^2 \omega_7 \omega_4^2 + 36\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4 - 6\omega_5^3 v_1^4 \omega_7^2 \omega_4^2 - 19\omega_5^2 v_1^2 \omega_7^2 \omega_4^3 - 36\omega_5 v_1^4 \omega_7 \omega_4^3 + 36\omega_5^2 v_1^2 \omega_4^3 - 12\omega_5 c_s^4 \omega_7^2 \omega_4^3 + 6\omega_5^2 c_s^2 \omega_7 \omega_4^3 - 18\omega_5^3 c_s^4 \omega_7 \omega_4^2 + 36\omega_5^3 v_1^4 \omega_4^3 - 99\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^3 + 12\omega_5^2 c_s^2 \omega_7^2 \omega_4^3 - 306\omega_5 v_1^2 c_s^2 \omega_7^2 \omega_4^3 + 12\omega_5^3 c_s^4 \omega_7 \omega_4 + 12\omega_5^2 c_s^4 \omega_7 \omega_4^2 - 6\omega_5^3 c_s^2 \omega_7 \omega_4^3 + 18\omega_5^2 v_1^2 c_s^2 \omega_7^2 \omega_4^2 - 12c_s^2 \omega_7^2 \omega_4^3 + 19\omega_5^2 v_1^4 \omega_7^2 \omega_4^3 + 6\omega_5^3 v_1^2 \omega_7^2 \omega_4^2 + 36\omega_5 v_1^4 \omega_7 \omega_4^3 - 36\omega_5 v_1^2 c_s^2 \omega_7^2 \omega_4^2 + 252v_1^2 c_s^2 \omega_7^2 \omega_4^3 + 12\omega_5^2 c_s^2 \omega_7^2 \omega_4^2 + 54\omega_5^3 v_1^2 c_s^2 \omega_7 \omega_4^2 - 4\omega_5^3 v_1^2 \omega_7^2 \omega_4^3 + 18\omega_5^3 c_s^2 \omega_7 \omega_4^2 + 60\omega_5^2 v_1^2 c_s^2 \omega_7^2 \omega_4^2 - 6\omega_5^2 c_s^4 \omega_7 \omega_4^3 + 90\omega_5 v_1^4 \omega_7^2 \omega_4^3 - 36\omega_5^2 v_1^4 \omega_4^3 - 3\omega_5^3 v_1^2 c_s^2 \omega_7^2 \omega_4^2 + 39\omega_5^3 v_1^2 \omega_7 \omega_4^3 - 72v_1^2 \omega_7^2 \omega_4^3 + \omega_5^2 c_s^4 \omega_7^2 \omega_4^3 - 108\omega_5^2 v_1^2 c_s^2 \omega_4^3 + 198\omega_5^2 v_1^2 c_s^2 \omega_7 \omega_4^3 - 5\omega_5^3 c_s^2 \omega_7^2 \omega_4^2 - 36\omega_5^3 v_1^4 \omega_4^3 + 12\omega_5^3 v_1^2 c_s^2 \omega_7^2 \omega_4^3 - 24\omega_5^3 c_s^4 \omega_7^2 \omega_4 - 108\omega_5 v_1^2 c_s^2 \omega_7 \omega_4^3 + 36\omega_5^2 v_1^2 c_s^2 \omega_7 \omega_4^2 + 36\omega_5^3 v_1^4 \omega_4^2 + 6\omega_5^2 c_s^4 \omega_7^2 \omega_4^2 - 36\omega_5^3 v_1^2 \omega_7 \omega_4^2 + 72\omega_5^2 v_1^4 \omega_7 \omega_4^3 - 90\omega_5 v_1^4 \omega_7^2 \omega_4^3 - 39\omega_5^3 v_1^4 \omega_7 \omega_4^3 + 72v_1^4 \omega_7^2 \omega_4^3 - 108\omega_5^3 v_1^2 c_s^2 \omega_4^2 + 13\omega_5^3 c_s^4 \omega_7^2 \omega_4^2 - \omega_5^2 c_s^2 \omega_7^2 \omega_4^3 + 6\omega_5^3 c_s^2 \omega_7^2 \omega_4 - 6\omega_5^2 c_s^2 \omega_7^2 \omega_4^2 - \omega_5^3 c_s^4 \omega_7^2 \omega_4^3 + 108\omega_5^3 v_1^2 c_s^2 \omega_4^3 - 72\omega_5^2 v_1^4 \omega_7 \omega_4^3 + 36\omega_5^3 v_1^4 \omega_7 \omega_4^2 - 18\omega_5^3 v_1^2 c_s^2 \omega_7^2 \omega_4) \frac{\rho}{12\omega_5^3 \omega_7^2 \omega_4^3}$$

$$C_{D_x^3 D_y v_2}^{(1), \text{CLBM2}} = C_{D_x^3 D_y v_2}^{(1), \text{CLBM1}}$$

$$C_{D_x^3 D_y v_2}^{(1), \text{CuLBM1}} = (-99v_1^2 \omega_3^2 c_s^2 \omega_4 \omega_1^3 - 39v_1^4 \omega_3 \omega_4 \omega_1^3 - 6\omega_3^3 c_s^4 \omega_4 \omega_1^2 + 252v_1^2 \omega_3^3 c_s^2 \omega_4^2 + 18\omega_3^2 c_s^2 \omega_4 \omega_1^3 - 18v_1^2 \omega_3^2 c_s^2 \omega_4^2 \omega_1^3 + 108v_1^2 \omega_3^3 c_s^2 \omega_1^3 + 198v_1^2 \omega_3^2 c_s^2 \omega_4 \omega_1^2 + 72v_1^4 \omega_3 \omega_4 \omega_1^2 - 12\omega_3^3 c_s^2 \omega_4^2 - 24\omega_3 c_s^4 \omega_4^2 \omega_1^3 + 90v_1^2 \omega_3^3 \omega_4^2 \omega_1 - 108v_1^2 \omega_3^3 c_s^2 \omega_1^2 - 12\omega_3^2 c_s^2 \omega_4 \omega_1^2 - 36v_1^2 \omega_3^2 \omega_4 \omega_1^3 + 6\omega_3^3 c_s^4 \omega_4 \omega_1^3 + 36v_1^2 \omega_3^2 c_s^2 \omega_4 \omega_1^2 - 19v_1^2 \omega_3^3 \omega_4^2 \omega_1^2 - 18\omega_3^2 c_s^4 \omega_4 \omega_1^3 + 6\omega_3^3 c_s^2 \omega_4 \omega_1^2 - 6v_1^4 \omega_3^2 \omega_4 \omega_1^3 - 36v_1^4 \omega_3^3 \omega_4 \omega_1 - 108v_1^2 \omega_3^3 c_s^2 \omega_4 \omega_1 + 54v_1^2 \omega_3^2 c_s^2 \omega_4 \omega_1^3 +$$

$$C_{D_3 \text{ up}}^{(1), \text{CuLB2M2}} = (144\omega_2^3 v_1^4 \omega_3 \omega_1^2 + 6\omega_2 \omega_3 c_s^4 \omega_1^3 - 72\omega_2^3 v_1^2 \omega_3 \omega_1^3 - 54\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1 + 6\omega_2^3 \omega_3 c_s^4 \omega_1 - 18\omega_2^3 \omega_3 c_s^2 \omega_1 + 18\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1 - 324\omega_2^3 v_1^2 c_s^2 \omega_1^2 + 36\omega_2^3 v_1^4 \omega_3 \omega_1^2 - 18\omega_2^3 \omega_3 \omega_1^3 - 72\omega_2^3 v_1^2 v_2^2 \omega_3 \omega_1^2 - 24\omega_2^3 \omega_3 c_s^2 \omega_1^2 + 29\omega_2^3 \omega_3 c_s^4 \omega_1^2 + 9\omega_2^3 v_1^2 \omega_3^2 + 36\omega_2^3 v_1^2 \omega_3 \omega_1^2 + 54\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1 - 18\omega_2^3 v_2^2 \omega_3 \omega_1 - 78\omega_2^3 v_1^4 \omega_3 \omega_1^3 + 135v_1^2 v_2^2 \omega_3 \omega_1^3 + 18\omega_2^3 c_s^4 \omega_1^3 - \omega_2^3 \omega_3 \omega_1^2 + 3\omega_2^3 v_2^2 \omega_3 \omega_1^2 + 18\omega_2^3 v_1^2 \omega_3 \omega_1 + 18\omega_2^3 \omega_3 c_s^2 \omega_1^3 - 2\omega_2^3 \omega_3 c_s^4 \omega_1^3 + 216\omega_2^3 v_1^2 c_s^2 \omega_1^3 + 99\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1 - 63\omega_2^3 v_1^4 \omega_3 \omega_1^3 + 63\omega_2^3 v_1^2 \omega_3 c_s^2 \omega_1^3 + 72\omega_2^3 v_1^2 \omega_3 \omega_1^3 + 6\omega_2^3 \omega_3 \omega_1^2 + 36\omega_2^3 \omega_3 c_s^2 \omega_1^2 - 144\omega_2^3 v_2^2 \omega_3 \omega_1^2 - 18\omega_2^3 v_2^2 \omega_3 \omega_1 - 6\omega_2^3 \omega_3 \omega_1^2 - 6\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^3 - 108\omega_2^3 v_1^2 c_s^2 \omega_1^3 - 54\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^2 - 297\omega_2^3 v_1^2 \omega_3 c_s^2 \omega_1^2 - 12\omega_2^3 \omega_3 c_s^2 \omega_1^3 - 45\omega_2^3 v_2^2 \omega_3 \omega_1^2 - \omega_2^3 c_s^4 \omega_1^3 + 306\omega_2^3 v_1^2 \omega_3 c_s^2 \omega_1^2 - 36\omega_2^3 v_1^4 \omega_3^2 + 108\omega_2^3 v_1^2 \omega_3^2 + 84\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^3 + 18\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^2 - 57\omega_2^3 \omega_3 c_s^4 \omega_1 - 6\omega_2 \omega_3 \omega_1^3 + 216\omega_2^3 v_1^2 v_2^2 \omega_3 \omega_1^2 + 78\omega_2^3 v_1^2 \omega_3 \omega_1^3 + 21\omega_2^3 \omega_3 c_s^2 \omega_1^3 - 36\omega_2^3 v_1^4 \omega_3 \omega_1^2 - 18\omega_2^3 v_1^2 \omega_3 \omega_1 - 198\omega_2^3 v_1^2 \omega_3 c_s^2 \omega_1^3 - 3\omega_2^3 v_2^2 \omega_3 \omega_1^2 + 36\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^2 + 30\omega_2^3 \omega_3 c_s^2 \omega_1^3 + 12\omega_2^3 \omega_3 c_s^4 \omega_1^2 + 135\omega_2^3 v_1^2 v_2^2 \omega_3 \omega_1^2 - 91\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^3 + 81\omega_2^3 v_1^2 \omega_3 c_s^2 \omega_1^2 + 30\omega_2^3 \omega_3 c_s^2 \omega_1^3 - 135\omega_2^3 v_1^2 v_2^2 \omega_3 \omega_1^2 + 18\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^3 - 72\omega_2^3 v_1^2 \omega_3 c_s^2 \omega_1^2 - 36\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^3 - 81v_1^2 \omega_3 \omega_1^3 + 36\omega_2^3 v_1^4 \omega_3^2 - 54\omega_2^3 v_1^4 \omega_3 \omega_1^2 - 8\omega_2^3 v_1^2 \omega_3 \omega_1^2 - 24\omega_2^3 c_s^2 \omega_1^3 - 63\omega_2^3 v_1^2 \omega_3 \omega_1 + 30\omega_2^3 v_1^2 \omega_3 c_s^2 \omega_1^2 + 6\omega_2^3 \omega_1^3 - 18\omega_2^3 \omega_3 c_s^4 \omega_1 - 54\omega_2^3 v_1^2 \omega_3 c_s^2 \omega_1^2 + 18\omega_2^3 v_1^2 \omega_3 \omega_1^2 - 6\omega_2 \omega_3 c_s^2 \omega_1^3 + 17\omega_2^3 v_1^2 \omega_3 \omega_1^2 + 19\omega_2^3 v_1^4 \omega_3 \omega_1^3 + 18\omega_2^3 \omega_3 c_s^4 \omega_1 + 18\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^2 + \omega_2^3 \omega_3 \omega_1^3 - 12\omega_2^3 \omega_3 c_s^2 \omega_1 + 189v_1^2 \omega_3 c_s^2 \omega_1^3 + 198\omega_2^3 v_1^2 \omega_3 c_s^2 \omega_1^2 + 36\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^2 + 18\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^3 - 12\omega_2^3 \omega_3 c_s^2 \omega_1^2 + 24\omega_2^3 \omega_3 c_s^4 \omega_1^2 + 24\omega_2^3 v_1^2 \omega_3 c_s^2 \omega_1^2 + 6\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^2 - 27\omega_2^3 v_1^2 \omega_3 \omega_1 - 27\omega_2^3 v_1^2 v_2^2 \omega_3 \omega_1^2 + 36v_1^4 \omega_3 \omega_1^3 + 18\omega_2^3 v_2^2 \omega_3 \omega_1^2 - 108\omega_2^3 v_1^2 \omega_3^2 + 15\omega_2^3 v_2^2 \omega_3 \omega_1^2 + 8\omega_2^3 v_1^4 \omega_3 \omega_1^2 - 9\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^3 - 15\omega_2^3 \omega_3 c_s^4 \omega_1 + 54\omega_2^3 v_1^2 \omega_3 \omega_1^2 - 216\omega_2^3 v_1^2 v_2^2 \omega_3 \omega_1^2 - 9\omega_2^3 v_1^2 \omega_3 \omega_1 - 18\omega_2^3 v_1^4 \omega_3 \omega_1^3 - 12\omega_2^3 \omega_3 c_s^2 \omega_1^2 - 30\omega_2^3 \omega_3 c_s^4 \omega_1 + 72\omega_2^3 v_1^2 v_2^2 \omega_3 \omega_1^2 - 43\omega_2^3 v_1^2 \omega_3 \omega_1^3 - 30\omega_2^3 \omega_3 c_s^4 \omega_1 - 171\omega_2^3 v_1^2 \omega_3 c_s^2 \omega_1^2 + 7\omega_2^3 v_1^4 \omega_3 \omega_1^2 - 6\omega_2^3 \omega_3 + 18\omega_2^3 v_2^2 \omega_3 \omega_1^3 + 72\omega_2^3 v_1^4 \omega_3 \omega_1^3 + 18\omega_2^3 v_2^2 \omega_3^2 + 12\omega_2^3 \omega_3 c_s^4 \omega_1^3 + 6\omega_2^3 \omega_3 \omega_1 + 36\omega_2^3 v_1^4 \omega_3 \omega_1^2 + 36\omega_2^3 v_1^2 \omega_3 \omega_1 + 27\omega_2^3 v_1^2 v_2^2 \omega_3 \omega_1^2) \frac{\rho}{24\omega_2^3 \omega_3 \omega_1^3}$$

$$C_{\text{D}_x^2\text{D}_y^2}^{(1),\text{SRT}} = (24 - 46c_s^2\omega^2 - 24v_1^2 + 5c_s^2\omega^3 - 72c_s^2 + 108c_s^2\omega + 36v_1^2\omega - 36\omega + v_1^2\omega^3 + 14\omega^2 - 14v_1^2\omega^2 - \omega^3) \frac{v_1 c_s^2}{12\omega^3}$$

$$C_{D_x^2 D_y^2 \rho}^{(1), \text{MRT2}} = C_{D_x^2 D_y^2 \rho}^{(1), \text{MRT1}}$$

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$$C_{D_2^2 D_2^2 y_1}^{(1), CLMB1} = (-36\omega_8 \omega_5^2 v_1^2 \omega_4^3 + 6\omega_8 \omega_5 \omega_9 \omega_7 \omega_4^3 - 36\omega_8 v_1^2 \omega_9 \omega_7 \omega_4^3 + 18\omega_8 \omega_5^2 \omega_9 c_s^2 \omega_7 \omega_4 + 12\omega_5^2 c_s^2 \omega_7 \omega_4^2 - 72\omega_8 \omega_5^2 v_1^2 \omega_9 \omega_4^2 - 12\omega_8 \omega_5^2 \omega_9 \omega_4^3 -$$

 $24\omega_8 \omega_5 \omega_9 \omega_7 \omega_4^2 + 18\omega_8 v_1^2 \omega_9 \omega_7 \omega_4^3 + 12\omega_8 \omega_5 c_s^2 \omega_4^3 + 36\omega_8 \omega_5^2 v_1^2 \omega_4^3 + 36\omega_8 \omega_5^2 v_1^2 \omega_9 \omega_4^3 + 24\omega_8 \omega_5^2 \omega_9 \omega_4^2 - 18\omega_8 \omega_5 v_1^2 \omega_7 \omega_4^3 - 6\omega_5^2 c_s^2 \omega_7 \omega_4^3 -$
 $12\omega_8 \omega_5 \omega_9 c_s^2 \omega_7 \omega_4 - 12\omega_8 \omega_5^2 c_s^2 \omega_7 \omega_4^2 + 12\omega_8 \omega_9 \omega_7 \omega_4^2 - 12\omega_8 \omega_5^2 \omega_9 \omega_4 + 24\omega_8 \omega_5 \omega_9 c_s^2 \omega_7 \omega_4^2 + 12\omega_8 \omega_5 \omega_9 c_s^2 \omega_4^3 + 6\omega_8 \omega_5 \omega_7 \omega_4^3 - 6\omega_5 \omega_9 \omega_7 \omega_4^3 -$
 $12\omega_8 \omega_9 c_s^2 \omega_7 \omega_4^3 - 12\omega_8 \omega_5 \omega_9 c_s^2 \omega_7 \omega_4 + 12\omega_8 \omega_5 \omega_9 \omega_7 \omega_4 + 18\omega_5 v_1^2 \omega_9 \omega_7 \omega_4^3 - 6\omega_8 \omega_5^2 c_s^2 \omega_7 \omega_4^3 - 12\omega_8 \omega_5 \omega_9 c_s^2 \omega_4^3 - 6\omega_8 \omega_9 \omega_7 \omega_4^3 + 36\omega_8 \omega_5^2 v_1^2 \omega_9 \omega_4^2 +$
 $6\omega_8 \omega_5^2 c_s^2 \omega_7 \omega_4^3 - 36\omega_5 v_1^2 \omega_9 \omega_7 \omega_4^3 - 12\omega_8 \omega_5 \omega_4^3 - 4\omega_8 \omega_5^2 c_s^2 \omega_7 \omega_4^2 + 12\omega_5 \omega_9 \omega_7 \omega_4^2 - 6\omega_8 v_1^2 \omega_7 \omega_4^3 + 36\omega_8 \omega_5 v_1^2 \omega_9 \omega_4^2 - 36\omega_8 \omega_5^2 v_1^2 \omega_7 \omega_4^2 - 12\omega_8 \omega_5^2 \omega_9 \omega_4^2)$

$$12\omega_8\omega_9c_s^2\omega_7\omega_4^2 + 12\omega_8\omega_5^2\omega_4^3 + 12\omega_8\omega_5^2\omega_9c_s^2\omega_4 + 6\omega_5^2\omega_7\omega_4^3 + 12\omega_8\omega_5^2\omega_7\omega_4^2 - 36\omega_8\omega_5v_1^2\omega_9\omega_4^3 - 36\omega_8\omega_5v_1^2\omega_9\omega_7\omega_4 + 18\omega_8\omega_5^2v_1^2\omega_7\omega_4^3 - 12\omega_8\omega_5^2\omega_4^2 + 6\omega_8\omega_9c_s^2\omega_7\omega_4^3 + 6\omega_5\omega_9c_s^2\omega_7\omega_4^3 + 36\omega_8\omega_5v_1^2\omega_4^3 + 12\omega_8\omega_5^2c_s^2\omega_4^2 - 24\omega_8\omega_5^2\omega_9c_s^2\omega_4^2 + 36\omega_5^2v_1^2\omega_7\omega_4^2 + 12\omega_8\omega_5^2v_1^2\omega_9\omega_7\omega_4^2 - 12\omega_8\omega_5^2c_s^2\omega_4^3 - 12\omega_5\omega_9c_s^2\omega_7\omega_4^2 - 18\omega_8\omega_5v_1^2\omega_9\omega_7\omega_4^3 - 6\omega_8\omega_5c_s^2\omega_7\omega_4^3 - 18\omega_5^2v_1^2\omega_7\omega_4^3 - 12\omega_8\omega_5\omega_9\omega_4^2 + 12\omega_8\omega_5^2\omega_9c_s^2\omega_4^3) \frac{\rho c_s^2}{12\omega_8\omega_5^2\omega_9\omega_7\omega_4^3}$$

$$C_{D_x^2 D_y^2 v_1}^{(1), \text{CLBM2}} = C_{D_x^2 D_y^2 v_1}^{(1), \text{CLBM1}}$$

$$C_{D_x^2 D_y^2 v_1}^{(1), \text{CuLBM1}} = (12\omega_3^2c_s^2\omega_1^2 - 4\omega_3^2c_s^2\omega_4\omega_1^2 + 12\omega_3^2c_s^2\omega_4\omega_1 - 12c_s^2\omega_4\omega_1^2 + 36v_1^2\omega_3\omega_1^2 - 12\omega_3\omega_1^2 - \omega_3^2c_s^2\omega_4\omega_1^2 - 12\omega_3^2\omega_4\omega_1 + 36v_1^2\omega_3^2\omega_4\omega_1 + 12\omega_3^2c_s^2 + 12\omega_3c_s^2\omega_1^2 - 24\omega_3^2c_s^2\omega_1 - 72v_1^2\omega_3^3\omega_1 + 36v_1^2\omega_3^3 + 12\omega_3\omega_4\omega_1 + 24\omega_3^3\omega_1 + 18\omega_3c_s^2\omega_4\omega_1^2 - 12\omega_3^3 - 72v_1^2\omega_3^2\omega_1 - 36v_1^2\omega_3\omega_4\omega_1 + 24\omega_3^3\omega_1^2 - 24\omega_3^2c_s^2\omega_1^2 - 24\omega_3^2\omega_1 + 24\omega_3^2c_s^2\omega_1 - 12\omega_3c_s^2\omega_4\omega_1 + 72v_1^2\omega_3^2\omega_1 + 12\omega_3^2\omega_4 - 12\omega_3^2c_s^2\omega_4 - 12\omega_3^3\omega_1^2 + 36v_1^2\omega_3^2\omega_1^2 - 36v_1^2\omega_3^2\omega_4) \frac{\rho c_s^2}{12\omega_3^3\omega_4\omega_1^2}$$

$$C_{D_x^2 D_y^2 v_1}^{(1), \text{CuLBM2}} = (6\omega_2\omega_3c_s^4\omega_1^3 + 45\omega_3^2v_4^2\omega_3\omega_1 + 18\omega_3^2v_2^2\omega_3c_s^2\omega_1 - 30\omega_2^2\omega_3^2c_s^4\omega_1 + 36\omega_3^2v_4^2\omega_1^2 - 30\omega_3^2\omega_3c_s^2\omega_1 + 18\omega_2v_2^2\omega_3\omega_1^3 - 45\omega_2^2v_2^2\omega_3^2c_s^2\omega_1 - 81v_2^2\omega_3^2\omega_1^3 - 60\omega_2^2\omega_3^2\omega_1^2 - 10\omega_2^2\omega_3^2c_s^4\omega_1^2 - 18\omega_3^2v_1^2\omega_3^2 + 126\omega_3^2v_1^2\omega_3c_s^2\omega_1 + 45\omega_2^2v_2^2\omega_3^2\omega_1 + 135v_1^2v_2^2\omega_3^2\omega_1^3 + 18\omega_3^2c_s^4\omega_1^3 + 36\omega_2^2v_2^2\omega_1^3 + 9\omega_2^2v_2^4\omega_3\omega_1^3 + 9\omega_3^2v_2^2\omega_3^2\omega_1^2 + 36\omega_2^2\omega_3^2c_s^2\omega_1^3 - 2\omega_2^2\omega_3^2c_s^4\omega_1^3 - 90\omega_2^2v_1^2\omega_3^2c_s^2\omega_1 - 18\omega_3^2v_1^2\omega_3^2c_s^2 - 6\omega_3^2\omega_3\omega_1 + 36\omega_2\omega_3^2c_s^2\omega_1^2 - 6\omega_2\omega_3\omega_1^2 - 36\omega_3^2v_4^2\omega_3^2 + 72\omega_2^2v_1^2\omega_3^2c_s^2\omega_1^2 - 18\omega_2v_4^2\omega_3\omega_1^3 + 30\omega_2^2v_2^2\omega_3^2c_s^2\omega_1^3 - 81\omega_2v_2^2\omega_3^2c_s^2\omega_1^2 - 9\omega_2v_1^2\omega_3^2c_s^2\omega_1^3 - 24\omega_2^2\omega_3c_s^2\omega_1^3 + 18\omega_2v_1^2\omega_3^2\omega_1^2 + 36v_1^4\omega_3\omega_1^3 + 2\omega_2^2\omega_3^2c_s^4\omega_1^3 - 90\omega_2^2v_2^2\omega_3^2\omega_1^2 - 198\omega_3^2v_1^2\omega_3^2c_s^2\omega_1^2 + 6\omega_2^2v_1^2\omega_3^2c_s^2\omega_1^3 + 108\omega_2^2v_2^2\omega_3^2c_s^2\omega_1^2 + 45\omega_2^2\omega_3^2c_s^4\omega_1 - 6\omega_2\omega_3^2\omega_1^3 - 135\omega_3^2v_1^2v_2^2\omega_3^2\omega_1 + 21\omega_2\omega_3^2c_s^2\omega_1^3 - 9\omega_2^2v_4^2\omega_3\omega_1^2 + 72\omega_3^2v_1^2\omega_3c_s^2\omega_1^3 - 9\omega_2^2v_2^2\omega_3^2\omega_1^3 - 144\omega_3^2v_2^2\omega_3c_s^2\omega_1^2 + 60\omega_2^2\omega_3c_s^2\omega_1^2 + 30\omega_2^2\omega_3^2c_s^4\omega_1^2 + 18\omega_2v_1^2\omega_3^2\omega_1^3 - 216\omega_2v_2^2\omega_3^2c_s^2\omega_1^3 - 54\omega_2v_1^2\omega_3^2c_s^2\omega_1^2 - 30\omega_3^2\omega_3c_s^4 + 135\omega_3^2v_1^2v_2^2\omega_3^2 + 189v_2^2\omega_3^2c_s^2\omega_1^3 + 144\omega_2^2v_1^2\omega_3c_s^2\omega_1^2 + 144\omega_2^2v_2^2\omega_3c_s^2\omega_1^3 + 12\omega_2^2\omega_3^2\omega_1^2 - 18v_1^2\omega_3^2\omega_1^3 + 108\omega_2^2v_2^2c_s^2\omega_1^2 - 54\omega_2^2v_2^2\omega_3\omega_1^3 - 36\omega_2^2v_4^2\omega_1^3 - 24\omega_3^2c_s^2\omega_1^3 - 6\omega_3^2v_1^2\omega_3^2c_s^2\omega_1^2 - 54\omega_3^2v_4^2\omega_3\omega_1^2 + 6\omega_3^2\omega_1^3 - 36\omega_2^2\omega_3c_s^4\omega_1^3 + 18\omega_2v_1^2\omega_3^2c_s^2\omega_1^3 - 6\omega_2\omega_3^2\omega_1^2 - 18\omega_3^2v_2^2\omega_3\omega_1 + \omega_3^2v_1^2\omega_3^2\omega_1^2 + \omega_2^2v_1^4\omega_3\omega_1^3 + 30\omega_2^2\omega_3c_s^2\omega_1 - 45\omega_2v_2^2\omega_3^2\omega_1^2 - 63\omega_3^2v_2^2\omega_3^2c_s^2 - 36\omega_3^2v_2^2\omega_1^3 + 36\omega_2^2\omega_3^2c_s^2\omega_1 + 18v_1^2\omega_3^2c_s^2\omega_1^3 - 90\omega_2^2v_1^2\omega_3^2c_s^2\omega_1^2 + 36\omega_2^2v_2^2\omega_3c_s^2\omega_1^3 - 54\omega_2v_2^2\omega_3c_s^2\omega_1^2 + 2\omega_2^2\omega_3^2c_s^2\omega_1^2 + 60\omega_2^2\omega_3c_s^4\omega_1^2 - 30\omega_3^2v_2^2\omega_3^2c_s^2\omega_1^2 + 18\omega_2^2v_1^2\omega_3\omega_1 - 135\omega_2^2v_1^2v_2^2\omega_3^2\omega_1 + 45\omega_2v_2^2\omega_3^2\omega_1^2 - 108\omega_2^2v_2^2c_s^2\omega_1^3 - 3\omega_2^2\omega_3^2c_s^2\omega_1 + 108\omega_3^2v_2^2\omega_3^2c_s^2\omega_1 - 15\omega_2^2\omega_3^2c_s^2\omega_1^3 - 36\omega_2^2v_1^2\omega_3^2\omega_1^2 + 270\omega_2^2v_1^2v_2^2\omega_3^2\omega_1^2 - 135\omega_2v_1^2v_2^2\omega_3^2\omega_1^3 + 18\omega_3^2v_1^2\omega_3\omega_1 + 54\omega_2^2v_2^2\omega_3\omega_1^2 + 54\omega_2^2v_4^2\omega_3\omega_1^3 - 42\omega_2^2\omega_3^2c_s^2\omega_1^2 - 60\omega_2^2\omega_3c_s^4\omega_1^2 - \omega_2^2v_1^2\omega_3^2\omega_1^3 - 30\omega_2\omega_3^2c_s^4\omega_1^2 + 18\omega_2^2v_4^2\omega_3\omega_1 + 9\omega_2^2v_1^2\omega_3^2c_s^2\omega_1 - \omega_2^2v_4^2\omega_3\omega_1^2 + 6\omega_2^2\omega_3 + 90\omega_2v_2^2\omega_3\omega_1^3 - 9\omega_2^2v_2^2\omega_3^2 - 2\omega_2^2\omega_3^2c_s^2\omega_1^3 + 24\omega_2^2\omega_3c_s^2\omega_1^3 - 6\omega_2^2\omega_3\omega_1 - 135\omega_2v_1^2v_2^2\omega_3^2\omega_1^2) \frac{\rho}{24\omega_3^2\omega_3^2\omega_1^3}$$

$$\text{coefficient } C_{D_x^2 D_y^2 v_2}^{(1)} \text{ at } \frac{\partial^4 v_2}{\partial x_1^4 \partial x_2^2} :$$

$$C_{D_x^2 D_y^2 v_2}^{(1), \text{SRT}} = 0$$

$$C_{D_x^2 D_y^2 v_2}^{(1), \text{MRT1}} = (-16\omega_8^2\omega_3^3\omega_9\omega_6c_s^2\omega_7^2\omega_2^2 - 4\omega_8^2\omega_5^2\omega_6\omega_7\omega_4^3 - \omega_8^2\omega_5^3v_1^2\omega_9\omega_6\omega_7^2\omega_4^2 - 24\omega_8\omega_5^3\omega_9\omega_6c_s^2\omega_7^2\omega_2^2 - 5\omega_8^2\omega_5^3\omega_9\omega_6c_s^2\omega_7\omega_4^3 + 4\omega_8^2\omega_5^2v_1^2\omega_6\omega_7\omega_4^3 - 6\omega_8^2\omega_5^2\omega_9\omega_6c_s^2\omega_7^2\omega_4 - 8\omega_8^2\omega_5^2\omega_9\omega_6c_s^2\omega_4^3 - 2\omega_8^2\omega_5^3\omega_9\omega_6\omega_7^2\omega_4 + 4\omega_8\omega_3^3\omega_6c_s^2\omega_7^2\omega_4^2 - 4\omega_8^2\omega_5^3v_1^2\omega_6\omega_7^2\omega_4^2 + 2\omega_8\omega_5^3\omega_9\omega_6\omega_7^2\omega_4^2 + 2\omega_8^2\omega_5^3\omega_9c_s^2\omega_7^2\omega_4^2 + 11\omega_8^2\omega_5^3\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 8\omega_8\omega_5^3\omega_9\omega_6c_s^2\omega_7^2\omega_4^3 + 12\omega_8^2\omega_9\omega_6c_s^2\omega_7^2\omega_4^3 - 15\omega_8^2\omega_5\omega_9\omega_6c_s^2\omega_7^2\omega_4^3 + 4\omega_8^2\omega_3^3v_1^2\omega_9\omega_6\omega_4^3 + 2\omega_8^2\omega_5^3v_1^2\omega_6\omega_7^2\omega_4^3 - 2\omega_8\omega_5^3\omega_6c_s^2\omega_7^2\omega_4^3 - \omega_8\omega_5^3\omega_9\omega_6\omega_7^2\omega_4^3 - 4\omega_8^2\omega_9\omega_6\omega_7^2\omega_4^3 - 4\omega_8^2\omega_5^3\omega_6\omega_7\omega_4^2 + 2\omega_8^2\omega_5^3\omega_9\omega_6\omega_7\omega_4^2 + 4\omega_8\omega_3^3\omega_9c_s^2\omega_7^2\omega_4^2 - 4\omega_8^2\omega_5^3\omega_9c_s^2\omega_7^2\omega_4 + 2\omega_8^2\omega_5^3\omega_6c_s^2\omega_7^2\omega_4^3 - 2\omega_8\omega_5^3v_1^2\omega_6\omega_7^2\omega_4^3 - 2\omega_8^2\omega_5^3v_1^2\omega_9\omega_6\omega_7\omega_4^2 + 3\omega_8^2\omega_5^3\omega_9\omega_6c_s^2\omega_7^2\omega_4^3 - 6\omega_8^2\omega_5^3\omega_9\omega_6c_s^2\omega_7\omega_4 + 4\omega_8^2\omega_5^3\omega_9\omega_6\omega_7\omega_4 + 4\omega_8^2\omega_9\omega_6c_s^2\omega_7^2\omega_4^2 - 2\omega_8\omega_5^3\omega_9c_s^2\omega_7^2\omega_4^3 + 4\omega_8^2\omega_5^3\omega_6\omega_7\omega_4^3 + 4\omega_8^2\omega_5^3\omega_6c_s^2\omega_7\omega_4^3 - 7\omega_8^2\omega_5^3\omega_9\omega_6\omega_7\omega_4^3 - 2\omega_8^2\omega_9\omega_6c_s^2\omega_7^2\omega_4^3 + 2\omega_8^2\omega_5^3v_1^2\omega_9\omega_6\omega_7^2\omega_4 + 12\omega_8\omega_5^3\omega_9\omega_6c_s^2\omega_7^2\omega_4 - \omega_8^2\omega_5^3\omega_9\omega_6c_s^2\omega_7^2\omega_4^2 + 4\omega_8\omega_5^3v_1^2\omega_6\omega_7^2\omega_4 - 4\omega_8^2\omega_5^3\omega_6\omega_7^2\omega_4^2 + 7\omega_8^2\omega_5^3v_1^2\omega_9\omega_6\omega_7\omega_4 + \omega_8^2\omega_5^3\omega_9\omega_6\omega_7^2\omega_4 - \omega_8^2\omega_5^3\omega_9\omega_6\omega_7^2\omega_4^2 - \omega_8^2\omega_5^3\omega_6\omega_7^2\omega_4^3 - 2\omega_8^2\omega_5^3\omega_6\omega_7^2\omega_4^3 - 2\omega_8^2\omega_5^3\omega_6c_s^2\omega_7^2\omega_4^3 - 4\omega_8^2\omega_5^3\omega_9\omega_6\omega_4^3 - 4\omega_8\omega_5^3\omega_6\omega_7^2\omega_4^2 - 6\omega_8^2\omega_5^3v_1^2\omega_9\omega_6\omega_7\omega_4 - 2\omega_8^2\omega_5^3\omega_9\omega_6\omega_7^2\omega_4^2 + \omega_8^2\omega_5^3v_1^2\omega_9\omega_6\omega_7^2\omega_4^3 - 9\omega_8^2\omega_5^3\omega_9\omega_6\omega_7\omega_4^2 + 4\omega_8^2\omega_5^3\omega_6c_s^2\omega_7\omega_4^2 + 2\omega_8^2\omega_5^3\omega_9\omega_6c_s^2\omega_7\omega_4^3 - 4\omega_8\omega_5^3\omega_9\omega_6c_s^2\omega_7^2\omega_4^3 + 4\omega_8^2\omega_5^3\omega_9\omega_6\omega_4^2 + 3\omega_8^2\omega_5^3\omega_9\omega_6\omega_7^2\omega_4^2 + 4\omega_8^2\omega_5^3\omega_6\omega_7^2\omega_4^2 + 8\omega_8\omega_5^3\omega_9\omega_6c_s^2\omega_7^2\omega_4^2 + 13\omega_8^2\omega_5^3\omega_9\omega_6c_s^2\omega_7\omega_4^3 + 3\omega_8^2\omega_5^3\omega_9\omega_6\omega_7\omega_4^3 - 3\omega_8^2\omega_5^3v_1^2\omega_9\omega_6\omega_7^2\omega_4^2 - 4\omega_8^2\omega_5^3\omega_6c_s^2\omega_7\omega_4^3 + 2\omega_8\omega_5^3\omega_6\omega_7^2\omega_4^3 + 5\omega_8^2\omega_5^3\omega_9\omega_6\omega_7^2\omega_4^3 + 26\omega_8^2\omega_5^3\omega_9\omega_6c_s^2\omega_7^2\omega_4 + 2\omega_8^2\omega_5^3v_1^2\omega_9\omega_6\omega_7^2\omega_4 - 3\omega_8^2\omega_5^3v_1^2\omega_9\omega_6\omega_7\omega_4^3 - 2\omega_8\omega_5^3v_1^2\omega_9\omega_6\omega_7^2\omega_4^2 - 8\omega_8^2\omega_5^3\omega_9\omega_6c_s^2\omega_7^2\omega_4^2 + 2\omega_8\omega_5^3v_1^2\omega_9\omega_6\omega_7^2\omega_4^3 - 4\omega_8^2\omega_5^3v_1^2\omega_6\omega_7\omega_4^3 - 2\omega_8\omega_5^2\omega_9\omega_6\omega_7^2\omega_4 - 8\omega_8^2\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4^3 + 8\omega_8^2\omega_5^3\omega_9\omega_6c_s^2\omega_4^3 + 4\omega_8^2\omega_5^3\omega_9\omega_6\omega_4^3 + \omega_8\omega_5^3v_1^2\omega_9\omega_6\omega_7^2\omega_4^3 + 9\omega_8^2\omega_5^3v_1^2\omega_9\omega_6\omega_7\omega_4^2 + 4\omega_8^2v_1^2\omega_9\omega_6\omega_7^2\omega_4^3 + 2\omega_8^2\omega_5^3\omega_6\omega_7^2\omega_4^3 - 4\omega_8^2\omega_5^2v_1^2\omega_9\omega_6\omega_4^3 + 6\omega_8^2\omega_5^3\omega_9\omega_6\omega_7\omega_4 - 2\omega_8^2\omega_5^3v_1^2\omega_6\omega_7^2\omega_4^3 - 8\omega_8^2\omega_5^3\omega_9\omega_6c_s^2\omega_4^2 - 5\omega_8^2\omega_5^3v_1^2\omega_9\omega_6\omega_7^2\omega_4^3 + 4\omega_8^2\omega_5^3v_1^2\omega_6\omega_7\omega_4^2) \frac{v_1\rho v_2}{2\omega_8^2\omega_5^3\omega_9\omega_6\omega_7^2\omega_4^3}$$

$$C_{D_x^2 D_y^2 v_2}^{(1), \text{MRT2}} = C_{D_x^2 D_y^2 v_2}^{(1), \text{MRT1}}$$

$$C_{D_x^2 D_y^2 v_2}^{(1), \text{CLBM1}} = 0$$

$$C_{D_x^2 D_y^2 v_2}^{(1), \text{CLBM2}} = 0$$

$$C_{D_x^2 D_y^2 v_2}^{(1), \text{CuLBM1}} = 0$$

$$C_{D_x^2 D_y^2 v_2}^{(1), \text{CuLBM2}} = (135\omega_2^3\omega_3c_s^2\omega_1 - 216\omega_2v_2^2\omega_3\omega_1^3 - 18\omega_2^2\omega_3\omega_1 - 18\omega_3^2v_1^2\omega_1 + 162\omega_2^2\omega_3c_s^2\omega_1^2 - 162\omega_2v_2^2\omega_3\omega_1^2 + 54\omega_2^2v_1^2\omega_3\omega_1^2 + 108\omega_2^2c_s^2\omega_1^2 - 18\omega_2v_1^2\omega_1^3 - 27\omega_2^2v_1^2\omega_3\omega_1 + 18\omega_2\omega_1^3 + 84\omega_2^2\omega_3c_s^2\omega_1^3 - 54\omega_2^3\omega_3c_s^2 - 46\omega_2^2\omega_3\omega_1^3 + 36\omega_2^3v_1^2\omega_3 - 36\omega_2^2v_1^2\omega_3\omega_1 + 270\omega_3c_s^2\omega_1^3 - 54\omega_2^2\omega_3\omega_1^2 + 36v_1^2\omega_3\omega_1^3 - 54\omega_2^2\omega_3c_s^2\omega_1 - 84\omega_2^3\omega_3c_s^2\omega_1 - 126\omega_3\omega_1^3 + 135\omega_2\omega_3\omega_1^3 - 162\omega_2\omega_3c_s^2\omega_1^2 + 100\omega_2^2v_2^2\omega_3\omega_1^3 - 198\omega_2^3v_2^2\omega_3 - 27\omega_2v_1^2\omega_3\omega_1^3 + 54\omega_2^3\omega_3 - 297\omega_2\omega_3c_s^2\omega_1^3 + 216\omega_2^3v_2^2\omega_3\omega_1 - 81\omega_3^3\omega_3\omega_1 + 90\omega_2^3\omega_3\omega_1^2 - 36\omega_2v_1^2\omega_3\omega_1^2 + 162\omega_2^2v_2^2\omega_3\omega_1 - 100\omega_3^2v_2^2\omega_3\omega_1^2 + 46\omega_2^3\omega_3\omega_1^2 + 36\omega_2^2v_1^2\omega_1^2 -$$

$$54\omega_2 c_s^2 \omega_1^3 + 18\omega_2^3 \omega_1 - 36\omega_2^2 \omega_1^2 - 54\omega_2^3 c_s^2 \omega_1 + 198v_2^2 \omega_3 \omega_1^3) \frac{v_1 \rho v_2}{24\omega_2^2 \omega_3 \omega_1^3}$$

$$\text{coefficient } C_{D_x D_y^3 \rho}^{(1)} \text{ at } \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} :$$

$$C_{D_x D_y^3 \rho}^{(1), \text{SRT}} = (24 - 42c_s^2\omega^2 + v_2^2\omega^3 - 14v_2^2\omega^2 + 3c_s^2\omega^3 + 36v_2^2\omega - 72c_s^2\omega + 108c_s^2\omega - 24v_2^2 - 36\omega + 14\omega^2 - \omega^3) \frac{v_2 c_s^2}{12\omega^3}$$

$$\begin{aligned} C_{D_x D_y^3 \rho}^{(1), \text{MRT1}} = & (6\omega_8^2 v_1^2 \omega_9 \omega_6 \omega_7 \omega_4^3 - 12\omega_8 \omega_5 \omega_9 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4 + 72\omega_2^2 \omega_5 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4 - 72\omega_8 \omega_5 v_1^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4 + 3\omega_2^2 \omega_5 \omega_9 \omega_6^2 c_s^4 \omega_7 \omega_4 + \\ & 24\omega_2^2 v_1^2 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4^2 - 24\omega_8 v_1^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4^2 - 12\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4 - 18\omega_8^2 \omega_6^2 c_s^4 \omega_7 \omega_4^3 + 9\omega_2^2 \omega_5 v_1^2 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4^3 - 36\omega_8^2 \omega_9 \omega_6 c_s^4 \omega_4^3 - \\ & 12\omega_8^2 \omega_5 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4 - 6\omega_2^2 \omega_5 v_1^2 \omega_9 v_2^2 \omega_7 \omega_4^3 - 6\omega_8 \omega_5 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - 72\omega_8 v_1^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 + 12\omega_8^2 \omega_5 \omega_9 \omega_6 c_s^2 \omega_4^2 - 24\omega_8 \omega_5 v_1^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4 + \\ & 12\omega_8^2 v_1^2 v_2^2 \omega_6 \omega_4^3 + 36\omega_8^2 v_1^2 \omega_6^2 c_s^2 \omega_4^3 - 9\omega_2^2 \omega_5 v_1^2 \omega_9 \omega_6 \omega_7 \omega_4^3 - 12\omega_8^2 \omega_5 \omega_6^2 c_s^2 \omega_4^3 + 15\omega_8 \omega_5 v_1^2 \omega_9 \omega_6 \omega_7 \omega_4^3 + 12\omega_8 v_1^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4^3 - \\ & 6\omega_8^2 v_1^2 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4^3 - 60\omega_8^2 \omega_5 \omega_9 \omega_6^2 c_s^4 \omega_7 \omega_4^2 - 24\omega_8^2 v_1^2 \omega_9 \omega_6 \omega_7 \omega_4^2 - 48\omega_8 \omega_5 v_1^2 \omega_9 \omega_6^2 \omega_7 \omega_4^2 + 12\omega_8^2 v_2^2 \omega_6^2 c_s^2 \omega_4^3 + 36\omega_8^2 \omega_5 v_1^2 \omega_9 \omega_6 \omega_7 \omega_4^2 + \\ & 12\omega_8^2 \omega_5 \omega_6^2 c_s^2 \omega_4^3 + 36\omega_8 v_1^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + 12\omega_8^2 \omega_5 v_1^2 \omega_9 v_2^2 \omega_7 \omega_4^2 + 12\omega_8 \omega_5 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^2 - 36\omega_2^2 \omega_5 v_1^2 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4^2 - 12\omega_8^2 \omega_5 \omega_9 \omega_6 c_s^2 \omega_4^3 + \\ & 24\omega_8 \omega_5 v_1^2 \omega_9 \omega_6^2 \omega_7 \omega_4 + 72\omega_8^2 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4^2 - 6\omega_5 v_1^2 \omega_9 \omega_6^2 \omega_7 \omega_4^3 - 24\omega_8^2 \omega_5 v_1^2 \omega_9 \omega_6 \omega_7 \omega_4 - 12\omega_2^2 v_1^2 \omega_9 v_2^2 \omega_6 \omega_4^3 - 18\omega_8^2 \omega_5 v_1^2 \omega_9 c_s^2 \omega_7 \omega_4^3 - \\ & 18\omega_8 \omega_5 v_1^2 \omega_9 c_s^2 \omega_7 \omega_4^3 - 15\omega_8 \omega_5 v_1^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4^3 + 6\omega_8^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + 24\omega_2^2 \omega_5 v_1^2 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4 - \omega_8^2 \omega_5 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - 5\omega_2^2 \omega_5 \omega_9 v_2^2 \omega_6 c_s^2 \omega_7 \omega_4^3 + \\ & 36\omega_2^2 \omega_6^2 c_s^4 \omega_4^3 + 156\omega_8^2 \omega_5 \omega_9 \omega_6^2 c_s^4 \omega_7 \omega_4 - 36\omega_2^2 \omega_5 v_1^2 \omega_9 \omega_6 c_s^2 \omega_4^3 + 12\omega_2^2 v_1^2 \omega_9 \omega_6 \omega_4^3 - 6\omega_8 \omega_5 v_1^2 v_2^2 \omega_6^2 \omega_7 \omega_4^3 - 45\omega_8 \omega_5 v_1^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + \\ & 27\omega_8^2 \omega_5 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4^3 + 12\omega_8^2 \omega_5 v_1^2 \omega_9 v_2^2 \omega_6 \omega_4^3 - 3\omega_8 \omega_5 \omega_9 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - 36\omega_8^2 v_1^2 \omega_9 \omega_6 c_s^2 \omega_4^3 + 12\omega_8^2 \omega_5 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 + 6\omega_8^2 \omega_5 v_2^2 \omega_6 c_s^2 \omega_7 \omega_4^3 + \\ & 48\omega_8 \omega_5 v_1^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4^2 + 36\omega_8 \omega_5 v_1^2 \omega_9 c_s^2 \omega_7 \omega_4^2 - 12\omega_8^2 v_1^2 \omega_6^2 \omega_4^3 + 12\omega_5 v_2^2 \omega_9 \omega_7 \omega_4^2 + 36\omega_2^2 \omega_5 v_1^2 \omega_9 c_s^2 \omega_7 \omega_4^2 - 18\omega_2^2 v_1^2 \omega_9 c_s^2 \omega_7 \omega_4^3 - \\ & 12\omega_8^2 \omega_5 v_1^2 \omega_9 v_2^2 \omega_6 \omega_4^2 + 18\omega_8 \omega_5 \omega_9 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^2 - 108\omega_2^2 \omega_5 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4^2 + 36\omega_8^2 \omega_5 v_1^2 \omega_9 \omega_6 c_s^2 \omega_4^3 + 6\omega_2^2 v_1^2 \omega_9 \omega_7 \omega_4^3 + 144\omega_8 \omega_5 v_1^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 + \\ & 12\omega_8 \omega_5 v_1^2 v_2^2 \omega_6^2 \omega_7 \omega_4^2 + 18\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_6 c_s^2 \omega_7 \omega_4^2 + 18\omega_8^2 \omega_5 v_1^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + 18\omega_2^2 \omega_5 \omega_6^2 c_s^4 \omega_7 \omega_4^3 - 12\omega_8^2 \omega_5 v_1^2 \omega_6^2 \omega_4^2 + 3\omega_8 \omega_5 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + \\ & 24\omega_8 v_1^2 \omega_9 \omega_6^2 \omega_7 \omega_4^2 - 6\omega_2^2 v_1^2 v_2^2 \omega_6^2 \omega_7 \omega_4^3 + 6\omega_8 \omega_5 v_1^2 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4^3 + 18\omega_8^2 \omega_9 \omega_6 c_s^4 \omega_7 \omega_4^3 + 12\omega_8^2 \omega_5 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4 - 12\omega_8 \omega_5 \omega_6^2 c_s^2 \omega_7 \omega_4^2 + \\ & 6\omega_8 \omega_5 v_1^2 \omega_6^2 \omega_7 \omega_4^3 - 15\omega_8^2 \omega_5 \omega_9 \omega_6 c_s^4 \omega_7 \omega_4^3 + 6\omega_8^2 \omega_5 v_1^2 \omega_9 \omega_7 \omega_4^3 - 36\omega_8^2 \omega_5 \omega_6^2 c_s^4 \omega_4^3 - 36\omega_2^2 \omega_5 v_1^2 \omega_6^2 \omega_4^3 + 12\omega_8^2 \omega_5 v_2^2 \omega_6^2 c_s^2 \omega_4^3 + \\ & 12\omega_8^2 \omega_5 \omega_9 v_2^2 c_s^2 \omega_7 \omega_4 - 6\omega_5 \omega_9 \omega_6^2 c_s^4 \omega_7 \omega_4^3 - 18\omega_8^2 v_1^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - 12\omega_2^2 \omega_5 v_1^2 v_2^2 \omega_6^2 \omega_4^3 + 36\omega_8^2 \omega_5 \omega_9 \omega_6 c_s^4 \omega_4^3 + 6\omega_8^2 \omega_5 v_1^2 v_2^2 \omega_6^2 \omega_7 \omega_4^3 + \\ & 12\omega_8 \omega_5 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4 + 6\omega_8 \omega_5 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - 12\omega_8 v_1^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - 18\omega_8 \omega_5 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 - 12\omega_8 \omega_5 v_1^2 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4^3 - 36\omega_8^2 \omega_5 v_1^2 \omega_6^2 c_s^2 \omega_7 \omega_4^2 - \\ & 36\omega_8^2 \omega_5 \omega_9 c_s^4 \omega_7 \omega_4^2 + 12\omega_8^2 \omega_5 v_1^2 \omega_6^2 \omega_4^3 - 12\omega_8^2 \omega_5 v_1^2 v_2^2 \omega_6^2 \omega_7 \omega_4^2 - 36\omega_8^2 \omega_5 \omega_9 \omega_6 c_s^4 \omega_4^3 + 12\omega_8^2 \omega_9 \omega_6 c_s^2 \omega_4^3 + 12\omega_5 \omega_9 \omega_6^2 c_s^4 \omega_7 \omega_4^2 + 12\omega_8^2 \omega_5 v_2^2 v_2^2 \omega_6^2 \omega_4^2 + \\ & 36\omega_8^2 \omega_5 v_1^2 \omega_6^2 \omega_4^3 - 12\omega_8^2 \omega_5 v_2^2 \omega_6^2 c_s^2 \omega_4^3 + 36\omega_2^2 \omega_5 \omega_6^2 c_s^4 \omega_4^3 - 12\omega_8^2 \omega_5 v_1^2 \omega_9 \omega_7 \omega_4^2 + 54\omega_8^2 \omega_5 \omega_9 \omega_6 c_s^4 \omega_7 \omega_4^2 - 12\omega_8 \omega_5 v_1^2 \omega_6^2 \omega_7 \omega_4^2 - 6\omega_8^2 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + \\ & 15\omega_8^2 \omega_5 \omega_9 \omega_6^2 c_s^4 \omega_7 \omega_4^3 - 12\omega_5 \omega_9 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^2 - 6\omega_8^2 \omega_5 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + \omega_8^2 \omega_5 \omega_9 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - 36\omega_5 v_1^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 + 36\omega_8 \omega_5 \omega_6^2 c_s^4 \omega_7 \omega_4^2 - \\ & 36\omega_2^2 \omega_5 \omega_9 \omega_6 c_s^4 \omega_7 \omega_4 + 6\omega_2^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + 18\omega_8 \omega_5 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4^3 - 6\omega_8 \omega_5 v_1^2 \omega_9 \omega_6 \omega_7 \omega_4^3 - 12\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_6 c_s^2 \omega_4^2 + 5\omega_8^2 \omega_5 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4^3 - \\ & 12\omega_5 v_1^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4^2 - 12\omega_8^2 \omega_5 v_2^2 \omega_9 \omega_6 \omega_4^3 + 12\omega_2^2 \omega_5 v_1^2 \omega_6^2 \omega_7 \omega_4^2 + 12\omega_8 \omega_5 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4 + 6\omega_8^2 \omega_5 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - 6\omega_5 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - \\ & 36\omega_8 \omega_5 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4^2 - 12\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_6 c_s^2 \omega_7 \omega_4^2 - 12\omega_8^2 \omega_9 v_2^2 \omega_6 c_s^2 \omega_4^3 - 18\omega_8 \omega_5 \omega_6^2 c_s^4 \omega_7 \omega_4^3 + 18\omega_5 v_1^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + 12\omega_2^2 \omega_5 \omega_6^2 c_s^2 \omega_7 \omega_4^2 - \\ & 42\omega_8 \omega_5 \omega_9 \omega_6^2 c_s^4 \omega_7 \omega_4^2 + 6\omega_5 \omega_9 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - 12\omega_2^2 \omega_6^2 c_s^2 \omega_4^3 + 12\omega_5 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 - 96\omega_2^2 \omega_5 \omega_9 \omega_6^2 c_s^4 \omega_7 + 12\omega_2^2 \omega_5 v_1^2 \omega_9 \omega_6 \omega_4^2 - 12\omega_2^2 \omega_5 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^2 - \\ & 6\omega_8^2 \omega_5 v_1^2 \omega_6^2 \omega_7 \omega_4^3 + 6\omega_5 v_1^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4^3 - 18\omega_8^2 \omega_5 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + 12\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_6 c_s^2 \omega_4^3 + 12\omega_8 \omega_5 v_1^2 \omega_9 \omega_6 \omega_7 \omega_4^2) \frac{v_2 c_s^2}{12\omega_8^2 \omega_5 \omega_9 \omega_6 \omega_7 \omega_4^3} \end{aligned}$$

$$C_{D_x D_y^3 \rho}^{(1), \text{MRT2}} = C_{D_x D_y^3 \rho}^{(1), \text{MRT1}}$$

$$\begin{aligned} C_{D_x D_y^3 \rho}^{(1), \text{CLBM1}} = & (-36\omega_2^2 \omega_5 \omega_6 c_s^2 \omega_7 \omega_4 + 12\omega_8^2 \omega_5 \omega_9 \omega_6 \omega_7 \omega_4 + 18\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_7 \omega_4 + 18\omega_8 \omega_5 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4 - 36\omega_2^2 \omega_5 \omega_9 c_s^2 \omega_4 + 12\omega_8^2 \omega_5 \omega_9 \omega_4 + \\ & 36\omega_2^2 \omega_6 c_s^2 \omega_4^2 + 3\omega_8 \omega_5 \omega_9 \omega_6 \omega_7 \omega_4 - 18\omega_8 \omega_5 \omega_6 c_s^2 \omega_7 \omega_4 + 5\omega_2^2 \omega_5 \omega_9 \omega_7 \omega_4 - 36\omega_5 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4 - 12\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_4 + 6\omega_5 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4 - \\ & 15\omega_8^2 \omega_5 \omega_9 c_s^2 \omega_7 \omega_4 + 12\omega_8 \omega_5 v_2^2 \omega_6 \omega_7 \omega_4 - 9\omega_8 \omega_5 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4 + 12\omega_2^2 v_2^2 \omega_6 \omega_4 + 6\omega_8^2 \omega_5 v_2^2 \omega_6 \omega_7 \omega_4 + 54\omega_8 \omega_5 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4 - 12\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4 - \\ & 12\omega_5 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4 + 12\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_4 - 12\omega_2^2 \omega_6 \omega_4 - 6\omega_8 \omega_5 v_2^2 \omega_6 \omega_7 \omega_4 + 54\omega_2^2 \omega_5 \omega_9 c_s^2 \omega_7 \omega_4 - 12\omega_8^2 \omega_5 \omega_9 \omega_4 + 36\omega_8 \omega_5 \omega_6 c_s^2 \omega_7 \omega_4 - \\ & 18\omega_8 \omega_5 \omega_9 \omega_6 \omega_7 \omega_4 + 18\omega_5 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4 - 18\omega_8^2 \omega_5 \omega_9 \omega_7 \omega_4 - \omega_8^2 \omega_5 \omega_9 \omega_6 \omega_7 \omega_4 + 18\omega_8^2 \omega_5 \omega_6 c_s^2 \omega_7 \omega_4 + 6\omega_2^2 \omega_6 \omega_7 \omega_4 + 36\omega_2^2 \omega_5 \omega_9 c_s^2 \omega_4 - \\ & 3\omega_8 \omega_5 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4 - 5\omega_2^2 \omega_5 \omega_9 v_2^2 \omega_7 \omega_4 + 12\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_6 \omega_4 + 6\omega_8 \omega_5 \omega_6 \omega_7 \omega_4 - 36\omega_8^2 \omega_5 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4 - 12\omega_2^2 \omega_9 v_2^2 \omega_4 + \\ & 36\omega_2^2 \omega_5 \omega_9 \omega_6 c_s^2 \omega_7 + 18\omega_8^2 \omega_9 c_s^2 \omega_7 \omega_4 + 36\omega_2^2 \omega_5 \omega_6 c_s^2 \omega_4 + \omega_8^2 \omega_5 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4 + 12\omega_8^2 \omega_5 \omega_6 \omega_4 - 6\omega_8^2 \omega_5 \omega_6 \omega_7 \omega_4 - 6\omega_8^2 v_2^2 \omega_6 \omega_7 \omega_4 - \\ & 6\omega_5 \omega_9 \omega_6 \omega_7 \omega_4 + 12\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_6 \omega_7 - 36\omega_8^2 \omega_9 c_s^2 \omega_4^2 - 12\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4 + 12\omega_8^2 \omega_5 \omega_6 \omega_7 \omega_4 - 12\omega_8^2 \omega_5 \omega_6 \omega_4 + 12\omega_5 \omega_9 \omega_6 \omega_7 \omega_4 - \\ & 36\omega_2^2 \omega_5 \omega_6 c_s^2 \omega_4^2 - 12\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_7 - 12\omega_8 \omega_5 \omega_9 v_2^2 \omega_6 \omega_7 - 6\omega_8^2 \omega_9 \omega_7 \omega_4 - 12\omega_8^2 \omega_5 \omega_9 \omega_6 \omega_7 + 12\omega_8^2 \omega_5 \omega_9 \omega_7 + 12\omega_8 \omega_5 \omega_9 \omega_6 \omega_7 - 36\omega_8^2 \omega_5 \omega_9 c_s^2 \omega_7 - \\ & 12\omega_2^2 \omega_5 v_2^2 \omega_6 \omega_4^2 - 12\omega_8 \omega_5 \omega_6 \omega_7 \omega_4 + 6\omega_2^2 \omega_9 v_2^2 \omega_7 \omega_4 + 3\omega_2^2 \omega_5 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4 - 18\omega_8^2 \omega_6 c_s^2 \omega_7 \omega_4 - 36\omega_8 \omega_5 \omega_9 \omega_6 c_s^2 \omega_7) \frac{v_2 c_s^2}{12\omega_8^2 \omega_5 \omega_9 \omega_6 \omega_7 \omega_4^2} \end{aligned}$$

$$C_{D_x D_y^3 \rho}^{(1), \text{CLBM2}} = C_{D_x D_y^3 \rho}^{(1), \text{CLBM1}}$$

$$\begin{aligned} C_{D_x D_y^3 \rho}^{(1), \text{CuLBM1}} = & (v_2^2 \omega_6^2 \omega_3^2 + 12\omega_2 \omega_6^2 \omega_3 - 3\omega_2 v_2^2 \omega_6 \omega_3^2 - 12\omega_6 \omega_3 + 12\omega_2 \omega_6 + 12\omega_2 \omega_3 - 36\omega_2 \omega_3 c_s^2 - 36\omega_2 \omega_6 c_s^2 + 36\omega_6 \omega_3 c_s^2 - 36\omega_6^2 c_s^2 - 6\omega_2 \omega_3^2 - \\ & 12v_2^2 \omega_6^2 - 9\omega_2 \omega_6 \omega_3^2 c_s^2 + 3\omega_6^2 \omega_3^2 c_s^2 + 18\omega_2 v_2^2 \omega_6 \omega_3 - 12\omega_2 \omega_6^2 + 6\omega_6 \omega_3^2 + 6v_2^2 \omega_6^2 \omega_3 - \omega_2 \omega_6^2 \omega_3^2 - 36\omega_2 \omega_6^2 \omega_3 c_s^2 + 18\omega_2 \omega_3 c_s^2 - \omega_6^2 \omega_3^2 - \\ & 12\omega_2 v_2^2 \omega_6^2 + 3\omega_2 \omega_6 \omega_3^2 + 6\omega_2 v_2^2 \omega_6 \omega_3 + 12\omega_2 \omega_6 \omega_3 + 3\omega_2 \omega_6^2 \omega_3^2 c_s^2 + 12\omega_2 v_2^2 \omega_6^2 - 18\omega_6 \omega_3^2 c_s^2 + 36\omega_2 \omega_6^2 c_s^2 + 54\omega_2 \omega_6 \omega_3 c_s^2 + 12\omega_6^2 + 18\omega_2 \omega_3^2 c_s^2 - \\ & 12\omega_2 v_2^2 \omega_6^2 - 18\omega_2 \omega_6 \omega_3 - 12\omega_2 v_2^2 \omega_3 - 6v_2^2 \omega_6 \omega_3^2 - 6\omega_6^2 \omega_3 + \omega_2 v_2^2 \omega_6^2 \omega_3^2) \frac{v_2 c_s^2}{12\omega_2 \omega_6^2 \omega_3^2} \end{aligned}$$

$$\begin{aligned} C_{D_x D_y^3 \rho}^{(1), \text{CuLBM2}} = & (18\omega_2 \omega_3 c_s^4 \omega_1^3 + 90\omega_2^3 v_4^2 \omega_3^2 \omega_1 - 30\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1 - 90\omega_2^2 \omega_3^2 c_s^4 \omega_1 + 30\omega_2^3 \omega_3 c_s^2 \omega_1 + 36\omega_2^3 c_s^4 \omega_1^3 - 165\omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_1 - 51v_2^2 \omega_3^2 \omega_1^3 - \\ & 24\omega_2^3 \omega_3^2 c_s^2 \omega_1^3 - 138\omega_2^3 \omega_3^2 c_s^4 \omega_1^3 - 72\omega_2^3 c_s^4 \omega_1^3 + 51\omega_2^3 v_2^2 \omega_3^2 \omega_1 + 90\omega_2^3 c_s^4 \omega_1^3 + 39\omega_2^3 v_4^2 \omega_3^2 \omega_1^3 - 7\omega_2^3 \omega_3^2 \omega_1^3 + 46\omega_2^3 v_2^2 \omega_3^2 \omega_1^3 + 18\omega_2^3 \omega_3^2 c_s^2 \omega_1^3 + 6\omega_2^3 \omega_3^2 c_s^4 \omega_1^3 + \\ & 12\omega_2^3 \omega_3^2 \omega_1 - 24\omega_2 \omega_3^2 c_s^2 \omega_1^3 - 102\omega_2^3 v_2^2 \omega_3^2 \omega_1 + 6\omega_2 \omega_3^2 \omega_1^3 - 45\omega_2^3 v_4^2 \omega_3^2 + 197\omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_1^3 + 141\omega_2 v_2^2 \omega_3^2 c_s^2 \omega_1^2 + 6\omega_2^3 \omega_3^2 c_s^2 \omega_1^3 + 45v_2^4 \omega_3^2 \omega_1^3 + 72\omega_2^3 \omega_3^2 c_s^4 \omega_1^3 - \\ & 6\omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_1^3 + 9\omega_2^3 v_2^2 \omega_3^2 c_s^4 \omega_1^3 + 12\omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_1^2 + 225\omega_2^3 \omega_3^2 c_s^4 \omega_1 - 12\omega_2 \omega_3^2 \omega_1^3 + 123\omega_2 \omega_3^2 c_s^2 \omega_1^3 - 45\omega_2^3 v_4^2 \omega_3^2 \omega_1 - 39\omega_2^3 v_2^2 \omega_3^2 \omega_1^2 - 46\omega_2^3 v_2^2 \omega_3^2 \omega_1^3 + \\ & 42\omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_1^2 - 42\omega_2^3 \omega_3^2 c_s^2 \omega_1^2 + 36\omega_2^3 \omega_3^2 c_s^4 \omega_1^2 - 465\omega_2 v_2^2 \omega_3^2 c_s^2 \omega_1^3 - 90\omega_2^3 \omega_3^2 c_s^4 \omega_1 + 24\omega_2^3 c_s^2 \omega_1^2 + 261v_2^2 \omega_3^2 c_s^2 \omega_1^3 - 18\omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_1^3 + 45\omega_2 v_2^2 \omega_3^2 \omega_1^2 - \end{aligned}$$

$$\begin{aligned}
& 36\omega_5^2\omega_1^2v_2^2\omega_6^2\omega_4^3 + 12\omega_8^2\omega_5\omega_9\omega_6c_s^4\omega_1^3 + 18\omega_8^2\omega_5v_1^2v_2^2\omega_6^2\omega_7\omega_4^3 + 12\omega_8\omega_5\omega_9\omega_6^2c_s^4\omega_7\omega_4 + 6\omega_8\omega_5\omega_6^2c_s^2\omega_7\omega_4^3 - 12\omega_8v_1^2\omega_9\omega_6^2\omega_7\omega_4^3 + \\
& 18\omega_8\omega_5\omega_9\omega_6^2c_s^2\omega_7\omega_4^2 - 36\omega_8\omega_5v_1^2\omega_9v_2^2\omega_6\omega_7\omega_4^2 - 12\omega_8^2\omega_5v_1^2\omega_6^2c_s^2\omega_7\omega_4^2 - 12\omega_8^2\omega_5\omega_6^2c_s^4\omega_7\omega_4^2 + 12\omega_8^2\omega_5v_1^2\omega_6^2\omega_4^3 - 36\omega_8^2\omega_5v_1^2v_2^2\omega_6^2\omega_7\omega_4^2 - \\
& 12\omega_8^2\omega_5\omega_9\omega_6c_s^4\omega_4^2 + 12\omega_8^2\omega_9\omega_6c_s^2\omega_4^3 + 36\omega_8^2\omega_5v_1^2v_2^2\omega_6^2\omega_4^2 + 12\omega_8^2\omega_5v_1^2\omega_6^2c_s^2\omega_4^2 - 36\omega_8^2\omega_5v_2^2\omega_6^2c_s^2\omega_4^3 + 12\omega_8^2\omega_5\omega_6^2c_s^4\omega_4^2 - 12\omega_8^2\omega_5v_1^2\omega_9\omega_7\omega_4^2 + \\
& 18\omega_8^2\omega_5\omega_9\omega_6c_s^4\omega_7\omega_4^2 - 12\omega_8\omega_5v_1^2\omega_6^2\omega_7\omega_4^2 - 18\omega_8^2v_2^2\omega_6^2c_s^2\omega_7\omega_4^3 + 6\omega_8\omega_5\omega_9\omega_6^2c_s^4\omega_7\omega_4^3 + 24\omega_5\omega_9v_2^2\omega_6^2c_s^2\omega_7\omega_4^2 - 6\omega_8^2\omega_5\omega_6^2c_s^2\omega_7\omega_4^3 - \\
& 12\omega_5v_1^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^2 + 12\omega_8\omega_5\omega_6^2c_s^4\omega_7\omega_4^2 - 12\omega_8^2\omega_5\omega_9\omega_6c_s^4\omega_7\omega_4 - 6\omega_8^2\omega_9\omega_6c_s^2\omega_7\omega_4^3 + 6\omega_8\omega_5v_1^2\omega_9\omega_6c_s^2\omega_7\omega_4^3 - 6\omega_8\omega_5v_1^2\omega_9\omega_6\omega_7\omega_4^3 - \\
& 36\omega_8^2\omega_5\omega_9v_2^2\omega_6c_s^2\omega_4^2 + 5\omega_8^2\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4^3 - 36\omega_5v_1^2\omega_9v_2^2\omega_6^2\omega_7\omega_4^2 - 12\omega_8^2\omega_5v_1^2\omega_9\omega_6\omega_4^3 + 12\omega_8^2\omega_5v_1^2\omega_6^2\omega_7\omega_4^2 - 12\omega_8\omega_5\omega_9\omega_6^2c_s^2\omega_7\omega_4 + \\
& 18\omega_8^2\omega_5v_2^2\omega_6^2c_s^2\omega_7\omega_4^2 - 12\omega_8\omega_5v_2^2\omega_9\omega_6c_s^2\omega_7\omega_4^2 - 15\omega_8^2\omega_5\omega_9v_2^2\omega_6^2c_s^2\omega_7\omega_4^2 - 36\omega_8^2\omega_9v_2^2\omega_6c_s^2\omega_4^3 - 6\omega_8\omega_5\omega_6^2c_s^4\omega_7\omega_4^3 + 6\omega_5v_1^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^3 + \\
& 12\omega_8^2\omega_5\omega_6^2c_s^2\omega_7\omega_4^2 - 18\omega_8\omega_5\omega_9\omega_6^2c_s^4\omega_7\omega_4^2 - 12\omega_5\omega_9v_2^2\omega_6^2c_s^2\omega_7\omega_4^2 - 12\omega_8^2\omega_6^2c_s^2\omega_4^3 - 12\omega_8^2\omega_5\omega_9\omega_6^2c_s^4\omega_7 + 12\omega_8^2\omega_5v_1^2\omega_9\omega_6\omega_4^2 - 36\omega_8^2\omega_5v_2^2\omega_6^2c_s^2\omega_7\omega_4^2 - \\
& 6\omega_8^2\omega_5v_1^2\omega_6^2\omega_7\omega_4^3 + 18\omega_5v_1^2\omega_9v_2^2\omega_6^2\omega_7\omega_4^3 - 18\omega_8^2\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 36\omega_8^2\omega_5\omega_9v_2^2\omega_6c_s^2\omega_4^3 + 12\omega_8\omega_5v_1^2\omega_9\omega_6\omega_7\omega_4^3) \frac{\rho}{12\omega_8^2\omega_5\omega_9\omega_6^2\omega_7\omega_4^3}
\end{aligned}$$

$$C_{D_x D_y^3 v_2}^{(1), \text{MRT}2} = C_{D_x D_y^3 v_2}^{(1), \text{MRT}1}$$

$$\begin{aligned}
C_{D_x D_y^3 v_2}^{(1), \text{CLBM}1} &= (12\omega_8\omega_5\omega_6\omega_4^3 + 5\omega_8\omega_5\omega_9\omega_7\omega_4^3 - 12\omega_8\omega_9c_s^2\omega_4^3 + 18\omega_5\omega_9v_2^2\omega_6\omega_7\omega_4^3 + \omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^2 - 12\omega_8\omega_5\omega_6c_s^2\omega_7\omega_4^2 + 12\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4 - \\
& \omega_8\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4^3 + 54\omega_8\omega_5\omega_9v_2^2\omega_7\omega_4^2 - 54\omega_5\omega_9v_2^2\omega_6\omega_7\omega_4^2 - 36\omega_8\omega_9v_2^2\omega_4^3 - 18\omega_8\omega_5\omega_9\omega_7\omega_4^2 - 12\omega_8\omega_5\omega_6\omega_4^2 - 5\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4^2 - \\
& 15\omega_8\omega_5\omega_9v_2^2\omega_7\omega_4^3 - 12\omega_8\omega_5\omega_9c_s^2\omega_7\omega_4^3 + 6\omega_8\omega_5\omega_6c_s^2\omega_7\omega_4^3 + 18\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4 + 6\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4^3 + 18\omega_8\omega_5\omega_9c_s^2\omega_7\omega_4^2 - 12\omega_8\omega_5\omega_9c_s^2\omega_4^2 + \\
& 36\omega_8\omega_5\omega_9v_2^2\omega_4^3 + 36\omega_5\omega_9v_2^2\omega_6\omega_7\omega_4 + 12\omega_8\omega_5\omega_9\omega_7\omega_4 - 36\omega_8\omega_5v_2^2\omega_6\omega_7\omega_4^2 + 12\omega_8\omega_9\omega_4^3 - 36\omega_8\omega_5\omega_9v_2^2\omega_4^2 - 5\omega_8\omega_5\omega_9c_s^2\omega_7\omega_4^3 + 12\omega_8\omega_5\omega_9c_s^2\omega_4^3 - \\
& 6\omega_8\omega_9\omega_7\omega_4^3 - 18\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4^2 - 36\omega_8\omega_5\omega_9v_2^2\omega_7\omega_4 + 18\omega_8\omega_5v_2^2\omega_6\omega_7\omega_4^3 - 3\omega_8\omega_5\omega_9v_2^2\omega_6\omega_7\omega_4^2 + 36\omega_8v_2^2\omega_6\omega_4^3 + 12\omega_8\omega_5\omega_6\omega_7\omega_4^2 - \\
& 6\omega_5\omega_9\omega_6\omega_7\omega_4^3 - 6\omega_5\omega_6c_s^2\omega_7\omega_4^3 + 12\omega_8\omega_6c_s^2\omega_4^3 - 18\omega_8v_2^2\omega_6\omega_7\omega_4^3 - 6\omega_8\omega_5\omega_6\omega_7\omega_4^3 - 12\omega_8\omega_6\omega_4^3 + 18\omega_5\omega_9\omega_6\omega_7\omega_4^2 + 12\omega_5\omega_6c_s^2\omega_7\omega_4^2 + 6\omega_8\omega_9c_s^2\omega_7\omega_4^3 + \\
& 36\omega_8\omega_5v_2^2\omega_6\omega_4^2 + 6\omega_8\omega_6\omega_7\omega_4^3 - 12\omega_8\omega_5\omega_6c_s^2\omega_4^3 - 12\omega_5\omega_9\omega_6\omega_7\omega_4 - 12\omega_5\omega_6\omega_7\omega_4^2 - 12\omega_8\omega_5\omega_9\omega_4^3 - 18\omega_5v_2^2\omega_6\omega_7\omega_4^3 - 6\omega_8\omega_6c_s^2\omega_7\omega_4^3 + \\
& 12\omega_8\omega_5\omega_6c_s^2\omega_4^2 + 18\omega_8\omega_9v_2^2\omega_7\omega_4^3 - 36\omega_8\omega_5v_2^2\omega_6\omega_4^3 + 36\omega_5v_2^2\omega_6\omega_7\omega_4^2 + 12\omega_8\omega_5\omega_9\omega_4^2 + 6\omega_5\omega_6\omega_7\omega_4^3 - 12\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7) \frac{\rho c_s^2}{12\omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^3}
\end{aligned}$$

$$C_{D_x D_y^3 v_2}^{(1), \text{CLBM}2} = C_{D_x v_y^3 v_2}^{(1), \text{CLBM}1}$$

$$\begin{aligned}
C_{D_x D_y^3 v_2}^{(1), \text{CuLBM}1} &= (-\omega_2\omega_6\omega_3c_s^2 - 3\omega_2v_2^2\omega_6\omega_3^2 + 12\omega_6\omega_3 - 12\omega_2\omega_3 + 12\omega_3^2c_s^2 + 12\omega_2\omega_3c_s^2 - \omega_6\omega_3^3 - 18v_2^2\omega_3^3 - 12\omega_2\omega_6c_s^2 - 12\omega_6\omega_3c_s^2 + \\
& 18\omega_2\omega_3^3 - 5\omega_2\omega_6\omega_3^2c_s^2 + 36v_2^2\omega_3^2 - 6\omega_6\omega_3^2 - 6\omega_3^3c_s^2 - 6\omega_2\omega_3^3 + 6\omega_2\omega_3^3c_s^2 - 12\omega_3^3 + \omega_2\omega_6\omega_3^2 - 54\omega_2v_2^2\omega_3^2 - 36v_2^2\omega_6\omega_3 + 6\omega_3^3 + 6\omega_6\omega_3^2c_s^2 + \\
& 18\omega_2\omega_6\omega_3c_s^2 + 18\omega_2v_2^2\omega_3^3 + 3v_2^2\omega_6\omega_3^3 - 18\omega_2\omega_3^3c_s^2 + 36\omega_2v_2^2\omega_3 + 18v_2^2\omega_6\omega_3^2 + \omega_6\omega_3^3c_s^2) \frac{\rho c_s^2}{12\omega_2\omega_6\omega_3^3}
\end{aligned}$$

$$\begin{aligned}
C_{D_x D_y^3 v_2}^{(1), \text{CuLBM}2} &= (-\omega_3^2v_1^4\omega_3\omega_1^2 - 33\omega_2\omega_3c_s^4\omega_1^3 - \omega_2^2v_1^2\omega_3\omega_1^3 + 351\omega_3^2v_2^2\omega_3c_s^2\omega_1 - 45\omega_3^2\omega_3c_s^2\omega_1 + 180\omega_2v_2^2\omega_3\omega_1^3 + 12\omega_3^3c_s^4\omega_1^3 - 6\omega_2^2\omega_3\omega_1 - \\
& 12\omega_2^2\omega_3c_s^2\omega_1^2 + 18\omega_2^2c_s^2\omega_1^3 - 30\omega_3^2c_s^4\omega_1^2 - 63\omega_2v_2^2\omega_3\omega_1^2 + 6\omega_2\omega_3c_s^4\omega_1^2 - 24\omega_3^2c_s^2\omega_1^2 - 24\omega_2^2\omega_3c_s^2\omega_1^3 - 171\omega_3^3v_2^4\omega_3 + \omega_2^2v_1^4\omega_3\omega_1^3 + \omega_2^2v_1^2\omega_3\omega_1^2 + \\
& 24\omega_3^2\omega_3c_s^2 + 207v_2^2\omega_3c_s^2\omega_1^3 - 324\omega_2v_2^2\omega_3\omega_1^3 - 207\omega_3^2v_2^2\omega_3c_s^2 + 7\omega_2^2\omega_3\omega_1^3 + 18\omega_3^3c_s^4\omega_1 - 3\omega_3^2v_1^2\omega_3c_s^2\omega_1^2 - 24\omega_3c_s^2\omega_1^3 + 18\omega_2v_2^2c_s^2\omega_1^3 + 135\omega_2v_2^2\omega_3\omega_1^2 + \\
& 36\omega_2^2\omega_3c_s^2\omega_1 - 153\omega_3^2v_2^2\omega_3c_s^2\omega_1^2 + 6\omega_2c_s^4\omega_1^3 + 24\omega_3^2\omega_3c_s^2\omega_1^2 + 6\omega_3\omega_1^3 + 30\omega_3^2c_s^2\omega_1^2 + 153\omega_2^2v_2^2\omega_3c_s^2\omega_1^3 - 12\omega_2\omega_3\omega_1^3 - 90\omega_3^2v_2^2c_s^2\omega_1^2 - 135\omega_2^2v_2^2\omega_3\omega_1 - \\
& 12\omega_3\omega_3c_s^2\omega_1^2 - 81\omega_5^2v_2^2\omega_3\omega_1^3 + 24\omega_2^2c_s^4\omega_1^2 - 138\omega_3^2v_2^4\omega_3\omega_1^2 - 42\omega_3^2\omega_3c_s^4 + 17\omega_2^2\omega_3c_s^4\omega_1^3 + 99\omega_3^2v_2^2\omega_3 + 63\omega_2v_2^2\omega_3c_s^2\omega_1^2 - 6\omega_3^2\omega_3 + 45\omega_2\omega_3c_s^2\omega_1^3 - \\
& 180\omega_3^2v_2^2\omega_3\omega_1 + 12\omega_3^2\omega_3\omega_1 + 69\omega_3^2\omega_3c_s^4\omega_1 + 36\omega_3^2v_2^2c_s^2\omega_1^3 - 12\omega_3^2c_s^2\omega_1^3 + 6\omega_2\omega_3\omega_1^2 + 3\omega_3^2v_1^2\omega_3c_s^2\omega_1^3 + 36\omega_3^2v_2^2\omega_3c_s^2\omega_1^2 + 171v_2^4\omega_3\omega_1^3 - \\
& 351\omega_2v_2^2\omega_3c_s^2\omega_1^3 + 12\omega_2^2\omega_3c_s^4\omega_1^2 - 18\omega_2^2c_s^4\omega_1^3 - 30\omega_2^2\omega_3c_s^4\omega_1 - 54\omega_2^2v_2^2c_s^2\omega_1^3 + 63\omega_3^2v_2^2\omega_3\omega_1 + 81\omega_3^2v_2^2\omega_3\omega_1^2 - 7\omega_3^2\omega_3\omega_1^2 - 6\omega_3^2\omega_3\omega_1^3 + 138\omega_2^2v_2^4\omega_3\omega_1^3 - \\
& 25\omega_3^2\omega_3c_s^4\omega_1^2 - 135\omega_2^2v_2^2\omega_3c_s^2\omega_1 + 324\omega_3^2v_2^4\omega_3\omega_1 + 72\omega_3^2v_2^2c_s^2\omega_1^2 - 18\omega_3^2c_s^2\omega_1 + 54\omega_3^2v_2^2c_s^2\omega_1 - 99v_2^2\omega_3\omega_1^3 - 2\omega_3^2\omega_3c_s^4\omega_1^3 + 18\omega_3^2c_s^4\omega_1^3) \frac{\rho}{24\omega_3^2\omega_3\omega_1^3}
\end{aligned}$$

$$\text{coefficient } C_{D_y^4 \rho}^{(1)} \text{ at } \frac{\partial^4 \rho}{\partial x^2}:$$

$$C_{D_y^4 \rho}^{(1), \text{SRT}} = (6v_2^4 + 3v_2^2\omega - 2c_s^2 + c_s^2\omega + 2c_s^4 + 24v_2^2c_s^2 - c_s^4\omega - 6v_2^2 - 12v_2^2c_s^2\omega - 3v_2^4\omega) \frac{v_1}{24\omega}$$

$$\begin{aligned}
C_{D_y^4 \rho}^{(1), \text{MRT}1} &= (-96\omega_8^2v_2^4\omega_6\omega_4 - 24\omega_8^2c_s^2\omega_4 - 30\omega_8v_2^4\omega_6^2\omega_4^2 - 144v_2^2\omega_6^2c_s^2\omega_4 - 14\omega_8^2\omega_6c_s^2\omega_4^2 + 24v_2^2\omega_6^2\omega_4 - 24\omega_8\omega_6^2c_s^4 - 12\omega_8^2v_2^2\omega_6^2c_s^2\omega_4^2 - \\
& 144\omega_8^2v_2^2c_s^2\omega_4^2 + 24\omega_6^2c_s^2\omega_4 - 72\omega_8^2v_2^2\omega_4 + 12v_2^4\omega_6^2\omega_4^2 - 126\omega_8v_2^2\omega_6^2c_s^2\omega_4^2 - 36\omega_8^2v_2^2\omega_6\omega_4^2 - 48\omega_8\omega_6^2c_s^2\omega_4 - 96\omega_8v_2^2\omega_6^2\omega_4 + 24\omega_8^2\omega_6c_s^4 + \\
& 12\omega_8\omega_6^2c_s^2\omega_4^2 + 96\omega_8^2v_2^2\omega_6\omega_4 + 30\omega_8v_2^2\omega_6^2\omega_4^2 + 36\omega_8^2v_2^2\omega_4^2 - 24v_2^4\omega_6^2\omega_4 + 432\omega_8v_2^2\omega_6^2c_s^2\omega_4 - 48\omega_8v_2^2\omega_6^2 - 12v_2^2\omega_6^2\omega_4^2 + 48\omega_8^2\omega_6c_s^2\omega_4 + \\
& 288\omega_8^2v_2^2c_s^2\omega_4 - 12\omega_6^2c_s^2\omega_4^2 + 36\omega_8^2v_2^4\omega_6\omega_4^2 + 12\omega_8^2c_s^2\omega_4^2 + 96\omega_8v_2^2\omega_6^2\omega_4 + 72v_2^2\omega_6^2c_s^2\omega_4^2 - 48\omega_8^2v_2^2\omega_6 - \omega_8^2\omega_6^2c_s^4\omega_4^2 + 72\omega_8^2v_2^4\omega_4 + 216\omega_8^2v_2^2\omega_6c_s^2 - \\
& 144\omega_8v_2^2\omega_6c_s^2\omega_4 + 48\omega_8\omega_6^2c_s^4\omega_4 - 3\omega_8^2v_2^4\omega_6^2\omega_4^2 - 48\omega_8v_2^4\omega_6\omega_4 - 432\omega_8^2v_2^2\omega_6c_s^2\omega_4 + 24\omega_8^2c_s^4\omega_4 - 24\omega_8v_2^2\omega_6\omega_4^2 + 14\omega_8^2\omega_6c_s^4\omega_4^2 - 24\omega_6^2c_s^4\omega_4 - \\
& 48\omega_8^2\omega_6c_s^4\omega_4 + 48\omega_8v_2^2\omega_6^2 + 24\omega_8\omega_6^2c_s^2 + 12\omega_6^2c_s^4\omega_4^2 + 3\omega_8^2v_2^2\omega_6^2\omega_4^2 + 150\omega_8^2v_2^2\omega_6c_s^2\omega_4^2 - 12\omega_8^2c_s^4\omega_4^2 + \omega_8^2\omega_6^2c_s^2\omega_4^2 + 48\omega_8v_2^2\omega_6\omega_4 + 48\omega_8^2v_2^4\omega_6 + \\
& 72\omega_8v_2^2\omega_6c_s^2\omega_4^2 - 12\omega_8\omega_6^2c_s^4\omega_4^2 + 24\omega_8v_2^2\omega_6\omega_4^2 - 24\omega_8^2\omega_6c_s^2 - 36\omega_8^2v_2^4\omega_4^2 - 216\omega_8v_2^2\omega_6^2c_s^2) \frac{v_1}{24\omega_8^2\omega_6^2\omega_4^2}
\end{aligned}$$

$$C_{D_y^4 \rho}^{(1), \text{MRT}2} = C_{D_y^4 \rho}^{(1), \text{MRT}1}$$

$$C_{D_y^4 \rho}^{(1), \text{CLBM}1} = (6v_2^4 + 3v_2^2\omega_6 - 12v_2^2\omega_6c_s^2 - 2c_s^2 - \omega_6c_s^4 - 3v_2^4\omega_6 + 2c_s^4 + 24v_2^2c_s^2 + \omega_6c_s^2 - 6v_2^2) \frac{v_1}{24\omega_6}$$

$$C_{D_y^4 \rho}^{(1), \text{CLBM}2} = C_{D_y^4 \rho}^{(1), \text{CLBM}1}$$

$$C_{D_y^4 \rho}^{(1), \text{CuLBM}1} = (6v_2^4 + 3\omega_2v_2^2 - 12\omega_2v_2^2c_s^2 - 2c_s^2 - \omega_2c_s^4 + 2c_s^4 + \omega_2c_s^2 + 24v_2^2c_s^2 - 6v_2^2 - 3\omega_2v_2^4) \frac{v_1}{24\omega_2}$$

$$C_{D_y^4 \rho}^{(1), \text{CuLBM2}} = (-3\omega_2 v_2^4 \omega_1 - 12\omega_2 v_2^2 c_s^2 \omega_1 - 3\omega_2 v_2^2 - \omega_2 c_s^4 \omega_1 + 12\omega_2 v_2^2 c_s^2 + \omega_2 c_s^4 - 3v_2^2 \omega_1 - c_s^2 \omega_1 - \omega_2 c_s^2 + \omega_2 c_s^2 \omega_1 + 3\omega_2 v_2^2 \omega_1 + 3\omega_2 v_2^4 + c_s^4 \omega_1 + 12v_2^2 c_s^2 \omega_1 + 3v_2^4 \omega_1) \frac{v_1}{24\omega_2 \omega_1}$$

coefficient $C_{D_y^4 v_1}^{(1)}$ **at** $\frac{\partial^4 v_1}{\partial x^4}$:

$$C_{D_y^4 v_1}^{(1), \text{SRT}} = (-14c_s^2 \omega^2 - 72v_2^4 - 3v_2^2 \omega^3 + 42v_2^2 \omega^2 + c_s^2 \omega^3 - 108v_2^2 \omega - 24c_s^2 + 36c_s^2 \omega + 48c_s^4 - 84v_2^2 c_s^2 \omega^2 - 42v_2^4 \omega^2 - 3c_s^4 \omega^3 + 30c_s^4 \omega^2 - 144v_2^4 c_s^2 + 3v_2^4 \omega^3 + 6v_2^2 c_s^2 \omega^3 - 72c_s^4 \omega + 72v_2^2 + 216v_2^2 c_s^2 \omega + 108v_2^4 \omega) \frac{\rho}{24\omega^3}$$

$$C_{D_y^4 v_1}^{(1), \text{MRT1}} = (12\omega_8^2 c_s^2 \omega_4 + 24\omega_8^2 c_s^4 + 24v_2^2 \omega_4^2 - 18\omega_8 v_2^4 \omega_4^3 - 24\omega_8 c_s^4 \omega_4^2 - 72\omega_8^2 v_2^2 c_s^2 \omega_4^2 - 24\omega_8^2 v_2^2 \omega_4 - 12v_2^2 \omega_4^3 + 6\omega_8^2 v_2^2 c_s^2 \omega_4^3 + 6\omega_8 c_s^4 \omega_4^3 + 72\omega_8 v_2^4 \omega_4^2 - 48\omega_8 v_2^4 \omega_4 + 24\omega_8^2 v_2^2 \omega_4^2 + \omega_8^2 c_s^2 \omega_4^3 + 24\omega_8 c_s^4 \omega_4 + 156\omega_8^2 v_2^2 c_s^2 \omega_4 - 8\omega_8^2 c_s^2 \omega_4^2 - 3\omega_8^2 v_2^2 \omega_4^3 + 48\omega_8 v_2^2 c_s^2 \omega_4^2 + 24\omega_8^2 v_2^2 \omega_4 + 12v_2^2 c_s^2 \omega_4^3 + 12v_2^4 \omega_4^3 - 6\omega_8 c_s^2 \omega_4^3 - 72\omega_8 v_2^2 \omega_4^2 - 48\omega_8^2 c_s^4 \omega_4 - 24v_2^4 \omega_4^2 - 24v_2^2 c_s^2 \omega_4^2 - 12\omega_8 v_2^2 c_s^2 \omega_4^3 + 18\omega_8 v_2^2 \omega_4^3 + 24\omega_8 c_s^2 \omega_4^2 - 24\omega_8^2 c_s^2 \omega_4 + 24\omega_8^2 c_s^4 \omega_4^2 + 3\omega_8^2 v_2^4 \omega_4^3 - 96\omega_8^2 v_2^2 c_s^2 + 48\omega_8 v_2^2 \omega_4 - 24\omega_8^2 v_2^4 \omega_4^2 - 24\omega_8 v_2^2 c_s^2 \omega_4 - 3\omega_8^2 c_s^4 \omega_4^3) \frac{\rho}{24\omega_8^2 \omega_4^3}$$

$$C_{D_y^4 v_1}^{(1), \text{MRT2}} = C_{D_y^4 v_1}^{(1), \text{MRT1}}$$

$$C_{D_y^4 v_1}^{(1), \text{CLBM1}} = (12\omega_8^2 c_s^2 \omega_4 + 24\omega_8^2 c_s^4 + 72v_2^2 \omega_4^2 - 30\omega_8 v_2^4 \omega_4^3 - 24\omega_8 c_s^4 \omega_4^2 - 12\omega_8^2 v_2^2 c_s^2 \omega_4^2 - 36v_2^2 \omega_4^3 + 6\omega_8^2 v_2^2 c_s^2 \omega_4^3 + 6\omega_8 c_s^4 \omega_4^3 + 72\omega_8 v_2^4 \omega_4^2 + 12\omega_8^2 v_2^2 \omega_4^2 + \omega_8^2 c_s^2 \omega_4^3 + 24\omega_8 c_s^4 \omega_4 - 36\omega_8^2 v_2^2 c_s^2 \omega_4 - 8\omega_8^2 c_s^2 \omega_4^2 - 3\omega_8^2 v_2^2 \omega_4^3 + 144\omega_8 v_2^2 c_s^2 \omega_4^2 + 108v_2^2 c_s^2 \omega_4^3 + 36v_2^4 \omega_4^3 - 6\omega_8 c_s^2 \omega_4^3 - 72\omega_8 v_2^2 \omega_4^2 - 48\omega_8^2 c_s^4 \omega_4 - 72v_2^4 \omega_4^2 - 216v_2^2 c_s^2 \omega_4^2 - 72\omega_8 v_2^2 c_s^2 \omega_4^3 + 30\omega_8 v_2^2 \omega_4^3 + 24\omega_8 c_s^2 \omega_4^2 - 24\omega_8^2 c_s^2 \omega_4 + 24\omega_8^2 c_s^4 \omega_4^2 + 3\omega_8^2 v_2^4 \omega_4^3 - 12\omega_8^2 v_2^2 \omega_4^2 + 72\omega_8 v_2^2 c_s^2 \omega_4 - 3\omega_8^2 c_s^4 \omega_4^3) \frac{\rho}{24\omega_8^2 \omega_4^3}$$

$$C_{D_y^4 v_1}^{(1), \text{CLBM2}} = C_{D_y^4 v_1}^{(1), \text{CLBM1}}$$

$$C_{D_y^4 v_1}^{(1), \text{CuLBM1}} = (12v_2^2 \omega_6^2 \omega_3^2 + 6\omega_6 \omega_3^3 c_s^4 - 3v_2^2 \omega_6^2 \omega_3^3 + 6v_2^2 \omega_6^2 \omega_3^3 c_s^2 + \omega_6^2 \omega_3^3 c_s^2 - 12v_2^4 \omega_6^2 \omega_3^2 - 36v_2^2 \omega_3^3 - 24\omega_6 \omega_3 c_s^2 - 48\omega_6^2 \omega_3 c_s^4 + 72v_2^2 \omega_6 \omega_3 c_s^2 - 8\omega_6^2 c_s^2 c_s^2 + 3v_2^4 \omega_6^2 \omega_3^3 + 72v_2^2 \omega_3^2 - 12v_2^2 \omega_6^2 \omega_3^2 c_s^2 - 24\omega_6 \omega_3^2 c_s^4 + 24\omega_6^2 c_s^4 + 12\omega_6^2 \omega_3 c_s^2 - 30v_2^4 \omega_6 \omega_3^3 - 36v_2^2 \omega_6^2 \omega_3 c_s^2 + 24\omega_6 \omega_3 c_s^4 - 216v_2^2 \omega_3^2 c_s^2 + 72v_2^4 \omega_6 \omega_3^3 + 24\omega_6 \omega_3^2 c_s^2 + 24\omega_6^2 \omega_3^2 c_s^4 + 144v_2^2 \omega_6 \omega_3^2 c_s^2 - 72v_2^2 \omega_3^2 + 30v_2^2 \omega_6 \omega_3^3 - 72v_2^2 \omega_6 \omega_3^3 c_s^2 + 36v_2^4 \omega_3^3 - 72v_2^2 \omega_6 \omega_3^2 - 3\omega_6^2 \omega_3^3 c_s^4 - 6\omega_6 \omega_3^3 c_s^2 + 108v_2^2 \omega_3^3 c_s^2) \frac{\rho}{24\omega_6^2 \omega_3^3}$$

$$C_{D_y^4 v_1}^{(1), \text{CuLBM2}} = (-24\omega_3 c_s^2 \omega_1 - 36v_2^2 \omega_1^3 + 72v_2^2 \omega_3 c_s^2 \omega_1 - 3v_2^2 \omega_3^2 \omega_1^3 + 24\omega_3^2 c_s^4 \omega_1^2 + 72v_2^2 \omega_1^2 - 3\omega_3^2 c_s^4 \omega_1^3 + 12v_2^2 \omega_3^2 \omega_1^2 - 72v_2^2 \omega_3 c_s^2 \omega_1^3 + 3v_2^4 \omega_3^2 \omega_1^3 - 6\omega_3 c_s^2 \omega_1^3 + 144v_2^2 \omega_3 c_s^2 \omega_1^2 - 48\omega_3^2 c_s^4 \omega_1 + 24\omega_3 c_s^2 \omega_1^2 - 12v_2^4 \omega_3^2 \omega_1^2 + 6v_2^2 \omega_3^2 c_s^2 \omega_1^3 - 216v_2^2 c_s^2 \omega_1^2 - 72v_2^2 \omega_1^2 + \omega_3^2 c_s^2 \omega_1^3 + 72v_2^4 \omega_3 \omega_1^2 + 36v_2^4 \omega_1^3 + 108v_2^2 c_s^2 \omega_1^3 + 24\omega_3 c_s^4 \omega_1 - 12v_2^2 \omega_3^2 c_s^2 \omega_1^2 - 30v_2^4 \omega_3 \omega_1^3 - 8\omega_3^2 c_s^2 \omega_1^2 + 12\omega_3^2 c_s^2 \omega_1 - 24\omega_3 c_s^4 \omega_1^2 - 72v_2^2 \omega_3 \omega_1^2 - 36v_2^2 \omega_3^2 c_s^2 \omega_1 + 30v_2^2 \omega_3 \omega_1^3 + 24\omega_3 c_s^4 + 6\omega_3 c_s^4 \omega_1^3) \frac{\rho}{24\omega_3^2 \omega_1^3}$$

coefficient $C_{D_y^4 v_2}^{(1)}$ **at** $\frac{\partial^4 v_2}{\partial x^4}$:

$$C_{D_y^4 v_2}^{(1), \text{SRT}} = (-4 - 5v_2^2 \omega + 6c_s^2 - 3c_s^2 \omega + 10v_2^2 + 2\omega) \frac{v_1 \rho v_2}{12\omega}$$

$$C_{D_y^4 v_2}^{(1), \text{MRT1}} = (72\omega_8^2 c_s^2 \omega_4 - 72\omega_8 \omega_6^2 \omega_4 - 48\omega_8^2 \omega_4 + 39\omega_8^2 \omega_6 c_s^2 \omega_4^2 - 48v_2^2 \omega_6^2 \omega_4 - 48\omega_6^2 c_s^2 \omega_4 + 120\omega_8^2 v_2^2 \omega_4 + 72\omega_8^2 \omega_6 \omega_4 + 61\omega_8^2 v_2^2 \omega_6 \omega_4^2 + 120\omega_8 \omega_6^2 c_s^2 \omega_4 + 168\omega_8 v_2^2 \omega_6^2 \omega_4 - 33\omega_8 \omega_6^2 c_s^2 \omega_4^2 - 168\omega_8^2 v_2^2 \omega_6 \omega_4 - 51\omega_8 v_2^2 \omega_6^2 \omega_4^2 - 36\omega_8^2 \omega_6 - 60\omega_8^2 v_2^2 \omega_4^2 - 25\omega_8^2 \omega_6 \omega_4^2 + 24v_2^2 \omega_6^2 \omega_4^2 - 120\omega_8^2 \omega_6 c_s^2 \omega_4 + 24\omega_8^2 \omega_4^2 + 24\omega_6^2 c_s^2 \omega_4^2 - 36\omega_8^2 c_s^2 \omega_4^2 + 84\omega_8^2 v_2^2 \omega_6 + 21\omega_8 \omega_6^2 \omega_4^2 + 12\omega_8 \omega_6 c_s^2 \omega_4^2 - 12\omega_8 \omega_6 \omega_4^2 + 24\omega_6^2 \omega_4 + 36\omega_8 v_2^2 \omega_6 \omega_4^2 + 2\omega_8^2 \omega_6^2 \omega_4^2 - 84\omega_8 v_2^2 \omega_6^2 - 60\omega_8 \omega_6^2 c_s^2 - 5\omega_8^2 v_2^2 \omega_6^2 \omega_4^2 + 36\omega_8 \omega_6^2 - 3\omega_8^2 \omega_6^2 c_s^2 \omega_4^2 - 72\omega_8 v_2^2 \omega_6 \omega_4 + 24\omega_8 \omega_6 \omega_4 - 12\omega_6^2 \omega_4^2 + 60\omega_8^2 \omega_6 c_s^2 - 24\omega_8 \omega_6 c_s^2 \omega_4) \frac{v_1 \rho v_2}{12\omega_8^2 \omega_6^2 \omega_4^2}$$

$$C_{D_y^4 v_2}^{(1), \text{MRT2}} = C_{D_y^4 v_2}^{(1), \text{MRT1}}$$

$$C_{D_y^4 v_2}^{(1), \text{CLBM1}} = (-4 - 5v_2^2 \omega_6 + 6c_s^2 + 2\omega_6 - 3\omega_6 c_s^2 + 10v_2^2) \frac{v_1 \rho v_2}{12\omega_6}$$

$$C_{D_y^4 v_2}^{(1), \text{CLBM2}} = C_{D_y^4 v_2}^{(1), \text{CLBM1}}$$

$$C_{D_y^4 v_2}^{(1), \text{CuLBM1}} = (-4 - 5\omega_2 v_2^2 + 2\omega_2 + 6c_s^2 - 3\omega_2 c_s^2 + 10v_2^2) \frac{v_1 \rho v_2}{12\omega_2}$$

$$C_{D_y^4 v_2}^{(1), \text{CuLBM2}} = (5\omega_2 v_2^2 - 2\omega_2 + 5v_2^2 \omega_1 + 2\omega_2 \omega_1 + 3c_s^2 \omega_1 + 3\omega_2 c_s^2 - 3\omega_2 c_s^2 \omega_1 - 5\omega_2 v_2^2 \omega_1 - 2\omega_1) \frac{v_1 \rho v_2}{12\omega_2 \omega_1}$$

3.3 Conservation of momentum: ρv_2

$$\begin{aligned}
& v_2 \frac{\partial \rho}{\partial t} + \rho \frac{\partial v_2}{\partial t} + v_1 v_2 \frac{\delta_L}{\delta t} \frac{\partial \rho}{\partial x_1} + \rho v_2 \frac{\delta_L}{\delta t} \frac{\partial v_1}{\partial x_1} + v_1 \rho \frac{\delta_L}{\delta t} \frac{\partial v_2}{\partial x_1} + (c_s^2 + v_2^2) \frac{\delta_L}{\delta t} \frac{\partial \rho}{\partial x_2} + 2\rho v_2 \frac{\delta_L}{\delta t} \frac{\partial v_2}{\partial x_2} + C_{D_x \rho, D_x v_2}^{(2)} \frac{\delta_L^2}{\delta t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1} + \\
& C_{D_x \rho, D_y v_1}^{(2)} \frac{\delta_L^2}{\delta t} \frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2} + C_{D_x v_1, D_y v_1}^{(2)} \frac{\delta_L^2}{\delta t} \frac{\partial v_1}{\partial x_1} \frac{\partial v_1}{\partial x_2} + C_{D_y \rho, D_x v_1}^{(2)} \frac{\delta_L^2}{\delta t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_1} + C_{D_y \rho, D_y v_2}^{(2)} \frac{\delta_L^2}{\delta t} \frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2} + \\
& C_{D_y v_2, D_y v_2}^{(2)} \frac{\delta_L^2}{\delta t} \left(\frac{\partial v_2}{\partial x_2} \right)^2 + C_{D_x^2 v_2}^{(2)} \frac{\delta_L^2}{\delta t} \frac{\partial^2 v_2}{\partial x_1^2} + C_{D_x D_y \rho}^{(2)} \frac{\delta_L^2}{\delta t} \frac{\partial^2 \rho}{\partial x_1 \partial x_2} + C_{D_x D_y v_1}^{(2)} \frac{\delta_L^2}{\delta t} \frac{\partial^2 v_1}{\partial x_1 \partial x_2} + C_{D_y^2 \rho}^{(2)} \frac{\delta_L^2}{\delta t} \frac{\partial^2 \rho}{\partial x_2^2} + C_{D_y^2 v_2}^{(2)} \frac{\delta_L^2}{\delta t} \frac{\partial^2 v_2}{\partial x_2^2} \\
& + C_{D_x^3 \rho}^{(2)} \frac{\delta_L^3}{\delta t} \frac{\partial^3 \rho}{\partial x_1^3} + C_{D_x^3 v_1}^{(2)} \frac{\delta_L^3}{\delta t} \frac{\partial^3 v_1}{\partial x_1^3} + C_{D_x^3 v_2}^{(2)} \frac{\delta_L^3}{\delta t} \frac{\partial^3 v_2}{\partial x_1^3} + C_{D_x^2 D_y \rho}^{(2)} \frac{\delta_L^3}{\delta t} \frac{\partial^3 \rho}{\partial x_1^2 \partial x_2} + C_{D_x^2 D_y v_1}^{(2)} \frac{\delta_L^3}{\delta t} \frac{\partial^3 v_1}{\partial x_1^2 \partial x_2} + C_{D_x^2 D_y v_2}^{(2)} \frac{\delta_L^3}{\delta t} \frac{\partial^3 v_2}{\partial x_1^2 \partial x_2} \\
& + C_{D_x D_y^2 \rho}^{(2)} \frac{\delta_L^3}{\delta t} \frac{\partial^3 \rho}{\partial x_1 \partial x_2^2} + C_{D_x D_y^2 v_1}^{(2)} \frac{\delta_L^3}{\delta t} \frac{\partial^3 v_1}{\partial x_1 \partial x_2^2} + C_{D_x D_y^2 v_2}^{(2)} \frac{\delta_L^3}{\delta t} \frac{\partial^3 v_2}{\partial x_1 \partial x_2^2} + C_{D_y^3 \rho}^{(2)} \frac{\delta_L^3}{\delta t} \frac{\partial^3 \rho}{\partial x_2^3} + C_{D_y^3 v_2}^{(2)} \frac{\delta_L^3}{\delta t} \frac{\partial^3 v_2}{\partial x_2^3} + C_{D_x^4 \rho}^{(2)} \frac{\delta_L^4}{\delta t} \frac{\partial^4 \rho}{\partial x_1^4} \\
& + C_{D_x^4 v_1}^{(2)} \frac{\delta_L^4}{\delta t} \frac{\partial^4 v_1}{\partial x_1^4} + C_{D_x^4 v_2}^{(2)} \frac{\delta_L^4}{\delta t} \frac{\partial^4 v_2}{\partial x_1^4} + C_{D_x^3 D_y \rho}^{(2)} \frac{\delta_L^4}{\delta t} \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} + C_{D_x^3 D_y v_1}^{(2)} \frac{\delta_L^4}{\delta t} \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} + C_{D_x^3 D_y v_2}^{(2)} \frac{\delta_L^4}{\delta t} \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} + \\
& C_{D_x^2 D_y^2 \rho}^{(2)} \frac{\delta_L^4}{\delta t} \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} + C_{D_x^2 D_y^2 v_1}^{(2)} \frac{\delta_L^4}{\delta t} \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} + C_{D_x^2 D_y^2 v_2}^{(2)} \frac{\delta_L^4}{\delta t} \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} + C_{D_x D_y^3 \rho}^{(2)} \frac{\delta_L^4}{\delta t} \frac{\partial^4 \rho}{\partial x_1 \partial x_2^3} + C_{D_x D_y^3 v_1}^{(2)} \frac{\delta_L^4}{\delta t} \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} + \\
& C_{D_x D_y^3 v_2}^{(2)} \frac{\delta_L^4}{\delta t} \frac{\partial^4 v_2}{\partial x_1 \partial x_2^3} + C_{D_y^4 \rho}^{(2)} \frac{\delta_L^4}{\delta t} \frac{\partial^4 \rho}{\partial x_2^4} + C_{D_y^4 v_2}^{(2)} \frac{\delta_L^4}{\delta t} \frac{\partial^4 v_2}{\partial x_2^4} = 0,
\end{aligned}$$

where:

coefficient $C_{D_x \rho, D_x v_2}^{(2)}$ **at** $\frac{\partial \rho}{\partial x_1} \frac{\partial v_2}{\partial x_1}$:

$$C_{D_x \rho, D_x v_2}^{(2), \text{SRT}} = (-2 + \omega) \frac{c_s^2}{2\omega}$$

$$C_{D_x \rho, D_x v_2}^{(2), \text{MRT1}} = (-2 + \omega_4) \frac{c_s^2}{2\omega_4}$$

$$C_{D_x \rho, D_x v_2}^{(2), \text{MRT2}} = C_{D_x \rho, D_x v_2}^{(2), \text{MRT1}}$$

$$C_{D_x \rho, D_x v_2}^{(2), \text{CLBM1}} = C_{D_x \rho, D_x v_2}^{(2), \text{MRT1}}$$

$$C_{D_x \rho, D_x v_2}^{(2), \text{CLBM2}} = C_{D_x \rho, D_x v_2}^{(2), \text{MRT1}}$$

$$C_{D_x \rho, D_x v_2}^{(2), \text{CuLBM1}} = (-2 + \omega_3) \frac{c_s^2}{2\omega_3}$$

$$C_{D_x \rho, D_x v_2}^{(2), \text{CuLBM2}} = (-2 + \omega_1) \frac{c_s^2}{2\omega_1}$$

coefficient $C_{D_x \rho, D_y v_1}^{(2)}$ **at** $\frac{\partial \rho}{\partial x_1} \frac{\partial v_1}{\partial x_2}$:

$$C_{D_x \rho, D_y v_1}^{(2), \text{SRT}} = (-2 + \omega) \frac{c_s^2}{2\omega}$$

$$C_{D_x \rho, D_y v_1}^{(2), \text{MRT1}} = (-2 + \omega_4) \frac{c_s^2}{2\omega_4}$$

$$C_{D_x \rho, D_y v_1}^{(2), \text{MRT2}} = C_{D_x \rho, D_y v_1}^{(2), \text{MRT1}}$$

$$C_{D_x \rho, D_y v_1}^{(2), \text{CLBM1}} = C_{D_x \rho, D_y v_1}^{(2), \text{MRT1}}$$

$$C_{D_x \rho, D_y v_1}^{(2), \text{CLBM2}} = C_{D_x \rho, D_y v_1}^{(2), \text{MRT1}}$$

$$C_{D_x \rho, D_y v_1}^{(2), \text{CuLBM1}} = (-2 + \omega_3) \frac{c_s^2}{2\omega_3}$$

$$C_{D_x \rho, D_y v_1}^{(2), \text{CuLBM2}} = (\omega_2 + 3c_s^2\omega_1 - 5\omega_2 c_s^2 + \omega_2 c_s^2\omega_1 + 3v_1^2\omega_1 - \omega_1 - 3\omega_2 v_1^2) \frac{1}{2\omega_2\omega_1}$$

coefficient $C_{D_x v_1, D_y v_1}^{(2)}$ **at** $\frac{\partial v_1}{\partial x_1} \frac{\partial v_1}{\partial x_2}$:

$$C_{D_x v_1, D_y v_1}^{(2), \text{SRT}} = 0$$

$$C_{D_x v_1, D_y v_1}^{(2), \text{MRT1}} = 0$$

$$C_{D_x v_1, D_y v_1}^{(2), \text{MRT}2} = 0$$

$$C_{D_x v_1, D_y v_1}^{(2), \text{CLBM}1} = 0$$

$$C_{D_x v_1, D_y v_1}^{(2), \text{CLBM}2} = 0$$

$$C_{D_x v_1, D_y v_1}^{(2), \text{CuLBM}1} = 0$$

$$C_{D_x v_1, D_y v_1}^{(2), \text{CuLBM}2} = (-\omega_2 + \omega_1) \frac{3v_1 \rho}{\omega_2 \omega_1}$$

coefficient $C_{D_y \rho, D_x v_1}^{(2)}$ **at** $\frac{\partial \rho}{\partial x_2} \frac{\partial v_1}{\partial x_1}$:

$$C_{D_y \rho, D_x v_1}^{(2), \text{SRT}} = 0$$

$$C_{D_y \rho, D_x v_1}^{(2), \text{MRT}1} = 0$$

$$C_{D_y \rho, D_x v_1}^{(2), \text{MRT}2} = 0$$

$$C_{D_y \rho, D_x v_1}^{(2), \text{CLBM}1} = 0$$

$$C_{D_y \rho, D_x v_1}^{(2), \text{CLBM}2} = 0$$

$$C_{D_y \rho, D_x v_1}^{(2), \text{CuLBM}1} = 0$$

$$C_{D_y \rho, D_x v_1}^{(2), \text{CuLBM}2} = (\omega_2 + c_s^2 \omega_1 - \omega_2 c_s^2 + 3v_1^2 \omega_1 - \omega_1 - 3\omega_2 v_1^2) \frac{1}{2\omega_2 \omega_1}$$

coefficient $C_{D_y \rho, D_y v_2}^{(2)}$ **at** $\frac{\partial \rho}{\partial x_2} \frac{\partial v_2}{\partial x_2}$:

$$C_{D_y \rho, D_y v_2}^{(2), \text{SRT}} = (-2 - 3v_2^2 \omega + 4c_s^2 - 2c_s^2 \omega + 6v_2^2 + \omega) \frac{1}{\omega}$$

$$C_{D_y \rho, D_y v_2}^{(2), \text{MRT}1} = (-2 - 3v_2^2 \omega_6 + 4c_s^2 + \omega_6 - 2\omega_6 c_s^2 + 6v_2^2) \frac{1}{\omega_6}$$

$$C_{D_y \rho, D_y v_2}^{(2), \text{MRT}2} = C_{D_y \rho, D_y v_2}^{(2), \text{MRT}1}$$

$$C_{D_y \rho, D_y v_2}^{(2), \text{CLBM}1} = C_{D_y \rho, D_y v_2}^{(2), \text{MRT}1}$$

$$C_{D_y \rho, D_y v_2}^{(2), \text{CLBM}2} = C_{D_y \rho, D_y v_2}^{(2), \text{MRT}1}$$

$$C_{D_y \rho, D_y v_2}^{(2), \text{CuLBM}1} = (-2 - 3\omega_2 v_2^2 + \omega_2 + 4c_s^2 - 2\omega_2 c_s^2 + 6v_2^2) \frac{1}{\omega_2}$$

$$C_{D_y \rho, D_y v_2}^{(2), \text{CuLBM}2} = (3\omega_2 v_2^2 - \omega_2 + 3v_2^2 \omega_1 + \omega_2 \omega_1 + 2c_s^2 \omega_1 + 2\omega_2 c_s^2 - 2\omega_2 c_s^2 \omega_1 - 3\omega_2 v_2^2 \omega_1 - \omega_1) \frac{1}{\omega_2 \omega_1}$$

coefficient $C_{D_y v_2, D_y v_2}^{(2)}$ **at** $\left(\frac{\partial v_2}{\partial x_2}\right)^2$:

$$C_{D_y v_2, D_y v_2}^{(2), \text{SRT}} = (2 - \omega) \frac{3\rho v_2}{\omega}$$

$$C_{D_y v_2, D_y v_2}^{(2), \text{MRT}1} = (2 - \omega_6) \frac{3\rho v_2}{\omega_6}$$

$$C_{D_y v_2, D_y v_2}^{(2), \text{MRT}2} = C_{D_y v_2, D_y v_2}^{(2), \text{MRT}1}$$

$$C_{D_y v_2, D_y v_2}^{(2), \text{CLBM}1} = C_{D_y v_2, D_y v_2}^{(2), \text{MRT}1}$$

$$C_{D_y v_2, D_y v_2}^{(2), \text{CLBM}2} = C_{D_y v_2, D_y v_2}^{(2), \text{MRT}1}$$

$$C_{D_y v_2, D_y v_2}^{(2), \text{CuLBM}1} = (2 - \omega_2) \frac{3\rho v_2}{\omega_2}$$

$$C_{D_y v_2, D_y v_2}^{(2), \text{CuLBM}2} = (\omega_2 - \omega_2 \omega_1 + \omega_1) \frac{3\rho v_2}{\omega_2 \omega_1}$$

coefficient $C_{D_x^2 v_2}^{(2)}$ **at** $\frac{\partial^2 v_2}{\partial x_1^2}$:

$$C_{D_x^2 v_2}^{(2), \text{SRT}} = (-2 + \omega) \frac{\rho c_s^2}{2\omega}$$

$$C_{D_x^2 v_2}^{(2), \text{MRT1}} = (-2 + \omega_4) \frac{\rho c_s^2}{2\omega_4}$$

$$C_{D_x^2 v_2}^{(2), \text{MRT2}} = C_{D_x^2 v_2}^{(2), \text{MRT1}}$$

$$C_{D_x^2 v_2}^{(2), \text{CLBM1}} = C_{D_x^2 v_2}^{(2), \text{MRT1}}$$

$$C_{D_x^2 v_2}^{(2), \text{CLBM2}} = C_{D_x^2 v_2}^{(2), \text{MRT1}}$$

$$C_{D_x^2 v_2}^{(2), \text{CuLBM1}} = (-2 + \omega_3) \frac{\rho c_s^2}{2\omega_3}$$

$$C_{D_x^2 v_2}^{(2), \text{CuLBM2}} = (-2 + \omega_1) \frac{\rho c_s^2}{2\omega_1}$$

coefficient $C_{D_x D_y \rho}^{(2)}$ **at** $\frac{\partial^2 \rho}{\partial x_1 \partial x_2}$:

$$C_{D_x D_y \rho}^{(2), \text{SRT}} = 0$$

$$C_{D_x D_y \rho}^{(2), \text{MRT1}} = 0$$

$$C_{D_x D_y \rho}^{(2), \text{MRT2}} = 0$$

$$C_{D_x D_y \rho}^{(2), \text{CLBM1}} = 0$$

$$C_{D_x D_y \rho}^{(2), \text{CLBM2}} = 0$$

$$C_{D_x D_y \rho}^{(2), \text{CuLBM1}} = 0$$

$$C_{D_x D_y \rho}^{(2), \text{CuLBM2}} = (\omega_2 + 3c_s^2 \omega_1 - 3\omega_2 c_s^2 + v_1^2 \omega_1 - \omega_1 - \omega_2 v_1^2) \frac{v_1}{2\omega_2 \omega_1}$$

coefficient $C_{D_x D_y v_1}^{(2)}$ **at** $\frac{\partial^2 v_1}{\partial x_1 \partial x_2}$:

$$C_{D_x D_y v_1}^{(2), \text{SRT}} = (-2 + \omega) \frac{\rho c_s^2}{2\omega}$$

$$C_{D_x D_y v_1}^{(2), \text{MRT1}} = (-2 + \omega_4) \frac{\rho c_s^2}{2\omega_4}$$

$$C_{D_x D_y v_1}^{(2), \text{MRT2}} = C_{D_x D_y v_1}^{(2), \text{MRT1}}$$

$$C_{D_x D_y v_1}^{(2), \text{CLBM1}} = C_{D_x D_y v_1}^{(2), \text{MRT1}}$$

$$C_{D_x D_y v_1}^{(2), \text{CLBM2}} = C_{D_x D_y v_1}^{(2), \text{MRT1}}$$

$$C_{D_x D_y v_1}^{(2), \text{CuLBM1}} = (-2 + \omega_3) \frac{\rho c_s^2}{2\omega_3}$$

$$C_{D_x D_y v_1}^{(2), \text{CuLBM2}} = (\omega_2 + c_s^2 \omega_1 - 3\omega_2 c_s^2 + \omega_2 c_s^2 \omega_1 + 3v_1^2 \omega_1 - \omega_1 - 3\omega_2 v_1^2) \frac{\rho}{2\omega_2 \omega_1}$$

coefficient $C_{D_y^2 \rho}^{(2)}$ **at** $\frac{\partial^2 \rho}{\partial x_2^2}$:

$$C_{D_y^2 \rho}^{(2), \text{SRT}} = (-2 - v_2^2 \omega + 6c_s^2 - 3c_s^2 \omega + 2v_2^2 + \omega) \frac{v_2}{2\omega}$$

$$C_{D_y^2 \rho}^{(2), \text{MRT1}} = (-2 - v_2^2 \omega_6 + 6c_s^2 + \omega_6 - 3\omega_6 c_s^2 + 2v_2^2) \frac{v_2}{2\omega_6}$$

$$C_{D_y^2 \rho}^{(2), \text{MRT2}} = C_{D_y^2 \rho}^{(2), \text{MRT1}}$$

$$C_{D_y^2 \rho}^{(2), \text{CLBM1}} = C_{D_y^2 \rho}^{(2), \text{MRT1}}$$

$$C_{D_y^2 \rho}^{(2), \text{CLBM2}} = C_{D_y^2 \rho}^{(2), \text{MRT1}}$$

$$C_{D_y^2 \rho}^{(2), \text{CuLBM1}} = (-2 - \omega_2 v_2^2 + \omega_2 + 6c_s^2 - 3\omega_2 c_s^2 + 2v_2^2) \frac{v_2}{2\omega_2}$$

$$C_{D_y^2 \rho}^{(2), \text{CuLBM2}} = (\omega_2 v_2^2 - \omega_2 + v_2^2 \omega_1 + \omega_2 \omega_1 + 3c_s^2 \omega_1 + 3\omega_2 c_s^2 - 3\omega_2 c_s^2 \omega_1 - \omega_2 v_2^2 \omega_1 - \omega_1) \frac{v_2}{2\omega_2 \omega_1}$$

coefficient $C_{D_y^2 v_2}^{(2)}$ **at** $\frac{\partial^2 v_2}{\partial x_2^2}$:

$$C_{D_y^2 v_2}^{(2), \text{SRT}} = (-2 - 3v_2^2 \omega + 2c_s^2 - c_s^2 \omega + 6v_2^2 + \omega) \frac{\rho}{2\omega}$$

$$C_{D_y^2 v_2}^{(2), \text{MRT1}} = (-2 - 3v_2^2 \omega_6 + 2c_s^2 + \omega_6 - \omega_6 c_s^2 + 6v_2^2) \frac{\rho}{2\omega_6}$$

$$C_{D_y^2 v_2}^{(2), \text{MRT2}} = C_{D_y^2 v_2}^{(2), \text{MRT1}}$$

$$C_{D_y^2 v_2}^{(2), \text{CLBM1}} = C_{D_y^2 v_2}^{(2), \text{MRT1}}$$

$$C_{D_y^2 v_2}^{(2), \text{CLBM2}} = C_{D_y^2 v_2}^{(2), \text{MRT1}}$$

$$C_{D_y^2 v_2}^{(2), \text{CuLBM1}} = (-2 - 3\omega_2 v_2^2 + \omega_2 + 2c_s^2 - \omega_2 c_s^2 + 6v_2^2) \frac{\rho}{2\omega_2}$$

$$C_{D_y^2 v_2}^{(2), \text{CuLBM2}} = (3\omega_2 v_2^2 - \omega_2 + 3v_2^2 \omega_1 + \omega_2 \omega_1 + c_s^2 \omega_1 + \omega_2 c_s^2 - \omega_2 c_s^2 \omega_1 - 3\omega_2 v_2^2 \omega_1 - \omega_1) \frac{\rho}{2\omega_2 \omega_1}$$

coefficient $C_{D_x^3 \rho}^{(2)}$ **at** $\frac{\partial^3 \rho}{\partial x_1^3}$:

$$C_{D_x^3 \rho}^{(2), \text{SRT}} = (-1 + v_1^2 + 3c_s^2) \frac{v_1 v_2}{12}$$

$$C_{D_x^3 \rho}^{(2), \text{MRT1}} = (-12\omega_5 v_1^2 + 12\omega_5 - 6v_1^2 \omega_7 \omega_4 + 6\omega_7 \omega_4 - \omega_5 \omega_7 \omega_4 + \omega_5 v_1^2 \omega_7 \omega_4 + 12v_1^2 \omega_7 + 18\omega_5 c_s^2 \omega_4 - 36\omega_5 c_s^2 + 36c_s^2 \omega_7 - 12\omega_7 - 6\omega_5 \omega_4 + 6\omega_5 v_1^2 \omega_4 - 18c_s^2 \omega_7 \omega_4 + 3\omega_5 c_s^2 \omega_7 \omega_4) \frac{v_1 v_2}{12\omega_5 \omega_7 \omega_4}$$

$$C_{D_x^3 \rho}^{(2), \text{MRT2}} = C_{D_x^3 \rho}^{(2), \text{MRT1}}$$

$$C_{D_x^3 \rho}^{(2), \text{CLBM1}} = C_{D_x^3 \rho}^{(2), \text{SRT}}$$

$$C_{D_x^3 \rho}^{(2), \text{CLBM2}} = C_{D_x^3 \rho}^{(2), \text{SRT}}$$

$$C_{D_x^3 \rho}^{(2), \text{CuLBM1}} = C_{D_x^3 \rho}^{(2), \text{SRT}}$$

$$C_{D_x^3 \rho}^{(2), \text{CuLBM2}} = C_{D_x^3 \rho}^{(2), \text{SRT}}$$

coefficient $C_{D_x^3 v_1}^{(2)}$ **at** $\frac{\partial^3 v_1}{\partial x_1^3}$:

$$C_{D_x^3 v_1}^{(2), \text{SRT}} = (-1 + 3v_1^2 + c_s^2) \frac{\rho v_2}{12}$$

$$C_{D_x^3 v_1}^{(2), \text{MRT1}} = (-36\omega_5 v_1^2 + 12\omega_5 - 18v_1^2 \omega_7 \omega_4 + 6\omega_7 \omega_4 - \omega_5 \omega_7 \omega_4 + 3\omega_5 v_1^2 \omega_7 \omega_4 + 36v_1^2 \omega_7 + 6\omega_5 c_s^2 \omega_4 - 12\omega_5 c_s^2 + 12c_s^2 \omega_7 - 12\omega_7 - 6\omega_5 \omega_4 + 18\omega_5 v_1^2 \omega_4 - 6c_s^2 \omega_7 \omega_4 + \omega_5 c_s^2 \omega_7 \omega_4) \frac{\rho v_2}{12\omega_5 \omega_7 \omega_4}$$

$$C_{D_x^3 v_1}^{(2), \text{MRT2}} = C_{D_x^3 v_1}^{(2), \text{MRT1}}$$

$$C_{D_x^3 v_1}^{(2), \text{CLBM1}} = C_{D_x^3 v_1}^{(2), \text{SRT}}$$

$$C_{D_x^3 v_1}^{(2), \text{CLBM2}} = C_{D_x^3 v_1}^{(2), \text{SRT}}$$

$$C_{D_x^3 v_1}^{(2), \text{CuLBM1}} = C_{D_x^3 v_1}^{(2), \text{SRT}}$$

$$C_{D_x^3 v_1}^{(2), \text{CuLBM2}} = C_{D_x^3 v_1}^{(2), \text{SRT}}$$

coefficient $C_{D_x^3 v_2}^{(2)}$ **at** $\frac{\partial^3 v_2}{\partial x_1^3}$:

$$C_{D_x^3 v_2}^{(2), \text{SRT}} = (6 - 3c_s^2 \omega^2 - 6v_1^2 - 18c_s^2 + 18c_s^2 \omega + 6v_1^2 \omega - 6\omega + \omega^2 - v_1^2 \omega^2) \frac{v_1 \rho}{6\omega^2}$$

$$C_{D_x^3 v_2}^{(2), \text{MRT1}} = (3c_s^2 \omega_4^2 + 3v_1^2 \omega_7 \omega_4 - 3\omega_7 \omega_4 + \omega_7 \omega_4^2 - 6c_s^2 \omega_4 - v_1^2 \omega_7 \omega_4^2 - 3c_s^2 \omega_7 \omega_4^2 - 12c_s^2 \omega_7 - 6v_1^2 \omega_4 + 3v_1^2 \omega_4^2 + 6\omega_4 - 3\omega_4^2 + 15c_s^2 \omega_7 \omega_4) \frac{v_1 \rho}{6\omega_7 \omega_4^2}$$

$$C_{D_x^3 v_2}^{(2), \text{MRT2}} = C_{D_x^3 v_2}^{(2), \text{MRT1}}$$

$$C_{D_x^3 v_2}^{(2), \text{CLBM1}} = (6 - 6v_1^2 - v_1^2 \omega_7 \omega_4 + \omega_7 \omega_4 + 3v_1^2 \omega_7 + 9c_s^2 \omega_4 - 18c_s^2 + 9c_s^2 \omega_7 + 3v_1^2 \omega_4 - 3\omega_7 - 3\omega_4 - 3c_s^2 \omega_7 \omega_4) \frac{v_1 \rho}{6\omega_7 \omega_4}$$

$$C_{D_x^3 v_2}^{(2), \text{CLBM2}} = C_{D_x^3 v_2}^{(2), \text{CLBM1}}$$

$$C_{D_x^3 v_2}^{(2), \text{CuLBM1}} = (6 + \omega_3 \omega_4 - v_1^2 \omega_3 \omega_4 - 3\omega_3 c_s^2 \omega_4 - 6v_1^2 + 9c_s^2 \omega_4 - 18c_s^2 - 3\omega_3 + 3v_1^2 \omega_4 - 3\omega_4 + 9\omega_3 c_s^2 + 3v_1^2 \omega_3) \frac{v_1 \rho}{6\omega_3 \omega_4}$$

$$C_{D_x^3 v_2}^{(2), \text{CuLBM2}} = (6 - 3\omega_3 c_s^2 \omega_1 + \omega_3 \omega_1 - 6v_1^2 - v_1^2 \omega_3 \omega_1 - 18c_s^2 + 9c_s^2 \omega_1 - 3\omega_3 + 3v_1^2 \omega_1 + 9\omega_3 c_s^2 + 3v_1^2 \omega_3 - 3\omega_1) \frac{v_1 \rho}{6\omega_3 \omega_1}$$

coefficient $C_{D_x^2 D_y \rho}^{(2)}$ **at** $\frac{\partial^3 \rho}{\partial x_1^2 \partial x_2}$:

$$C_{D_x^2 D_y \rho}^{(2), \text{SRT}} = (-12 + 12\omega - \omega^2) \frac{c_s^4}{6\omega^2}$$

$$C_{D_x^2 D_y \rho}^{(2), \text{MRT1}} = (-12 + 12\omega_4 - \omega_4^2) \frac{c_s^4}{6\omega_4^2}$$

$$C_{D_x^2 D_y \rho}^{(2), \text{MRT2}} = C_{D_x^2 D_y \rho}^{(2), \text{MRT1}}$$

$$C_{D_x^2 D_y \rho}^{(2), \text{CLBM1}} = C_{D_x^2 D_y \rho}^{(2), \text{MRT1}}$$

$$C_{D_x^2 D_y \rho}^{(2), \text{CLBM2}} = C_{D_x^2 D_y \rho}^{(2), \text{MRT1}}$$

$$C_{D_x^2 D_y \rho}^{(2), \text{CuLBM1}} = (-12 + 12\omega_3 - \omega_3^2) \frac{c_s^4}{6\omega_3^2}$$

$$C_{D_x^2 D_y \rho}^{(2), \text{CuLBM2}} = (-6c_s^2 \omega_1^2 + 9\omega_2^2 v_1^4 \omega_1 + 45\omega_2^2 v_1^2 c_s^2 \omega_1 + 9\omega_2 v_1^2 \omega_1^2 + 45v_1^2 c_s^2 \omega_1^2 + 9\omega_2^2 v_1^2 - 6\omega_2^2 c_s^2 \omega_1 + 9v_1^4 \omega_1^2 - 45\omega_2^2 v_1^2 c_s^2 - 6\omega_2 c_s^4 \omega_1^2 + 6\omega_2^2 c_s^2 - 2\omega_2^2 c_s^4 \omega_1^2 - 9\omega_2^2 v_1^2 \omega_1 + 6c_s^4 \omega_1^2 - 9\omega_2 v_1^4 \omega_1^2 - 30\omega_2^2 c_s^4 - 9\omega_2^2 v_1^4 - 45\omega_2 v_1^2 c_s^2 \omega_1^2 - 9v_1^2 \omega_1^2 + 30\omega_2^2 c_s^4 \omega_1 + 6\omega_2^2 c_s^2 \omega_1^2) \frac{1}{12\omega_2^2 \omega_1^2}$$

coefficient $C_{D_x^2 D_y v_1}^{(2)}$ **at** $\frac{\partial^3 v_1}{\partial x_1^2 \partial x_2}$:

$$C_{D_x^2 D_y v_1}^{(2), \text{SRT}} = 0$$

$$C_{D_x^2 D_y v_1}^{(2), \text{MRT1}} = (\omega_7 \omega_4 - 2\omega_7 + 2\omega_4 - \omega_4^2) \frac{v_1 \rho c_s^2}{\omega_7 \omega_4^2}$$

$$C_{D_x^2 D_y v_1}^{(2), \text{MRT2}} = C_{D_x^2 D_y v_1}^{(2), \text{MRT1}}$$

$$C_{D_x^2 D_y v_1}^{(2), \text{CLBM1}} = 0$$

$$C_{D_x^2 D_y v_1}^{(2), \text{CLBM2}} = 0$$

$$C_{D_x^2 D_y v_1}^{(2), \text{CuLBM1}} = 0$$

$$C_{D_x^2 D_y v_1}^{(2), \text{CuLBM2}} = (9c_s^2\omega_1^2 + 5\omega_2\omega_1^2 - 11\omega_2v_1^2\omega_1^2 - 11\omega_2^2v_1^2 + 5\omega_2^2 + 9\omega_2^2c_s^2\omega_1 - 9\omega_2^2c_s^2 - 5\omega_2^2\omega_1 + 11\omega_2^2v_1^2\omega_1 - 5\omega_1^2 + 11v_1^2\omega_1^2 - 9\omega_2c_s^2\omega_1^2) \frac{v_1\rho}{4\omega_2^2\omega_1^2}$$

coefficient $C_{D_x^2 D_y v_2}^{(2)}$ **at** $\frac{\partial^3 v_2}{\partial x_1^2 \partial x_2}$:

$$C_{D_x^2 D_y v_2}^{(2), \text{SRT}} = \frac{-\rho v_2 c_s^2}{6}$$

$$C_{D_x^2 D_y v_2}^{(2), \text{MRT1}} = (-12\omega_6\omega_4^2 + 12\omega_6\omega_4 - \omega_8\omega_6\omega_4^2 - 12\omega_8\omega_6 + 12\omega_8\omega_6\omega_4 + 12\omega_4^2 - 12\omega_8\omega_4) \frac{\rho v_2 c_s^2}{6\omega_8\omega_6\omega_4^2}$$

$$C_{D_x^2 D_y v_2}^{(2), \text{MRT2}} = C_{D_x^2 D_y v_2}^{(2), \text{MRT1}}$$

$$C_{D_x^2 D_y v_2}^{(2), \text{CLBM1}} = C_{D_x^2 D_y v_2}^{(2), \text{SRT}}$$

$$C_{D_x^2 D_y v_2}^{(2), \text{CLBM2}} = C_{D_x^2 D_y v_2}^{(2), \text{SRT}}$$

$$C_{D_x^2 D_y v_2}^{(2), \text{CuLBM1}} = C_{D_x^2 D_y v_2}^{(2), \text{SRT}}$$

$$C_{D_x^2 D_y v_2}^{(2), \text{CuLBM2}} = C_{D_x^2 D_y v_2}^{(2), \text{SRT}}$$

coefficient $C_{D_x D_y^2 \rho}^{(2)}$ **at** $\frac{\partial^3 \rho}{\partial x_1 \partial x_2^2}$:

$$C_{D_x D_y^2 \rho}^{(2), \text{SRT}} = 0$$

$$C_{D_x D_y^2 \rho}^{(2), \text{MRT1}} = (v_2^2\omega_6^2\omega_4 + 3\omega_6^2c_s^2\omega_4 + 3\omega_8\omega_6c_s^2 - 3\omega_6^2c_s^2 - v_2^2\omega_6^2 + \omega_6\omega_4 - \omega_6^2\omega_4 - \omega_8\omega_6 + \omega_8v_2^2\omega_6 + \omega_6^2 + 3\omega_8c_s^2\omega_4 - 3\omega_6c_s^2\omega_4 - \omega_8v_2^2\omega_6\omega_4 + \omega_8\omega_6\omega_4 + \omega_8v_2^2\omega_4 - v_2^2\omega_6\omega_4 - \omega_8\omega_4 - 3\omega_8\omega_6c_s^2\omega_4) \frac{v_1v_2}{\omega_8\omega_6^2\omega_4}$$

$$C_{D_x D_y^2 \rho}^{(2), \text{MRT2}} = C_{D_x D_y^2 \rho}^{(2), \text{MRT1}}$$

$$C_{D_x D_y^2 \rho}^{(2), \text{CLBM1}} = 0$$

$$C_{D_x D_y^2 \rho}^{(2), \text{CLBM2}} = 0$$

$$C_{D_x D_y^2 \rho}^{(2), \text{CuLBM1}} = 0$$

$$C_{D_x D_y^2 \rho}^{(2), \text{CuLBM2}} =$$

$$(6c_s^2\omega_1^2 + \omega_2\omega_1^2 - \omega_2v_1^2\omega_1^2 - \omega_2^2v_1^2 + v_2^2\omega_1^2 + 3\omega_2^2c_s^2\omega_1 + 2\omega_2\omega_1 - \omega_2^2\omega_1 - 6\omega_2c_s^2\omega_1 - 2\omega_2v_2^2\omega_1 + \omega_2^2v_1^2\omega_1 - 2\omega_1^2 + v_1^2\omega_1^2 - 3\omega_2c_s^2\omega_1^2 + \omega_2^2v_2^2) \frac{3v_1v_2}{4\omega_2^2\omega_1^2}$$

coefficient $C_{D_x D_y^2 v_1}^{(2)}$ **at** $\frac{\partial^3 v_1}{\partial x_1 \partial x_2^2}$:

$$C_{D_x D_y^2 v_1}^{(2), \text{SRT}} = (12 - 11c_s^2\omega^2 - 3v_2^2\omega^2 + 12v_2^2\omega - 36c_s^2 + 36c_s^2\omega - 12v_2^2 - 12\omega + 3\omega^2) \frac{\rho v_2}{12\omega^2}$$

$$C_{D_x D_y^2 v_1}^{(2), \text{MRT1}} = (-6\omega_8\omega_6^2\omega_4 + 12\omega_6\omega_4^2 - 12v_2^2\omega_6^2\omega_4 - 12\omega_6^2c_s^2\omega_4 + 42\omega_8\omega_6^2c_s^2\omega_4 + 6\omega_8v_2^2\omega_6^2\omega_4 - 11\omega_8\omega_6^2c_s^2\omega_4^2 - 3\omega_8v_2^2\omega_6^2\omega_4^2 + 12v_2^2\omega_6^2\omega_4^2 + 12\omega_6^2c_s^2\omega_4^2 + 3\omega_8\omega_6^2\omega_4^2 - 12\omega_8\omega_4^2 - 12v_2^2\omega_6\omega_4^2 - 18\omega_8\omega_6c_s^2\omega_4^2 + 6\omega_8\omega_6\omega_4^2 + 12\omega_6^2\omega_4 + 12\omega_8v_2^2\omega_4^2 - 12\omega_6c_s^2\omega_4^2 - 6\omega_8v_2^2\omega_6\omega_4^2 + 36\omega_8c_s^2\omega_4^2 - 24\omega_8\omega_6^2c_s^2 - 12\omega_6^2\omega_4^2 - 24\omega_8\omega_6c_s^2\omega_4) \frac{\rho v_2}{12\omega_8\omega_6^2\omega_4^2}$$

$$C_{D_x D_y^2 v_1}^{(2), \text{MRT2}} = C_{D_x D_y^2 v_1}^{(2), \text{MRT1}}$$

$$C_{D_x D_y^2 v_1}^{(2), \text{CLBM1}} = (3\omega_8\omega_6^2\omega_4 + 12v_2^2\omega_6^2\omega_4 + 36\omega_6^2c_s^2\omega_4 - 11\omega_8\omega_6^2c_s^2\omega_4 - 3\omega_8v_2^2\omega_6^2\omega_4 - 36\omega_6^2c_s^2 - 12v_2^2\omega_6^2 + 12\omega_6\omega_4 - 12\omega_6^2\omega_4 + 12\omega_2^2 + 6\omega_8v_2^2\omega_6^2 + 18\omega_8\omega_6^2c_s^2 + 36\omega_8c_s^2\omega_4 - 36\omega_6c_s^2\omega_4 - 6\omega_8\omega_6^2 - 6\omega_8v_2^2\omega_6\omega_4 + 6\omega_8\omega_6\omega_4 + 12\omega_8v_2^2\omega_4 - 12v_2^2\omega_6\omega_4 - 12\omega_8\omega_4 - 18\omega_8\omega_6c_s^2\omega_4) \frac{\rho v_2}{12\omega_8\omega_6^2\omega_4}$$

$$C_{D_x D_y^2 v_1}^{(2), \text{CLBM2}} = C_{D_x D_y^2 v_1}^{(2), \text{CLBM1}}$$

$$C_{D_x D_y^2 v_1}^{(2), \text{CuLBM1}} = (6\omega_2^2 v_2^2 \omega_6 - 12\omega_6 \omega_3 + 12\omega_2 \omega_3 - 11\omega_2^2 \omega_6 \omega_3 c_s^2 + 12\omega_2^2 v_2^2 \omega_3 + 12\omega_2^2 + 3\omega_2^2 \omega_6 \omega_3 - 36\omega_2 \omega_3 c_s^2 + 36\omega_6 \omega_3 c_s^2 - 6\omega_2 v_2^2 \omega_6 \omega_3 - 36\omega_2^2 c_s^2 + 18\omega_2^2 \omega_6 c_s^2 + 12v_2^2 \omega_6 \omega_3 - 18\omega_2 \omega_6 \omega_3 c_s^2 - 12\omega_2^2 \omega_3 + 6\omega_2 \omega_6 \omega_3 - 12\omega_2 v_2^2 \omega_3 - 3\omega_2^2 v_2^2 \omega_6 \omega_3 - 6\omega_2^2 \omega_6 - 12\omega_2^2 v_2^2 + 36\omega_2^2 \omega_3 c_s^2) \frac{\rho v_2}{12\omega_2^2 \omega_6 \omega_3}$$

$$C_{D_x D_y^2 v_1}^{(2), \text{CuLBM2}} = (-12\omega_2^2 \omega_3 \omega_1 - 11\omega_2^2 \omega_3 c_s^2 \omega_1^2 + 6\omega_2 \omega_1^2 + 12\omega_2^2 v_2^2 \omega_1^2 - 3\omega_2 v_2^2 \omega_3 \omega_1^2 + 6\omega_2^2 v_2^2 \omega_3 + 36\omega_2^2 c_s^2 \omega_1^2 - 54\omega_2^2 c_s^2 \omega_1 + 27v_1^2 \omega_3 \omega_1^2 - 15\omega_3 \omega_1^2 + 27\omega_2^2 v_1^2 \omega_3 \omega_1 + 3\omega_2^2 \omega_3 \omega_1^2 + 18\omega_2^2 \omega_3 c_s^2 \omega_1 - 18\omega_2^2 v_2^2 \omega_1 + 27\omega_3 c_s^2 \omega_1^2 + 18\omega_2^2 \omega_1 - 18\omega_2 \omega_3 c_s^2 \omega_1^2 + 12\omega_2 \omega_3 \omega_1^2 - 27\omega_2 v_1^2 \omega_3 \omega_1^2 - 3\omega_2^2 v_2^2 \omega_3 \omega_1^2 + 3\omega_2^2 \omega_3 + 3\omega_2^2 v_2^2 \omega_3 \omega_1 - 6\omega_2 v_2^2 \omega_1^2 + 6v_2^2 \omega_3 \omega_1^2 - 27\omega_2^2 v_1^2 \omega_3 - 12\omega_2^2 \omega_1^2 - 18\omega_2 c_s^2 \omega_1^2 + 9\omega_2^2 \omega_3 c_s^2) \frac{\rho v_2}{12\omega_2^2 \omega_3 \omega_1^2}$$

coefficient $C_{D_x D_y^2 v_2}^{(2)}$ **at** $\frac{\partial^3 v_2}{\partial x_1 \partial x_2^2}$:

$$C_{D_x D_y^2 v_2}^{(2), \text{SRT}} = 0$$

$$C_{D_x D_y^2 v_2}^{(2), \text{MRT1}} = (3v_2^2 \omega_6^2 \omega_4 + \omega_6^2 c_s^2 \omega_4 + \omega_8 \omega_6 c_s^2 - \omega_6^2 c_s^2 - 3v_2^2 \omega_6^2 + \omega_6 \omega_4 - \omega_6^2 \omega_4 - \omega_8 \omega_6 + 3\omega_8 v_2^2 \omega_6 + \omega_6^2 + \omega_8 c_s^2 \omega_4 - \omega_6 c_s^2 \omega_4 - 3\omega_8 v_2^2 \omega_6 \omega_4 + \omega_8 \omega_6 \omega_4 + 3\omega_8 v_2^2 \omega_4 - 3v_2^2 \omega_6 \omega_4 - \omega_8 \omega_4 - \omega_8 \omega_6 c_s^2 \omega_4) \frac{v_1 \rho}{\omega_8 \omega_6^2 \omega_4}$$

$$C_{D_x D_y^2 v_2}^{(2), \text{MRT2}} = C_{D_x D_y^2 v_2}^{(2), \text{MRT1}}$$

$$C_{D_x D_y^2 v_2}^{(2), \text{CLBM1}} = 0$$

$$C_{D_x D_y^2 v_2}^{(2), \text{CLBM2}} = 0$$

$$C_{D_x D_y^2 v_2}^{(2), \text{CuLBM1}} = 0$$

$$C_{D_x D_y^2 v_2}^{(2), \text{CuLBM2}} = (-\omega_2^2 \omega_3 \omega_1 + 2\omega_2 \omega_1^2 - 2\omega_2 v_1^2 \omega_1^2 + 9\omega_2^2 v_2^2 \omega_3 + 6\omega_2^2 c_s^2 \omega_1 + 2v_1^2 \omega_3 \omega_1^2 - 5\omega_3 \omega_1^2 - 18\omega_2 v_2^2 \omega_3 \omega_1 + \omega_2^2 v_1^2 \omega_3 \omega_1 + 3\omega_2^2 \omega_3 c_s^2 \omega_1 + 9\omega_3 c_s^2 \omega_1^2 - 2\omega_2^2 \omega_1 - 3\omega_2 \omega_3 c_s^2 \omega_1^2 + 2\omega_2^2 v_1^2 \omega_1 + \omega_2 \omega_3 \omega_1^2 - \omega_2 v_1^2 \omega_3 \omega_1^2 - \omega_2^2 \omega_3 + 9v_2^2 \omega_3 \omega_1^2 + 6\omega_2 \omega_3 \omega_1 - 2\omega_2^2 v_1^2 \omega_3 - 6\omega_2 c_s^2 \omega_1^2 - 6\omega_2 \omega_3 c_s^2 \omega_1 - 3\omega_2^2 \omega_3 c_s^2) \frac{v_1 \rho}{4\omega_2^2 \omega_3 \omega_1^2}$$

coefficient $C_{D_y^3 \rho}^{(2)}$ **at** $\frac{\partial^3 \rho}{\partial x_2^3}$:

$$C_{D_y^3 \rho}^{(2), \text{SRT}} = (-c_s^2 \omega^2 + 36v_2^4 - 7v_2^2 \omega^2 + 36v_2^2 \omega - 12c_s^2 + 12c_s^2 \omega + 12c_s^4 + 24v_2^2 c_s^2 \omega^2 + 7v_2^4 \omega^2 + c_s^4 \omega^2 + 144v_2^2 c_s^2 - 12c_s^4 \omega - 36v_2^2 - 144v_2^2 c_s^2 \omega - 36v_2^4 \omega) \frac{1}{12\omega^2}$$

$$C_{D_y^3 \rho}^{(2), \text{MRT1}} = (36v_2^4 + 36v_2^2 \omega_6 - 144v_2^2 \omega_6 c_s^2 - \omega_6^2 c_s^2 - 12c_s^2 - 7v_2^2 \omega_6^2 - 12\omega_6 c_s^4 + \omega_6^2 c_s^4 - 36v_2^4 \omega_6 + 12c_s^4 + 144v_2^2 c_s^2 + 12\omega_6 c_s^2 - 36v_2^2 + 24v_2^2 \omega_6^2 c_s^2 + 7v_2^4 \omega_6^2) \frac{1}{12\omega_6^2}$$

$$C_{D_y^3 \rho}^{(2), \text{MRT2}} = C_{D_y^3 \rho}^{(2), \text{MRT1}}$$

$$C_{D_y^3 \rho}^{(2), \text{CLBM1}} = C_{D_y^3 \rho}^{(2), \text{MRT1}}$$

$$C_{D_y^3 \rho}^{(2), \text{CLBM2}} = C_{D_y^3 \rho}^{(2), \text{MRT1}}$$

$$C_{D_y^3 \rho}^{(2), \text{CuLBM1}} = (36v_2^4 + 36\omega_2 v_2^2 + 7\omega_2^2 v_2^4 - 144\omega_2 v_2^2 c_s^2 - 12c_s^2 - 12\omega_2 c_s^4 - \omega_2^2 c_s^2 + 24\omega_2^2 v_2^2 c_s^2 + 12c_s^4 + 12\omega_2 c_s^2 + 144v_2^2 c_s^2 + \omega_2^2 c_s^4 - 36v_2^2 - 36\omega_2 v_2^4 - 7\omega_2^2 v_2^2) \frac{1}{12\omega_2^2}$$

$$C_{D_y^3 \rho}^{(2), \text{CuLBM2}} = (18\omega_2 v_2^4 \omega_1 + 54\omega_2 v_2^2 c_s^2 \omega_1 - 6c_s^2 \omega_1^2 - 7\omega_2^2 v_2^2 \omega_1^2 + 9\omega_2^2 v_2^4 - 9v_2^2 \omega_1^2 - \omega_2^2 c_s^2 \omega_1^2 + 6\omega_2^2 c_s^2 \omega_1 - 6\omega_2 c_s^4 \omega_1^2 - 6\omega_2^2 c_s^2 + 18\omega_2^2 v_2^2 \omega_1 - 72\omega_2 v_2^2 c_s^2 \omega_1^2 - 18\omega_2 v_2^4 \omega_1^2 + 45\omega_2^2 v_2^2 c_s^2 + 45v_2^2 c_s^2 \omega_1^2 + 9v_2^4 \omega_1^2 + \omega_2^2 c_s^4 \omega_1^2 - 72\omega_2^2 v_2^2 c_s^2 \omega_1 - 18\omega_2 v_2^2 \omega_1 + 6c_s^4 \omega_1^2 + 6\omega_2^2 c_s^4 + 7\omega_2^2 v_2^4 \omega_1^2 - 18\omega_2^2 v_2^4 \omega_1 + 18\omega_2 v_2^2 \omega_1^2 - 6\omega_2^2 c_s^4 \omega_1 + 24\omega_2^2 v_2^2 c_s^2 \omega_1^2 + 6\omega_2 c_s^2 \omega_1^2 - 9\omega_2^2 v_2^2) \frac{1}{12\omega_2^2 \omega_1^2}$$

coefficient $C_{D_y^3 v_2}^{(2)}$ **at** $\frac{\partial^3 v_2}{\partial x_2^3}$:

$$C_{D_y^3 v_2}^{(2), \text{SRT}} = (-24 + 5c_s^2 \omega^2 + 11v_2^2 \omega^2 - 60v_2^2 \omega + 36c_s^2 - 36c_s^2 \omega + 60v_2^2 + 24\omega - 4\omega^2) \frac{\rho v_2}{6\omega^2}$$

$$C_{D_y^3 v_2}^{(2), \text{MRT}1} = (-24 - 60v_2^2\omega_6 + 5\omega_6^2c_s^2 + 36c_s^2 + 11v_2^2\omega_6^2 + 24\omega_6 - 4\omega_6^2 - 36\omega_6c_s^2 + 60v_2^2)\frac{\rho v_2}{6\omega_6^2}$$

$$C_{D_y^3 v_2}^{(2), \text{MRT}2} = C_{D_y^3 v_2}^{(2), \text{MRT}1}$$

$$C_{D_y^3 v_2}^{(2), \text{CLBM}1} = C_{D_y^3 v_2}^{(2), \text{MRT}1}$$

$$C_{D_y^3 v_2}^{(2), \text{CLBM}2} = C_{D_y^3 v_2}^{(2), \text{MRT}1}$$

$$C_{D_y^3 v_2}^{(2), \text{CuLBM}1} = (-24 - 60\omega_2v_2^2 + 24\omega_2 - 4\omega_2^2 + 36c_s^2 + 5\omega_2^2c_s^2 - 36\omega_2c_s^2 + 60v_2^2 + 11\omega_2^2v_2^2)\frac{\rho v_2}{6\omega_2^2}$$

$$C_{D_y^3 v_2}^{(2), \text{CuLBM}2} = (27c_s^2\omega_1^2 + 24\omega_2\omega_1^2 + 22\omega_2^2v_2^2\omega_1^2 - 15\omega_1^2 + 33v_2^2\omega_1^2 + 10\omega_2^2c_s^2\omega_1^2 - 36\omega_2^2c_s^2\omega_1 - 18\omega_2\omega_1 + 27\omega_2^2c_s^2 - 60\omega_2^2v_2^2\omega_1 + 24\omega_2^2\omega_1 + 18\omega_2c_s^2\omega_1 + 54\omega_2v_2^2\omega_1 - 15\omega_1^2 - 60\omega_2v_2^2\omega_1^2 - 8\omega_2^2\omega_1^2 - 36\omega_2c_s^2\omega_1^2 + 33\omega_2^2v_2^2)\frac{\rho v_2}{12\omega_2^2\omega_1^2}$$

coefficient $C_{D_x^4 \rho}^{(2)}$ **at** $\frac{\partial^4 \rho}{\partial x_1^4}$:

$$C_{D_x^4 \rho}^{(2), \text{SRT}} = (-3v_1^4\omega - 6v_1^2 - 2c_s^2 + c_s^2\omega + 24v_1^2c_s^2 - 12v_1^2c_s^2\omega + 3v_1^2\omega + 2c_s^4 - c_s^4\omega + 6v_1^4)\frac{v_2}{24\omega}$$

$$C_{D_x^4 \rho}^{(2), \text{MRT}1} = (-12\omega_5^2v_1^2\omega_4^2 + 24\omega_5v_1^4\omega_7\omega_4^2 - 24c_s^2\omega_7^2\omega_4 + 12\omega_5^2c_s^2\omega_7\omega_4^2 - 432\omega_5v_1^2c_s^2\omega_7\omega_4 + 48\omega_5c_s^2\omega_7\omega_4 + 48\omega_5^2c_s^4\omega_7\omega_4 - 48\omega_5^2v_1^4\omega_7 + 14\omega_5c_s^4\omega_7^2\omega_4^2 - 48\omega_5v_1^2\omega_7^2\omega_4^2 + 3\omega_5^2v_1^2\omega_7^2\omega_4^2 - 24\omega_5^2c_s^4\omega_4 + 288v_1^2c_s^2\omega_7^2\omega_4 + 48\omega_5v_1^2\omega_7\omega_4 - 12c_s^4\omega_7^2\omega_4^2 - 216\omega_5^2v_1^2c_s^2\omega_7 - 144v_1^2c_s^2\omega_7^2\omega_4^2 + 12\omega_5^2c_s^4\omega_4^2 + 24c_s^4\omega_7^2\omega_4 - 24\omega_5v_1^2\omega_7\omega_4^2 - 12\omega_5^2c_s^4\omega_7\omega_4^2 - 12\omega_5^2v_1^2c_s^2\omega_7^2\omega_4^2 - 48\omega_5c_s^4\omega_7^2\omega_4 + 150\omega_5v_1^2c_s^2\omega_7^2\omega_4^2 - 48\omega_5^2c_s^2\omega_7\omega_4 - 14\omega_5c_s^2\omega_7^2\omega_4^2 + 24\omega_5^2c_s^2\omega_7 + 24\omega_5c_s^4\omega_7^2 + 24\omega_5^2v_1^4\omega_4 - 3\omega_5^2v_1^4\omega_7^2\omega_4^2 + 12c_s^2\omega_7^2\omega_4^2 - 48\omega_5v_1^4\omega_7\omega_4 + 72\omega_5v_1^2c_s^2\omega_7\omega_4^2 + 48\omega_5^2v_1^2\omega_7 + 72v_1^4\omega_7^2\omega_4 + 48\omega_5v_1^4\omega_7^2 - 30\omega_5^2v_1^4\omega_7\omega_4^2 + 24\omega_5^2c_s^2\omega_4 - 96\omega_5v_1^4\omega_7^2\omega_4 + 12\omega_5^2v_1^4\omega_4^2 - 96\omega_5^2v_1^2\omega_7\omega_4 - 36\omega_5v_1^2\omega_7^2\omega_4^2 - 126\omega_5^2v_1^2c_s^2\omega_7\omega_4^2 + 72\omega_5^2v_1^2c_s^2\omega_4^2 - \omega_5^2c_s^4\omega_7^2\omega_4^2 + 36v_1^2\omega_7^2\omega_4^2 + 432\omega_5^2v_1^2c_s^2\omega_7\omega_4 - 144\omega_5^2v_1^2c_s^2\omega_4 - 72v_1^2\omega_7^2\omega_4 - 24\omega_5^2c_s^4\omega_7 - 24\omega_5c_s^2\omega_7^2 + 30\omega_5^2v_1^2\omega_7\omega_4^2 + 216\omega_5v_1^2c_s^2\omega_7^2 - 24\omega_5^2v_1^4\omega_4 + 96\omega_5v_1^2\omega_7^2\omega_4 + 96\omega_5^2v_1^4\omega_7\omega_4 - 12\omega_5^2c_s^2\omega_4^2 + 36\omega_5v_1^4\omega_7^2\omega_4 - 144\omega_5v_1^2c_s^2\omega_7\omega_4 + \omega_5^2c_s^2\omega_7^2\omega_4^2 - 36v_1^4\omega_7^2\omega_4^2)\frac{v_2}{24\omega_5^2\omega_7^2\omega_4^2}$$

$$C_{D_x^4 \rho}^{(2), \text{MRT}2} = C_{D_x^4 \rho}^{(2), \text{MRT}1}$$

$$C_{D_x^4 \rho}^{(2), \text{CLBM}1} = (3\omega_5v_1^2 - 6v_1^2 - 2c_s^2 - 12\omega_5v_1^2c_s^2 + \omega_5c_s^2 + 24v_1^2c_s^2 + 2c_s^4 - \omega_5c_s^4 - 3\omega_5v_1^4 + 6v_1^4)\frac{v_2}{24\omega_5}$$

$$C_{D_x^4 \rho}^{(2), \text{CLBM}2} = C_{D_x^4 \rho}^{(2), \text{CLBM}1}$$

$$C_{D_x^4 \rho}^{(2), \text{CuLBM}1} = (-6v_1^2 - 3v_1^4\omega_1 - 2c_s^2 - 12v_1^2c_s^2\omega_1 + c_s^2\omega_1 + 24v_1^2c_s^2 + 2c_s^4 + 3v_1^2\omega_1 - c_s^4\omega_1 + 6v_1^4)\frac{v_2}{24\omega_1}$$

$$C_{D_x^4 \rho}^{(2), \text{CuLBM}2} = (3\omega_2v_1^4 - \omega_2c_s^4\omega_1 + 3v_1^4\omega_1 + \omega_2c_s^4 + 3\omega_2v_1^2\omega_1 + 12v_1^2c_s^2\omega_1 - c_s^2\omega_1 + 12\omega_2v_1^2c_s^2 - \omega_2c_s^2 + \omega_2c_s^2\omega_1 - 3v_1^2\omega_1 - 12\omega_2v_1^2c_s^2\omega_1 - 3\omega_2v_1^4\omega_1 + c_s^4\omega_1 - 3\omega_2v_1^2)\frac{v_2}{24\omega_2\omega_1}$$

coefficient $C_{D_x^4 v_1}^{(2)}$ **at** $\frac{\partial^4 v_1}{\partial x_1^4}$:

$$C_{D_x^4 v_1}^{(2), \text{SRT}} = (-4 + 10v_1^2 + 6c_s^2 - 3c_s^2\omega - 5v_1^2\omega + 2\omega)\frac{v_1\rho v_2}{12\omega}$$

$$C_{D_x^4 v_1}^{(2), \text{MRT}1} = (24\omega_5^2v_1^2\omega_4^2 + 72c_s^2\omega_7^2\omega_4 - 33\omega_5^2c_s^2\omega_7\omega_4^2 - 120\omega_5c_s^2\omega_7^2\omega_4 - 12\omega_5^2\omega_4^2 - 36\omega_5\omega_7^2 + 84\omega_5v_1^2\omega_7^2 - 5\omega_5^2v_1^2\omega_7^2\omega_4^2 + 24\omega_5\omega_7\omega_4 - 72\omega_5v_1^2\omega_7\omega_4 - 12\omega_5\omega_7\omega_4^2 + 36\omega_5v_1^2\omega_7\omega_4^2 + 24\omega_5^2\omega_4 + 2\omega_5^2\omega_7^2\omega_4^2 + 120\omega_5^2c_s^2\omega_7\omega_4 + 39\omega_5c_s^2\omega_7^2\omega_4^2 - 60\omega_5^2c_s^2\omega_7 - 48\omega_5^2v_1^2\omega_4 - 36c_s^2\omega_7^2\omega_4^2 + 21\omega_5^2\omega_7\omega_4^2 + 12\omega_5c_s^2\omega_7\omega_4^2 - 84\omega_5^2v_1^2\omega_7 - 48\omega_5^2c_s^2\omega_4 + 24\omega_7^2\omega_4^2 + 36\omega_5^2\omega_7 + 168\omega_5^2v_1^2\omega_7\omega_4 + 61\omega_5v_1^2\omega_7^2\omega_4^2 - 25\omega_5\omega_7^2\omega_4^2 - 60v_1^2\omega_7^2\omega_4^2 + 120v_1^2\omega_7^2\omega_4 + 60\omega_5c_s^2\omega_7^2 - 51\omega_5^2v_1^2\omega_7\omega_4^2 - 48\omega_7^2\omega_4 - 168\omega_5v_1^2\omega_7^2\omega_4 + 72\omega_5\omega_7^2\omega_4 + 24\omega_5^2c_s^2\omega_4^2 - 3\omega_5^2c_s^2\omega_7^2\omega_4^2 - 72\omega_5^2\omega_7\omega_4 - 24\omega_5c_s^2\omega_7\omega_4)\frac{v_1\rho v_2}{12\omega_5^2\omega_7^2\omega_4^2}$$

$$C_{D_x^4 v_1}^{(2), \text{MRT}2} = C_{D_x^4 v_1}^{(2), \text{MRT}1}$$

$$C_{D_x^4 v_1}^{(2), \text{CLBM}1} = (-4 - 5\omega_5v_1^2 + 2\omega_5 + 10v_1^2 + 6c_s^2 - 3\omega_5c_s^2)\frac{v_1\rho v_2}{12\omega_5}$$

$$C_{D_x^4 v_1}^{(2), \text{CLBM}2} = C_{D_x^4 v_1}^{(2), \text{CLBM}1}$$

$$C_{D_x^4 v_1}^{(2), \text{CuLBM}1} = (-4 + 10v_1^2 + 6c_s^2 - 3c_s^2\omega_1 - 5v_1^2\omega_1 + 2\omega_1)\frac{v_1\rho v_2}{12\omega_1}$$

$$C_{D_x^4 v_1}^{(2), \text{CuLBM2}} = (-2\omega_2 + 2\omega_2\omega_1 - 5\omega_2 v_1^2\omega_1 + 3c_s^2\omega_1 + 3\omega_2 c_s^2 - 3\omega_2 c_s^2\omega_1 + 5v_1^2\omega_1 - 2\omega_1 + 5\omega_2 v_1^2) \frac{v_1 \rho v_2}{12\omega_2\omega_1}$$

$$\text{coefficient } C_{D_x^4 v_2}^{(2)} \text{ at } \frac{\partial^4 v_2}{\partial x_1^4} :$$

$$C_{D_x^4 v_2}^{(2), \text{SRT}} = (-84v_1^2 c_s^2 \omega^2 - 14c_s^2 \omega^2 + 108v_1^4 \omega + 6v_1^2 c_s^2 \omega^3 + 72v_1^2 + c_s^2 \omega^3 - 42v_1^4 \omega^2 - 24c_s^2 + 36c_s^2 \omega - 144v_1^2 c_s^2 + 216v_1^2 c_s^2 \omega + 3v_1^4 \omega^3 - 108v_1^2 \omega + 48c_s^4 - 3c_s^4 \omega^3 + 30c_s^4 \omega^2 - 72c_s^4 \omega - 3v_1^2 \omega^3 + 42v_1^2 \omega^2 - 72v_1^4) \frac{\rho}{24\omega^3}$$

$$C_{D_x^4 v_2}^{(2), \text{MRT1}} = (12c_s^2 \omega_7^2 \omega_4 - 3c_s^4 \omega_7^2 \omega_4^3 + 12v_1^2 c_s^2 \omega_4^3 + 72v_1^4 \omega_7 \omega_4^2 + 48v_1^2 \omega_7 \omega_4 - 18v_1^4 \omega_7 \omega_4^3 + 156v_1^2 c_s^2 \omega_7^2 \omega_4 - 24v_1^2 c_s^2 \omega_4^2 + 24c_s^4 \omega_7^2 \omega_4^2 - 72v_1^2 c_s^2 \omega_7^2 \omega_4^2 - 48c_s^4 \omega_7^2 \omega_4 + 12v_1^4 \omega_4^3 + c_s^2 \omega_7^2 \omega_4^3 - 72v_1^2 \omega_7 \omega_4^2 - 24v_1^4 \omega_4^2 + 6v_1^2 c_s^2 \omega_7^2 \omega_4^3 - 48v_1^4 \omega_7 \omega_4 + 18v_1^2 \omega_7 \omega_4^3 + 24c_s^4 \omega_7^2 - 8c_s^2 \omega_7^2 \omega_4^2 + 24v_1^4 \omega_7^2 \omega_4 - 12v_1^2 c_s^2 \omega_7 \omega_4^3 - 3v_1^2 \omega_7^2 \omega_4^3 + 24c_s^2 \omega_7 \omega_4^2 + 48v_1^2 c_s^2 \omega_7 \omega_4^2 + 24c_s^4 \omega_7 \omega_4 - 6c_s^2 \omega_7 \omega_4^3 + 24v_1^2 \omega_7^2 \omega_4^2 + 24v_1^2 \omega_4^2 - 96v_1^2 c_s^2 \omega_7^2 - 24v_1^2 \omega_7^2 \omega_4 - 24v_1^2 c_s^2 \omega_7 \omega_4 + 3v_1^4 \omega_7^2 \omega_4^3 - 24c_s^2 \omega_7 \omega_4^2 - 12v_1^2 \omega_4^3 - 24c_s^2 \omega_7 \omega_4 + 6c_s^2 \omega_7 \omega_4^3 - 24v_1^4 \omega_7^2 \omega_4^2) \frac{\rho}{24\omega_7^2 \omega_4^3}$$

$$C_{D_x^4 v_2}^{(2), \text{MRT2}} = C_{D_x^4 v_2}^{(2), \text{MRT1}}$$

$$C_{D_x^4 v_2}^{(2), \text{CLBM1}} = (12c_s^2 \omega_7^2 \omega_4 - 3c_s^4 \omega_7^2 \omega_4^3 + 108v_1^2 c_s^2 \omega_4^3 + 72v_1^4 \omega_7 \omega_4^2 - 30v_1^4 \omega_7 \omega_4^3 - 36v_1^2 c_s^2 \omega_7^2 \omega_4 - 216v_1^2 c_s^2 \omega_4^2 + 24c_s^4 \omega_7^2 \omega_4^2 - 12v_1^2 c_s^2 \omega_7^2 \omega_4^2 - 48c_s^4 \omega_7^2 \omega_4 + 36v_1^4 \omega_4^3 + c_s^2 \omega_7^2 \omega_4^3 - 72v_1^2 \omega_7 \omega_4^2 - 72v_1^4 \omega_4^2 + 6v_1^2 c_s^2 \omega_7^2 \omega_4^3 + 30v_1^2 \omega_7 \omega_4^3 + 24c_s^4 \omega_7^2 - 8c_s^2 \omega_7^2 \omega_4^2 - 72v_1^2 c_s^2 \omega_7 \omega_4^3 - 3v_1^2 \omega_7^2 \omega_4^3 + 24c_s^2 \omega_7 \omega_4^2 + 144v_1^2 c_s^2 \omega_7 \omega_4^2 + 24c_s^2 \omega_7 \omega_4 - 6c_s^2 \omega_7 \omega_4^3 + 12v_1^2 \omega_7^2 \omega_4^2 + 72v_1^2 \omega_4^2 + 72v_1^2 c_s^2 \omega_7 \omega_4 + 3v_1^4 \omega_7^2 \omega_4^3 - 24c_s^4 \omega_7 \omega_4^2 - 36v_1^2 \omega_4^3 - 24c_s^2 \omega_7 \omega_4 + 6c_s^2 \omega_7 \omega_4^3 - 12v_1^4 \omega_7^2 \omega_4^2) \frac{\rho}{24\omega_7^2 \omega_4^3}$$

$$C_{D_x^4 v_2}^{(2), \text{CLBM2}} = C_{D_x^4 v_2}^{(2), \text{CLBM1}}$$

$$C_{D_x^4 v_2}^{(2), \text{CuLBM1}} = (-72v_1^4 \omega_3^2 + 6v_1^2 \omega_3^3 c_s^2 \omega_4^2 + 72v_1^2 \omega_3 c_s^2 \omega_4 - 24\omega_3 c_s^2 \omega_4 + \omega_3^3 c_s^2 \omega_4^2 + 36v_1^4 \omega_3^3 - 216v_1^2 \omega_3^2 c_s^2 + 24\omega_3^3 c_s^4 \omega_4^2 - 24\omega_3^3 c_s^4 \omega_4 + 12\omega_3 c_s^2 \omega_4^2 - 6\omega_3^3 c_s^2 \omega_4 - 72v_1^2 \omega_3^3 c_s^2 \omega_4 - 36v_1^2 \omega_3 c_s^2 \omega_4^2 + 108v_1^2 \omega_3^3 c_s^2 + 24c_s^4 \omega_4^2 + 24\omega_3 c_s^4 \omega_4 - 36v_1^2 \omega_3^3 + 12v_1^2 \omega_3^2 \omega_4^2 - 3\omega_3^3 c_s^4 \omega_4^2 + 3v_1^4 \omega_3^3 \omega_4^2 - 8\omega_3^3 c_s^2 \omega_4^2 + 72v_1^2 \omega_3^3 - 12v_1^2 \omega_3^3 c_s^2 \omega_4^2 + 30v_1^2 \omega_3^3 \omega_4 + 72v_1^2 \omega_3^2 \omega_4 - 12v_1^4 \omega_3^2 \omega_4^2 - 3v_1^4 \omega_3^2 \omega_4^2 + 144v_1^2 \omega_3^3 c_s^2 \omega_4 - 30v_1^4 \omega_3^3 \omega_4 + 24\omega_3^3 c_s^2 \omega_4 - 48\omega_3 c_s^4 \omega_4^2 + 6\omega_3^3 c_s^4 \omega_4 - 72v_1^2 \omega_3^2 \omega_4) \frac{\rho}{24\omega_3^3 \omega_4^3}$$

$$C_{D_x^4 v_2}^{(2), \text{CuLBM2}} = (-24\omega_3 c_s^2 \omega_1 + 72v_1^4 \omega_3 \omega_1^2 - 216v_1^2 c_s^2 \omega_1^2 + 24\omega_3^3 c_s^4 \omega_1^2 - 30v_1^4 \omega_3 \omega_1^3 - 3\omega_3^3 c_s^4 \omega_1^3 + 108v_1^2 c_s^2 \omega_1^3 + 72v_1^2 \omega_3 c_s^2 \omega_1 - 72v_1^4 \omega_1^2 + 144v_1^2 \omega_3 c_s^2 \omega_1^2 - 72v_1^2 \omega_3 \omega_1^2 - 6\omega_3^3 c_s^2 \omega_1^3 - 72v_1^2 \omega_3^2 c_s^2 \omega_1^3 + 30v_1^2 \omega_3 \omega_1^3 + 36v_1^4 \omega_1^3 - 48\omega_3^3 c_s^4 \omega_1 + 24\omega_3 c_s^2 \omega_1^2 - 12v_1^2 \omega_3^2 c_s^2 \omega_1^2 - 3v_1^2 \omega_3^2 \omega_1^3 + \omega_3^2 c_s^2 \omega_1^3 + 24\omega_3^3 c_s^4 \omega_1 + 12v_1^2 \omega_3^2 \omega_1^2 + 6v_1^2 \omega_3^2 c_s^2 \omega_1^3 - 8\omega_3^3 c_s^2 \omega_1^2 + 3v_1^4 \omega_3^2 \omega_1^3 - 36v_1^2 \omega_1^3 + 12\omega_3^2 c_s^2 \omega_1 - 24\omega_3 c_s^4 \omega_1^2 + 72v_1^2 \omega_1^2 - 12v_1^4 \omega_3^2 \omega_1^2 - 36v_1^2 \omega_3^2 c_s^2 \omega_1 + 24\omega_3^3 c_s^4 + 6\omega_3^3 c_s^4 \omega_1^3) \frac{\rho}{24\omega_3^3 \omega_1^3}$$

$$\text{coefficient } C_{D_x^3 D_y \rho}^{(2)} \text{ at } \frac{\partial^4 \rho}{\partial x_1^3 \partial x_2} :$$

$$C_{D_x^3 D_y \rho}^{(2), \text{SRT}} = (24 - 42c_s^2 \omega^2 - 24v_1^2 + 3c_s^2 \omega^3 - 72c_s^2 + 108c_s^2 \omega + 36v_1^2 \omega - 36\omega + v_1^2 \omega^3 + 14\omega^2 - 14v_1^2 \omega^2 - \omega^3) \frac{v_1 c_s^2}{12\omega^3}$$

$$C_{D_x^3 D_y \rho}^{(2), \text{MRT1}} = (-12\omega_8 \omega_5^2 \omega_9 v_2^2 \omega_7 \omega_4^3 + 18\omega_8 \omega_5 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4^2 - 3\omega_8 \omega_5^2 v_2^2 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4^3 - 36\omega_5^2 \omega_6 c_s^4 \omega_7^2 \omega_4^3 + 6\omega_8 \omega_9 v_2^2 \omega_6 \omega_7^2 \omega_4^3 - 12\omega_5^2 v_2^2 \omega_6 \omega_7^2 \omega_4^3 + 6\omega_8 \omega_5^2 \omega_6 c_s^2 \omega_7 \omega_4^3 - 6\omega_8 v_2^2 \omega_9 v_2^2 \omega_6 \omega_7^2 \omega_4^3 + 18\omega_8 \omega_5^2 v_2^2 \omega_6 c_s^2 \omega_7^2 \omega_4^3 + 12\omega_5^2 v_2^2 \omega_6 \omega_7^2 \omega_4^3 - 108\omega_8 \omega_5 \omega_9 v_2^2 \omega_6 c_s^2 \omega_7^2 \omega_4^3 - 45\omega_8 \omega_5^2 \omega_9 v_2^2 \omega_6 c_s^2 \omega_7 \omega_4^3 - 42\omega_8 \omega_5^2 \omega_9 \omega_6 c_s^4 \omega_7 \omega_4^2 + 24\omega_8 \omega_5 v_1^2 \omega_9 v_2^2 \omega_7^2 \omega_4^2 - 24\omega_8 \omega_5^2 v_1^2 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4 + 6\omega_8 \omega_5^2 v_1^2 v_2^2 \omega_6 \omega_7^2 \omega_4^3 - 48\omega_8 \omega_5^2 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4^2 - 96\omega_8 \omega_5^2 \omega_9 \omega_6 c_s^4 \omega_7^2 + 144\omega_8 \omega_5^2 \omega_9 v_2^2 \omega_6 c_s^2 \omega_7 \omega_4^2 + 27\omega_8 \omega_5 \omega_9 v_2^2 \omega_6 c_s^2 \omega_7^2 \omega_4^3 - 12\omega_8 \omega_5^2 \omega_9 c_s^2 \omega_7 \omega_4^2 + 12\omega_5^2 v_1^2 \omega_6 c_s^2 \omega_7^2 \omega_4^2 - 12\omega_5^2 v_2^2 \omega_6 \omega_7^2 \omega_4^2 + 12\omega_8 \omega_5^2 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4 - 36\omega_8 \omega_5^2 v_2^2 \omega_6 c_s^2 \omega_7^2 \omega_4^2 + 12\omega_8 v_1^2 \omega_9 v_2^2 \omega_6 \omega_7^2 \omega_4^2 + 18\omega_8 \omega_5^2 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4^2 - 5\omega_8 \omega_5 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4^3 + 36\omega_5^2 \omega_6 c_s^2 \omega_7^2 \omega_4^2 - 12\omega_8 \omega_9 v_2^2 \omega_6 \omega_7^2 \omega_4^2 + 12\omega_5^2 v_1^2 v_2^2 \omega_7^2 \omega_4^3 + 24\omega_8 \omega_5^2 \omega_9 v_2^2 \omega_7 \omega_4^2 + 12\omega_5 \omega_9 c_s^2 \omega_7^2 \omega_4^3 + 15\omega_8 \omega_5^2 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4^3 - 12\omega_5 v_2^2 \omega_9 c_s^2 \omega_7^2 \omega_4^3 - 12\omega_8 \omega_5^2 v_1^2 v_2^2 \omega_6 \omega_7^2 \omega_4^2 - 6\omega_8 \omega_5 v_1^2 \omega_9 v_2^2 \omega_7^2 \omega_4^3 + 15\omega_8 \omega_5^2 \omega_9 \omega_6 c_s^4 \omega_7 \omega_4^3 + 24\omega_8 \omega_5 v_1^2 \omega_9 v_2^2 \omega_6 \omega_7^2 \omega_4 - 12\omega_5^2 v_1^2 v_2^2 \omega_6 \omega_7^2 \omega_4^3 - 72\omega_8 \omega_5^2 \omega_9 v_2^2 c_s^2 \omega_7 \omega_4^2 + 6\omega_8 \omega_5^2 v_2^2 \omega_6 \omega_7 \omega_4^3 + 12\omega_5^2 \omega_6 c_s^2 \omega_7^2 \omega_4^3 + 12\omega_8 \omega_5^2 \omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4^2 - 18\omega_8 \omega_9 v_2^2 \omega_6 c_s^2 \omega_7^2 \omega_4^3 - 36\omega_8 \omega_5 v_2^2 \omega_9 v_2^2 \omega_6 \omega_7^2 \omega_4^2 - 18\omega_8 \omega_5^2 \omega_6 c_s^4 \omega_7 \omega_4^3 - 6\omega_8 \omega_5^2 \omega_9 v_2^2 \omega_6 \omega_4^3 - 15\omega_8 \omega_5^2 v_1^2 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4^3 + 6\omega_8 \omega_5^2 v_1^2 \omega_6 c_s^2 \omega_7^2 \omega_4^3 - 72\omega_8 \omega_5^2 \omega_9 v_2^2 \omega_6 c_s^2 \omega_7 \omega_4 - 36\omega_5^2 v_2^2 \omega_6 c_s^2 \omega_7^2 \omega_4^3 + 18\omega_8 \omega_5^2 \omega_9 v_2^2 \omega_6 c_s^2 \omega_4^3 - 6\omega_8 \omega_5 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4^3 - 18\omega_8 \omega_5^2 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4^2 + 36\omega_5^2 v_2^2 c_s^2 \omega_7^2 \omega_4^3 - 12\omega_8 \omega_5^2 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4 + 48\omega_8 \omega_5^2 v_1^2 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4^2 + 12\omega_8 \omega_5^2 \omega_9 v_2^2 \omega_6 \omega_4^2 + 36\omega_8 \omega_5^2 \omega_9 \omega_6 c_s^4 \omega_7 \omega_4^2 + 9\omega_8 \omega_5 v_1^2 \omega_9 v_2^2 \omega_6 \omega_7^2 \omega_4^3 + 12\omega_8 \omega_5^2 \omega_9 \omega_6 c_s^4 \omega_7 \omega_4 - 12\omega_5 v_1^2 \omega_9 v_2^2 \omega_7^2 \omega_4^3 + 36\omega_8 \omega_9 v_2^2 \omega_6 c_s^2 \omega_7^2 \omega_4^2 - 6\omega_8 \omega_5^2 \omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4^3 - 36\omega_5 \omega_9 c_s^4 \omega_7^2 \omega_4^3 + 6\omega_8 \omega_5 v_1^2 \omega_9 c_s^2 \omega_7^2 \omega_4^3 - 12\omega_8 \omega_5^2 v_2^2 \omega_6 \omega_7 \omega_4^2 + 36\omega_8 \omega_5^2 \omega_9 v_2^2 c_s^2 \omega_7 \omega_4^3 + 12\omega_5^2 v_1^2 v_2^2 \omega_6 \omega_7^2 \omega_4^2 - 12\omega_5^2 \omega_6 c_s^2 \omega_7^2 \omega_4^2 + 24\omega_8 \omega_5^2 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4 - 12\omega_8 \omega_5^2 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4^2 + 36\omega_5^2 v_2^2 \omega_6 c_s^2 \omega_7^2 \omega_4^2 - 6\omega_8 \omega_5^2 v_1^2 v_2^2 \omega_7^2 \omega_4^3 - 6\omega_8 \omega_5 \omega_9 c_s^2 \omega_7^2 \omega_4^3 + 156\omega_8 \omega_5^2 \omega_9 \omega_6 c_s^4 \omega_7^2 \omega_4 - 36\omega_8 \omega_5^2 \omega_9 v_2^2 \omega_6 c_s^2 \omega_7^2 \omega_4 + 72\omega_8 \omega_5 \omega_9 v_2^2 \omega_6 c_s^2 \omega_7^2 \omega_4 + 36\omega_5^2 v_2^2 \omega_6 c_s^2 \omega_7^2 \omega_4^2 + 6\omega_8 \omega_5 v_1^2 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4^3 + 6\omega_8 \omega_5^2 c_s^2 \omega_7^2 \omega_4^2 - 12\omega_5^2 v_2^2 \omega_6 \omega_7^2 \omega_4^3 + 12\omega_5 \omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4 + 36\omega_5 \omega_9 v_2^2 \omega_6 c_s^2 \omega_7^2 \omega_4^2 + 24\omega_8 \omega_5^2 v_1^2 \omega_9 v_2^2 \omega_6 \omega_7^2 \omega_4^2 + 54\omega_8 \omega_5 \omega_9 \omega_6 c_s^4 \omega_7^2 \omega_4^2 + 12\omega_5 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4^3 + 12\omega_8 \omega_5^2 v_1^2 \omega_6 c_s^2 \omega_7 \omega_4^2 + 36\omega_5^2 c_s^4 \omega_7^2 \omega_4^3 - \omega_8 \omega_5^2 \omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4^3 + 36\omega_8 \omega_5 \omega_9 v_2^2 \omega_6 \omega_7^2 \omega_4^2 - 12\omega_5 \omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4^3 + 12\omega_8 \omega_5 \omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4 - 6\omega_8 \omega_5^2 v_2^2 \omega_6 \omega_7^2 \omega_4^3 - 12\omega_8 \omega_5 v_1^2 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4^2 + 18\omega_8 \omega_5^2 \omega_6 c_s^4 \omega_7^2 \omega_4^3 - 9\omega_8 \omega_5 \omega_9 v_2^2 \omega_6 \omega_7^2 \omega_4^3 + 12\omega_8 \omega_5^2 \omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4^2 - 12\omega_5 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4^2 - 6\omega_8 \omega_5^2 v_1^2 \omega_9 c_s^2 \omega_7 \omega_4^3 - 36\omega_5 \omega_9 v_2^2 c_s^2 \omega_7^2 \omega_4^3 + 12\omega_8 \omega_5^2 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4 - 36\omega_5 \omega_9 v_2^2 \omega_6 c_s^2 \omega_7^2 \omega_4^2 - 15\omega_8 \omega_5 \omega_9 \omega_6 c_s^4 \omega_7^2 \omega_4^3 + 12\omega_8 \omega_5^2 v_1^2 \omega_9 v_2^2 \omega_7 \omega_4^3 - 12\omega_8 \omega_5^2 \omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4 + 72\omega_8 \omega_5 \omega_9 v_2^2 c_s^2 \omega_7^2 \omega_4^2 + 12\omega_8 \omega_5^2 \omega_6 c_s^2 \omega_7^2 \omega_4^2 + 18\omega_8 \omega_5 \omega_9 v_2^2 \omega_6 c_s^2 \omega_7 \omega_4^3 + 12\omega_5 \omega_9 v_2^2 \omega_6 \omega_7^2 \omega_4^2 + 18\omega_8 \omega_5 \omega_9 c_s^4 \omega_7^2 \omega_4^3 - 36\omega_5 \omega_9 \omega_6 c_s^4 \omega_7^2 \omega_4^2 - 18\omega_8 \omega_5^2 c_s^4 \omega_7^2 \omega_4^3 - 12\omega_8 \omega_5^2 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4^2 + 12\omega_5 v_1^2 \omega_9 v_2^2 \omega_6 \omega_7^2 \omega_4^3 + 6\omega_8 \omega_5 \omega_9 v_2^2 \omega_7^2 \omega_4^3 - 18\omega_8 \omega_5 \omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4^2 - 6\omega_8 \omega_5^2 v_1^2 c_s^2 \omega_7^2 \omega_4^3 + 6\omega_8 \omega_5^2 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4^3 + 3\omega_8 \omega_5^2 \omega_9 \omega_6 c_s^4 \omega_7^2 \omega_4^3 - 12\omega_5^2 c_s^2 \omega_7^2 \omega_4^3 - 12\omega_8 \omega_5^2 v_1^2 \omega_9 v_2^2 \omega_6 \omega_4^2 + 6\omega_8 \omega_5^2 v_2^2 \omega_7^2 \omega_4^3 - 36\omega_8 \omega_5 \omega_9 \omega_6 c_s^4 \omega_7^2 \omega_4 + \omega_8 \omega_5^2 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4^3 + 36\omega_5 \omega_9 \omega_6 c_s^4 \omega_7^2 \omega_4^3 + 12\omega_8 \omega_5^2 \omega_9 \omega_6 c_s^4 \omega_7^2 \omega_4^2 - 12\omega_5 \omega_9 v_2^2 \omega_6 \omega_7^2 \omega_4^3 - 36\omega_8 \omega_5 \omega_9 v_2^2 \omega_6 c_s^2 \omega_7^2 \omega_4^2 - 24\omega_8 \omega_5 \omega_9 v_2^2 \omega_6 \omega_7^2 \omega_4 - 18\omega_8 \omega_5^2 v_2^2 \omega_6 c_s^2 \omega_7^2 \omega_4^3 - 6\omega_8 \omega_5^2 \omega_9 v_2^2 c_s^2 \omega_7^2 \omega_4^3 + 6\omega_8 \omega_5^2 v_1^2 \omega_9 v_2^2 \omega_6 \omega_4^3 - 60\omega_8 \omega_5^2 \omega_9 \omega_6 c_s^2 \omega_7^2 \omega_4^2 - 24\omega_8 \omega_5 \omega_9 v_2^2 \omega_7^2 \omega_4^2$$

$$18\omega_8\omega_5^2v_2^2c_s^2\omega_7^3\omega_4^3 - 12\omega_8\omega_5^2v_1^2\omega_9\omega_6c_s^2\omega_4^2 + 5\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7^3\omega_4^3 - 12\omega_5v_1^2\omega_9v_2^2\omega_6\omega_7^2\omega_4^2 - 6\omega_8\omega_5^2v_1^2v_2^2\omega_6\omega_7\omega_4^3) \frac{v_1}{12\omega_8\omega_5^2\omega_9\omega_6\omega_7^2\omega_4^3}$$

$$C_{D_x^3D_y\rho}^{(2),\text{MRT}2} = C_{D_x^3D_y\rho}^{(2),\text{MRT}1}$$

$$C_{D_x^3D_y\rho}^{(2),\text{CLBM}1} = (12\omega_8\omega_5v_1^2\omega_6\omega_7\omega_4 + 12v_1^2\omega_9\omega_6\omega_7^2\omega_4^2 - 36\omega_8\omega_5\omega_9\omega_6c_s^2\omega_4 + 12\omega_9\omega_6\omega_7^2\omega_4 + 54\omega_8\omega_9\omega_6c_s^2\omega_7^2\omega_4 + 12\omega_8\omega_5\omega_6\omega_7^2\omega_4 + 3\omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^2 + 12\omega_8\omega_5\omega_9\omega_6\omega_4 - 18\omega_8\omega_5\omega_6c_s^2\omega_7\omega_4^2 - 12\omega_8\omega_5v_1^2\omega_9\omega_6\omega_7 + \omega_8\omega_5v_1^2\omega_9\omega_6\omega_7^2\omega_4^2 - 36\omega_9c_s^2\omega_7^2\omega_4^2 + 5\omega_8\omega_9\omega_6\omega_7^2\omega_4^2 + 36\omega_5\omega_6c_s^2\omega_7^2\omega_4 - 9\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 12\omega_5\omega_6\omega_7^2\omega_4^2 - 12\omega_5v_1^2\omega_6\omega_7^2\omega_4^2 - 18\omega_8\omega_5c_s^2\omega_7^2\omega_4^2 + 54\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4 - 12\omega_5\omega_6\omega_7^2\omega_4 + 12\omega_5v_1^2\omega_6\omega_7^2\omega_4 - 36\omega_8\omega_9\omega_6c_s^2\omega_7^2 - 18\omega_8\omega_9\omega_6\omega_7^2\omega_4 - 36\omega_5\omega_6c_s^2\omega_7^2\omega_4^2 - 6\omega_8\omega_5\omega_6\omega_7^2\omega_4^2 - 15\omega_8\omega_9\omega_6c_s^2\omega_7^2\omega_4^2 - 6\omega_8\omega_5\omega_9\omega_6\omega_4^2 + 36\omega_8\omega_5\omega_6c_s^2\omega_7\omega_4 - 18\omega_8\omega_5\omega_9\omega_6\omega_7\omega_4 - 12\omega_8\omega_5v_1^2\omega_9\omega_6\omega_7^2\omega_4 + 12\omega_8\omega_5v_1^2\omega_9\omega_6\omega_7^2 + 36\omega_5c_s^2\omega_7^2\omega_4^2 - 12v_1^2\omega_9\omega_6\omega_7^2\omega_4 - 6\omega_8\omega_5v_1^2\omega_6\omega_7\omega_4^2 + 18\omega_8\omega_9c_s^2\omega_7^2\omega_4^2 + 18\omega_8\omega_5\omega_9\omega_6c_s^2\omega_4^2 - 12\omega_9\omega_6\omega_7^2\omega_4^2 + 12\omega_8\omega_5\omega_9\omega_6\omega_7^2\omega_4 + 6\omega_8\omega_5v_1^2\omega_9\omega_6\omega_4^2 + 6\omega_8\omega_5v_1^2\omega_9\omega_6\omega_7^2\omega_4^2 - 36\omega_8\omega_5\omega_6c_s^2\omega_7\omega_4 - 6\omega_8\omega_5\omega_7^2\omega_4^2 + 6\omega_8\omega_5\omega_7^2\omega_4^2 + 36\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7^2\omega_4 + 12\omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^2 + 12\omega_5v_1^2\omega_7^2\omega_4^2 - 12\omega_5\omega_7^2\omega_4^2 - 5\omega_8v_1^2\omega_9\omega_6\omega_7^2\omega_4^2 - 12\omega_8v_1^2\omega_9\omega_6\omega_7^2 - 12v_1^2\omega_9\omega_7^2\omega_4^2 - 36\omega_9\omega_6c_s^2\omega_7^2\omega_4 + 36\omega_9\omega_6c_s^2\omega_7^2\omega_4^2 + 3\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7^2\omega_4^2 + 18\omega_8v_1^2\omega_9\omega_6\omega_7^2\omega_4 + 6\omega_8v_1^2\omega_9\omega_7^2\omega_4^2 - 12\omega_8\omega_5v_1^2\omega_9\omega_6\omega_4 - 12\omega_8\omega_5v_1^2\omega_6\omega_7^2\omega_4 + 12\omega_8\omega_5\omega_9\omega_6\omega_7 + 18\omega_8\omega_5\omega_6c_s^2\omega_7\omega_4^2 - \omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^2 + 12\omega_8\omega_9\omega_6\omega_7^2 - 12\omega_8\omega_5\omega_6\omega_7\omega_4 - 6\omega_8\omega_5v_1^2\omega_7^2\omega_4^2 - 3\omega_8\omega_5v_1^2\omega_9\omega_6\omega_7\omega_4^2 - 36\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7) \frac{v_1c_s^2}{12\omega_8\omega_5\omega_9\omega_6\omega_7^2\omega_4^3}$$

$$C_{D_x^3D_y\rho}^{(2),\text{CLBM}2} = C_{D_x^3D_y\rho}^{(2),\text{CLBM}1}$$

$$C_{D_x^3D_y\rho}^{(2),\text{CuLBM}1} = (-36\omega_3c_s^2\omega_1 - 12\omega_4^2\omega_1 - 12\omega_3\omega_4 + 12v_1^2\omega_4^2\omega_1 + 12v_2^2\omega_3\omega_4 - 36c_s^2\omega_4^2 + 12\omega_3\omega_1 + 36\omega_3c_s^2\omega_4 - 36\omega_3c_s^2\omega_4^2\omega_1 - 12v_2^2\omega_3\omega_1 - 36c_s^2\omega_4\omega_1 - 9\omega_3^2c_s^2\omega_4\omega_1 - 12v_2^2\omega_3\omega_4^2\omega_1 + 18\omega_3c_s^2\omega_4^2 + 3\omega_3^2\omega_4\omega_1 - 3v_1^2\omega_3^2\omega_4\omega_1 + 6v_1^2\omega_3\omega_4^2 + 12\omega_3\omega_4^2\omega_1 - 6\omega_3\omega_4^2 + v_1^2\omega_3^2\omega_4^2\omega_1 + v_2^2\omega_3^2\omega_4^2 - \omega_3^2\omega_4^2 - 18\omega_3\omega_4\omega_1 + 3\omega_3^2c_s^2\omega_4 + 18v_1^2\omega_3\omega_4\omega_1 + 3\omega_3^2c_s^2\omega_4^2\omega_1 - \omega_3^2\omega_4^2\omega_1 - 12v_1^2\omega_4^2 - 6\omega_3^2\omega_1 + 18\omega_3^2c_s^2\omega_1 + 54\omega_3c_s^2\omega_4\omega_1 + 36c_s^2\omega_4^2\omega_1 + 6v_1^2\omega_3^2\omega_1 + 6\omega_3^2\omega_4 + 12\omega_4\omega_1 - 18\omega_3^2c_s^2\omega_4 + 12\omega_4^2 - 6v_1^2\omega_3^2\omega_4 - 12v_1^2\omega_4\omega_1) \frac{v_1c_s^2}{12\omega_3^2\omega_4^2\omega_1}$$

$$C_{D_x^3D_y\rho}^{(2),\text{CuLBM}2} = (18\omega_2\omega_3c_s^4\omega_1^3 - 90\omega_2^2\omega_3c_s^4\omega_1 + 30\omega_3^2\omega_3c_s^2\omega_1 + 36\omega_3^3c_s^4\omega_1^3 - 24\omega_3^2v_1^2c_s^2\omega_1^2 + 45\omega_2v_1^4\omega_3^2\omega_1^2 - 24\omega_2^2\omega_3c_s^2\omega_1^2 - 138\omega_3^2\omega_3^2c_s^4\omega_1^2 - 72\omega_3^3c_s^4\omega_1^2 + 51\omega_3^2v_1^2\omega_3^2 - 30\omega_3^2v_1^2\omega_3^2\omega_1 + 90\omega_3^3c_s^4\omega_1^3 + 3\omega_2^2v_1^4\omega_3^2\omega_1^3 - 7\omega_3^2\omega_3^2\omega_1^2 + 3\omega_3^2v_2^2\omega_3^2\omega_1^2 + 18\omega_2^2\omega_3c_s^2\omega_1^3 + 6\omega_3^2\omega_3^2c_s^4\omega_1^3 + 12\omega_3^2v_1^2c_s^2\omega_1^3 - 165\omega_2^2v_1^2\omega_3^2c_s^2\omega_1 - 90\omega_2v_1^4\omega_3^2\omega_1^3 - 261\omega_3^2v_1^2\omega_3^2c_s^2\omega_1^2 + 12\omega_3^2\omega_3^2\omega_1 - 24\omega_2\omega_3^2c_s^2\omega_1^2 + 6\omega_2\omega_3^2\omega_1^2 + 12\omega_2^2v_1^2\omega_3^2c_s^2\omega_1^2 + 9\omega_2^2v_2^2\omega_3^2c_s^2\omega_1^3 - 465\omega_2v_1^2\omega_3^2c_s^2\omega_1^3 + 6\omega_3^2\omega_3^2\omega_1^3 - 51\omega_2v_1^4\omega_3^2\omega_1^3 + 72\omega_2^2\omega_3^2c_s^4\omega_1^3 + 42\omega_2^2v_1^2\omega_3^2c_s^2\omega_1^2 + 197\omega_2^2v_1^2\omega_3^2c_s^2\omega_1^3 + 225\omega_2^2\omega_3^2c_s^4\omega_1 - 12\omega_2\omega_3^2\omega_1^3 + 123\omega_2\omega_3^2c_s^2\omega_1^3 - 3\omega_3^2v_1^4\omega_3^2\omega_1^2 - 6\omega_3^2v_1^2\omega_3^2\omega_1^3 - 3\omega_2^2v_2^2\omega_3^2\omega_1^3 - 42\omega_2^2\omega_3^2c_s^2\omega_1^2 + 36\omega_2^2\omega_3^2c_s^4\omega_1^2 + 102\omega_2v_1^2\omega_3^2\omega_1^3 + 141\omega_2v_1^2\omega_3^2c_s^2\omega_1^2 - 90\omega_2^2\omega_3^2c_s^4 + 24\omega_3^2c_s^2\omega_1^2 + 24\omega_2^2v_1^2\omega_3^2c_s^2\omega_1^2 - 51v_1^2\omega_3^2\omega_1^3 - 45\omega_3^2v_1^4\omega_3^2 - 72\omega_2^2c_s^2\omega_1^3 + 90\omega_2^2v_1^4\omega_3^2\omega_1 - 219\omega_3^2v_1^2\omega_3^2c_s^2\omega_1^2 - 2\omega_3^2\omega_3^2c_s^2\omega_1^3 + 6\omega_3^2\omega_1^3 - 54\omega_2\omega_3^2c_s^4\omega_1^3 + 6\omega_2v_1^2\omega_3^2c_s^2\omega_1^3 - 6\omega_2\omega_3^2c_s^2\omega_1^3 + 46\omega_3^2v_1^2\omega_3^2\omega_1^2 + 39\omega_2^2v_1^4\omega_3^2\omega_1^3 - 90\omega_2^2\omega_3^2c_s^4\omega_1 + 7\omega_2^2\omega_3^2\omega_1^3 + 48\omega_2^2\omega_3^2c_s^2\omega_1 + 261v_1^2\omega_3^2c_s^2\omega_1^3 - 12\omega_2^2c_s^2\omega_1^3 - 18\omega_2^2v_1^2\omega_3^2c_s^2\omega_1^3 + 72\omega_3^2\omega_3^2c_s^2 + 81\omega_2^2\omega_3^2c_s^2\omega_1^2 + 72\omega_2^2\omega_3^2c_s^4\omega_1^2 + 2\omega_2^2v_1^2\omega_3^2c_s^2\omega_1^3 - 9\omega_2^2v_2^2\omega_3^2c_s^2\omega_1^2 + 51\omega_2^2v_1^2\omega_3^2\omega_1 + 45v_1^4\omega_3^2\omega_1^3 - 147\omega_2^2\omega_3^2c_s^2\omega_1 - 153\omega_2\omega_3^2c_s^4\omega_1^3 - 102\omega_2^2v_1^2\omega_3^2\omega_1 - 12\omega_2^2\omega_3^2c_s^2\omega_1^2 + 126\omega_2^2\omega_3^2c_s^4\omega_1^2 - 46\omega_2^2v_1^2\omega_3^2\omega_1^3 + 18\omega_2\omega_3^2c_s^4\omega_1^2 + 489\omega_2^2v_1^2\omega_3^2c_s^2\omega_1 - 39\omega_2^2v_1^2\omega_3^2\omega_1^2 - 6\omega_2^2\omega_3^2 - 59\omega_2^2\omega_3^2c_s^2\omega_1^3 - 18\omega_2^2\omega_3^2c_s^4\omega_1^3 - 6\omega_2^2\omega_3^2\omega_1 - 45\omega_2^2v_1^4\omega_3^2\omega_1) \frac{v_1}{24\omega_2^2\omega_3^2\omega_1^3}$$

$$\text{coefficient } C_{D_x^3D_yv_1}^{(2)} \text{ at } \frac{\partial^4 v_1}{\partial x_1^3 \partial x_2} :$$

$$C_{D_x^3D_yv_1}^{(2),\text{SRT}} = (-12 + 2c_s^2\omega^2 + 36v_1^2 - c_s^2\omega^3 - 54v_1^2\omega + 18\omega - 6\omega^2 + 18v_1^2\omega^2) \frac{\rho c_s^2}{12\omega^3}$$

$$C_{D_x^3D_yv_1}^{(2),\text{MRT}1} = (-48\omega_8\omega_5^2v_1^2\omega_9\omega_6c_s^2\omega_7^2 - 12\omega_8\omega_5^2\omega_9v_2^2\omega_7\omega_4^3 + 54\omega_8\omega_5v_1^2\omega_9\omega_6c_s^2\omega_7^2\omega_4^2 + 30\omega_8\omega_5^2v_1^2\omega_9\omega_6c_s^2\omega_7\omega_4^3 - 12\omega_5^2\omega_6c_s^4\omega_7^2\omega_4^3 + 6\omega_8\omega_9v_2^2\omega_6\omega_7^2\omega_4^3 - 36\omega_5^2v_1^2\omega_6c_s^2\omega_7^2\omega_4^3 + 6\omega_8\omega_5^2\omega_6c_s^2\omega_7\omega_4^3 - 18\omega_8v_1^2\omega_9v_2^2\omega_6\omega_7^2\omega_4^3 + 6\omega_8\omega_5^2v_2^2\omega_6c_s^2\omega_7^2\omega_4^3 + 12\omega_5^2v_2^2\omega_6\omega_7^2\omega_4^3 - 36\omega_8\omega_5\omega_9v_2^2\omega_6c_s^2\omega_7^2\omega_4^2 - 15\omega_8\omega_5^2\omega_9v_2^2\omega_6c_s^2\omega_7\omega_4^3 - 18\omega_8\omega_5^2\omega_9\omega_6c_s^4\omega_7\omega_4^2 + 72\omega_8\omega_5v_1^2\omega_9v_2^2\omega_6\omega_7^2\omega_4^2 - 72\omega_8\omega_5^2v_1^2\omega_9v_2^2\omega_6\omega_7\omega_4 + 18\omega_8\omega_5^2v_2^2\omega_6\omega_7^2\omega_4^3 - 48\omega_8\omega_5^2\omega_9v_2^2\omega_6\omega_7\omega_4^3 - 12\omega_8\omega_5^2\omega_9\omega_6c_s^4\omega_7^2 + 48\omega_8\omega_5^2\omega_9v_2^2\omega_6c_s^2\omega_7\omega_4^2 + 9\omega_8\omega_5\omega_9v_2^2\omega_6c_s^2\omega_7^2\omega_4^3 - 12\omega_8\omega_5^2\omega_6c_s^2\omega_7\omega_4^2 + 36\omega_5^2v_1^2\omega_6c_s^2\omega_7^2\omega_4^2 - 12\omega_5^2v_2^2\omega_6\omega_7^2\omega_4^2 - 12\omega_8\omega_5^2\omega_9\omega_6c_s^2\omega_7\omega_4 - 12\omega_8\omega_5^2v_2^2\omega_6c_s^2\omega_7^2\omega_4^2 + 36\omega_8v_1^2\omega_9v_2^2\omega_6\omega_7^2\omega_4^2 - 102\omega_8\omega_5^2v_1^2\omega_9\omega_6c_s^2\omega_7\omega_4^2 - 15\omega_8\omega_5v_1^2\omega_9\omega_6c_s^2\omega_7^2\omega_4^3 + 12\omega_5^2\omega_6c_s^4\omega_7^2\omega_4^2 - 12\omega_8\omega_9v_2^2\omega_6\omega_7^2\omega_4^2 + 36\omega_5^2v_1^2v_2^2\omega_6\omega_7^2\omega_4^2 + 24\omega_8\omega_5^2\omega_9v_2^2\omega_6\omega_7\omega_4^2 + 12\omega_5\omega_9c_s^2\omega_7^2\omega_4^3 + 15\omega_8\omega_5^2\omega_9v_2^2\omega_6\omega_7\omega_4^3 - 36\omega_5v_1^2\omega_9c_s^2\omega_7^2\omega_4^3 - 36\omega_8\omega_5^2v_1^2v_2^2\omega_6\omega_7^2\omega_4^2 - 18\omega_8\omega_5v_1^2\omega_9v_2^2\omega_7^2\omega_4^3 + 6\omega_8\omega_5^2\omega_9\omega_6c_s^4\omega_7\omega_4^3 + 72\omega_8\omega_5v_1^2\omega_9v_2^2\omega_6\omega_7^2\omega_4 - 36\omega_5^2v_1^2v_2^2\omega_6\omega_7^2\omega_4^3 - 24\omega_8\omega_5^2\omega_9v_2^2c_s^2\omega_7\omega_4^2 + 6\omega_8\omega_5^2v_2^2\omega_6\omega_7\omega_4^3 + 12\omega_5^2\omega_6c_s^2\omega_7^2\omega_4^3 - 6\omega_8\omega_9v_2^2\omega_6c_s^2\omega_7^2\omega_4^3 - 108\omega_8\omega_5v_1^2\omega_9v_2^2\omega_6\omega_7^2\omega_4^2 - 6\omega_8\omega_5^2\omega_6c_s^4\omega_7\omega_4^3 - 6\omega_8\omega_5^2\omega_9v_2^2\omega_6\omega_4^3 - 45\omega_8\omega_5^2v_1^2\omega_9v_2^2\omega_6\omega_7\omega_4^3 + 18\omega_8\omega_5^2v_1^2\omega_6c_s^2\omega_7^2\omega_4^3 - 24\omega_8\omega_5^2\omega_9v_2^2\omega_6c_s^2\omega_7\omega_4 - 12\omega_5^2v_2^2\omega_6c_s^2\omega_7^2\omega_4^3 + 6\omega_8\omega_5^2\omega_9v_2^2\omega_6c_s^2\omega_4^3 - 6\omega_8\omega_5\omega_9v_2^2\omega_6\omega_7\omega_4^3 + 18\omega_8\omega_5^2\omega_9v_2^2\omega_6\omega_7\omega_4^2 + 12\omega_5\omega_9c_s^4\omega_7\omega_4^3 + 12\omega_8\omega_5^2\omega_9v_2^2\omega_6\omega_7\omega_4^2 + 12\omega_8\omega_5^2\omega_9\omega_6c_s^4\omega_7\omega_4 - 36\omega_5v_1^2\omega_9v_2^2\omega_6\omega_7^2\omega_4^3 + 12\omega_8\omega_5^2\omega_9\omega_6c_s^4\omega_7\omega_4 - 36\omega_5v_1^2\omega_9v_2^2\omega_6\omega_7^2\omega_4^3 + 12\omega_5\omega_9v_2^2\omega_7^2\omega_4^3 + 12\omega_8\omega_5\omega_9v_2^2c_s^2\omega_7\omega_4^3 + 36\omega_5^2v_1^2v_2^2\omega_6\omega_7^2\omega_4^2 - 12\omega_5^2\omega_6c_s^2\omega_7^2\omega_4^2 + 24\omega_8\omega_5\omega_9v_2^2\omega_6\omega_7\omega_4^3 - 6\omega_8\omega_5^2\omega_9v_2^2\omega_6\omega_7\omega_4^3 + 12\omega_5\omega_9c_s^4\omega_7\omega_4^3 + 36\omega_8\omega_5\omega_9v_2^2\omega_6\omega_7\omega_4^2 - 12\omega_5\omega_9\omega_6c_s^2\omega_7^2\omega_4^3 + 12\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7^2\omega_4^2 - 6\omega_8\omega_5^2v_2^2\omega_6\omega_7^2\omega_4^3 - 36\omega_8\omega_5v_1^2\omega_9v_2^2\omega_6\omega_7\omega_4^2 + 6\omega_8\omega_5^2\omega_6c_s^4\omega_7^2\omega_4^3 - 9\omega_8\omega_5\omega_9v_2^2\omega_6\omega_7^2\omega_4^3 + \omega_8\omega_5^2\omega_9\omega_6c_s^2\omega_7^2\omega_4^2 - 36\omega_5v_1^2\omega_9\omega_6c_s^2\omega_7^2\omega_4^2 - 18\omega_8\omega_5^2v_2^2\omega_6c_s^2\omega_7\omega_4^3 - 12\omega_5\omega_9v_2^2c_s^2\omega_7^2\omega_4^3 + 60\omega_8\omega_5^2v_1^2\omega_9\omega_6c_s^2\omega_7^2\omega_4 - 12\omega_5\omega_9v_2^2\omega_6c_s^2\omega_7^2\omega_4^2 - 5\omega_8\omega_5\omega_9\omega_6c_s^4\omega_7^2\omega_4^3 + 36\omega_8\omega_5^2v_1^2\omega_9v_2^2\omega_6\omega_7\omega_4^3 + 24\omega_8\omega_5\omega_9v_2^2c_s^2\omega_7^2\omega_4^2 + 12\omega_8\omega_5^2\omega_6c_s^2\omega_7^2\omega_4^2 + 6\omega_8\omega_5\omega_9v_2^2\omega_6c_s^2\omega_7\omega_4^3 + 12\omega_5\omega_9v_2^2\omega_6\omega_7^2\omega_4^2 + 6\omega_8\omega_5\omega_9c_s^4\omega_7^2\omega_4^3 - 12\omega_5\omega_9\omega_6c_s^4\omega_7^2\omega_4^2 - 6\omega_8\omega_5^2c_s^4\omega_7^2\omega_4^3 - 15\omega_8\omega_5^2v_1^2\omega_9\omega_6c_s^2\omega_7^2\omega_4^2 + 36\omega_5v_1^2\omega_9v_2^2\omega_6\omega_7^2\omega_4^3 + 36\omega_8\omega_5^2v_1^2v_2^2\omega_6\omega_7\omega_4^2 +$$

$$6\omega_8\omega_5\omega_9v_2^2\omega_4^3 - 18\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7^2\omega_4^2 - 18\omega_8\omega_5^2v_1^2c_s^2\omega_7\omega_4^3 - 12\omega_8\omega_5^2v_1^2\omega_9\omega_6c_s^2\omega_4^3 - \omega_8\omega_5^2\omega_9\omega_6c_s^4\omega_7^2\omega_4^3 - 12\omega_5^2c_s^2\omega_7\omega_4^3 - 36\omega_8\omega_5^2v_1^2\omega_9v_2^2\omega_6\omega_4^2 + 6\omega_8\omega_5^2v_2^2\omega_7^3 - 12\omega_8\omega_5\omega_9\omega_6c_s^4\omega_7^2\omega_4 - 12\omega_5\omega_9\omega_6c_s^4\omega_7^2\omega_4^3 - 12\omega_5\omega_9v_2^2\omega_6\omega_7^2\omega_4^3 - 12\omega_8\omega_5\omega_9v_2^2\omega_6c_s^2\omega_7\omega_4^3 - 24\omega_8\omega_5\omega_9v_2^2\omega_6\omega_7^2\omega_4 - 6\omega_8\omega_5^2v_2^2\omega_6c_s^2\omega_7\omega_4^3 - 6\omega_8\omega_5\omega_9v_2^2c_s^2\omega_7^2\omega_4^3 + 18\omega_8\omega_5^2v_1^2\omega_9v_2^2\omega_6\omega_4^3 - 5\omega_8\omega_5^2\omega_9\omega_6c_s^4\omega_7^2\omega_4^3 - 24\omega_8\omega_5\omega_9v_2^2\omega_7^2\omega_4^3 - 6\omega_8\omega_5^2v_2^2c_s^2\omega_7^2\omega_4^3 + 24\omega_8\omega_5^2v_1^2\omega_9\omega_6c_s^2\omega_7^2\omega_4^3 + 5\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7^2\omega_4^3 - 36\omega_5v_1^2\omega_9v_2^2\omega_6\omega_7^2\omega_4^3 - 18\omega_8\omega_5^2v_1^2v_2^2\omega_6\omega_7\omega_4^3) \frac{\rho}{12\omega_8\omega_5^2\omega_9\omega_6\omega_7^2\omega_4^3}$$

$$C_{D_x^3 D_y v_1}^{(2), \text{MRT2}} = C_{D_x^3 D_y v_1}^{(2), \text{MRT1}}$$

$$C_{D_x^3 D_y v_1}^{(2), \text{CLBM1}} = (6\omega_8\omega_5\omega_6\omega_4^3 + 12\omega_9\omega_6c_s^2\omega_7\omega_4^3 + 12\omega_8\omega_5\omega_9\omega_6c_s^2\omega_4 + \omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^2 - 12\omega_8\omega_5\omega_9\omega_6\omega_4 - 12\omega_8\omega_5\omega_6c_s^2\omega_7\omega_4^2 - \omega_8\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4^3 - 12\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 18\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7\omega_4 + 54\omega_8v_1^2\omega_9\omega_6\omega_7\omega_4^2 - 6\omega_8\omega_5\omega_9\omega_6\omega_4^3 + 6\omega_8\omega_5\omega_7\omega_4^3 + 36\omega_8\omega_5v_1^2\omega_6\omega_4^2 + 6\omega_8\omega_5\omega_9\omega_6c_s^2\omega_4^3 + 18\omega_8\omega_5v_1^2\omega_6\omega_7\omega_4^3 + 36\omega_5v_1^2\omega_7\omega_4^3 + 18\omega_8\omega_5\omega_9\omega_6\omega_4^2 - 15\omega_8v_1^2\omega_9\omega_6\omega_7\omega_4^3 - 12\omega_5\omega_7\omega_4^3 - 6\omega_8\omega_5\omega_7\omega_4^3 - 36\omega_8\omega_5v_1^2\omega_6\omega_7\omega_4^2 - 18\omega_8\omega_5\omega_9\omega_6c_s^2\omega_4^2 - 18\omega_8\omega_5v_1^2\omega_6\omega_4^3 - 36v_1^2\omega_9\omega_7\omega_4^3 + 18\omega_8\omega_9\omega_6c_s^2\omega_7\omega_4^2 + 12\omega_8\omega_5\omega_6\omega_7\omega_4^2 - 54\omega_8\omega_5v_1^2\omega_9\omega_6\omega_4^2 - 12\omega_5\omega_6c_s^2\omega_7\omega_4^3 + 12\omega_9\omega_6\omega_7\omega_4^2 + 12\omega_5c_s^2\omega_7\omega_4^3 - 5\omega_8\omega_9\omega_6c_s^2\omega_7\omega_4^3 - 6\omega_8\omega_5\omega_6\omega_7\omega_4^3 - 12\omega_9\omega_6\omega_7\omega_4^3 + 12\omega_5\omega_6c_s^2\omega_7\omega_4^2 + 18\omega_8\omega_5v_1^2\omega_9\omega_6\omega_4^3 + 12\omega_8\omega_9\omega_6\omega_7\omega_4 + 6\omega_8\omega_9c_s^2\omega_7\omega_4^3 - 6\omega_8\omega_5\omega_6c_s^2\omega_4^3 + 36v_1^2\omega_9\omega_6\omega_7\omega_4^2 - 18\omega_8\omega_9\omega_6\omega_7\omega_4^2 - 12\omega_5\omega_6\omega_7\omega_4^2 + 36\omega_5v_1^2\omega_6\omega_7\omega_4^2 + 5\omega_8\omega_9\omega_6\omega_7\omega_4^3 + 36\omega_8\omega_5v_1^2\omega_9\omega_6\omega_4 - 36v_1^2\omega_9\omega_6\omega_7\omega_4^2 + 12\omega_8\omega_5\omega_6c_s^2\omega_4^2 - 12\omega_9c_s^2\omega_7\omega_4^3 - 36\omega_5v_1^2\omega_6\omega_7\omega_4^3 - 12\omega_8\omega_9\omega_6c_s^2\omega_7\omega_4 - 6\omega_8\omega_5c_s^2\omega_7\omega_4^3 + 12\omega_5\omega_6\omega_7\omega_4^3 - 3\omega_8\omega_5v_1^2\omega_9\omega_6\omega_7\omega_4^2 - 12\omega_8\omega_5\omega_9\omega_6c_s^2\omega_7) \frac{\rho c_s^2}{12\omega_8\omega_5\omega_9\omega_6\omega_7\omega_4^3}$$

$$C_{D_x^3 D_y v_1}^{(2), \text{CLBM2}} = C_{D_x^3 D_y v_1}^{(2), \text{CLBM1}}$$

$$C_{D_x^3 D_y v_1}^{(2), \text{CuLBM1}} = (12\omega_3c_s^2\omega_1 + 12\omega_3\omega_4 - 36v_1^2\omega_3\omega_4 - 12\omega_3\omega_1 - 12\omega_3c_s^2\omega_4 - \omega_3^3c_s^2\omega_4\omega_1 + 12\omega_3^2c_s^2 + 36v_1^2\omega_3\omega_1 - 12c_s^2\omega_4\omega_1 - 5\omega_3^2c_s^2\omega_4\omega_1 + \omega_3^3\omega_4\omega_1 + \omega_3^3c_s^2\omega_4 - 3v_1^2\omega_3^2\omega_4\omega_1 - 6\omega_3^3c_s^2 + 6\omega_3^3c_s^2\omega_1 + 18v_1^2\omega_3^3\omega_1 - 18v_1^2\omega_3^3 - 12\omega_3^3 - 6\omega_3^3\omega_1 + 36v_1^2\omega_3^2 + 6\omega_3^3 + 3v_1^2\omega_3^3\omega_4 - \omega_3^3\omega_4 + 18\omega_3^2\omega_1 - 18\omega_3^2c_s^2\omega_1 + 18\omega_3c_s^2\omega_4\omega_1 - 54v_1^2\omega_3^2\omega_1 - 6\omega_3^2\omega_4 + 6\omega_3^2c_s^2\omega_4 + 18v_1^2\omega_3^2\omega_4) \frac{\rho c_s^2}{12\omega_3^3\omega_4\omega_1}$$

$$C_{D_x^3 D_y v_1}^{(2), \text{CuLBM2}} = (-138\omega_3^2v_1^4\omega_3\omega_1^2 - 33\omega_2\omega_3c_s^4\omega_1^3 - 81\omega_2^2v_1^2\omega_3\omega_1^3 - 45\omega_3^2\omega_3c_s^2\omega_1 + 12\omega_3^2c_s^4\omega_1^3 - 90\omega_3^2v_1^2c_s^2\omega_1^2 - 6\omega_3^2\omega_3\omega_1 - 12\omega_2^2\omega_3c_s^2\omega_1^2 + 18\omega_3^2c_s^2\omega_1^3 - 135\omega_2^2v_1^4\omega_3\omega_1 - 30\omega_3^2c_s^4\omega_1^2 + 171v_1^4\omega_3\omega_1^3 + 351\omega_3^2v_1^2\omega_3c_s^2\omega_1 + 6\omega_2\omega_3c_s^4\omega_1^2 - 24\omega_2^2c_s^2\omega_1^2 - 180\omega_3^2v_1^2\omega_3\omega_1 - 24\omega_2^2\omega_3c_s^2\omega_1^3 + 36\omega_3^2v_1^2c_s^2\omega_1^3 + 138\omega_2^2v_1^4\omega_3\omega_1^3 + 81\omega_2^2v_1^2\omega_3\omega_1^2 + 24\omega_2^2\omega_3c_s^2 + 7\omega_2^2\omega_3\omega_1^3 - 54\omega_2^2v_1^2c_s^2\omega_1^3 + 18\omega_3^2c_s^4\omega_1 + 99\omega_3^2v_1^2\omega_3 - 153\omega_2^2v_1^2\omega_3c_s^2\omega_1^2 + 63\omega_2^2v_1^2\omega_3\omega_1 - 24\omega_3c_s^2\omega_1^3 + 207v_1^2\omega_3c_s^2\omega_1^3 - 99v_1^2\omega_3\omega_1^3 + 54\omega_3^2v_1^2c_s^2\omega_1 + 36\omega_2^2\omega_3c_s^2\omega_1 + 324\omega_3^2v_1^4\omega_3\omega_1 - 3\omega_3^2v_2^2\omega_3c_s^2\omega_1^2 + 6\omega_2c_s^4\omega_1^3 + 24\omega_2^2\omega_3c_s^2\omega_1^2 + 6\omega_3\omega_1^3 + 72\omega_2^2v_1^2c_s^2\omega_1^2 + 30\omega_2^2c_s^2\omega_1^2 + 36\omega_2^2v_1^2\omega_3c_s^2\omega_1^2 + 3\omega_2^2v_2^2\omega_3c_s^2\omega_1^3 - 12\omega_2\omega_3\omega_1^3 - 12\omega_2\omega_3c_s^2\omega_1^2 - \omega_2^2v_2^2\omega_3\omega_1^3 + 24\omega_2^2c_s^4\omega_1^2 - \omega_3^3v_2^2\omega_3\omega_1^2 - 42\omega_3^2\omega_3c_s^4 + 17\omega_2^2\omega_3c_s^4\omega_1^3 - 351\omega_2v_1^2\omega_3c_s^2\omega_1^3 + 180\omega_2v_1^2\omega_3\omega_1^3 - 6\omega_2^2\omega_3 + 45\omega_2\omega_3c_s^2\omega_1^3 + 12\omega_2^2\omega_3\omega_1 + 69\omega_2^2\omega_3c_s^4\omega_1 - 12\omega_2^2c_s^2\omega_1^3 + 6\omega_2\omega_3\omega_1^2 + 153\omega_2^2v_1^2\omega_3c_s^2\omega_1^3 - 63\omega_2v_1^2\omega_3\omega_1^2 + 63\omega_2v_2^2\omega_3c_s^2\omega_1^2 + 12\omega_2^2\omega_3c_s^4\omega_1^2 - 18\omega_2^2c_s^4\omega_1^3 - 30\omega_2^2\omega_3c_s^4\omega_1 + 18\omega_2v_2^2c_s^2\omega_1^3 + \omega_3^3v_2^2\omega_3\omega_1^2 - 7\omega_2^2\omega_3\omega_1^2 - 6\omega_2c_s^2\omega_1^3 + \omega_2^2v_2^4\omega_3\omega_1^3 - 324\omega_2v_1^2\omega_3\omega_1^3 - 207\omega_2^2v_1^2\omega_3c_s^2 - 25\omega_2^2\omega_3c_s^2\omega_1^2 - 135\omega_2^2v_1^2\omega_3c_s^2\omega_1 - 18\omega_2^2c_s^2\omega_1 + 135\omega_2v_1^2\omega_3\omega_1^2 - 171\omega_2^2v_1^4\omega_3 - 2\omega_2^2\omega_3c_s^4\omega_1^3 + 18\omega_3c_s^4\omega_1^3) \frac{\rho}{24\omega_2^2\omega_3\omega_1^3}$$

$$\text{coefficient } C_{D_x^3 D_y v_2}^{(2)} \text{ at } \frac{\partial^4 v_2}{\partial x_1^3 \partial x_2} :$$

$$C_{D_x^3 D_y v_2}^{(2), \text{SRT}} = (2 - 2v_1^2 - 6c_s^2 + 3c_s^2\omega + v_1^2\omega - \omega) \frac{v_1 \rho v_2}{12\omega}$$

$$C_{D_x^3 D_y v_2}^{(2), \text{MRT1}} = (-12\omega_8^2\omega_5^2\omega_9\omega_7^2\omega_4^3 + 24\omega_5^2\omega_5^2\omega_6^2\omega_7^2\omega_4^2 + 90\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^2 + 12\omega_5^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 60\omega_8\omega_5^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^3 + 12\omega_5^2\omega_5^2v_1^2\omega_9\omega_6\omega_7^2\omega_4 - 72\omega_8^2\omega_5^2\omega_9\omega_6c_s^2\omega_7^2\omega_4 + 12\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 12\omega_5^2\omega_5^2v_1^2\omega_9\omega_6^2\omega_4^3 - 24\omega_5^2\omega_5^2v_1^2\omega_6^2\omega_7^2\omega_4^2 + 12\omega_5^2\omega_5^2\omega_6^2c_s^2\omega_7^2\omega_4^3 - 96\omega_5^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2 + 24\omega_8\omega_5^2\omega_6^2\omega_7^2\omega_4^3 - 12\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4 - 18\omega_8\omega_5^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - 12\omega_8\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 - 24\omega_8^2\omega_5^2v_1^2\omega_6^2\omega_7^2\omega_4^3 - 12\omega_5^2\omega_5^2v_1^2\omega_6^2\omega_7^2\omega_4^3 - 12\omega_5^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 24\omega_5^2\omega_5^2\omega_9\omega_6^2\omega_4^2 - 132\omega_8\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 - 24\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^3 - 6\omega_8^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^3 - 12\omega_5^2\omega_5^2\omega_6^2\omega_7^2\omega_4^3 + 4\omega_8^2\omega_5^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^3 - 24\omega_8\omega_5^2\omega_6^2\omega_7^2\omega_4^2 - 24\omega_8\omega_5^2\omega_6^2c_s^2\omega_7^2\omega_4^2 + 24\omega_5^2\omega_5^2\omega_9\omega_6\omega_7\omega_4^2 - \omega_5^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 - 24\omega_8\omega_5^2v_1^2\omega_6^2\omega_7^2\omega_4^3 + 6\omega_8^2\omega_5\omega_9\omega_6^2c_s^2\omega_7\omega_4^3 - 12\omega_8\omega_5^2v_1^2\omega_9\omega_6\omega_7^2\omega_4^3 - 24\omega_5^2\omega_5^2v_1^2\omega_9\omega_6\omega_7\omega_4^2 + \omega_5^2\omega_5^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^3 + 72\omega_8\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4 + 24\omega_8\omega_5^2\omega_6^2c_s^2\omega_7^2\omega_4^2 + 18\omega_5^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - 24\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 - 12\omega_5^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^2 + 12\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7\omega_4^2 - 4\omega_5^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4 - 4\omega_5^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 - 24\omega_8\omega_5^2\omega_6^2c_s^2\omega_7^2\omega_4^3 + 84\omega_5^2\omega_5^2\omega_9\omega_6c_s^2\omega_7^2\omega_4^2 - 12\omega_5^2\omega_5^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^2 + 12\omega_8^2\omega_5^2v_1^2\omega_9\omega_6\omega_7\omega_4^3 - 42\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 + 24\omega_8\omega_5^2v_1^2\omega_6^2\omega_7^2\omega_4^3 - 12\omega_8^2\omega_5^2\omega_6^2c_s^2\omega_7^2\omega_4^3 - 66\omega_5^2\omega_5^2\omega_9\omega_6^2\omega_7\omega_4^2 - 6\omega_8^2\omega_5\omega_9\omega_6^2\omega_7\omega_4^3 + 156\omega_5^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4 - 24\omega_8^2\omega_5^2\omega_9\omega_6c_s^2\omega_7\omega_4^2 - 84\omega_8\omega_5^2\omega_9\omega_6c_s^2\omega_7^2\omega_4^3 + 36\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 + 66\omega_5^2\omega_5^2v_1^2\omega_9\omega_6^2\omega_7\omega_4^2 + 12\omega_8\omega_5^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^3 + 18\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7\omega_4^3 - 12\omega_8^2\omega_5^2\omega_9\omega_6\omega_7^2\omega_4^2 + 12\omega_8\omega_5^2\omega_9\omega_6c_s^2\omega_7^2\omega_4^3 + 24\omega_8\omega_5^2\omega_9c_s^2\omega_7^2\omega_4^3 - 12\omega_8\omega_5^2v_1^2\omega_9\omega_6^2\omega_7^2\omega_4^2 - 18\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^3 - 18\omega_5^2\omega_5^2v_1^2\omega_9\omega_6^2\omega_7\omega_4^3 + 48\omega_8\omega_5^2\omega_9\omega_6c_s^2\omega_7^2\omega_4^2 + 12\omega_8^2\omega_5^2\omega_9\omega_6c_s^2\omega_7\omega_4^3 + 12\omega_5^2\omega_5^2v_1^2\omega_9\omega_6\omega_7^2\omega_4^2 + 12\omega_8^2\omega_5\omega_9\omega_6^2\omega_7\omega_4^2 + 24\omega_5^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^3 - 12\omega_8^2\omega_5^2v_1^2\omega_6^2\omega_7\omega_4^3 + 3\omega_5^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^3 + 12\omega_8^2\omega_5^2\omega_6^2\omega_7\omega_4^3 - 24\omega_8\omega_5^2\omega_6\omega_7^2\omega_4^3 - 12\omega_8^2\omega_5^2\omega_6\omega_7^2\omega_4^3 - 24\omega_8\omega_5^2\omega_6\omega_7^2\omega_4^3 - 66\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_7^2\omega_4^2 + 12\omega_8^2\omega_5^2\omega_6\omega_7^2\omega_4^3 + 24\omega_8^2\omega_5^2v_1^2\omega_6^2\omega_7\omega_4^2 - 36\omega_5^2\omega_5^2v_1^2\omega_9\omega_6^2\omega_7\omega_4^2 + 12\omega_8\omega_5^2\omega_9\omega_6\omega_7^2\omega_4^3 - 48\omega_8^2\omega_5^2\omega_9\omega_6^2c_s^2\omega_4^2 + 24\omega_8\omega_5^2\omega_6c_s^2\omega_7^2\omega_4^3 - 12\omega_5^2\omega_5^2v_1^2\omega_6\omega_7^2\omega_4^3 - 12\omega_5^2\omega_5^2\omega_6^2c_s^2\omega_7\omega_4^3 + 12\omega_8^2\omega_5\omega_9\omega_6^2c_s^2\omega_7^2\omega_4 - 24\omega_8^2\omega_5^2\omega_9c_s^2\omega_7^2\omega_4^2 - 12\omega_8^2\omega_5^2v_1^2\omega_9\omega_6^2\omega_7\omega_4^2 + 6\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3 + 36\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7\omega_4) \frac{v_1 \rho v_2}{12\omega_8^2\omega_5^2\omega_9\omega_6^2\omega_7^2\omega_4^3}$$

$$C_{D_x^3 D_y v_2}^{(2), \text{MRT2}} = C_{D_x^3 D_y v_2}^{(2), \text{MRT1}}$$

$$C_{D_x^3 D_y v_2}^{(2), \text{CLBM1}} = (-3\omega_5v_1^2 + 3\omega_5 + \omega_5v_1^2\omega_7 - \omega_5\omega_7 + v_1^2\omega_7 - 9\omega_5c_s^2 + 3\omega_5c_s^2\omega_7 + 3c_s^2\omega_7 - \omega_7) \frac{v_1 \rho v_2}{12\omega_5\omega_7}$$

$$C_{D_x^3 D_y v_2}^{(2), \text{CLBM2}} = C_{D_x^3 D_y v_2}^{(2), \text{CLBM1}}$$

$$C_{D_x^3 D_y v_2}^{(2), \text{CuLBM1}} = (3c_s^2 \omega_4 \omega_1 + 3c_s^2 \omega_4 - 9c_s^2 \omega_1 - 3v_1^2 \omega_1 + v_1^2 \omega_4 - \omega_4 + 3\omega_1 - \omega_4 \omega_1 + v_1^2 \omega_4 \omega_1) \frac{v_1 \rho v_2}{12 \omega_4 \omega_1}$$

$$C_{D_x^3 D_y v_2}^{(2), \text{CuLBM2}} = (6\omega_3 c_s^2 \omega_1 + 2\omega_2 \omega_3 - 4\omega_3 \omega_1 + \omega_2 v_1^2 \omega_3 + v_1^2 \omega_3 \omega_1 + 6\omega_2 \omega_1 - 6\omega_2 v_1^2 \omega_1 - 18\omega_2 c_s^2 \omega_1 + 9v_2^2 \omega_3 \omega_1 + 2\omega_2 v_1^2 \omega_3 \omega_1 - 2\omega_2 \omega_3 \omega_1 - 9\omega_2 v_2^2 \omega_3 + 6\omega_2 \omega_3 c_s^2 \omega_1) \frac{v_1 \rho v_2}{24 \omega_2 \omega_3 \omega_1}$$

$$\text{coefficient } C_{D_x^2 D_y \rho}^{(2)} \text{ at } \frac{\partial^4 \rho}{\partial x_1^2 \partial x_2^2} :$$

$$C_{D_x^2 D_y \rho}^{(2), \text{SRT}} = (24 - 46c_s^2 \omega^2 + v_2^2 \omega^3 - 14v_2^2 \omega^2 + 5c_s^2 \omega^3 + 36v_2^2 \omega - 72c_s^2 \omega + 108c_s^2 \omega - 24v_2^2 - 36\omega + 14\omega^2 - \omega^3) \frac{v_2 c_s^2}{12 \omega^3}$$

$$\begin{aligned} C_{D_x^2 D_y \rho}^{(2), \text{MRT1}} = & (18\omega_8^2 v_1^2 \omega_9 \omega_6 \omega_7 \omega_4^3 + 12\omega_9 v_2^2 \omega_6^3 c_s^2 \omega_7 \omega_4^3 + 12\omega_8 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4 + 12v_1^2 \omega_9 \omega_6^3 \omega_7 \omega_4^3 - 12\omega_8 \omega_9 v_2^2 \omega_6^3 c_s^2 \omega_7 \omega_4 - 12\omega_8^2 \omega_9 v_2^2 \omega_6^3 c_s^2 \omega_4^3 + \\ & 36v_1^2 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^3 + 12\omega_8^2 v_1^2 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4^3 + 12\omega_8^2 \omega_6^3 c_s^2 \omega_7 \omega_4^2 - 12\omega_8 v_1^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4^2 - 12\omega_8^2 \omega_6^3 c_s^2 \omega_7 \omega_4^2 + 12\omega_8^2 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^2 + 30\omega_8 \omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4^3 - \\ & 12\omega_8^2 v_1^2 \omega_9 \omega_7 \omega_4^3 - 12\omega_8^2 v_1^2 v_2^2 \omega_6^3 \omega_7 \omega_4^3 - 12\omega_8^2 v_1^2 \omega_6^3 \omega_4^3 + 12\omega_8 v_1^2 \omega_9 \omega_6^3 \omega_7 \omega_4^3 + 24\omega_8 v_1^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4^3 + 180\omega_8^2 \omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4 - 36v_1^2 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^3 - \\ & 6\omega_8^2 \omega_6^3 c_s^2 \omega_7 \omega_4^3 - 18\omega_8^2 v_1^2 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4^3 + 6\omega_8^2 \omega_9 v_2^2 \omega_6^2 c_s^2 \omega_4^3 - 12\omega_8^2 v_1^2 \omega_9 \omega_6 \omega_7 \omega_4^2 - 12\omega_9 v_2^2 \omega_6^3 c_s^2 \omega_7 \omega_4^2 + 6\omega_8^2 v_1^2 v_2^2 \omega_6^3 \omega_7 \omega_4^3 - 18\omega_8 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^3 - \\ & 36\omega_8 v_1^2 \omega_9 \omega_6^3 \omega_7 \omega_4^2 + 6\omega_8^2 v_1^2 \omega_6^3 \omega_4^3 - 42\omega_8 \omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4^2 + 18\omega_8 \omega_9 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - \omega_8^2 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^3 - 12\omega_8 \omega_6^3 c_s^2 \omega_7 \omega_4^2 + 36\omega_8 \omega_6^3 c_s^4 \omega_7 \omega_4^3 + \\ & 72\omega_8 v_1^2 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^3 + 150\omega_8^2 \omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4^2 + 24\omega_8 v_1^2 \omega_9 \omega_6^3 \omega_7 \omega_4 + 36\omega_8^2 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4^2 + 6\omega_8^2 \omega_6^3 c_s^2 \omega_4^3 + 12\omega_8 \omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4 - 12\omega_8 v_1^2 \omega_6^2 \omega_7 \omega_4^3 + \\ & 6\omega_8^2 \omega_6^3 c_s^2 \omega_7 \omega_4^3 - 6\omega_8^2 v_1^2 v_2^2 \omega_6^3 \omega_4^3 - 36\omega_8^2 \omega_6^3 c_s^4 \omega_7 \omega_4^2 - 84\omega_8^2 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4 - 88\omega_8^2 \omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4^2 - 36\omega_8 v_1^2 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^3 - 36\omega_8 \omega_6^3 c_s^4 \omega_7 \omega_4^3 + \\ & 12\omega_8^2 \omega_9 v_2^2 c_s^2 \omega_4^3 + 2\omega_8^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - 12\omega_8 v_1^2 v_2^2 \omega_6^3 \omega_7 \omega_4^3 - 6\omega_8 \omega_9 v_2^2 \omega_6^3 c_s^2 \omega_7 \omega_4^3 + 6\omega_8 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^3 + 36\omega_8^2 v_1^2 \omega_9 c_s^2 \omega_7 \omega_4^3 + 12\omega_8^2 v_1^2 v_2^2 \omega_6^3 \omega_4^3 + \\ & 18\omega_8^2 \omega_6^3 c_s^4 \omega_7 \omega_4^3 - 36v_1^2 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^3 + 12\omega_8 v_1^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + 12\omega_8 v_1^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - 12\omega_8^2 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4 - 12\omega_8^2 \omega_6^3 c_s^2 \omega_4^2 - 12\omega_9 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - \\ & 54\omega_8^2 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4^3 - 18\omega_8 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^2 - 42\omega_8 \omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4^3 + 18\omega_8 \omega_9 v_2^2 \omega_6^3 c_s^2 \omega_7 \omega_4^2 - 12v_1^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4^3 + 12\omega_8 v_1^2 v_2^2 \omega_6^3 \omega_7 \omega_4^3 - \\ & 18\omega_8^2 \omega_9 v_2^2 c_s^2 \omega_7 \omega_4^2 - 12\omega_8 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + 6\omega_8^2 v_1^2 \omega_6^2 \omega_7 \omega_4^3 + 108\omega_8 v_1^2 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^2 + 36\omega_8 \omega_6^3 c_s^4 \omega_7 \omega_4^2 + 5\omega_8^2 \omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4^3 - 6\omega_8^2 \omega_9 \omega_6^2 c_s^2 \omega_4^3 - \\ & 12\omega_8^2 v_2^2 \omega_6^3 c_s^2 \omega_7 \omega_4^2 + 36\omega_8^2 v_1^2 \omega_6^3 c_s^2 \omega_4^2 - 36\omega_8^2 v_1^2 \omega_9 \omega_6^2 c_s^2 \omega_4^2 - 6\omega_8^2 v_2^2 \omega_6^3 c_s^2 \omega_4^3 + 12\omega_8 v_1^2 \omega_9 \omega_6^2 \omega_7 \omega_4^2 - 6\omega_8^2 v_1^2 v_2^2 \omega_6^2 \omega_7 \omega_4^3 - 36\omega_8^2 \omega_9 \omega_6 c_s^4 \omega_7 \omega_4^3 - \\ & 12\omega_8^2 \omega_9 c_s^2 \omega_7 \omega_4^3 - 12\omega_8^2 v_1^2 \omega_9 v_2^2 \omega_6^2 \omega_4^2 - 24\omega_8^2 v_1^2 \omega_9 \omega_6^2 \omega_7 \omega_4 - 36\omega_8^2 v_1^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + \omega_8^2 \omega_9 v_2^2 \omega_6^3 c_s^2 \omega_7 \omega_4^3 - 12\omega_8 v_1^2 \omega_9 v_2^2 \omega_6^3 \omega_7 \omega_4^3 + 12\omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4^2 + \\ & 18\omega_8^2 v_1^2 \omega_6^3 c_s^2 \omega_7 \omega_4^3 + 12\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + 12v_1^2 \omega_9 \omega_6^3 \omega_7 \omega_4^2 + 12\omega_8^2 v_1^2 \omega_9 \omega_6^2 \omega_4^2 - 12\omega_8^2 \omega_9 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + 6\omega_8^2 v_1^2 \omega_9 v_2^2 \omega_6^2 \omega_4^3 + 12\omega_8 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - \\ & 36\omega_8 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4^3 - 48\omega_8^2 \omega_9 \omega_6 c_s^4 \omega_7 \omega_4^2 + 12\omega_8 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4^3 - 24\omega_8 v_1^2 \omega_9 \omega_6^2 \omega_7 \omega_4^3 + 72\omega_8^2 v_1^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4 - 12\omega_8 \omega_9 v_2^2 \omega_6 c_s^2 \omega_7 \omega_4^3 + \\ & 6\omega_8^2 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - 18\omega_8^2 v_1^2 \omega_6^3 c_s^2 \omega_4^3 + 18\omega_8^2 v_1^2 \omega_9 \omega_6^2 c_s^2 \omega_4^3 + 12\omega_8^2 v_2^2 \omega_6^3 c_s^2 \omega_4^2 - 6\omega_8^2 v_1^2 \omega_9 \omega_6^2 \omega_4^3 - 12v_1^2 \omega_9 \omega_6^3 \omega_7 \omega_4^3 + 24\omega_8^2 v_1^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4 - \\ & 12\omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4^3 + 36\omega_8 v_1^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4^2 - 12\omega_8^2 \omega_9 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^2 + 36\omega_8 v_1^2 \omega_6^3 c_s^2 \omega_7 \omega_4^3 - 6\omega_8^2 v_1^2 \omega_9 \omega_6^2 \omega_7 \omega_4^3 - 6\omega_8^2 v_1^2 \omega_6^2 \omega_7 \omega_4^3 + 12\omega_8^2 \omega_9 v_2^2 \omega_6^3 c_s^2 \omega_7 \omega_4 + \\ & 12\omega_8^2 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4^3 + 12v_1^2 \omega_9 v_2^2 \omega_6^3 \omega_7 \omega_4^3 - 96\omega_8^2 \omega_9 \omega_6^3 c_s^4 \omega_7 + 18\omega_8^2 \omega_9 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^2 - 12\omega_8 v_1^2 \omega_6^3 \omega_7 \omega_4^2 + 12\omega_9 \omega_6^2 c_s^4 \omega_7 \omega_4^3 + 12\omega_8^2 \omega_9 v_2^2 c_s^2 \omega_7 \omega_4^3 - \\ & 72\omega_8^2 v_1^2 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4^2 - 36\omega_8^2 v_1^2 \omega_6^3 c_s^2 \omega_7 \omega_4^2 + 12\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 + 18\omega_8^2 \omega_9 \omega_6^2 c_s^4 \omega_4^3 + 24\omega_8^2 v_1^2 \omega_9 \omega_6^2 \omega_7 \omega_4^2 + 12\omega_8^2 v_1^2 \omega_6^2 \omega_7 \omega_4^2 - 12v_1^2 \omega_9 v_2^2 \omega_6^3 \omega_7 \omega_4^2 + \\ & 12\omega_8 v_1^2 v_2^2 \omega_6^3 \omega_7 \omega_4^3 - 18\omega_8^2 \omega_6^3 c_s^4 \omega_4^3 + 12\omega_8 v_2^2 \omega_6^3 c_s^2 \omega_7 \omega_4^2 + 6\omega_8^2 v_1^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4^3 - 12\omega_8 v_1^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4^3 + 12\omega_8 \omega_9 \omega_6^2 c_s^4 \omega_7 \omega_4^3 - 36\omega_8^2 \omega_9 \omega_6^2 c_s^4 \omega_4^2 - \\ & 12\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + 18\omega_8^2 v_1^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + 18\omega_8^2 v_1^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - 2\omega_8^2 \omega_9 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + 12\omega_8 v_1^2 \omega_6^2 \omega_7 \omega_4^3 + 36\omega_8 v_1^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3) \frac{v_2 c_s^2}{12 \omega_8^2 \omega_9 \omega_6^3 \omega_7 \omega_4^2} \end{aligned}$$

$$C_{D_x^2 D_y \rho}^{(2), \text{MRT2}} = C_{D_x^2 D_y \rho}^{(2), \text{MRT1}}$$

$$\begin{aligned} C_{D_x^2 D_y \rho}^{(2), \text{CLBM1}} = & (18\omega_8 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4^2 - 18\omega_8 \omega_9 \omega_6^2 \omega_7 \omega_4^2 + 54\omega_8 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4 + 12\omega_8^2 \omega_9 \omega_6^2 \omega_4 - 6\omega_8^2 \omega_6^3 \omega_7 \omega_4^2 + 54\omega_8^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4 + 18\omega_8^2 \omega_6^3 c_s^2 \omega_7 \omega_4^2 + \\ & 5\omega_8^2 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^2 + 18\omega_8^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4 + 36\omega_8^2 \omega_6^3 c_s^2 \omega_4 + 12\omega_9 v_2^2 \omega_6^3 \omega_7 \omega_4^2 + 54\omega_8 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 - 36\omega_8^2 \omega_9 \omega_6^2 c_s^2 \omega_4 - 12\omega_8 \omega_6^2 \omega_7 \omega_4^2 - 12\omega_8^2 \omega_9 v_2^2 \omega_6^2 \omega_4 + \\ & 2\omega_8^2 \omega_9 \omega_6^2 \omega_7 \omega_4^2 - 36\omega_8 \omega_6^3 c_s^2 \omega_7 \omega_4^2 + 12\omega_8 \omega_9 \omega_6^3 \omega_7 - 18\omega_8^2 \omega_9 \omega_6^2 \omega_7 \omega_4 + 12\omega_8 v_2^2 \omega_6^2 \omega_7 \omega_4^2 + 36\omega_8 \omega_6^3 c_s^2 \omega_7 \omega_4 + 18\omega_8^2 \omega_9 \omega_6^2 c_s^2 \omega_4 + 36\omega_8 \omega_6^2 c_s^2 \omega_7 \omega_4^2 + \\ & 12\omega_9 \omega_6^2 \omega_7 \omega_4^2 + 6\omega_8^2 \omega_9 v_2^2 \omega_6^2 \omega_4 - 36\omega_8 \omega_9 \omega_6^3 c_s^2 \omega_7 - 2\omega_8^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4^2 - 18\omega_8^2 \omega_6^2 c_s^2 \omega_7 \omega_4^2 - 40\omega_8^2 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4 - 12\omega_9 v_2^2 \omega_6^3 \omega_7 \omega_4 - 18\omega_8^2 \omega_6^3 c_s^2 \omega_4^2 + \\ & 36\omega_8^2 \omega_9 \omega_6^3 c_s^2 \omega_7 - 18\omega_8 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^2 - 6\omega_8^2 \omega_9 \omega_6^2 \omega_4 - 36\omega_8^2 \omega_6^3 c_s^2 \omega_7 \omega_4 - 6\omega_8^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 - 6\omega_8^2 v_2^2 \omega_6^2 \omega_7 \omega_4^2 + 12\omega_8^2 \omega_6^3 \omega_7 \omega_4 - 36\omega_8 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 - \\ & \omega_8^2 \omega_9 \omega_6^2 \omega_7 \omega_4^2 - 6\omega_8^2 v_2^2 \omega_6^2 \omega_4^2 + 12\omega_8 v_2^2 \omega_6^3 \omega_7 \omega_4 + 12\omega_8^2 \omega_9 v_2^2 \omega_6^2 \omega_7 + 12\omega_8 \omega_6^2 \omega_7 \omega_4^2 + 12\omega_9 \omega_6^2 \omega_7 \omega_4 - 12\omega_8^2 \omega_6^2 \omega_4 - 12\omega_8^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4 + \\ & 36\omega_8^2 \omega_9 c_s^2 \omega_7 \omega_4^2 - 12\omega_8^2 \omega_9 \omega_6^3 \omega_7 - 12\omega_8^2 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4^2 - 12\omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4^2 - 36\omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4 - 36\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 + 6\omega_8 \omega_9 \omega_6^3 \omega_7 \omega_4^2 - 6\omega_8 \omega_9 v_2^2 \omega_6^3 \omega_7 \omega_4^2 - \\ & 12\omega_8 \omega_9 v_2^2 \omega_6^3 \omega_7 + 6\omega_8^2 \omega_6^2 \omega_7 \omega_4^2 - 12\omega_8^2 v_2^2 \omega_6^3 \omega_7 \omega_4 + 18\omega_8 \omega_9 v_2^2 \omega_6^3 \omega_7 \omega_4 - 18\omega_8 \omega_9 \omega_6^3 \omega_7 \omega_4 - 12\omega_8 \omega_9 v_2^2 \omega_6 \omega_7 \omega_4^2 + 6\omega_8^2 v_2^2 \omega_6^3 \omega_7 \omega_4^2 + 12\omega_8 \omega_9 \omega_6^2 \omega_7 \omega_4^2 + \\ & \omega_8^2 \omega_9 v_2^2 \omega_6^3 \omega_7 \omega_4^2 - 12\omega_8^2 \omega_9 v_2^2 \omega_6^2 \omega_7 - 36\omega_8^2 \omega_9 \omega_6^2 c_s^2 \omega_7 - 12\omega_8^2 \omega_9 \omega_7 \omega_4^2 + 36\omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 - 36\omega_8^2 \omega_9 \omega_6 c_s^2 \omega_7 \omega_4^2 - 12\omega_9 \omega_6^3 \omega_7 \omega_4^2 - 12\omega_8 \omega_6^3 \omega_7 \omega_4 + \\ & 6\omega_8^2 \omega_6^2 \omega_4^2 + 12\omega_8^2 \omega_9 \omega_6^3 \omega_7 \omega_4 + 12\omega_8^2 \omega_9 v_2^2 \omega_7 \omega_4^2 + 12\omega_8^2 v_2^2 \omega_6^3 \omega_4 + 12\omega_8^2 \omega_9 \omega_6 \omega_7 \omega_4^2 - 12\omega_8 v_2^2 \omega_6^3 \omega_7 \omega_4^2 + 12\omega_8^2 \omega_9 \omega_6^2 \omega_7) \frac{v_2 c_s^2}{12 \omega_8^2 \omega_9 \omega_6^3 \omega_7 \omega_4^2} \end{aligned}$$

$$C_{D_x^2 D_y \rho}^{(2), \text{CLBM2}} = C_{D_x^2 D_y \rho}^{(2), \text{CLBM1}}$$

$$\begin{aligned} C_{D_x^2 D_y \rho}^{(2), \text{CuLBM1}} = & (12v_2^2 \omega_6^2 \omega_3^2 + 12\omega_2^2 \omega_6^2 \omega_3^2 c_s^2 + \omega_2^3 v_2^2 \omega_6^2 \omega_3^2 - \omega_2^3 \omega_6^2 \omega_3^2 + 6\omega_2^2 v_2^2 \omega_6^2 \omega_3 - 36\omega_2^2 \omega_3^2 c_s^2 + 36\omega_2^2 \omega_6 \omega_3 c_s^2 - 12\omega_2^2 \omega_6 \omega_3 - 36\omega_2^3 \omega_6 c_s^2 + \\ & 54\omega_2^3 \omega_6 \omega_3 c_s^2 - 36\omega_2^3 \omega_3 c_s^2 - 12\omega_2^3 v_2^2 \omega_3^2 - 6\omega_2^3 \omega_6 \omega_3^2 - 36\omega_2^2 \omega_6^2 c_s^2 - 12\omega_2^2 v_2^2 \omega_6^2 + 36\omega_2^2 \omega_3^2 c_s^2 + 4\omega_2^3 v_2^2 \omega_6^2 \omega_3^2 + 5\omega_2^3 \omega_6^2 \omega_3^2 c_s^2 + 18\omega_2 \omega_6^2 \omega_3^2 - \\ & 12\omega_2^3 v_2^2 \omega_6^2 \omega_3 + 12\omega_2^3 \omega_6^2 \omega_3 + 6\omega_2^3 v_2^2 \omega_6 \omega_3^2 - 12\omega_6^2 \omega_3^2 - 40\omega_2^3 \omega_6^2 \omega_3 c_s^2 + 12\omega_2^2 \omega_6^2 - 18\omega_2^2 \omega_6 \omega_3 - 54\omega_2 \omega_6^2 \omega_3^2 c_s^2 - 12\omega_2^3 v_2^2 \omega_3 + 12\omega_2^3 \omega_3 + \\ & 18\omega_2^3 v_2^2 \omega_6 \omega_3 - 12\omega_2^3 v_2^2 \omega_6 + 12\omega_2^3 \omega_6 - 18\omega_2^3 \omega_6 \omega_3^2 c_s^2 + 36\omega_2^3 \omega_3^2 c_s^2 + 12\omega_2^2 \omega_3^2 - 4\omega_2^2 \omega_6^2 \omega_3^2 - 6\omega_2^2 \omega_6^2 \omega_3 + 18\omega_2^2 \omega_6 \omega_3^2 c_s^2 - 12\omega_2^3 \omega_6^2 + 12\omega_2^3 v_2^2 \omega_6^2 + \\ & 36\omega_2^3 \omega_6^2 c_s^2 + 18\omega_2^2 \omega_6^2 \omega_3 c_s^2 + 6\omega_2^2 \omega_6 \omega_3^2 - 12\omega_2^3 \omega_3^2 - 6\omega_2^3 v_2^2 \omega_6 \omega_3^2 + 12\omega_2^3 v_2^2 \omega_3^2 + 12\omega_2^2 v_2^2 \omega_6 \omega_3 - 18\omega_2 v_2^2 \omega_6^2 \omega_3^2) \frac{v_2 c_s^2}{12 \omega_2^3 \omega_6^2 \omega_3^2} \end{aligned}$$

$$C_{D_x^2 D_y^2 \rho}^{(2), \text{CuLBM2}} = (-18\omega_2 \omega_3 c_s^4 \omega_1^3 - 30\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1 - 90\omega_2^2 \omega_3^2 c_s^4 \omega_1 + 30\omega_2^3 \omega_3 c_s^2 \omega_1 + 72\omega_2^3 c_s^4 \omega_1^3 - 30\omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_1 - 54\omega_2 v_1^4 \omega_3^2 \omega_1^2 - 6v_2^2 \omega_2^3 \omega_1^3 - 36\omega_2^3 \omega_3^2 c_s^2 \omega_1^2 - 91\omega_2^3 \omega_3^2 c_s^4 \omega_1^2 + 12\omega_2^3 c_s^2 \omega_1^3 - 108\omega_2^3 c_s^4 \omega_1^2 + 9\omega_2^3 v_1^2 \omega_3^2 + 6\omega_2^3 v_2^2 \omega_3^2 \omega_1 + 45v_1^2 v_2^2 \omega_3^2 \omega_1^3 + 90\omega_2^3 c_s^4 \omega_1^3 + 10\omega_2^3 \omega_3^2 c_s^4 \omega_1^3 + 27\omega_2^3 v_1^2 \omega_3^2 c_s^2 \omega_1 - 54\omega_2 v_1^4 \omega_3^2 \omega_1^3 - 135\omega_2^3 v_1^2 \omega_3^2 c_s^2 \omega_1^2 - 6\omega_2^3 \omega_3^2 \omega_1 + 24\omega_2 \omega_3^2 c_s^2 \omega_1^2 + 6\omega_2^3 v_2^2 \omega_3^2 \omega_1 - 6\omega_2 \omega_3^2 \omega_1^2 + 270\omega_2^2 v_1^2 \omega_3^2 c_s^2 \omega_1^3 + 8\omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_1^3 - 6\omega_2 v_2^2 \omega_3^2 c_s^2 \omega_1^2 - 459\omega_2 v_1^2 \omega_3^2 c_s^2 \omega_1^3 + 12\omega_2^3 \omega_3^2 c_s^2 \omega_1^3 + 99\omega_2 v_1^2 \omega_3^2 \omega_1^3 + 35\omega_2^3 \omega_3^2 c_s^4 \omega_1^3 - 12\omega_2^3 v_2^2 \omega_3^2 \omega_1^2 - 12\omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_1^3 + 138\omega_2^3 v_1^2 \omega_3^2 c_s^2 \omega_1^3 + 18\omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_1^3 + 63\omega_2^3 \omega_3^2 c_s^4 \omega_1 - 6\omega_2 \omega_3^2 \omega_1^3 - 45\omega_2^3 v_1^2 v_2^2 \omega_3^2 \omega_1 + 93\omega_2 \omega_3^2 c_s^2 \omega_1^3 + 48\omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_1^2 - 48\omega_2^3 \omega_3^2 c_s^2 \omega_1^2 + 54\omega_2^3 \omega_3^2 c_s^4 \omega_1^2 + 99\omega_2 v_1^2 \omega_3^2 \omega_1^3 - 21\omega_2 v_2^2 \omega_3^2 c_s^2 \omega_1^3 - 297\omega_2 v_1^2 \omega_3^2 c_s^2 \omega_1^2 + 18\omega_2^3 \omega_3^2 c_s^4 + 45\omega_2^3 v_1^2 v_2^2 \omega_3^2 + 36\omega_2^3 c_s^2 \omega_1^2 + 18v_2^2 \omega_3^2 c_s^2 \omega_1^3 + 12\omega_2^3 \omega_3^2 \omega_1^2 - 99v_1^2 \omega_3^2 \omega_1^3 - 36\omega_2^3 v_2^2 c_s^2 \omega_1^2 - 54\omega_2^3 v_1^2 \omega_3^2 - 72\omega_2^3 c_s^2 \omega_1^3 + 54\omega_2^3 v_1^4 \omega_3^2 \omega_1 - 138\omega_2^3 v_1^2 \omega_3^2 c_s^2 \omega_1^2 + 2\omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_1^3 - 2\omega_2^3 \omega_3^2 c_s^2 \omega_1^3 + 6\omega_2^3 \omega_1^3 + 6\omega_2 \omega_3^2 c_s^2 \omega_1^3 + 27\omega_2^3 v_1^2 \omega_3^2 \omega_1^2 + 27\omega_2^3 v_1^4 \omega_3^2 \omega_1^3 - 90\omega_2^3 \omega_3^2 c_s^4 \omega_1 + 18\omega_2^3 v_2^2 \omega_3^2 c_s^2 + 48\omega_2^3 \omega_3^2 c_s^2 \omega_1 + 24\omega_2^3 v_2^2 c_s^2 \omega_1^3 + 405v_1^2 \omega_3^2 c_s^2 \omega_1^3 - 24\omega_2^3 c_s^2 \omega_1^3 + 36\omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_1^2 - 6\omega_2 v_2^2 \omega_3^2 c_s^2 \omega_1^3 + 41\omega_2^3 \omega_3^2 c_s^2 \omega_1^2 + 108\omega_2^3 \omega_3^2 c_s^4 \omega_1^2 - 24\omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_1^2 - 36\omega_2^3 c_s^4 \omega_1^3 - 9\omega_2^3 v_1^2 \omega_3^2 \omega_1 - 45\omega_2^3 v_1^2 v_2^2 \omega_3^2 \omega_1 + 54v_1^4 \omega_3^2 \omega_1^3 + 6\omega_2 v_2^2 \omega_3^2 \omega_1^2 - 12\omega_2^3 v_2^2 c_s^2 \omega_1^3 - 39\omega_2^3 \omega_3^2 c_s^2 \omega_1 + 3\omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_1 - 117\omega_2 \omega_3^2 c_s^4 \omega_1^3 - 90\omega_2^3 v_1^2 \omega_3^2 \omega_1^2 + 90\omega_2^3 v_1^2 v_2^2 \omega_3^2 \omega_1^2 - 45\omega_2 v_1^2 v_2^2 \omega_3^2 \omega_1^3 - 9\omega_2^3 v_1^2 \omega_3^2 \omega_1 - 54\omega_2^2 \omega_3^2 c_s^2 \omega_1^2 + 144\omega_2^2 \omega_3^2 c_s^4 \omega_1^2 - 27\omega_2^3 v_1^2 \omega_3^2 \omega_1^3 - 18\omega_2 \omega_3^2 c_s^4 \omega_1^2 + 189\omega_2^3 v_1^2 \omega_3^2 c_s^2 \omega_1 - 27\omega_2^3 v_1^4 \omega_3^2 \omega_1^2 + 6\omega_2^3 \omega_3^2 + 6\omega_2 v_2^2 \omega_3^2 \omega_1^3 - 6\omega_2^3 v_2^2 \omega_3^2 - 25\omega_2^2 \omega_3^2 c_s^2 \omega_1^3 - 36\omega_2^3 \omega_3^2 c_s^4 \omega_1^3 - 6\omega_2^3 \omega_3^2 \omega_1 + 54\omega_2^3 v_1^4 \omega_3^2 \omega_1 - 45\omega_2 v_1^2 v_2^2 \omega_3^2 \omega_1^2) \frac{v_2}{24\omega_2^3 \omega_3^2 \omega_1^3}$$

$$\text{coefficient } C_{D_x^2 D_y^2 v_1}^{(2)} \text{ at } \frac{\partial^4 v_1}{\partial x_1^2 \partial x_2^2} :$$

$$C_{D_x^2 D_y^2 v_1}^{(2), \text{SRT}} = 0$$

$$C_{D_x^2 D_y^2 v_1}^{(2), \text{MRT1}} = (-2\omega_2^2 \omega_5 \omega_6^3 \omega_7^2 \omega_4^3 + 4\omega_8 \omega_5 \omega_6^3 c_s^2 \omega_7^2 \omega_4^2 - 4\omega_5 \omega_9 \omega_6^3 \omega_7^2 \omega_4^3 + 2\omega_8^2 \omega_5 \omega_9 \omega_6^3 \omega_7 \omega_4^2 - 24\omega_8^2 \omega_5 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^2 - 5\omega_8 \omega_5 \omega_9 \omega_6^3 c_s^2 \omega_7^2 \omega_4^3 - 7\omega_8 \omega_5 \omega_9 \omega_6^2 \omega_7^2 \omega_4^3 + 4\omega_8^2 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^2 - 4\omega_8 \omega_5 \omega_6^3 \omega_7^2 \omega_4^2 + 2\omega_8^2 \omega_5 \omega_9 \omega_6^2 \omega_7^2 \omega_4^2 - 2\omega_8^2 \omega_5 \omega_6^2 c_s^2 \omega_7^2 \omega_4^3 + 11\omega_8 \omega_5 \omega_9 \omega_6^3 c_s^2 \omega_7^2 \omega_4^2 + 8\omega_8^2 \omega_5 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^3 + 4\omega_5 \omega_9 \omega_6^2 \omega_7^2 \omega_4^2 - 4\omega_5 \omega_9 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 - \omega_8^2 \omega_5 \omega_9 \omega_6^3 \omega_7 \omega_4^3 - 2\omega_8^2 \omega_5 \omega_9 \omega_6^2 \omega_7^2 \omega_4^2 - 4\omega_8^2 \omega_5 \omega_6^3 c_s^2 \omega_7^2 \omega_4^3 + 4\omega_8^2 \omega_5 \omega_6^3 \omega_7^2 \omega_4^3 - 2\omega_8^2 \omega_5 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^3 - 8\omega_5 \omega_9 \omega_6^2 c_s^2 \omega_7^2 \omega_4^3 - 2\omega_8^2 \omega_5 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 + 2\omega_8 \omega_5 \omega_9 \omega_6^2 \omega_7^2 \omega_4^2 + 4\omega_8^2 \omega_5 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 + 4\omega_8^2 \omega_5 \omega_6^2 c_s^2 \omega_7^2 \omega_4^3 + 4\omega_8 \omega_5 \omega_6^2 c_s^2 \omega_7^2 \omega_4^3 + 2\omega_8^2 \omega_5 \omega_6^2 c_s^2 \omega_7^2 \omega_4^3 - 4\omega_2^3 \omega_5 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + 2\omega_8 \omega_5 \omega_9 \omega_6^2 c_s^2 \omega_7^2 \omega_4^2 - 6\omega_8 \omega_5 \omega_9 \omega_6^3 c_s^2 \omega_7^2 \omega_4 - 2\omega_8 \omega_5 \omega_9 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 + 2\omega_2^3 \omega_5 \omega_9 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 + 8\omega_5 \omega_9 \omega_6^3 c_s^2 \omega_7^2 \omega_4^3 + 4\omega_8 \omega_5 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 + 3\omega_8^2 \omega_5 \omega_9 \omega_6^2 \omega_7^2 \omega_4^2 + 2\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_6^2 \omega_7^2 \omega_4 - 4\omega_8^2 \omega_5 \omega_6^3 c_s^2 \omega_7^2 \omega_4^3 - \omega_8^2 \omega_5 \omega_9 \omega_6^2 \omega_7^2 \omega_4^3 - 5\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 - 8\omega_5 \omega_9 \omega_6^3 c_s^2 \omega_7^2 \omega_4^3 + 12\omega_8^2 \omega_5 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^3 + 7\omega_8 \omega_5 \omega_9 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 - 4\omega_8 \omega_5 \omega_6^2 \omega_7^2 \omega_4^3 - 4\omega_8^2 \omega_5 \omega_6^3 c_s^2 \omega_7^2 \omega_4^3 - \omega_8^2 \omega_5 \omega_9 \omega_6^2 c_s^2 \omega_7^2 \omega_4^2 + 4\omega_8^2 \omega_5 \omega_6^3 c_s^2 \omega_7 \omega_4^3 - 9\omega_8 \omega_5 \omega_9 \omega_6^3 \omega_7^2 \omega_4^2 + 2\omega_8^2 \omega_5 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 + 4\omega_5 \omega_9 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 - 16\omega_8^2 \omega_5 \omega_9 \omega_6^3 c_s^2 \omega_7^2 - 6\omega_8 \omega_5 \omega_9 v_2^2 \omega_6^2 \omega_7^2 \omega_4 - 2\omega_8^2 \omega_5 \omega_9 \omega_6^2 \omega_7^2 \omega_4^3 - 3\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_6^2 \omega_7^2 \omega_4^2 + 26\omega_8^2 \omega_5 \omega_9 \omega_6^2 c_s^2 \omega_7^2 \omega_4^2 - 2\omega_8^2 \omega_5 \omega_9 \omega_6^3 \omega_7^2 \omega_4^3 + 3\omega_8 \omega_5 \omega_9 \omega_6^2 \omega_7^2 \omega_4^3 - 4\omega_8^2 \omega_5 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 + 3\omega_8^2 \omega_5 \omega_9 \omega_6^2 c_s^2 \omega_7^2 \omega_4^3 - 2\omega_8^2 \omega_5 \omega_6^3 c_s^2 \omega_7 \omega_4^3 + 4\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 + 2\omega_8^2 \omega_5 \omega_6^2 \omega_7^2 \omega_4^3 - 3\omega_8 \omega_5 \omega_9 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 - 2\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 - 4\omega_8^2 \omega_5 \omega_9 \omega_6^2 \omega_7^2 \omega_4^3 - 4\omega_8^2 \omega_5 \omega_9 \omega_6^2 \omega_7^2 \omega_4^3 + 4\omega_8 \omega_5 \omega_9 \omega_6^2 \omega_7^2 \omega_4^3 + 4\omega_5 \omega_9 \omega_6^2 \omega_7^2 \omega_4^3 + 4\omega_8 \omega_5 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 - 3\omega_8 \omega_5 \omega_9 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 - 2\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 - 4\omega_8^2 \omega_5 \omega_9 \omega_6^2 \omega_7^2 \omega_4^3 - 4\omega_8^2 \omega_5 \omega_9 \omega_6^2 \omega_7^2 \omega_4^3 + 4\omega_8 \omega_5 \omega_9 \omega_6^2 \omega_7^2 \omega_4^3 + \omega_8^2 \omega_5 \omega_9 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 + 9\omega_8 \omega_5 \omega_9 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 + 12\omega_8^2 \omega_5 \omega_9 c_s^2 \omega_7^2 \omega_4^3 - 8\omega_8^2 \omega_5 \omega_9 \omega_6^3 c_s^2 \omega_7^2 \omega_4^2 + 2\omega_8^2 \omega_5 \omega_9 v_2^2 \omega_6^2 \omega_7^2 \omega_4 + \omega_8^2 \omega_5 \omega_9 \omega_6^2 \omega_7^2 \omega_4^3 - 4\omega_8 \omega_5 v_2^2 \omega_6^2 \omega_7^2 \omega_4^3 + 2\omega_8^2 \omega_5 \omega_9 \omega_6^3 c_s^2 \omega_7^2 \omega_4^3 - 6\omega_8^2 \omega_5 \omega_9 \omega_6^2 c_s^2 \omega_7^2 \omega_4^3 + 6\omega_8 \omega_5 \omega_9 \omega_6^3 \omega_7^2 \omega_4^3 - 8\omega_8 \omega_5 \omega_9 \omega_6^2 c_s^2 \omega_7^2 \omega_4^3) \frac{v_1 \rho v_2}{2\omega_8^2 \omega_5 \omega_9 \omega_6^3 \omega_7^2 \omega_4^3}$$

$$C_{D_x^2 D_y^2 v_1}^{(2), \text{MRT2}} = C_{D_x^2 D_y^2 v_1}^{(2), \text{MRT1}}$$

$$C_{D_x^2 D_y^2 v_1}^{(2), \text{CLBM1}} = 0$$

$$C_{D_x^2 D_y^2 v_1}^{(2), \text{CLBM2}} = 0$$

$$C_{D_x^2 D_y^2 v_1}^{(2), \text{CuLBM1}} = 0$$

$$C_{D_x^2 D_y^2 v_1}^{(2), \text{CuLBM2}} = (100\omega_2^2 v_1^2 \omega_3 \omega_1^3 + 135\omega_2^3 \omega_3 c_s^2 \omega_1 - 27\omega_2 v_2^2 \omega_3 \omega_1^3 - 18\omega_2^2 \omega_3 \omega_1 + 162\omega_2^2 \omega_3 c_s^2 \omega_1^2 + 36\omega_2^2 v_2^2 \omega_1^2 - 36\omega_2 v_2^2 \omega_3 \omega_1^2 + 108\omega_2^2 c_s^2 \omega_1^2 + 216\omega_2^3 v_1^2 \omega_3 \omega_1 + 18\omega_2 \omega_1^3 + 84\omega_2^2 \omega_3 c_s^2 \omega_1^3 - 100\omega_2^3 v_1^2 \omega_3 \omega_1^2 - 54\omega_2^3 \omega_3 c_s^2 - 46\omega_2^2 \omega_3 \omega_1^3 - 198\omega_2^3 v_1^2 \omega_3 + 162\omega_2^2 v_1^2 \omega_3 \omega_1 + 270\omega_3 c_s^2 \omega_1^3 - 54\omega_2^2 \omega_3 \omega_1^2 + 198v_1^2 \omega_3 \omega_1^3 - 54\omega_2^2 \omega_3 c_s^2 \omega_1 - 84\omega_2^3 \omega_3 c_s^2 \omega_1^2 - 126\omega_3 \omega_1^3 + 135\omega_2 \omega_3 \omega_1^3 - 162\omega_2 \omega_3 c_s^2 \omega_1^2 + 36\omega_2^3 v_2^2 \omega_3 - 216\omega_2 v_1^2 \omega_3 \omega_1^3 + 54\omega_2^3 \omega_3 - 297\omega_2 \omega_3 c_s^2 \omega_1^3 - 27\omega_2^3 v_2^2 \omega_3 \omega_1 - 81\omega_2^2 \omega_3 \omega_1 + 90\omega_2 \omega_3 \omega_1^2 - 162\omega_2 v_1^2 \omega_3 \omega_1^2 + 54\omega_2^2 v_2^2 \omega_3 \omega_1^2 - 36\omega_2^2 v_2^2 \omega_3 \omega_1 + 46\omega_2^3 \omega_3 \omega_1^2 - 54\omega_2 c_s^2 \omega_1^3 - 18\omega_2^3 v_2^2 \omega_1 + 18\omega_2^3 \omega_1 - 36\omega_2^2 \omega_1^2 - 54\omega_2^3 c_s^2 \omega_1 + 36v_2^2 \omega_3 \omega_1^3 - 18\omega_2 v_2^2 \omega_1^3) \frac{v_1 \rho v_2}{24\omega_2^3 \omega_3^2 \omega_1^3}$$

$$\text{coefficient } C_{D_x^2 D_y^2 v_2}^{(2)} \text{ at } \frac{\partial^4 v_2}{\partial x_1^2 \partial x_2^2} :$$

$$C_{D_x^2 D_y^2 v_2}^{(2), \text{SRT}} = (-24 + 8c_s^2 \omega^2 + 36v_2^2 \omega^2 - c_s^2 \omega^3 - 108v_2^2 \omega + 12c_s^2 - 18c_s^2 \omega + 72v_2^2 + 36\omega - 12\omega^2) \frac{\rho c_s^2}{12\omega^3}$$

$$C_{D_x^2 D_y^2 v_2}^{(2), \text{MRT1}} = (18\omega_8^2 v_1^2 \omega_9 \omega_6 \omega_7 \omega_4^3 - 24\omega_9 v_2^2 \omega_6^3 c_s^2 \omega_7 \omega_4^3 - 12\omega_8 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4 + 12v_1^2 \omega_9 \omega_6^2 \omega_7 \omega_4^3 + 60\omega_8 \omega_9 v_2^2 \omega_6^3 c_s^2 \omega_7 \omega_4 - 36\omega_8^2 \omega_9 v_2^2 \omega_6^2 c_s^2 \omega_4^2 + 12v_1^2 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^3 + 36\omega_8^2 v_1^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4^2 + 12\omega_8^2 \omega_6^3 c_s^2 \omega_7 \omega_4^2 - 36\omega_8 v_1^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + 12\omega_8 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 + 12\omega_8 \omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4^3 - 12\omega_8^2 v_1^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 - 36\omega_8^2 v_1^2 v_2^2 \omega_6^2 \omega_7 \omega_4^3 - 12\omega_8^2 v_1^2 \omega_6^3 \omega_4^2 + 12\omega_8 v_1^2 \omega_9 \omega_6^3 \omega_7 \omega_4^3 + 72\omega_8 v_1^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4^3 + 18\omega_8^2 \omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4 - 12v_1^2 \omega_9 \omega_6^3 c_s^2 \omega_7 \omega_4^2 - 6\omega_8^2 \omega_6^3 c_s^2 \omega_7 \omega_4^3 - 54\omega_8^2 v_1^2 \omega_9 v_2^2 \omega_6^2 \omega_7 \omega_4^3 + 18\omega_8^2 \omega_9 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - 12\omega_8^2 v_1^2 \omega_9 \omega_6^2 \omega_7 \omega_4^2 + 24\omega_9 v_2^2 \omega_6^3 c_s^2 \omega_7 \omega_4^2 + 18\omega_8^2 v_1^2 v_2^2 \omega_6^3 \omega_7 \omega_4^3 + 12\omega_8 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - 36\omega_8 v_1^2 \omega_9 \omega_6^3 \omega_7 \omega_4^2 + 6\omega_8^2 v_1^2 \omega_6^3 \omega_4^3 - 24\omega_8 \omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4^2 - 144\omega_8 \omega_9 v_2^2 \omega_6^2 c_s^2 \omega_7 \omega_4^3 - 48\omega_8^2 \omega_9 v_2^2 \omega_6^3 c_s^2 \omega_7 - 12\omega_8 \omega_6^3 c_s^2 \omega_7 \omega_4^2 + 12\omega_8 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + 24\omega_8 v_1^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^3 + 24\omega_8^2 \omega_9 \omega_6^2 c_s^4 \omega_7 \omega_4^2 + 24\omega_8 v_1^2 \omega_9 \omega_6^3 \omega_7 \omega_4 + 12\omega_8^2 v_1^2 \omega_9 \omega_6^2 c_s^2 \omega_7 \omega_4^2 + 6\omega_8^2 \omega_6^3 c_s^2 \omega_4^3 + 12\omega_8 \omega_9 \omega_6^3 c_s^4 \omega_7 \omega_4 -$$

$$4\omega_8\omega_6c_s^4\omega_4 + 24v_2^2\omega_6^2c_s^2\omega_4^2 - 4\omega_6^2c_s^4\omega_4^2 - 13\omega_8v_2^2\omega_6^2\omega_4^2 - 144\omega_2^2v_2^2\omega_6c_s^2\omega_4^2 + 4\omega_8^2\omega_2^2c_s^4 - 4\omega_6^3c_s^2\omega_1^2 + 8\omega_8^2c_s^4\omega_4^2 - 4\omega_8\omega_6^2c_s^2\omega_4^2 - 20\omega_8v_2^4\omega_6^2\omega_4 - 72\omega_8v_2^2\omega_6c_s^2\omega_4^2 + 8\omega_8\omega_6^2c_s^4\omega_4^2 - 20\omega_8v_2^4\omega_6\omega_4^2 + 84\omega_8v_2^2\omega_6^2c_s^2\omega_4 + 8\omega_8v_2^2\omega_6^3 + 24\omega_8^2v_2^4\omega_4^2 + 4\omega_8\omega_6^3c_s^2\omega_4^2) \frac{v_1}{4\omega_8^2\omega_6^3\omega_4^2}$$

$$C_{D_x D_y^3 \rho}^{(2), \text{MRT2}} = C_{D_x D_y^3 \rho}^{(2), \text{MRT1}}$$

$$C_{D_x D_y^3 \rho}^{(2), \text{CLBM1}} = 0$$

$$C_{D_x D_y^3 \rho}^{(2), \text{CLBM2}} = 0$$

$$C_{D_x D_y^3 \rho}^{(2), \text{CuLBM1}} = 0$$

$$C_{D_x D_y^3 \rho}^{(2), \text{CuLBM2}} = (-18\omega_2\omega_3c_s^4\omega_1^3 - 54\omega_2^3v_1^4\omega_3^2\omega_1 - 18\omega_2^2\omega_3^2c_s^4\omega_1 - 6\omega_2^3\omega_3c_s^2\omega_1 - 297\omega_2^2v_2^2\omega_3^2c_s^2\omega_1 + 12\omega_2^3v_1^2c_s^2\omega_1^2 - 99v_2^2\omega_3^2\omega_1^3 - 24\omega_2^3v_1^2v_2^2\omega_3^2\omega_1^2 - 6\omega_2^3\omega_3^2c_s^4\omega_1^2 + 12\omega_2^2c_s^2\omega_1^3 + 108\omega_2^2v_2^4\omega_3^2\omega_1^2 + 36\omega_2^3c_s^4\omega_1^2 + 6\omega_2^3v_1^2\omega_3^2 + 6\omega_2^3v_1^2\omega_3c_s^2\omega_1 + 63\omega_2^2v_2^2\omega_3^2\omega_1 + 45v_1^2v_2^2\omega_3^2\omega_1^3 + 90\omega_2^3c_s^4\omega_1^3 - \omega_2^3\omega_3^2\omega_1^2 + 24\omega_2^3v_2^2\omega_3^2\omega_1^2 - 18\omega_2^2\omega_3c_s^2\omega_1^3 + 6\omega_2^2v_1^2\omega_3^2c_s^2\omega_1 - 18\omega_2^3v_1^2\omega_3^2c_s^2 + 6\omega_2^3\omega_3^2\omega_1 + 60\omega_2\omega_3^2c_s^2\omega_1^2 - 18\omega_2^3v_2^2\omega_3^2\omega_1 - 6\omega_2\omega_3^2\omega_1^3 + 54\omega_2^3v_2^4\omega_3^2 + 72\omega_2^2v_2^2\omega_3^2c_s^2\omega_1^3 - 12\omega_2^3v_1^2c_s^2\omega_1^3 - 243\omega_2v_2^2\omega_3^2c_s^2\omega_1^2 - 21\omega_2v_1^2\omega_3^2c_s^2\omega_1^3 + 6\omega_2v_1^2\omega_3^2\omega_1^2 + 54v_2^4\omega_3^2\omega_1^3 + 6\omega_2^2\omega_3^2c_s^4\omega_1^3 - 108\omega_2^2v_2^2\omega_3^2\omega_1^2 - 18\omega_2^3v_1^2\omega_3c_s^2\omega_1^2 + 2\omega_2^2v_1^2\omega_3^2c_s^2\omega_1^3 + 540\omega_2^2v_2^2\omega_3^2c_s^2\omega_1^2 + 27\omega_2^3\omega_3^2c_s^4\omega_1 - 6\omega_2\omega_3^2\omega_1^3 + 72\omega_2^3v_1^2v_2^2\omega_3^2\omega_1 + 75\omega_2\omega_3^2c_s^2\omega_1^3 - 54\omega_2^2v_2^2\omega_3^2\omega_1 - 24\omega_2^2v_2^2\omega_3^2\omega_1^3 + 18\omega_2^3\omega_3c_s^2\omega_1^2 + 72\omega_2^2\omega_3^2c_s^4\omega_1^2 + 6\omega_2v_1^2\omega_3^2\omega_1^3 - 486\omega_2v_2^2\omega_3^2c_s^2\omega_1^3 - 6\omega_2v_1^2\omega_3^2c_s^2\omega_1^2 - 18\omega_2^3\omega_3^2c_s^4 - 45\omega_2^3v_1^2v_2^2\omega_3^2 - 12\omega_2^2c_s^2\omega_1^2 + 405v_2^2\omega_3^2c_s^2\omega_1^3 - 6v_1^2\omega_3^2\omega_1^3 - 54\omega_2v_2^4\omega_3^2\omega_1^2 - 72\omega_2^3c_s^2\omega_1^3 - 2\omega_2^2v_1^2\omega_3^2c_s^2\omega_1^2 + 6\omega_2^3\omega_3^2c_s^4\omega_1^3 - 6\omega_2v_1^2\omega_3c_s^2\omega_1^3 + 6\omega_2\omega_3c_s^2\omega_1^3 + \omega_2^3v_1^2\omega_3^2\omega_1^2 + 18\omega_2^3\omega_3c_s^4\omega_1 - 54\omega_2v_2^4\omega_3^2\omega_1^3 + 135\omega_2^3v_2^2\omega_3^2c_s^2 + \omega_2^3\omega_3^2\omega_1^3 + 12\omega_2^2\omega_3^2c_s^2\omega_1 + 18v_1^2\omega_3^2c_s^2\omega_1^3 + 18\omega_2^2v_1^2\omega_3c_s^2\omega_1^3 + 5\omega_2^3\omega_3^2c_s^2\omega_1^2 - 72\omega_2^3v_2^2\omega_3^2c_s^2\omega_1^2 - 36\omega_2^2c_s^4\omega_1^3 - 6\omega_2^2v_1^2\omega_3\omega_1 - 9\omega_2^2v_1^2v_2^2\omega_3^2\omega_1 + 45\omega_2^3v_2^2\omega_3^2\omega_1^2 - 3\omega_2^3\omega_3^2c_s^2\omega_1 - 54\omega_2^3v_2^2\omega_3^2c_s^2\omega_1 - 99\omega_2\omega_3^2c_s^4\omega_1^3 - 72\omega_2v_1^2v_2^2\omega_3^2\omega_1 - 6\omega_2^3v_1^2\omega_3^2\omega_1 - 72\omega_2^2\omega_3^2c_s^2\omega_1^2 - 54\omega_2^3\omega_3c_s^4\omega_1^2 + 24\omega_2^2v_1^2v_2^2\omega_3^2\omega_1^3 - \omega_2^2v_2^2\omega_3^2\omega_1^3 - 54\omega_2\omega_3^2c_s^4\omega_1^2 + 21\omega_2^2v_1^2\omega_3^2c_s^2\omega_1 - 6\omega_2^3\omega_3^2 + 126\omega_2v_2^2\omega_3^2\omega_1^3 - 9\omega_2^3v_2^2\omega_3^2 - 5\omega_2^2\omega_3^2c_s^2\omega_1^3 + 6\omega_2^2\omega_3^2\omega_1 + 9\omega_2v_1^2v_2^2\omega_3^2\omega_1^2) \frac{v_1}{24\omega_2^3\omega_3^2\omega_1^3}$$

$$\text{coefficient } C_{D_x D_y^3 v_1}^{(2)} \text{ at } \frac{\partial^4 v_1}{\partial x_1 \partial x_2^3} :$$

$$C_{D_x D_y^3 v_1}^{(2), \text{SRT}} = (-12c_s^2\omega^2 - 36v_2^4 - 4v_2^2\omega^3 + 26v_2^2\omega^2 - 54v_2^2\omega - 24c_s^2 + 36c_s^2\omega + 36c_s^4 - 42v_2^2c_s^2\omega^2 - 26v_2^4\omega^2 - c_s^4\omega^3 + 20c_s^4\omega^2 - 36v_2^2c_s^2 + 4v_2^4\omega^3 + 12v_2^2c_s^2\omega^3 - 54c_s^4\omega + 36v_2^2 + 54v_2^2c_s^2\omega + 54v_2^4\omega) \frac{\rho}{12\omega^3}$$

$$C_{D_x D_y^3 v_1}^{(2), \text{MRT1}} = (54\omega_8v_2^2\omega_6^2c_s^2\omega_4^3 - 12v_2^4\omega_6^2\omega_4^3 - 24\omega_8v_2^4\omega_6^2\omega_4^2 - 48\omega_8v_2^2\omega_6^2\omega_4^2 + 12\omega_8^2\omega_6c_s^2\omega_4^2 - 12v_2^2\omega_6^3\omega_4^3 + 162\omega_8^2v_2^2\omega_6^2c_s^2\omega_4^2 + 90\omega_8^2v_2^2\omega_6\omega_4^3 + 6\omega_8^2\omega_6^2c_s^2\omega_4 + 60\omega_8v_2^4\omega_6^2\omega_4^3 + 12\omega_8\omega_6^3c_s^4\omega_4 - 24\omega_8v_2^4\omega_6^3\omega_4 - 12\omega_8v_2^2\omega_6^2c_s^2\omega_4^2 - 48\omega_8^2v_2^2\omega_6^3c_s^2 + 60\omega_8^2v_2^2\omega_6^2c_s^2\omega_4^3 + 252\omega_8^2v_2^2c_s^2\omega_4^3 + 12v_2^2\omega_6^3\omega_4^3 + 12\omega_8^2\omega_6^2c_s^2\omega_4^2 + 27\omega_8v_2^4\omega_6^3\omega_4 - 12\omega_8\omega_6^2c_s^2\omega_4^2 + 24\omega_8v_2^2\omega_6^2\omega_4^2 + 12v_2^2\omega_6^3\omega_4 - 18\omega_8\omega_6^3c_s^4\omega_4 - 12v_2^2\omega_6^2c_s^2\omega_4^3 + \omega_8^2\omega_6^2c_s^4\omega_4^3 + 12v_2^2\omega_6^3\omega_4^3 - 12\omega_2^3c_s^4\omega_4^3 + 48\omega_8v_2^4\omega_6^3\omega_4 - 90\omega_8^2v_2^4\omega_6\omega_4^3 + 24\omega_8v_2^2\omega_6^3\omega_4 - 60\omega_8v_2^2\omega_6^3\omega_4 + 6\omega_8\omega_6^2c_s^2\omega_4^3 - 48\omega_2^2v_2^2\omega_6^2c_s^2\omega_4 - 5\omega_8^2\omega_6^3c_s^2\omega_4^2 - 27\omega_8v_2^4\omega_6^3\omega_4^3 + 6\omega_8\omega_6^3c_s^4\omega_4 - 72\omega_8^2v_2^2\omega_4^3 + 12v_2^4\omega_6^2\omega_4^2 + 6\omega_8^2\omega_6^2c_s^4\omega_4^2 - 12\omega_8\omega_6^3c_s^2\omega_4^2 + 36\omega_8v_2^2\omega_6\omega_4^3 + 12\omega_8^2\omega_6^3c_s^4 + 12\omega_8^2v_2^2\omega_6^2c_s^2\omega_4^3 + 18\omega_8^2v_2^2\omega_6^3\omega_4^2 + 12\omega_2^2v_2^4\omega_6^2\omega_4^2 + 30\omega_8v_2^4\omega_6^3c_s^2\omega_4^2 - 12\omega_2^2\omega_6^4c_s^4\omega_4^3 - 4\omega_8^2v_2^2\omega_6^3\omega_4^3 - 81\omega_8^2v_2^2\omega_6^3c_s^2\omega_4^2 - 21\omega_8v_2^2\omega_6^3c_s^2\omega_4^3 - 12\omega_8^2\omega_6^4c_s^4\omega_4^2 - 24\omega_8^2\omega_6^3c_s^4\omega_4 + 19\omega_8^2v_2^2c_s^4\omega_4^3 + 12\omega_8^2v_2^2\omega_6^2\omega_4 - 36\omega_8v_2^2\omega_6\omega_4^3 + 13\omega_8^2\omega_6^3c_s^4\omega_4^2 - 12v_2^2\omega_6^3c_s^2\omega_4^3 - 6\omega_8\omega_6^3c_s^4\omega_4^3 - 36\omega_8v_2^2\omega_6^2c_s^2\omega_4^3 - 18\omega_8^2v_2^2\omega_6^3\omega_4^2 - 12\omega_8^2v_2^2\omega_6^2\omega_4^2 - 108\omega_8^2v_2^2\omega_6^2c_s^2\omega_4^2 - 6\omega_2^2\omega_6^2c_s^2\omega_4^2 + 102\omega_8^2v_2^2\omega_6^3c_s^2\omega_4 + 72\omega_8^2v_2^4\omega_4^3 - 6\omega_8\omega_6^3c_s^2\omega_4^3 + 4\omega_8^2v_2^4\omega_6^3\omega_4 - \omega_8^2\omega_6^3c_s^4\omega_4^3 + 12\omega_8\omega_6^2c_s^4\omega_4^2 + 12v_2^2\omega_6^3c_s^2\omega_4^3 - 12\omega_8v_2^2\omega_6^3c_s^2\omega_4 - \omega_8^2\omega_6^2c_s^2\omega_4^3 + 18\omega_8\omega_6^2c_s^2\omega_4^2 + 12\omega_8^2c_s^4\omega_4^3 - 306\omega_8^2v_2^2\omega_6^2c_s^2\omega_4^3 - 12\omega_8^2v_2^2\omega_6^3\omega_4 - 19\omega_8^2v_2^2\omega_6^2\omega_4^2) \frac{\rho}{12\omega_8^2\omega_6^3\omega_4^3}$$

$$C_{D_x D_y^3 v_1}^{(2), \text{MRT2}} = C_{D_x D_y^3 v_1}^{(2), \text{MRT1}}$$

$$C_{D_x D_y^3 v_1}^{(2), \text{CLBM1}} = (198\omega_8v_2^2\omega_6^2c_s^2\omega_4^3 - 36v_2^4\omega_6^2\omega_4^3 - 36\omega_8v_2^2\omega_6^3\omega_4^2 + 12\omega_8\omega_6c_s^2\omega_4^2 - 36v_2^2\omega_6^3\omega_4^3 + 18\omega_8^2v_2^2\omega_6^2c_s^2\omega_4^2 + 90\omega_8^2v_2^2\omega_6\omega_4^3 + 6\omega_8^2\omega_6^3c_s^2\omega_4 + 72\omega_8v_2^4\omega_6^2\omega_4^3 + 12\omega_8\omega_6^3c_s^4\omega_4 + 36\omega_8v_2^2\omega_6^2c_s^2\omega_4^2 + 60\omega_8^2v_2^2\omega_6^2c_s^2\omega_4^3 + 252\omega_8^2v_2^2c_s^2\omega_4^3 + 36v_2^2\omega_6^3\omega_4^2 + 12\omega_8^2\omega_6^2c_s^2\omega_4^3 + 39\omega_8v_2^2\omega_6^3\omega_4^3 - 12\omega_8\omega_6^2c_s^2\omega_4^2 + 36v_2^2\omega_6^3\omega_4^3 - 18\omega_8\omega_6^3c_s^4\omega_4^2 - 108v_2^2\omega_6^2c_s^2\omega_4^3 + \omega_8^2\omega_6^2c_s^4\omega_4^3 + 36v_2^4\omega_6^3\omega_4^3 - 12\omega_8^2c_s^2\omega_4^3 + 36\omega_8v_2^4\omega_6^3\omega_4^2 - 90\omega_8^2v_2^4\omega_6\omega_4^3 - 72\omega_8v_2^2\omega_6^2\omega_4^3 + 6\omega_8\omega_6^2c_s^2\omega_4^3 - 5\omega_8^2\omega_6^3c_s^2\omega_4^2 - 39\omega_8v_2^4\omega_6^3\omega_4^3 + 6\omega_8\omega_6^3c_s^4\omega_4^3 - 72\omega_8^2v_2^2\omega_4^3 - 36v_2^4\omega_6^3\omega_4^2 + 6\omega_8^2\omega_6^2c_s^4\omega_4^2 - 12\omega_8\omega_6^3c_s^2\omega_4^3 + 36\omega_8v_2^2\omega_6\omega_4^3 + 12\omega_8^2\omega_6^3c_s^4 + 12\omega_2^2v_2^4\omega_6^2\omega_4^2 + 6\omega_8^2v_2^2\omega_6^3\omega_4^2 + 54\omega_8v_2^2\omega_6^3c_s^2\omega_4^2 - 12\omega_2^2\omega_6^4c_s^4\omega_4^3 - 4\omega_8^2v_2^2\omega_6^3\omega_4^3 - 3\omega_8^2v_2^2\omega_6^3c_s^2\omega_4^2 - 99\omega_8v_2^2\omega_6^3c_s^2\omega_4^3 - 12\omega_8^2\omega_6^4c_s^4\omega_4^2 - 24\omega_8^2\omega_6^3c_s^4\omega_4 + 19\omega_8^2v_2^4\omega_6^2\omega_4^2 - 36\omega_8v_2^4\omega_6\omega_4^3 + 13\omega_8^2\omega_6^3c_s^4\omega_4^2 - 108v_2^2\omega_6^3c_s^2\omega_4^2 - 6\omega_8\omega_6^2c_s^4\omega_4^3 - 108\omega_8v_2^2\omega_6^2c_s^2\omega_4^3 - 6\omega_8^2v_2^2\omega_6^3\omega_4^2 - 36\omega_2^2v_2^4\omega_6^2\omega_4^2 - 6\omega_8^2\omega_6^2c_s^2\omega_4^2 - 18\omega_8^2v_2^2\omega_6^3c_s^2\omega_4 + 72\omega_8^2v_2^4\omega_4^3 - 6\omega_8\omega_6^3c_s^2\omega_4^3 + 4\omega_8^2v_2^4\omega_6^3\omega_4^3 - \omega_8^2\omega_6^3c_s^4\omega_4^3 + 12\omega_8\omega_6^2c_s^4\omega_4^2 + 108v_2^2\omega_6^3c_s^2\omega_4^3 + 36\omega_8v_2^2\omega_6^3c_s^2\omega_4 - \omega_8^2\omega_6^2c_s^2\omega_4^3 + 18\omega_8\omega_6^2c_s^2\omega_4^2 + 12\omega_8^2c_s^4\omega_4^3 - 306\omega_8^2v_2^2\omega_6^2c_s^2\omega_4^3 - 19\omega_8^2v_2^2\omega_6^3\omega_4 - 19\omega_8^2v_2^2\omega_6^2\omega_4^2) \frac{\rho}{12\omega_8^2\omega_6^3\omega_4^3}$$

$$C_{D_x D_y^3 v_1}^{(2), \text{CLBM2}} = C_{D_x D_y^3 v_1}^{(2), \text{CLBM1}}$$

$$C_{D_x D_y^3 v_1}^{(2), \text{CuLBM1}} = (-6\omega_2^2\omega_6^2\omega_3^2c_s^2 + 12\omega_2^3\omega_6^2c_s^4 + 6\omega_2^3v_2^2\omega_6^2\omega_3^2 + 19\omega_2^2v_2^4\omega_6^2\omega_3^3 - 12\omega_2v_2^2\omega_6^2\omega_3^3c_s^4 - 108\omega_2v_2^2\omega_6\omega_3^3c_s^2 + 12\omega_2^2\omega_6\omega_3^2c_s^4 - 4\omega_2^3v_2^2\omega_6^2\omega_3^3 + 6\omega_2^3\omega_6\omega_3^2c_s^4 - 72v_2^2\omega_6^2\omega_3^3 + 252v_2^2\omega_6^2\omega_3^3c_s^2 + 198\omega_2^2v_2^2\omega_6\omega_3^3c_s^2 + 36\omega_2v_2^2\omega_6\omega_3^3 + 12\omega_2^3v_2^2\omega_6^2\omega_3^3c_s^2 - 12\omega_6^2\omega_3^3c_s^2 - 6\omega_2^2\omega_6\omega_3^3c_s^4 - 12\omega_2^3\omega_6\omega_3^2c_s^2 + 36\omega_2^3v_2^2\omega_6\omega_3^2c_s^2 - 12\omega_2\omega_6^2\omega_3^2c_s^4 - 19\omega_2^3v_2^2\omega_6^2\omega_3^3 - 36\omega_2^3v_2^4\omega_6^2\omega_3^3 - 6\omega_2^3v_2^4\omega_6^2\omega_3^2 - 24\omega_2^3\omega_6^2\omega_3^2c_s^4 - \omega_2^2\omega_6^2\omega_3^2c_s^2 + 36\omega_2^3v_2^2\omega_6^2\omega_3^2c_s^2 + 4\omega_2^3v_2^4\omega_6^2\omega_3^3 + 72v_2^4\omega_6^2\omega_3^3 - 5\omega_2^3\omega_6^2\omega_3^2c_s^2 - 36\omega_2v_2^4\omega_6\omega_3^3 - 18\omega_2^3\omega_6\omega_3^2c_s^4 + 36\omega_2^3v_2^2\omega_6\omega_3^2c_s^2 + 36\omega_2^2v_2^2\omega_6\omega_3^2c_s^2 - 18\omega_2^3v_2^2\omega_6\omega_3^2c_s^2 - 18\omega_2^3\omega_6\omega_3^2c_s^2 + 12\omega_2^3\omega_6\omega_3^2c_s^4 + 6\omega_2^3\omega_6\omega_3^2c_s^2 - 36\omega_2v_2^2\omega_6^2\omega_3^2c_s^2 + 12\omega_2^3\omega_6\omega_3^2c_s^4 + 6\omega_2^2\omega_6\omega_3^2c_s^2 - 90\omega_2v_2^4\omega_6^2\omega_3^3 + 12\omega_2\omega_6^2\omega_3^2c_s^2 - 108\omega_2^2v_2^2\omega_3^2c_s^2 + 18\omega_2^3\omega_6\omega_3^2c_s^2 +$$

$$36\omega_3^3 v_2^4 \omega_6 \omega_3^2 - 108\omega_2^3 v_2^2 \omega_3^2 c_s^2 - 72\omega_2^2 v_2^2 \omega_6 \omega_3^3 + 54\omega_3^3 v_2^2 \omega_6 \omega_3^2 c_s^2 + 18\omega_2^2 v_2^2 \omega_6^2 \omega_3^2 c_s^2 + 13\omega_2^3 \omega_6^2 \omega_3^2 c_s^4 + 12\omega_2 \omega_6^2 \omega_3^2 c_s^2 + 39\omega_3^3 v_2^2 \omega_6 \omega_3^3 - 36\omega_2^3 v_2^2 \omega_3^3 - 12\omega_2^2 \omega_6 \omega_3^2 c_s^2 - 306\omega_2 v_2^2 \omega_6^2 \omega_3^2 c_s^2 + 6\omega_2^2 \omega_6^2 \omega_3^2 c_s^4 + 90\omega_2 v_2^2 \omega_6^2 \omega_3^3 - 36\omega_2^2 v_2^4 \omega_3^3 + 60\omega_2^2 v_2^2 \omega_6^2 \omega_3^2 c_s^2 - \omega_2^2 \omega_6^2 \omega_3^2 c_s^4 + 72\omega_2^2 v_2^4 \omega_6 \omega_3^3 - 36\omega_2^3 v_2^2 \omega_6 \omega_3^2 + 36\omega_2^3 v_2^2 \omega_3^3 + 12\omega_6^2 \omega_3^3 c_s^4 + 108\omega_2^2 v_2^2 \omega_3^2 c_s^2 - 99\omega_3^3 v_2^2 \omega_6 \omega_3^2 c_s^2 - 6\omega_2^3 \omega_6 \omega_3^2 c_s^2) \frac{\rho}{12\omega_2^3 \omega_6^2 \omega_3^3}$$

$$C_{D_x D_y^3 v_1}^{(2), \text{CuLBM2}} = (6\omega_2 \omega_3 c_s^4 \omega_1^3 - 63\omega_2^3 v_2^4 \omega_3^2 \omega_1 + 54\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1 + 6\omega_2^2 \omega_3^2 c_s^4 \omega_1 - 108\omega_2^3 v_2^4 \omega_1^2 - 18\omega_2^3 \omega_3 c_s^2 \omega_1 + 18\omega_2 v_2^2 \omega_3 \omega_1^3 + 99\omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_1 - 81v_2^2 \omega_3^2 \omega_1^3 - 72\omega_2^3 v_1^2 v_2^2 \omega_3^2 \omega_1^2 - 24\omega_2^2 \omega_3 c_s^2 \omega_1^2 + 29\omega_2^3 \omega_3^2 c_s^4 \omega_1^2 - 8\omega_2^3 v_2^2 \omega_3^2 \omega_1^3 - 54\omega_2^2 v_2^4 \omega_3^2 \omega_1^2 + 18\omega_2^3 v_1^2 \omega_3^2 + 72\omega_3^3 v_2^4 \omega_1^3 - 54\omega_2^3 v_1^2 \omega_3 c_s^2 \omega_1 - 27\omega_2^3 v_2^2 \omega_3^2 \omega_1 + 135v_1^2 v_2^2 \omega_3^2 \omega_1^3 + 18\omega_3^3 c_s^4 \omega_1^3 + 36\omega_2^2 v_2^2 \omega_3^3 + 19\omega_2^2 v_2^4 \omega_3^2 \omega_1^3 - \omega_2^3 \omega_3^2 \omega_1^2 + 17\omega_2^3 v_2^2 \omega_3^2 \omega_1^2 + 18\omega_2^2 \omega_3 c_s^2 \omega_1^3 - 2\omega_2^3 \omega_3^2 c_s^4 \omega_1^3 + 18\omega_2^3 v_1^2 \omega_3^2 c_s^2 \omega_1 + 18\omega_3^3 v_1^2 \omega_3^2 c_s^2 + 6\omega_2^3 \omega_3^2 \omega_1 + 36\omega_2 \omega_3^2 c_s^2 \omega_1^2 - 9\omega_2^3 v_2^2 \omega_3^2 \omega_1 - 6\omega_2 \omega_3^2 \omega_1^2 + 36\omega_2^3 v_2^2 \omega_3^2 + 18\omega_2^3 v_1^2 \omega_3^2 c_s^2 \omega_1^2 - 18\omega_2 v_2^4 \omega_3 \omega_1^3 + 84\omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_1^3 + 81\omega_2 v_2^2 \omega_3^2 c_s^2 \omega_1^2 - 9\omega_2 v_1^2 \omega_3^2 c_s^2 \omega_1^3 - 12\omega_2^3 \omega_3 c_s^2 \omega_1^3 + 18\omega_2 v_1^2 \omega_3^2 \omega_1^2 + 36v_2^4 \omega_3^2 \omega_1^3 - \omega_2^2 \omega_3^2 c_s^4 \omega_1^3 + 54\omega_2^2 v_2^2 \omega_3^2 \omega_1^2 + 36\omega_2^3 v_1^2 \omega_3 c_s^2 \omega_1^2 - 198\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^3 + 8\omega_2^3 v_2^4 \omega_3^2 \omega_1^3 - 6\omega_2^2 v_1^2 \omega_3^2 c_s^2 \omega_1^2 - 144\omega_2^2 v_2^2 \omega_3^2 c_s^2 \omega_1^2 - 57\omega_2^3 \omega_3^2 c_s^4 \omega_1 - 6\omega_2 \omega_3^2 \omega_1^3 + 216\omega_2^3 v_1^2 v_2^2 \omega_3^2 \omega_1 + 21\omega_2 \omega_3^2 c_s^2 \omega_1^3 + 36\omega_2^3 v_2^4 \omega_3^2 \omega_1 + 7\omega_2^3 v_2^4 \omega_3^2 \omega_1^2 - 43\omega_2^3 v_2^2 \omega_3^2 \omega_1^3 + 306\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^2 + 30\omega_2^3 \omega_3 c_s^2 \omega_1^2 + 12\omega_2^2 \omega_3^2 c_s^4 \omega_1^2 + 18\omega_2 v_1^2 \omega_3^2 \omega_1^3 - 297\omega_2 v_2^2 \omega_3^2 c_s^2 \omega_1^3 - 54\omega_2 v_1^2 \omega_3^2 c_s^2 \omega_1^2 + 30\omega_2^3 \omega_3^2 c_s^4 - 135\omega_2^3 v_2^2 v_2^2 \omega_3^2 + 189v_2^2 \omega_3^2 c_s^2 \omega_1^3 + 36\omega_2^2 v_1^2 \omega_3 c_s^2 \omega_1^2 + 198\omega_2^2 v_2^2 \omega_3 c_s^2 \omega_1^3 - 18v_1^2 \omega_3^2 \omega_1^3 + 36\omega_2 v_2^2 \omega_3^2 \omega_1^2 - 324\omega_2^3 v_2^2 c_s^2 \omega_1^2 - 72\omega_2^3 v_2^2 \omega_3^3 - 72\omega_2^2 v_2^2 \omega_3 \omega_1^3 - 36\omega_2^2 v_2^2 \omega_1^3 - 24\omega_2^3 c_s^2 \omega_1^3 + 6\omega_2^3 v_1^2 \omega_3^2 c_s^2 \omega_1^2 + 24\omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_1^2 + 144\omega_2^3 v_2^4 \omega_3 \omega_1^2 + 6\omega_2^3 \omega_1^3 - 18\omega_2^2 \omega_3 c_s^4 \omega_1^3 + 18\omega_2 v_1^2 \omega_3 c_s^2 \omega_1^3 - 6\omega_2 \omega_3 c_s^2 \omega_1^3 + 18\omega_3^3 v_2^2 \omega_3 \omega_1 + 3\omega_2^3 v_1^2 \omega_3^2 \omega_1^2 + 18\omega_2^3 \omega_3 c_s^4 \omega_1 - 63\omega_2 v_2^4 \omega_3^2 \omega_1^3 + 63\omega_2^3 v_2^2 \omega_3^2 c_s^2 + \omega_2^2 \omega_3^2 \omega_1^3 + 108\omega_2^3 v_2^2 \omega_1^2 - 12\omega_2^2 \omega_3^2 c_s^2 \omega_1 + 216\omega_2^3 v_2^2 c_s^2 \omega_1^3 + 18v_1^2 \omega_3^2 c_s^2 \omega_1^3 - 36\omega_2^2 v_1^2 \omega_3 c_s^2 \omega_1^2 - 72\omega_2^2 v_2^2 \omega_3 c_s^2 \omega_1^2 - 54\omega_2 v_2^2 \omega_3 c_s^2 \omega_1^3 - 12\omega_2^3 \omega_3^2 c_s^2 \omega_1^2 + 24\omega_2^2 \omega_3 c_s^4 \omega_1^2 + 30\omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_1^2 - 78\omega_2^3 v_2^4 \omega_3 \omega_1^3 - 18\omega_2^2 v_1^2 \omega_3^2 \omega_1 - 27\omega_2^3 v_1^2 v_2^2 \omega_3 \omega_1 + 36\omega_2^2 v_2^2 \omega_3 \omega_1^2 - 45\omega_2 v_2^2 \omega_3^2 \omega_1^2 - 108\omega_2^2 v_2^2 c_s^2 \omega_1^3 + 15\omega_2^3 \omega_3^2 c_s^2 \omega_1 - 171\omega_2^3 v_2^2 \omega_3^2 c_s^2 \omega_1 - 15\omega_2 \omega_3^2 c_s^4 \omega_1^3 - 216\omega_2 v_1^2 v_2^2 \omega_3^2 \omega_1^3 - 18\omega_2^3 v_1^2 \omega_3^2 \omega_1 - 144\omega_2^3 v_2^2 \omega_3 \omega_1^2 + 72\omega_2^2 v_2^4 \omega_3 \omega_1^3 - 12\omega_2^2 \omega_3^2 c_s^2 \omega_1^2 - 30\omega_2^3 \omega_3 c_s^4 \omega_1^2 + 72\omega_2^2 v_1^2 v_2^2 \omega_3^2 \omega_1^3 - 3\omega_2^2 v_1^2 \omega_3^2 \omega_1^3 - 30\omega_2 \omega_3^2 c_s^4 \omega_1^2 - 18\omega_2^3 v_2^4 \omega_3 \omega_1 - 9\omega_2^3 v_1^2 \omega_3^2 c_s^2 \omega_1 - 6\omega_2^2 \omega_3^2 + 135\omega_2 v_2^2 \omega_3^2 \omega_1^3 + 9\omega_2^3 v_2^2 \omega_3^2 + 12\omega_2^2 \omega_3 c_s^4 \omega_1^3 + 6\omega_2^2 \omega_3^2 \omega_1 - 36\omega_2^2 v_2^4 \omega_3 \omega_1^2 + 27\omega_2 v_1^2 v_2^2 \omega_3^2 \omega_1^2 + 78\omega_2^3 v_2^2 \omega_3 \omega_1^3) \frac{\rho}{24\omega_2^3 \omega_3^2 \omega_1^3}$$

coefficient $C_{D_x D_y^3 v_2}^{(2)}$ **at** $\frac{\partial^4 v_2}{\partial x_1 \partial x_2^2}$:

$$C_{D_x D_y^3 v_2}^{(2), \text{SRT}} = 0$$

$$C_{D_x D_y^3 v_2}^{(2), \text{MRT1}} = (20\omega_8^2 \omega_6^2 c_s^2 + 28\omega_8^2 v_2^2 \omega_6^2 + 16\omega_8 \omega_6^2 \omega_4 - 43\omega_8 v_2^2 \omega_6^2 \omega_4^2 - 72\omega_8^2 \omega_6 c_s^2 \omega_4^2 - 24\omega_8^2 \omega_6 \omega_4 + 17\omega_8 \omega_6^3 \omega_4^2 - 120\omega_8^2 v_2^2 \omega_6 \omega_4^2 - 16\omega_8 \omega_6^2 c_s^2 \omega_4 - 48\omega_8 v_2^2 \omega_6^2 \omega_4 + 16v_2^2 \omega_6^2 \omega_4^2 - 12\omega_8^2 \omega_6^2 + 56\omega_8 \omega_6^2 c_s^2 \omega_4^2 + 64\omega_8^2 v_2^2 \omega_6 \omega_4 - 16v_2^2 \omega_6^3 \omega_4 + 104\omega_8 v_2^2 \omega_6^2 \omega_4^2 + 80\omega_8^2 v_2^2 \omega_4^2 - 28\omega_8 \omega_6^3 \omega_4 + 48\omega_8^2 \omega_6 \omega_4^2 - 16v_2^2 \omega_6^2 \omega_4^2 + 68\omega_8 v_2^2 \omega_6^2 \omega_4 + 32\omega_8^2 \omega_6 c_s^2 \omega_4 - 32\omega_8^2 \omega_4^2 - 16\omega_6^2 c_s^2 \omega_4^2 + 48\omega_8^2 c_s^2 \omega_4^2 - 40\omega_8 \omega_6^2 \omega_4^2 + 44\omega_8 \omega_6^3 c_s^2 \omega_4 - 32\omega_8 \omega_6 c_s^2 \omega_4^2 + 24\omega_8 \omega_6 \omega_4^2 - 20\omega_8 \omega_6^3 c_s^2 - 68\omega_8^2 v_2^2 \omega_6^2 \omega_4 - 16\omega_6^3 c_s^2 \omega_4 - 64\omega_8 v_2^2 \omega_6 \omega_4^2 - 44\omega_8^2 \omega_6^2 c_s^2 \omega_4 - 17\omega_8^2 \omega_6^2 \omega_4^2 - 8\omega_6^3 \omega_4 + 8\omega_6^3 \omega_4 + 28\omega_8^2 \omega_6^2 \omega_4 + 43\omega_8^2 v_2^2 \omega_6^2 \omega_4 + 16\omega_6^3 c_s^2 \omega_4^2 + 25\omega_8^2 \omega_6^2 c_s^2 \omega_4^2 + 8\omega_6^2 \omega_4^2 - 28\omega_8 v_2^2 \omega_6^2 - 25\omega_8 \omega_6^2 c_s^2 \omega_4^2 + 12\omega_8 \omega_6) \frac{v_1 \rho v_2}{4\omega_8^3 \omega_6^3 \omega_4^2}$$

$$C_{D_x D_y^3 v_2}^{(2), \text{MRT2}} = C_{D_x D_y^3 v_2}^{(2), \text{MRT1}}$$

$$C_{D_x D_y^3 v_2}^{(2), \text{CLBM1}} = 0$$

$$C_{D_x D_y^3 v_2}^{(2), \text{CLBM2}} = 0$$

$$C_{D_x D_y^3 v_2}^{(2), \text{CuLBM1}} = 0$$

$$C_{D_x D_y^3 v_2}^{(2), \text{CuLBM2}} = (10\omega_2^2 v_1^2 \omega_3 \omega_1^3 - 27\omega_2^3 \omega_3 c_s^2 \omega_1 - 198\omega_2 v_2^2 \omega_3 \omega_1^3 + 90\omega_2^2 \omega_3 \omega_1 + 18\omega_2^3 v_1^2 \omega_1 + 324\omega_2^2 \omega_3 c_s^2 \omega_1^2 + 54\omega_2^2 c_s^2 \omega_1^3 - 198\omega_2 v_2^2 \omega_3 \omega_1^2 - 18\omega_2 v_1^2 \omega_1^3 + 45\omega_2^3 v_1^2 \omega_3 \omega_1 + 18\omega_2 \omega_1^3 + 30\omega_2^2 \omega_3 c_s^2 \omega_1^2 - 10\omega_2^3 v_1^2 \omega_3 \omega_1^2 + 54\omega_2^3 \omega_3 c_s^2 - 10\omega_2^2 \omega_3 \omega_1^3 - 36\omega_2^3 v_1^2 \omega_3 + 270\omega_3 c_s^2 \omega_1^3 - 18\omega_2^3 v_1^2 \omega_1^2 - 180\omega_2^2 \omega_3 \omega_1^2 + 36v_1^2 \omega_3 \omega_1^3 - 162\omega_2^2 \omega_3 c_s^2 \omega_1 - 30\omega_2^3 \omega_3 c_s^2 \omega_1^2 - 126\omega_3 \omega_1^3 - 54\omega_2^3 c_s^2 \omega_1^2 + 135\omega_2 \omega_3 \omega_1^3 - 162\omega_2 \omega_3 c_s^2 \omega_1^2 + 198\omega_2^3 v_2^2 \omega_3 - 45\omega_2 v_1^2 \omega_3 \omega_1^3 - 54\omega_2^2 \omega_3 - 297\omega_2 \omega_3 c_s^2 \omega_1^3 - 198\omega_2^2 v_2^2 \omega_3 \omega_1 + 45\omega_2^3 \omega_3 \omega_1 + 18\omega_2^2 \omega_1^2 + 90\omega_2 \omega_3 \omega_1^2 + 396\omega_2^2 v_2^2 \omega_3 \omega_1^2 - 198\omega_2^2 v_2^2 \omega_3 \omega_1 + 10\omega_2^3 \omega_3 \omega_1^2 - 54\omega_2 c_s^2 \omega_1^3 - 18\omega_2^3 \omega_1 - 18\omega_2^2 \omega_1^3 + 54\omega_2^2 c_s^2 \omega_1 + 198v_2^2 \omega_3 \omega_1^3 + 18\omega_2^2 v_1^2 \omega_1^3) \frac{v_1 \rho v_2}{24\omega_2^3 \omega_3 \omega_1^3}$$

coefficient $C_{D_y^4 \rho}^{(2)}$ **at** $\frac{\partial^4 \rho}{\partial x_2^2}$:

$$C_{D_y^4 \rho}^{(2), \text{SRT}} = (12 - 78c_s^2 \omega^2 + 144v_2^4 + 10v_2^2 \omega^3 - 98v_2^2 \omega^2 + 6c_s^2 \omega^3 + 234v_2^2 \omega - 132c_s^2 + 198c_s^2 \omega + 144c_s^4 + 404v_2^2 c_s^2 \omega^2 + 90v_2^4 \omega^2 - 5c_s^4 \omega^3 + 82c_s^4 \omega^2 + 672v_2^2 c_s^2 - 9v_2^4 \omega^3 - 34v_2^2 c_s^2 \omega^3 - 216c_s^4 \omega - 156v_2^2 - 18\omega + 8\omega^2 - 1008v_2^2 c_s^2 \omega - 216v_2^4 \omega - \omega^3) \frac{v_2}{12\omega^3}$$

$$C_{D_y^4 \rho}^{(2), \text{MRT1}} = (12 + 6\omega_6^3 c_s^2 + 144v_2^4 + 234v_2^2 \omega_6 - 1008v_2^2 \omega_6 c_s^2 - 78\omega_6^2 c_s^2 - 132c_s^2 - 98v_2^2 \omega_6^2 - 18\omega_6 - 216\omega_6 c_s^4 + 10v_2^2 \omega_6^3 + 82\omega_6^2 c_s^4 - 34v_2^2 \omega_6^3 c_s^2 - 216v_2^4 \omega_6 + 144c_s^4 - \omega_6^3 + 8\omega_6^2 + 672v_2^2 c_s^2 + 198\omega_6 c_s^2 - 156v_2^2 - 9v_2^4 \omega_6^3 + 404v_2^2 \omega_6^2 c_s^2 - 5\omega_6^3 c_s^4 + 90v_2^4 \omega_6^2) \frac{v_2}{12\omega_6^3}$$

$$C_{D_y^4 \rho}^{(2), \text{MRT2}} = C_{D_y^4 \rho}^{(2), \text{MRT1}}$$

$$C_{D_y^4 \rho}^{(2), \text{CLBM1}} = C_{D_y^4 \rho}^{(2), \text{MRT1}}$$

$$C_{D_y^4 \rho}^{(2), \text{CLBM2}} = C_{D_y^4 \rho}^{(2), \text{MRT1}}$$

$$C_{D_y^4 \rho}^{(2), \text{CuLBM1}} = (12 - \omega_2^3 + 144v_2^4 + 234\omega_2 v_2^2 - 18\omega_2 + 90\omega_2^2 v_2^4 + 8\omega_2^2 + 6\omega_2^3 c_s^2 - 34\omega_2^3 v_2^2 c_s^2 - 1008\omega_2 v_2^2 c_s^2 - 132c_s^2 - 216\omega_2 c_s^4 - 78\omega_2^2 c_s^2 - 9\omega_2^3 c_s^4 + 404\omega_2^2 v_2^2 c_s^2 + 144c_s^4 + 198\omega_2 c_s^2 + 10\omega_2^3 v_2^2 + 672v_2^2 c_s^2 + 82\omega_2^2 c_s^4 - 156v_2^2 - 216\omega_2 v_2^4 - 5\omega_2^3 c_s^4 - 98\omega_2^2 v_2^2) \frac{v_2}{12\omega_2^3}$$

$$C_{D_y^4 \rho}^{(2), \text{CuLBM2}} = (-171\omega_2 \omega_3 c_s^4 \omega_1^3 - 600\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1 + 141\omega_2^3 \omega_3 c_s^2 \omega_1 + 129\omega_2 v_2^2 \omega_3 \omega_1^3 + 6\omega_2^2 \omega_3 \omega_1 + 114\omega_2^2 \omega_3 c_s^2 \omega_1^2 - 105\omega_2 v_2^2 \omega_3 \omega_1^2 + 54\omega_2 \omega_3 c_s^4 \omega_1^2 + 12\omega_2^2 c_s^2 \omega_1^2 - 78\omega_2^2 \omega_3 c_s^2 \omega_1^3 + 45\omega_2^3 v_2^4 \omega_3 - 72\omega_2^3 \omega_3 c_s^2 - 261v_2^2 \omega_3 c_s^2 \omega_1^3 - 117\omega_2 v_2^4 \omega_3 \omega_1^3 + 261\omega_2^3 v_2^2 \omega_3 c_s^2 + 8\omega_2^2 \omega_3 \omega_1^3 + 18\omega_2^3 c_s^4 \omega_1 + 12\omega_2^3 \omega_3 c_s^2 \omega_1^3 - 68\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^3 - 72\omega_2^3 c_s^2 \omega_1^3 + 6\omega_2 v_2^2 c_s^2 \omega_1^3 - 12\omega_2^2 \omega_3 \omega_1^2 + 99\omega_2 v_2^4 \omega_3 \omega_1^2 - 60\omega_2^2 \omega_3 c_s^2 \omega_1 + 404\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^2 + 18\omega_2 c_s^4 \omega_1^3 - 78\omega_2^3 \omega_3 c_s^2 \omega_1^2 + 6\omega_2 \omega_1^3 + 404\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^3 - 12\omega_2 \omega_3 \omega_1^3 + 99\omega_2^2 v_2^4 \omega_3 \omega_1 - 60\omega_2 \omega_3 c_s^2 \omega_1^2 - 98\omega_2^2 v_2^2 \omega_3 \omega_1^3 - 36\omega_2^2 c_s^4 \omega_1^2 + 90\omega_2^3 v_2^4 \omega_3 \omega_1^2 + 90\omega_2^3 \omega_3 c_s^4 + 82\omega_2^2 \omega_3 c_s^4 \omega_1^3 - 51\omega_2^3 v_2^2 \omega_3 + 411\omega_2 v_2^2 \omega_3 c_s^2 \omega_1^2 + 6\omega_2^3 \omega_3 + 141\omega_2 \omega_3 c_s^2 \omega_1^3 + 129\omega_2^3 v_2^2 \omega_3 \omega_1 - 12\omega_2^3 \omega_3 \omega_1 - 171\omega_2^3 \omega_3 c_s^4 \omega_1 + 6\omega_2 \omega_3 \omega_1^2 - 816\omega_2^2 v_2^2 \omega_3 c_s^2 \omega_1^2 + 45v_2^4 \omega_3 \omega_1^3 - 600\omega_2 v_2^2 \omega_3 c_s^2 \omega_1^3 - 90\omega_2^2 \omega_3 c_s^4 \omega_1^2 - 18\omega_2^3 v_2^4 \omega_3 \omega_1^3 + 210\omega_2^2 v_2^2 \omega_3 \omega_1^2 + 54\omega_2^2 \omega_3 c_s^4 \omega_1 - 105\omega_2^2 v_2^2 \omega_3 \omega_1 - 98\omega_2^3 v_2^2 \omega_3 \omega_1^2 + 8\omega_2^3 \omega_3 \omega_1^2 - 6\omega_2 c_s^2 \omega_1^3 + 90\omega_2^2 v_2^4 \omega_3 \omega_1^3 + 82\omega_2^3 \omega_3 c_s^4 \omega_1^2 + 411\omega_2^2 v_2^2 \omega_3 c_s^2 \omega_1 - 117\omega_2^3 v_2^4 \omega_3 \omega_1 - 12\omega_2^2 v_2^2 c_s^2 \omega_1^2 - 6\omega_2^3 c_s^2 \omega_1 + 6\omega_2^3 v_2^2 c_s^2 \omega_1 - 51v_2^2 \omega_3 \omega_1^3 - 10\omega_2^2 \omega_3 c_s^4 \omega_1^3 - 2\omega_2^2 \omega_3 \omega_1^3 - 198\omega_2^2 v_2^4 \omega_3 \omega_1^2 + 90\omega_2^3 c_s^4 \omega_1^3 + 20\omega_2^3 v_2^2 \omega_3 \omega_1^3) \frac{v_2}{24\omega_2^3 \omega_3 \omega_1^3}$$

coefficient $C_{D_y^4 v_2}^{(2)}$ at $\frac{\partial^4 v_2}{\partial x_2^4}$:

$$C_{D_y^4 v_2}^{(2), \text{SRT}} = (12 - 22c_s^2 \omega^2 + 504v_2^4 + 14v_2^2 \omega^3 - 154v_2^2 \omega^2 + 2c_s^2 \omega^3 + 378v_2^2 \omega - 36c_s^2 + 54c_s^2 \omega + 24c_s^4 + 252v_2^2 c_s^2 \omega^2 + 310v_2^4 \omega^2 - c_s^4 \omega^3 + 14c_s^4 \omega^2 + 432v_2^2 c_s^2 - 29v_2^4 \omega^3 - 18v_2^2 c_s^2 \omega^3 - 36c_s^4 \omega - 252v_2^2 - 18\omega + 8\omega^2 - 648v_2^2 c_s^2 \omega - 756v_2^4 \omega - \omega^3) \frac{\rho}{12\omega^3}$$

$$C_{D_y^4 v_2}^{(2), \text{MRT1}} = (12 + 2\omega_6^3 c_s^2 + 504v_2^4 + 378v_2^2 \omega_6 - 648v_2^2 \omega_6 c_s^2 - 22\omega_6^2 c_s^2 - 36c_s^2 - 154v_2^2 \omega_6^2 - 18\omega_6 - 36\omega_6 c_s^4 + 14v_2^2 \omega_6^3 + 14\omega_6^2 c_s^4 - 18v_2^2 \omega_6^3 c_s^2 - 756v_2^4 \omega_6 + 24c_s^4 - \omega_6^3 + 8\omega_6^2 + 432v_2^2 c_s^2 + 54\omega_6 c_s^2 - 252v_2^2 - 29v_2^4 \omega_6^3 + 252v_2^2 \omega_6^2 c_s^2 - \omega_6^3 c_s^4 + 310v_2^4 \omega_6^2) \frac{\rho}{12\omega_6^3}$$

$$C_{D_y^4 v_2}^{(2), \text{MRT2}} = C_{D_y^4 v_2}^{(2), \text{MRT1}}$$

$$C_{D_y^4 v_2}^{(2), \text{CLBM1}} = C_{D_y^4 v_2}^{(2), \text{MRT1}}$$

$$C_{D_y^4 v_2}^{(2), \text{CLBM2}} = C_{D_y^4 v_2}^{(2), \text{MRT1}}$$

$$C_{D_y^4 v_2}^{(2), \text{CuLBM1}} = (12 - \omega_2^3 + 504v_2^4 + 378\omega_2 v_2^2 - 18\omega_2 + 310\omega_2^2 v_2^4 + 8\omega_2^2 + 2\omega_2^3 c_s^2 - 18\omega_2^3 v_2^2 c_s^2 - 648\omega_2 v_2^2 c_s^2 - 36c_s^2 - 36\omega_2 c_s^4 - 22\omega_2^2 c_s^2 - 29\omega_2^3 v_2^4 + 252\omega_2^2 v_2^2 c_s^2 + 24c_s^4 + 54\omega_2 c_s^2 + 14\omega_2^3 v_2^2 + 432v_2^2 c_s^2 + 14\omega_2^2 c_s^4 - 252v_2^2 - 756\omega_2 v_2^4 - \omega_2^3 c_s^4 - 154\omega_2^2 v_2^2) \frac{\rho}{12\omega_2^3}$$

$$C_{D_y^4 v_2}^{(2), \text{CuLBM2}} = (-33\omega_2 \omega_3 c_s^4 \omega_1^3 - 432\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1 + 45\omega_2^3 \omega_3 c_s^2 \omega_1 + 225\omega_2 v_2^2 \omega_3 \omega_1^3 + 6\omega_2^2 \omega_3 \omega_1 + 18\omega_2^2 \omega_3 c_s^2 \omega_1^2 - 153\omega_2 v_2^2 \omega_3 \omega_1^2 + 6\omega_2 \omega_3 c_s^4 \omega_1^2 + 12\omega_2^2 c_s^2 \omega_1^2 - 22\omega_2^2 \omega_3 c_s^2 \omega_1^3 + 171\omega_2^3 v_2^4 \omega_3 - 24\omega_2^3 \omega_3 c_s^2 + 207v_2^2 \omega_3 c_s^2 \omega_1^3 - 423\omega_2 v_2^4 \omega_3 \omega_1^3 + 207\omega_2^3 v_2^2 \omega_3 c_s^2 + 8\omega_2^2 \omega_3 \omega_1^3 + 6\omega_2^3 c_s^4 \omega_1 + 4\omega_2^3 \omega_3 c_s^2 \omega_1^3 - 36\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^3 - 24\omega_2^3 c_s^2 \omega_1^3 + 18\omega_2 v_2^2 c_s^2 \omega_1^3 - 12\omega_2^2 \omega_3 \omega_1^2 + 333\omega_2 v_2^4 \omega_3 \omega_1^2 - 12\omega_2^2 \omega_3 c_s^2 \omega_1 + 252\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1^2 + 6\omega_2 c_s^4 \omega_1^3 - 22\omega_2^3 \omega_3 c_s^2 \omega_1^2 + 6\omega_3 \omega_1^3 + 252\omega_2^2 v_2^2 \omega_3 c_s^2 \omega_1^3 - 12\omega_2 \omega_3 \omega_1^3 + 333\omega_2^2 v_2^4 \omega_3 \omega_1 - 12\omega_2 \omega_3 c_s^2 \omega_1^2 - 154\omega_2^2 v_2^2 \omega_3 \omega_1^3 - 12\omega_2^2 c_s^4 \omega_1^2 + 310\omega_2^3 v_2^4 \omega_3 \omega_1^2 + 18\omega_2^3 \omega_3 c_s^4 + 14\omega_2^2 \omega_3 c_s^4 \omega_1^3 - 99\omega_2^3 v_2^2 \omega_3 + 225\omega_2 v_2^2 \omega_3 c_s^2 \omega_1^2 + 6\omega_2^3 \omega_3 + 45\omega_2 \omega_3 c_s^2 \omega_1^3 + 225\omega_2^3 v_2^2 \omega_3 \omega_1 - 12\omega_2^3 \omega_3 \omega_1 - 33\omega_2^3 \omega_3 c_s^4 \omega_1 + 6\omega_2 \omega_3 \omega_1^2 - 432\omega_2^2 v_2^2 \omega_3 c_s^2 \omega_1^2 + 171v_2^4 \omega_3 \omega_1^3 - 432\omega_2 v_2^2 \omega_3 c_s^2 \omega_1^3 - 6\omega_2^2 \omega_3 c_s^4 \omega_1^2 - 58\omega_2^3 v_2^4 \omega_3 \omega_1^3 + 306\omega_2^2 v_2^2 \omega_3 \omega_1^2 + 6\omega_2^2 \omega_3 c_s^4 \omega_1 - 153\omega_2^3 v_2^2 \omega_3 \omega_1 - 154\omega_2^3 v_2^2 \omega_3 \omega_1^2 + 8\omega_2^3 \omega_3 \omega_1^2 - 6\omega_2 c_s^2 \omega_1^3 + 310\omega_2^2 v_2^2 \omega_3 \omega_1^3 + 14\omega_2^3 \omega_3 c_s^4 \omega_1^2 + 225\omega_2^3 v_2^2 \omega_3 c_s^2 \omega_1 - 423\omega_2^3 v_2^4 \omega_3 \omega_1 - 36\omega_2^2 v_2^2 c_s^2 \omega_1^2 - 6\omega_2^3 c_s^2 \omega_1 + 18\omega_2^3 v_2^2 c_s^2 \omega_1 - 99v_2^2 \omega_3 \omega_1^3 - 2\omega_2^2 \omega_3 c_s^4 \omega_1^3 - 2\omega_2^2 \omega_3 \omega_1^3 - 666\omega_2^2 v_2^4 \omega_3 \omega_1^2 + 18\omega_2^3 c_s^4 \omega_1^3 + 28\omega_2^3 v_2^2 \omega_3 \omega_1^3) \frac{\rho}{24\omega_2^3 \omega_3 \omega_1^3}$$

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